2016

Brine Shrimp Lab

Brynja Kohler
Utah State University

Rebecca Swank
Utah State Univerisity

Jim Haefner
Utah State University

Jim Powell
Utah State University

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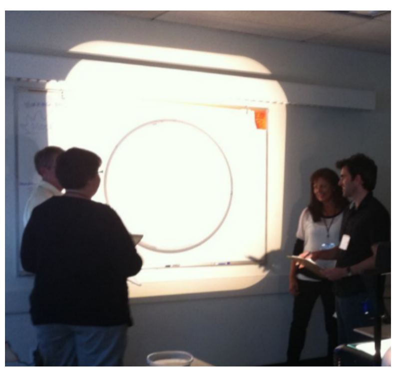
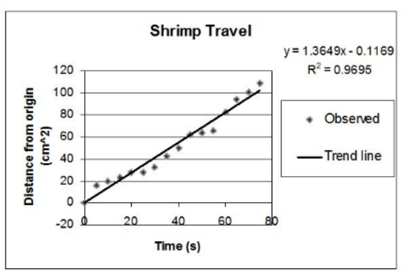
Laboratory Experiences in Mathematical Biology

Overview: Students measure and record the distances brine shrimp travel in a petri dish and determine whether the brine shrimp move in a random walk. Concepts as simple as distance and linear relationships and as complex as non-linear fitting and modeling with PDEs are illustrated through the lab.

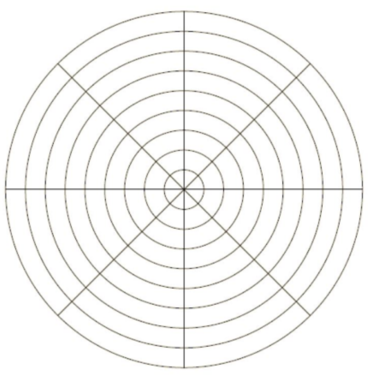


Lesson Outline: Students decide whether brine shrimp swimming in shallow water in a petri dish are moving randomly. In algebra and statistics courses, the lab requires students to create a line-of-best-fit to match displacement data. The expectations and lab agenda below are geared towards an introductory mathematical biology course comprised of under-class mathematics, statistics, biology and biological engineering students with some calculus experience.

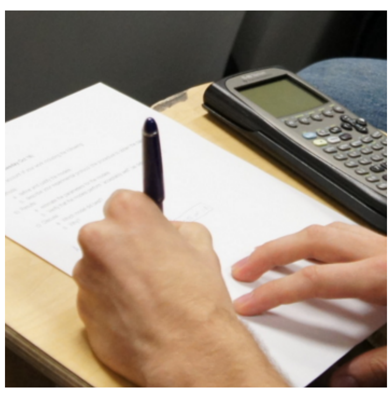
Lab Setup: Students study the movement of brine shrimp using readily available materials.



Data and Examples: Data observed and recorded by students along with their results from fitting the diffusion equation to the data is presented in order to highlight students' reasoning and methods.



Background and Extensions: Introductory material describing the basic mathematics of diffusion is discussed.



Assessment Items: The following assessment items were written to target learning objectives in the Brine Shrimp Lab in a beginning PDE or Applied Mathematics in Biology course setting.

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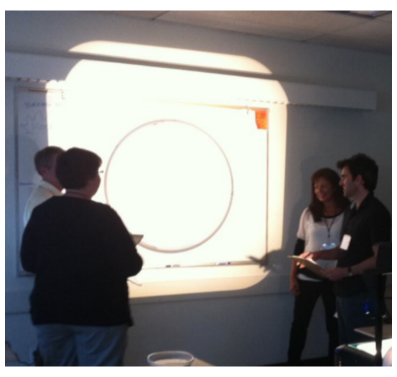

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Expectations

Each group will complete a homework assignment which will include

- A brief executive summary of approaches and results.
- As estimate of the diffusion parameter for your brine shrimp population, accompanied by a graph of mean squared displacements and the regression line used to estimate D.
- Presentation of predictions and observations for arrival times, with some `goodness of fit' parameter(s).
- Presentation of predictions and observations for rates of dispersal into annuli, with some `goodness of fit' parameter(s).
- Brief concluding discussion of whether or not the diffusion equation is an appropriate model for dispersal of brine shrimp, supported by the above results.

Lab Agenda

The in-class portion of the Brine Shrimp Lab proceeds as follows:

1. Lecture: Introduction to Brine Shrimp Lab (5 minutes)
2. Lecture: Class discussion on random walks and diffusion (20 minutes)
3. Data Collection: Groups measure mean squared displacement for a variety of individual brine shrimp to estimate the diffusion constant(25 minutes)
4. Class Discussion: As a class discuss prediction arrival times using the diffusion constants the groups estimated and the diffusion equation (10 minutes)
5. Data Collection: Groups observe arrival times to test the validity of the diffusion model for brine shrimp (30 minutes)
6. Class Discussion: Groups present their results on whether their data suggests brine shrimp move in a random walk (30 minutes)

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Lab Setup: Students study the movement of brine shrimp using readily available materials.

Materials

Each group will need:

- Two small dishes with tap/``Instant Ocean'' water in 5/1 proportions; place several brine shrimp in one dish.
- An eye dropper for releasing individuals.
- A light table, overhead projector or document camera.
- Datasheets and pencils for recording the location of the shrimp at each `mark.'
- A large petri dish with 50 ml of tap/``Instant Ocean'' water in 5/1 proportions.
- Graph paper (document camera) or translucent graph paper (light table) or graph paper copied onto transparency (overhead projector), with 10 lines per inch.
- A stopwatch or timer.

Methods

We recommend the following experimental procedure for the Brine Shrimp Lab:

1. Divide into groups of 3-4. Each group will need at least one person to manage the stopwatch (Timer), observe the location of the brine shrimp (Observer) and record data (Recorder).
2. In the center of the graph paper indicate an origin and relative to that origin annotate coordinates at one inch intervals to aid in position estimation.
3. Place the graph paper on the document camera (or similar), place the large petri dish on top, and center the coordinate system approximately at the center of the petri dish.
4. Place a little brine in one small glass bowl.
5. Suck up a number of brine shrimp with the eye dropper and gently squirt them into the brined bowl.
6. Observe brine shrimp movement
   a) The observer must suck up an individual shrimp with the eye dropper and carefully release them at the origin. Be careful; it's easy to give the shrimp(and the water) a big push upon release!
   b) Let the water settle and, if needed, reposition the petri dish so the brine shrimp is at the origin.
   c) When the brine shrimp is in position, the Observer should say ``Start'' and the timer will start timing.
   d) Every five seconds the Timer will call out ``Mark,'' the Observer will estimate and call out the shrimp's x, y position for the Recorder to write down (Alternatively, the brine shrimp setup can be projected onto a whiteboard and the positions can be noted on the board and the x, y coordinates can be recorded at the conclusion of the experiment.)
   e) Continue each data track until the shrimp finds the edge of the petri dish or 90 seconds, whichever comes first.
   f) To avoid confusion as well as sample across the shrimp population, suck the shrimp out of the petri dish and expel it into a second small bowl.
7. Repeat the observation sequence switching roles so that every group member has a chance to observe.

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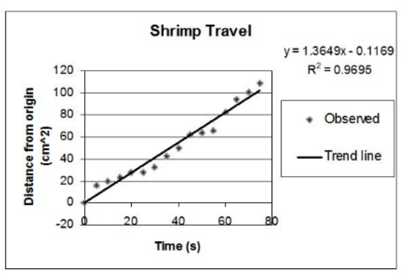


Data and Examples: Data observed and recorded by students along with their results from fitting the diffusion equation to the data is presented in order to highlight students' reasoning and methods.

Sample Data

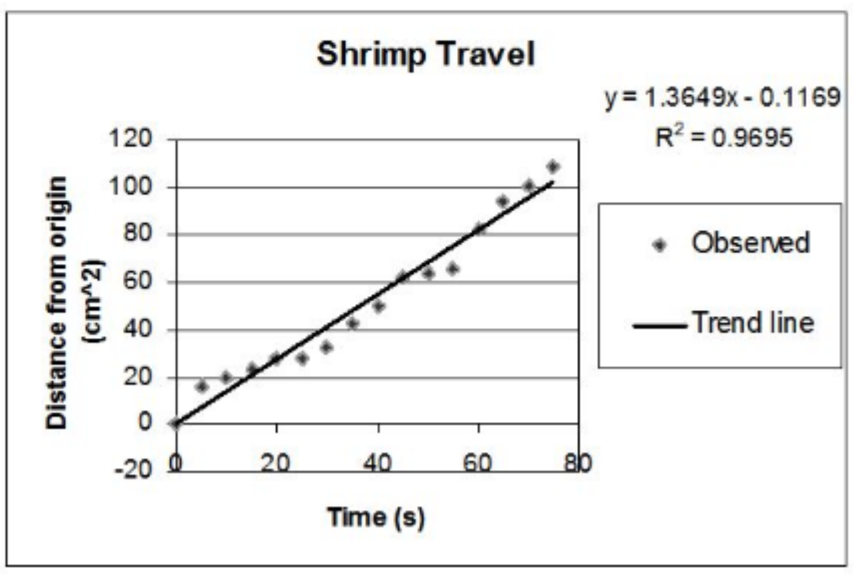


Figure 1: Students graph of the mean displacement squared of brine shrimp. The shrimp started at the center of the dish placed on a piece of graph paper on a light box. Every 5 seconds the shrimp's location was recorded during ten 1-minute trials. The resulting averages of distance removed from the origin show shrimp disperse from the origin at a dependable rate.

Examples

From the data plotted above (Figure 1), students were able to estimate the diffusion constant of their shrimp by dividing the slope of the fitted line by four. Hence, $D \approx 0.865$. To test whether the diffusion PDE is a good model for brine shrimp movement, the students used their cell phone to film the arrival of shrimp at each of the 6 inner-most annuli pictured in (Figure 2). They released ~200 shrimp and counted the number of shrimp in each annuli at 5, 10 and 15 seconds.

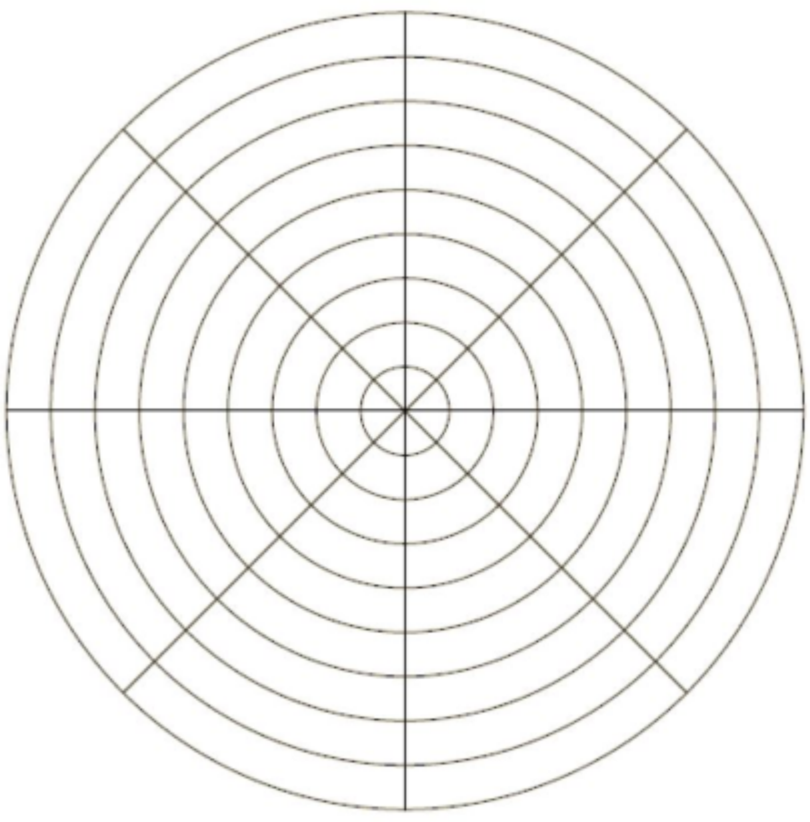
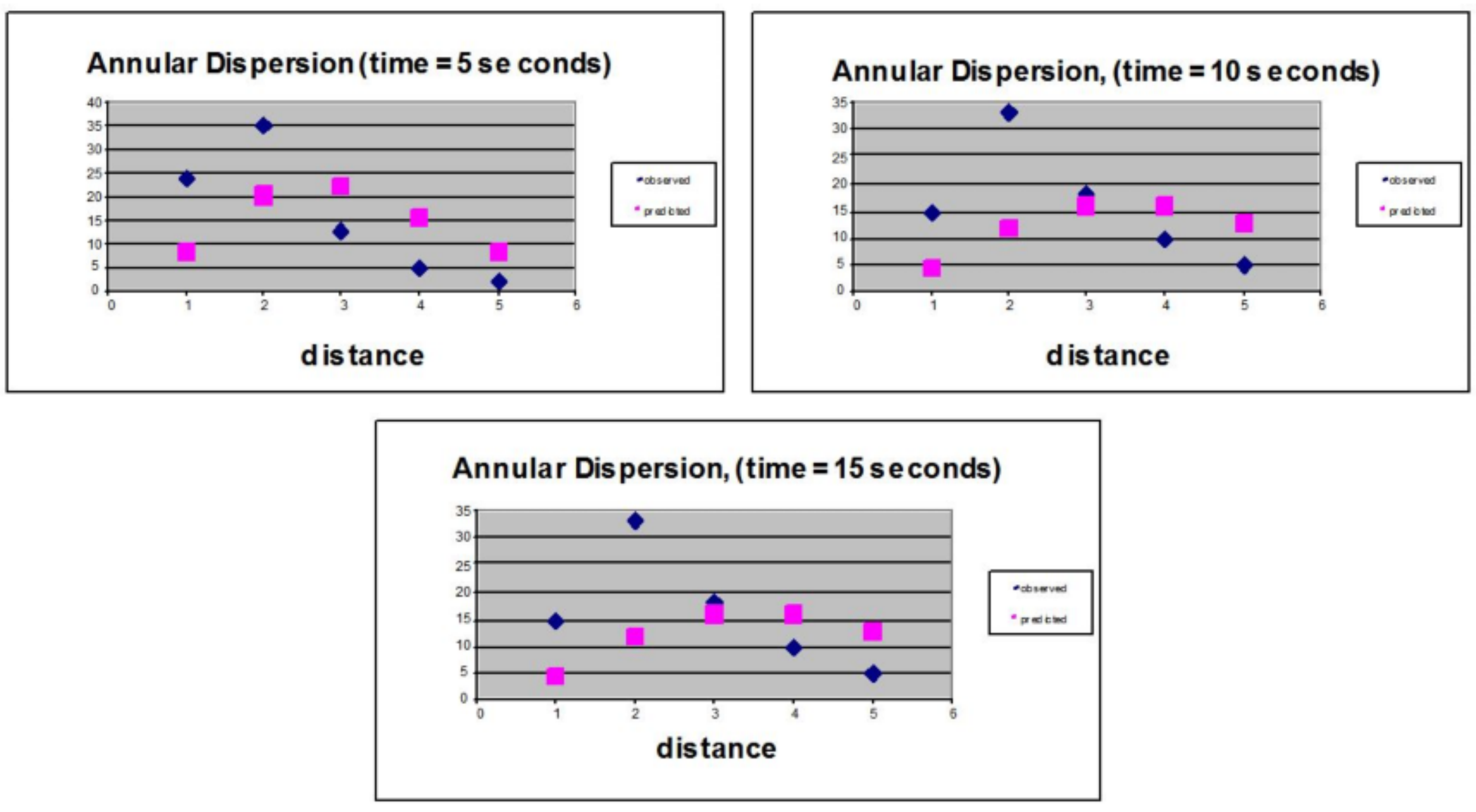


Figure 2: Students use the annuli paper to observe shrimp arrival times at various distances from the origin.



Graphs created by students depicting the number of shrimp in each annuli of the mean displacement squared of brine shrimp. The students concluded the diffusion model broadly captures the movement of brine shrimp. Furthermore, they surmised biological factors (e.g., foraging for food) are likely the cause it does not perform better.

While the students felt the diffusion model matched the general shape of the shrimp distribution over time, they surmised that biological factors (specifically, foraging) caused the model to perform quite poorly.

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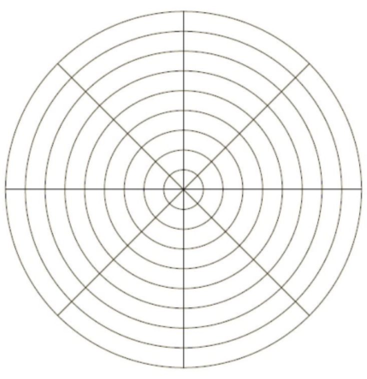

Background and Extensions: Introductory material describing the basic mathematics of diffusion is discussed.

One way to think about an organism that searches its environment randomly is that it performs a random walk. Of, on average, organisms make a `run' of distance Δs over a time interval of Δt, after which they re-orient and choose a new direction at random, it can be shown that the population density, $P(x,y,t)$, for a number of such organisms obeys the diffusion equation

$$\frac{\partial P}{\partial t} = D\left[\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right]. \quad (1)$$

The *diffusion constant*, D, is related to the random-walk parameters by

$$D = \frac{\Delta s^2}{4\Delta t}.$$

The solution to (1) corresponding to predicted population densities following a point release of N organisms at the origin at $t=0$ is

$$P(r,t) = \frac{N}{4\pi D t} e^{-r^2/4Dt} = N \cdot \frac{1}{\sqrt{4\pi Dt}} e^{-x^2/4Dt} \cdot \frac{1}{\sqrt{4\pi Dt}} e^{-y^2/4Dt} \quad (2)$$

where $r = \sqrt{x^2 + y^2}$ is the distance from the origin. Note that (2) depends on x and y independently through the two normal distributions, each with variance $\sigma^2 = 2Dt$.

The mean square expectations, $\langle x^2 \rangle, \langle y^2 \rangle$, for the two coordinates grow linearly with t, that is

$$\langle x^2 \rangle, \langle y^2 \rangle = 2Dt$$

because the density function can be written as the product of two independent Normal probability distributions (as in (2)). Since each coordinate is independent, we have

$$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle = 2Dt + 2Dt = 4Dt.$$

This gives a method for estimating the diffusion constant for a brine shrimp population tested in this lab. Individuals can be tracked, with coordinates measured at several instants in time. The mean of the *square* displacement for all individuals can be calculated at each point in time and then fitted to a line. One quarter of the slope of that line is an estimate of the diffusion constant.

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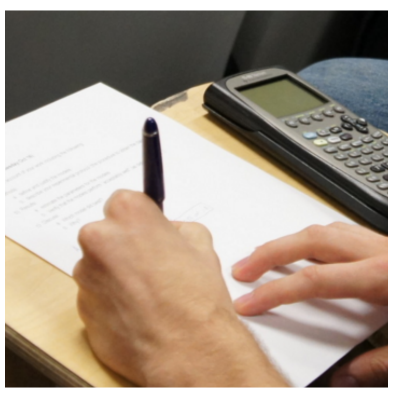

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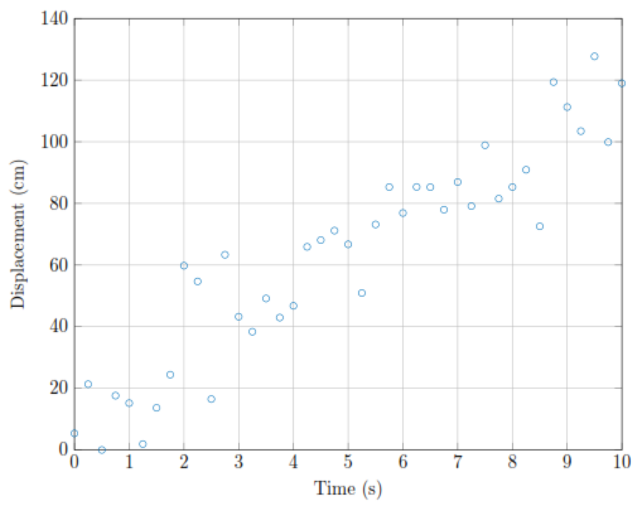
1. Simple Knowledge: State the PDE for diffusion and the correct units for the diffusion coefficient.

2. Algorithmic Skill: Solve the PDE for diffusion
$u_t = D u_{xx}$, $0<x<l$, $-\infty<t<\infty$, $k>0$
$u(x,0)=f(x)$, $u(0,t)=0$, $u(l,t)=0$

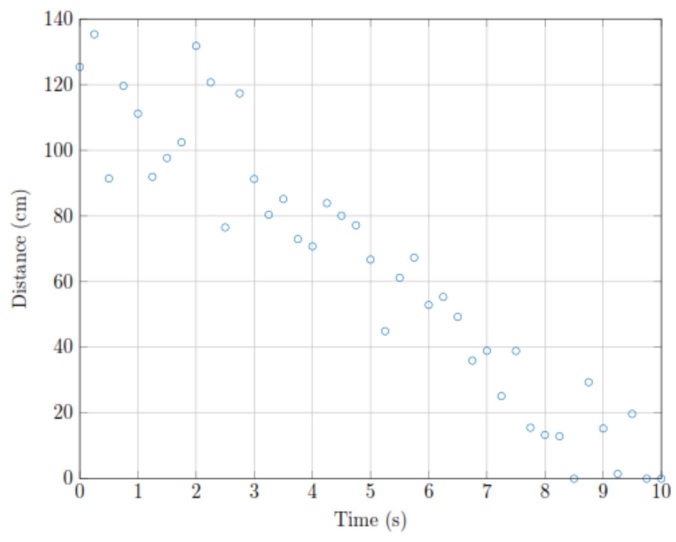
3. Construct a Concept: Provide 5 examples of diffusion or random motion in nature.

4. Algorithmic Skill: From the given data, estimate the diffusion coefficient.



Displacement of brine shrimp over time

5. Comprehension and Communication: Explain the behavior of the brine shrimp based on the data provided.



Distance of brine shrimp from a center point over time