Magnetic Substance Disturbance Torque Caused by Shape Magnetic Anisotropy and its Applications in Small-Sized Satellites

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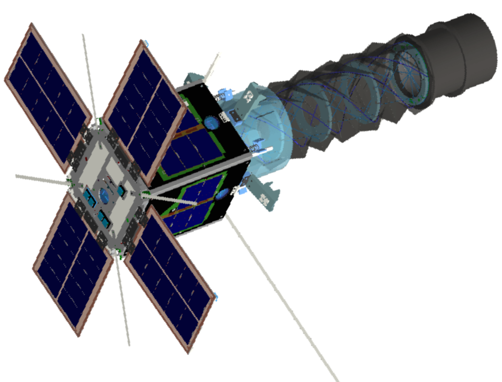

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30th Annual AIAA/USU Conference on Small Satellites · Logan, UT, USA · Utah State University

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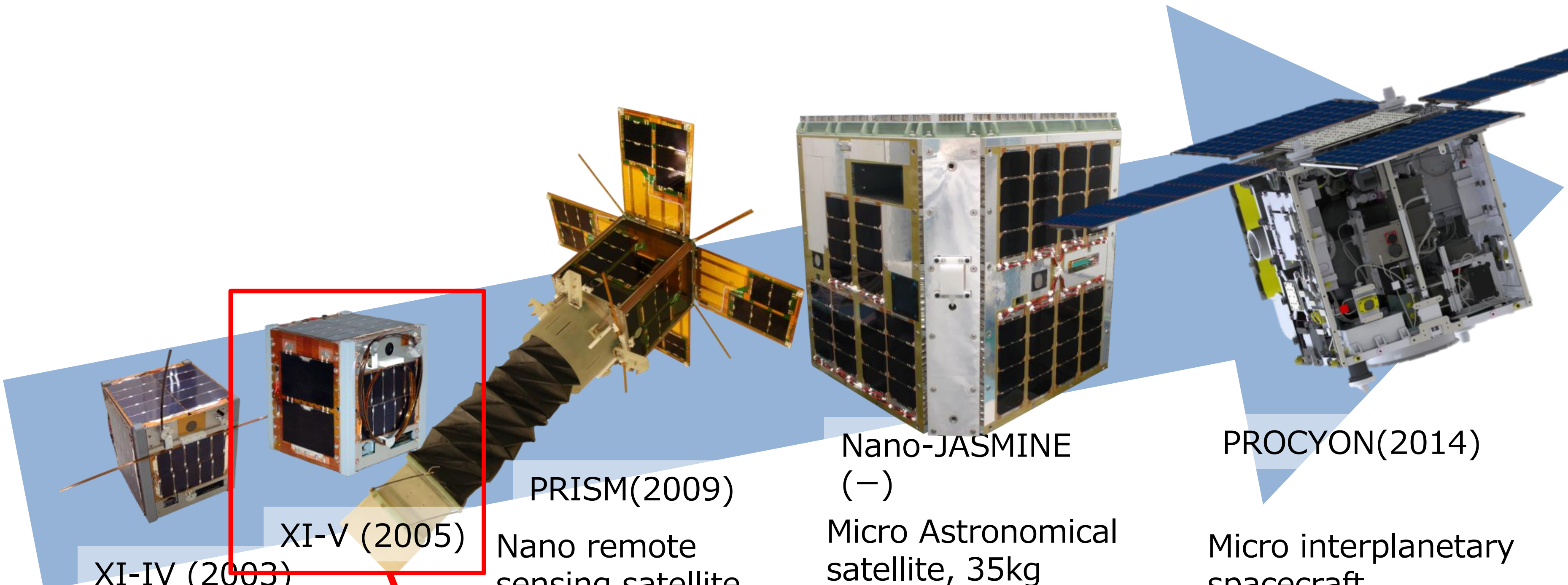
- Small satellites in the University of Tokyo
- Attitude disturbance caused by magnetic substances
  - Ellipsoid shape
  - Arbitrary shape
- Conclusions

Nano- and micro-satellites (University of Tokyo)



XI-IV (2003)
Technical demonstration satellite , 1 kg

XI-V (2005)

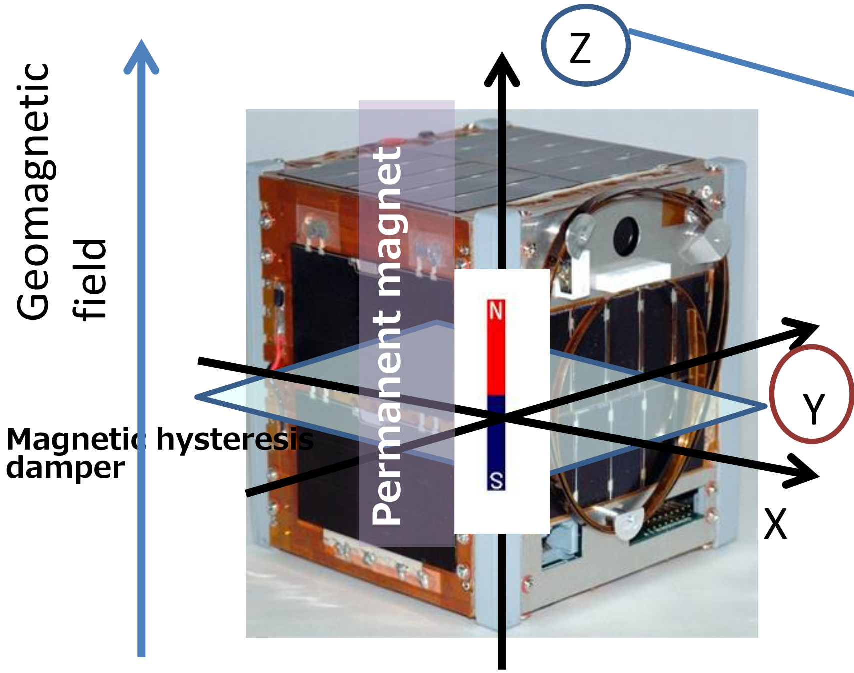
PRISM(2009)
Nano remote sensing satellite, 8.5 kg

Nano-JASMINE (－)
Micro Astronomical satellite, 35kg

PROCYON(2014)
Micro interplanetary spacecraft

Cubesat XI-V (2005)
10cm × 10cm × 10cm ,
1kg
Magnetic passive attitude control system

Coarse attitude estimation by thermometers



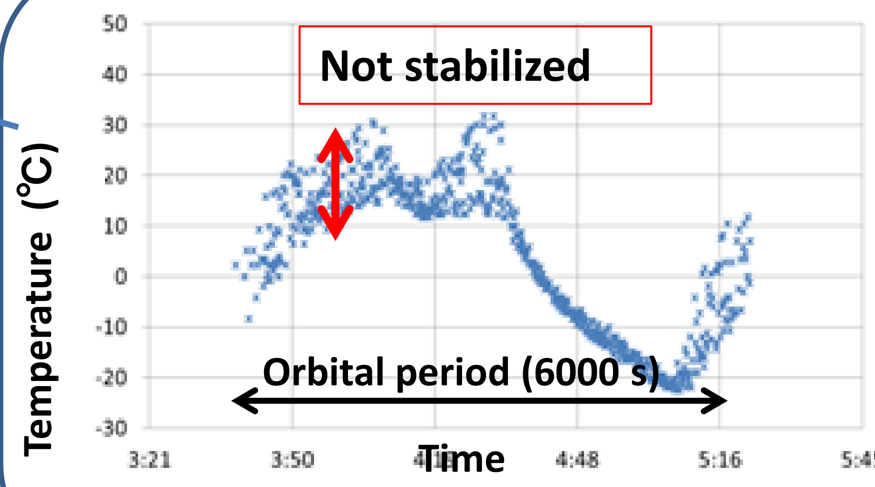


Fig. 1 +Z thermometer

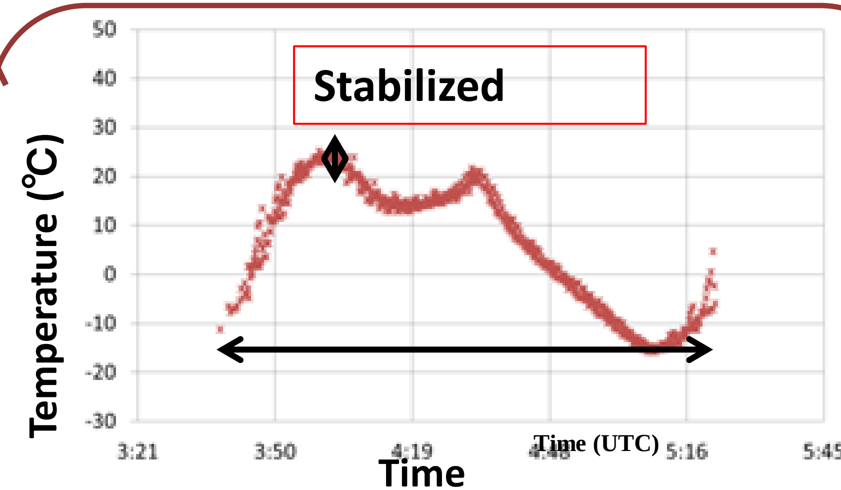


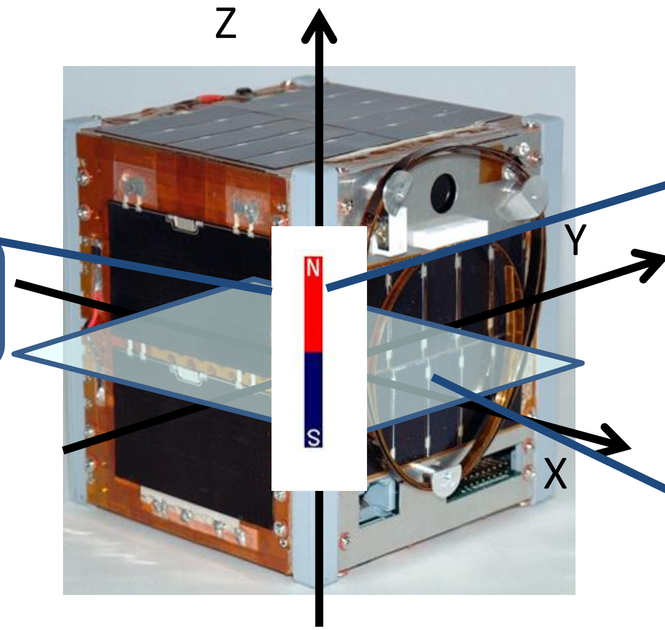
Fig. 2 +Y thermometer

- Although the z axis is not stabilized, the Y axis is aligned with the geomagnetic field vector and stabilized.
- A strong disturbance may affect the satellite attitude.

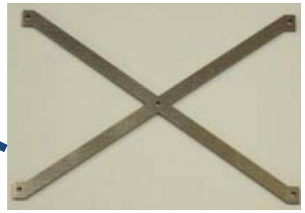
Attitude disturbances in Cubesat XI-V

Table 1 Attitude disturbances in XI-IV

Attitude disturbance	(Nm)
Magnetic disturbance	1×10^{-6}
Aerodynamic disturbance	2×10^{-10}
Solar pressure disturbance	1×10^{-9}
Gravity gradient disturbance	1×10^{-8}



Permanent magnet



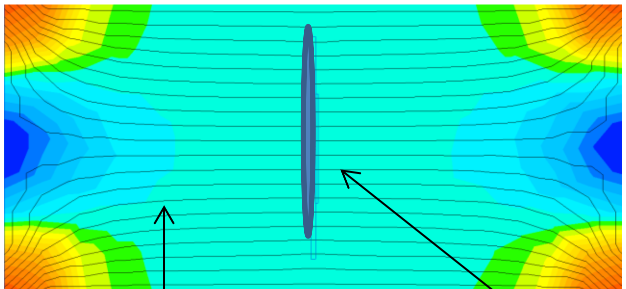
Magnetic hysteresis damper

- The strongest attitude torque is the magnetic torque by the permanent magnet.
- An unexpected disturbance affects the satellite attitude.

Attitude torque caused by a magnetic substance

Perpendicular to magnetic filed

Magnetically unstable



Uniform magnetic field

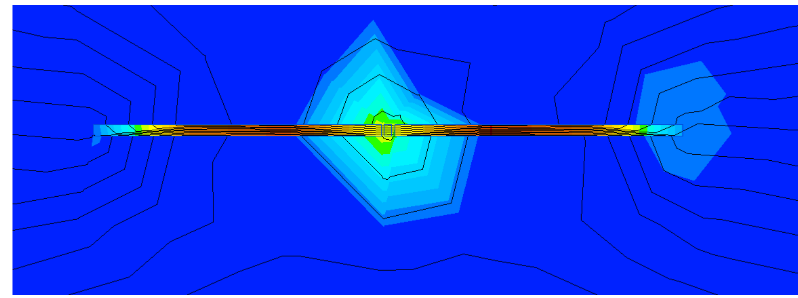
Iron rod

Attitude torque

$$u = \frac{\mathbf{B} \cdot \mathbf{H}}{2}$$

Parallel to magnetic filed

Magnetically stable



- The magnetic energies in Fig. 1 and Fig .2 are not the same.
- The rod rotates to more stable configuration.
  →Magnetic torque by shape magnetic anisotropy

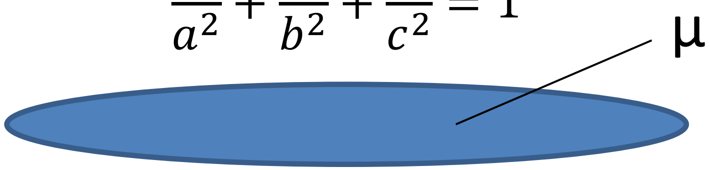
In a uniform magnetic field, a magnetic substance with nonsymmetrical body causes a magnetic torque.

(Attitude disturbance torque caused by shape magnetic anisotropy of on-board magnetic substances)

Magnetic field in an ellipsoid magnetic substance

Magnetic field in an ellipsoid magnetic substance can be solved with the Laplace equation ($\nabla^2\Phi=0$) for an ellipsoid coordinate system.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$



$$(\eta-\zeta)R_\xi\frac{\partial}{\partial\xi}\left(R_\xi\frac{\partial\phi}{\partial\xi}\right)+(\zeta-\xi)R_\eta\frac{\partial}{\partial\eta}\left(R_\eta\frac{\partial\phi}{\partial\eta}\right)+(\xi-\eta)R_\zeta\frac{\partial}{\partial\zeta}\left(R_\zeta\frac{\partial\phi}{\partial\zeta}\right)=0$$

$$R_s=\sqrt{(s+a^2)(s+b^2)(s+c^2)}\quad (s=\xi,\eta,\zeta)$$

Analytical solution

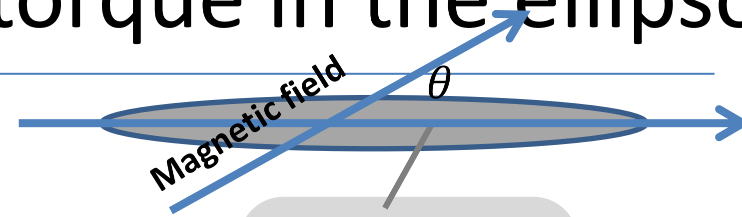
$$H_x=\frac{H_{0x}}{1+X(-1+\mu_r)}\qquad X=\frac{abc}{2}\int_0^\infty\frac{ds}{(a^2+s)R}$$

$$H_y=\frac{H_{0y}}{1+Y(-1+\mu_r)}\qquad Y=\frac{abc}{2}\int_0^\infty\frac{ds}{(b^2+s)R}$$

$$H_z=\frac{H_{0z}}{1+Z(-1+\mu_r)}\qquad Z=\frac{abc}{2}\int_0^\infty\frac{ds}{(c^2+s)R}$$

$$R=\sqrt{(a^2+s)(b^2+s)(c^2+s)}$$

Magnetic substance torque in the ellipsoid



Uniform magnetic field in the x-y plane

$$H_{0x} = H_0 cos\theta$$
$$H_{0y} = H_0 sin\theta$$
$$H_{0z} = 0$$

$$H_x = \frac{H_{0x}}{1 + X(-1+\mu_r)}$$
$$H_y = \frac{H_{0y}}{1 + Y(-1+\mu_r)}$$
$$H_z = \frac{H_{0z}}{1 + Z(-1+\mu_r)}$$

$$X = \frac{abc}{2}\int_0^\infty \frac{ds}{(a^2+s)R}$$
$$Y = \frac{abc}{2}\int_0^\infty \frac{ds}{(b^2+s)R}$$
$$Z = \frac{abc}{2}\int_0^\infty \frac{ds}{(c^2+s)R}$$
$$R = \sqrt{(a^2+s)(b^2+s)(c^2+s)}$$

Magnetic torque:

$$T = \frac{\partial U}{\partial \theta}$$

$$T = \frac{V\mu}{\mu_0^2}\left(-\frac{1}{(1+X(-1+\mu_r))^2} + \frac{1}{(1+Y(-1+\mu_r))^2}\right)B^2 \cos\theta \sin\theta$$

Parameter depending on magnetic permeability and shape Ms.

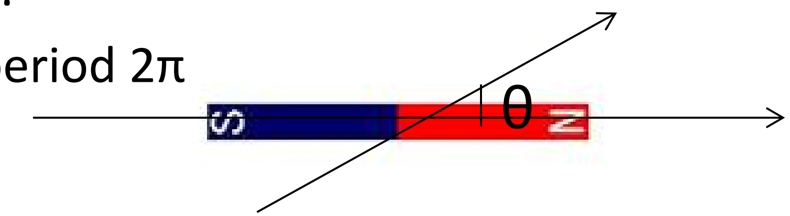
$$T = M_s B^2 \cos\theta \sin\theta$$
$$= M_n B_x B_y$$

V: Volume
μ0: Magnetic permeability
μr: Relative permeability
θ: Angle

①The magnitude is proportional to the square of the geomagnetic field strength.
②The torque line can be expressed with a sine curve with period π.

Dipole and magnetic substance attitude disturbances

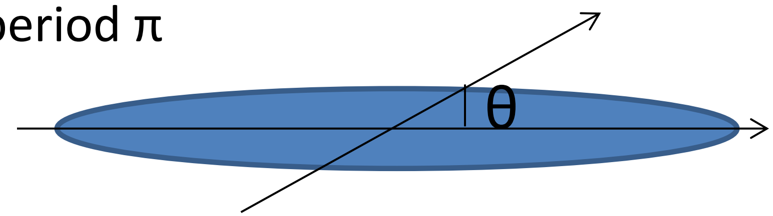
- Attitude disturbance caused in a magnet.
  - Proportional to an outer magnetic field.
  - can be expressed as a sine curve with period 2π



$$T = M_d B \sin\theta$$

M_d: Residual magnetic moment

- Attitude disturbance caused in magnetic substances (in ellipsoid body)
  - Proportional to the square of an outer magnetic field.
  - can be expressed as a sine curve with period π



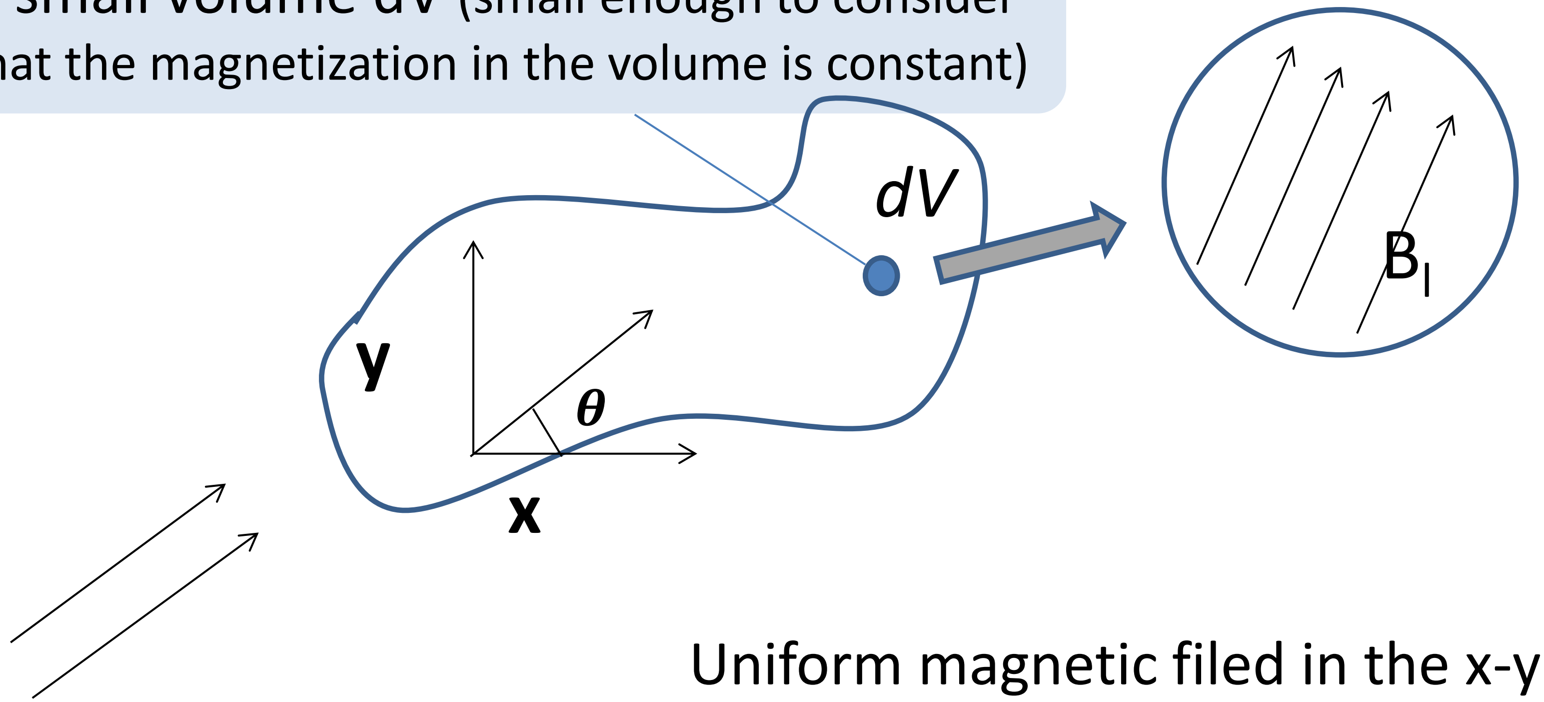
$$T = M_n B^2 \cos\theta \sin\theta$$

M_n: Magnetic substance disturbance coefficient.

- Can we express the torque in arbitrary magnetic substance with the same equation $\mathbf{T} = \boldsymbol{M_s B^2 sin2\theta}$?

Magnetic substance torque in an arbitrary-shape magnetic substance

A small volume dV (small enough to consider that the magnetization in the volume is constant)



$$\boldsymbol{B_0} = \begin{pmatrix} B_0 \cos\theta \\ B_0 \sin\theta \\ 0 \end{pmatrix}$$

Uniform magnetic filed in the x-y plane.

(small enough to consider that the B-H curve is linear, and the magnetic permeability is constant)

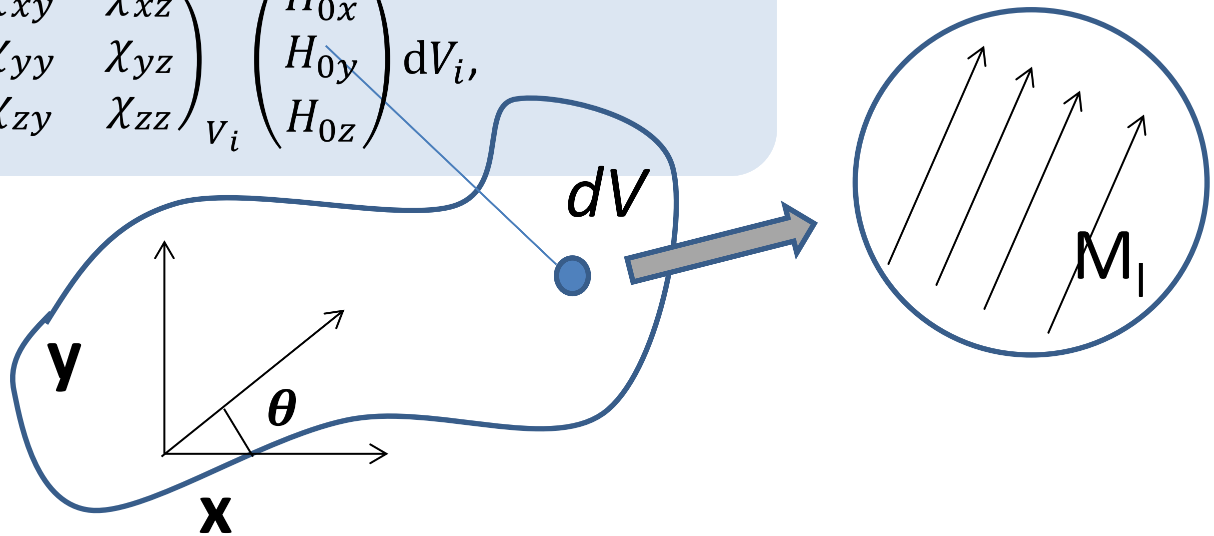
Magnetization in an arbitrary-shape magnetic substance

Magnetization in a small volume

$$\boldsymbol{M_i} = \boldsymbol{\chi H_0} \mathrm{d}V_i,$$

$$\boldsymbol{M_{v_i}} = \begin{pmatrix} \chi_{xx} & \chi_{xy} & \chi_{xz} \\ \chi_{yx} & \chi_{yy} & \chi_{yz} \\ \chi_{zx} & \chi_{zy} & \chi_{zz} \end{pmatrix}_{V_i} \begin{pmatrix} H_{0x} \\ H_{0y} \\ H_{0z} \end{pmatrix} \mathrm{d}V_i,$$

$$\boldsymbol{B_0} = \begin{pmatrix} B_0 \cos\theta \\ B_0 \sin\theta \\ 0 \end{pmatrix}$$



Magnetic energy in a small volume

$$\mathrm{d}U_{V_i} = \frac{1}{2} \boldsymbol{M_{v_i}} \cdot \boldsymbol{H_0} \mu_0$$

$$= \frac{1}{2}\mu_0 \begin{pmatrix} X_{xx} & X_{xy} & X_{xz} \\ X_{yx} & X_{yy} & X_{yz} \\ X_{zx} & X_{zy} & X_{zz} \end{pmatrix} \begin{pmatrix} H_{0x} \\ H_{0y} \\ H_{0z} \end{pmatrix} \cdot \begin{pmatrix} H_{0x} \\ H_{0y} \\ H_{0z} \end{pmatrix},$$

$$X_{jk} = \mu_0 \int_{V_i} \chi_{jk} \, \mathrm{d}V_i \ .$$

Magnetic energy in the whole body

$$U = \frac{\mu_0}{2} \int_V \boldsymbol{M} \cdot \boldsymbol{H_0} dV$$

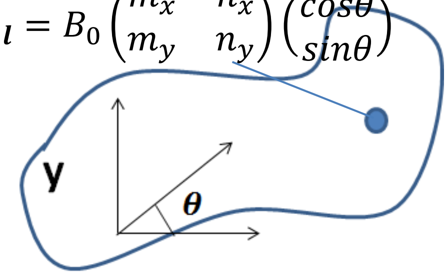
Magnetic substance torque in an arbitrary-shape magnetic substance

- Magnetic energy

$$U = \frac{1}{2}\mu_0 \begin{pmatrix} X_{xx} & X_{xy} & X_{xz} \\ X_{yx} & X_{yy} & X_{yz} \\ X_{zx} & X_{zy} & X_{zz} \end{pmatrix} \begin{pmatrix} H_{0x} \\ H_{0y} \\ H_{0z} \end{pmatrix} \cdot \begin{pmatrix} H_{0x} \\ H_{0y} \\ H_{0z} \end{pmatrix},$$

$$U = \frac{1}{2}\mu_0 H_0^2 (A + B\cos 2\theta + C\sin 2\theta)$$

$$\boldsymbol{B_l} = B_0 \begin{pmatrix} m_x & n_x \\ m_y & n_y \end{pmatrix} \begin{pmatrix} cos\theta \\ sin\theta \end{pmatrix}$$



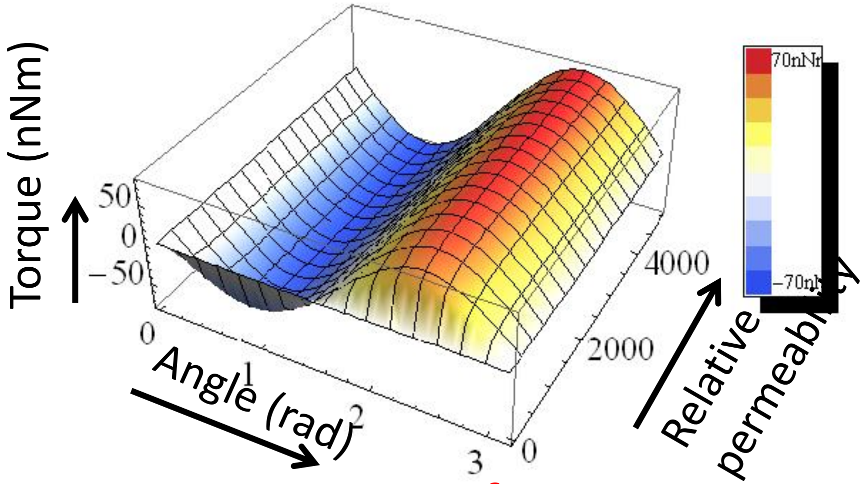
$$H_{\mathbf{0}} = \begin{pmatrix} H_0\cos\theta \\ H_0\sin\theta \\ 0 \end{pmatrix}$$

- Magnetic torque

$$T = \frac{\mathrm{d}U}{\mathrm{d}\theta}$$
$$= \mu_0 H_0^{\mathbf{2}}(-B\sin 2\theta + C\cos 2\theta)$$
$$= \mu_0 H_0^{\mathbf{2}} A' \sin(2\theta + \delta).$$

Magnetic substance disturbance in Cubesat XI-V

- Hysteresis damper
- 5×10^{-8} Nm



- Antenna 4×10^{-6} Nm

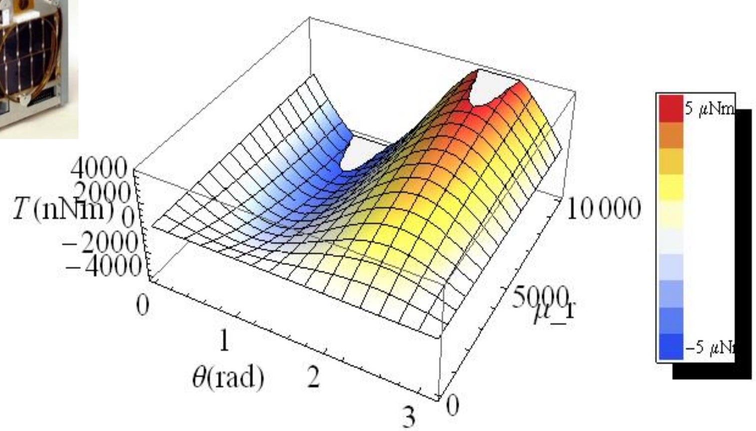
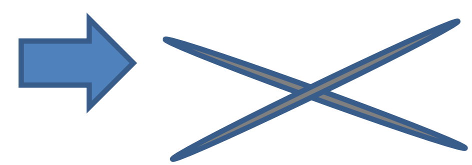


Table 1 Attitude disturbance XI−V

	Torque (Nm)
Residual magnetic (permanent magnet)	**1×10^{-6}**
Aerodynamic	**2×10^{-10}**
Solar radiation	**1×10^{-9}**
Gravity gradient	**1×10^{-8}**



The torque is coarsely calculated using the ellipsoid model.

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FEM analysis

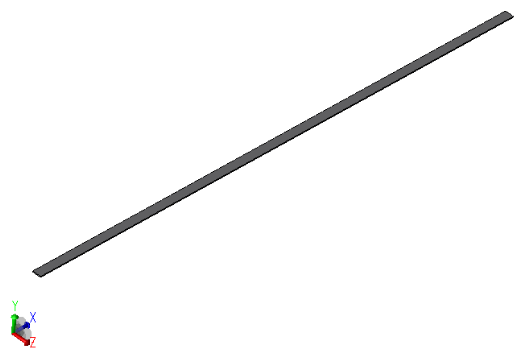

Fig. 1 Antenna

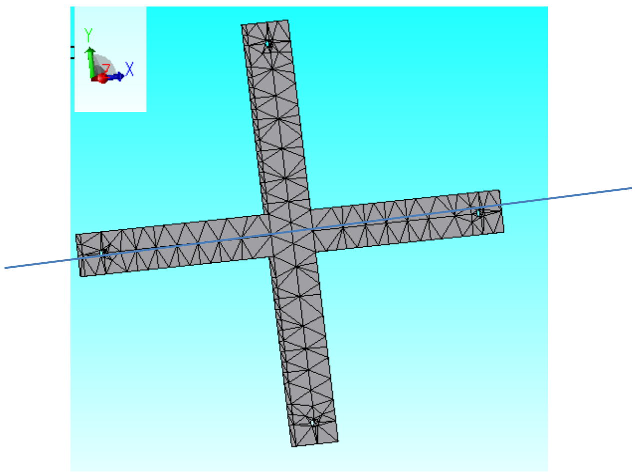

Fig. 2 Hysteresis damper

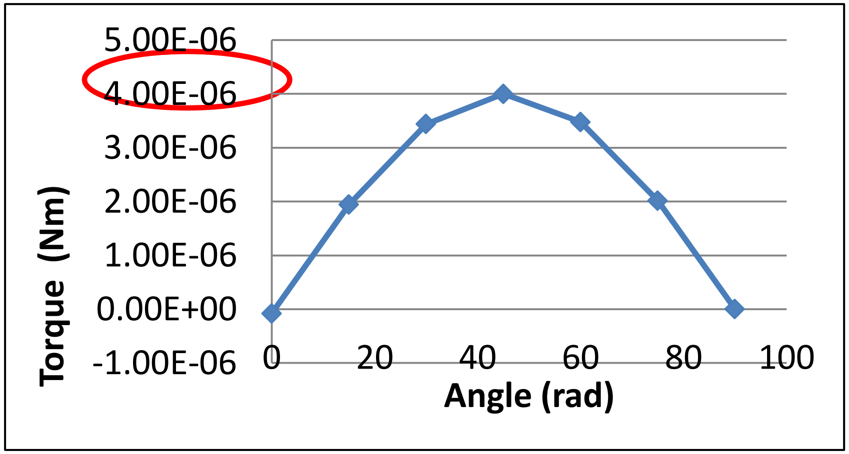

Fig. 3 Magnetic torque(Antenna)

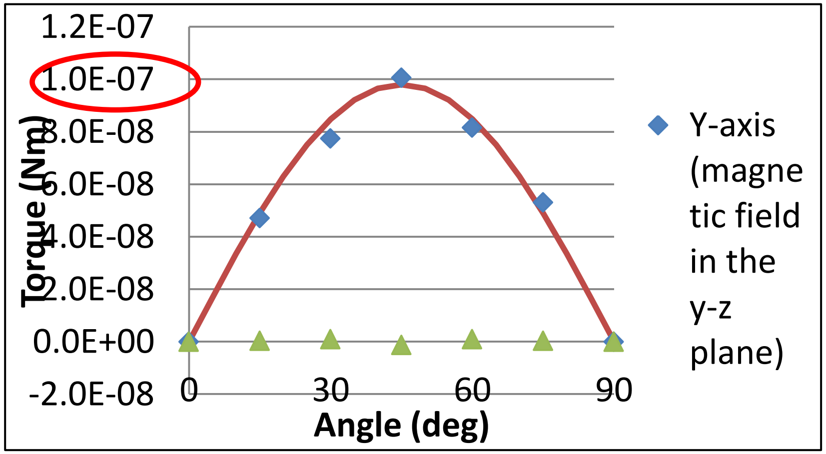

Fig. 4 Magnetic toque (Hysteresis damper)

Conclusions

- Nonsymmetrical magnetic substances in a uniform magnetic field cause a torque.
- Magnetic substances in a satellite causes attitude disturbance torque.
- The magnitude of the disturbance is almost the same magnitude of the other attitude disturbances in the worst case scenario.