Virtual Manipulatives Used by K-8 Teachers for Mathematics Instruction: Considering Mathematical, Cognitive, and Pedagogical Fidelity

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Abstract

This study examined teachers' uses of virtual manipulatives across grades K-8 after participating in a professional development institute in which manipulatives and technology were the major resources used throughout all of the activities. Researchers analyzed 95 lesson summaries in which classroom teachers described their uses of virtual manipulatives during school mathematics instruction. The findings indicated that the content in a majority of the lessons focused on two National Council of Teachers of Mathematics (2000a) standards: Number & Operations and Geometry. Virtual geoboards, pattern blocks, base-10 blocks, and tangrams were the applets used most often by teachers. The ways teachers used the virtual manipulatives most frequently focused on investigation and skill solidification. It was common for teachers to use the virtual manipulatives alone or to use physical manipulatives first, followed by virtual manipulatives. One important finding of this study was that teachers used the virtual manipulatives during the main portion of their lessons when students were learning mathematics content. These results represent an initial exploration of teachers' current use of virtual manipulatives in K-8 classrooms.

The release of the most recent National Council of Teachers of Mathematics standards (NCTM, 2000a) gave prominence to *representation* as a significant area of mathematics education research. Although this was the first appearance of representation as a standard, teachers have used a variety of representations during mathematics instruction for many years. Representations commonly used in school mathematics include physical or concrete representations (e.g., manipulatives and geometric models), visual or pictorial representations (e.g., pictures, graphs, and diagrams), symbolic or abstract representations (e.g., letters, operation signs, and numerals), and dynamic electronic representations that merge characteristics of all of these modalities (e.g., dynamic geometry software and virtual manipulatives (Moyer, Bolyard, & Spikell, 2002). This study focuses on one of these forms of representation by examining teachers' uses of virtual manipulatives in mathematics lessons across grades K-8.

Research on Virtual Manipulatives, Representation, and Multimedia Learning

A virtual manipulative is defined as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, et al., 2002, p. 373). They have also been defined as "computer based renditions of common mathematics manipulatives and tools" (Dorward, 2002, p. 329). Virtual manipulatives are often dynamic visual/pictorial replicas of physical manipulatives (such as pattern blocks, base-10 blocks, geometric solids, tangrams, or geoboards). They are placed on the Internet as *applets*, or smaller stand-alone versions of application programs. Users move the computer mouse to manipulate these dynamic, visual objects. The ability to manipulate virtual manipulatives makes them particularly useful in teaching mathematics interactively.

Virtual manipulatives can be thought of as *cognitive technological tools* (Zbiek, Heid, Blume, & Dick, 2007). Their characteristics as a cognitive tool are evident in the capability that allows users to act on the virtual manipulatives as representations of objects, with the consequences of the user's actions resulting in visual on-screen feedback from the virtual tool. Although virtual manipulatives have some similarities with their physical manipulative counterparts, as cognitive tools, virtual manipulatives have unique characteristics that go beyond the capabilities of physical manipulatives. Their potential is thus increased for mathematically meaningful actions by users and influences the user's learning.

For example, some virtual manipulatives include links among enactive, iconic and symbolic notations, thereby, supporting learners in making connections among these forms of representation. Other virtual manipulatives have the capability to be altered, including changing the shape of the onscreen object or marking the object with mathematical notations. In addition, virtual manipulatives are readily available with unlimited access to many copies of an electronic object through the click of the mouse (Moyer, Bolyard, & Spikell, 2001; Moyer, Niezgoda, & Stanley, 2005).

Several collections of virtual manipulatives are available on the Web, including The National Council of Teachers of Mathematics Illuminations Web site, the National Library of Virtual Manipulatives Web site, and the Shodor Education Foundation Curriculum Materials Web site. (***Editors note***: See Resources section for Web site URLs.) Among these collections are a variety of virtual manipulatives applets with a range of characteristics. These characteristics include applets that present dynamic electronic: (a) pictorial images only, (b) combined pictorial and numeric images, (c) simulations, and (d) concept tutorials, which include pictorial and numeric images with directions and feedback. When selecting a virtual manipulative for instructional use, it is important to consider these characteristics in terms of the mathematical fidelity, cognitive fidelity, pedagogical fidelity, and externalized representations of each individual tool or applet (Zbiek, Heid, Blume, & Dick, 2007). The mathematical fidelity of virtual manipulative tools refers to the degree to which the mathematical object is faithful to the underlying mathematical properties of that object in the virtual environment; while the *cognitive fidelity* refers to how well the virtual tools reflect the user's cognitive actions and possible choices while using the tool in the virtual environment. Zbiek et al. described pedagogical fidelity as "the extent to which teachers (as well as students) believe that a tool allows students to act mathematically in ways that correspond to the nature of mathematical learning that underlies a teachers practice..." (p. 1187).

Virtual manipulatives may be thought of as a unique externalized representational form. Because a representation is thought of as a configuration of signs, characters, icons, or objects that represents something else (Goldin, 2003), virtual manipulatives may be considered a unique form of representation or a combination of several representations. Students' capacity to translate among multiple representational systems influences their abilities to model and understand mathematical constructs (Goldin & Shteingold, 2001). Virtual manipulatives are unique because they provide a visual image like a pictorial model, they can be moved and manipulated like a physical model, and unlike physical models, they feature linked verbal and symbolic notations. The power of virtual manipulatives is in combining several representations in ways that support the learner in connecting multiple aspects of mathematical concepts and ideas.

Essentially, the virtual manipulative brings together the visual or pictorial representation of a mathematics concept, along with symbolic notation for that concept, or even a demonstration of the procedure one follows for a particular algorithm. Students do not always make connections among representations (for example, a rectangular shape with one fourth shaded, a circular shape with one fourth shaded, the numerical representation .25, or the numerical representation 2/8); therefore, combining multiple representations in a virtual environment allows students to manipulate and change the representations to develop their relational thinking and to generalize mathematical ideas.

Virtual manipulatives are also a powerful cognitive tool for learners because they constrain the user's actions on the mathematical object in the virtual environment, directing the user to focus on the mathematics in the environment; they react to user input with visual and verbal/symbolic feedback showing the user the results of their actions on the object; and, they enforce mathematical rules of behavior (Zbiek et al., 2007). As the NCTM Technology Principle indicates, "Work with virtual manipulatives.... can allow young children to extend physical experience and to develop an initial understanding of sophisticated ideas like the use of algorithms" (NCTM, 2000a, pp. 26-27). In essence, virtual manipulatives have some of the advantageous properties of several different forms of representation, as well as some additional advantages brought about by their technological properties.

Cognitive science has influenced educational research by proposing theoretical models that explain the encoding of information among representational systems. For example, Dual Coding Theory (DCT), proposed by researchers in the field of educational psychology and based on Cognitive Information Processing Theory, is the assumption that information for memory is processed and stored by two interconnected systems and sets of codes (Clark & Paivio, 1991). These sets of codes include visual codes and verbal codes, which can represent letters, numbers, or words. According to DCT, presenting learners with both visual and verbal codes, which are functionally independent, has additive effects on their recall.

A common design structure for virtual manipulative applets is to include verbal codes (i.e., letters, numbers, and words) and visual codes (i.e., pictures, movable objects represented in two and three dimensions) presented simultaneously, which Mayer and Anderson's (1992) Contiguity Principle purports to increase the effectiveness of multimedia instruction. Applying DCT to instruction when virtual manipulatives are used suggests that mathematics environments that activate multiple systems of codes have a greater potential for improving learning, because two mental representations are available for use by the learner rather than one. In addition, Rieber's (1994) research showed that students can more easily recall information from visual processing codes than from verbal codes because visual information is accessed using synchronous processing rather than sequential processing. Virtual manipulative applets, which are primarily visually based tools, can facilitate greater access to memory.

These principles and theories of learning through the use of media provide some potential insight into why researchers are finding positive initial results in studies on the use of virtual manipulatives in classrooms. Although this base of research on virtual manipulatives has been limited, classroom studies and dissertations have demonstrated the unique features of these tools for teaching mathematics. Overall, these results have indicated that students using virtual manipulatives, either alone or in combination with physical manipulatives, demonstrate gains in mathematics achievement and understanding (Bolyard, 2006; Moyer et al., 2005; Reimer & Moyer, 2005; Suh, 2005; Suh & Moyer, 2007; Suh, Moyer, & Heo, 2005) and appear to be more engaged and on task (Drickey, 2000).

In one study conducted in a kindergarten classroom, children created patterns using virtual pattern blocks, wooden pattern blocks, and drawings (Moyer & Niezgoda, 2003). When the patterns were analyzed comparing each form of media, the results indicated the children created a greater number of patterns, used more elements in their pattern stems, and exhibited more creative behaviors using the virtual pattern blocks, as compared to the wooden pattern blocks or drawings. Another study focusing on second-graders' ability to demonstrate the regrouping process of addition showed that students' interactions with the virtual base-10 blocks impacted their ability to create a pictorial and written representation, making them better able to express (both in written explanations and drawings) their conceptual understanding of the regrouping process (Moyer et al., 2005).

A study of third graders using several virtual manipulative fraction applets during a 2-week unit on fractions indicated a statistically significant improvement in students' conceptual knowledge and a significant relationship between students' scores on the posttests of conceptual knowledge and procedural knowledge (Reimer & Moyer, 2005). During interviews, students reported that the virtual manipulatives: helped them learn more about fractions, provided immediate and specific feedback, were easier and faster to use than paper-pencil methods, and enhanced their enjoyment while learning mathematics. Further, Suh's (2005) dissertation results in two third-grade classrooms showed a statistically significant difference in student achievement during a unit on fraction addition (favoring virtual manipulatives over physical manipulatives), but not during a unit on balancing algebraic equations. This research highlighted how different representations (i.e., physical versus virtual manipulatives) and even different individual applets (virtual algebra balance versus virtual fraction addition) can each have their own unique characteristics and affordances that promote different kinds of learning for different mathematical purposes.

Virtual manipulatives are one important element of mathematics teaching and learning as components of representational systems. Because this relatively new technology is now being used with greater frequency in mathematics classrooms, we designed this exploratory study to examine how teachers across grades K-8 are using virtual manipulatives. To focus on teachers' uses of various virtual manipulatives, we analyzed a large collection of lesson summaries where these tools were used for mathematics instruction. These lesson summaries were written by teachers who had participated in a professional development institute where the use of manipulatives and technology was a major component. The following research question guided our analysis of the summaries: What virtual manipulatives are used by teachers in mathematics lessons and how are they used?

Methods

A total of 116 teachers voluntarily participated in the study, with two sections at each of four grade-specific groups (K-2, 3-4, 5-6, 7-8), for a total of eight groups. Participants were kindergarten through eighth-grade teachers in eight different teacher professional development institutes that started during the summer and concluded during the spring of the following year. The eight groups remained intact within their grade-specific groups throughout their participation in the study. (In other words, all grades 3-4 teachers participated in the professional development and follow-up sessions together as a group over the course of data collection). All teachers worked in the same school system. Teaching experience in the group ranged from 1 to 32 years (mean = 12.3 yrs; mode = 8 yrs; median = 9 yrs), with the majority of teachers at the elementary school level (67%) and the remainder at the middle school level (33%). A Teacher Practice Survey was administered at the beginning and end of the professional development activities to gather background information on the teachers that included teachers' prior use of manipulatives and technology.

The eight groups of teacher participants attended 1-week summer mathematics institutes (40 hours) followed by four formal meetings during the fall and spring of the following academic year (8 hours). These were taught by four different instructors. The purpose of the professional development activities was to improve mathematics instruction through the use of readings, discussions, and hands-on experiences that focused on teaching and learning mathematics. Manipulatives and technology (e.g., calculators and computer resources including virtual manipulatives, spreadsheets, Geometer's Sketchpad, and graphing programs) were the major resources used throughout all of the activities and were used daily in some form with each of the grade-specific groups during the summer institutes.

Some of the virtual manipulatives applets used during instruction were open ended and exploratory, others were specific and focused concept tutorials, while others were games. These mathematical tools were used by the instructors to investigate concepts, solidify skills, and introduce mathematics content through individual and collaborative problem solving activities during the summer institutes. During the academic year, teachers had access to calculators and computer resources through their school system.

The primary source of data for the study was teacher-developed lesson summaries. These summaries were a compilation of the teacher's plan for the lesson, a list of the various elements included in the lesson when it was taught, and a self-reported description of what actually took place in the classroom when the teacher taught the lesson. In addition, teachers were required to provide a standard set of data for each lesson summary that included grade level, objectives, materials, procedures, and assessment.

Throughout the academic school year, teacher participants developed, taught, and wrote summaries for their mathematics lessons. In the written lesson summaries, teachers described their own instruction and student activities. This descriptive information provided evidence of and insight into how the virtual manipulatives were used by the teachers and by the students. Each of the 116 teachers participating in the summer institute was asked to design, teach, and write a summary for five mathematics lessons during the academic year, with the requirement that the lessons included the use of a mathematical tool, for a total of 580 lessons. Teachers were encouraged to include at least one lesson that used a virtual manipulative in some way for mathematics instruction. Researchers collected the written lesson summaries at the end of the academic year. Of the total lessons designed and taught by the teachers, 95 of the lessons included the use of a virtual manipulative (28% in grades K-2, 16% in grades 3-4, 32% in Grades 5-6, and 24% in grades 7-8). These 95 lesson summaries were the focus of the present analysis.

The first analysis of the lesson summaries examined the choice of mathematical content by the teachers, comparing this content with the NCTM Content Standards (2000a). The second analysis determined the types of virtual manipulatives used within grade-specific groups (K-2, 3-4, 5-6, 7-8). In the third analysis, we employed a categorical system (Fraenkel & Wallen, 1993) to determine how teachers used manipulatives, virtual manipulatives, and other tools during the lessons. To determine the categories for this classification, we used a constant comparative method to review the original 580 lessons and to classify how *all* mathematical tools were used in teachers' lessons (as recommended in Strauss & Corbin, 1998). This method of comparison produced seven categories describing how mathematical tools were used in the lessons.

These seven categories were used to classify teachers' uses of virtual manipulatives: (a) investigate concepts (students engaged in open-ended investigations/problem-solving activities to promote the development of concepts and relationships), (b) skill solidification (students developed and solidified specific mathematical concepts and skills to support procedural fluency through teacher guidance and practice; procedural fluency was defined as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately," National Research Council, 2001, p. 116), (c) introduce (teachers used the virtual manipulatives to introduce a new concept), (d) game (students played games with the virtual manipulatives), (e) aid (virtual manipulatives were used for remediation), (f) model (teachers demonstrated concepts, but students did not use the virtual manipulatives), and (g) extend (virtual manipulatives were used to extend concepts for students achieving above grade level). Ten percent of all lesson summaries were double coded for reliability using the seven categories, with an interrater reliability rating of 96%. In the final analysis, researchers determined whether the virtual manipulatives were used in connection with physical manipulatives based on the descriptive information in the lesson summaries.

Results

Teachers' Choices

In the majority of the virtual manipulative lessons, teachers' choice of mathematical content came from the Number & Operations Standard (35%), followed closely by Geometry (32%). A lesser portion of the lessons included Algebra (13%), Measurement (13%), and Data Analysis & Probability (7%) content.

Table 1 shows a complete list of the 31 different virtual manipulatives teachers reported in their 95 lesson summaries. Most frequently used of all virtual manipulatives across the grade levels were virtual geoboards (11% of all lessons), pattern blocks (11%), tangrams (9%), and base-10 blocks (8%). The most frequent use of virtual manipulatives within grade-specific groups included virtual pattern blocks in 22% and virtual tangrams in 19% of K-2 lessons, virtual base-10 blocks in 20% of grades 3-4 lessons, and virtual geoboards in 17% of grades 5-6 lessons and 17% of grades 7-8 lessons. These virtual manipulative applets were also used commonly by the instructors of the professional development institutes. Other virtual manipulatives were used with less frequency among the groups. When the groups were compared, some interesting patterns emerged. No virtual manipulative was used by all four grade-specific groups; however, eight virtual manipulatives were used by three of the four grade-specific groups: virtual base-10 blocks, fraction circles, fraction squares, geoboards, geometric solids, number lines, pattern blocks, and tangrams.

Table 1

Teachers' Uses of Virtual Manipulatives by Grade-Specific Groups

Virtual Manipulatives	**Grade-Specific Groups**				
	K-2 (*N* = 27)	**3-4 (*N* = 15)**	**5-6 (*N* = 30)**	**7-8 (*N* = 23)**	**All (*N* = 95)**
Virtual Angles	0 (0%)	0 (0%)	0 (0%)	1 (4%)	1 (1%)
Virtual Arrays	0 (0%)	0 (0%)	1 (3%)	0 (0%)	1 (1%)
Virtual Attribute Blocks	1 (4%)	1 (7%)	0 (0%)	0 (0%)	2 (2%)
Virtual Balance	2 (7%)	0 (0%)	0 (0%)	2 (9%)	4 (4%)
Virtual Balls in Bags	0 (0%)	1 (7%)	0 (0%)	0 (0%)	1 (1%)
Virtual Base-Ten Blocks	3 (11%)	3 (20%)	2 (7%)	0 (0%)	8 (8%)
Virtual Box Plots	0 (0%)	0 (0%)	1 (3%)	0 (0%)	1 (1%)
Virtual Clocks	0 (0%)	2 (13%)	0 (0%)	0 (0%)	2 (2%)
Virtual Color Chips	0 (0%)	0 (0%)	1 (3%)	1 (4%)	2 (2%)
Virtual Color Tiles	0 (0%)	0 (0%)	1 (3%)	0 (0%)	1 (1%)
Virtual Factor Trees	0 (0%)	0 (0%)	2 (7%)	3 (13%)	5 (5%)
Virtual Fractals	0 (0%)	0 (0%)	1 (3%)	0 (0%)	1 (1%)
Virtual Fraction Bars	1 (4%)	1 (7%)	0 (0%)	0 (0%)	2 (2%)
Virtual Fraction Circles	0 (0%)	1 (7%)	3 (10%)	1 (4%)	5 (5%)
Virtual Fraction Squares	1 (4%)	0 (0%)	2 (7%)	1 (4%)	4 (4%)
Virtual Geoblocks	0 (0%)	0 (0%)	0 (0%)	3 (13%)	3 (3%)
Virtual Geoboards	1 (4%)	0 (0%)	5 (17%)	4 (17%)	10 (11%)
Virtual Geometric Solids	0 (0%)	1 (7%)	1 (3%)	1 (4%)	3 (3%)
Virtual Histograms	0 (0%)	0 (0%)	1 (3%)	0 (0%)	1 (1%)
Virtual Let's Make a Deal	0 (0%)	0 (0%)	1 (3%)	0 (0%)	1 (1%)
Virtual Money	4 (15%)	1 (7%)	0 (0%)	0 (0%)	5 (5%)
Virtual Number Lines	0 (0%)	1 (7%)	1 (3%)	1 (4%)	3 (3%)
Virtual Pattern Blocks	6 (22%)	2 (13%)	0 (0%)	2 (9%)	10 (11%)
Virtual Peg Puzzles	0 (0%)	0 (0%)	1 (3%)	0 (0%)	1 (1%)
Virtual Pentominoes	1 (4%)	0 (0%)	0 (0%)	1 (4%)	2 (2%)

Virtual Percent Bars	0 (0%)	0 (0%)	1 (3%)	1 (4%)	2 (2%)
Virtual Protractors	0 (0%)	0 (0%)	1 (3%)	0 (0%)	1 (1%)
Virtual Quilt	0 (0%)	0 (0%)	1 (3%)	0 (0%)	1 (1%)
Virtual Spinners	1 (4%)	0 (0%)	0 (0%)	1 (4%)	2 (2%)
Virtual Tangrams	5 (19%)	1 (7%)	3 (10%)	0 (0%)	9 (9%)
Virtual Triangles	1 (4%)	0 (0%)	0 (0%)	0 (0%)	1 (1%)
Note: Because groups contain different *N*s, data are presented in numeric and percent formats for comparison purposes.					

How Teachers Used the Virtual Manipulatives

Table 2 shows *how* virtual manipulatives were used in the lessons. As Table 2 indicates, most of the use of the virtual manipulatives focused on investigation and skill solidification (45% and 37%, respectively). There were a greater number of lessons that included open-ended investigations at grades K-2 (52%) and 5-6 (47%). A greater number of lessons at grades 3-4 were designed to develop skill solidification and procedural fluency (47%), while an equal number of lessons at grades 7-8 asked students to investigate concepts and develop skill solidification (43%).

Table 2
Ways Virtual Manipulatives Were Used in the Lessons

Virtual Manipulative Use	**Grade-Specific Groups**				
	K-2 (*N* = 27)	**3-4 (*N* = 15)**	**5-6 (*N* = 30)**	**7-8 (*N* = 23)**	**All (*N* = 95)**
Investigation	14 (52)	5 (33%)	14 (47%)	10 (43%)	43 (45%)
Skill Solidification	8 (29%)	7 (47%)	10 (33%)	10 (43%)	35 (37%)
Introduction	3 (11%)	3 (20%)	4 (13%)	3 (13%)	13 (14%)
Game	1 (4%)	0 (0%)	1 (3%)	0 (0%)	2 (2%)
Other (Aid, Model, Extend)	1 (4%)	0 (0%)	1 (3%)	0 (0%)	2 (2%)
Note: Because groups contain different *N*s, data are presented in numeric and percent formats for comparison purposes.					

The use of the virtual manipulatives for investigation and skill solidification was consistent with the way the instructors of the summer institutes used the virtual manipulatives during a large portion of their instruction with the teachers. However, the instructors of the summer institutes also integrated the use of the virtual manipulatives for introducing lessons and games and to aid, model, or extend ideas. Yet, these instructional uses were not as prevalent among the teachers' lesson summaries and, in some grade-specific groups, were not present at all. This finding indicates that the virtual manipulatives were used during the core part of the instructional sequence in the lesson for student investigation and skill solidification, rather than focusing the use of the virtual manipulatives on peripheral activities in the instructional sequence, such as the introduction, as an aid, or as an extension.

Table 3 shows the relationship of virtual manipulatives with other mathematical tools used in the lessons (e.g., physical manipulatives). An approximately equal number of lessons used virtual manipulatives alone (49 lessons) as used virtual manipulatives together with physical manipulatives (46 lessons). A larger portion of lessons at grades K-2 (59%), 3-4 (53%), and 7-8 (52%) used both virtual manipulatives and physical manipulatives, using the physical materials first, and then using the virtual manipulatives during the second part of the lesson. The majority of grades 5-6 lessons used the virtual manipulatives alone (77%).

Table 3
The Relationship of Virtual Manipulatives With Other Mathematical Tools Used in the Lessons.

Relationships	**Grade-Specific Groups**				
	K-2 (*N* = 27)	**3-4 (*N* = 15)**	**5-6 (*N* = 30)**	**7-8 (*N* = 23)**	**All (*N* = 95)**
Used Only VM	10 (37%)	6 (40%)	23 (77%)	10 (44%)	49 (52%)
Used PM, Then VM	16 (59%)	8 (53%)	5 (17%)	12 (52%)	41 (42%)
Used VM, Then PM	0 (0%)	0 (0%)	2 (7%)	0 (0%)	2 (2%)
Simultaneous Use (VM & PM)	1 (4%)	1 (7%)	0 (0%)	1 (4%)	3 (3%)

Note: VM = virtual manipulatives and PM = physical manipulatives. Because groups contain different *N*s, data are presented in numeric and percent formats for comparison purposes.

An example of a first-grade lesson that included both physical manipulatives and virtual manipulatives asked students to investigate transformations by combining and subdividing shapes. Students used plastic tangrams to create animals. Then students recreated their animals using virtual tangrams. While using the virtual tangrams, students focused on transformations (i.e., slides, flips, and turns) to move the shapes and position them accurately in their animal picture. Another lesson that used physical manipulatives before using virtual manipulatives was designed to help third graders solidify skills for telling time. Students used clock faces with movable hands to show different times given by the teacher. Then students practiced telling time by moving the hands on virtual clocks to show times given by the computer.

A lesson developed to help third graders solidify place value skills used dice and virtual base-10 blocks simultaneously. Students rolled the dice and created the largest possible number with the digits rolled. They modeled the number with the virtual base-10 blocks and wrote the number in standard form, expanded form, and words. The virtual base-10 blocks provided a visual model for their written work.

Another lesson that used virtual manipulatives and physical manipulatives simultaneously was designed to help eighth graders investigate equivalent fractions. Students worked with partners. One student created a variety of pictorial models of equivalent areas to represent equivalent fractions using virtual pattern blocks. The other student completed the same activity with wooden pattern blocks. Students were then asked to discuss relationships among the different models they had created and to determine patterns among the representations that helped them to identify the representations as equivalent.

In a lesson using only virtual manipulatives, seventh graders investigated how to develop an area rule for triangles with virtual geoboards. Students created different right triangles on the virtual geoboards and looked at patterns of height, area, and lengths of sides. They compared the areas of the right triangles to the areas of related rectangles. Discussions around their findings led students to discover a formula for the area of a right triangle. The investigation continued as students created different non-right triangles and inscribed those triangles in rectangles with the same bases and heights, ultimately discovering the formula for the area of any triangle.

In another example, sixth graders investigated probability on a virtual manipulative that showed three closed doors. Behind one of the doors was a prize like a new car, movie tickets, or money. Behind the other two doors were less desirable prizes, such as pretzels, plungers, or clothespins. Students chose one door by clicking on it. One of the doors not selected by the student was opened showing one of the less desirable prizes. At that point, students were given the opportunity to “stick” with their original door or “switch” to the other closed door. Students repeated this investigation several times, experimenting with the two strategies. Discussions with their classmates about the percentages of finding a more desirable prize helped students to determine the better strategy. Students also calculated the odds of winning associated with each strategy.

Limitations

There are several limitations to consider in reviewing these results. We did not ask the teachers to report in their lesson summaries *why* they chose particular virtual manipulatives for instruction or why they selected specific content. We also did not ask them to report why they selected the five lessons that were submitted. This information will be useful to gather in future analyses. Because our goal was to look at the selection and use of virtual manipulatives broadly, we did not observe individual classroom use of the virtual manipulatives. In future studies, this more focused examination of classroom practice could provide further insights about teachers’ implementation practices.

Discussion

This study provides an initial examination of what and how virtual manipulatives are used by K-

8 teachers in their mathematics instruction. The findings provide evidence of the following teacher practices: (a) the choice of mathematical content that was addressed by K-8 teachers when virtual manipulatives were used, (b) the specific virtual manipulatives used and the grade levels where they were used most frequently, (c) the ways virtual manipulative use was related to other mathematical tools used during the lesson, and (d) the ways the virtual manipulatives were used during the instructional sequence.

Teachers’ Selection of Virtual Manipulatives and Mathematics Content

As the results showed, the K-8 teachers in this study used 31 different virtual manipulatives for mathematics instruction. The most common virtual manipulatives used were geoboards, pattern blocks, tangrams, and base-10 blocks. We were not surprised to find these specific virtual manipulatives used most frequently across the grade levels, as these are commonly used physical manipulatives with which many teachers are familiar. In fact, the Teacher Practice Survey administered to the participants at the beginning of this study indicated that this group of teachers was familiar with a variety of physical manipulatives and different technologies for use in teaching mathematics. These virtual manipulatives were also used frequently by the instructors of the mathematics professional development institutes.

In addition to having experiences in the use of these common virtual manipulatives during the professional development institutes, teachers may have selected these particular virtual manipulatives most frequently for two possible reasons. First, these virtual manipulatives have physical counterparts, making it more likely that the teachers using them had prior familiarity with them in their teaching. In fact, the following physical materials were distributed to the grade-specific groups and used by instructors during the summer institutes: geoboards class set (grades 5-6) and overhead geoboards (grades 3-4 and 5-6); pattern blocks class set (grades K-2) and overhead pattern blocks (grades K-2, 3-4, & 7-8); tangrams class set (grades K-2 & 5-6) and overhead tangrams (grades 3-4); and base-10 blocks class set (grades 5-6) and overhead base-10 blocks (grades K-2, 3-4, & 5-6).

Second, teachers may have previously designed successful lessons using the physical versions of these virtual manipulatives and, therefore, had developed a pedagogical model for the use of the particular tool during mathematics instruction. This level of familiarity was likely to influence the selection of virtual manipulatives with physical counterparts.

The most common virtual manipulatives used within the grade-specific groups included virtual pattern blocks and tangrams in grades K-2 (students at this grade level explored characteristics of shapes and creating patterns with the geometric shapes); virtual base-10 blocks in grades 3-4 (students used these blocks to model the base-10 numeration system and computational procedures); and virtual geoboards in grades 5-6 and 7-8, (students investigated properties of shapes, concepts of area and perimeter, and characteristics of angles).

Because these applets were selected most commonly within the grade-specific groups by teachers, the tools themselves may have been perceived by the teachers as having greater mathematical or cognitive fidelity, and teachers’ beliefs about specific virtual manipulatives applets in relation to student learning may have been significant in this selection. In cases where teachers used virtual manipulatives that were also available in a physical form, teachers may have believed that the choice of these tools allowed their students to act mathematically in a way that was closely aligned with teachers’ previous practices and the features and affordances available within particular virtual manipulative applets.

In the majority of K-8 lessons, teachers chose to address content in the Number and Geometry Standards (NCTM, 2000a). The NCTM expectations for the emphasis of the content standards in grades K-8 are consistent with these findings. As the NCTM expectations propose, Number and Geometry are two areas that should receive the greatest emphasis in grades K-8, in relation to the other content standards (p. 30). The use of virtual manipulatives to teach concepts in these standards shows that the teachers in this study used the virtual manipulatives to teach content that was aligned with NCTM's expectations.

An additional consideration in terms of Geometry is teachers may have used virtual manipulatives to teach these topics because of the visual and pictorial aspects of the content of geometry studied in grades K through 8. The topic of geometry may show the alignment among teachers' beliefs about the use of tools for teaching geometry, the mathematical fidelity of the tools themselves in teaching geometric concepts, and teachers' beliefs about how the technology tools facilitate students' mathematical learning.

Other research on teachers' uses of physical manipulatives has indicated the prevalence of geometry concepts among the lessons observed in middle school teachers' classrooms (35% of the observed lessons focused on geometry concepts; Moyer, 2001). In both Moyer's classroom observation study and our present study, teachers themselves selected the mathematical content to be taught when using the physical and virtual manipulatives. In the present study, virtual manipulatives were used to teach geometry concepts in 32% of the reported lessons, consistent with Moyer's research. Taken together, these studies reflect a possible preference for the use of both physical and virtual manipulatives when teachers select instructional tools for the teaching of concepts in geometry.

Meira (1998) argued that different mathematical tools have varying degrees of transparency, or access, mediated by users' participation in activities with those tools. Inherent in the study of some content, such as geometry, are visual and pictorial aspects, aspects that may lend studying such content more to the use of physical and virtual manipulatives.

The Relationship Between Virtual Manipulatives and Other Tools

About the same number of lessons used virtual manipulatives alone as those combining the use of virtual and physical manipulatives. Research has indicated the positive influence of using different tools for different purposes. For example, Terry's (1995) examination of 102 students in grades 2 through 5 using base-10 blocks and attribute blocks found that when students used a combination of both physical and virtual manipulatives, they showed significant gains between the pretest and posttest when compared to students using only physical manipulatives or only virtual manipulatives.

When working with groups of third-grade students learning algebraic concepts, Suh and Moyer (2007) reported that unique features, both in the physical and virtual environment, encouraged relational thinking and promoted algebraic reasoning. For example, the activities using virtual algebra applets promoted the understanding of the fundamental algebraic idea of equality using the dynamic feature of the tilting balance scales. The physical manipulatives, Hands-On Equations®, encouraged students' invented methods and mental mathematics. Takahashi's (2002) dissertation, using a physical geoboard and a virtual geoboard with middle school students, also indicated that students benefited from instruction when using both physical and virtual tools. These studies show that importance of considering the characteristics of different learning environments and how those characteristics influence different types of learning experiences for students.

In particular, virtual manipulatives present a somewhat unique learning environment that combines the characteristics of several representational forms within the virtual applet. This allows teachers and students to use not only one representational form but several representational forms in ways that are dynamic and responsive to the learner. The NCTM (2000b) Representation standard stated:

> Representations are necessary to students' understanding of mathematical concepts and relationships. Representations allow students to communicate mathematical approaches, arguments, and understanding to themselves and to others. They allow students to recognize connections among related concepts and apply mathematics to realistic problems. (p. 14)

When one considers the connections that can be made in the virtual environment among different representational forms, many virtual manipulatives applets provide support for this process by making these connections among representations explicit. It is often the role of the teacher to help students make connections among representational forms. A well-designed virtual manipulatives applet can support the role of the teacher and allow the learner to freely explore these connections individually by manipulating features of the applet. The power for translating among representations is then placed in the hands of the student.

Using Virtual Manipulatives in Mathematics Teaching

One of the most important implications of this study is in the findings on how teachers used the virtual manipulatives in these lessons. The examination of *how* the virtual manipulatives were used indicated that, more frequently, virtual manipulatives were used in the lesson for investigation or skill solidification. In contrast, they were used less frequently or not at all to introduce the lesson, for a game, as an aid, as a model, or to *extend* ideas. From a pedagogical point of view, we propose that the investigation and skill solidification portions of the instructional sequence are the core of the lesson. For example, if a lesson is an instructional sequence with multiple parts, in most simplistic terms there is a beginning, middle, and end. The portion in the middle of the lesson, where students are investigating concepts and relationships and solidifying their skills to promote procedural fluency, is essentially the heart or core of the lesson sequence. It was during these activities (investigation and skill solidification) that teachers reported the engagement of the students with the virtual manipulatives.

The use of virtual manipulatives during the heart or core of the mathematics lesson contrasts with the way other researchers have characterized teachers' uses of physical manipulatives. For example, Moyer's research described teachers associating the use of physical manipulatives with "having fun" and not focusing on the "real mathematics" in the lesson. While Moyer's (2001) classroom observations reported that 30% of the lessons in the study used physical manipulatives for games, games were reported in only 2 of the 95 (2%) virtual manipulatives lessons in this study. This difference represents a significant gap between the way teachers in this study chose to use virtual manipulatives in their lessons and the way teachers in Moyer's (2001) study used physical manipulatives.

A simple explanation could be that the teachers participated in activities focused on investigation and skill solidification while using the virtual manipulatives during the mathematics institutes, and they transferred this example directly to their classroom instruction. However, in other mathematics teacher institutes where physical manipulatives were used for investigation and skill solidification, instructional practices with physical manipulatives in the institute did not directly transfer to teachers' classroom practices (Moyer, 2001).

When physical manipulatives are used, there is some amount of effort on the part of the student to extract the mathematical ideas from the student's actions on the physical objects. In contrast, built-in constraint systems in many virtual tools provide support for sense-making and make mathematics ideas explicit as the user interacts with the tool. In addition, physical tools do not provide specific and directed feedback and interaction with the student; while virtual tools react to the user's actions, providing prompts and guidance that assist the user in focusing on the mathematics in the task.

Physical manipulatives must also be manually linked to other representations, such as pictures or symbolic notations. On the other hand, virtual manipulatives may include connected representations in which the manipulation of one representation also produces a matched change in another representation. Both Dual Coding Theory (Clark & Pavio, 1991) and Multimedia Principles (Mayer & Anderson, 1992) support the notion that when learners are presented with visual and verbal codes, the effects of multimedia instruction and students' recall of information are increased. As visual information is accessed by the learner (i.e., the onscreen virtual manipulatives) the processing required to interpret this information facilitates greater access to memory (Rieber, 1994). Finally, the transformative nature of many virtual tools simply allows students to explore ideas flexibly, modeling the fluidity of the brain's activity and human thinking in ways that cannot be done in a physical space.

Conclusions

An interesting finding in this study was the way virtual manipulatives were used by teachers as cognitive technological tools to support learning during K-8 mathematics instruction. As the results showed, the virtual manipulatives were central to the mathematics learning and content development and were often used in combination with physical manipulatives. Seemingly, teachers' chose to use the virtual manipulatives when they were central to the *lesson* and to the learning and development of the *mathematics* in the lesson.

Future research should examine how teachers' perceptions of the mathematical, cognitive, and pedagogical fidelity of using virtual manipulatives influences their choices of how and when to use them in instruction. For example, it is important to determine if this group of teachers, influenced by this particular professional development, are the only teachers using virtual manipulatives in ways central to the lesson and to the mathematics in the lesson or if other teachers are using virtual manipulatives in the same ways.

With respect to the teachers in the present study, the findings suggest that teachers' choices about which virtual manipulatives to use, what content to teach using them, and whether to use virtual manipulatives in combination with physical manipulatives were potentially influenced by familiarity with similar physical manipulatives and beliefs about the mathematical, cognitive, and pedagogical fidelity of virtual manipulative use. Further examinations, using in-depth interviews with teachers and observations of classroom implementation, have the potential to reveal additional insights into these results.

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Resources

Base-10 blocks - http://nlvm.usu.edu/en/nav/frames_asid_154_g_1_t_1.html

National Council of Teachers of Mathematics Illuminations - http://illuminations.nctm.org/

National Library of Virtual Manipulatives - http://nlvm.usu.edu/en/nav/vlibrary.html

Pattern blocks - http://nlvm.usu.edu/en/nav/frames_asid_169_g_1_t_2.html?open=activities

Shodor Education Foundation Curriculum Materials - http://www.shodor.org/curriculum/

Tangrams - http://nlvm.usu.edu/en/nav/frames_asid_268_g_1_t_3.html?open=activities

Virtual clocks - http://nlvm.usu.edu/en/nav/frames_asid_317_g_1_t_4.html

Virtual geoboards - http://nlvm.usu.edu/en/nav/frames_asid_277_g_1_t_3.html?open=activities

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