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Piano Key Weir for Enlargement of the West Fork of Eno River Reservoir

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ABSTRACT

The West Fork of Eno River Reservoir Dam is located near Hillsborough, North Carolina, USA. The earthen embankment dam provides an impoundment used by the town for water supply. The project was originally completed in 2000 with forethought by the owner for an expansion, which was anticipated in the original design. Currently, the design of that expansion to raise the reservoir level for increased water supply is underway. Following a detailed analysis of viable upgrades, a piano key (PK) weir (anticipated as the first to be constructed in the USA) was selected by the owner due to its effectiveness in meeting project requirements. Presented in this paper are the site-specific characteristics of the existing auxiliary chute spillway, the proposed PK weir, and additional modifications to the chute. This paper also presents site-specific information including the existing auxiliary chute spillway, the new PK weir, and additional modifications to the auxiliary chute. The results include the comparison of four PK weir hydraulic design methods for sizing the spillway. Details and results of a CFD model are also included. Finally, additional significant design considerations are presented, which include anticipated hydraulic conditions in the downstream chute and identified existing structure deficiencies; estimated spillway design flood hydraulic force distributions taken from the CFD model; the PK key weir stability summary; and other important project aspects including embankment modifications and seepage control measures. It is anticipated that this project will be of interest to those concerned with dam safety and will further encourage the consideration of PK weirs in the USA.

Keywords: *Piano Key Weir, Spillway Upgrade, Water Supply, Lateral Spillway, CFD*

1. INTRODUCTION

It is not uncommon for passive spillways (i.e., various weir types) to be implemented as a risk reduction measure to address dam safety issues related to spillways. Such concerns may be related to spillway capacity, hydrologic loadings on the system including hydraulic structures, or operations and maintenance. New dams, levee networks, and upgrades related to water supply also regularly make use of these hydraulic structures. Weirs may be among the oldest and most ubiquitous control structures used in water supply systems.

In North Carolina, USA, the Town of Hillsborough uses the West Fork of Eno River Reservoir (surface area of 0.9 km^2) for water supply. Originally constructed in 2000, water is impounded by a 760-m (2,500 ft) long, 18.9-m (64-ft) high earthen embankment. The dam features two spillways; the primary spillway is the intake tower. The auxiliary spillway (see Figures 1 and 2) is an atypical lateral or side-channel spillway – the control section is a 65.5-m (215-ft) long, 4.6-m (15-ft) wide broad-crested weir immediately followed by a converging chute with a 70° bend left (oriented downstream) that reduces the chute width by over 60%. The 24.4-m (80-ft) wide chute terminates with a small hydraulic jump stilling basin.

Figure 1. The Existing Auxiliary Spillway for West Fork of Eno River Reservoir Dam looking upstream at control section (A), control section (B), and chute (C)

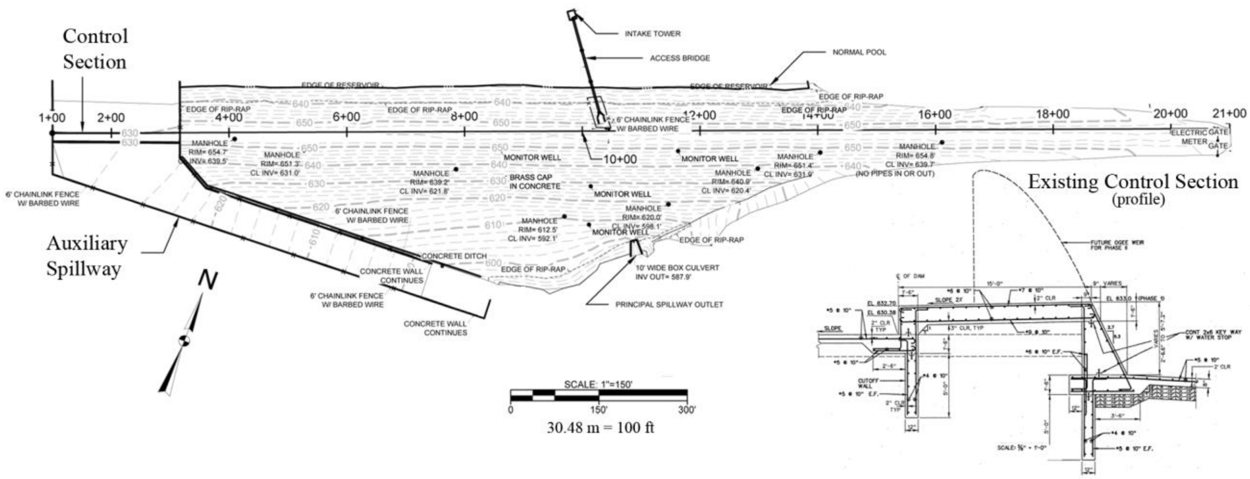

Figure 2. Schematic of existing auxiliary spillway for West Fork of Eno River Reservoir Dam and detail of broad crest and originally proposed ogee for raise

1.1. Project Requirements

The original design of the dam and necessary permits included a two-phased development plan. Phase 1 included constructing the existing dam and spillway structures. Phase 2 was planned to include construction of a 3.05-m (10-ft) high ogee weir in the auxiliary spillway to allow the normal reservoir pool to be raised, thereby clearing vegetation along the reservoir rim between the Phase 1 and Phase 2 normal pool elevations and raising roadway embankments to provide adequate freeboard between the reservoir surface and adjacent roadways. It was anticipated that construction of Phase 2 would be completed by the end of 2018.

During the preliminary engineering phase of Phase 2, a number of design details (shown on Record Drawings) that could result in potential distress during flood loading or less than desirable performance over the life of the structure were identified on Record Drawings. One deficiency related to the spillway hydraulics was the height of the auxiliary spillway chute walls downstream of the control section. The change in direction of flow in the chute combined with the reduction in spillway width is expected to create both flow bulking on the right side of the spillway channel where the chute changes directions and standing waves. In addition, overtopping of the chute walls could lead to erosion of the soil adjacent to the spillway chute, which in its current condition, would likely lead to a structural failure.

The other deficiencies included insufficient slab thickness and improper configuration of seepage cutoffs. The Record Drawings show that the typical thickness of the spillway slab is generally a mere 20 cm (8 inches). The impacts of high-velocity flows at the base of the proposed ogee weir section and in the stilling basin are expected to provide a significant stress to the spillway chute slab. The stress associated with these high-velocity flows impacting the slab would be expected to damage a concrete slab of this thickness. The current configuration for seepage mitigation below the control section includes two reinforced concrete cutoff walls, one installed at the upstream edge of the control section and one installed at the downstream edge of the control section. The downstream cutoff wall extends about 1 m (3.5 ft) lower than the upstream cutoff wall. This configuration could result in water pressures building below the control section, i.e., between the two cutoff walls, which is undesirable.

As a result of the multiple deficiencies identified and to more effectively and cost-efficiently address these deficiencies, alternative spillway configurations to the originally planned ogee weir were considered. The two other options considered were a labyrinth and PK weir. The labyrinth weir was eliminated as a cost-effective alternative due to the anticipated upstream-downstream depth of the weir being much larger than the existing control section depth [12.2 m (40 ft) versus 4.6 m (15 ft)]. A PK weir could be constructed within the footprint of the existing control section, and the existing spillway width could be narrowed by 45%. A narrower control section would result in construction cost savings due to less concrete being required for spillway slab overlays and a narrower seepage cut-off (Robblee et al. 2016).

2. PIANO KEY WEIR HYDRAULICS

The PK weir is a type of labyrinth weir with a non-linear weir geometry to increase the crest length within a channel (Crookston and Tullis 2013). The additional length in a compact spillway footprint provides additional discharge capacity relative to a broad-crested weir for the same channel width. Therefore, the ability to reduce the channel width while still safely passing the design flow with non-linear spillways (e.g. traditional labyrinth weirs) often results in economically attractive, viable solutions and, consequently, is commonly implemented in the USA for rehabilitations, upgrades, and new dams (Crookston et al. 2015). The authors have no knowledge of any PK weirs having been constructed in the USA to date.

PK weirs are characterized by rectangular ‘keys’ and ramps and commonly have a sidewall angle (α) of 0° (i.e., parallel sidewalls); however, a number of geometric variations exist that are grouped into four general types (Lempérière et al. 2011). A representation of a Type-A PK weir is presented in Figure 3. Since 1998, many studies have been performed in various laboratories such as in France, Belgium, Algeria, India, China, Vietnam, Switzerland, and the USA, among others, to characterize hydraulic performance and behaviors of PK weirs.

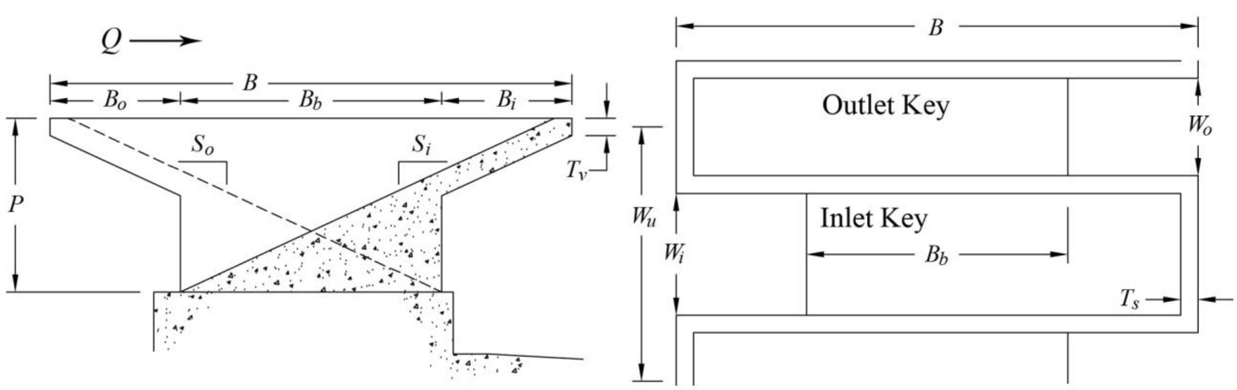

Figure 3. PK weir geometric parameters (Pralong et al. 2011)

2.1. Proposed PK Weir Geometry

The reservoir, spillways, and downstream river were analyzed as a system to select an appropriate PK weir geometry. This included maximum allowable pool elevation in the reservoir, inflow hydrographs, and attenuation provided by reservoir storage. It also allowed examination of potential downstream hazards associated with larger discharges and facilitated a comparison of anticipated peak outflows (during a given flood event) of the existing auxiliary spillway and the proposed PK weir. Depending on risks assumed by the owner, a non-linear spillway outflow hydrograph can be modified by placing weir crest segments at two or more elevations. These ‘staged’ labyrinth spillways (Dabling et al. 2013) can generally match existing peak outflows in an outflow hydrograph for more frequently occurring storms while still providing the needed discharge capacity for the spillway design storm (e.g., the probable maximum flood or an approved ratio thereof). However, a staged labyrinth spillway with the higher stage above normal pool frequently requires additional crest length compared to a single-stage labyrinth weir (crest set at normal pool) and, therefore, incurs additional construction expenses.

In the instance of West Fork of Eno River Reservoir Dam, the hydrologic and hydraulic routings indicated that a two-stage 9-key PK weir (0.3 m vertical offset) would generally match peak outflows of the existing auxiliary spillway (with broad-crested weir) for the 100-year frequency flood. For the same event, a single-stage 8-key PK weir would increase peak outflows by about 25% (equivalent upstream head) from the two-stage 9-key weir; however, no additional hazards downstream (i.e., due to additional flow) were observed in the river model. Also, the 100-year peak discharge of the 8-key PK weir was less than the estimated outflow for the 100-year frequency flood by FEMA for the West Fork of Eno River (FEMA 2007). For the spillway design flood (SDF), which is estimated as the ¾ probable maximum precipitation (PMP), the PK weir peak discharge would be 15% less than the peak discharge of the existing auxiliary spillway. This is due to the increased hydraulic efficiency of the proposed weir (more flow is conveyed at a given pool elevation during the rising limb of the hydrograph, reserving reservoir storage when the flood is most intense). Based upon these results and supporting information, the owner selected a single-stage PK weir configuration. Estimation of the PK weir rating curve is discussed in the following section.

The PK weir will be installed on an upgraded concrete platform where the existing broad-crested weir is located, with a bottom elevation at 192.69 m a.s.l. The PK weir crest has a “broad-crested” shape and is installed at elevation 195.74 m a.s.l., so that a height of P=3.05 m (10.0 ft) results. It consists of eight full cycles. The characteristic geometrical parameters are the linear transversal channel width W=35.60 m (116.8 ft), the developed PK weir crest length L=177.02 m (580.8 ft), the streamwise PK weir length B=9.14 m (30.0 ft), the inlet key width W_i=2.13 m (7.0 ft), the outlet key width W_o=1.71 m (5.6 ft), and the streamwise overhang lengths $B_{i,o}$=2.29 m (7.5 ft). Accordingly, the magnification ratio is L/W=4.97, the key depth ratio is B/P=3.00, the key width ratio is W_i/W_o=1.25, and the overhang ratio is $B_{i,o}/B$=0.25.

2.2. PK Weir Discharge Rating Curve

The rating curve was estimated with two approaches. First, four empirical PK weir hydraulic design methods provided in literature were considered. Second, data derived from a sectional physical model test (Anderson 2011) were used, taking into account that the model did not exactly represent the considered PK weir geometry. Note that the design point of H_d=2.26 m (7.4 ft) is based upon allowable maximum reservoir pool elevation.

For the first approach, the rating curves according to Kabiri-Samani and Javaheri (2012), Leite Ribeiro et al. (2012), Machiels et al. (2014) Machiels et al. (2015) with a correction, and Anderson and Tullis (2013, personal communication 2015) were computed. However, these involve different crest types; namely, a sharp-crested type for Kabiri-Samani and Javaheri (2012), a cylindrical crest for Leite Ribeiro et al. (2012), and a broad-crested type for Machiels et al. (2014) and Anderson and Tullis (2013). The rating curves were thus computed according to the proposed equations of the afore-mentioned studies and then adapted as required to the broad-crested weir, as proposed for the discussed prototype. This adjustment was done by multiplying the discharge for a certain head with the discharge coefficient ratio of the considered crest types with the broad-crested type as reference. The discharge coefficients C_d were derived per crest type based on

- Hager and Schwalt (1994) for the broad-crested weir, where C_d tends from C_d=0.33 for H<0.2 m to C_d=0.42 for H>1 m,
- Hager and Schleiss (2009) for the sharp-crested weir, where C_d=0.42 for all H, and
- Castro-Orgaz (2012) for cylindrical weir crests if H<0.5 m (C_d=0.54 at H=0.5 m), then linearly approaching C_d=0.42 at H=4.4 m as observed by Ramamurthy and Vo (1993), and finally C_d=0.42 for all H>4.4 m.

For the second approach, the data points derived from a particular physical model tests performed by Anderson and Tullis (2013) were analyzed. The aforementioned rating curve estimation included a series of tests with different PK weir geometries, whereas for the second approach only that closest to the prototype PK weir was retained. Given that the model set-up (L/W=5.06, B/P=3.0, W_i/W_o=1.25 and B_i/B=0.25) was not identical with the PK weir prototype configuration, two discharge conversions were used to up-scale the model values to prototype discharges:

- The C_d development in function of H/P of the model was applied on the prototype, and
- A geometrical scale factor referring to P was defined, resulting in λ=15.5. The discharge was then up-scaled based on the Froude similitude. Considering other characteristic lengths (e.g., W_i or B) would result in slightly different values of λ. Following Leite Ribeiro et al. (2012), P has a dominant effect on the rating curve, whereas the other lengths are of secondary importance.

A comparison of both approaches to estimate the rating curve of the considered prototype is shown in Figure 4. Most of the rating curves collapse and, for a design head of H_d=2.26 m (7.4 ft), closely predict the design discharge Q_d=445 m^3/s (15,720 ft^3/s). The curve of Machiels et al. (2014) predicts smaller discharges for a given head if H>1.5 m, given that this equation explicitly takes the effect of the approach flow depth into account. None of the other approaches do. Nevertheless, this effect becomes relevant if H_d>0.5P, i.e. for H>1.5 m being similar to Q>300 m^3/s. The equation of Kabiri-Samani and Javaheri (2012) clearly overestimates the discharge capacity, as observed previously (Pfister et al. 2012).

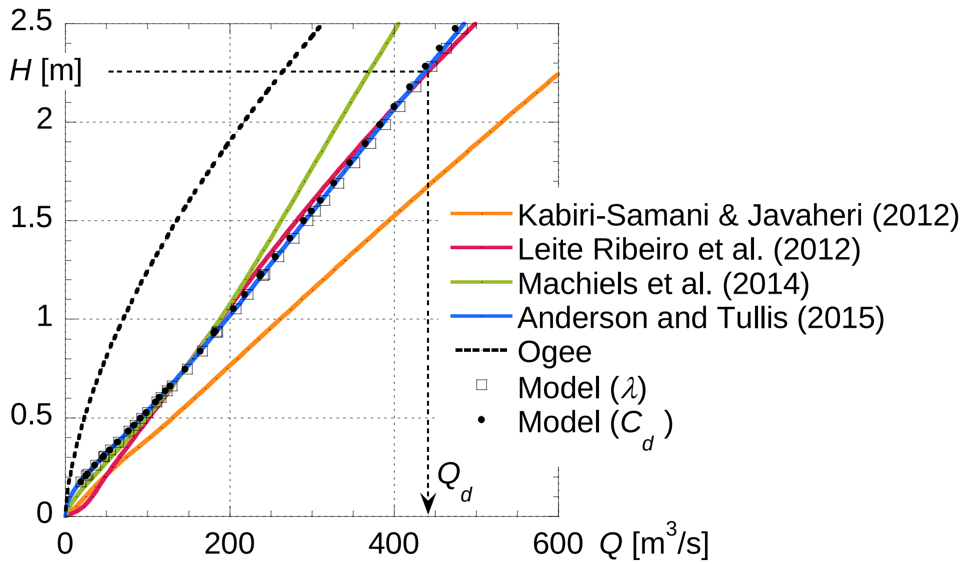

Figure 4. Comparison of Rating Curves from Literature and based upon Physical Model Tests

2.3. Numerical Modeling of Spillway

The primary objective of the numerical model was to simulate site-specific conditions with a focus on the PK weir and the rapidly varied flow region immediately downstream of the control section. A computational fluid dynamics (CFD) model of the control section and chute, basin, and portion of the reservoir was developed using a commercially available CFD solver. The majority of simulations utilized the Reynolds-Averaged Navier-Stokes (RANS) equations with the Renormalized-Group (RNG) turbulence model (Yakhot et al. 1992). This solver uses a finite volume method with conservation of mass and momentum via Eqs. (1) and (2).

$$V_F \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u A_x) + \frac{\partial}{\partial y}(\rho v A_y) + \frac{\partial}{\partial z}(\rho w A_z) + \xi \frac{\rho u A_x}{s} = R_{DIF} \tag{1}$$

$$\frac{\partial U_i}{\partial t}+\frac{1}{V_F}\left(U_j A_j \frac{\partial u_i}{\partial x_j}\right)=-\frac{1}{\rho}\frac{\partial P'}{\partial x_i}+g_i+f_i \quad (2)$$

The simulation matrix included existing conditions, an ogee crest alternative (not selected for design; results not included herein), and the proposed PK weir. The domain was discretized into hexahedral cells with select mesh planes defined based upon geometries, which were drafted as three-dimensional solids. To improve simulation efficiency, solids that represented domain-removing volumes or initial water geometry (i.e., conditions within reservoir and adjacent to control section) were included.

This project did not include a Froude-scaled hydraulic model of the spillway and site-specific conditions. However, calibration efforts utilized experimental data provided by Utah State University for PK weir geometries tested in a laboratory flume. Limitations and uncertainties are associated with hydraulic physical and numerical models (Heller 2011, Pfister and Chanson 2012, Freitas 1993); results require experience and expertise for interpretation (Knight 2013). Simulations are summarized in Tables 1 and 2.

Table 1. CFD Model Discharge Calibration Results

Simulation	*H/P*	Mesh Size		$C_{d\text{-}cfd}$	$C_{d\text{-}cfd}/C_{d\text{-}exp}$	$C_{d\text{-}cfd}/C_{d\text{-}emp}$	Turbulence Model
		(mm)	(ft)				
7	0.2	8.47	1/36	0.501	7.27%	6.47%	RNG / 0.07 *P*
8	0.2	8.47	1/36	0.501	7.27%	6.47%	RNG / 0.07 *H*
9	0.2	8.47	1/36	0.497	6.28%	5.49%	RNG / Dynamic
10	0.2	6.35	1/48	0.504	7.78%	6.98%	RNG / Dynamic
11	0.2	5.08	1/60	0.494	5.70%	4.91%	RNG / Dynamic
12	0.2	8.47	1/36	0.500	6.97%	6.18%	LES
13	0.2	5.08	1/48	0.501	7.30%	6.50%	LES
14	0.3	8.47	1/36	0.414	7.57%	6.30%	RNG / Dynamic
15	0.3	5.08	1/60	0.400	3.99%	2.76%	RNG / Dynamic
16	0.4	8.47	1/36	0.351	4.99%	3.81%	RNG / Dynamic
17	0.4	5.08	1/60	0.345	3.13%	1.98%	RNG / Dynamic
18	0.5	8.47	1/36	0.316	6.53%	5.18%	RNG / Dynamic
19	0.5	5.08	1/60	0.305	2.65%	1.34%	RNG / Dynamic
20	0.7	8.47	1/36	0.256	1.24%	0.82%	RNG / Dynamic
21	0.7	5.08	1/60	0.253	0.10%	-0.31%	RNG / Dynamic
22	0.9	8.47	1/36	0.229	3.98%	2.32%	RNG / Dynamic
23	0.9	6.35	1/48	0.231	4.64%	2.97%	RNG / Dynamic
24	0.9	5.08	1/60	0.229	3.89%	2.23%	RNG / Dynamic

As can be seen in Figures 5 and 6, good agreement regarding discharge capacity was established between the experimental results of Anderson (2011) and a geometrically duplicate numerical model. The RNG turbulence model with dynamically computed turbulence mixing length and second order monotonicity preserving momentum advection produced the closest agreement, with final simulations accurate from 0.1% to 5.7%, with a 3.2% mean. Figure 6 shows the general matching of flow structure, with some differences occurring primarily where the flow is more turbulent and highly aerated.

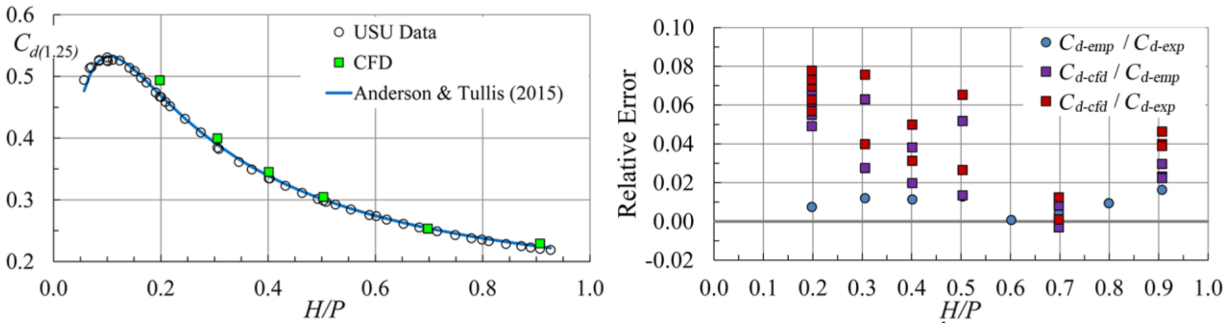

Figure 5. Agreement between Numerical Results and USU Experimental Results for Type-A PK Weir

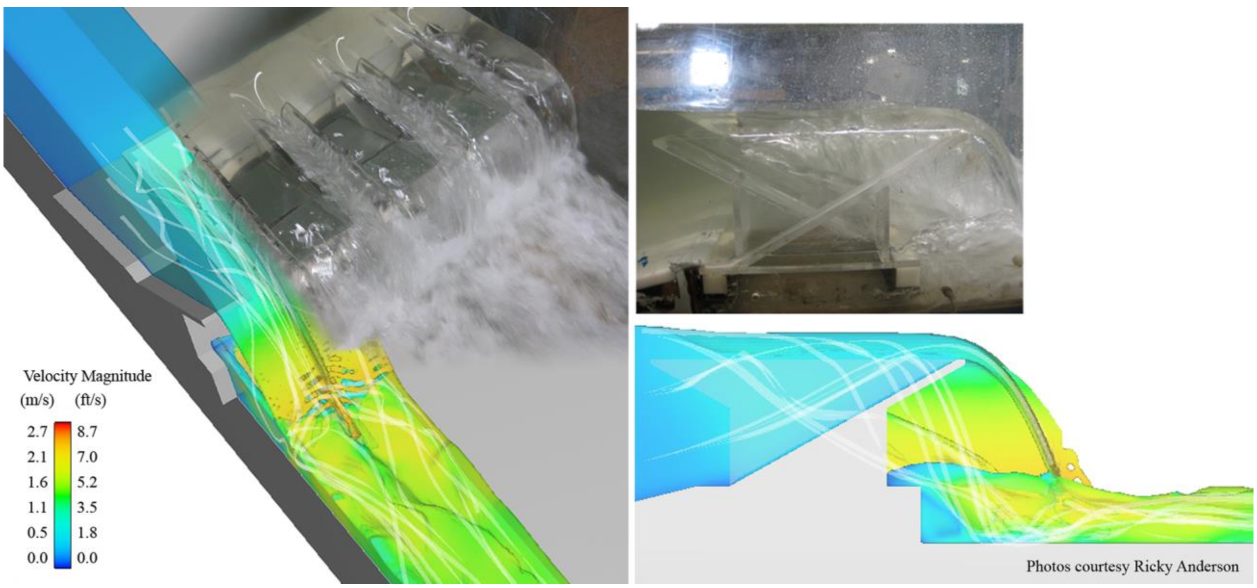

Figure 6. Comparison of physical and numerical model flow structure

Table 2. CFD Modeling Test Matrix – Hillsborough

Hillsborough Model Simulation		Turbulence Model
1	Existing – 100 yr	RNG / Dynamic
2	Existing – ¾ PMP	RNG / Dynamic
3	PK weir – 100 yr	RNG / Dynamic
4	PK weir – ¾ PMP	RNG / Dynamic
5	Ogee – 100 yr	RNG / Dynamic
6	Ogee – ¾ PMP	RNG / Dynamic

With an increased understanding of the numerical model performance regarding PK weirs, a location model of the existing proposed PK weir and original ogee-weir concept were simulated for the 3/4 PMP and the 100-year frequency storm. Although not verified, it was understood that the CFD model could provide qualitative and quantitative information and incremental quantities useful to the designers, such as estimated flow depths downstream of the three control sections (Figure 7), the pressure field acting on the PK weir (Figure 8), and also identify potential problematic areas as the flow enters the lateral chute and is conveyed to the stilling basin. Please note that in Figure 7a, flow overtops the existing chute but is partially confined due to the domain limits.

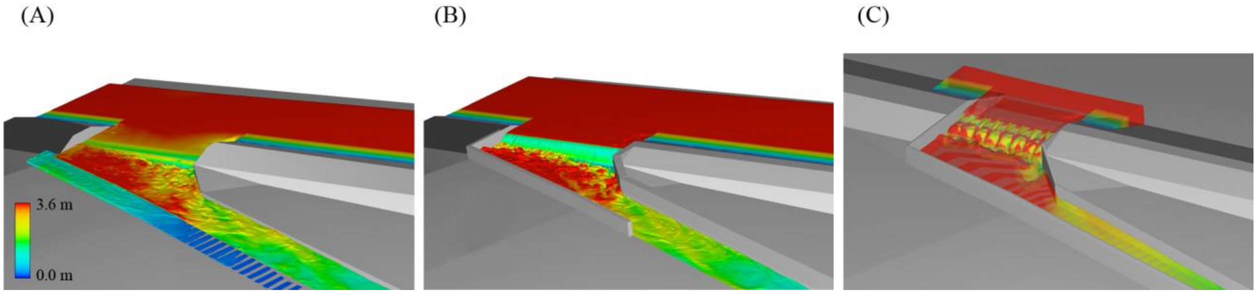

Figure 7. Estimated flow depths for SDF for existing conditions (A), ogee weir (b), and proposed PK weir (C)

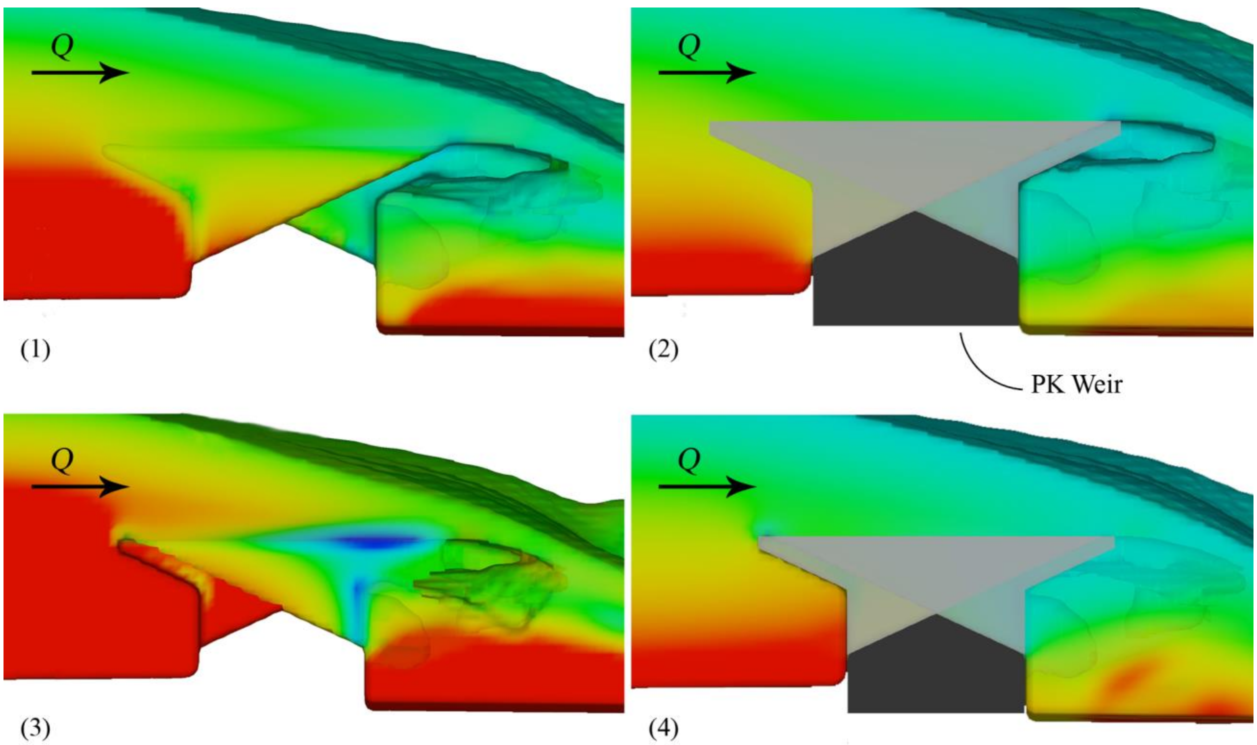

Figure 8. Estimated pressure field at four locations across the PK weir at Q_d=445 m^3/s (15,720 ft^3/s)

For the existing broad-crested weir, the downstream chute wall was estimated to overtop by about 2.4 m (8 ft). The numerical results predict a hydraulic jump in all cases immediately downstream of the control sections; however, the PK weir provides additional energy dissipation relative to the ogee-weir, and the additional turbulence appears to provide some benefit within the transition portion of the chute. A comparison to the numerical results was performed via a short estimation of the hydraulic jump and shock wave. Assuming no energy dissipation exclusive to the hydraulic jump (a computed F_1=2.95 assuming a 2D channel), the sequent depth at the impact wall is approximately 4.9 m for stagnant water. Furthermore, an estimation of the shockwave height via Hager (1992) indicates that that a hydraulic jump is expected with no shockwave (θ=70°, β_s=90°), which agrees with the numerical simulations. An oblique standing wave forms at the end of the transition or chute entrance followed by anticipated shockwave patterns in the supercritical flowing chute.

Replacement of the existing 2.4-m high (8-ft) wall with a 5-m high wall (about 16 ft) in the downstream chute is required to adequately contain the flow from the SDF. During such an extreme event, some local splash and spray may exit the channel. The maximum headwater ratio for this PK weir is H/P=0.74, which is approximately twice the typical ratios for PK weirs built in France. To ensure stability, additional cutoff walls will be constructed at the control section and drainage added to reduce uplift pressures. Also, the PK weir will be structurally connected to the new base slab.

3. CONCLUSIONS

The design of the West Fork of Eno River Reservoir Dam expansion is currently underway, and preliminary engineering analyses identified several design inadequacies related to spillway capacity and structural integrity of the existing chute. An alternatives analysis identified a PK weir to be the optimal economic solution that met all project requirements. To estimate the spillway rating curve, four empirical methods were evaluated with close agreement found between Leite Ribeiro et al. (2012) and Anderson and Tullis (2012, personal communication 2015). Further hydraulic analyses used CFD modeling, which was validated using experimental results by Anderson (2011). The CFD provided qualitative and quantitative information that assisted with design of the chute transition region, including chute wall heights and stability of the PK weir.

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5. REFERENCES

Anderson R.M. (2011). "Piano Key Weir Head Discharge Relationships." M.S. *Thesis*, Utah State University, Logan, Utah.

Anderson, R.M. Tullis, B.P. (2013). Piano Key Hydraulics and Labyrinth Weir Comparisons. *J. Irrig. Drain. Engng*., 139(3):246-253.

Castro-Orgaz, O. (2012). Discussion Overflow characteristics of circular-crested weirs. *J. Hydraulic Res.* 50(2): 241–243.

Crookston, B.M. Tullis, B.P. (2013). Hydraulic Design and Analysis of Labyrinth Weirs. 1: Discharge Relationships. *J. Irrig. Drain. Engng*., 139(5):363-370

Freitas, C.J. (1993). Journal of fluids engineering editorial policy statement on the control of numerical accuracy. *Journal of Fluids Engineering*-Transactions of the ASME 115(3): 339-340.

Crookston, B.M., Mortensen, D., Stanard, T., Tullis, B.P., Vasquez, V. (2015). Debris and Maintenance of Labyrinth Spillways." *Proc. of the 35th Annual USSD Conference*, Louisville, Kentucky. CD-ROM.

Dabling, M.R., Tullis, B.P., Crookston, B.M. (2013). Staged Labyrinth Weir Hydraulics. *J. Irrig. Drain. Engng*., 139(11):955-960.

FEMA (20017). Federal Emergency Management Agency, Flood Insurance Study: A Report of Flood Hazards in Orange County, North Carolina and Incorporated Areas (FIS Study Number 37135CV001A), February, 2, 2007.

Hager, W.H. (1992). Spillways-shockwaves and air entrainment. ICOLD Bulletin, Vol. 81 International Commission on Large Dams, Paris, France.

Hager, W.H., Schleiss, A.J. (2009). Constructions hydrauliques, Ecoulements stationnaires. *Traité de Génie Civil*, Vol. 15, Presses Polytechniques et Universitaires Romandes, Lausanne, Switzerland.

Hager, W.H., Schwalt, M. (1994). Broad-crested weir. *J. Irrig. Drain. Engng*. 120(1): 13-26.

Heller, V. (2011). Scale effects in physical hydraulic engineering models. *J. Hydraulic Res.* 49(3), 293-306.

Kabiri-Samani, A., Javaheri, A. (2012). Discharge coefficient for free and submerged flow over Piano Key weirs. *J. Hydraulic Res.* 50(1), 114-120.

Knight, D.W. (2013). River hydraulics—A view from midstream. *J. Hydraulic Res.* 51(1):2–18

Leite Ribeiro, M., Pfister, M., Schleiss, A.J., Boillat, J.-L. (2012). Hydraulic design of A-type Piano Key weirs. *J. Hydraulic Res.* 50(4): 400–408.

Lempérière, F., Vigny, J.P., Ouamane, A. (2011). General comments on Piano Kew Weirs: The past and present. Proc. Intl *Workshop on Labyrinths and Piano Key Weirs* PKW 2011, CRC Press, 17-25.

Machiels, O., Pirotton, M., Archambeau, P., Dewals, B., Erpicum, S. (2014). Experimental parametric study and design of Piano Key Weirs. *J. Hydraulic Res.* 52(3): 326–335.

Machiels, O., Pirotton, M., Archambeau, P., Dewals, B., Erpicum, S. (2015). Closure to Experimental parametric study and design of Piano Key Weirs. *J. Hydraulic Res.* 53(4): 545.

Pfister, M., Chanson, H. (2012). Scale Effects in Physical Hydraulic Engineering Models. Discussion. *J. Hydraulic Res*. 50(2): 244-246

Pfister, M., Erpicum, S., Machiels, O., Schleiss, A., Pirotton, M. (2012). Discussion to Discharge coefficients for free and submerged flow over Piano Key Weirs. *J. Hydraulic Res.* 50(6): 642-645.

Pralong, J., Vermeulen, J., Blancher, B., Laugier, F., Machiels, O., Erpicum S., Pirotton, M. Leite Ribeiro, M., Boillat, J.L. and Schleiss, A.J. (2011). “A naming convention for the piano key weirs and geometrical parameters.” in Labyrinth and piano key weirs. Erpicum, S., Laugier, F., Boillat, J.-L., Pirotton, M., Reverchon, B., Schleiss, A.J. (eds.) CRC Press, Boca Raton, FL.

Ramamurthy, A.R., Vo, N.D. (1993). Characteristics of circular-crested weir. *J. Hydraulic Engng.* 119(9), 1055-1062

Yakhot, V., Orszag, S.A., Thangam, S., Gatski, T.B. & Speziale, C.G. (1992), Development of turbulence models for shear flows by a double expansion technique, *Physics of Fluids A*, 4:7 pp1510-1520.