Near-optimal Energy-efficient Joint Resource Allocation for Multi-hop MIMO-AF Systems

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***Abstract*—Energy efficiency (EE) is becoming an important performance indicator for ensuring both the economical and environmental sustainability of the next generation of communication networks. Equally, cooperative communication is an effective way of improving communication system performances. In this paper, we propose a near-optimal energy-efficient joint resource allocation algorithm for multi-hop multiple-input-multiple-output (MIMO) amplify-and-forward (AF) systems. We first show how to simplify the multivariate unconstrained EE-based problem, based on the fact that this problem has a unique optimal solution, and then solve it by means of a low-complexity algorithm. We compare our approach with classic optimization tools in terms of energy efficiency as well as complexity, and results indicate the near-optimality and low-complexity of our approach. As an application, we use our approach to compare the EE of multi-hop MIMO-AF with MIMO systems and our results show that the former outperforms the latter mainly when the direct link quality is poor.**

I. Introduction

Network operators not only require the next generation of communication systems to be more spectrally efficient, which has been the trend for the last decade, but also to be more energy-efficient in order to ensure both the economical and environmental sustainability of their activity. Consequently, energy efficiency (EE) is gradually becoming an important performance indicator, which is currently extensively studied for both power-limited [1], [2], e.g. mobile device, and power-unlimited [3], [4], e.g. cellular system, applications.

Cooperative communication has proved in the past to be an effective solution for increasing the spectral efficiency (SE) or/and the coverage of cellular networks [5] as well as reducing the cost of network deployment [6]. More recently, it has been indicated in [7] that cooperative communication can also be deployed to improve EE by using relays for reducing the transmit power at the base station (BS). Due to its simplicity, amplify-and-forward (AF) remains one of the most popular schemes for implementing cooperative multi-input multi-output (MIMO) communication. As far as resource (power or/and rate) allocation for multi-hop MIMO-AF system is concerned, most of the existing methods for jointly allocating resources at the source node (SN) and relay nodes (RNs) are based on SE maximization or mean square error minimization [8], [9] and, thus, do not take into account the EE. With the growing importance of EE in communication, EE-based resource allocation is becoming popular such that EE-optimal resource allocation schemes for the uplink and downlink of MIMO systems over a frequency selective channel have been recently proposed in [1], [2] and [4], respectively. Meanwhile, energy-efficient resource allocation methods for the two-hop MIMO-AF and multi-hop MIMO systems have been provided in [10] and [11], respectively.

In this paper, we propose an energy-efficient joint resource allocation method for the multi-hop MIMO-AF system by a considering a realistic multi-hop MIMO power model. Contrarily to [10], we propose a joint optimization of all the transmitting nodes and generalize the problem to N hops instead of two hops. In addition, contrarily to [11], we consider AF relaying, a realistic multi-hop MIMO power model and our objective function is the closed-form expression of the EE and not an approximated bound of it. In Section II, we first describe the multi-hop MIMO-AF system as well as power models and, second, formulate the energy-per-bit consumption of the multi-hop MIMO-AF system based on these models. Given that the energy-per-bit consumption function has a unique minimum, we simplify the unconstrained energy-efficient joint optimization problem in Section III and provide a low-complexity algorithm for solving this optimization problem. In Section IV, we numerically show the near-optimality and low-complexity of our approach in comparison with a traditional convex optimization method. We then use our approach to compare the EE of multi-hop MIMO-AF with MIMO systems and our results show that multi-hop MIMO-AF can be beneficial for reducing the energy-per-bit consumption when the direct link quality is poor. Conclusions are drawn in Section V.

II. Multi-hop MIMO AF system and power models

A. System model

We consider in this paper a N-hop MIMO AF system with $N+1$ nodes, i.e. including one SN with t_1 antennas, $N-1$ nonregenerative RN with t_i antennas, $i \in \{2, \ldots, N\}$, and one destination node (DN) with t_{N+1} antennas, as it is depicted in Fig. 1. Moreover, we assume that all the nodes operate in half-duplex mode as in [11], such that all the transmission phases have equal duration. In each transmission phase, the transmit signal is linearly precoded at the i-th node by using a precoding matrix $\mathbf{F}_i \in \mathbb{C}^{t_i \times t_i}$ and is then transmitted towards the $i+1$-th node such that $\mathbf{y}_i = \mathbf{H}_i \mathbf{F}_i \mathbf{y}_{i-1} + \mathbf{n}_i$ for any $i \in \mathcal{N} = \{1, \ldots, N\}$. Any matrix $\mathbf{H}_i \in \mathbb{C}^{t_{i+1} \times t_i}$ represents the MIMO channel between the i-th and $i+1$-th node. In addition, $\mathbf{n}_i \in \mathbb{C}^{t_{i+1} \times 1}$ is a vector of independent zero-mean complex Gaussian noise entries with a variance of σ_i^2. Accordingly, the

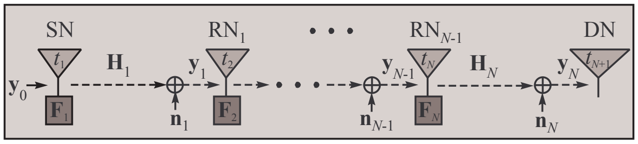

Fig. 1. N-hop MIMO AF system model .

mutual information (bit/s) over N time slots of this N-hop MIMO-AF system is straightforwardly given by

$$I(\mathbf{y}_N;\mathbf{y}_0)=W\log_2\left|\mathbf{I}_{t_{N+1}}+\prod_{i=1}^{N}\mathbf{H}_i\mathbf{F}_i\mathbf{F}_i^{\dagger}\mathbf{H}_i^{\dagger}\mathbf{R}_N^{-1}\right|, \quad (1)$$

where W is the channel bandwidth, $\mathbf{R}_i=\sigma_i^2\mathbf{I}_{t_{i+1}}+\mathbf{H}_i\mathbf{F}_i\mathbf{R}_{i-1}\mathbf{F}_i^{\dagger}\mathbf{H}_i^{\dagger}$ is the i-th noise covariance matrix, for any $i\in\mathcal{N}$, and $\mathbf{R}_0=\mathbf{0}_{t_1}$. In addition, $\mathbf{I}_x$ is a $x\times x$ identity matrix, $\mathbf{0}_x$ is a $x\times x$ matrix of zeros, $|.|$ is the matrix determinant and $(.)^{\dagger}$ denotes the conjugate transpose.

The Hadamard determinant theorem [12] states that an optimal precoder structure is the one that diagonalizes the matrix within the determinant in (1). In the two-hop scenario, such a structure has been proved to be optimal for maximizing the SE, minimizing the transmit power, and optimizing the EE in [13], [14] and [10], respectively. Assuming that each node knows its previous and next links' channel state information (CSI) (only next link CSI for the SN), an optimal precoder structure is of the form [15]

$$\mathbf{F}_i=\mathbf{V}_i\widehat{\mathbf{F}}_i\mathbf{U}_{i-1}^{\dagger}, \quad (2)$$

for any $i\in\mathcal{N}$, where $\mathbf{V}_i$ and $\mathbf{U}_{i-1}$ are unitary matrices that contain the t_i right-singular vectors of $\mathbf{H}_i$ and t_i left-singular vectors of $\mathbf{H}_{i-1}$, respectively, with $\mathbf{U}_0=\mathbf{I}_{t_1}$. In addition $\widehat{\mathbf{F}}_i=\mathrm{diag}(\sqrt{p_{i,1}},\ldots,\sqrt{p_{i,t_i}})$ is a $t_i\times t_i$ diagonal matrix. Inserting (2) into (1), the latter simplifies to $I(\mathbf{y}_N;\mathbf{y}_0)=$

$$R_{\Sigma}(\mathbf{P})=W\sum_{m=1}^{M}\log_2\left(1+\prod_{i=1}^{N}p_{i,m}\lambda_{i,m}(r_{N,m}(\mathbf{P}))^{-1}\right), \quad (3)$$

where $\lambda_{i,m}$ denotes the non-zeros eigenvalues of $\mathbf{H}_i$, $r_{i,m}(\mathbf{P})=\sigma_i^2+p_{i,m}\lambda_{i,m}r_{i-1,m}(\mathbf{P})$ with $r_{0,m}(\mathbf{P})=0$, for any $m\in\mathcal{M}=\{1,\ldots,M\}$ and $\mathbf{P}=[p_{1,1},\ldots,p_{i-1,M},p_{i,1},\ldots,p_{N,M}]\succeq 0$. Moreover, M is the number of orthogonal subchannels, such that $M=t$ or $M=Kt$ in a flat or frequency selective channel scenario, respectively, where $t\triangleq\min_{i\in\mathcal{N}}\{\mathrm{rk}\{\mathbf{H}_i\}\}$ is the total number of spatial subchannels with $\mathrm{rk}\{.\}$ being the rank operator and K denotes the number of frequency-flat subchannels [16].

B. Power consumption model

Even though BS, relay and user equipment (UE) are quite different in terms of architecture, it has been shown in [3], [17] and [1], respectively, that their power consumption models are similar, i.e. there exists a linear relation between their respective consumed and transmit powers, such as

$$P_{\text{in}}=\Delta P+P^{Ci}, \quad (4)$$

where Δ and P^{Ci} accounts for the radio frequency (RF) dependent and circuit (fixed) power consumptions, respectively. Given that each antenna has its own RF chain [3], this model has been refined for the MIMO setting in [18] as

$$P_{\text{in}}=t(\Delta P+P^{CipA})+P^{Ci}, \quad (5)$$

where P^{CipA} is the circuit power per antenna and t is the number of transmit antennas. In the N-hop MIMO-AF system of Fig. 1, the total transmit power of the i-th node, $P_i(\mathbf{P})$, can be expressed as $P_i(\mathbf{P})=\mathrm{E}\{\|\mathbf{F}_i\mathbf{y}_{i-1}\|_F^2\}$ [15], where $\mathrm{E}\{.\}$ stands for the expectation and $\|.\|_F^2$ denotes the Frobenius norm. Inserting (2) into $P_i(\mathbf{P})$, the latter can be re-expressed as

$$P_i(\mathbf{P})=\sum_{m=1}^{M}\sigma_i^2\lambda_{i,m}^{-1}\left(r_{i,m}(\mathbf{P})\sigma_i^{-2}-1\right). \quad (6)$$

According to Fig. 1, during the propagation of the signal $\mathbf{y}_0$ from the SN to the DN via the $N-1$ relays, each of these nodes will either transmit, receive or be inactive, except for the SN which does not receive and the DN which does not transmit. Accordingly, these different types of power consumptions should be reflected in the total power consumption of the system. Let us define $P_{.}^{Tx}$, $P_{.}^{Rx}$, $P_{.}^{Sl}$ as the transmit, receive and sleep mode powers of the nodes, then, the total power consumption over N time slots of the N-hop MIMO-AF system of Fig. 1 can be expressed as

$$\begin{aligned}P_{\Sigma}&=P_{\text{SN}}^{Tx}+P_{\text{DN}}^{Rx}+t(N-1)\left(P_{\text{SN}}^{Sl}+P_{\text{DN}}^{Sl}\right)+\sum_{i=1}^{N-1}P_{\text{RN}_i}^{Tx}\\&+P_{\text{RN}_i}^{Rx}+t(N-2)P_{\text{RN}_i}^{Sl},\end{aligned} \quad (7)$$

when assuming that the N transmission phases have equal duration. The power components $P_{\text{RN}_i}^{Tx}$ and $P_{\text{RN}_i}^{Rx}$ in (7) can be further detailed as $P_{\text{RN}_i}^{Tx}=\Delta_{\text{RN}_i}P_i(\mathbf{P})+tP_{\text{RN}_i}^{CipA}+P_{\text{RN}_i}^{Ci}$ and $P_{\text{RN}_i}^{Rx}=\varsigma(tP_{\text{RN}_i}^{CipA}+P_{\text{RN}_i}^{Ci})$, respectively, according to equation (5), where ς characterizes the ratio between transmission and reception overhead powers with $0\le\varsigma\le 1$. Intuitively, less overhead power is necessary for receiving than transmitting signals. Similarly, $P_{\text{SN}}^{Tx}=\Delta_{\text{SN}}P_1(\mathbf{P})+tP_{\text{SN}}^{CipA}+P_{\text{SN}}^{Ci}$ and $P_{\text{DN}}^{Rx}=\varsigma(tP_{\text{DN}}^{CipA}+P_{\text{DN}}^{Ci})$. Relying on these more detailed expressions for P_{SN}^{Tx}, P_{DN}^{Rx}, $P_{\text{RN}_i}^{Tx}$ and $P_{\text{RN}_i}^{Rx}$, P_{Σ} in (7) can be reformulated as

$$P_{\Sigma}(\mathbf{P})=\sum_{i=1}^{N}\Delta_iP_i(\mathbf{P})+P_c, \quad (8)$$

where $\Delta_1=\Delta_{\text{SN}}$, $\Delta_i=\Delta_{\text{RN}_{i-1}}$, for any $i\in\{2,\ldots,N\}$, and $P_c=tP_{\text{SN}}^{CipA}+P_{\text{SN}}^{Ci}+\varsigma(tP_{\text{DN}}^{CipA}+P_{\text{DN}}^{Ci})+t(N-1)\left(P_{\text{SN}}^{Sl}+P_{\text{DN}}^{Sl}\right)+\sum_{i=1}^{N-1}(1+\varsigma)(tP_{\text{RN}_i}^{CipA}+P_{\text{RN}_i}^{Ci})+t(N-2)P_{\text{RN}_i}^{Sl}$.

C. EE formulation

The existence of a trade-off between EE and SE [19] implies that these two quantities can only be jointly optimized by using the explicit expression of this trade-off as an objective function. In the general case, it has been shown in [19] that an explicit expression of this trade-off can be obtained through the ratio between the total rate and total consumed power, which are respectively given in (3) and (8) as a function of the transmit power. However, since this trade-off is between EE and SE, it is generally defined as a function of the SE, $\boldsymbol{\mathcal{C}}$, such that

$$E_b(\boldsymbol{\mathcal{C}}) = \frac{P_\Sigma(\boldsymbol{\mathcal{C}})}{R_\Sigma(\boldsymbol{\mathcal{C}})}, \tag{9}$$

where E_b stands for the energy-per-bit, i.e. 1/EE, and $\boldsymbol{\mathcal{C}} = [\mathcal{C}_{1,1}, \ldots, \mathcal{C}_{i-1,M}, \mathcal{C}_{i,1}, \ldots, \mathcal{C}_{N,M}] \succeq 0$ in the N-hop MIMO-AF scenario. In addition, we define the SE of the m-th subchannel of node i as $\mathcal{C}_{i,m} = \log_2(r_{i,m}(\mathbf{P})\sigma_i^{-2})$. We can then reformulate (3) and (8) by inserting $\mathcal{C}_{i,m}$ into them, such that

$$\begin{aligned} R_\Sigma(\boldsymbol{\mathcal{C}}) &= W \sum_{m=1}^{M} \sum_{i=1}^{N} \mathcal{C}_{i,m} - \log_2 \left(\prod_{i=1}^{N} 2^{\mathcal{C}_{i,m}} \right. \\ &\quad \left. - \prod_{i=1}^{N} \left(2^{\mathcal{C}_{i,m}} - 1\right) \right) \quad \text{and} \end{aligned} \tag{10a}$$

$$P_\Sigma(\boldsymbol{\mathcal{C}}) = P_c + \sum_{i=1}^{N} \sum_{m=1}^{M} A_{i,m} \left(2^{\mathcal{C}_{i,m}} - 1\right), \tag{10b}$$

respectively, where $A_{i,m} \triangleq \Delta_i \sigma_i^2 \lambda_{i,m}^{-1}$.

III. Multi-hop MIMO-AF EE Unconstrained Optimization

In this section, we first demonstrate how to reformulate the NM-variable function $E_b(\boldsymbol{\mathcal{C}})$ in (9) into a M-variable function. We then rely on a one-dimensional root finding method for obtaining the near-optimal energy-efficient joint resource allocation in a low-complexity manner by solving

$$\begin{aligned} \min_{\boldsymbol{\mathcal{C}}} \quad & E_b(\boldsymbol{\mathcal{C}}) \\ \text{s.t.} \quad & \boldsymbol{\mathcal{C}} \succeq 0. \end{aligned} \tag{11}$$

Proposition 1: Assuming that $E_b^\star \triangleq E_b(\boldsymbol{\mathcal{C}} = \boldsymbol{\mathcal{C}}^\star)$ is the unique minimum of $E_b(\boldsymbol{\mathcal{C}})$ in (9) over its domain, i.e. for any $\boldsymbol{\mathcal{C}} \succeq 0$, $E_b^\star$ can be well-approximated by

$$E_b^\star \approx \frac{\ln(2) A_{i,m}(4z_m^2 - \alpha_{i,m}^2)}{4W\alpha_{i,m}^2} \left(\frac{\prod_{j=1}^{N}(2z_m + \alpha_{j,m})}{\prod_{j=1}^{N}(2z_m - \alpha_{j,m})} - 1 \right), \tag{12}$$

for any $i \in \mathcal{N}$ and $m \in \mathcal{M}^\star$, where

$$z_m \triangleq \left(2^{\mathcal{C}_{k,m}} - 1/2\right) \alpha_{k,m}, \tag{13}$$

for any $i \in \mathcal{N}$ and $m \in \mathcal{M}^\star$. In addition, $\mathcal{M}^\star = \{m \in \mathcal{M} | \mathcal{C}_m^\star > 0\}$ is the optimal set of allocated subchannel indices and $\alpha_{k,m} = \sqrt{A_{k,m}} \sum_{j=1}^{N} \sqrt{A_{j,m}}$. Thus, the NM-variable function in (9) simplifies into a M-variable function in (12) when $\boldsymbol{\mathcal{C}} = \boldsymbol{\mathcal{C}}^\star$.

Proof: The full proof for this proposition is detailed in the Appendix. ■

Equation (12) not only shows that (9) can be simplified but also indicates that any z_m variables can be approximated as a function of $\mu = E_b^\star$ by solving this polynomial equation

$$\begin{aligned} &\left(4z_m^2 - \frac{\sum_{i=1}^{N} A_{i,m}\alpha_{i,m}^2}{\sum_{i=1}^{N} A_{i,m}} \right) \left(\prod_{i=1}^{N}(2z_m + \alpha_{i,m}) - \prod_{i=1}^{N}(2z_m \right. \\ &\left. - \alpha_{i,m}) \right) - \frac{4W\mu}{\ln(2)} \left(\sum_{i=1}^{N} \alpha_{i,m} \right) \prod_{i=1}^{N}(2z_m - \alpha_{i,m}) = 0 \end{aligned} \tag{14}$$

such that $z_m = \max\{\max_{i \in \mathcal{N}}\{\alpha_{i,m}/2\}, z_m^+\}$, for any $m \in \mathcal{M}^\star$, where z_m^+ is the largest real root of (14). For instance, z_m^+ can be given in closed-form for $N = 2$ and $N = 3$ as

$$z_m^+ = -a_{1,m} + \sqrt{a_{1,m}^2 - a_{0,m}} \quad \text{and} \tag{15a}$$

$$\begin{aligned} z_m^+ &= -\frac{b_{2,m}}{3} + \frac{1 - i\sqrt{3}}{6} \sqrt[3]{\frac{1}{2}\left[\Theta_m + \sqrt{-27\Lambda_m}\right]} \\ &\quad + \frac{1 + i\sqrt{3}}{6} \sqrt[3]{\frac{1}{2}\left[\Theta_m - \sqrt{-27\Lambda_m}\right]}, \end{aligned} \tag{15b}$$

respectively, with $a_{0,m} = \frac{W\mu}{2\ln(2)} \frac{\prod_{i=1}^{N}\alpha_{i,m}}{\sum_{i=1}^{N} A_{i,m}}$, $a_{1,m} = \frac{\sum_{i=1}^{N} A_{i,m}\alpha_{i,m}}{4\sum_{i=1}^{N} A_{i,m}} - \frac{W\mu}{2\ln(2)}$, $\Theta_m = 2b_{2,m}^3 - 9b_{2,m}b_{1,m} + 27b_{0,m}$, and $\Lambda_m = 18b_{2,m}b_{1,m}b_{0,m} - 4b_{2,m}^2 b_{1,m}^2 - 4b_{1,m}^3 - 27b_{0,m}^2$. In addition, $b_{0,m} = \frac{\prod_{i=1}^{N}\sqrt{A_{i,m}} \sum_{i=1}^{N} A_{i,m}\alpha_{i,m}^2 - 2W\mu/\ln(2)\prod_{i=1}^{N}\alpha_{i,m}}{8\sum_{i=1}^{N} A_{i,m}}$, $b_{1,m} = \frac{\prod_{i=1}^{N}\alpha_{i,m}}{4\sum_{i=1}^{N} A_{i,m}} + \frac{W\mu}{2\ln(2)} \frac{\sum_{i=1}^{N}\alpha_{i,m}^2 \sum_{j=1, j\neq i}^{N}\alpha_{j,m}}{\left(\sum_{i=1}^{N} A_{i,m}\right)\left(\sum_{i=1}^{N}\alpha_{i,m}\right)}$ and $b_{2,m} = 2a_{1,m}$. For $N \geq 3$, z_m^+ can be obtained numerically by using a classic root finding method, e.g. Laguerre's method.

It can be remarked that E_b in (9) can be re-expressed as a function of z_m such that

$$E_b(\mathbf{z}) \approx \frac{P_c - \frac{1}{2}\sum_{i=1}^{N}\sum_{m\in\mathcal{M}^\star} A_{i,m} + \sum_{m\in\mathcal{M}^\star} z_m}{-W\sum_{m\in\mathcal{M}^\star}\log_2\left(1 - \frac{\prod_{j=1}^{N}(2z_m - \alpha_{j,m})}{\prod_{j=1}^{N}(2z_m + \alpha_{j,m})}\right)} \tag{16}$$

by inserting (13) into R_Σ and P_Σ in (10). It can also be observed that (12) and (16) must be approximately equal when $\mu = E_b^\star$. Given that E_b has a unique minimum over its domain, we can apply a one-dimensional root finding method for obtaining an approximation of this minimum based on equations (14) and (16) as it is fully detailed in Algorithm 1. Indeed, we can first obtain z_m by inserting $\mu > 0$ in (14) or (15), for any $m \in \mathcal{M}^\star$; the latter is then used in (16) for computing an updated version of μ, until $|E_b(\mathbf{z}) - \mu| \ll 1$.

Our near-optimal energy-efficient joint resource allocation algorithm for multi-hop MIMO-AF systems is based on a one-dimensional root finding method, and, consequently, exhibits by design a low-computational complexity. However, extra computational complexity is required when $N > 3$ for computing the roots of the $N+1$ degree polynomial in (14). We assume as in [10], [13] that the eigenvalues of each link, $\lambda_{i,m}$, are sorted in descending order prior to using our algorithm.

IV. Numerical results and discussion

A. Accuracy and complexity results

In order to demonstrate the accuracy and low-complexity of our algorithm for jointly optimizing the unconstrained EE of all the nodes in a multi-hop MIMO-AF systems, i.e. Algorithm

Algorithm 1

Inputs: $P_c, W, N, M, \alpha_{i,m}$ and $A_{i,m}$, for any $i \in \mathcal{N}$ and $m \in \mathcal{M}$
Set $\varepsilon = 10^{-6}, U = M, F = 1$;
Set $z_m = \max\left\{\sum_{i=1}^{N} A_{i,m}/2 - P_c/M, \max_{i\in\mathcal{N}}\{\alpha_{i,m}\}\right\}$, for any $m \in \mathcal{M}$;
Compute $\varepsilon = 10^{-10}, U = K, F = 1$;
while $F > \varepsilon$ **do**
  $\mu = E_b(\mathbf{z})$ in (16);
  Set $z_m = \alpha_{1,m}/2$, for any $m \in \mathcal{M}$;
  Obtain z_m^+ via (15a) if $N = 2$, (15b) if $N = 3$ or by solving (14) if $N > 3$, for any $m \in \{1, \ldots, U\}$;
  Set $z_m = \max\{\max_{i\in\mathcal{N}}\{\alpha_{i,m}/2\}, z_m^+\}$, for any $m \in \{1, \ldots, U\}$;
  Set $U = M - \sum_{m=1}^{M}(z_m == \max_{i\in\mathcal{N}}\{\alpha_{i,m}/2\})$;
  Set $F = |E_b(\mathbf{z}) - \mu|$;
end while
Set $\mathcal{C}_{i,m}^{\star} \approx W\left(\log_2\left(1 + \sqrt{1 + \frac{A_{1,m}}{A_{i,m}}\left(-1 + 4z_m^2\alpha_{1,m}^{-2}\right)}\right) - 1\right)$, for any $i \in \mathcal{N}$ and $m \in \mathcal{M}$
Obtain $\Sigma_{E_b}^{\star}$ by inserting $\mathcal{C}_{i,m}^{\star}$ in (9)
Outputs: $\mathcal{C}_{i,m}^{\star}$ and $E_b^{\star}$.

1, we compare it in Figs. 2 and 3 against the Matlab "fmincon" function in terms of energy-per-bit performances (upper graph) as well as relative computational complexity (lower graph). We define the relative computational complexity between these two methods as the ratio of "fmincon" to Algorithm 1 execution time. Both figures are plotted by assuming a MIMO Rayleigh fading channel between each node, for $W = 1$, $\varsigma = 1/2$, various values of N, M as well as $\sigma_i^2 = 0$ and ± 20 dB in Figs. 2 and 3, respectively, for any $i \in \{2, \ldots, N\}$. In addition, the values of Table I have been used for setting the power parameters of Section II-C with $P_{\mathrm{RN}_i}^{Ci} = P_{\mathrm{RN}}^{Ci}$ and $P_{\mathrm{RN}_i}^{Sl} = P_{\mathrm{RN}}^{Sl}$, for any $i \in \{1, \ldots, N-1\}$.

The energy-per-bit results in both Figs. 2 and 3 clearly show the tight match between our algorithm and the "fmincon" function performances in any scenario, which confirms the near-optimality of our algorithm. Indeed, the "fmincon" function returns optimal results since E_b is quasiconvex. In addition, the relative computational complexity results show that our algorithm can at least reduce the computational complexity by two orders of magnitude, i.e. 100 times lower, in comparison with "fmincon". Moreover, the larger is M or N, the larger is the relative reduction in complexity. For instance in Fig. 2, our algorithm is more than 1000 times faster than the "fmincon" method for $N = 2$ and $M = 16$.

B. Discussion

From an intuitive point of view, having extra nodes to convey data is likely to increase the overall power consumption, since it can easily be seen that the fixed power term, P_c, in (8) increases linearly with N. Another drawback of multi-hop communication in comparison with direct communication is that the aggregate subchannel rate, $C_m = \sum_{i=1}^{N} \mathcal{C}_{i,m} - \log_2\left(\prod_{i=1}^{N} 2^{\mathcal{C}_{i,m}} - \prod_{i=1}^{N}\left(2^{\mathcal{C}_{i,m}} - 1\right)\right)$, can only be as good as the worst of the N links' subchannel rate, i.e.

$$\min_{i\in\mathcal{N}}\{\mathcal{C}_{i,m}\} - 1 \le \mathcal{C}_m \le \min_{i\in\mathcal{N}}\{\mathcal{C}_{i,m}\}. \quad (17)$$

TABLE I
POWER PARAMETER VALUES

Parameters	Δ	P^{CipA} (W)	P^{Ci} (W)	P^{Sl} (W)
SN (BS)	4.7 [3]	100	180	75 [3]
RN	6.3 [20]	4	4.9	3.45
DN (UE)	–	0.03	0.07	0.02

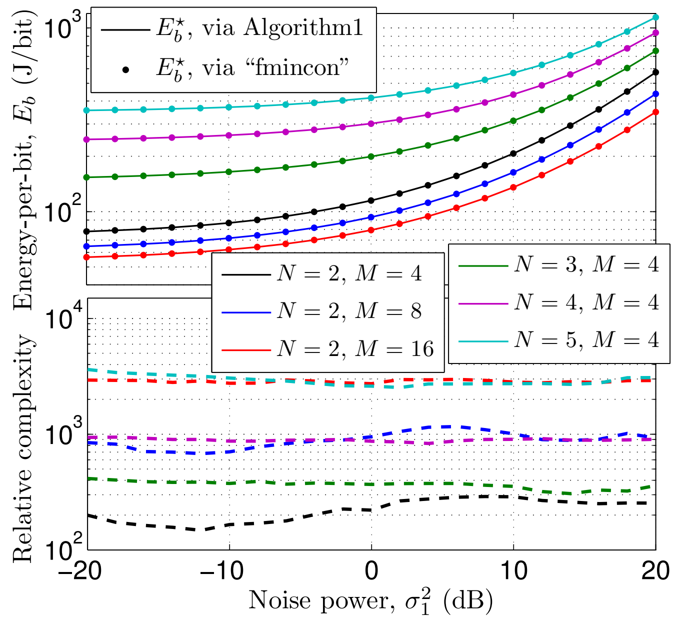

Fig. 2. Comparison of the optimal energy-per-bit consumption obtained via Algorithm 1 and "fmincon" for various values of N and M with $\sigma_i^2 = 0$ dB, $\forall i \in \{2, \ldots, N\}$.

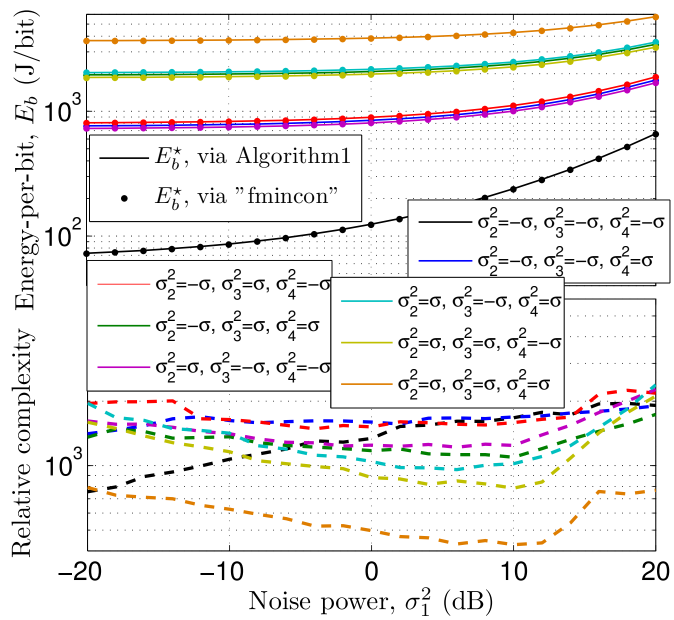

Fig. 3. Comparison of the optimal energy-per-bit consumption obtained via Algorithm 1 and "fmincon" for $N = M = 4$ and various values of σ_i^2, with $\sigma = 20$ dB.

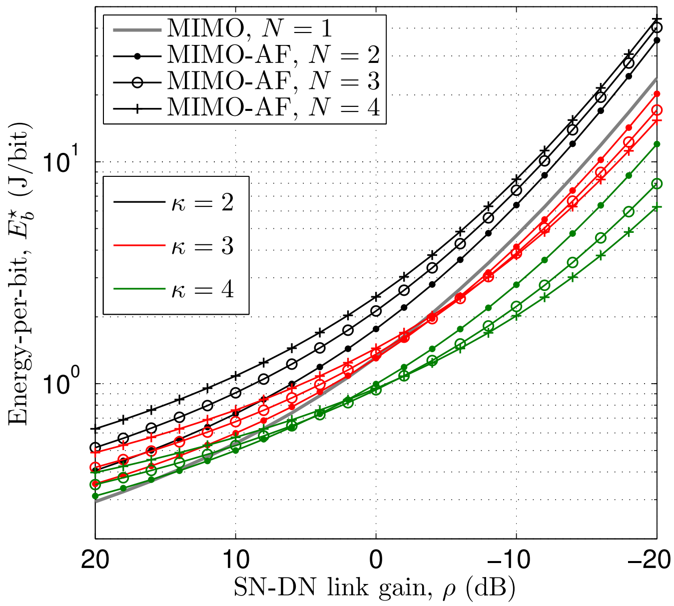

Fig. 4. Comparison of the optimal energy-per-bit consumption of N-hops MIMO-AF and MIMO systems for various pathloss exponents.

Consequently, these two handicaps make a multi-hop communication always surely less energy-efficient than a direct communication if adding extra nodes does node improve the channel quality. On the other hand, the inherent purpose of having relays is to reduce the communication distance, which in turn improves the channel quality. Let d be the distance between the SN and DN, having $N-1$ relays will reduce at best the inter-distance between each node to d/N. Considering a simple distant-dependent pathloss model such that the channel gain of the SN-DN link is given by $\rho = 10^{C-10\kappa\log_{10}(d)}$, then the maximum channel gain improvement provided by having $N-1$ relays can be quantified as $10\kappa\log_{10}(N)$ dB, where κ is the pathloss exponent, C is a constant. Hence, more EE gain can be achieved for high values of κ, i.e. when the channel propagation conditions are poor.

In order to illustrate this statement, we compare in Fig. 4, the optimal energy-per-bit consumption of N-hops MIMO-AF and MIMO systems when considering both path-loss and small scale fading, the power model values of Table I, $W = 1$, $\varsigma = 1/2$, $M = 256$, i.e. $t = 2$ & $K = 128$, $\sigma_i^2 = 0$ dB, $\forall i \in \mathcal{N}$, and $\rho_i = \rho + 10\kappa\log_{10}(N)$ dB, $\forall i \in \mathcal{N}$, for the multi-hop system. As we expected, multi-hoping is mainly beneficial in terms of energy-per-bit consumption when the pathloss exponent is high, or in other words, when the channel quality between the SN and DN is poor. For instance, using 2, 3 or 4 hops instead of direct transmission can reduce the energy-per-bit consumption by 30% when $\rho = 0$ dB and $\kappa = 4$.

V. Conclusion

In this paper, a near-optimal energy-efficient joint resource allocation method has been designed for the multi-hop MIMO-AF system when considering that full CSI is available at the RNs and transmit CSI is available at the SN. We have demonstrated how to simplify the multivariate unconstrained EE-based problem by proving the existence of a unique optimal solution. We have then provided a low-complexity algorithm for solving this problem. The performances of our algorithm have been compared against a traditional convex optimization method and results have confirmed the near-optimality and low-complexity of our approach. As an application, we have used our method to compare the EE of multi-hop MIMO-AF with MIMO systems and our results have showed that multi-hoping is mainly beneficial in terms of energy-per-bit consumption when the channel quality between the SN and DN is poor. In the future, we would like to extend our algorithm for the case of power or/and rate constraint.

Appendix

1) Uniqueness of the minimum of E_b: The function E_b in (9) being continuous and twice differentiable, the gradient and Hessian of E_b can be formulated as

$$\nabla E_b(\boldsymbol{\mathcal{C}}) = \frac{\nabla P_\Sigma(\boldsymbol{\mathcal{C}})R_\Sigma(\boldsymbol{\mathcal{C}}) - \nabla R_\Sigma(\boldsymbol{\mathcal{C}})P_\Sigma(\boldsymbol{\mathcal{C}})}{R_\Sigma(\boldsymbol{\mathcal{C}})^2} \quad \text{and} \tag{18a}$$

$$\nabla^2 E_b(\boldsymbol{\mathcal{C}}) = \frac{\nabla^2 P_\Sigma(\boldsymbol{\mathcal{C}})R_\Sigma(\boldsymbol{\mathcal{C}}) - \nabla^2 R_\Sigma(\boldsymbol{\mathcal{C}})P_\Sigma(\boldsymbol{\mathcal{C}})}{R_\Sigma(\boldsymbol{\mathcal{C}})^2} + \frac{\nabla R_\Sigma(\boldsymbol{\mathcal{C}})^{\mathrm{T}}\nabla E_b(\boldsymbol{\mathcal{C}}) + \nabla E_b(\boldsymbol{\mathcal{C}})^{\mathrm{T}}R_\Sigma(\boldsymbol{\mathcal{C}})}{R_\Sigma(\boldsymbol{\mathcal{C}})}, \tag{18b}$$

respectively, where $\{.\}^{\mathrm{T}}$ is the transpose operator. Let $\mathbf{w} \in \mathbb{R}^{NM}$, we know from (3.21) of [21] that if E_b satisfies

$$\nabla E_b(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}} = 0 \Rightarrow \mathbf{w}\nabla^2 E_b(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}} \geq 0 \tag{19}$$

then E_b is quasiconvex.

Firstly, $\nabla E_b(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}} = 0$ implies that $\mathbf{w}\nabla^2 E_b(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}} =$

$$\frac{\mathbf{w}\nabla^2 P_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}R_\Sigma(\boldsymbol{\mathcal{C}}) - \mathbf{w}\nabla^2 R_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}P_\Sigma(\boldsymbol{\mathcal{C}})}{R_\Sigma(\boldsymbol{\mathcal{C}})^2} \tag{20}$$

according to (18b). Given that according to (10b) the gradient and Hessian of P_Σ are expressed as

$$\nabla P_\Sigma(\boldsymbol{\mathcal{C}}) = \ln(2)[A_{1,1}2^{\mathcal{C}_{1,1}}, \ldots, A_{N,M}2^{\mathcal{C}_{N,M}}], \tag{21a}$$

$$\nabla^2 P_\Sigma(\boldsymbol{\mathcal{C}}) = \ln(2)\,\mathrm{diag}\{\nabla P_\Sigma(\boldsymbol{\mathcal{C}})\}, \tag{21b}$$

respectively, then $\mathbf{w}\nabla^2 P_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}R_\Sigma(\boldsymbol{\mathcal{C}}) = \ln(2)\mathbf{w} \cdot \nabla P_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}R_\Sigma(\boldsymbol{\mathcal{C}})$, where $\cdot$ denotes the dot product. Secondly, $\nabla E_b(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}} = 0$ also implies that

$$\nabla P_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}R_\Sigma(\boldsymbol{\mathcal{C}}) = \nabla R_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}P_\Sigma(\boldsymbol{\mathcal{C}}), \tag{22}$$

according to (18a), such that $\ln(2)\mathbf{w} \cdot \nabla P_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}R_\Sigma(\boldsymbol{\mathcal{C}}) = \ln(2)\mathbf{w} \cdot \nabla R_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}P_\Sigma(\boldsymbol{\mathcal{C}})$. By substituting $\mathbf{w}\nabla^2 P_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}R_\Sigma(\boldsymbol{\mathcal{C}})$ with the latter in (28), we obtain that $\mathbf{w}\nabla^2 E_b(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}} =$

$$\frac{E_b(\boldsymbol{\mathcal{C}})}{R_\Sigma(\boldsymbol{\mathcal{C}})}\left(\ln(2)\mathbf{w} \cdot \nabla R_\Sigma(\boldsymbol{\mathcal{C}}) - \mathbf{w}\nabla^2 R_\Sigma(\boldsymbol{\mathcal{C}})\right)\mathbf{w}^{\mathrm{T}}. \tag{23}$$

Knowing that the (i,m)-th element of the gradient of R_Σ can be expressed as

$$\frac{\partial R_\Sigma(\boldsymbol{\mathcal{C}})}{\partial \mathcal{C}_{i,m}} = W\frac{\prod_{\substack{j=1\\j\neq i}}^{N}\left(2^{\mathcal{C}_{j,m}}-1\right)}{\prod_{j=1}^{N}2^{\mathcal{C}_{j,m}}-\prod_{j=1}^{N}\left(2^{\mathcal{C}_{j,m}}-1\right)}, \tag{24}$$

it can be easily proved that $\ln(2)\mathbf{w}\cdot\nabla R_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}} \geq \mathbf{w}\nabla^2 R_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}$ and, hence, E_b is quasiconvex, i.e. unimodal.

Let $\boldsymbol{\mathcal{C}}^\star$ be one of the stationary point of E_b, accordingly, $\nabla E_b(\boldsymbol{\mathcal{C}}=\boldsymbol{\mathcal{C}}^\star)=0$. Moreover, we know from (18a) that if $\nabla E_b(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}=0$ then $E_b(\boldsymbol{\mathcal{C}})=\frac{\nabla P_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}}{\nabla R_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}}$ such that $E_b(\boldsymbol{\mathcal{C}}+\mathbf{w})-E_b(\boldsymbol{\mathcal{C}})=$

$$\frac{\nabla P_\Sigma(\boldsymbol{\mathcal{C}})(2^{\mathbf{w}}-\mathbf{1})^{\mathrm{T}}}{\ln(2)R_\Sigma(\boldsymbol{\mathcal{C}}+\mathbf{w})}-\frac{\left[R_\Sigma(\boldsymbol{\mathcal{C}}+\mathbf{w})-R_\Sigma(\boldsymbol{\mathcal{C}})\right]\nabla P_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}}{R_\Sigma(\boldsymbol{\mathcal{C}}+\mathbf{w})\nabla R_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}}. \tag{25}$$

In addition, let $F:\mathbf{X}\in\mathbb{R}^{2M}\mapsto\mathbb{R}$ and $\|\mathbf{w}\|\ll\mathbf{1}$, then the gradient of F is similar to

$$\nabla F(\boldsymbol{X})\mathbf{w}^{\mathrm{T}}\simeq F(\boldsymbol{X}+\mathbf{w})-F(\boldsymbol{X}). \tag{26}$$

Given that $\nabla P_\Sigma(\boldsymbol{\mathcal{C}})(2^{\mathbf{w}}-\mathbf{1})^{\mathrm{T}}>\ln(2)\nabla P_\Sigma(\boldsymbol{\mathcal{C}})\mathbf{w}^{\mathrm{T}}$, for $\mathbf{w}\neq\mathbf{0}$, it implies with (25) and (26) that $E_b(\boldsymbol{\mathcal{C}}^\star+\mathbf{w})>E_b(\boldsymbol{\mathcal{C}}^\star)$. Hence, E_b reaches a local minimum in $\boldsymbol{\mathcal{C}}^\star$, which happens to be a unique and global minimum since E_b is unimodal and $E_b(\boldsymbol{\mathcal{C}}^\star+\mathbf{w})>E_b(\boldsymbol{\mathcal{C}}^\star)$.

2) $E_b(\boldsymbol{\mathcal{C}})$: from a NM to a N-variable function: According to equation (18a), solving $\nabla E_b(\boldsymbol{\mathcal{C}}=\boldsymbol{\mathcal{C}}^\star)=0$ yields

$$E_b^\star=\frac{P_\Sigma(\boldsymbol{\mathcal{C}}^\star)}{R_\Sigma(\boldsymbol{\mathcal{C}}^\star)}=\frac{\partial P_\Sigma(\boldsymbol{\mathcal{C}}^\star)}{\partial\mathcal{C}_{i,m}}\left[\frac{\partial R_\Sigma(\boldsymbol{\mathcal{C}}^\star)}{\partial\mathcal{C}_{i,m}}\right]^{-1}. \tag{27}$$

Inserting $\frac{\partial P_\Sigma(\boldsymbol{\mathcal{C}})}{\partial\mathcal{C}_{i,m}}=\ln(2)A_{i,m}2^{\mathcal{C}_{i,m}}$ and $\frac{\partial R_\Sigma(\boldsymbol{\mathcal{C}})}{\partial\mathcal{C}_{i,m}}$ in (24) into (27), the latter can be reformulated as

$$E_b^\star=\frac{\ln(2)A_{i,m}2^{\mathcal{C}_{i,m}}\left(2^{\mathcal{C}_{i,m}}-1\right)}{W}\left[\frac{\prod_{j=1}^{N}2^{\mathcal{C}_{j,m}}}{\prod_{j=1}^{N}\left(2^{\mathcal{C}_{j,m}}-1\right)}-1\right], \tag{28}$$

for any $i\in\mathcal{N}$ and $m\in\mathcal{M}^\star$, which in turn implies that

$$A_{i,m}2^{\mathcal{C}_{i,m}}\left(2^{\mathcal{C}_{i,m}}-1\right)=A_{k,m}2^{\mathcal{C}_{k,m}}\left(2^{\mathcal{C}_{k,m}}-1\right), \tag{29}$$

for any $(i,k)\in\mathcal{N}^2$ and $m\in\mathcal{M}^\star$, when $\boldsymbol{\mathcal{C}}=\boldsymbol{\mathcal{C}}^\star$. Let k be a fixed index, then any $\mathcal{C}_{i,m}$ variables can be related to the variable $\mathcal{C}_{k,m}$ such that

$$\mathcal{C}_{i,m}=\log_2\left(1+\sqrt{1+4\frac{A_{k,m}}{A_{i,m}}2^{\mathcal{C}_{k,m}}\left(2^{\mathcal{C}_{k,m}}-1\right)}\right)-1, \tag{30}$$

for any $i\in\mathcal{N}$ and $m\in\mathcal{M}^\star$ with $i\neq k$. Hence, the NM-variable function $E_b(\boldsymbol{\mathcal{C}})$ in (9) can be reformulated into a M-variable function when $\boldsymbol{\mathcal{C}}=\boldsymbol{\mathcal{C}}^\star$ by substituting $\mathcal{C}_{i,m}$ in (10a) and (10b) with (30). Moreover, since $2^x-0.5$ is a good approximation of $\sqrt{2^x\left(2^x-1\right)}$, i.e. they differ by less than 1% for $x\geq 2$, a simplified but approximated relation between any $\mathcal{C}_{i,m}$ and $\mathcal{C}_{k,m}$ is given by

$$\mathcal{C}_{i,m}\approx\log_2\left(1+\sqrt{\frac{A_{k,m}}{A_{i,m}}}\left(2^{\mathcal{C}_{k,m}+1}-1\right)\right)-1. \tag{31}$$

Equation (12) can then be obtained by inserting (31) into (28) and using the change of variables defined in (13).

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