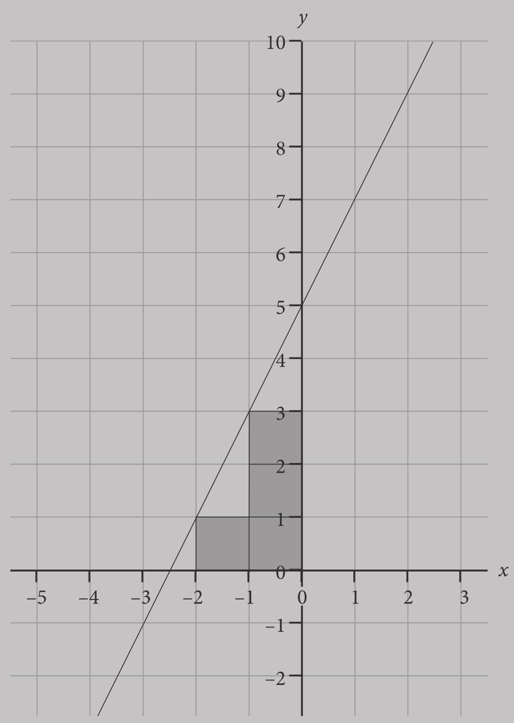
How do you describe mathematics tasks?

Colin Foster and Matthew Inglis ask what it means to describe a task as 'rich'.

The diagram below shows the graph of y = 2x + 5.

There are four whole unit squares 'trapped' between the line and the axes.

Find the number of trapped squares for other graphs.



Taken from Foster (2011, p. 185).

What do you think of this task (Foster, 2011)? Would you use it with learners? How would you describe it? Would you say it was a 'rich' task?

A lot of different adjectives are used to describe mathematics tasks, such as 'open', 'inquiry-based', 'procedural', and so on, but what do they mean? Do teachers understand them in broadly similar ways or in a variety of different ways? The National Curriculum suggests that learners should be offered "rich and sophisticated problems" (DfE, 2014, p. 3). But what does that mean? How do you interpret such language? In a recent piece of research (Foster & Inglis, 2017 – available open access if you want to read more detail), we carried out two studies to investigate how mathematics teachers use adjectives to describe mathematics tasks.

The two studies

In the first study, we made a list of 84 adjectives which have been used to describe mathematics tasks (you can see the full list in the article, Foster & Inglis [2017]). We then created an internet-based survey where we asked secondary mathematics teachers to think of any mathematics task that they had used recently with learners, or saw another teacher use. Then they were asked to rate how accurately each of our 84 adjectives described the task. A total of 360 teachers completed the study. (Perhaps you were one of them? We are very grateful to ATM, among others, for publicising it, and to all the teachers for slogging through the 84 words!)

We put the data into an exploratory factor analysis, which is a statistical approach that attempts to represent a large number of variables using a smaller set of factors, while accounting for as much of the original variance as possible. This gave us seven factors, which altogether accounted for 44% of the variance. We named the seven factors as shown in Table 1, which also shows the adjectives that were most representative of each factor. Table 2 shows the linear correlations between each pair of factors, where 1 indicates perfect positive correlation, −1 indicates perfect negative correlation and zero indicates no linear correlation at all.

Engagement	*Demand*	*Routineness*	*Strangeness*	*Inquiry*	*Context*	*Interactivity*
enjoyable	difficult	routine	strange	open	real-life	hands-on
fun	complicated	repetitive		inquiry-based	realistic	cooperative
pleasing	demanding	procedural		deep	context-based	collaborative
appealing	perplexing	formal		exploratory	applied	practical
attention-grabbing	easy*	mechanical		investigative		
motivating	challenging	rule-based		rich		
stimulating	simple*			thought-provoking		
memorable	problematic			closed*		
boring*	puzzling			analytical		
interesting						
absorbing						
exciting						
inspiring						
dull*						
engaging						

Table 1. The adjectives most representative of each of our seven factors.
* *indicates an adjective that loaded negatively*

	Engagement	Demand	Routineness	Strangeness	Inquiry	Context	Interactivity
Engagement	1.00	.08	–.20	–.04	.32	.26	.29
Demand		1.00	.02	–.06	.28	.03	.01
Routineness			1.00	.06	–.10	.05	–.13
Strangeness				1.00	.09	–.06	–.13
Inquiry					1.00	.30	.20
Context						1.00	.27

Table 2. The correlations between each pair of factors.

You can see from the small absolute values of the correlations between pairs of factors in Table 2 that the factors were fairly independent of each other, which is what you would hope to obtain from a factor analysis. This means, for example, that how engaging a task was perceived to be was largely independent of how demanding it was perceived to be. There are some weak relationships present, however, which are worth thinking about. Routine tasks were less likely to be engaging than non-routine tasks, as you might expect. You can also see that inquiry tasks were a little more likely to be engaging, but this relationship was also weak, suggesting that there is no automatic link between the use of inquiry tasks and learner engagement. Likewise, tasks which rated highly on the context factor were slightly more likely to rate highly on the engagement factor and the inquiry factor. Tasks which were rated highly on the interactive factor were more likely also to be rated highly on the engagement, inquiry and context factors, although all of these relationships were weak.

The main message is that teachers positioned mathematics tasks along seven relatively independent dimensions, which could be interpreted as meaning that there are seven fairly separate features of mathematics tasks.

In our second study, we looked at whether different teachers agreed about how particular adjectives related to the same task. In other words, if one teacher believed that a task was 'rich', would another teacher agree? To do this, we had to present teachers with a given task and ask them to rate how well they felt it represented each factor. This time we found that teachers disagreed quite a lot. For example, some teachers felt that the 'Trapped Squares' task was engaging and inquiry-based (the signature characteristics of 'rich' tasks according to our first study), but many regarded it as neither engaging nor inquiry-based. We found more agreement concerning the context factor, but that might simply have been because all of the tasks we used were fairly pure. We did find that teachers interpreted routineness quite consistently, however.

Implications

One way to interpret the finding of seven factors is that if you are choosing or designing a mathematics task, these are seven separate things that you might want to think about. In other words, don't assume that dealing with one of them will automatically take care of any of the others. For example, contrary to some suggestions (e.g., Kitchen, 2010), engagement and inquiry are perceived by teachers to be only weakly related. So while an inquiry task might be engaging, it might not – the link between these two features is not strong.

Previously, lots of people have tried to characterise what makes a 'rich' task, and there is some agreement but also some disagreement (e.g., Griffin, 2009). Our results offer a possible reason for this: a task's richness seems to depend on at least two largely independent properties, because the word 'rich' loaded strongly onto both the inquiry and the engagement factors, suggesting that richness is a multidimensional notion. That could be one reason why it is hard to characterise.

A final implication concerns teacher agreement and disagreement. Teachers were quite internally consistent in their ratings. For example, if a teacher felt that a given task was 'appealing', they were also extremely likely to believe that it was 'pleasing' (two words from the same 'engaging' factor). However, there was very little *between-teacher* agreement. This means that we should not assume that teachers will interpret words like 'rich' in the same way as each other. This is a problem, because it limits how effective it can be talking in general terms about, say, 'rich' tasks. If you want someone to know what you mean, you really need to give examples and not rely too much on adjectives.

Conclusion

One problem with our study is the relationship between the *task* (what you ask learners to do) and the *activity* (what actually happens as a result when you use the task with particular learners). It may even be that "There are no rich mathematical tasks, only tasks used richly" (Mason, 2015, p. 15). Does it make sense to try to judge a task in isolation? Maybe it's only sensible to say that a task is 'rich' if you are thinking of somebody who finds it so? However, it seems to us that we have to talk about tasks using some kind of language, and we need some basis for choosing one task rather than another to use with learners. So we think that exploring the way that teachers talk about tasks is an important thing to do. And we hope that the seven dimensions might help when thinking about designing or selecting tasks to use with your learners.

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Colin Foster works at the University of Nottingham and Matthew Inglis works at Loughborough University.

email: colin.foster@nottingham.ac.uk