Computational Methods in Modeling Fusion Plasmas

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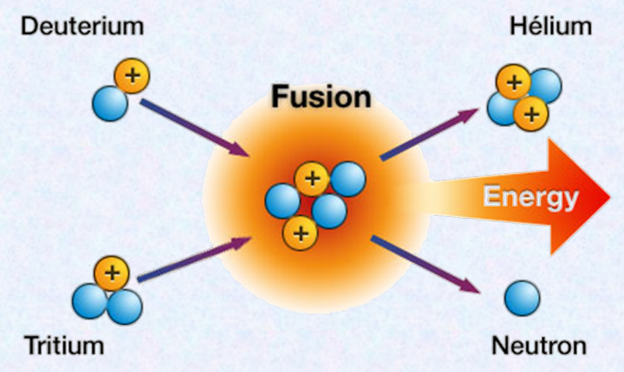
STUDENT RESEARCH SYMPOSIUM

Fusion Basics

- ~ 67% of energy produced in the United States is from fossil fuels (a cost of around $29 billion dollars per year) [1].
- Producing energy from fossil fuels requires vast amounts of fuel, and contributes undesirable waste products to the environment.
- Fusion is a potential viable alternative energy source.
- Fusion is the method by which the sun produces its energy.
- "If used to fuel a fusion power station, the lithium in one laptop battery, complemented with half a bath of water, would produce the same amount of electricity as burning 44 tons of coal" [2].
- There is enough easily accessible Deuterium and Lithium on the earth to supply the world's current energy needs for hundreds of years [2].



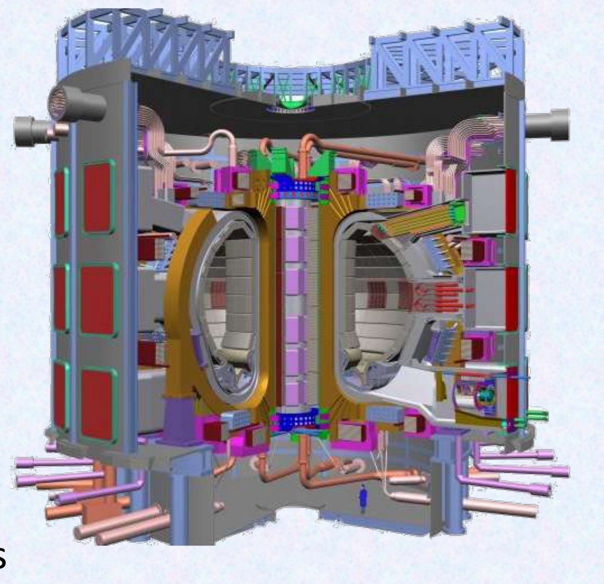
Deuterium and Tritium combine to form Helium, a Neutron, and ENERGY [2]

Confinement – Fusion's Biggest Challenge

- How to confine the plasma long enough for energy output to exceed energy input.
- When fully heated, the electrons in the neutral Deuterium and Tritium atoms become separated from the nuclei, and the gas becomes a "soup" of energetic ions and electrons (i.e. a plasma).
- Temperatures in fusion plasmas can reach 200 million degrees Celsius [3].
- Highest melting point currently hypothesized for a material is 4,137 degrees Celsius (**we have a problem here** . . .) [4].

ITER

- To a first approximation, charged particles will follow magnetic field lines.
- The Tokamak is a device that seeks to confine a plasma using strong magnetic fields.
- ITER (Latin for "the way") is a large-scale Tokamak fusion device that is currently under construction in Cadarache, France.
- The goal of this device is to demonstrate "the scientific and technical feasibility of fusion as an energy source" [2].
- Assembly of the ITER Tokamak is expected to be completed by the year 2019, with first plasma expected by 2020 (or at the latest, 2025).
- ITER is an international effort consisting of 35 different countries, including the United States.
- Key to preparing for the completion of ITER is to increase our understanding of the physical processes that we anticipate will occur inside its fusion plasmas.



Artist's Rendering of ITER- Credit: ITER

References

1. US Energy Information Administration, 2015. [http://www.eia.gov/electricity/data.cfm]
2. Fusion For Energy, 2016. [http://fusionforenergy.europa.eu/understandingfusion]
3. EUROfusion, 2016. [https://www.euro-fusion.org/fusion/fusion-conditions/]
4. Q.-J. Hong and A. van de Walle, Physical Review B **92** (2), 020104 (2015).
5. J. J. Ramos, Physics of Plasmas **22** (7), 070702 (2015).
6. E. C. Howell and C. R. Sovinec, "Solving the Grad-Shafranov equation with spectral elements," Computer Physics Communications **185**, 1416, (2014), and J. R. King of Tech-X Corporation.
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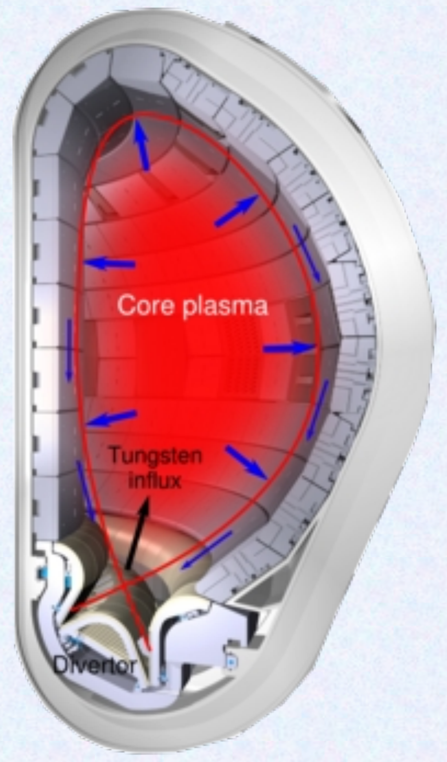
Modeling Confinement Scenarios in ITER

- One of the most important regions of ITER to understand is the area of the confinement region closest to the edge of the core plasma, (see diagram).
- Near this edge region, strong gradients develop in pressure and temperature, which help to confine the plasma and keep it from impacting the walls of the container.
- Most important issues relevant to confinement deal with the physics governing the plasma dynamics in this edge region.



Cross Section of ITER Confinement Chamber – Credit: Foundation for Fundamental Research on Matter

A Test of NIMROD's Ability to Accurately Model Plasma Confinement in ITER

- NIMROD is a computer code that is used to simulate plasma dynamics in a wide variety of fusion confinement scenarios.
- This code is developed and worked on by many individuals across North America (including Eric D. Held here at USU, among others).
- My research here consisted in testing the numerical results of an ion transport term that analytically is equal to 0, but consists of two non-zero terms added together (see equation below) [5].
- The smoothness of the final result, especially closer to the edge region of ITER, would be indicative of NIMROD's ability to accurately model plasma confinement in these regions of ITER.

Re-solving the Grad-Shafranov Equation

- The specific aspect of NIMROD that was tested was its ability to refine its interpolation of essential plasma input parameters (i.e. pressure, current density, and poloidal flux).
- This refining process is accomplished through re-solving, through successive iterations, something called the Grad-Shafranov equation [6].
- The Grad-Shafranov Equation is, in effect, a manipulation and combination of a Newton's Second Law force balance relation, and Ampere's Law from electromagnetism (see equations below). *It is a statement that the sum of the forces on the plasma in equilibrium must sum to be 0.*

Vanishing Ion Transport Term

$$\Gamma(\Psi) = 2\frac{d}{d\Psi}\int dl\,\boldsymbol{B}\cdot\nabla R^2 + \int \frac{dl}{B}\left[2\boldsymbol{b}\cdot\nabla\left(\nabla\Psi\cdot\nabla\ln R^2\right) + \nabla\Psi\cdot\nabla\left(\boldsymbol{b}\cdot\nabla\ln B\right)\right]$$

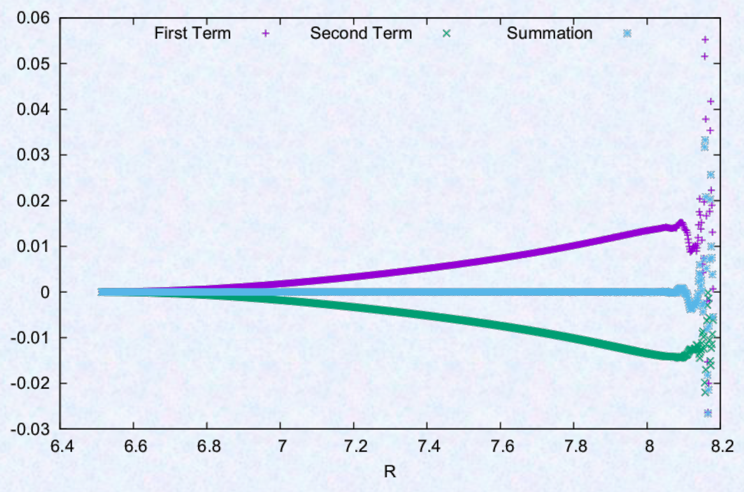
Force Balance Relation **Ampere's Law**

Implementing The Transport Term in NIMROD

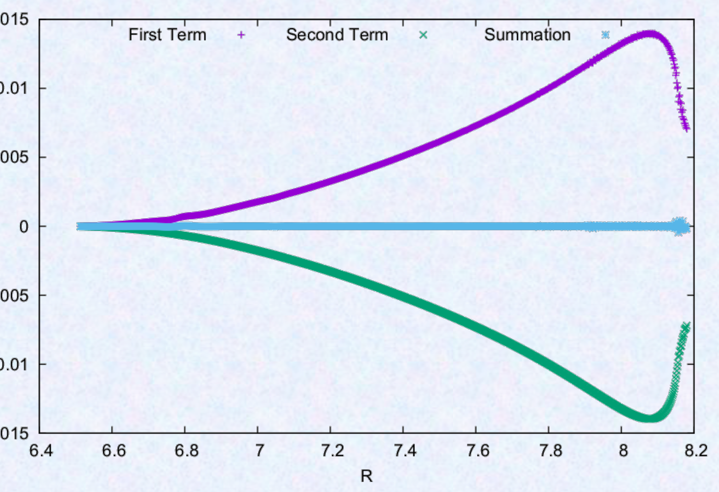
- First, a numerically determined set of plasma parameters from ITER were loaded into NIMROD.
- Using these parameters, the transport term was coded into NIMROD's finite element basis , computed, and then graphed.
- To determine the effectiveness of the Grad-Shafranov re-solve capabilities of NIMROD, the term was computed twice: first it was computed without any re-solves of the Grad-Shafranov equation, and second it was computed after many re-solves of the Grad-Shafranov equation.
- A cubic spline program, which I coded, was used to take the derivatives of the first term with respect to .
- Finding that small numerical noise in the data was amplified when taking the derivative of the first term using a cubic spline, a "smooth noise-robust differentiator" functionality was added to the cubic spline program I wrote to allow for a more accurate numerical derivative of that term [7].

Graph of Vanishing Transport Term

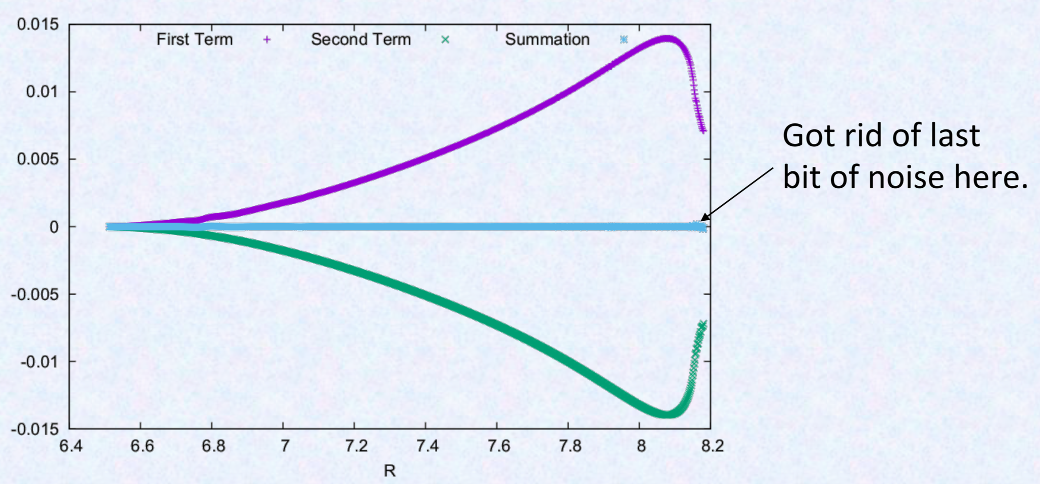
No Re-solves of Grad –Shafranov Equation



Many Re-solves of Grad –Shafranov Equation



Many Re-solves with "Smooth Noise-Robust Differentiator"



Summary and Future Work

- It is clearly seen from the above results that re-solving the Grad-Shafranov equation helped contribute to the smoothness and accuracy of the calculated vanishing ion transport term.
- The benefits of the Grad-Shafranov re-solves are more clearly seen towards the edge region of ITER (where the gradients in pressure and temperature are the steepest).
- In addition, using the "Smooth Noise-Robust Differentiator" helped to smooth out that last bit of noise towards the edge region, giving overall a very smooth zero result for the vanishing transport term.
- The importance of using the Grad-Shafranov re-solve capabilities of NIMROD can be clearly seen from these results, especially when modeling plasma phenomena closer to the edge region of ITER.
- Future work would involve further study of the plasma dynamics close to the edge region of ITER, using the Grad-Shafranov re-solve capabilities of NIMROD to first improve the quality of the input data that is read into NIMROD.