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LOCAL: LOCATION-ALLOCATION MODELS FOR ESTABLISHING FACILITIES

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Abstract

Two models and accompanying computer programs are presented which were developed to locate facilities for the timber harvesting industry. Addressed is a case of the facility location-allocation problem. The optimum number and location of landings, facilities to which timber is transshipped, are determined so as to minimize a cost function. This function is composed of the following components: (1) cost of transporting material from its original location to a landing, (2) cost of establishing a landing (facility cost), and (3) cost of transporting material from the landings to some designated point termed the origin. Output of the computer programs is designed to facilitate production planning and includes plotter-drawn graphics. Although both models have been developed under a grant from the U.S. Forest Service for the forest products industry, the models have broader application and hence, other industrial applications are addressed.

Introduction

The process of harvesting timber and transporting it to a mill can be defined as consisting of five stages or subsystems: (1) felling, (2) bucking, (3) skidding (yarding) and decking, (4) loading, and (5) hauling. Felling consists of downing the trees and is accomplished with a chainsaw or a mechanized feller. Bucking consists of limbing the felled trees and then cutting the trunks into predetermined lengths. Transporting the timber to consolidation areas, termed landings, is the next stage of a logging system. If logs are dragged on the ground, the operation is generally referred to as skidding. However, if the logs are transported in the air, the operation is called yarding. Once logs arrive at a landing, they are arranged in a group called a deck. From the deck, the logs are transferred to a tractor-trailer rig or railcar in the loading operation and then transported to a mill in the hauling operation.

In planning a logging operation, the area to be logged is generally divided into subareas called units on the basis of timber and terrain characteristics. The number of landings, the location of landings, and the allocation of timber in the various units to landings must be determined. For a given landing situated in a specific location, several costs can be identified: (1) the cost to skid (yard) timber to the landing, (2) the cost to clear an area for the landing, and (3) the cost to haul from the landing. Also the cost to build a spur road from an existing road to the landing is sometimes incurred. For a given area being logged, the

number and location of landings affect each of these costs. As the number of landings increases, the skidding (yarding) costs generally decrease; however, the cost to establish landings increases simultaneously. Conversely, if the cost to establish landings is minimized, skidding or yarding costs become large. Hence, in order to minimize total cost, an optimum balance must be achieved between opposing cost components.

Location-Allocation Problems

The problem described in the preceding section is analogous to the facility location problem familiar to most industrial engineers. Where and how many landings to establish is analogous to the problem of, for example, determining where and how many warehouses should be built for a given distribution system. More specifically, the problem belongs to a class of problems referred to as location-allocation problems, as defined by Cooper² because timber must be allocated to various landings.

The general location-allocation problem is often described in terms of the "warehouse location problem" and stated as follows. Given the location of various consuming centers and all relevant information about them, determine: how many warehouses should be established, where the warehouses should be located, which consuming centers should be serviced by which warehouse, what volume each warehouse should handle, and what modes of distribution should be used so as to minimize some overall cost function.⁵

A large number of variations of the problem exist. For example, alternative methods of measuring distance (e.g., Euclidean, squared Euclidean, rectilinear) lead to alternative formulations. Also the consideration of various cost components in the overall cost function yields a number of different formulations. Whether the solution space is considered continuous or discrete results in a further subdivision of the problem. The general problem has defied solution because of its dimensions and complexities. Solutions to special cases do exist and many notable treatments can be found in the literature. These are not reviewed here, but may be found in a number of references such as Cabot, et al¹ and Francis and White⁶.

This paper presents another variation of the problem. The various units can be considered as consuming centers, or existing facilities, and potential landings are analogous to the new

facilities. Gibson and Egging^{4,7} have developed two formulations and accompanying solutions for application in the forest harvesting industry. One formulation applies to yarding timber by helicopter while the other relates to skidding by means of either a rubber-tired skidder (RTS)--a four-wheeled drive, articulated, tractor-type vehicle--or a crawler tractor. In several respects both formulations are unique compared to those in the literature. First, although Euclidean distances are used to determine shipping costs, obstacles may be present. That is, a shipping route may be a series of linear line segments routed around an obstacle, as opposed to a single straight-line path. Secondly, the cost of establishing a facility has two components: a fixed cost and also a variable cost which is dependent upon the amount of material shipped to it. Also, a second shipping cost is included in the helicopter model--the cost of transporting material from facilities to another designated point.

Several features distinguish the two models: (1) the treatment of shipping costs, (2) location constraints, and (3) solution technique. The RTS model utilizes multiple regression analysis to relate costs to characteristics of the route traveled, whereas the helicopter model utilizes a more traditional and deterministic methodology. Facilities are constrained to be located on an existing transportation network in the helicopter model. In the RTS model, a finite number of potential locations are independent of the present road systems. Relative to solution technique, the helicopter model utilizes an iterative technique to enumerate solutions until locations are found to be within a specified increment of the optimum, whereas the RTS model utilizes a truncated enumeration technique to find the optimum solution.

Helicopter Model

Problem

Increased awareness of the environmental impact of logging has directed more attention towards aerial systems such as yarding by helicopter. Helicopter yarding minimizes road building and disturbance to soil and standing timber. In addition, the great demand for wood products has made helicopter logging attractive since the technique has made it possible to harvest timber that had heretofore been inaccessible to conventional methods. As a result, the volume of timber harvested by helicopter systems has been increasing steadily each year.

Once timber is felled and marked for turns (groups of logs designated to be yarded together) in a helicopter logging operation, it is yarded to landings that are generally located on or in close proximity to an existing road. Decks are formed and the timber is then subsequently loaded on trucks and hauled to the mill. Location and number of landings affect: (1) yarding costs, (2) landing construction costs, and (3) hauling costs. Topographical features often constrain flight paths and areas suitable for landings.

Figure 1 illustrates an area to be logged by

helicopter where there are four units (centroids on the basis of timber volume shown) and a haul road represented by eight linear line segments. Note that associated with two of the units are ridges that may serve as flight obstacles. Two portions of the road designated by slashes are not suitable for a landing. Contour lines are also shown.

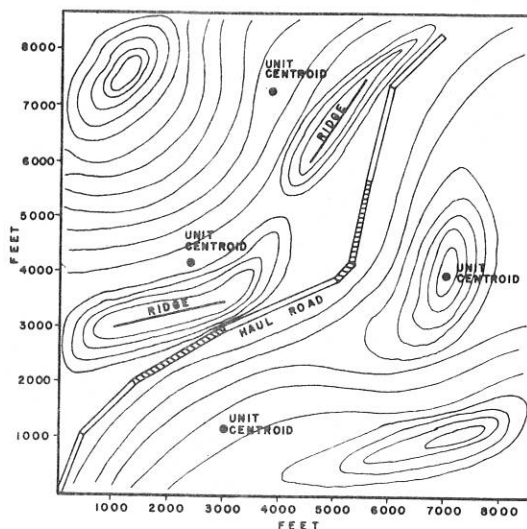


FIGURE 1.-- TOPOGRAPHICAL ILLUSTRATION OF LANDING LOCATION PROBLEM.

A formal statement of the problem is presented below:

Given:

1. a haul road which can be represented by a set of linear line segments,
2. a set of units to be logged, together with their respective volumes and centroids,
3. road segments infeasible for landing locations,
4. ridges or other topographical obstacles which can be represented by linear line segments that may preclude direct flight paths between units and landings, and
5. pertinent system costs and operating characteristics.

Find:

1. the number of landings,
2. the location of landings on the haul road, and
3. the allocation of units to landings,

so as to minimize cost.

Cost has three components, those associated with yarding, landing construction, and hauling. Yarding cost is the cost to transport material from the units to the landing and includes the average speed when loaded and the average speed when the aircraft is empty. Unit centroids are used to calculate average flight distances. Distances between unit centroids and the haul road are Euclidean (including elevation) except when an obstacle is encountered. In such a case, the flight path is determined to be the shortest distance composed of two linear line segments around the obstacle. If a ridge or other topographical obstacle runs through a unit, the unit may be redefined as two units. In this way, part of the material will be routed around the obstacle and the remainder can be transported directly to the landing. Landing construction cost can vary with location (road segment) and may have both fixed and variable components. The latter component varies with weight yarded to the landing. Hauling cost is the cost to transport material from the landings to some designated point such as the origin of the layout.

Solution Procedure

Several features distinguish this problem from those commonly found in the literature: (1) facilities are constrained to be on an existing transportation network (the haul road), (2) obstacles may be present, (3) facility construction cost (location dependent) has not only a fixed component, but a variable one as well, which is dependent upon the amount of material shipped to it, and (4) a second transportation cost is included--the hauling cost. Thus, no previously developed algorithm proves appropriate. To solve the problem the haul road is searched in increments specified by the analyst. Such a search is made for each possible allocation of units to landings. All feasible solutions are enumerated in order to find the optimal. The size of the typical helicopter landing problem (generally less than 8 units) facilitates this approach.

Figure 2 gives a simplified flow chart of the solution procedure which is employed in the model's accompanying computer code. Block 1 represents the operation of reading input relative to all pertinent parameters of the problem, including information concerning the units logged, the haul road, operating characteristics of the logging system, landing construction costs, hauling and operating costs, and topography of the area.

Block 2 represents the establishment of an allocation scheme; that is, specifying how units are to be assigned to landings. For example, if three units are to be logged, timber in the first unit may be yarded to one landing and timber in the other two may be yarded to a second landing. The program has the facility to search all possible allocation schemes or only a particular subset of special interest to the user.

Once an allocation scheme is fixed, the program finds the best location for the landings. The program can search along the entire road for these

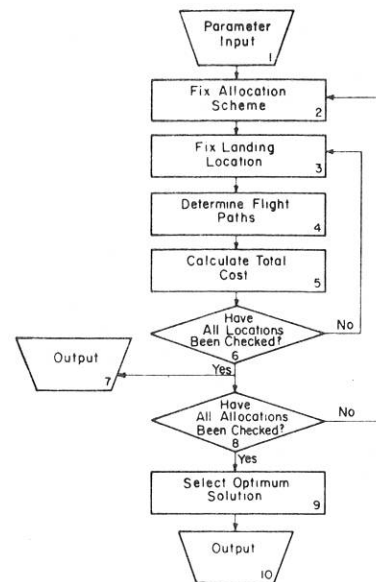


FIGURE 2.-- SIMPLIFIED FLOW CHART OF SOLUTION PROCEDURE.

locations or will search only specified points. In either case, it begins the search by fixing a location. This operation is represented by Block 3.

Next, flight paths and distances are determined between the units and the landings. This entails checking to see if direct flight paths are precluded because of ridges or other obstacles. As mentioned previously, if an obstacle is encountered, the flight path is determined to be the shortest distance composed of two linear line segments around the obstacle.

Total costs, computed for the allocation scheme and location(s) at hand, are then calculated as illustrated by Block 5.

Block 6 of figure 2 signifies the program's check to see if all locations have been evaluated. If they have not, the cost of the next location is calculated. The next location can be at a specified increment up the road or at a particular location specified by the user. This process continues until all locations have been evaluated for a given allocation scheme.

Intermediate output of suboptimal solutions can be obtained as shown by Block 7. That is, the program can, at the discretion of the user, print out the best solution for a given allocation scheme.

Once all locations have been evaluated for the current allocation and the minimum cost location selected, the program checks to see if all allocation schemes have been examined. This operation is represented by Block 8. If all schemes have not been evaluated, the process cycles back

to Block 2, otherwise the overall optimal solution is selected from previous calculations as shown in Block 9. Block 10 illustrates the final output which may be in several forms at the option of the user, including graphical plotting.

Example

Figure 3 gives the layout showing in figure 1 with coordinates, elevations, and other input data that are required by the model. Note that two sections of the roadway cannot accommodate a landing. Output from the program has been designed to accommodate the user. Hence, output can be of several forms on a printout from a line printer and also a plot graphically portraying the solution. Figure 4 shows the plot of the optimal solution for this example. Three landings are indicated with x, y, & z coordinates given for each. The minimum cost is \$17,535.76 (yarding = \$12,730.80, landing building = \$4,760.00, hauling = \$44.96).

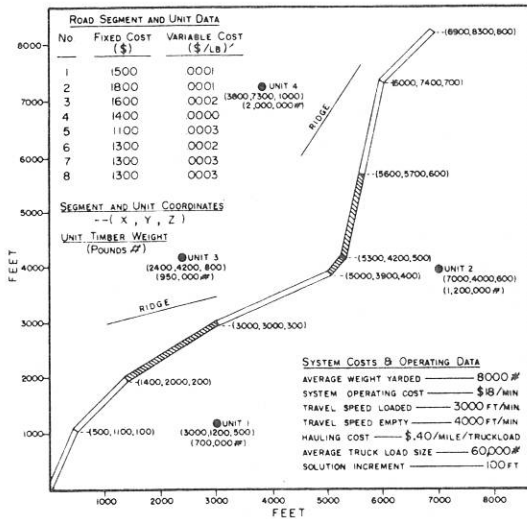
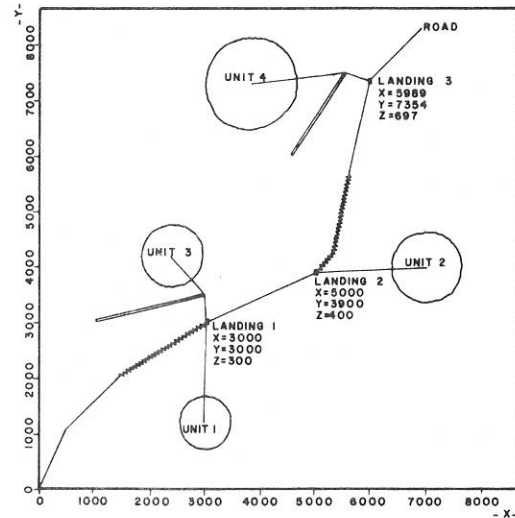


FIGURE 3.-- LAYOUT OF AREA TO BE YARDED WITH HELICOPTER.

Problem

Consider the layout shown in figure 5. This irregularly shaped area is to be clearcut and skidded by means of a rubber-tired skidder. It has been subdivided into nine units via timber and terrain characteristics. Unit centroids are identified (U1, U2, ..., U9). Eleven points have been selected as possible locations of landings (L1, L2, ..., L11). Also, three topographical constraints over which the skidders are unable to traverse have been identified.

It is desired to find the optimal number and location of landings together with the allocation of units to decks so as to minimize cost. Cost is defined as having three components: skidding cost,



HELICOPTER LANDING LOCATION MODEL
AIE EXAMPLE PROBLEM

FIGURE 4.-- PLOT OF OPTIMAL SOLUTION TO HELICOPTER YARDING PROBLEM.

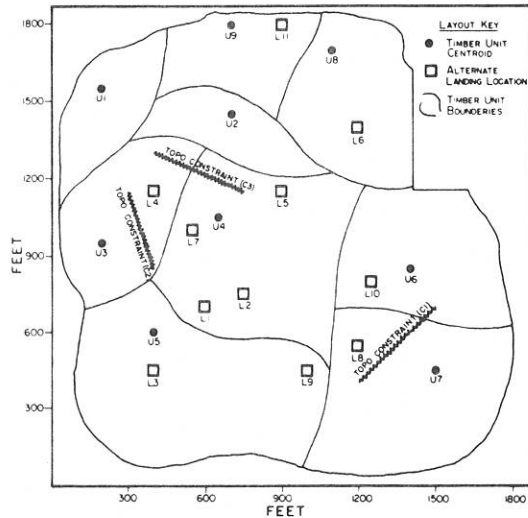


FIGURE 5.-- LAYOUT OF AREA TO BE SKIDDED WITH RUBBER-TIRED SKIDDER.

landing building cost, and spur road building cost. Skidding cost, the cost to transport material from woods to landing, is computed as the product of skidding time per trip, number of trips, and cost per unit time. The skidding time per trip is computed from a regression equation which relates characteristics of the route traveled such as slope, distance, weight skidded, etc., to travel time. Ray⁸ also used regression equations in his location analyses. The equations used in

conjunction with this model have been developed on the basis of a comprehensive work measurement program. Landing building cost represents the cost to clear an area for a deck and, as with the helicopter model, is location dependent and has both fixed and variable components. The variable component is proportional to the volume of timber handled at that landing. Spur road building cost represents the cost of establishing a road from an existing road to the deck.

Solution Procedure

The problem described in the preceding section is analogous to the discrete facility location-allocation problem. However, because of the unique characteristics of its cost function and routing features, existing formulations are not appropriate for solution of the complete problem. The model consists of four main phases: parameter input, location phase, allocation phase, and solution output. Limitations of space preclude a complete presentation of the solution algorithm. Such a presentation is to be given in a subsequent paper. However, an outline of each phase of the model follows.

Parameter Input. Four main sets of data are required as input: general characteristics (number of units, number of alternative landing locations, etc.), regression coefficients, unit information, and landing information. Any regression equation of the form

$$Y = A + B_1 X_1^{C_1} + B_2 X_2^{C_2} + \dots + B_N X_N^{C_N} \quad (1)$$

where X_1, X_2, \dots, X_N are independent variables and $A, B_1, B_2, \dots, B_N, C_1, C_2, \dots, C_N$ are constants, can be used. Unit information consists of data such as centroid locations, topographical features, timber characteristics, and other information which may be required by the regression equation. Landing information includes data concerning location and costs. Spur road building costs are assumed to be included in the fixed costs.

Location Phase. Given I units, there are

$$LC(I) = \sum_{k=1}^I \binom{I}{k} \quad (2)$$

different ways (combinations) these units can be combined with other units. In the location phase of the model, alternative landing locations are searched to determine the best location for each unit combination. For example, if there are five units, the optimal location for each of

$$\sum_{k=1}^5 \binom{5}{k} = \frac{5!}{1!4!} + \frac{5!}{2!3!} + \frac{5!}{3!2!} + \frac{5!}{4!1!} + \frac{5!}{5!0!} = 31 \quad (3)$$

unit combinations is determined. The 31 possible unit combinations are enumerated below.

1	1,2	2,4	1,2,3	1,4,5	1,2,3,4	1,2,3,4,5
2	1,3	2,5	1,2,4	2,3,4	1,2,3,5	
3	1,4	3,4	1,2,5	2,3,5	1,2,4,5	
4	1,5	3,5	1,3,4	2,4,5	1,3,4,5	
5	2,3	4,5	1,3,5	3,4,5	2,3,4,5	

Allocation Phase. Define an allocation scheme as a specification of how a given number of objects (units) are to be assigned to different sets (landings). Allocation matrices can be employed to illustrate allocation schemes. One possible allocation matrix for five units and three landings is illustrated below.

Units	Landings		
	1	2	3
1	1	0	0
2	0	0	1
3	0	1	0
4	0	0	1
5	0	1	0

A 0 or 1 entry for the element in the i^{th} row and j^{th} column indicates that unit i is not or is, respectively, assigned to landing j .

To find the minimum cost solution associated with this matrix, one could combine the solutions found in the previous stage for the unit combinations: 1; 3,5; 2,4.

The problem is clearly combinatorial. Given I units and J landings, there exists

$$AS(I,J) = \frac{1}{J!} \sum_{k=0}^J \binom{J}{k} (-1)^k (J-k)^I \quad (4)$$

allocation schemes. Assuming that a unit would be assigned to at most one landing (possibility of redefining units permits this assumption), there are

$$TAS(I) = \sum_{J=1}^I AS(I,J) \quad (5)$$

total possible allocation schemes to consider given I units. Shown below is the number of location combinations (equation 2) and allocation schemes (equation 5) that exist for various numbers of units.

No. of Units	No. of Location Combinations	No. of Allocation Matrices
1	1	1
2	3	2
3	7	5
4	15	15
5	31	52
6	63	203
7	127	877
8	255	4,140
9	511	21,147
10	1,023	115,975
11	2,047	678,570
12	4,095	3,229,936
13	8,191	7,434,656
14	16,383	13,226,835
15	32,767	37,582,186

It is desired to determine which allocation scheme, together with the designation of landings to be established, yields the minimum cost solution. Cooper³ and others have discussed the computational difficulty of finding the optimal solution to such problems. Heuristic methods are often used to solve large problems. Because most of the problems encountered in the area of application at hand are relatively small, it was decided to develop an optimal method. Future work is proposed to test and extend its rationale for larger problems.

The least-cost solution for any allocation scheme can be found by combining various unit combination solutions derived in the location phase of the model. Allocation schemes are divided into subsets of allocation patterns. An allocation pattern is defined to be a set of numbers, summing to the number of units being considered, that represent the number of units at each of some designated number of landings. For example, if there are five units, one possible allocation pattern would be 3-2. This signifies that three units are going to be skidded to one landing and two units are going to be skidded to another landing. Several allocation schemes would be associated with this allocation pattern. An example of another allocation pattern for five units is 2-2-1. Again, several allocation schemes would be associated with this allocation pattern.

Allocation patterns are classified into three groups. Solutions associated with the first group, those patterns of the form I and 1-(I-1), where I is the number of units being considered, are enumerated utilizing unit combinations. This is done in order to establish a current feasible lower bound. Next, solutions associated with the second group of patterns, those of the form $1_1 - 1_2 - \dots - 1_N - (I-N)$ and $N - (I-N)$ (where $N = 1, 2, \dots, I/2$ if I even or $N = 1, 2, \dots, [I-1]/2$ if I odd), is searched employing dynamic programming to attempt to improve the current solution. Finally, solutions associated with the remaining patterns are searched using a branch-and-bound methodology. Upon completion of this

process, the current solution is the optimal solution.

Solution Output. Output of the coded model can include both printed solutions from the line printer and plotted schematics of the optimal solution, or solutions of specific allocation schemes. A number of output options are available to the user.

Example

Consider the layout illustrated in figure 5. It is desired to determine the number and location of landings together with the allocation of units to landings so as to minimize costs. Pertinent data for the problem is given in Table 1. A round trip between a unit and a landing is called a "turn." The "average per turn" figures relate to the amount of material that is skidded from a unit during each trip. This information is required because of the form of the regression equation. As mentioned previously, the regression equation used in conjunction with the model was derived from time-and-motion studies. This particular equation has an R^2 of approximately .85.

Searching the three sets of allocation patterns yields the solutions shown in Table 2. In this example, each stage of the search yields successively an improved solution. Figure 6 shows the optimal solution plotted by the computer code. Circle diameters are proportional to the amount of timber in each unit.

TABLE 1.--DATA FOR EXAMPLE PROBLEM
UNIT DATA

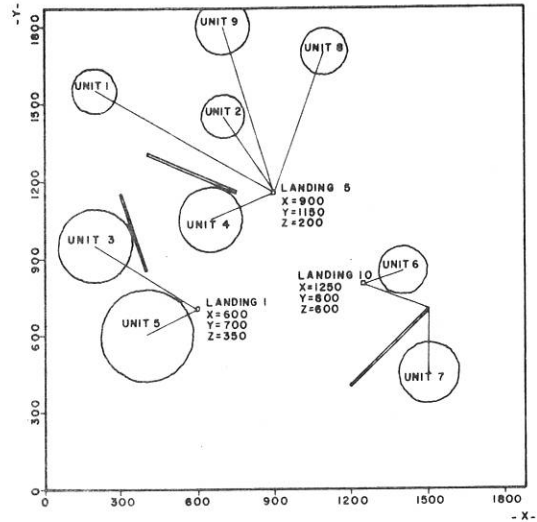
UNIT	X	Y	Z	VOLUME, M ³ /BM	AVERAGE PER TURN		CONSTRAINT
					NO. LOGS	VOLUME, HEIGHT, FBM LR	
1	200	1550	100	23.0	6	800 4000	
2	700	1450	150	25.2	5	1200 4500	C3
3	200	950	300	72.0	8	1000 3500	C2
4	650	1950	200	55.0	7	1400 5200	C3
5	400	600	300	112.0	3	2000 6500	
6	1400	350	600	30.5	5	1100 4300	
7	1500	450	500	43.2	9	600 3300	C1
8	1100	1700	100	30.0	7	1000 5000	
9	700	1800	100	40.0	7	900 4800	
TOTAL					440.9		

LANDING	X	Y	Z	LANDING DATA	
				FIXED COST, DOLLARS	VARIABLE COST, DOLLARS/M ³ BM
1	600	700	350	150	0.40
2	750	750	500	250	.45
3	400	450	500	300	.50
4	400	1150	100	150	.50
5	900	1150	200	100	.30
6	1200	1400	150	350	.55
7	550	1000	300	200	.40
8	1200	550	600	150	.45
9	1000	450	600	200	.50
10	1250	800	600	100	.25
11	900	1800	100	150	.30

CONSTRAINT DATA						
No.	x ₁	y ₁	z ₁	x ₂	y ₂	z ₂
C1	1200	400	600	1500	700	500
C2	300	1150	100	400	850	500
C3	400	1300	100	750	1150	500

REGRESSION EQUATION (MIN/TURN) $TT = 2.73846 + 0.72634(\text{No. LOGS})$
 $+ 0.00363(\text{VOL.}) - 0.00020(\text{WT.}) + 0.00777(\text{DISTANCE, FT.})$
 $+ 0.00313(\% \text{ SLOPE})^2$
 SKIDDER COST = \$19.50/HR.

The set numbers shown in Table 2 refer to the sets or groups into which the allocation patterns are subdivided during the allocation phase. Each set is searched utilizing a different technique. Since there are 9 units in this example, set 1 contains the allocation patterns of the forms 9 and 1-8. All such patterns are enumerated and the one with the least cost is chosen as the best current feasible solution. Next patterns of the forms 2-7, 1-1-7, 3-6, 1-1-1-6, 4-5, and 1-1-1-1-5 are searched. This is done by taking the minimum cost pattern (utilizing values derived in the location phase) of $1-1-2-\dots-1_N$ or N plus the cost of the associated $(I-N)$ pattern. If at any time such a search yields a better solution, this solution becomes the best current feasible solution. Finally patterns in the third set are searched using a branch-and-bound methodology. In order to speed the searching process both infeasible and feasible solutions may be generated. However if a solution is found that is better than the best current feasible solution, branching continues until feasibility is verified or until the branch's value exceeds the value of the best current feasible solution. The optimum solution in this example was found while searching those patterns of the form 2-2-5.



DECK LOCATION MODEL
A11E EXAMPLE PROBLEM

FIGURE 6.-- PLOT OF OPTIMAL SOLUTION TO SKIDDING PROBLEM.

Table 2.--Successive improvement to solution in allocation phase

Set 1 - Enumeration	
<u>Units</u>	<u>Landing</u>
3	1
1-2-4-5-6-7-8-9	5
Skid cost	= \$2,688.02
Landing cost	= <u>396.67</u>
Total cost	= \$3,084.69
Set 2 - Dynamic Programming	
<u>Units</u>	<u>Landing</u>
3-5-6-7	5
1-2-4-8-9	1
Skid cost	= \$2,706.90
Landing cost	= <u>158.54</u>
Total cost	= \$2,865.44
Set 3 - Branch and Bound	
<u>Units</u>	<u>Landing</u>
1-2-4-8-9	5
3-5	1
6-7	10
Skid cost	= \$2,344.93
Landing cost	= <u>496.74</u>
Total cost	= \$2,841.67

Other Applications

The location models presented have been developed as part of a comprehensive study of logging harvesting, transportation, and distribution systems. Although the problem addressed belongs to a general class of location-allocation problems, unique characteristics of the timber-harvesting system precluded the adoption of existing formulations. These same characteristics may likewise prevent the application of the models presented in this paper to certain other areas. However, there are most likely transportation and distribution situations of similar nature in other industries. For example, the landings may be thought of as any facility to be located, such as industrial and municipal facilities that have a fixed-cost component and a variable component proportional to the amount of work, material, etc. processed. Logging units may be thought of as any type of source or sink, such as a warehouse or a demand center. Drawing analogies to these and other aspects of the problem would lead to other applications. To illustrate such cases, two hypothetical location problems relating to the transportation and distribution of material for the energy producing industry are now presented.

Locating Pumping Stations for "Mine-Mouth" Generating Plants

Located in southeastern Montana are vast deposits of low sulfur coal that can be removed by stripmining. Currently, plans are underway to utilize this resource in coal-fired electrical generating plants located at the mine site, called "mine-mouth" generating plants. The mine-mouth process requires large amounts of water for cooling. It is proposed to obtain water rights and pump

water from the Yellowstone River for this purpose.

Water is to be pumped to one or more mine-mouth facilities by a system of single high-pressure stations. The single station system eliminates the need for auxiliary pumping stations along pipeline routes, thus reducing costs for power line extensions, access roads, and maintenance. Since a single pumping station will supply water to one or more generating plants, each requiring upwards from 50,000 gallons per minute (gpm), station installation requirements are relatively large. Closed conduit flow under pressure will be the method used to transport the water from the source to plants. Exact pumping station size will depend on the allocation of mines to stations. Variable cost of each station will reflect this allocation and include, based on gpm capacity, the costs of: earthwork and excavation, structure, pumps, motors, controls, electrical system, pipes within the structure, mechanical accessories, and devices to remove silt, sand, and sediment. Pumping station fixed costs include: land purchase, access roads, improvements, and utility hookups.

Figure 7 shows the layout of three areas of coal deposit in relation to the Yellowstone River and two towns in southeastern Montana. Mountain Power Company has plans to build a mine-mouth plant at each site. Given the location of these facilities the company must determine the number and location of pumping stations together with the allocation of mines to stations so as to minimize cost. Two segments of the river are not suited for stations. One segment is restricted to recreational uses only, and the other segment does not have sufficient depth. The presence of a town and a restricted land section prohibit pipeline crossing. Pertinent costs are shown in figure 7.

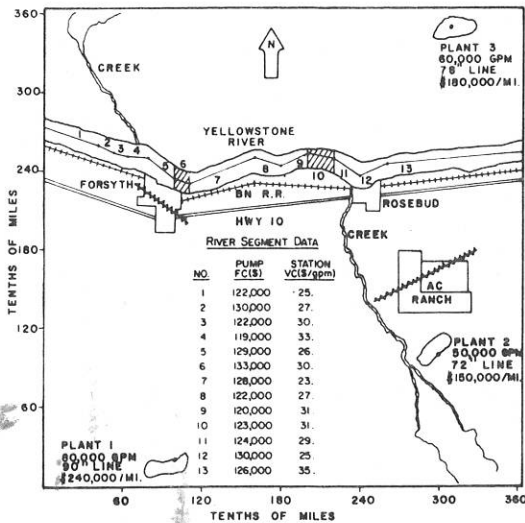


FIGURE 7.-- LAYOUT OF PUMPING STATION - GENERATING PLANT AREA.

This problem has the same basic structure as that of the helicopter landing problem. Locating pumping stations along the river is analogous to locating landings along the haul road. Cost analogies between the two problems can be easily made. Figure 8 shows the computer plot of the optimal solution together with its associated costs. The helicopter model can also be used to find the cost of specific location-allocation combinations that may interest management.

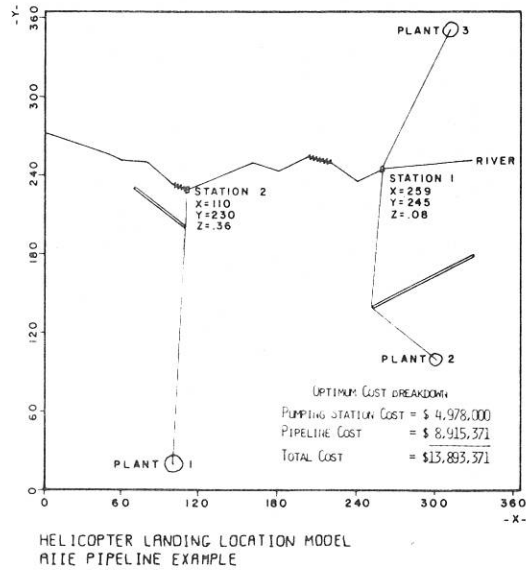


FIGURE 8.-- PLOT OF OPTIMAL SOLUTION TO PUMPING STATION - GENERATING PLANT PROBLEM.

Locating Tank Farms for Oil Shale Production

Locked in the Rocky Mountains' Green River formation is perhaps one of the most promising solutions to our Nation's current oil shortage. Here large deposits of oil shale--a marlstone-type inorganic component mixed with an organic polymer called kerogen--exist in a 17,000 square mile region that encompasses parts of Wyoming, Utah, and Colorado. Several processes exist for recovering oil from oil shale and involve heating to decompose the kerogen to volatile oil and gas followed by condensation and recovery of vapors. It has been estimated that there is enough oil shale to produce 30 to 70 times the amount of oil the United States has produced since the Civil War. Although economics in the past have precluded use of this resource, recent market conditions have changed this situation.

A major U.S. oil corporation has been awarded a \$210 million, 20-year lease for six tracts totaling 5,120 acres along the Green River in Wyoming (fig. 9). It has been decided to use the "in situ" method of extraction in which oil is re-torted in the ground and then pumped to the surface. As opposed to stripmining, crushing, and then retorting, this method eliminates waste disposal and other ecological and economic problems.