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Maximising Average Energy Efficiency for Two-user AWGN Broadcast Channel

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Abstract: Energy consumption has become an increasingly important aspect of wireless communications, from both an economical and environmental point of view. New enhancements are being placed on mobile networks to reduce the power consumption of both mobile terminals and base stations. This paper studies the achievable rate region of AWGN broadcast channels under Time-division, Frequency-division and Superposition coding, and locates the optimal energy-efficient rate-pair according to a comparison metric based on the average energy efficiency of the system. In addition to the transmit power, circuit power and signalling power are also incorporated in the energy efficiency function, with simulation results verifying that the Superposition coding scheme achieves the highest energy efficiency in an ideal, but non-realistic scenario, where the signalling power is zero. With moderate signalling power, the Frequency-division scheme is the most energy-efficient, with Superposition coding and Time-division becoming second and third best. Conversely, when the signalling power is high, both Time-division and Frequency-division schemes outperform Superposition coding. On the other hand, the Superposition coding scheme also incorporates rate-fairness into the system, which allows both users to transmit whilst maximising the energy efficiency.

Keywords: Broadcast Channel, Capacity Region, Energy Efficiency.

1. Introduction

With the global exhaustion of natural resources and increasing greenhouse gas emissions, new targets are being set to increase the energy efficiency of mobile networks and reduce CO_2 emissions. The *information and communication technologies* (ICT) sector contributes to around 2% of the world-wide CO_2 emissions [1], with wireless communications only contributing to approximately 15% of this figure, but its fast recent growth sets a challenge for mobile operators to stop these figures from growing.

New energy saving designs in mobile networks are closely linked to the reduction of power consumption in both mobile terminals (MT) and base stations (BS). Current mobile terminals feature large screens and offer multimedia applications, leading to a substantial increase in their power consumption. On the other hand, battery technology is improving at a much slower rate compared to the energy demand, which is why the gap between energy demand and battery capacity is increasing exponentially [2]. Conversely, due to the mobile nature of mobile terminals and rigid constraints on their available power supply, over 80% of the power in a mobile network is consumed by the base stations [3]. It is fair to point out that, equipment manufacturers have also made very good progress to reduce the energy consumption of base stations, by increasing the power efficiency of the transceivers and replacing active air conditioners with fresh-air cooling systems in indoor sites [4]. The work in [3]-[5] investigates different methods for improving the energy efficiency of base stations in cellular networks.

Current radio access comparison metrics mainly consider conventional optimization criterions such as data throughput and spectral efficiency, with limited work covering energy efficiency. The most common metric used for defining energy efficiency is introduced in [6], and further extended in [7], where, the capacity in bits-per-joule is derived for a single link on flat fading and frequency selective *Additive White Gaussian Noise* (AWGN) channels. In terms of a system-level view, the energy efficiency of a single cell is studied in [8] and [9] for flat fading, and frequency selective channels respectively, where the authors investigated uplink energy-efficient communications in *Orthogonal Frequency Division Multiple Access* (OFDMA) systems by improving the utilization of the mobile energy with the assumption of having a fixed circuit power (energy needed to keep nodes active). Our previous work in [10] studied the achievable capacity region of a single carrier, 2-user uplink channel and defined average energy efficiency as the optimisation metric to find the optimal rate pair based on different system requirements. Results verified the achievability of the optimal energy-efficient rate pair within the capacity region, and provided a trade-off for energy efficiency, fairness and maximum sum-rate.

In this paper, we start by revisiting the known formulation in literature on the set of achievable rate vectors of a 2-user AWGN broadcast channel (BC) under *Time division* (TD), *Frequency division* (FD), and *Superposition coding* (SPC), and locate the optimal rate-pair from an energy efficiency point of view. The rest of the paper is organised as follows: Section 2 briefly describes the system model and revisits the AWGN capacity region for the three schemes of interest to this work; TD, FD and SPC. Section 3 defines a system level metric based on the average energy efficiency of the system to investigate the performance of the three schemes defined in section 2. And finally, the paper is concluded in section 4.

2. System Model and Evaluation Framework

Consider a single carrier broadcast channel consisting of a single transmitter and two distant receivers (shown in Figure 1), with the channel power gain and power constraint of user k defined as g_k and p_k respectively, where $k = 1,2$. The received signal is corrupted by AWGN with power spectral density (PSD) $N_0/2$, and it is arbitrarily assumed that $g_1 > g_2$. The total transmit power and bandwidth are denoted as P and B respectively. Throughout this paper, boldface is used to denote vectors.

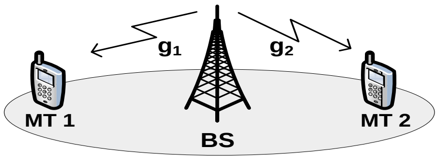

Figure 1: Downlink network model

In a standard point-to-point transmission, the capacity of a channel acts as an upper-bound (i.e. reliable communication is possible at rates less than the capacity). With two users in the system, this is extended to a capacity region, which contains the set of rate pairs such that both users can simultaneously transmit at rates R_1 and R_2 with arbitrarily small error probability.

The rate of user k was shown in [11] to be

$$R_k \leq B\, log_2\left(1 + \frac{p_k g_k}{N_0 B}\right), \qquad k = 1,2 \tag{1}$$

The set of achievable rates includes $(R_1, 0)$ and $(0, R_2)$, which corresponds to two extreme scenarios where one user transmits at its maximum rate, with the other remaining silent. This section characterizes the 2-user AWGN BC using equal power Time division, Frequency division and Superposition coding, which are discussed in subsequent sections.

2.1 Equal Power Time Division (TD)

In this scheme, the entire transmit power and bandwidth are allocated to user 1 for a fraction α of the total transmission time, and the remainder to user 2. The union of all achievable rate pairs provide the TD rate region, which is a straight line connecting the single user bound of points R_1 and R_2, as derived in (1). The TD rate region was shown in [12] to be:

$$\mathcal{C}_{TD} = \bigcup_{0\le\alpha\le1} \left(R_1 = \alpha B\, log_2\left(1+\frac{Pg_1}{N_0B}\right), R_2 = (1-\alpha)B\, log_2\left(1+\frac{Pg_2}{N_0B}\right)\right) \qquad (2)$$

In cases where, as well as allocating a fraction of the time slot to each user, the transmit power of each user is also varied based on an average total power constraint, the average power constraints of each user becomes:

$$P = \alpha p_1 + (1-\alpha)p_2 \qquad (3)$$

This will define the rate region of the un-equal power TD, where it was shown in [13] to equate to the rate region of the frequency division schemes (discussed in next section).

2.2 – Frequency Division (FD)

In frequency division, the total transmit power and bandwidth are allocated to both users subject to total power and bandwidth constraints, i.e. $P = p_1 + p_2$ and $B = B_1 + B_2$. The union of the rate regions of the fixed FD over all possible bandwidth divisions provides the FD achievable rate region [12].

$$\mathcal{C}_{FD} = \bigcup_{\substack{P=p_1+p_2\\ B=B_1+B_2}} \left(R_1 = B_1\, log_2\left(1+\frac{p_1g_1}{N_0B_1}\right), R_2 = B_2\, log_2\left(1+\frac{p_2g_2}{N_0B_2}\right)\right) \qquad (4)$$

As shown in [12], this achievable rate region exceeds the rate region of the equal power TD. On the other hand, when $g_1 = g_2$, the FD rate region reduces to the equal power TD.

2.3 – Superposition Coding (SPC)

The Superposition coding scheme exploits scenarios where the channel power gain of one user is greater than the other, and the transmitter has knowledge of the channel gains of both receivers. Since $g_1 > g_2$, user 1 can correctly receive user 2's signal, which it will then decode and subtract out to decode its own signal. On the other hand, user 2 cannot decode user 1's message, which is why the received signal of user 2 will have an added noise term originating from user 1's signal. The SPC rate region was shown in [13] to be:

$$\mathcal{C}_{SPC} = \bigcup_{P=p_1+p_2} \left(R_1 = B\, log_2\left(1+\frac{p_1g_1}{N_0B}\right), R_2 = B\, log_2\left(1+\frac{p_2g_2}{N_0B+p_1g_2}\right)\right) \qquad (5)$$

The work in [14] showed that when $g_1 > g_2$, the rate region of SPC exceeds that of FD and TD. To be more precise, (5) gives the maximum achievable rate pair, which is why it is also referred to as the BC capacity region ($\mathcal{C}_{BC}$).

3. Energy-Efficient Design

For a channel with an average transmit power of p_k Watts and a channel capacity of R_k bits/sec, energy efficiency is defined as R_k/p_k bits/joule. In addition to the transmit power, some power is consumed in the circuitry or dissipated in the form of heat, which is defined as the circuit power (P_c). Throughout this work, it is assumed that P_c is a fixed value, independent of the transmission state and equal for both users. When the transmitter requires full knowledge of the channel gains of both users, some additional power is consumed in the system for the signalling overhead, which will be refereed to as P_{CSI}, and is applicable to the SPC scheme.

The overall energy efficiency of user k is defined as:

$$EE_k = \frac{R_k}{(P_c + P_{CSI}) + p_k}, \quad k = 1,2 \tag{6}$$

The comparison metric used in this study is based on the average energy efficiency of the system which is defined as:

$$EE_{AV} = \frac{EE_1 + EE_2}{2} \tag{7}$$

where, EE_1 and EE_2 denote the energy efficiency of user one and two respectively.

The average energy efficiency metric can be characterized using all three capacity schemes discussed in section 2. With reference to the frequency division scheme, the bandwidth allocation can be shown such that $B_1 = \alpha B$ and $B_2 = (1-\alpha)B$. The results in Figure 2 investigate the behaviour of the FD energy efficiency based on varying α.

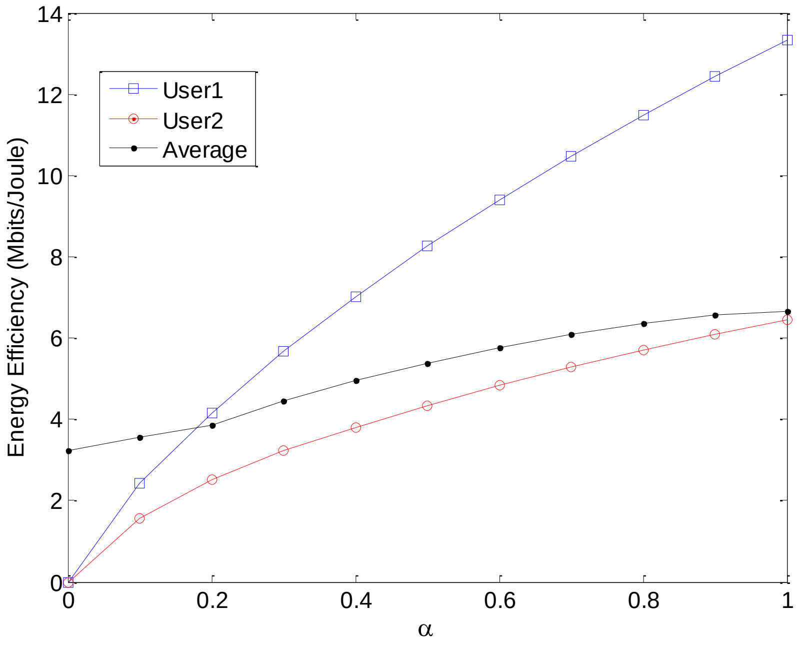

Figure 2: Energy efficiency of the FD scheme based on variable bandwidth allocation

$P = 0.1$ W, $P_c = 0.02$ W, $g_1 = 0.15$, $g_2 = 0.025$, $B = 100$ KHz, $N_0 = 10^{-9}$ W/Hz

The single user energy efficiency bounds of user 1 and 2 can be seen in Figure 2, where, an increase in the bandwidth share, increases the single user energy efficiency. Using (7), the average energy efficiency of the 2-user case is also depicted in Figure 2, where the same increasing pattern can be seen. The maximum achievable energy efficiency in this case is to allocate all bandwidth to the stronger user (user 1), but the fairness of the system will be compromised. It should also be noted that in the FD scheme, since the transmitter does not require knowledge of both users channel gains, $P_{CSI} = 0$.

3.1 –Maximum Energy Efficiency

In the case of the equal power TD scheme, the entire power and bandwidth are allocated to each user for a fraction of the total transmission time, therefore, the maximum energy-efficient rate-pair is simply chosen as the point that has the maximum average energy efficiency over the entire time frame of the transmission. In the case of the SPC scheme, the system is designed such that the total transmit power (P) is divided between both users, and the power division that achieves the highest average energy efficiency value is chosen. For the FD scheme, the same approach as the SPC scheme is employed, with the difference that this time, both the total transmit power and bandwidth are divided between users. The maximization problem is defined as follows:

$$\max_{R_1, R_2}(EE_{AV}) \tag{8}$$

$$s.t.\ (R_1, R_2) \in C_{BC}(P;\ \boldsymbol{g}) \tag{9}$$

where, constraint (9) specifies that all rate pairs should be chosen within the capacity region boundary (based on the chosen scheme, i.e. TD, FD or SPC), and $\boldsymbol{g} = [\,g_1, g_2]$.

As explained in section 2.3, the rate region of the SPC scheme exceeds that of TD and FD. On the other hand, the SPC scheme requires full knowledge of the channel gains of both users, which means that P_{CSI} is no longer zero. The results in Figure 3 investigate the behaviour of the energy efficiency function of the three schemes by increasing P_{CSI}.

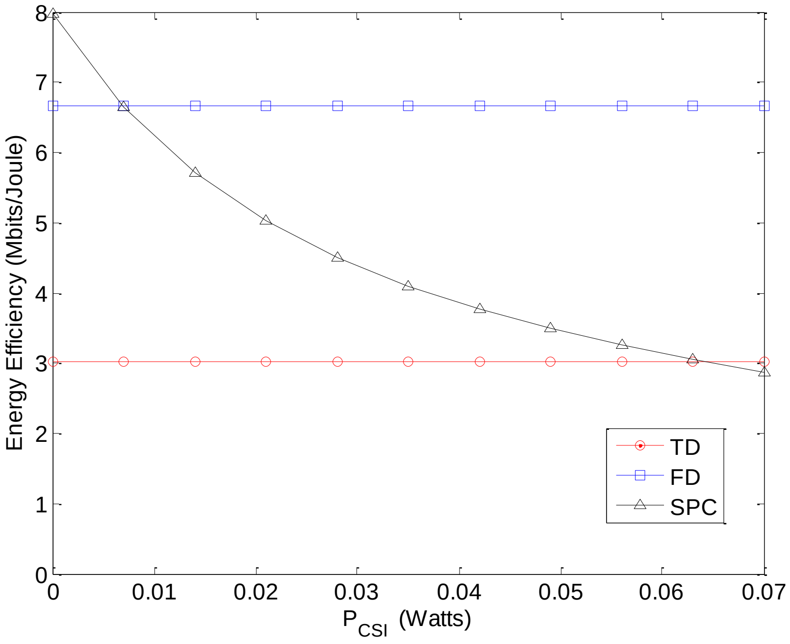

Figure 3: Comparison of the energy efficiency functions with variable signalling power

$P = 0.1$ W, $P_c = 0.02$ W, g_1= 0.15, g_2= 0.025, B= 100 KHz, $N_0 = 10^{-9}$ W/Hz

It is clearly seen that the SPC scheme achieves the highest energy efficiency when the signalling power is zero. On the other hand, as P_{CSI} increases, the SPC energy efficiency decreases whilst the TD and FD energy efficiencies remain unchanged. In cases where the signalling power is high, both FD and TD schemes outperform SPC, which shows the importance of P_{CSI} in the system design.

As shown in Figure 4, increasing the transmit power will lead to a higher energy efficiency. However, this is only true up to a certain point, after which the energy efficiency of the system decreases. This is the main idea behind energy-efficient design, where, a continuous increase in the transmit power will decrease the energy efficiency. Therefore, maximum energy efficiency is achieved by tuning the power according to the rate requirements of the system. Without the presence of P_{CSI}, and at low transmit powers, all three schemes perform the same. However, as soon as the transmit power is increased, SPC proves to be the most energy-efficient technique. On the other hand, when signalling power is considered in the system design, the FD technique can be shown to achieve the highest energy efficiency, which accentuates the results presented in Figure 3.

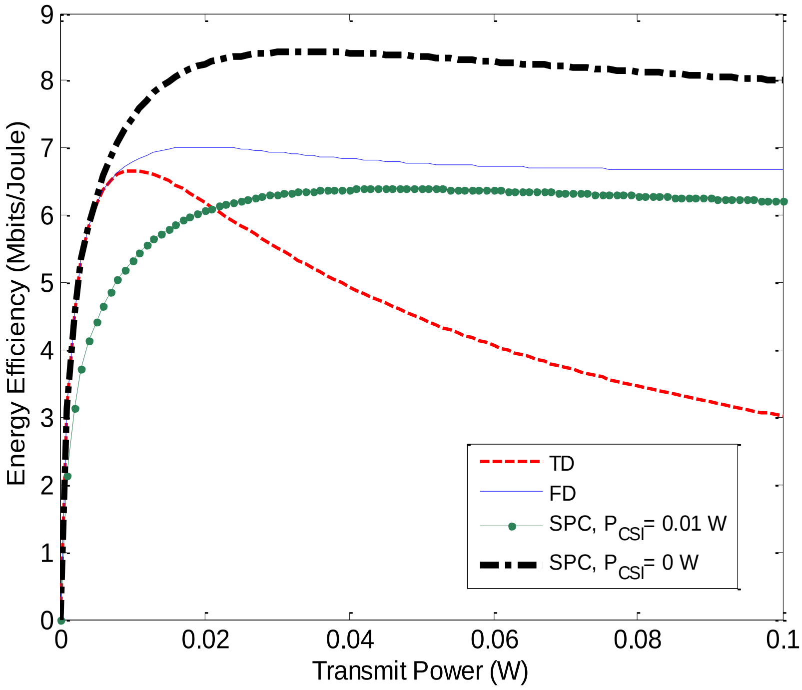

Figure 4: Comparison of the energy efficiency of the TD, FD, and SPC schemes

$P_c = 0.02$ *W*, g_1*= 0.15*, g_2*= 0.025, B= 100 KHz,* N_0 *=*10^{-9} *W/Hz*

By considering the rate regions from the three schemes identified in section 2, and the maximization problem defined in (8), the points with the highest energy efficiency for each scheme can be identified, which are depicted in Figure 5. The channel gain of user 1 has be chosen to be much higher than user 2's channel gain, which is why the achievable rate region of the FD scheme exceeds that of the equal power TD. Once again, the results show the superiority of the SPC scheme in scenarios where there is no signalling power involved. In terms of the most energy-efficient rate-pair, it can be seen that in the TD and FD schemes, maximum energy efficiency is achieved when the stronger user (user 1 in this case) transmits at its single user bound with the weaker user remaining silent. Although this method maximises the energy efficiency, the rate-fairness is compromised. Conversely, the SPC scheme maximises the energy efficiency whilst both users are able to transmit.

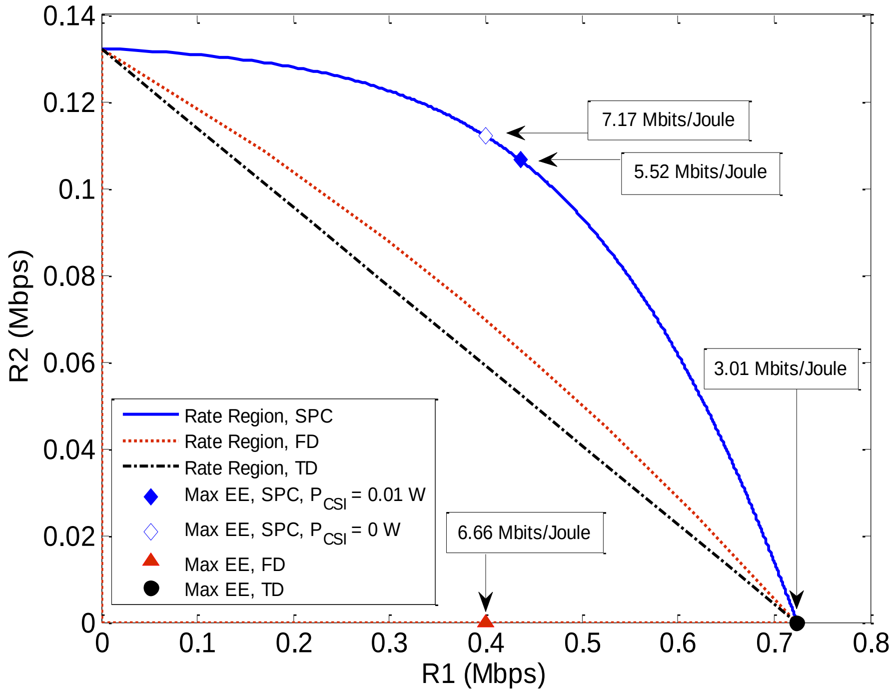

Figure 5: Rate region and maximum Energy-Efficient points for the TD, FD and SPC schemes

$P = 0.1\ W,\ P_c = 0.02\ W,\ g_1 = 0.15,\ g_2 = 0.0015,\ B = 100\ KHz,\ N_0 = 10^{-9}\ W/Hz$

Using the same approach as in Figure 5, but with a higher channel gain for the weak user (user 2 in this case), the TD and FD capacity regions can be shown to be almost identical. This can be seen in Figure 6, where the TD scheme still shows to be unfair, with only the stronger user transmitting. On the other hand, the FD scheme is no longer unfair, with user 2 being able to transmit, but only at a fraction of user 1's rate. And finally, the SPC scheme still proves to be the most rate-fair scheme.

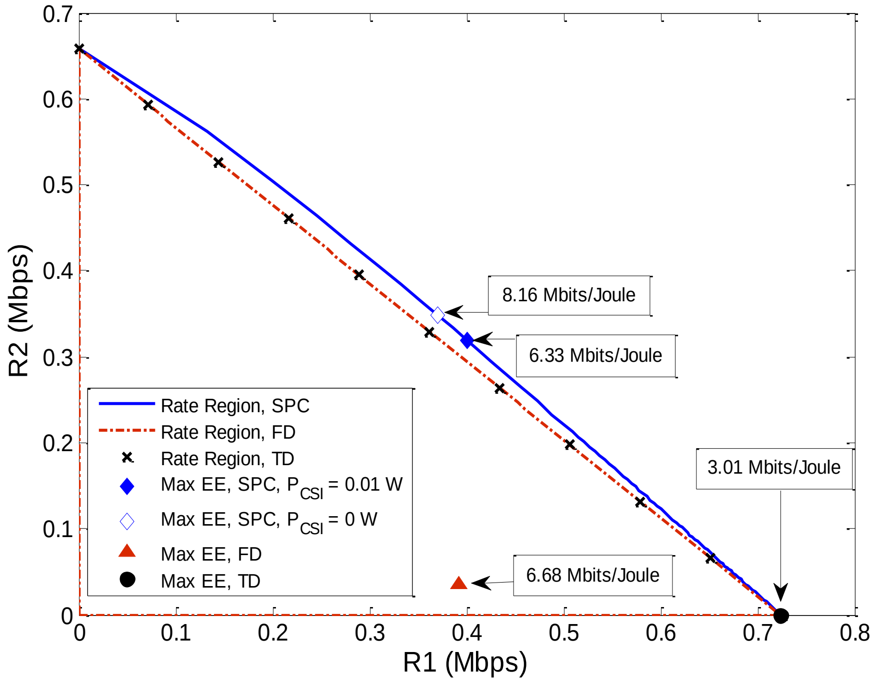

Figure 6: Rate region and maximum Energy-Efficient points for the TD, FD and SPC schemes

$P = 0.1\ W,\ P_c = 0.02\ W,\ g_1 = 0.15,\ g_2 = 0.095,\ B = 100\ KHz,\ N_0 = 10^{-9}\ W/Hz$

4. Conclusions

This paper studies the achievable rate region of a 2-user AWGN downlink channel under Time-division, Frequency-division and Superposition coding, and locates the optimal energy-efficient rate-pair. The comparison metric used in this study is based on the average energy efficiency of the system, which in addition to the transmit power, also considers circuit power and signalling power. Simulation results verify that the Superposition coding schemes achieves the highest energy efficiency in scenarios where the signalling power is zero. On the other hand, when the signalling power is high, the Time-division and Frequency-division schemes can outperform Superposition coding. The Superposition coding scheme also incorporates rate-fairness into the system, which allows both users to transmit whilst maximising the energy efficiency, whereas in the TD scheme, only the stronger user transmits, and in the FD scheme, depending on the channel gains, the weaker user may only transmit at a fraction of the stronger user's rate. As part of future work, the work presented in this paper will be further extended to the $k > 2$ scenario.

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