A Teacher's Story

A Penny's Worth of Principles and Standards Using Scientific Notation

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A Penny For Your Problem

"How much is a penny worth? Have you ever thrown away a penny? If you had enough pennies to fill your bedroom, how much money would you have?" These were just a few of the questions that were asked as a high school Algebra I class completed the study of exponents and scientific notation. The teacher was in search of real life applications of scientific notation, something students could put their fingers on, to get a feel for the mathematics. What evolved was a three-day lesson entitled *Pennies, Pennies, Everywhere!!!* (Staley & Walls, 1999). During this multimedia interdisciplinary lesson students utilized ratios, proportions, measurement and conversion skills, perimeter, area, volume, and scientific notation to determine the value of penny-filled cubes the size of their desks, the classroom, the earth and other celestial objects. The initial purpose of the lesson was to determine the amount of money that could be contained in a classroom full of pennies. This had direct relevance to the students, because each week the school held a penny collection to raise funds for a charitable cause.

The Principles and Standards in Action

The lesson *Pennies, Pennies, Everywhere!!!* directly incorporates the NCTM Principles (Equity, Teaching, Learning, Assessment, and Technology), the Content Standards (Number & Operations, Algebra, Geometry, Measurement, and Problem Solving), and the Process Standards (Communications and Connections) (NCTM, 2000). This article discusses how the Principles and Standards played an integral role in the design and implementation of the lesson in an Algebra I high school class.

Lesson Overview

This three-day lesson consists of several classroom activities and an assessment in which students work collaboratively to complete a variety of tasks involving the number of pennies that can fit into different objects. They calculate the penny value for perimeter, area, and volume of various objects.

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Day one's activities, *Sizing Things Up* uses a brief video clip to review ratios and proportions, and Penny Statistics allows students to gather measurements on pennies, a desk and the classroom. The second day's activity, To Infinity and Beyond…, utilizes the Internet to gather data about celestial objects that are compared to the size of the classroom. Day three's activity is the assessment that involves students in determining the penny value of a football field and writing a letter explaining the process used to calculate the penny value of their bedrooms.

We use the following vocabulary for penny measurements.

- Penny Statistic - the conversion of an object's length, width or height into a quantity of pennies.
- Penny Width (pennyw) - the diameter of one penny, it is used for length and width measurement conversions. 1 pennyw = .75 inches or 1.33 pennyw = 1 inch. Area calculations use squares the width of one pennyw.
- Penny Height (pennyh) - the thickness of a penny is used to measure height. 1 pennyh = .10 inch or 10 pennyh = 1 inch. Note pennyh is used to convert the height of an object into its penny statistic.
- Penny Value - the dollar worth of an object after its length, width or height has been converted into penny statistics.

We assume that all pennies are neatly placed in rows and columns when being counted for area and perimeter. For example, the penny value of a one inch stack of 3 by 5 cards can be calculated for the perimeter and area of a card or the volume of the stack of cards.

Videos Captivate Students

To begin the first activity, we showed *Sizing Things Up*, an episode from the Numbers Alive! TV series produced by Maryland Public Television (1995). This episode documents a rock band's adventures while performing at Camden Yards, where the band members use ratios and proportions to make a variety of estimations.

The *Sizing Things Up* activity sheet following this article focuses students on the following tasks: recording the capacity of Camden yards; calculating the number of stadiums, the same size as Camden Yards, that it would take to seat one million people; and determining the amount of money a hot dog vendor would donate to a charity.

As the class viewed the video segments, students actively participated in the discussion while completing the worksheet. Pausing the video allowed ample opportunity for students to record information, perform calculations, and share their answers with classmates. More importantly, it allowed the teacher to check student readiness for the lesson by assessing prerequisite skills of number computations, ratios, and proportions. The teacher used the following series of questions:

(1) How many people can Camden Yards seat?
(2) How might you determine the capacity of the stadium?
(3) Do you think this is an accurate way to estimate the seating capacity for the stadium?
(4) How many stadiums of this size would you need to seat one million people?

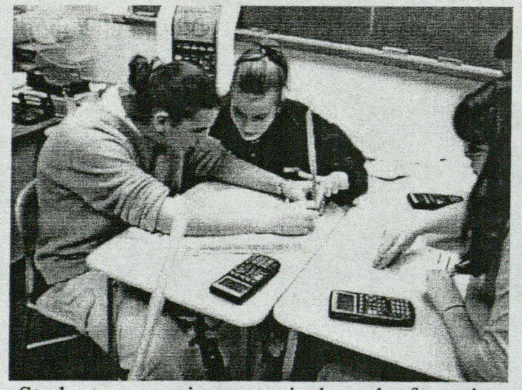
During the discussion about the number of fans that could fit into Camden Yards, several students began to see the connection between the teacher's questions from a prior lesson: How many pennies do you think it would take to fill a box the size of your desk or the classroom? They used this connection along with their understanding of ratios and proportions when making estimations.

During the second set of questions, the teacher asked:

(1) How much money will Ben, the owner of the concession stand, donate for each hot dog sold at the game?
(2) What is the ratio of hot dogs sold to people in attendance?
(3) If the stadium holds approximately 50,000 people, how many hot dogs will Ben sell?
(4) How much money will he donate?

After viewing the video, the class quickly transitioned into *Penny Statistics*—the second activity sheet—where students gathered various measurements related to pennies, their desk, and the classroom. Students applied the Measurement Standards, including understanding measurable attributes of objects and the units, systems, and processes of measurement, and applying appropriate techniques, tools, and formulas to determine measurements, to complete the activities (NCTM, 2000). They used materials from their packets: a standard ruler, 5 yards of string knotted at yard intervals, a yardstick, and a handful of pennies. The teacher began the lesson by defining a penny height as the thickness of one penny used to measure height and the penny value as the dollar worth of an object after its length, width or height have been converted into penny statistics and the perimeter, area or volume calculated. A student then read the directions aloud and the teacher stated that the first goal was to gather all measurements before beginning the calculations.

The collaborative groups of four students began their adventure of gathering measurements of pennies, stacks of pennies, desks, and the classroom as the teacher circulated around the classroom observing their progress. Students first estimated the number of pennies in a one-inch stack, the number of pennies needed to cover their desks, and the number of pennies in a stack that would reach from the floor to the ceiling of the classroom. Next they measured the diameter of a penny, the thickness of a penny and the number of pennies in a one-inch stack.



Students measuring a one-inch stack of pennies

After completing all the measurements students used ratios and proportions to determine the number of pennies in a one-foot stack, a one-yard stack, and a yard of pennies lying flat on a table. Students then measured their desk and the classroom in inches and converted these measurements into the appropriate penny equivalent before calculating perimeter, area, volume, and determining the penny value of each object.

Student work during each part of the lesson was monitored with the use of built in checks on the worksheets that required the teacher's initials before proceeding to the next section. For example, when students completed Part B of the *Penny Statistics* worksheet, they obtained the teacher's initials then proceeded to measure their desks. A second check required students to get the teacher's initials before gathering classroom measurements. Students of varying ability levels and grades (9th-11th) were very involved in the activity. Students discussed different strategies to complete the various measurements and utilized different tools at their disposal to gather measurements. Most of the groups were able to gather the measurements without any assistance from the teacher.

Interdisciplinary Activities Enrich The Lesson

On the second day, students completed the *To Infinity and Beyond...* activity, which involves research on the Internet and extensive use of scientific notation to perform various calculations. Students used the first 15 minutes of class to gather and record the diameter of several celestial objects from pre-selected Internet sites. For many of the students this activity incorporated material they had studied in their science class, and it applied scientific notation skills recently studied in mathematics class. Once they completed gathering and recording the data they began a series of calculations that allowed them to compare the size of their classroom with an earth-sized cube and other cubes the size of celestial objects.

Students first converted the room volume, from the previous activity Penny Statistics, from cubic inches to cubic kilometers, using their ratio and proportion skills. Next students determined the volume of a cube with sides the length of the earth's diameter, approximately 12756.3 km. They also determined the number of classrooms that could fit inside of an earth cube, which they labeled N = volume earth cube / volume of classroom.

During the remainder of the class students calculated the volume of cubes with sides equal to the diameters of the following celestial objects: Venus, Mars, Jupiter, Saturn, Neptune, and Pluto. They compared each volume to the size of the earth cube and determined the number of classrooms that could fit inside it. Their last calculation involved determining the penny value for each object. This allowed students to compare the various objects in the context of dollars and cents, which gave them a referent for understanding the magnitude of the different sizes of each object.

Alternative Assessment Leads To Community Involvement

The assessment activity consisted of two parts, a written assessment modeled after the Maryland assessment test and making fliers or brochures. Students first calculated the number of pennies needed to cover a football field. Students then wrote a letter to a friend explaining the process used to determine the number of pennies needed to fill their bedrooms. This writing activity provided opportunities for students to practice the necessary writing skills while communicating about mathematical concepts. The use of familiar test formats, in our case the Maryland mathematics assessment, allowed for review and reinforcement of test taking skills, incorporated writing into the mathematics curriculum, and served as a summative assessment for the lesson.

The enduring understanding students gained from the lessons about penny measurements and planets was revealed several days later when students created and presented a flier or brochure for the school's penny collection program. The flier had to be informational (the dates, times of penny collection and general purpose) and educational (information learned during the lesson). The use of technology was not required but highly recommended and several students used a desk top publishing tool to create their projects. One student created a tri-fold brochure stating several facts about pennies and the information about the penny collection day. Another student's flier compared the weight of an F-16 jet to the jet's penny value. Many of the projects were displayed in classes throughout the school to promote penny collection day.

Pennies Add Up

As we reflected on the development of the lesson and how it was implemented we focused on two Process Standards: Communications and Connections (NCTM, 2000). The contextual nature of the lesson allowed students to make connections between mathematics and science content, enabling them to recognize and connect among mathematical ideas, and apply mathematics in contexts outside of school mathematics (NCTM, 2000). A major benefit of the lesson context was that it gave students a purpose for performing various calculations that connected to the school's fund raising activity. Students were no longer just doing the mathematics for the purpose of practice but they were highly engaged in trying to determine the amount of money that could be contained in each object.

Students were initially engaged out of curiosity for the number of pennies needed to fill the classroom, which led to their motivation for conducting the various measurements and calculations. The use of celestial objects was a direct connection to material they had recently studied in their science class. This allowed students to connect the performance of standard and scientific notation calculations in mathematics class with the same concepts studied in science.

Communication played a key role during the *Sizing Things Up* video and *Penny Statistics* activity. These experiences allowed students to communicate their understanding of concepts, develop negotiation and interpersonal skills, and construct meaning through interaction with peers. Listening and participating in the various student group discussions was one of the most meaningful parts of the lesson. Students organized and consolidated their mathematical thinking, communicated ideas with peers and the teacher, analyzed their thinking and the thinking strategies of others, and used the language of mathematics to express ideas (NCTM, 2000). By interacting and posing several open-ended questions with the groups, the teacher was able to extend, enrich, and assess student understanding.

Pennies, Pennies, Everywhere!!! allowed students multiple opportunities to develop a deeper understanding of very large numbers and the various representations of them, reinforcing the 9th-12th grade Number and Operations Standard (NCTM, 2000). The calculations and conversions developed computational estimation skills using mental and paper-and-pencil computations, as well as technology for more complicated situations.

The Algebra and Geometry standards were an underlying thread in the lesson as students represented and analyzed mathematical situations by using algebraic symbols and language in their verbal and written expression of relations. Students used equations for converting money, metric and standard measurement conversions, and the use of geometric formulas. The connection between the use of the algebraic equations, formulas, visualization, spatial reasoning, and geometric modeling, all Geometry Standards, allowed students to develop their problem solving skills. It also provided opportunities for them to solve problems in other contexts, apply and adapt a variety of appropriate strategies, and monitor and reflect on the process of mathematical problem solving as outlined in the Problem Solving Standards (NCTM, 2000).

The role of assessment is key in any lesson and may often be the driving force when selecting learning activities for classroom instruction. Assessment during the lesson was on-going and occurred as students completed various learning activities. The built-in check points, requiring the teacher's initials, served as formative assessments and allowed the teacher to assess student performance and provide individualized or small group instruction as needed. The summative assessment activity allowed the teacher to assess students' understanding of key mathematical concepts addressed in the lesson and focused on their ability to communicate these ideas in writing. The true assessment of enduring learning was when students were able to communicate what they learned in their penny collection projects. After completing this lesson many students developed a different outlook on scientific notation, the value of a penny and the value of participating in the school's penny collections. Several students commented that they couldn't wait to hear someone say "What's a penny worth anyway" or "It's only a penny" so that they could tell them all about the true value of a penny.

Author's Note: A copy of the complete lesson plan can be obtained at the Thinkport website. The following activities have been renamed or revised with the revised name in parenthesis: *Penny Statistics* (How Many Pennies…?), *To Infinity and Beyond…*(Pennies in Outer Space) and *Assessment* (Penny Donation Box). The *Numbers Alive!* series videos are available from the NETA Educational Resources website <http://www.netaonline.org>.

References

Maryland Public Television (Producer). (1995). *Numbers Alive! Sizing Things Up* [Television Series Episode]. Owings Mills: MPT.

National Council of Teachers of Mathematics. (2002). *Principles and standards for school mathematics.* Reston, VA: NCTM.

Staley, J. & Walls, E. (1999). *Pennies, Pennies, Everywhere!!!* [On-line]. Available: http://www.thinkport.org/Tools/ContentViewer/ContentPreview.aspx?ContentID=c6d2d31f-7ef2-46e2-9d52-5371666019ab

Mini-Grants from NCCTM

NCCTM is pleased to be able to offer Mini-Grants to teachers who need some support to implement an innovative idea at their schools. There are no preconceived criteria for projects except that students should benefit from the grant. Possible projects for consideration include math clubs, field days, contests, workshops for parents, math activities, math laboratories, and research topics.

Each of the three NCCTM state regions has $5000 to award to teachers in their area. The average mini-grant is about $600 but some have been awarded for as little as $100 or as much as $2000. Applications will be accepted only from persons who are NCCTM members as of 1 September 2005. Don't let the application process intimidate you! See the sample application on the website and use it as a guide!

Completed applications must be received by 15 September 2006 to be considered. For more information and submission guidelines, see the website <http://www.ncctm.org> or contact Phyllis W. Johnson <pwjohnson210@earthlink.net, 252-752-1796>.

The NCCTM Materials Marketplace

The Marketplace will be back this fall, and the organizers need your help! Please consider donating materials to the marketplace. We are looking for new or gently used supplies such as manipulatives, posters, books, professional development materials—anything that would be useful to new teachers.

Preservice and new inservice teachers will be invited to come and purchase at rock bottom prices all sorts of materials—textbooks, technology, supplies, etc.—to start building their resource base.

Please contact coordinators Kim Aiello and Shana Runge if you have materials to contribute.

<ncctmmarketplace@hotmail.com>

Sizing Things Up Activity Sheet

Use this space to record numbers and numerical terms mentioned in the video.

______________ ______________
______________ ______________
______________ ______________
______________ ______________
______________ ______________
______________ ______________

Video Segment 1:

1. How many people can Camden Yard seat? ______________

Video Segment 2:

2. How many stadiums the size of Camden Yards would it take to seat 1 million people? Show your work, and explain how you got your answer on the lines below.

____

____

Video Segment 3:

3. How much money will Ben donate for each hot dog sold at the game? ________

4. What is the ratio of hot dogs sold to fans in attendance at the game? ________

$$\frac{\text{hot dogs}}{\text{fans}} = \_\_\_\_$$

5. What will the total of Ben's donation be? Show your work in the space below and explain your answer.

Penny Statistics (How Many Pennies...?) Activity Sheet

Directions: Today, you will use ratios and proportions to help you calculate the penny capacity of several objects. Perform all necessary measurements and calculations with the materials given to you in your kit. Now you are ready to begin your penny experience. Have fun counting!

Part A: Estimation

Use your estimation skills to fill in the blanks to the following statements.

1. There are __________ pennies in a one-inch stack.
2. It takes ___________ pennies to cover the top of your desk.
3. It would take _______ stacked pennies to reach from the floor to the ceiling of your classroom.

Part B: Data Collection

4. Find the diameter of one penny. ___________ inches
   We will call this "penny width" or $penny_w$. Use this information to complete the following ratios.

$$\frac{1\ penny_w}{\_\_\_\ inch} \qquad \frac{\_\_\_\ penny_w}{1\ inch}$$

$$\frac{\_\_\_\ penny_w}{1\ foot} \qquad \frac{\_\_\_\ penny_w}{1\ yard}$$

5. Find the height of one penny. ______ inches. We will call this "penny height" or $penny_h$. Use this information to complete the following ratios.

$$\frac{1\ penny_w}{\_\_\_\ inch} \qquad \frac{\_\_\_\ penny_w}{1\ inch}$$

$$\frac{\_\_\_\ penny_w}{1\ foot} \qquad \frac{\_\_\_\ penny_w}{1\ yard}$$

Check Point: Get your teacher's initials before you continue. ______________

Part C: Box Capacity

Draw a sketch of the cardboard box your teacher has provided. Be sure to include accurate measurements of the length, width, and height. Convert your measurements into inches and $penny_w$ or $penny_{st}$.

length = ___________ inches = __________ $penny_w$

width = ___________ inches = __________ $penny_w$

height = ___________ inches = __________ $penny_h$

Find the perimeter, area, and volume for the box. Record your answers in inches, pennies, and dollar value. Show your work.

6. Box top perimeter = ________ inches = __________ $penny_w$. $________

   Box top area = ____________ $inches^2$ = ___________ $penny_w^2$. $________

   Box volume = ____________ $inches^3$ = ___________ $penny_w$. $________

7. Find the box volume in $pennies_v$ ("penny volumes"),

   Box volume = ________ $penny_w^2$ x _________ $penny_h$ = ___________ $penny_v$
   $___________

Check point: Get your teacher's initials before you continue. _______

Part D: Room Capacity

Draw a picture of your room. Be sure to include accurate measurements of the length, width, and height of the room. (If the room is not rectangular, use the area the teacher has marked as the "room area.") Convert your measurements into inches and $penny_w$, $penny_h$, and $penny_v$

room length = _______ inches = __________ $penny_w$

room width = _______ inches = __________ $penny_w$

room height = _______ inches = __________ $penny_h$

Find the perimeter, area, and volume for the room. Record your answers in inches, pennies, and dollar value. Show your work.

8. floor perimeter = __________ inches = __________ $penny_w$. $________

   floor area = ____________ $inches^2$ = __________ $penny_w^2$. $________

   floor volume = ____________ $inches^3$ = __________ $penny_w$. $________

9. Find the room volume in $pennies_v$ ("penny volumes")

   Room volume = ______ $penny_w^2$ x _________ $penny_h$ = ___________ $penny_v$
   $___________

Check point: Get your teacher's initials. ____________________

To Infinity and Beyond... (Pennies in Outer Space) Activity Sheet

Directions: Today, we will find a ratio to compare the size of our classroom to several celestial objects. Data for this activity can be found at http://www.seds.org/billa/tnp/overview.html. You should perform your calculations in scientific notation, since most of your data involves numbers of great magnitude.

Part A: First Things First

1. We must first convert our room volume from cubic inches to cubic kilometers. Use the data you gathered in yesterday's activity.

   Room Volume Conversion

   $$(\underline{\hspace{3cm}}\ \text{in}^3) \times \left[\frac{1\text{ m}}{\underline{\hspace{0.8cm}}\text{ inch}}\right]^3 \times \left[\frac{1\text{ km}}{\underline{\hspace{0.8cm}}\text{ m}}\right]^3 = \underline{\hspace{2.5cm}}\ \text{km}^3$$

2. How many classrooms would it take to fill a cube with sides the length of the diameter of the Earth?

   Volume of an Earth Cube = $(\text{diameter})^3 = (\underline{\hspace{2.5cm}})^3 = \underline{\hspace{2.5cm}}\ \text{km}^3$

   Number of classrooms that would fit in the Earth Cube $= \dfrac{\text{volume of Earth Cube}}{\text{volume of classroom}} =$

Part B: Celestial Volumes

Directions: Now use the calculations from Part A and data from http://www.seds.org/billa/tnp/asteroids.html for asteroid data to complete the chart. Note: to compute the size of an object compared to Earth, use a ratio of the object's radius to the Earth's radius.

Celestial Object	Ratio of Object's Radius to Earth's Radius (E)	Number of Classrooms that would Fit in Object	Measurement of object in penny$_v$
Earth			
Venus			
Mars			
Jupiter			
Saturn			
Neptune			
Pluto			