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VIBRATION AND FORCE ANALYSIS OF LOWER ARM OF SUSPENSION SYSTEM

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To Mom and Dad

Who always picked me up on time

And encouraged me to go on every adventure

Especially this one

Abstract

This study describes the analysis of suspension system and its lower arm. These analyses were specifically applied on lower suspension arm to determine its vibration and stress behaviour during its operation. The suspension system is one of the most important components of vehicle, which directly affects the safety, performance, noise level and style of it. The main objectives are to have less stress on lower suspension arm system as well as to reduce imposed load by optimization.

The optimization of imposed load on lower suspension arm system is directly related to imposed force on the vehicle created by the road. This load has a direct effect on the value of imposed force on lower arm, where the lower arm force will be minimized by reducing the applied load. Hence, genetic algorithms for optimization and MATLAB optimization toolbar are used, as well as, specific M-file codes have been developed into MATLAB for optimization.

By determining optimized design values of suspension system for reducing road force, it is possible to survey vibration condition of lower arm according to frequency respond of suspension system and its natural frequency. Therefore, frequency response of its acceleration has been determined according to the whole mass of suspension system. Using FFT technique and making transfer function for frequency response of suspension system, will present responded frequency of suspension system which is using for vibration analysis.

For stress analysis, load condition of lower suspension arm system must be determined in advance. Hence, a typical model of McPherson suspension system has been selected for analysis. According to the road profile considered for analysis and the velocity of vehicle, it is possible to obtain both velocity and acceleration equations for whole components of McPherson suspension system. These values are used to determine dynamic force condition of lower arm suspension system during its operation. By using dynamic forces which are governing on lower arm of suspension system, in ABAQUS, the stress condition of lower arm can be determined during its operation.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Vibration is an important part in mechanical analysis systems, which can be found in every automotive engineering problem. Hence, it is obvious to have a better mastery over problems of mechanical systems; vibration analysis will be helpful to get precise solution for reducing both vibrations and forces. Vibration is a mechanical phenomenon whereby oscillations occur through an equilibrium point. It can be occurred in two ways, either, when a mechanical system is set off with an initial input and then allowed to vibrate freely that is called free vibration or when an alternating force or motion is applied to a mechanical system that is called force vibration [1–3].

Vibration force is the main factor in the analysis of moving vehicles, because, the forces which are imposed throughout the chassis by the road cause vibration. Suspension system of a vehicle has an important role in damping vibration of a vehicle. Due to the linkage between the tires and chassis, hence, every force which is imposed to the tires by the humps of the road will be transferred to the chassis. This fact has been made suspension system as an important part during vibration analysis of the vehicle [4,5].

During operation of suspension system, when the vehicle is moving, all parts of suspension system are vibrating with specific angular velocity and acceleration. This dynamic condition in suspension system causes dynamic forces which cause stress on different parts of suspension system, especially on lower arm of suspension system. In order to determine this stress on the lower arm, dynamic analysis must be done on suspension system [6]. It must be mentioned that each suspension system has contained different parts, where one of them is the lower arm which has important role in operation of suspension system.

There are many aspects of suspension system that can be analysed for reducing the vibration due to road forces, such as different types of optimized designing of a suspension system; however, it was focused in this thesis about vibration and force behavior of a lower drive arm of suspension system during its operation. Furthermore, the use of aluminium in automotive industry, specifically in different parts of suspension system, leads to analyse the behavior of the lower drive arm in order to qualify strength and vibration properties of it [4,7–9].

In general, this research has three main parts which are analytical analysis, finite element analysis and experimental analysis. Analytical analysis has been done in order to make an objective function for reducing dynamic forces on lower arm, as well as, for determining velocity, acceleration and dynamic forces of lower arm. The outcome of analytical analysis can be used for vibration analysis by using MATLAB, FFT and doing both stress and vibration analysis by using ABAQUS which is finite element software. Finally, it aims to validate determined values by finite element method by using some experimental methods for vibration analysis.

1.2 Problem definition

The rough surface road profiles and its influence on vehicle, causing unwilling vibrations due to kinematic excitations, have been made destructive problems on suspension system of vehicle. Respectively, every part of the suspension system will be affected by destructive created vibration. The lower drive arm of a suspension system is mainly influenced by this condition [10].

Moreover, Increasing in load condition on the lower arm of suspension system can change distribution of stress. Hence, these unpleasant vibration and force conditions will reduce life time of lower arm, by creating physical damage due to stress [11]. Therefore, vibration and force analysis of this part will be helpful to make a better operation condition for suspension system. The main problems due to vibration and load conditions on lower arm of suspension system can be defined as follow:

- In a moving vehicle, resonance and beating are not unavoidable in suspension system and its components because of some compatible frequencies. These phenomena are destructive in suspension system and its components and may cause to increase load on suspension system and respectively cause increasing stress.

- Important frequencies which are caused making resonance and beating can be listed as follow, natural frequency of suspension system, imposed frequency to the vehicle by the road and frequency response of acceleration of suspension system, as well as, natural frequency of suspension system which has great impact. Therefore, these frequencies cause vibration problems in suspension system and its lower arm.
- Imposed force to the vehicle by the road has important role in load condition of suspension system, specially, on its lower arm. Increasing in this load may cause to increase stress on lower arm. Hence, trying to reduce this force is considerable. The value of road force depends on specific conditions such as quality of the road, velocity and acceleration of vehicle. There are other conditions that can affect this force; these conditions are related to physical condition of suspension system and vehicle tire such as: suspension stiffness, suspension damping coefficient, tire stiffness and suspension system mass.
- Reaction forces on suspension system, also, play an important role in load and stress conditions of lower arm. These forces are imposing to lower arm from the chassis (the lower arm is fixed to the chassis) and from other parts which are connected to it. Figure 1.1 describes the problem conditions on lower arm of suspension system.

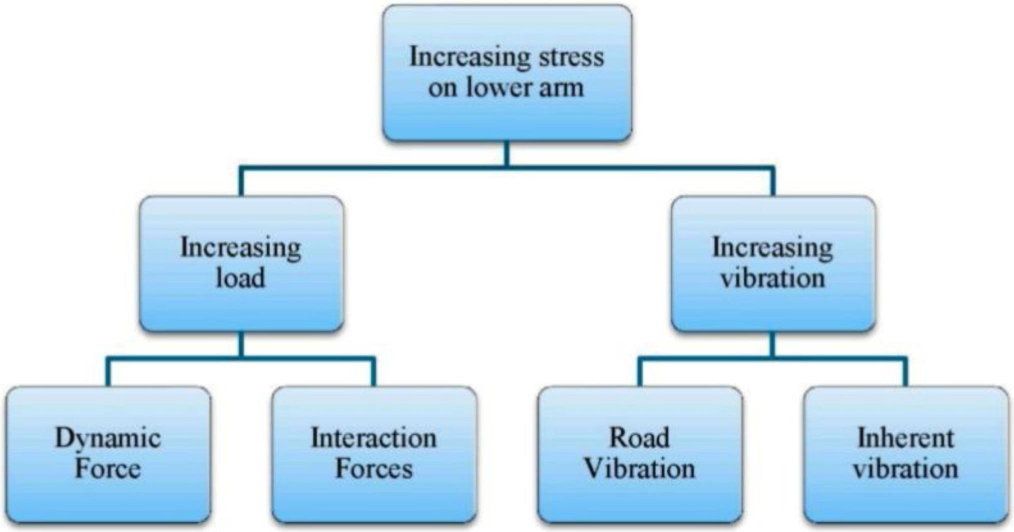


Figure 1.1: Problem condition of lower arm of suspension system

General problems of suspension system of a moving vehicle can be divided in two categories. The first one is related to problems of stress on different parts of suspension system and the second one is related to destructive vibration on suspension system.

1.3 Objectives

The main goal of this study is to reduce the stresses applied on lower arm suspension system. During the operation of suspension system, distribution of stresses reduce the life time of different parts, especially on the regions which have been fixed to the chassis or connected to other parts [12,13]. In order to catch this aim, different processes must be applied on suspension system and its lower arm, moreover, use of aluminum in manufacturing of lower arm imposes specific factors to our analysis. For instance, using of aluminium is decreasing amount of usage mass and respectively reducing natural frequency of this part. Hence, the following goals can be considered as main objectives of this study.

- Determining the objective function for minimizing imposed force on the suspension system by the road. This objective function must be comprised by different effective parameters of suspension system, tire of the vehicle and profile of the road.
- Vibration analysis of frequency response of suspension system is relating to different determined optimized values. In order to determine relation between road force and vibration resistance of suspension system, it must be focused on vibration of lower arm of suspension system.
- Applying mathematical modeling for a typical suspension system by a simplified model, all dimensions of real model must be kept constant. This model is used for applied velocity, acceleration in whole parts of suspension system and dynamic force analysis on lower arm of suspension system. Hence, dynamic analysis must be done on the whole parts of suspension system in advance.
- Stress analysis must be applied on lower arm of suspension system; therefore, determined forces of dynamic analysis must be used in ABAQUS. By this methos, optimum distribution of stress on lower arm during operation of suspension system can be obtained by using finite element methods.
- Vibration analysis must be done on lower arm of suspension system; hence, by using ABAQUS we can determine the natural frequency of the lower arm. Moreover, by modeling of suspension system, vibration behaviour of lower arm can be surveyed.
- Finally, in order to validate natural frequencies of lower arm of suspension system, specific experimental methods must be applied on lower arm. In this study, two different methods will be compared to each other.

1.4 Methodology

The methodology in this study can be divided in three main methods which are: analytical analysis, finite element analysis, by using related software, and finally experimental analysis. The main focus in this study has been on analytical and finite element analysis [9,14–18].

In order to reduce the amount of imposed force on the vehicle by the road, we have to optimize imposed force. Hence, an objective function according to different parameters of suspension system and road profile must be determined. At first step, according to the study aims, a proper road profile must be considered for the road. By assigning a proper vibration model to the vehicle and its suspension system, the equations of motion governing to vibration model can be obtained. Finally, by determining frequency response of the vibration model and using power spectral density of suspension system's frequency response, the objective function can be achieved [19][10].

According to determined objective function for minimizing optimization, genetic algorithm is used as an effective optimization method for minimizing imposed road force. There are different characters that can be optimized. Important factors in suspension system are unsprung mass, suspension stiffness, suspension damping coefficient and tire stiffness.

Optimized values are used for vibration analysis of lower arm; hence, frequency response of acceleration of unsprung mass has been used. The optimized values for reducing road force have been used for vibration analysis and for comparing these results with un-optimized values. Therefore, time domain of acceleration of unsprung mass has been determined. By using Fast Fourier Transform (FFT), vibration behavior of unsprung mass can be determined. The surveying of unsprung mass is related to lower arm of suspension system. In modeling for vehicle and its suspension system to a vibration models, mass of lower arm is defined as unsprung mass. Therefore, by analyzing unsprung mass, vibration behavior of lower arm can be determined [4,9,20,21].

In second step, stress analysis must be applied on lower arm of suspension system; hence, dynamic analysis must be applied on it. McPherson suspension system is selected as a typical suspension system for analysis. At first step, a simplified model of McPherson suspension system will be modeled according to the real dimensions of McPherson suspension system. Afterward, velocity and acceleration analysis will be applied on the whole parts of suspension system. By using these results, dynamic force analysis will be applied on lower arm of suspension system. The results obtained can be used to determine load condition of lower arm of suspension system [6,14,22,23].

By determining load forces, stress analysis on lower arm is possible. Finite element method is used for stress analysis. The stress behavior of lower arm during operation of suspension system can be determined by using ABAQUS and determining different steps for analysis such as part properties, load condition, boundary condition and mesh condition. Vibration analysis, in order to determine the natural frequency of lower arm, can be applied by ABAQUS. Finally, experimental analysis for validating natural frequency must be applied by using acceleration meters in order to determine displacement of free vibration of lower arm; catching data can be analyzed by FFT analysis [24].

Suspension system is used for minimizing annoying vibration of the road, as well as, for steering vehicle. The main problem in suspension system yields by destructive distribution of stress. An important part in suspension system is lower arm; in this study our focus is on lower arm. Hence, force and vibration conditions of lower arm will be surveyed, because vibration and load are two important factors that increase stress on lower arm.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

There are eight main areas of interest in literature that are studied in the following sections. These parts of interest are; suspension system, road profile, vibration model, frequency response, optimization, vibration analysis, dynamic analysis and finite element analysis. According to what has already been discussed, parametric objective function is critical for optimization of imposed force on the vehicle by the road. Hence, by making a model of vehicle and its suspension system, it is possible to determine frequency response of the system and make desired objective function for optimization in order to minimize the road force.

Vibration analysis on suspension system according to optimized value is not unavoidable. Finite element is a numerical technique for finding approximate solutions to boundary value problems for differential equations. This method can be used for determining distribution of stress on lower arm according to its load condition. Dynamic analysis is an effective way for determining velocity, acceleration and force in moving mechanical systems. Each of these topics will be presented in this chapter to explain the main methods and foundation of vibration and force analysis on lower arm of suspension system.

2.2 suspension system

Suspension is the term given to the system of springs, shock absorbers and linkages that connects a vehicle to its wheels and allows relative motion between the two parts. Suspension systems serve a dual purpose contributing to the vehicle's road holding, handling and braking for active safety and driving pleasure. In addition, Suspension systems are used for keeping vehicle occupants comfortable and reasonably well isolated from road noise, bumps, and vibrations. Figure 2.1 shows simplified model of suspension system and its different components [25,26].

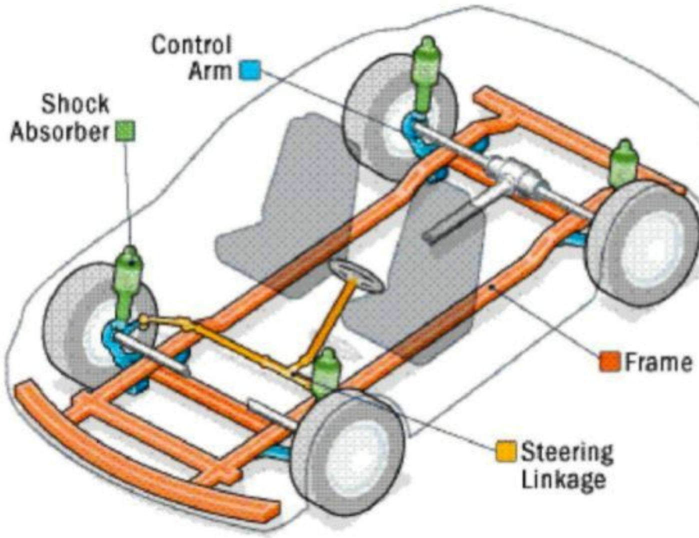


Figure 2.1: Suspension system and its components

These goals are generally at odds, so the tuning of suspensions involves finding the right compromise. It is important for the suspension to keep the road wheel in contact with the road surface as much as possible, because the road forces are acting on the vehicle through the contact patches of the tires. The main functions of suspension system are as follows:

- To protect vehicle from road shocks.
- To safeguard passengers from shocks.
- To prevent pitching or rolling.

According to the design, suspension system is mainly classified into two types, which are dependant suspension system and independent one. Regarding to dependant suspension system, a beam holds the wheels parallel to each other and perpendicular to the axle. When the camber of one wheel changes, the similarly camber of opposite wheel also changes. In contrary, independent suspension points to each wheel having its own suspension; it won't upset the wheel of the axle if the opposite wheel has been experienced. In other words, it can be mentioned that both the front and the rear wheel are utilized; when one wheel goes down, the other wheel does not have much effect. This study focuses on independent suspension systems; Figure 2.2 shows three different types of independent suspension system. Basic classifications of the design are divided to McPherson strut, double wishbone and multi link.

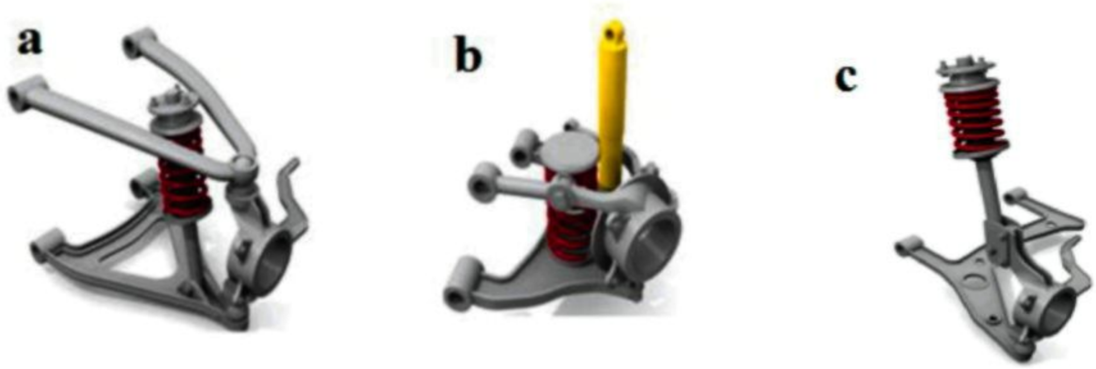


Figure 2.2: Different types of independent suspension system a) Double wishbone b) Multi linkages c) McPherson

Comfort and control aspects are majors in field of design and manufacturing. Spring and damper systems are used as shock absorber in automobiles. As concerned to comfort and control aspects, their systems are lagging to provide optimum level of performance. The geometry of suspension system does this optimum level of performance by providing automatic compensation that minimizes deviations caused by external forces [27,28]. The automobile chassis is generally mounted on the axle through springs. These springs are used to prevent the vehicle body from shock which refers to bounce, pitch and roll, etc. Due to these shock frames, body is affected by additional stresses which affect indirectly on rider to feel some discomfort.

In a moving vehicle, a proper suspension system must satisfy some duties which are considered to prevent vehicle from road shocks, to safeguard passengers from shocks and to prevent pitching or rolling. Springs are placed between the wheels and body; when wheels come across the bumps on road, it rises and deflects the spring. Thus, energy is stored and released when the spring rebounds due to its elasticity. Gradually the amplitude decreases due to the friction between spring and joints. Also, It must keep tires in contact with road.

2.3 Road profile

Profiles taken along a lateral line show the super elevation and crown of the road design, plus rutting and other distress. Longitudinal profiles show the design grade, roughness, and texture. The road profile is shown in Figure 2.3 [29,30].

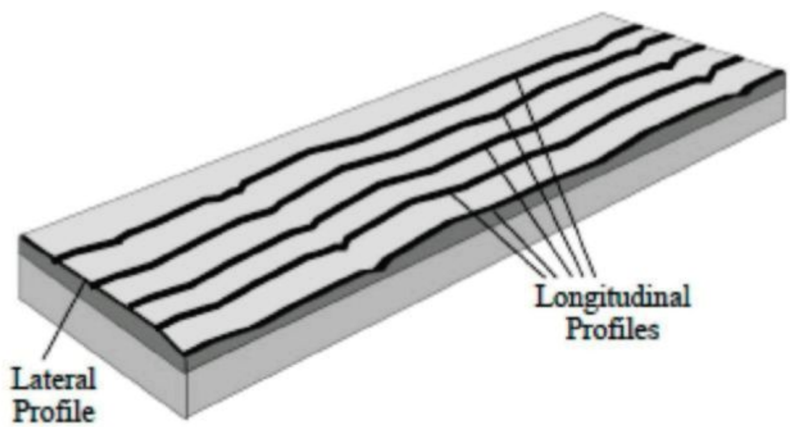


Figure 2.3: Lateral and longitudinal profile of road

In road analysis, the main aspect that must be considered is longitudinal condition of the road. Road classification has been based on the International Organization for Standardization (ISO 8606). The ISO has proposed road roughness classification using the power spectral density (PSD) [31,32]. It means that classification of the roads have been done according to their PSD. In general, a typical road is characterized by the existence of large isolated irregularities, such as potholes or bumps, which are superposed to smaller but continuously distributed profile irregularities. Usually for a vehicle, travelling over road profiles that are characterized by random fields must be considered. These random fields are real-valued, zero mean, stationary, and Gaussian. Therefore, for its complete statistical description, it is sufficient to specify their second-order moment. Hence, this requirement is fulfilled by assuming that the road irregularities possess a known single-sided power spectral density.

The road profile can be represented by a PSD function. The power spectral densities of roads show a characteristic drop in magnitude with the wave number. To determine the power spectral density function, or PSD, it is necessary to measure the surface profile with respect to a reference plane. The ISO has proposed road roughness classification using the power spectral density (PSD) values as shown in Table 2.1 and Table 2.2. Random road profiles can be approximated by a PSD in the form of:

$$\Phi(\Omega) = \Phi(\Omega_0)(\frac{\Omega}{\Omega_0})^{-\omega} \quad \text{Or} \quad \Phi(n) = \Phi(n_0)(\frac{n}{n_0})^{-\omega} \tag{2.1}$$

Where:

$\Omega = \frac{2\pi}{L}$ $in\ rad/m$, denotes the angular spatial frequency, L is the wavelength, $\Phi_0 = \Phi_0(\Omega_0)$ $in\ m^2/(rad/m)$ Describes the values of the PSD at the reference wave number $\Omega_0 = 1 rad/m$, $n = \frac{\Omega}{2\pi}$ is the spatial frequency $n_0 = 0.1 cycle/m$, ω is the waviness, for most of the road surface $\omega = 2$.

Table 2.1: Road roughness values classification by ISO			
Degree of roughness $\Phi(n_0)(10^{-6}\ m^2/(cycle/m))$ where $n_0 = 0.1 cycle/m$			
Road class	Lower limit	Geometric mean	Upper limit
A (very good)	_	16	32
B (good)	32	64	128
C (average)	128	256	512
D (poor)	512	1024	2048
E (very poor)	2048	4096	8192

For a rough and quick estimation of the roughness quality, the following guidance can be considered:

- New roadway layers, such as, for example, asphalt or concrete layers, can be assumed to have a good or even a very good roughness quality.
- Old roadway layers which are not maintained may be classified as having a medium roughness.
- Roadway layers consisting of cobblestones or similar material may be classified as medium (average) or bad (poor, very poor).

Table 2.2: Degree of roughness expressed in terms of Ω			
Degree of roughness $\Phi(\Omega_0)(10^{-6}\ m^2/(cycle/m))$ where $\Omega_0 = 0.1 cycle/m$			
Road class	Lower limit	Geometric mean	Upper limit
A (very good)	_	1	2
B (good)	2	4	8
C (average)	8	16	32
D (poor)	32	64	128
E (very poor)	128	256	512

There are two of the most commonly adopted methods, namely shaping filter and sinusoidal approximation, for generating one-dimensional random road profiles that are used in the simulation of a quarter car vehicle suspension system control. For the shaping filter method, the first order transfer function generating the road profile is independent of the grade of road. While for the sinusoidal approximation method, a detail derivation of the amplitude of each sinusoidal function is re-derived for completeness.

2.4 Vibration model

In general, there are two kinds of approaches in dynamic analysis of a vehicle. The first approach relies on experimental analysis and field test; the second one utilizes computer simulation to conduct a numerical analysis. The application of the first approach is not as expanded as the second approach; the first approach is known by its high cost and the required equipments are so expensive. In contrast, the second approach has been popular, because of its low cost and flexibility in testing different scenarios of a model. However, it must be mentioned that to have a computer analysis of a vehicle, the former of a vehicle must be available which can be expensive [4,33,34]. Hence, in order to use the second approach, a vehicle needs to be simplified to develop a vehicle model for simulating the real operation conditions. Based on the vehicle model, the dynamic response at any position in the vehicle can be approximated numerically. A vibration diagram for the whole body of the vehicle has been shown on the Figure 2.4.

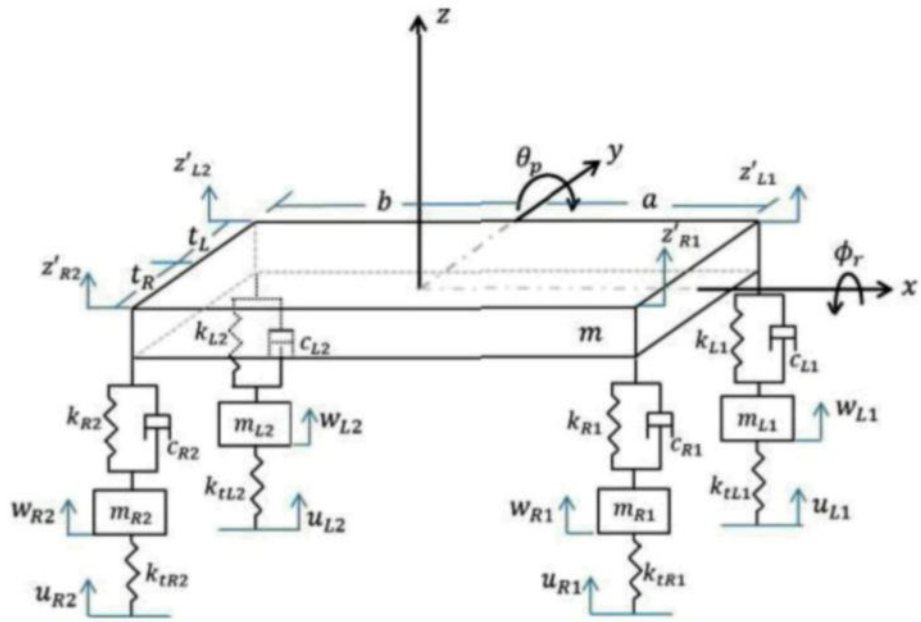


Figure 2.4: Vibration model of whole body of vehicle

It is noted that vibration system has ten degrees of freedom, which are three in translational and three in rotation, as well as, four degrees of freedom for the wheel masses. The last ones are shown in individual springs, since such a large number of degrees of freedom complicate the solution, the model is simplified to a half of vehicle [21][35].

Governing equations in suspension system can be used for formulating the desired parameters in suspension system. Hence, for studying the imposed force on the suspension system and respectively on lower arm, a proper system must be assigned to suspension system. The regular way for simulating suspension system has been done by vibration systems. In other words, a set of mass, dampers and spring collected as a vibration system for simulating the operation of suspension system.

These vibration models are usually used according to the desired properties of suspension system and its operation. For instance, degrees of freedom in the vibration model have direct influence on equations of motions. The other important factor has been related to the part of vehicle that has been assigned to vibrating system. It is possible to assign a vibrating model to whole body of vehicle, half of vehicle or quarter of vehicle. Figure 2.5 shows quarter and half vibration models which are popular in assigning to vehicle and its suspension system in analysis.

a b

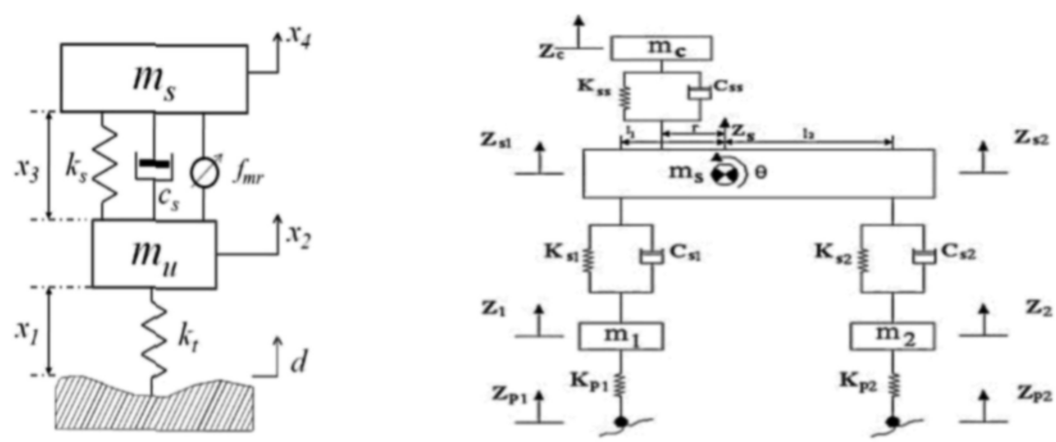


Figure 2.5: Two common vibrating model assigned to vehicle and its suspension system a) quarter model b) half model

The quarter vibration models with 2 degrees of freedom are commonly used for studying the vehicle and its suspension system. Such a quarter vehicle model is based on the following assumptions: constant vehicle velocity, no vehicle body or axle roll, rigid vehicle bodies, linear suspension and tire characteristics, point tire to road contact, and small pitch angles. By determining the proper vibration model for the vehicle, equations of motion for vibrating system must be determined; the desired objective function can be achieved by solving the determined equation [36,37].

The quarter vibration model m_s is the mass for quarter of vehicle body and m_u is considered as unsprung mass which is defined as mass of quarter of vehicle located under the vehicle body (such as suspension system, tire and etc). The general equation of motion for any vibration model assigned to vehicle can be defined as follow:

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}=\{P\} \tag{2.2}$$

Where: $[M]$ is mass matrix, $[C]$ is damping coefficient matrix, $[K]$ is stiffness matrix and $\{P\}$ is defined as imposed force on the vehicle by the road.

2.5 Frequency response

Frequency responses the quantitative measurement of the output spectrum of a system or device in response to a stimulus, and is used to characterise the dynamics of the system. It is a measure of magnitude and phase of the output as a function of frequency, in comparison to the input. In simple terms, if a sine wave is injected into a system at a given frequency, a linear system will respond at that same frequency with a certain magnitude and a certain phase angle relative to the input. Furthermore, for a linear system, double the amplitude of the input leads to double the amplitude of the output. In addition, if the system is time-invariant, then the frequency response also will not vary with time [38].

Transfer function can be used for presenting frequency response of the system. It is an effective way for determining response of the system to its input, as well as, it is a common way for vibration analysis. In other words, it is a mathematical representation, in terms of spatial or temporal frequency, of the relation between the input and output of a linear time-invariant system with zero initial conditions and zero-point equilibrium. In order to make a transfer function of a system, a mathematical transformation employed to transform signals between time (or spatial) domain and frequency domain. There are different ways for applying this approach. These methods are classified to traditional such as Fourier Transform and modern methods such as Pseudo excitation method [19].

2.5.1 Pseudo excitation method

In the design of Long-span bridges, the spatial effects of earthquakes, including the wave passage effect, the incoherence effect, and the local site effect, must be taken into account. The random vibration method can fully account for the statistical nature as well as the spatial effects of earthquakes; so it has been widely regarded as a very promising method. Unfortunately, the very low computational efficiency has become the bottle-neck of its practical use [39,40].

In the last 30 years, a great number of civil engineering projects, dams and long-spam bridges, have been carried out in China; many of these projects are located in earthquake regions. Over the last 20 years, a very efficient method, known as the pseudo-excitation method (PEM) to cope with the above computational difficulty, has been developed. This method can easily compute the 3D random seismic responses of long-span bridges using finite element models. It can be used with up to thousands of degrees of freedom on a small personal computer, in which the seismic spatial effect is accounted for accurately. This method is now being applied and developed in China by a great number of scholars [41].

The method is also well introduced by a whole chapter in another work, namely Vibration and Shock Handbook, published by CRC press (US) in 2005. Owing to its extensive applications in the Chinese civil engineering industry, the PEM has become an important part of random vibration courses taught in some Chinese universities and colleges.

It is considered a linear system subjected to a zero-mean stationary random excitation with a given PSD which is $S_{xx}(\omega)$. Suppose that for two arbitrarily selected responses $y(t)$ and $z(t)$, the auto-PSD $S_{yy}(\omega)$ and cross-PSD $S_{yz}(\omega)$ are desired. Considering that $H_y(\omega)$ and $H_z(\omega)$ are the corresponding frequency response functions and $x(t)$ is replaced by a sinusoidal form as follows:

$$\tilde{x} = \sqrt{S_{xx}(\omega)}\exp(i\omega t) \tag{2.3}$$

Consequently, the responses of $y(t)$ and $z(t)$ would be respectively $\tilde{y} = \sqrt{S_{xx}(\omega)}H_y(\omega)\exp(i\omega t)$ and $\tilde{z} = \sqrt{S_{xx}(\omega)}H_z(\omega)\exp(i\omega t)$. It can be readily verified that

$$\begin{aligned} \tilde{y}^*\tilde{y} &= \sqrt{S_{xx}(\omega)}H_y^*(\omega)\exp(-i\omega t).\sqrt{S_{xx}(\omega)}H_y(\omega)\exp(i\omega t) \\ &= \left|H_y(\omega)\right|^2 S_{xx}(\omega) = S_{yy}(\omega) \end{aligned} \tag{2.4}$$

$$\begin{aligned} \tilde{y}^*\tilde{z} &= \sqrt{S_{xx}(\omega)}H_y^*(\omega)\exp(-i\omega t).\sqrt{S_{xx}(\omega)}H_z(\omega)\exp(i\omega t) \\ &- H_y^*(\omega)S_{xx}(\omega)H_z(\omega) - S_{yz}(\omega) \end{aligned} \tag{2.5}$$

If $y(t)$ and $z(t)$ are two arbitrarily selected random response vectors of the structure, and $\tilde{y} = a_y \exp(i\omega t)$ and $\tilde{z} = a_z \exp(i\omega t)$ are the corresponding harmonic response vectors due to the pseudo excitation, it can also be proved that the PSD matrices of $y(t)$ and $z(t)$ are

$$S_{yy}(\omega) = \tilde{y}^*\tilde{y}^T = a_y^* a_y^T \tag{2.6}$$

$$S_{yz}(\omega) = \tilde{y}^*\tilde{z}^T = a_y^* a_z^T \tag{2.7}$$

This means that the auto- and cross-PSD function of two arbitrarily selected random responses can be computed using the corresponding pseudo harmonic responses. Consider that a linear structure is subjected to a number of stationary random excitations, which are denoted as an m dimensional stationary random process vector $x(t)$ with PSD matrix $S_{xx}(\omega)$. So, it is a Hermitian matrix and consequently it can be decomposed by using its Eigen pairs ψ_j and d_j, into

$$S_{xx}(\omega) = \sum_{j=1}^{r} d_j \psi_j^* \psi_j^T \quad (j = 1,2,...,r)\ (r \le m) \tag{2.8}$$

Where: r is the rank of $S_{xx}(\omega)$, next, constitute r pseudo harmonic excitations

$$\tilde{x}_j(t) = \sqrt{d_j}\psi_j \exp(i\omega t) \quad (j = 1, 2, \ldots, r) \tag{2.9}$$

By applying each of these pseudo harmonic excitations, two arbitrarily selected response vectors $y_j(t)$ and $z_j(t)$ of the structure, which can be displacements, internal forces or other linear responses, may be easily obtained and expressed as follows:

$$\tilde{y}_j(t) = a_{yj}(\omega)\exp(i\omega t) \tag{2.10}$$

$$\tilde{z}_j(t) = a_{zj}(\omega)\exp(i\omega t) \tag{2.11}$$

The corresponding PSD matrices can be computed by means of the following formulas:

$$S_{yy}(\omega) = \sum_{j=1}^{r} \tilde{y}_j^*(t)\tilde{y}_j^T(t) = \sum_{j=1}^{r} a_{yj}^*(\omega)a_{yj}^T(\omega) \tag{2.12}$$

$$S_{yz}(\omega) = \sum_{j=1}^{r} \tilde{y}_j^*(t)\tilde{z}_j^T(t) = \sum_{j=1}^{r} a_{yj}^*(\omega)a_{zj}^T(\omega) \tag{2.13}$$

The way used to decompose $S_{xx}(\omega)$ is not unique. In fact, the Cholesky scheme is perhaps the most efficient and convenient way to do it. $S_{xx}(\omega)$ is decomposed as follows:

$$S_{xx}(\omega) = L^* D L^T = \sum_{j=1}^{r} d_j l_j^* l_j^T \quad (r \le m) \tag{2.14}$$

Consider a linear system subjected to an evolutionary random excitation as follows:

$$f(t) = g(t)x(t) \tag{2.15}$$

In which $g(t)$ is a slowly varying modulation function, while $x(t)$ is a zero-mean stationary random process with auto-PSD $S_{xx}(\omega)$. The deterministic function $g(t)$ and

$S_{xx}(\omega)$ are both assumed to be given. In order to compute the PSD function of various linear responses due to action of $f(t)$, the pseudo excitation has the following form.

$$\tilde{f}(\omega,t) = g(t)\sqrt{S_{xx}(\omega)}\exp(i\omega t) \tag{2.16}$$

Suppose that $y(t)$ and $z(t)$ are two arbitrarily selected response vectors and $\tilde{y}(\omega,t)$ and $\tilde{z}(\omega,t)$ are the corresponding transient responses due to the pseudo excitation $f(\omega,t)$ with the structure initially at rest. It has been proved the following equations.

$$S_{yy}(\omega,t) = \tilde{y}^{*}(\omega,t)\tilde{y}^{T}(\omega,t) \tag{2.17}$$

$$S_{yz}(\omega,t) = \tilde{y}^{*}(\omega,t)\tilde{z}^{T}(\omega,t) \tag{2.18}$$

For cases with fully coherent excitations, partially coherent excitations, and non-uniformly modulated evolutionary random excitations, the corresponding pseudo-excitation algorithms are very similar to those for the stationary random excitation cases.

2.6 Optimization

Most increasing in the tire load will cause to increase stress of the suspension system during its operation. It would certainly benefit if the dynamic load generated by vehicle vibration can be reduced to a minimum value. Consequently, a method for optimum vehicle suspension design using dynamic tire load as a design criterion must be developed. The goal of optimization processes is to achieve the best possible load force by the road under various conditions. The procedure involves three major tasks; first to choose what measure should be minimized to best depict the problem under study. The next task is to decide which parameters are allowed to vary during the optimization. Finally, it has been to decide what constraints must be satisfied in order to avoid trivial solutions to the problem [21,42–44].

In mathematics, computer science, or management science, mathematical optimization (alternatively, optimization or mathematical programming) is the selection of a best element (with regard to some criteria) from some set of available alternatives. For the simple case, the optimization problem represents in maximizing or minimizing a real function by systematically selecting input values from available set and by computing the value of the function. The generalization of optimization theory and techniques to other formulations comprises a large area of applied mathematics. More generally, optimization includes finding "best available" values of specific objectives function given a defined domain (or a set of constraints), including a variety of different types of objective functions and different types of domains.

There are different methods for optimization which are based on the types of the problem and its complexity. These methods can be listed as: steepest descent, Newton and quasi Newton methods, Lagrange methods and Penalty and Barrier Methods, moreover, genetic algorithm which is an effective method for optimization. In the computer science field of artificial intelligence, genetic algorithm is a search heuristic that mimics the process of natural selection. This heuristic is routinely used to generate useful solutions to optimize and search problems. Genetic algorithms belong to the larger class of evolutionary algorithms which generate solutions to optimize problems using techniques inspired by natural evolution, such as inheritance, mutation, selection, and crossover [20,45,46].

Genetic Algorithms (GA) are direct, parallel, stochastic methods for global search and optimization, which imitates the evolution of the living beings, described by Charles Darwin. GA is part of the group of Evolutionary Algorithms (EA); GA is a subset of evolutionary algorithms that model and mimic biological processes to find optimal solutions of highly complex problems.

The evolutionary algorithms use the three main principles of the natural evolution; reproduction, natural selection and diversity of the species are maintained by the differences of each generation with the previous. Genetic Algorithm treats with a set of individuals, representing possible solutions of the task. The selection principle is applied by using a criterion, giving an evaluation for the individual with respect to the desired solution. The best-suited individuals create the next generation. The large variety of problems in the engineering sphere, as well as in other fields, requires the usage of algorithms from different types, with different characteristics and settings.

2.6.1 Genetic algorithm

Major component of GA is including chromosomes, selection, recombination and mutation operators; these elements are briefly explained by this section in the contest of the suspension system optimization.

- **Chromosomes**

During the division process of the human cells the chromatin (contained in the nucleus and built from DNA (deoxyribonucleic acid), proteins and RNA (ribonucleic acid)) become shorter and thicker and forms spiral strings – chromosomes. Chromosomes are the genes that carry inherited cell information. Every gene codes particular protein and is known as independent factor of the genetic information, which determines the appearance of different peculiarities.

For the genetic algorithms, the chromosomes represent set of genes, which code the independent variables. Every chromosome represents a solution of the given problem. Individual and vector of variables will be used as other words for chromosomes. In other hand, the genes could be Boolean, integers, floating point or string variables, as well as any combination of the above sets. A set of different chromosomes (individuals) forms a generation. By means of evolutionary operators, as selection, recombination and mutation, an offspring population is created.

- **Selection**

In the nature, the selection of individuals is performed by survival of the fittest. The more individual one is adapted to the environment; the bigger are its chances to survive and create an offspring and thus transfer its genes to the next population. In EA the selection of the best individuals is based on an evaluation of fitness function or fitness functions.

Examples for such fitness function are the sum of the square error between the required system response and the real one; it is known as the distance of the poles of the closed-loop system to the desired poles, etc. If the optimization problem is minimized, the individuals with small value of the fitness function will have bigger chances for recombination and respectively for generating offspring.

- **Recombination**

The first step in the reproduction process is the recombination (crossover). The genes of the parents are used to form an entirely new chromosome. The typical recombination for the GA is an operation requiring two parents; schemes with more parents' area also possible. Two of the most widely used algorithms are Conventional (Scattered) Crossover and Blending (Intermediate) Crossover.

- Conventional (scattered) Crossover
- Blending (intermediate) crossover

- **Mutation**

The new created means of selection and crossover population can be further applied to mutation. Mutation means, that some elements of the DNA are changed. Those changes are caused mainly by mistakes during the copy process of the parent's genes. In the terms of GA, mutation means random change of the value of a gene in the population.

2.6.2 Outline of genetic algorithm

The GA holds a population of individuals (chromosomes), which evolve means of selection and other operators like crossover and mutation. Every individual in the population gets an evaluation of its adaptation (fitness) to the environment. In the terms of optimization, this means that the function is maximized or minimized which is evaluated for every individual. The selection of the best gene combinations (individuals), which through crossover and mutation, should drive to better solutions in the next population. The procedure of operation genetic algorithm is as follow:

- Generate initial population in most of the algorithms; the first generation is randomly generated by selecting the genes of the chromosomes among the allowed alphabet for the gene. Due to the easier computational procedure, it is accepted that all populations have the same number (N) of individuals.
- Calculation of the values of the function that required to be minimized.
- Check for termination of the algorithm.
- Selection between all individuals in the current population, what will continue and by means of crossover and mutation will produce offspring population.
- Crossover, the individuals selected by recombine with each other and new individuals will be created. The aim is to get offspring individuals that inherit the best possible combination of the characteristics (genes) of their parents.
- Mutation, by means of random change of some of the genes, it is guaranteed that even if none of the individuals contain the necessary gene value for the minimum, it is still possible to reach the minimum.
- New generation, the elite individuals selected are combined with those passed the crossover and mutation, and form the next generation.

2.7 Vibration analysis

The mechanical and mathematical model of suspension systems is usually simplified as a multiple-mass and complicated vibration system. Due to road roughness, suspension system may come into complicated vibration, which is disadvantageous to its components such as lower arm. Therefore, it is important and necessary to control the suspension system's vibration within a limited grade in order to ensure proper operation of lower arm, safety steering and physical health of drivers and passengers, as well as the operating stability of man-vehicle-road system [47].

Regarding to vehicles movement, the random and changeable of road surface are the main factors to induce vehicle vibration. Therefore, investigation of stochastic vibration induced to suspension system by road roughness has been a significant problem of suspension system design and its performance simulation. In order to satisfy this problem, the Fourier transform analysis must be used to investigate the dynamic characteristics of vibrating problems of suspension system based on stationary random vibration theory. After completion of a vibration model for lower arm of McPherson suspension system, it is important to derive the frequency characteristic of lower arm vibration responses. It is also necessary to establish power spectrum density function of road excitation and lower arm vibration responses.

Vibration analysis can be used to analyze the influence of lower arm structural parameters and road excitation on lower arm random vibration. Although this method was relatively simple, its derivation process is too complicated. Hence, this method needs not only to derive the frequency response characteristics of lower arm of suspension system, but also to derive the frequency response characteristics of suspension system vibration response values [48,49].

However, in some circumstances, vehicles are running in changeable speeds, such as in accelerating starting period and decelerating stopping period. The road excitation and the lower arm of suspension system dynamic response in time domain are non-stationary. The stochastic vibration analysis method, based on Fourier transform and its inverse transform, has been used to study the changeable speed response of lower arm of suspension system; its computation work is enormous.

The pseudo excitation method can be used to analyze the stochastic vibration of structural systems. By pseudo excitation method, stochastic vibration analysis can carried on lower arm of suspension system vibration; the vibration response of lower arm with constant vehicle speed can be investigated to a stationary random road excitation. In this study, the time-space frequency relationship of lower arm vibration under variable speeds can be derived by pseudo excitation method. Consequently, the equation of transient power spectrum density of lower arm vibration response can be obtained [39–41].

The use of effective methods for monitoring the condition of mechanical systems and their component parts are essential prerequisites to the adoption of on-condition maintenance. The developments of monitoring techniques are currently considerable interest, especially in areas where machine availability is vital. These techniques also can be used for monitoring a mechanical system in experimental condition to predict condition of the system in real situation. Hence, monitoring can be used to detect performance deterioration, damage or incipient failure, to diagnose the source of the problem and to predict its sequence of development.

Data sources for condition monitoring include direct physical inspection, non-destructive material inspection techniques, examination of lubricating oil and oil borne wear debris, and the analysis of dynamic values of various parameters generated while the machine is in operation. Of the latter, the most commonly used parameter is machine vibration, although noise, torque and other parameters can be used when they provide significant data. Vibration analysis has been extensively used for evaluating the condition of every vibrating systems for instance lower arm of suspension system. Spectrum analysis using the Fourier Transform analyser has virtually become a standard technique for this purpose.

Vibration amplitude may be measured as a displacement, a velocity, or acceleration. Vibration amplitude measurements may either be relative or absolute. The absolute vibration measurement is relative to free space. Absolute vibration measurements are made with seismic vibration transducers. Seismic vibration transducers include swing coil velocity transducers, accelerometers and voltmeters. The relative vibration measurement is relative to a fixed point on the machine. Relative vibration measurements are generally limited to displacement measurements. Accelerometers can be used as practical sensors in determining the vibration measurement in mechanical systems. They are typically placed at key locations on the measuring system which are directly affected by imposed force.

2.8 Dynamic analysis

Vector analysis mostly has been used to express dynamic behavior of mechanical systems, as well as, suspension system. This method can develop the understanding of suspension operation and its effects on total vehicle performance. The calculations for dynamic analysis will include a series of analyses including:

- Velocity analysis
- Acceleration analysis
- Dynamic force analysis

Most practical vehicles have some form of suspension, particularly when there are four or more wheels. The suspension system in general must reduce the vertical wheel load variations which are imposed to the wheel by bumps of the road. However, the introduction of a suspension system introduces some tasks of its own; each additional interface and component brings some specific load condition for suspension system during its operation. These three categories are considered as an important load condition of suspension system that has been encountered: wheel load variation, handling load and component loading environment [6,50,51].

In order to determine the wheel load variation in a vehicle, it is possible to assign specific stiffness to each wheel and suspension of each tire and model vehicle as sprung and un-sprung mass to determine the load. It must be mentioned that the calculated load completely depends on the profile of the load. For handling force, the distribution of loads between sprung and un-sprung load paths have an important role. Therefore an anti-pitch angle will be defined in order to reduce the load between sprung and un-sprung in different manoeuvres of the vehicle. In the third part, component loading, each part of the suspension system due to both speed and acceleration of the vehicle and imposed forces will have interactions to each other. These interactions cause forces on each joints.

In order to analyze the lower arm of suspension system by finite element software, its load condition must be determined. Hence, imposed forces on the lower arm created by the other parts of the suspension system must be determined. Figure 2.6 shows McPherson suspension system. All of components of McPherson suspension system have been showed as well.

Figure 2.6: McPherson suspension system

The imposed forces on lower arm are due to interaction of other parts with lower arm. Hence the first step in force analysis has been related to both velocity and acceleration analysis on suspension system. For velocity analysis, a starting point will be set on contact point between the tire and the road in order to establish a boundary condition. The position, velocity and acceleration of starting point, respectively depends on the profile of the road and velocity and acceleration of the vehicle.

The usual method for analysis component of suspension system is vector analysis used to determine their velocity, acceleration and dynamic forces. For proceeding with velocity and acceleration analysis, it is necessary to identify the unknowns that define the problem and the same number of equations as unknowns leading to solution. The angular velocity will be assigned to the each rigid body of suspension system and a rotating velocity in the attachment of lower arm to the chassis of the vehicle.

There is also the same assumption for acceleration analysis of suspension system. By determining velocity and acceleration of lower arm and other components of suspension system, dynamic force analysis of suspension system is not unavoidable. Specifically for the lower arm of suspension system, six equations of motions will be set up. The dynamic analysis depends on the physical properties of suspension components like mass, mass moments of inertia and center of mass location.

2.9 Finite element analysis

Finite element analysis is a powerful numerical procedure used to get information about designed components that would be difficult, if not impossible to be determined analytically. In order to perform finite element analysis for every part, important information must be provided as the geometry of the part to be analyzed and the material properties of the part to be analyzed. Properties of part for finite element analysis can be listed as: elastic modulus, shear modulus, Poisson's ratio and the type for the material of the part for instance being homogenous.

Modeling for FEA requires a thorough understanding and accurate representation of the part to be analyzed. Accurate modeling is not easily to be done, particularly where loading and boundary conditions are concerned. The geometry of the part is divided into thousands of little pieces called "elements"; the vertex of every element is called a node.

Inside the software, there are equations called shape functions that tell the software how to vary the values of x across the element [52–54].

Average values of x are determined at the nodes. In fact, the place that access to values of stress and/or deflections has been possible only into the nodes. The finer the "mesh" of elements, the more accurate nodal values will be. In addition to supply the software by several kinds of loads imposed on the part and by type of material the part made of, it must supply the software how the part resists the loads imposed on it. It should to recall well that every loads acting on an object has an equal and opposite load acting on it. For example, a round cantilevered beam subjected to twisting will resist the external twisting moment with equal and opposite twisting at the wall. However, the way that the FEA modeller would communicate with this information to the FEA software would be through the use of "boundary conditions." Boundary conditions tell the FEA software how loading is resisted by constraining displacements and rotations of certain nodes.

2.10 Conclusion

It is going to do vibration and stress analysis on lower arm of suspension system. Hence, specific type of fields and subjects must be surveyed for this matter; moreover, by using related methods, desired goals can be achieved. Literature review has focused at first, on making an objective function for minimization of imposed force on the vehicle by the road. Hence, profile of the road and different effective conditions of road profile, different vibration models for assigning to the vehicle and its suspension system, and different possibilities for making an objective function for optimization, have been studied. Therefore, sinusoidal methods for road profile, 3d quarter vibration model, and road force criteria according to frequency response of suspension system have been selected.

In order to proceed in optimization for minimization of imposed force to the vehicle by the road, different possibilities for optimization have surveyed and genetic optimization has been selected. The goals for minimizing of road force can be used for vibration analysis of suspension system and its lower arm. These optimized values can be used for comparing frequency response of suspension system with non optimized values. It must be mentioned that for determining frequency response, Fast Fourier Transform and Pseudo Excitation methods can be effective.

Finally, for stress analysis on lower arm of suspension system, the load condition must be determined; it is common to use vector analysis for determining load condition of every dynamic system by determining velocity and acceleration and by using these two, determining dynamic loads. The most effective way of analysis for determining stress and strain, especially in complicated problem, is finite element analysis. Using finite element software is really effective method; hence, Using ABAQUS, as finite element software can be really effective. The stress analysis on lower arm of suspension system can be around using ABAQUS.

CHAPTER 3

MATHEMATIC OBJECTIVE FUNCTION AND OPTIMIZATION

3.1 Introduction

In order to reduce stress on lower arm of suspension system, reaction force by other components of suspension system on lower arm must be reduced. The reaction forces are directly related to force road which is imposed to the vehicle and suspension system by bumps of the road. Hence, in order to have minimum stress on lower arm of suspension system, the imposed force to the vehicle by the road must be in minimum value.

In first step, imposed force of the vehicle must be optimized to minimum value. Hence, an objective function according to frequency response of suspension system, profile of the road and mechanical properties of suspension system must be created. Therefore, a proper road profile must be assigned to the road and a vibration model must be assigned to the vehicle and its suspension system. Combination of road profile, frequency response and power spectral density method will lead to make an objective function for optimization. By using MATLAB optimization toolbar, the imposed force by the road can be minimized.

3.2 Road profile

The unevenness degree of road profile can be generally described by power spectrum density. As mentioned before, the power spectrum density of road profile is described by:

$$S_q(n) = S_q(n_0)(\frac{n}{n_0})^{-\omega} \tag{3.1}$$

Where $S_q(n_0)$, is the unevenness coefficient of road profile; n_0 is the referenced spatial frequency, $n_0 = 0.1(m^{-1})$, n is the spatial frequency (m^{-1}), ω is the frequency exponent of the graded road spectrum and generally selected as 2. The amount of $S_q(n_0)$ can be considered as grade B of the road condition which is $S_q(n_0) = 64\times10^{-6}\,(m^2/m^{-1})$.

When automobiles move in changeable speeds, the excitations of automobile systems are different in time domain and space domain. It is not stationary in space domain but in time domain. However, the automobile's mechanical responses are non-stationary. Using the inherent characteristics of frequency response function $H(\omega)$ of vehicle system in time domain, the relation of time frequency ω and space frequency n, the transient frequency response function $H(s,n)$ can be obtained. Consequently, it can solve the stochastic vibration of the vehicle system in changeable speed moving. The unevenness degree of roads in time domain is shown as follows:

$$q(t) = h_0 e^{j\omega t} \tag{3.2}$$

Where h_0, is the amplitude of unevenness degree of roads and the expression of unevenness degree of roads in space domain is as follows:

$$q = h_0 e^{j\Omega s} \tag{3.3}$$

$$\omega t = \Omega \upsilon \tag{3.4}$$

Where Ω, is the spatial angular frequency and when the automobile is moving in a constant speed $s = \upsilon t$, it has the following relation, $\omega = \Omega s$ or $f = n\upsilon$. When the automobile is moving in a changeable speed, it has

$$s = \upsilon_0 + \frac{at^2}{2} \tag{3.5}$$

Where υ_0 is the initial velocity of the automobile, and a is its acceleration, then (3.4) can be rewritten as

$$\omega\, dt = \Omega ds$$

$$\omega = \Omega \frac{ds}{dt} = 2n\pi(\upsilon_0 + at) = 2n\pi(2as + \upsilon_0^2)^{1/2} \tag{3.6}$$

Equation (3.6) reflects the time-space frequency relation of automobiles in an accelerated moving.

In general, a typical road is characterized by the existence of large isolated irregularities, such as potholes or bumps, which are superposed to smaller but continuously distributed profile irregularities. For studying the road profile in general, the latter type of road irregularities is considered. This section deals with the estimation of the second-order moment response characteristics of the vehicle models presented in the previous section.

Traveling over road profiles are characterized by random fields. These random fields are real-valued, zero mean, stationary, and Gaussian. Therefore, for their complete statistical description, it is sufficient to specify their second-order moment. This requirement is fulfilled by assuming that the road irregularities possess a known single-sided power spectral density, $S_q(\Omega)$, where $\Omega = 2\pi/\lambda$ and λ is a spatial frequency, corresponding to a harmonic irregularity with wavelength. The geometrical profile of typical roads fit accurately the following simple analytical form:

$$S_q(\Omega) = A_q\Omega^{-n} \tag{3.7}$$

In this way, the amplitude ratio of the roughness between two different road profiles is proportional to the square root ratio of the respective A_q values. Moreover, it is frequently quite accurate and analytically convenient to select the value n=2 for the exponent in equation (3.7).

It was mentioned that wheel hop during moving of vehicle causes nonlinearity. Hence, for nonlinear vehicle models, samples of the road profile are generated using the spectral representation method. More specifically, if the vehicle is assumed to travel with a constant horizontal speed v_0 over a given road, the forcing resulting from the road irregularities can be simulated by the following series

$$x_q(t)=\sum_{n=1}^{N} s_n \sin\left(n\omega_0 t+\varphi_n\right) \tag{3.8}$$

In the previous equation, the amplitudes $s_n=\sqrt{2s_q(n\Delta\Omega)\Delta\Omega}$ of the excitation harmonics are evaluated from the road spectra selected. Where $\Delta\Omega=2\pi/L$ and L is the length of the road segment considered. Moreover, the value of the fundamental temporal frequency ω_0 is determined from the following relation, $\omega_0=\frac{2\pi}{L}v_0$

The phases φ_n are treated as random variables, following a uniform distribution in the interval range of 0 to π. Then, the response of the vehicle to each sample road profile is computed by integrating the equations of motion.

Unevenness degree of road profile can be generally described by power spectrum density. As mentioned before, the power spectrum density of road profile is described by:

$$S_q(n)=S_q(n_0)\left(\frac{n}{n_0}\right)^{-\omega} \tag{3.9}$$

3.3 Vibration model

For symmetry and simplification, a quarter model of a passenger vehicle has been selected for modeling suspension system and body of the vehicle. During operation of a vehicle, three different forces can affect the main body of vehicle by making rotation angles around the axes of fixed coordinate system. These rotation angles are pitch angle which is α around X axe, roll angle which is β around Y axe and finally yaw angle which is γ around Z axe. These angles have been shown in Figure 3.1.

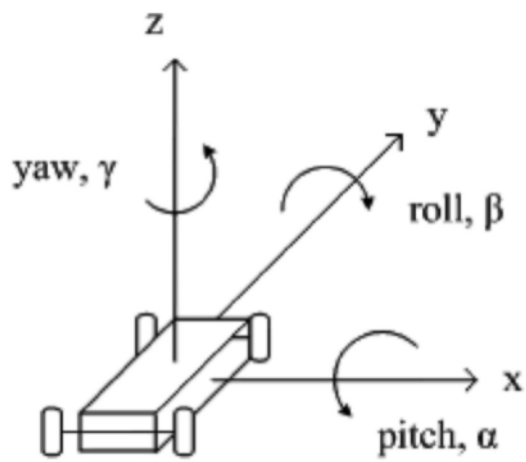


Figure 3.1: Rotation angles of vehicles

The yaw angle in a vehicle analysis can be considered as zero, because this kind of rotation angle does not occur in moving vehicles. Another important movement of the main body of the vehicle is along direction Z. Hence, our modeling system has four degree of freedom by considering 3 degrees of freedom for the quarter of whole body as sprung mass and 1 degree of freedom which is movement of un-sprung mass along Z direction. The other masses under suspension system and mass of suspension system can be considered as unsprung mass. Quarter vibration model of the vehicle and its suspension system has been depicted in Figure 3.2.

It must be mentioned normally that when quarter of vehicle gets modeled, the quarter vibration model will be in 2d, because of simplification. However, the model, in this case, is selected as quarter 3d vibration model, in order to have real condition of operation of vehicle and its suspension system. The spring and damping arm attached to the sprung mass has a ball joint; this ball joint gives an ability to sprung mass in 3d vibration model to rotate against x and y axis.

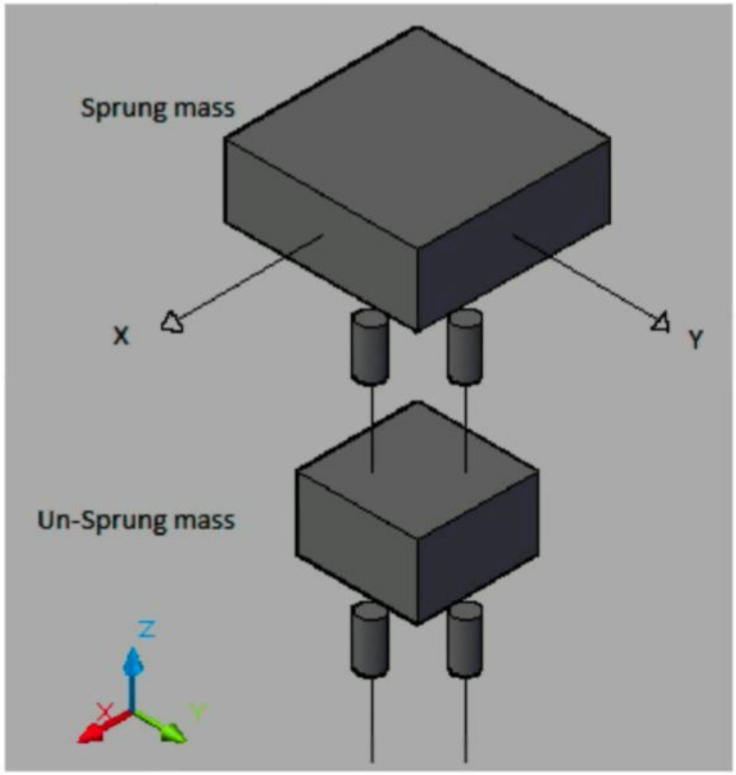


Figure 3.2: Quarter vibration model of vehicle

Relating to the equations of motion which are governed on the quarter vibration model of the vehicle, the following equations of motion have been distracted:

$$\begin{aligned}
&M_s\ddot{z}_s + K_s\left(z_s - a\theta_s + b\phi_s - z_u\right) + c_s\left(\dot{z}_s - a\dot{\theta}_s + b\dot{\phi}_s - \dot{z}_u\right) = 0 \\
&M_u\ddot{z}_u - K_s\left(z_s - a\theta_s + b\phi_s - z_u\right) - c_s\left(\dot{z}_s - a\dot{\theta}_s + b\dot{\phi}_s - \dot{z}_u\right) + K_t\left(z_u - Q\right) + C_t\left(\dot{z}_u - \dot{Q}\right) = 0 \\
&I_y\ddot{\theta}_s - aK_s\left(z_s - a\theta_s + b\phi_s - z_u\right) + ac_s\left(\dot{z}_s - a\dot{\theta}_s + b\dot{\phi}_s - \dot{z}_u\right) = 0 \\
&I_x\ddot{\phi}_s + bK_s\left(z_s - a\theta_s + b\phi_s - z_u\right) - bc_s\left(\dot{z}_s - a\dot{\theta}_s + b\dot{\phi}_s - \dot{z}_u\right) = 0
\end{aligned} \tag{3.10}$$

For multi degrees of freedom system, the equations of motions are expressed in matrix form as follows:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\} \tag{3.11}$$

If the equations of (3.10) are re-written in the form of equation (3.11) depending on the displacement vector,

$$\{x(t)\} = \begin{Bmatrix} z_s \\ z_u \\ \theta_s \\ \phi_s \end{Bmatrix} \tag{3.12}$$

then mass, stiffness, damping matrices, and force vector are written as follow:

$$[M] = \begin{bmatrix} M_s & 0 & 0 & 0 \\ 0 & M_u & 0 & 0 \\ 0 & 0 & I_y & 0 \\ 0 & 0 & 0 & I_x \end{bmatrix} \tag{3.13}$$

$$[C] = \begin{bmatrix} c_s & -c_s & -ac_s & bc_s \\ -c_s & c_s + c_t & ac_s & -bc_s \\ ac_s & -ac_s & -a^2c_s & abc_s \\ -bc_s & bc_s & abc_s & -b^2c_s \end{bmatrix} \tag{3.14}$$

$$[K] = \begin{bmatrix} k_s & -k_s & -ak_s & bk_s \\ -k_s & k_s + k_t & ak_s & -bk_s \\ -ak_s & ak_s & a^2k_s & -abk_s \\ bk_s & -bk_s & -abk_s & b^2k_s \end{bmatrix} \tag{3.15}$$

$$\{F(t)\} = \begin{bmatrix} 0 \\ -k_t Q \\ 0 \\ 0 \end{bmatrix} \tag{3.16}$$

3.4 Objective function

According to the pseudo excitation rules and its application in simulating the profile of the road, a pseudo excitation of road is as follows:

$$\tilde{q}(t) = \sqrt{S_q(n)}\, e^{i\omega t} \tag{3.17}$$

Therefore, the excitation input is written as follows:

$$\{\tilde{q}(t)\} = \{H_q(\omega)\}\tilde{q}(t) \tag{3.18}$$

For a multi degree of freedom system, its frequency response characteristic is the complex number ratio of response vector and excitation vector. For the quarter-car, four degree of freedom vehicle system is considered in this study; if supposed that its frequency response is $[H(\omega)]$, then the relation of pseudo response and pseudo excitation is as follows:

$$\{\tilde{z}(t)\} = [H(\omega)]\{\tilde{q}(t)\} \tag{3.19}$$

Substituting (3.18) into (3.19) gives the following relation.

$$\{\tilde{z}(t)\} = [H(\omega)]\{H_q(\omega)\}\tilde{q}(t) = \{h_g(\omega)\}\tilde{q}(t) \tag{3.20}$$

Since $\{h_g(\omega)\} = [H(\omega)]\{H_q(\omega)\}$, then the following relations are obtained.

$$\{\dot{\tilde{z}}(t)\} = \{h_g(\omega)\}\dot{\tilde{q}}(t) = i\omega\{h_g(\omega)\}\tilde{q}(t) \tag{3.21}$$

$$\{\ddot{\tilde{z}}(t)\} = -\omega^2\{h_g(\omega)\}\tilde{q}(t) \tag{3.22}$$

Substitute (3.21) and (3.22) into the system equation, then the system frequency response function can be obtained as follows:

$$[H(\omega)] = \left[[K] - \omega^2[M] + i\omega[C]\right]^{-1} F \tag{3.23}$$

Where

$$[H(\omega)]=\begin{bmatrix} H(\omega)_s \\ H(\omega)_u \\ H(\omega)_p \\ H(\omega)_r \end{bmatrix} \tag{3.24}$$

By substituting equations (3.13), (3.14), and (3.15) on the equation (3.23), the frequency response of the quarter-car will be determined as:

$$[H(\omega)]=\begin{bmatrix} \begin{bmatrix} k_s & -k_s & -ak_s & bk_s \\ -k_s & k_s+k_t & ak_s & -bk_s \\ -ak_s & ak_s & a^2k_s & -abk_s \\ bk_s & -bk_s & -abk_s & b^2k_s \end{bmatrix} - \omega^2 \begin{bmatrix} M_s & 0 & 0 & 0 \\ 0 & M_u & 0 & 0 \\ 0 & 0 & I_y & 0 \\ 0 & 0 & 0 & I_x \end{bmatrix} + \ldots \\ i\omega \begin{bmatrix} c_s & -c_s & -ac_s & bc_s \\ -c_s & c_s+c_t & ac_s & -bc_s \\ ac_s & -ac_s & -a^2c_s & abc_s \\ -bc_s & bc_s & abc_s & -b^2c_s \end{bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -k_tQ \\ 0 \\ 0 \end{bmatrix} \tag{3.25}$$

$$[H(\omega)]=\begin{bmatrix} k_s-\omega^2M_s+i\omega c_s & -k_s-i\omega c_s & -ak_s-i\omega ac_s & bk_s+i\omega bc_s \\ -k_s-i\omega c_s & k_s+k_t-\omega^2M_u+i\omega(c_s+c_t) & ak_s+i\omega ac_s & -bk_s-i\omega bc_s \\ -ak_s+i\omega ac_s & ak_s-i\omega ac_s & a^2k_s-\omega^2I_y-i\omega a^2c_s & -abk_s+i\omega abc_s \\ bk_s-i\omega bc_s & -bk_s+i\omega bc_s & -abk_s+i\omega abc_s & b^2k_s-\omega^2I_x-i\omega b^2c_s \end{bmatrix}^{-1} \ldots \begin{bmatrix} 0 \\ -k_tQ \\ 0 \\ 0 \end{bmatrix} \tag{3.26}$$

In order to determine the frequency matrix response of the quarter-vehicle model, the previous matrix must be inversed and be multiple to the matrix of the road excitation. As it is obvious, the matrix, that must be inversed, has 4 rows and 4 columns. Hence, its inversed matrix is very complicated with much of terms. For simplicity, it should to assign each element of the above matrix to its array addresses. Therefore, the following relations are got:

$$\begin{bmatrix} k_s-\omega^2M_s+i\omega c_s & -k_s-i\omega c_s & -ak_s-i\omega ac_s & bk_s+i\omega bc_s \\ -k_s-i\omega c_s & k_s+k_t-\omega^2M_u-i\omega(c_s+c_t) & ak_s+i\omega ac_s & -bk_s-i\omega bc_s \\ -ak_s+i\omega ac_s & ak_s-i\omega ac_s & a^2k_s-\omega^2I_y-i\omega a^2c_s & -abk_s+i\omega abc_s \\ bk_s-i\omega bc_s & -bk_s+i\omega bc_s & -abk_s+i\omega abc_s & b^2k_s-\omega^2I_x-i\omega b^2c_s \end{bmatrix}^{-1} = \tag{3.27}$$

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}^{-1}$$

By replacing equation (3.25) in the equation (3.24), the matrix of quarter-vehicle model will determine as follows:

$$[H(\omega)]=\begin{bmatrix} H(\omega)_s \\ H(\omega)_u \\ H(\omega)_p \\ H(\omega)_r \end{bmatrix} \tag{3.28}$$

Frequency response for the sprung mass has been determined as:

$$H(\omega)_s=\frac{H_{11}}{H'_{11}}\times k_tQ \tag{3.29}$$

Which H_{11} and H'_{11} are defined as:

$$H_{11}=A_{12}\times A_{33}\times A_{44}-A_{12}\times A_{34}\times A_{43}-A_{32}\times A_{13}\times A_{44}+A_{32}\times A_{14}\times A_{43}+A_{42}\times A_{13}\times A_{34}-A_{42}\times A_{14}\times A_{33}$$

$$\begin{aligned} H'_{11}=\;&A_{11}\times A_{22}\times A_{33}\times A_{44}-A_{11}\times A_{22}\times A_{34}\times A_{43}-A_{11}\times A_{32}\times A_{23}\times A_{44}+A_{11}\times A_{32}\times A_{24}\times A_{43} \\ &+A_{11}\times A_{42}\times A_{23}\times A_{34}-A_{11}\times A_{42}\times A_{24}\times A_{33}-A_{21}\times A_{12}\times A_{33}\times A_{44}+A_{21}\times A_{12}\times A_{34}\times A_{43} \\ &+A_{21}\times A_{32}\times A_{13}\times A_{44}-A_{21}\times A_{32}\times A_{14}\times A_{43}-A_{21}\times A_{42}\times A_{13}\times A_{34}+A_{21}\times A_{42}\times A_{14}\times A_{33} \\ &+A_{31}\times A_{12}\times A_{23}\times A_{44}-A_{31}\times A_{12}\times A_{24}\times A_{43}-A_{31}\times A_{22}\times A_{13}\times A_{44}+A_{31}\times A_{22}\times A_{14}\times A_{43} \\ &+A_{31}\times A_{42}\times A_{13}\times A_{24}-A_{31}\times A_{42}\times A_{14}\times A_{23}-A_{41}\times A_{12}\times A_{23}\times A_{34}+A_{41}\times A_{12}\times A_{24}\times A_{33} \\ &+A_{41}\times A_{22}\times A_{13}\times A_{34}-A_{41}\times A_{22}\times A_{14}\times A_{33}-A_{41}\times A_{32}\times A_{13}\times A_{24}+A_{41}\times A_{32}\times A_{14}\times A_{23} \end{aligned}$$

Frequency response for unsprung mass has been determined as follows:

$$H(\omega)_u = -\frac{H_{21}}{H'_{21}} \times k_t Q \tag{3.30}$$

Where H_{21} and H'_{21} are defined as:

$$H_{21} = A_{11} \times A_{33} \times A_{44} - A_{11} \times A_{34} \times A_{43} - A_{31} \times A_{13} \times A_{44} + A_{31} \times A_{14} \times A_{43} + A_{41} \times A_{13} \times A_{34} - A_{41} \times A_{14} \times A_{33}$$

$$\begin{aligned} H'_{21} = {} & A_{11} \times A_{22} \times A_{33} \times A_{44} - A_{11} \times A_{22} \times A_{34} \times A_{43} - A_{11} \times A_{32} \times A_{23} \times A_{44} + A_{11} \times A_{32} \times A_{24} \times A_{43} \\ & + A_{11} \times A_{42} \times A_{23} \times A_{34} - A_{11} \times A_{42} \times A_{24} \times A_{33} - A_{21} \times A_{12} \times A_{33} \times A_{44} + A_{21} \times A_{12} \times A_{34} \times A_{43} \\ & + A_{21} \times A_{32} \times A_{13} \times A_{44} - A_{21} \times A_{32} \times A_{14} \times A_{43} - A_{21} \times A_{42} \times A_{13} \times A_{34} + A_{21} \times A_{42} \times A_{14} \times A_{33} \\ & + A_{31} \times A_{12} \times A_{23} \times A_{44} - A_{31} \times A_{12} \times A_{24} \times A_{43} - A_{31} \times A_{22} \times A_{13} \times A_{44} + A_{31} \times A_{22} \times A_{14} \times A_{43} \\ & + A_{31} \times A_{42} \times A_{13} \times A_{24} - A_{31} \times A_{42} \times A_{14} \times A_{23} - A_{41} \times A_{12} \times A_{23} \times A_{34} + A_{41} \times A_{12} \times A_{24} \times A_{33} \\ & + A_{41} \times A_{22} \times A_{13} \times A_{34} - A_{41} \times A_{22} \times A_{14} \times A_{33} - A_{41} \times A_{32} \times A_{13} \times A_{24} + A_{41} \times A_{32} \times A_{14} \times A_{23} \end{aligned}$$

As same as the above frequency response, the frequency response of the rotation against x and y axes of coordinate system can be determined.

The frequency response functions of the relative displacement of suspension and the dynamic loads of tires are as follows, respectively:

$$H_{sd}(\omega) = H_s - l_1 H_p - l_2 H_r - H_u \tag{3.31}$$

$$H_{tf}(\omega) = [H_u(\omega)](k_t + ic_t\omega) \tag{3.32}$$

Where, l_1 and l_2 are distances between the mass center of the sprung mass and the axes of the wheel. Due to the symmetry and geometry of our modeling in quarter car model (Figure 3.2), it is assumed that the mass center of the quarter-car model is completely under the axis of the wheel. Hence, the amount of l_1 and l_2 must be considered as zero; by rewriting the equation (3.25), the following frequency response for deflection of the suspension system will be obtained:

$$H_{sd}(\omega) = H_s - H_u \tag{3.33}$$

According to random vibration theory, it will be possible to determine the power spectrum density of the deflection of suspension system and imposed force to the wheels of the vehicle.

$$S_{sd} = \left|H_{sd}(\omega)\right|^2 S_q(\omega) \tag{3.34}$$

$$S_{tf}(\omega) = \left|H_{tf}(\omega)\right|^2 S_q(\omega) \tag{3.35}$$

In order to reduce the force imposed to the tire, the expression σ_{tf}^2 is considered as the variance of dynamic vehicle load. By optimizing the vehicle dynamic force for reducing the wheel load, the amount of stress on lower arm can be reduced. Hence, getting the following relation:

$$\sigma_{tf}^2 = \int_{-\infty}^{\infty} S_{tf}(\omega)\,d\omega \tag{3.36}$$

For calculating the above integral, in order to have an objective function for optimization, it should discrete the integral. If we consider the ω_1 as lower limit of integration and ω_n as upper limit of integration, then, for n data points $(\omega_1, S_{tf}(\omega_1))$, $(\omega_2, S_{tf}(\omega_2))$, $(\omega_3, S_{tf}(\omega_3))$, ... , $(\omega_n, S_{tf}(\omega_n))$, where, ω_1, ω_2, ... , ω_n are in an ascending order, the approximate value of the integral $\sigma_{tf}^2 = \int_{\omega_1}^{\omega_n} S_{tf}(\omega)\,d\omega$ is given by the followings:

$$\sigma_{tf}^2 = \int_{\omega_1}^{\omega_n} S_{tf}(\omega)d\omega = \int_{\omega_1}^{\omega_2} S_{tf}(\omega)d\omega + \int_{\omega_2}^{\omega_3} S_{tf}(\omega)d\omega + \ldots + \int_{\omega_{n-1}}^{\omega_n} S_{if}(\omega)d\omega \approx$$

$$\approx (\omega_2 - \omega_1)\frac{S_{tf}(\omega_1) + S_{tf}(\omega_2)}{2} + (\omega_3 - \omega_2)\frac{S_{tf}(\omega_2) + S_{tf}(\omega_3)}{2} + \ldots + (\omega_n - \omega_{n-1})\frac{S_{tf}(\omega_{n-1}) + S_{tf}(\omega_n)}{2}$$

This approach uses the trapezoidal rule in the intervals $\left[\omega_1, \omega_2\right], \left[\omega_2, \omega_3\right], \ldots, \left[\omega_{n-1}, \omega_n\right]$ and then adds the obtained values. Finally, in order to reduce the dynamic force on lower arm, the above equation according to the desire variables must be optimized to get minimized.

3.5 Optimization by using genetic algorithm

There are different types of optimization method that have been used for solving problems. Choosing these methods is related to the nature of the problem and the number of variables that must be optimized. An important factor, for selecting a method, has been related to precision of answers. It means that according to the accuracy of method, we will have precise answers.

In order to optimize a problem with more than two variables and due to the relation between variables, the values of variables can be changed or got wrong during the calculation. However, for problem with less complexity, using different types of optimization methods and using simple method with less details, can determine the optimized values of the problem.

The importance of Genetic Algorithm is depicted when a problem with more than two variables is occurred; this method and its procedure make a situation for determining precise amount for each variable for minimizing the problem. The objective function of our problem has been determined in equation (3.36) aiming to optimize for minimization this function on MATLAB optimization toolbar by using Genetic Algorithm. Before that, the objective function must be discrete and all constant variables and control variables must be defined. Hence, for generating our objective function, three types of M-File have been generated in MATLAB. Table (3.1) depicts related code for generating general form of objective function.

Table 3.1: General form of objective function with constant and shape variables

```
%% Unsprungmass_Force
% Determining the objective function of quarter model of vehicl in order to
% optimizing wheel force which is imposed to the veehicle by the road
function [A11, A12, A13, A14, A21, A22, A23, A24, A31,...
         A32,A33, A34,A41, A42, A43, A44, H1, h1, Sq]...
         = WheelForce(Ks, Kt, w, Cs, Mu)
 syms Ks Kt w Cs Mu
a = 1;
b = 0.5;
Ix = 208.3;
Iy = 177.1;
Ms = 500;
n = 2;
no = 0.1;
So = 64 * 10^-6;
Ct = 0;
A11 = Ks - w^2 * Ms + i * w * Cs;
A12 = -Ks - i * w * Cs;
A13 = -a * Ks - i * w * a * Cs;
A14 = b * Ks + i * w * b * Cs;
A21 = -Ks - i * w * Cs;
A22 = Ks + Kt - w^2 * Mu + i * w * (Cs + Ct)  ;
A23 = a * Ks + i * w * a * Cs;
A24 = -b * Ks - i * w * b * Cs;
A31 = -a * Ks + i * w * a * Cs;
A32 = a * Ks - i * w * a * Cs;
A33 = a^2 * Ks - w^2 * Iy - i * w * a^2 * Cs;
A34 = - a * b * Ks + i * w * a * b * Cs;
A41 = b * Ks - i * w * b * Cs;
A42 = -b * Ks + i * w * b * Cs;
A43 = -a * b * Ks + i * w * a * b * Cs;
A44 = b^2 * Ks - w^2 * Ix - i * w * b^2 * Cs;
H1 = A11 * A33 * A44 - A11 * A34 * A43 - A31 * A13 * A44...
    + A31 * A14 * A43 + A41 * A13 * A34 - A41 * A14 * A33;
h1 = A11 * A22 * A33 * A44 - A11 * A22 * A34 * A43- A11 * A32 * A23 * A44...
    + A11 * A32 * A24 * A43+ A11 * A42 * A23 * A34 - A11 * A42 * A24 * A33...
    - A21 * A12 * A33 * A44 + A21 * A12 * A34 * A43+ A31 * A12 * A23 * A44...
    - A31 * A12 * A24 * A43- A31 * A22 * A13 * A44 + A31 * A22 * A14 * A43...
    + A31 * A42 * A13 * A24 - A31 * A42 * A14 * A23- A41 * A12 * A23 * A34...
    + A41 * A12 * A24 * A33 + A41 * A22 * A13 * A34 - A41 * A22 * A13 *
A34...
    - A41 * A22 * A14 * A33 - A41 * A32 * A13 * A24...
    + A41 * A32 * A14 * A23;
% Determining power spectral density of the road. The road profile has been
% dependant on the quality of the road.
Sq = So * ( no / n )^w;
```

According to the Table (3.1), there are five variables which are important for us. It is shown that after generating discrete function, the optimize amount of each variable will be determined according to the variation range of frequency of imposed force by the road. Desired variable which are used as shape variables are: K_s: stiffness of suspension system, C_s: damping coefficient of suspension system, K_t: Stiffness of vehicle's wheel, M_u: unsprung mass and ω : frequency of imposed force by the road. The constant variables shown in the Table 3.2 are determined according to the real properties of vehicle and the quality of the road. For this specific example power spectrum density of the road, S_0 is considered for a road with good surface quality.

It should to determine the dimension of quarter model of vehicle, its mass and some other related parameters. In order to generate our final equation, the ingredient of Table 3.1 has been used in another code which is depicted on Table 3.2.

Table 3.2: Generating discrete form of objective function

```
%% main program
 syms Ks Kt Cs Mu w
 % constants
 Vo = 20;
L = 200;
Ct = 0;
K = 0;
n = 2;
 [A11, A12, A13, A14, A21, A22, A23, A24, A31,A32,A33, A34,A41, A42,...
 A43, A44, H1, h1, Sq] = WheelForce(Ks, Kt, w, Cs, Mu);
 B = 0;
 for t = 0.1:0.5:5
 % Determining the value of the irregularities of the road in order to
get Q
 Q=0;
        for f = 1:1:100
            p = Sq * sin(n * (((2*pi)/L)*Vo) * t + rand(1)*(2*pi));
            p = p+Q;
            Q = p;
        end
% Determining the frequency response of the Un-sprung mass of the quarter
% model of the vehicle
H = - ( H1 / h1) * Kt * Q;
 % Frequency response of the imposed force to the wheel
Htf = H * (Kt + i * Ct * w);
 % The following part must be getting integrad in order to specify the
% variance of the imposed force to the wheel of the vehicle
Stf = (abs(Htf))^2 * Sq;
n = 0.1;
m = 0.11;
for v = 0.1:0.5:15
    for w = n:0.1:m
    K = K + Stf;
    end
        n = m;
        m = m + 0.01;
        if m == 15.01
        end
end
B = B + K;
end
B
```

After making these two codes in MATLAB, the condition is proper to make our objective function; Table 3.3 depicts the objective function of our suspension system for reducing imposed force of the road. It must be mentioned that the frequency in this function is considered as a shape variable, but, in our calculation it will assign a specific amount to it and will optimize other variable. The reason for this approach is related to frequency of road force during moving of vehicle. The range of road frequency is changed from 0 to 20 hertz. Moreover, all of these frequencies can be occurred to the suspension system during moving vehicle.

Table 3.3: Objective function for optimizing imposed force to the vehicle by the road

```
T = (434714609901358511939722179605868337884615316129672l7*(1/20)^w*...
    abs((1/20)^w*((Ks/2 - (Cs*w*i)/2)^2*(Ks + Cs*w*i) - (Ks/2 - ...
    (Cs*w*i)/2)^2*(Cs*w*i - 500*w^2 + Ks) + (Ks/2 - (Cs*w*i)/2)*...
    (Ks/2 + (Cs*w*i)/2)*((1771*w^2)/10 + Cs*w*i - Ks) + ((1771*w^2)/10...
    + Cs*w*i - Ks)*((2083*w^2)/10 + Cs*w*(i/4) - Ks/4)*...
    (Cs*w*i - 500*w^2 + Ks) + (Ks - Cs*w*i)*(Ks + Cs*w*i)*...
    ((2083*w^2)/10 + Cs*w*(i/4) - Ks/4) + (Ks/2 - (Cs*w*i)/2)*...
    (Ks/2 + (Cs*w*i)/2)*(Ks - Cs*w*i)))^2*abs(Kt)^4)/...
    (21467531516653651510048140829603516632793088000000000000000000*abs...
    ((Ks/2 + Cs*w*(-i/2))^2*(Ks + Cs*w*i)*(Ks + Cs*w*i - 500*w^2) - ...
    (Ks/2 + Cs*w*(-i/2))^2*(Ks + Cs*w*i - 500*w^2)*(Ks + Kt +...
    Cs*w*i - Mu*w^2) - (Ks + Cs*w*i)^2*(Cs*w*i - Ks +...
    (1771*w^2)/10)*(Cs*w*(i/4) - Ks/4 + (2083*w^2)/10) - ...
    (Ks + Cs*w*(-i))*(Ks + Cs*w*i)^2*(Cs*w*(i/4) - Ks/4 +...
    (2083*w^2)/10) + (Ks/2 + Cs*w*(-i/2))*(Ks/2 + Cs*w*(i/2))*....
    (Ks + Cs*w*(-i))*(Ks + Kt + Cs*w*i - Mu*w^2) - (Ks/2 +...
    Cs*w*(-i/2))*(Ks/2 + Cs*w*(i/2))*(Ks + Cs*w*(-i))*(Ks + Cs*w*i)...
    + (Ks + Cs*w*(-i))*(Ks + Cs*w*i)*(Cs*w*(i/4) - Ks/4 + ...
    (2083*w^2)/10)*(Ks + Cs*w*i - 500*w^2) + (Ks/2 + ...
    Cs*w*(-i/2))*(Ks/2 + Cs*w*(i/2))*(Ks + Cs*w*(-i))*...
    (Ks + Cs*w*i - 500*w^2) + (Ks/2 + Cs*w*(-i/2))*...
    (Ks/2 + Cs*w*(i/2))*(Cs*w*i - Ks + (1771*w^2)/10)*...
    (Ks + Kt + Cs*w*i - Mu*w^2) + (Cs*w*i - Ks + (1771*w^2)/10)*...
    (Cs*w*(i/4) - Ks/4 + (2083*w^2)/10)*(Ks + Cs*w*i - 500*w^2)*...
    (Ks + Kt + Cs*w*i - Mu*w^2) - (Ks/2 + Cs*w*(-i/2))*...
    (Ks/2 + Cs*w*(i/2))*(Ks + Cs*w*i)*(Cs*w*i - Ks + (1771*w^2)/10) +...
    (Ks + Cs*w*(-i))*(Ks + Cs*w*i)*(Cs*w*(i/4) - Ks/4 +(2083*w^2)/10)*...
    (Ks + Kt + Cs*w*i - Mu*w^2) + (Ks/2 + Cs*w*(-i/2))*...
    (Ks/2 + Cs*w*(i/2))*(Cs*w*i - Ks + (1771*w^2)/10)*...
    (Ks + Cs*w*i - 500*w^2))^2)
```

The last program can generate the required objective function for optimization. Therefore, MATLAB optimization toolbar can be used for minimizing imposed force by the road. This minimization provides different values of suspension system in order to have minimum dynamic force on lower arm of suspension system and respectively minimum amount of stress on it. Table 3.4 depicts the non-constant characteristics of suspension system that have been used for optimizing our system. It must be mentioned that these values have been selected for a typical city car.

Table 3.4: Non-constant characteristics of Quarter vibration model of suspension system		
Variable	Lower level	Upper level
Un-sprung Mass $M_u (kg)$	20	80
Suspension damping $C_s (kg/s)$	1000	1200
Suspension Stiffness $K_s (N/m)$	17000	20000
Wheel Stiffness $K_t (N/m)$	100000	130000

Genetic Algorithm has different stages that have been used for optimization. Different values must be assigned to it; in general, these values are selective and depend on required time for calculation and preciseness of calculations. Table 3.5 depicts different characteristics of Genetic Algorithm for optimizing our problem.

Table 3.5: Characteristics of Genetic Algorithm for optimization	
Population	1000
Fitness scaling	rank
Elite count	2
Mutation	Constraint dependent
Initial range	[0;1]
Selection	Stochastic uniform
Crossover fraction	0.8
Crossover	Scattered

Table 3.6 depicts the optimum values for stiffness of suspension system, damping coefficient of suspension system, un-sprung mass and stiffness of vehicle wheel. By

determining these values in suspension system, the data for different purposes can be used. For instance, focusing on vibration, it is possible to determine the frequency response of our system with these values and to compare them with non optimized ones. Moreover, it is possible to use these optimum values for determining the natural frequency of our system and to compare it to the natural frequency of lower arm of suspension system. It must be mentioned that, as mentioned before, the frequency value of imposed force to the vehicle by the road has been changed numerically from one to twenty; hence the objective function for some values between these limitless will be studied.

Table 3.6: Optimized values for quarter model of vehicle and its suspension system

$\omega(Hz)$	$M_u(kg)$	$C_s(kg/s)$	$K_s(N/m)$	$K_t(N/m)$	Values of function
1	20.5	1133.5	19818.1	107144	5.7E-8
2	25.7	1176.5	19778.4	101773.7	1.7E-9
3	30.1	1167	19912.8	100398	6E-12
4	21.25	1199.3	19537.1	107608.4	1.6E-14
5	20.3	1199.9	19919.7	100736.9	5.4E-17
6	20	1000	17000	100000	3E-21
7	20	1000	17000	100000	2.2E-25
8	20	1000	17000	100000	2.2E-29
9	20	1000	17000	100000	2.6E-33
10	20	1066	17000	100000.3	2.9E-38
11	20	1196	17000	100000	2.6E-41
12	20	1000	17000	100000	2.2E-45
13	20	1000	17000	100000	2.1E-49
14	20	1000	19170.8	100000	2.3E-53
15	20	1000	18508.6	100000	2.8E-57
16	20	1000	19711.6	100000	3.4E-61
17	20	1000	19988.3	100000	4.4E-65
18	20	1039.2	18272.4	100001	6.2E-69
19	20	1189.1	18855.2	100000.1	7.8E-73
20	20	1147.3	18992.8	100000	1E-76

The data declared in Table 3.6 are related to different frequencies of road force. These data are approximately raw; hence, it is possible to do some analysis on them in order to determine some logical results in optimization of suspension system.

3.6 Conclusion

In this section of work, optimization for minimizing of road imposed force to the vehicle is done. First step is about studying different condition and properties of road profile in order to assign a proper model for our calculation. This part has a great impact, because it affects the terms of road force objective function, as well as, dynamic analysis might affect imposed velocity and acceleration to the vehicle by the road.

In next step, vehicle and its suspension system has been modeled by using a vibration model. It is common to use a quarter vibration models for analysis; a three dimensional quarter vibration model has been selected by using creativity. The main reason for selecting a three dimensional model is having a real work condition of vehicle and its suspension system. It must be mentioned that, in vibration model, the connection of damper and spring to sprung mass has ball joint (in order to have desired movement).

The governing equations of motion for vibration model and using Pseudo excitation method, frequency respond of vehicle can be determined. In order to achieve an objective function for minimization of road force, power spectral density of the road force can be used. Moreover, a set of codes have been developed in MATLAB for generating a proper format of objective function for optimization. Finally, MATLAB optimization toolbar has been used for minimizing road force; this optimization is done for all range of road frequencies in order to get proper values for design variable of suspension system.

CHAPTER 4
DYNAMIC ANALYSIS

4.1 Introduction

The important step for determining stress condition of a part is related to determine its load condition. These load conditions even the part be either in a static or dynamic condition are important, moreover, the first step is for doing finite element analysis in order to determine the stress condition. Furthermore, determining static loads condition of a part has less complexity rather than its dynamic loads condition. Hence, the static load condition of lower arm of suspension system is determined, therefore, dynamic analysis has greater importance. Hence, vector analysis is used to determine dynamic behavior of lower arm of suspension system. The whole parts of suspension system must be considered for dynamic analysis as there is a significant relation between velocity and acceleration of lower arm with other parts.

Vector analysis can develop an understanding of suspension operation and its effects on total vehicle performance, especially on lower arm of suspension system. The example selected is based on a typical McPherson suspension system. The whole dimensions are according to the real dimensions of suspension system, but, the geometry of the parts is simplified.

The calculations will include a series of analyses including:

- Velocity analysis
- Acceleration analysis
- Dynamic force analysis

For doing stress analysis for every mechanical part which has rotation and motion, velocity and acceleration must be determined. Hence, for this simplified model of McPherson suspension system, both angular velocity and acceleration of different parts should be determined. However, before analysis McPherson suspension system, the vector analysis should be explained.

4.2 Velocity analysis

Consider the rigid body, shown in Figure 4.1; In this case, it is initially only interested in motion in the $x_1 y_1$ plane. The body moves and rotates through an angle $\delta\gamma$, measured in radians, around the Z_1 axis. The vector $\{R_{PQ}\}_1$ moves with the body to a new position $\{R_{P'Q'}\}_1$. The new vector $\{R_{P'Q'}\}_1$ is defined by the transformation as follows:

$$\{R_{P'Q'}\}_1 = [A]\{R_{PQ}\}_1 \tag{4.1}$$

Where $[A]$ is the rotation matrix that rotates $\{R_{PQ}\}_1$ onto $\{R_{P'Q'}\}_1$; it is expanding to the following:

$$\begin{bmatrix} P'Q'_x \\ P'Q'_y \\ P'Q'_z \end{bmatrix} = \begin{bmatrix} \cos\delta\gamma & -\sin\delta\gamma & 0 \\ \sin\delta\gamma & \cos\delta\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} PQ_x \\ PQ_y \\ PQ_z \end{bmatrix} \tag{4.2}$$

Assuming that $\delta\gamma$ is small so we can take $\cos\delta\gamma = 1$ and $\sin\delta\gamma = \delta\gamma$ leads to the following:

$$\begin{bmatrix} P'Q'_x \\ P'Q'_y \\ P'Q'_z \end{bmatrix} = \begin{bmatrix} 1 & -\delta\gamma & 0 \\ \delta\gamma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} PQ_x \\ PQ_y \\ PQ_z \end{bmatrix} \tag{4.3}$$

The change in relative position is as follows:

$$\delta\{R_{PQ}\}_1 = \{R_{P'Q'}\}_1 - \{R_{PQ}\}_1 \tag{4.4}$$

Or

$$\begin{bmatrix} \delta PQ_x \\ \delta PQ_y \\ \delta PQ_z \end{bmatrix} = \begin{bmatrix} 1 & -\delta\gamma & 0 \\ \delta\gamma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} PQ_x \\ PQ_y \\ PQ_z \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} PQ_x \\ PQ_y \\ PQ_z \end{bmatrix} \tag{4.5}$$

$$\begin{bmatrix} \delta PQ_x \\ \delta PQ_y \\ \delta PQ_z \end{bmatrix} = \begin{bmatrix} 0 & -\delta\gamma & 0 \\ \delta\gamma & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} PQ_x \\ PQ_y \\ PQ_z \end{bmatrix} \tag{4.6}$$

If this change takes place in time δt then:

$$\frac{\delta}{\delta t}\begin{bmatrix} \delta PQ_x \\ \delta PQ_y \\ \delta PQ_z \end{bmatrix} = \begin{bmatrix} 0 & -\delta\gamma / \delta t & 0 \\ \delta\gamma / \delta t & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} PQ_x \\ PQ_y \\ PQ_z \end{bmatrix} \tag{4.7}$$

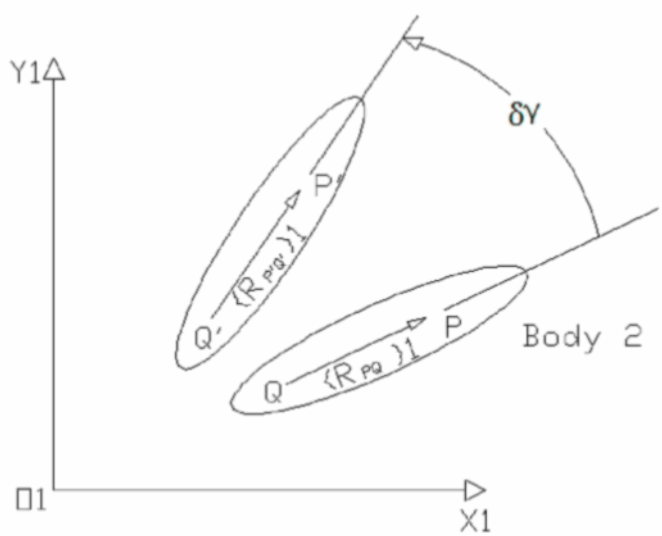


Figure 4.1: Motion of a vector attached to a rigid body

In the limit for δt approaches zero, the following relation can be obtained.

$$\frac{\delta}{dt}\begin{bmatrix}\delta PQ_x\\ \delta PQ_y\\ \delta PQ_z\end{bmatrix}=\begin{bmatrix}0 & -\delta\gamma/dt & 0\\ \delta\gamma/dt & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}PQ_x\\ PQ_y\\ PQ_z\end{bmatrix} \tag{4.8}$$

The previous relation can be rearranged as follows:

$$\begin{bmatrix}V_{PQ_x}\\ V_{PQ_y}\\ V_{PQ_z}\end{bmatrix}=\begin{bmatrix}0 & -\omega_z & 0\\ \omega_z & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}PQ_x\\ PQ_y\\ PQ_z\end{bmatrix}$$

(4.9)

Note that generally rotations cannot be represented as vector quantities unless they are very small, as in finite element programs. Hence angular velocities obtained by differencing rotations over very small time intervals, are in fact vector quantities. If the rigid link also undergoes by small rotations δ_α about the X-axis and δ_β about the Y-axis, then the full expression is as follows:

$$\begin{bmatrix}V_{PQ_x}\\ V_{PQ_y}\\ V_{PQ_z}\end{bmatrix}=\begin{bmatrix}0 & -\omega_z & \omega_y\\ \omega_z & 0 & -\omega_x\\ -\omega_y & \omega_x & 0\end{bmatrix}\begin{bmatrix}PQ_x\\ PQ_y\\ PQ_z\end{bmatrix} \tag{4.10}$$

Note that matrix is the skew-symmetric from of the angular velocity vector $[\omega x_4 \; \omega y_4 \; \omega z_4]$. In general terms, the following relation is obtained.

$$\{V_{PQ}\}_1=\{\omega_2\}_1\times\{R_{PQ}\}_1 \tag{4.11}$$

The direction of the relative velocity vector $\{V_{PQ}\}_1$ is perpendicular to the line of the relative vector $\{R_{PQ}\}_1$ as shown in Figure 4.2.

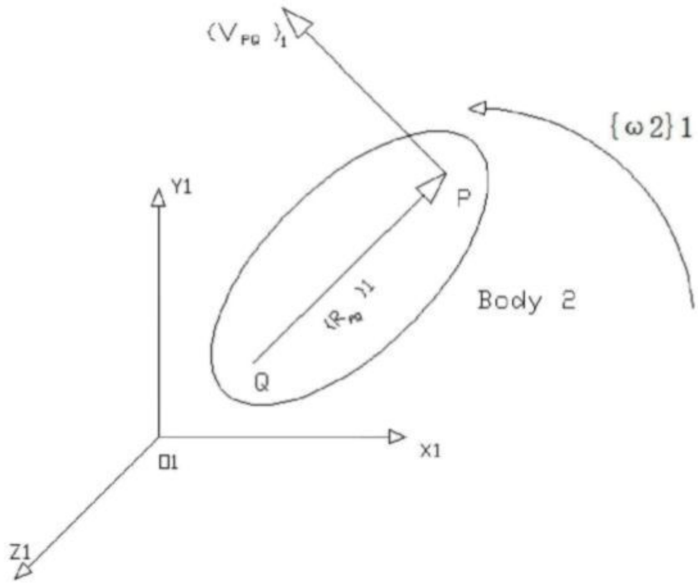


Figure 4.2: Relative velocity vectors

The two points P and Q are fixed in the same rigid body. As such there can be no component of motion along the line PQ and any relative motion must therefore be perpendicular to this line. This relationship can be expressed mathematically in two ways. The first way uses the vector dot product to enforce in perpendicular as shown in equation (4.12)

$$\{V_{PQ}\}_1 \cdot \{R_{PQ}\}_1 = 0 \tag{4.12}$$

The second method uses the vector cross product as shown in equation (4.13).

$$\{V_{PQ}\}_1 = \{\omega_2\}_1 \times \{R_{PQ}\}_1 \tag{4.13}$$

More often, the cross product is used to yield the angular velocity vector $\{\omega_2\}_1$; this may be required for a later acceleration analysis. By developing the equations to solve the velocities in a system of interconnected rigid bodies, the triangle law of vector addition can also be used as shown in equation (4.14).

$$\{V_P\}_1 = \{V_Q\}_1 + \{V_{PQ}\}_1 \tag{4.14}$$

4.3 Acceleration analysis

The acceleration is defined as the time rate of change of velocity; the equations required for an acceleration analysis can be developed using the vectors shown in Figure 4.3. It is possible to develop equations that would yield the relative acceleration vector $\{A_{PQ}\}_1$ using the angular velocity vector $\{\omega_2\}_1$ and the angular acceleration vector $\{\alpha_2\}_1$ as follows:

$$\{A_{PQ}\}_1 = \frac{d}{dt}\{V_{PQ}\}_1 \tag{4.15}$$

$$\{A_{PQ}\}_1 = \frac{d}{dt}\left\{\{\omega_2\}_1 \times \{R_{PQ}\}_1\right\} \tag{4.16}$$

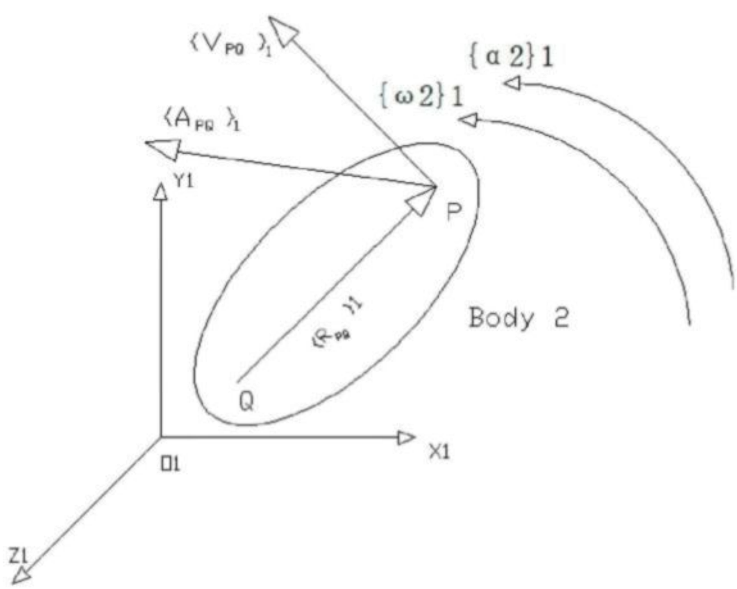


Figure 4.3: Relative acceleration vectors

$$\{A_{PQ}\}_1 = \{\omega_2\}_1 \times \frac{d}{dt}\{R_{PQ}\}_1 + \frac{d}{dt}\{\omega_2\}_1 \times \{R_{PQ}\}_1 \tag{4.17}$$

Since it is known that

$$\frac{d}{dt}\{R_{PQ}\}_1 = \{V_{PQ}\}_1 \tag{4.18}$$

$$\frac{d}{dt}\{\omega_2\}_1 = \{\alpha_2\}_1 \tag{4.19}$$

the following relation can be obtained.

$$\{A_{PQ}\}_1 = \{\omega_2\}_1 \times \{V_{PQ}\}_1 + \{\alpha_2\}_1 \times \{R_{PQ}\}_1 \tag{4.20}$$

Since it is also known that

$$\{V_{PQ}\}_1 = \{\omega_2\}_1 \times \{R_{PQ}\}_1 \tag{4.21}$$

this leads to the expression

$$\{A_{PQ}\}_1 = \{\omega_2\}_1 \times \{\{\omega_2\}_1 \times \{R_{PQ}\}_1\} + \{\alpha_2\}_1 \times \{R_{PQ}\}_1 \tag{4.22}$$

The acceleration vector $\{A_{PQ}\}_1$ can be considered to have a centripetal component $\{A_{PQ}^P\}_1$ and a transverse component $\{A_{PQ}^t\}_1$. This is illustrated in Figure 4.4, where lower arm of McPherson suspension system is shown. In this case the centripetal component of acceleration is given by

$$\{A_{PQ}^P\}_1 = \{\omega_2\}_1 \times \{\{\omega_2\}_1 \times \{R_{PQ}\}_1\} \tag{4.23}$$

Note that as the suspension arm is constrained to rotate about the axis NQ, ignoring at this stage any possible deflection due to compliance in the suspension bushes, the vectors $\{\omega_2\}_1$ for the angular velocity of body 2 and $\{\alpha_2\}_1$ for the angular acceleration would act along the axis of rotation through NQ. The components of these vectors would adopt signs consistent with producing a positive rotation about this axis as shown in Figure 4.4. When setting up the equations to solve a velocity or acceleration analysis, it may be desirable to reduce the number of unknowns based on the knowledge that a particular body is constrained to rotate about a known axis as shown in Figure 4.4.

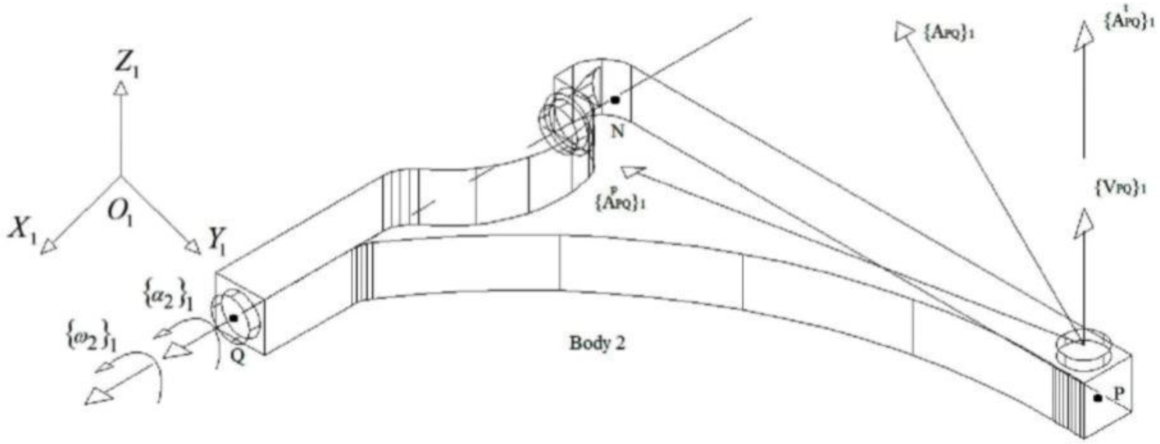


Figure 4.4: Centripetal and transverse components of acceleration vectors

The velocity vector $\{\omega_2\}_1$ could, for example, be represented as follow:

$$\{\omega_2\}_1 = f\omega_2 \{R_{QN}\}_1 \tag{4.24}$$

In this case, since $\{\omega_2\}_1$ is parallel to the relative position vector $\{R_{QN}\}_1$ a scale factor $f\omega_2$ can introduce. This would reduce the problem from the three unknown components, ωx_2, ωy_2 and ωz_2 of the vector $\{\omega_2\}_1$ to a single unknown $f\omega_2$. A similar approach could be used for an acceleration analysis; an example is as follows:

$$\{\alpha_2\}_1 = f\alpha_2 \{R_{QN}\}_1 \tag{4.25}$$

It can also be shown from Figure 4.4 that the centripetal acceleration acts perpendicularly towards the axis of rotation of the body. This relationship can be proved and used for having the centripetal acceleration by use of the dot product with the following relation.

$$\{A_{PQ}^P\}_1 \times \{R_{QN}\}_1 = 0 \tag{4.26}$$

The transverse component of acceleration is given by the following relation.

$$\{A_{PQ}^P\}_1 = \{\alpha_2\}_1 \times \{R_{PQ}\}_1 \tag{4.27}$$

The transverse component of acceleration is also perpendicular to, in this case, the vector $\{R_{PQ}\}_1$ as defined by the dot product.

$$\{A_{PQ}^t\}_1 \times \{R_{PQ}\}_1 = 0 \tag{4.28}$$

Although the vector $\{A_{PQ}^t\}_1$ is acting in the same direction as the vector $\{V_{PQ}\}_1$ in Figure 4.4; this may not necessarily be the case. A reversal of $\{A_{PQ}^t\}_1$ would correspond to a reversal of $\{\alpha_2\}_1$. This would indicate that point P is moving in a certain direction but in fact decelerating. The resultant acceleration vector $\{A_{PQ}^P\}_1$ is found to give the expression shown in equation (4.29) using the triangle law to add the centripetal and transverse components as follows:

$$\{A_{PQ}\}_1 = \{A^P_{PQ}\}_1 + \{A^t_{PQ}\}_1 \tag{4.29}$$

As shown in Figure 4.4, the analysis may often focus on suspension movement only and assume the vehicle body to be fixed and not moving. This would mean that the velocity or acceleration at point Q $\{A_Q\}_1$ would be zero. Based on the triangle law, the following relation can be obtained.

$$\{A_{PQ}\}_1 = \{A_P\}_1 - \{A_Q\}_1 \tag{4.30}$$

It therefore follows in this case that since Q is fixed:

$$\{A_P\}_1 = \{A_{PQ}\}_1 \tag{4.31}$$

Note that the same principle could be used when solving the velocities for this problem; hence the following relation can be obtained.

$$\{V_P\}_1 = \{V_{PQ}\}_1 \tag{4.32}$$

Finally it should be noted that, for the particular example shown in Figure 4.4, it could work from either point Q or point N when solving for the velocities or accelerations and obtain the same answers for the velocity or acceleration at point P.

4.4 Dynamic analysis

In order to understand the concept of dynamic analysis of a mechanical system, first of all, it is crucially important to know some definitions and concepts which have been directly related to this matter. Hence, this part introduces some conception in field of momentum and inertia relies on rigid bodies in motion. As well as, equations of motion will be introduced respectively. These are some subjects that are introduced as follow:

- Linear momentum of a rigid body
- Angular momentum of a rigid body
- Equations of motions

4.4.1 Linear momentum of a rigid body

As a body translates and rotates in space, it will have linear momentum $\{L\}_1$ associated with translation and angular momentum $\{H\}_1$ associated with rotation. For body 2 which is shown in Figure 4.5, the mass centre is located at G_2 by the vector $\{R_{G2}\}_1$ relative to the reference frame O_2.

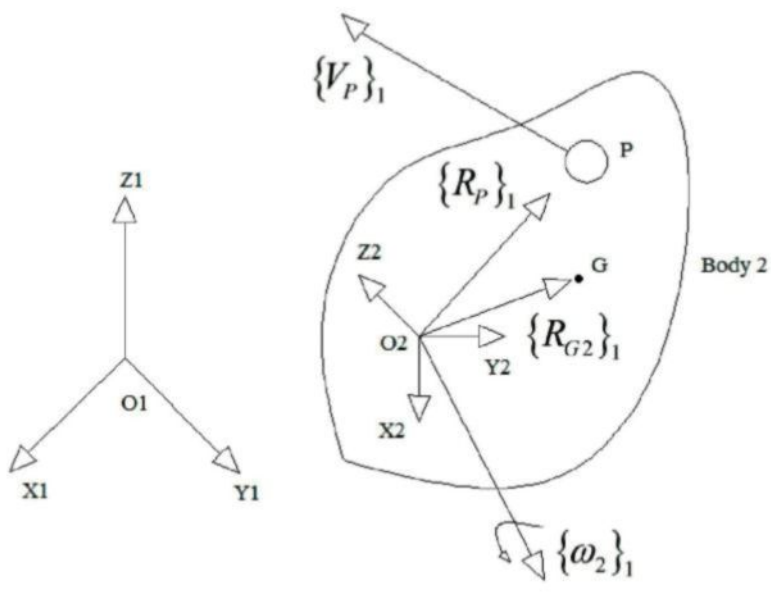


Figure 4.4: Linear momentum of a rigid body

The linear momentum $\{L_2\}_1$ of the body is the linear moment of the elements of mass that comprise the body. Therefore, the linear momentum of body 2 in terms of the overall mass m_2 and the velocity at the mass centre $\{V_{G2}\}_1$:

$$\{L_2\}_1 = m_2\{V_{G2}\}_1 \tag{4.33}$$

In this case, O_2 and G_2 would be coincident for body 2; so that $\{R_{G2}\}_1$ would be zero and the linear momentum would be obtained directly from the following relation.

$$\{L_2\}_1 = m_2\{V_{G2}\}_1 = m_2\{V_{O2}\}_1 \tag{4.34}$$

4.4.2 Angular momentum of a rigid body

The angular momentum ($\{H_P\}_1$) of the particle p with mass dm shown in Figure 4.5, can be obtained by taking the moment of the linear momentum of the particle about the frame O_2 as follows:

$$\{H_2\}_1 = \int_{vol}\{R_P\}_1 \times \{\{\omega_2\}_1 \times \{R_P\}_1\} dm \tag{4.35}$$

If we take the general case for any rigid body, according to the moments of inertia in a rigid body which are:

$$I_{xx} = \int (y^2 + z^2) dm,\ I_{yy} = \int (x^2 + z^2) dm,\ I_{zz} = \int (x^2 + y^2) dm$$

$$I_{xy} = I_{yx} = -\int xydm,\ I_{xz} = I_{zx} = -\int xzdm,\ I_{yz} = I_{zy} = -\int yzdm \tag{4.36}$$

By expanding the vectors into their full form, the angular momentum will be as follows:

$$\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} I_{xx} \; I_{xy} \; I_{xz} \\ I_{xy} \; I_{yy} \; I_{yz} \\ I_{xz} \; I_{yz} \; I_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{4.37}$$

If we return to our earlier consideration of body 2 shown in Figure 4.5, the matrix equation would lead to the following relation.

$$\{H_2\}_{1/1} = [I_2]_{2/1} \{\omega_2\}_{1/1} \tag{4.38}$$

It must be noted that by considering the equation for angular momentum of a body, it is preferable therefore to consider all quantities to be referred to a frame fixed in the body. Hence, in this case, the equation of angular momentum according to frame O_2 can be rewritten as follows:

$$\{H_2\}_{1/2} = [I_2]_{2/2} \{\omega_2\}_{1/2} \tag{4.39}$$

4.4.3 Equation of motion

If it is considered the rigid body 2, which is shown in Figure 4.6, it is possible to formulate six equation of motion corresponding with the six degrees of freedom resulting from unconstrained motion. If it is considered the expression for angular momentum, it is possible to obtain the equations of motion associated with rotational motion. For the rotational equations, it is convenient to refer the vectors to the reference frame O_2 fixed in and rotating with Body 2.

$$\sum \{M_{G2}\}_{1/2} = \frac{d}{dt}\{H_2\}_{1/2} = \frac{d}{dt}[I_2]\{\omega_2\}_{1/2} \tag{4.40}$$

$$\frac{d}{dt}\{H_2\}_{1/1} = \frac{d}{dt}\{H_2\}_{1/2} + \{\omega_2\}_{1/2} \times \{H_2\}_{1/2} \tag{4.41}$$

$$\{M_{G2}\}_{1/2} = [I_2]_{2/2}\{\alpha_2\}_{1/2} + \{\omega_2\}_{1/2} \times \{H_2\}_{1/2} \tag{4.42}$$

By expanding (4.42), it will give the following relation.

$$\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix} \begin{bmatrix} \alpha_{2x} \\ \alpha_{2y} \\ \alpha_{2z} \end{bmatrix} + \begin{bmatrix} 0 & -\omega_{2z} & \omega_{2y} \\ \omega_{2z} & 0 & -\omega_{2x} \\ -\omega_{2y} & \omega_{2x} & 0 \end{bmatrix} \begin{bmatrix} H_{2x} \\ H_{2y} \\ H_{2z} \end{bmatrix} \tag{4.43}$$

In summary the rotational equations of motion for body 2 may be written in vector form as follows:

$$\sum \{M_{G2}\}_{1/2} = [I_2]_{2/2}\{\alpha_2\}_{1/2} + \{\omega_2\}_{1/2}[I_2]_{2/2}\{\omega_2\}_{1/2} \tag{4.44}$$

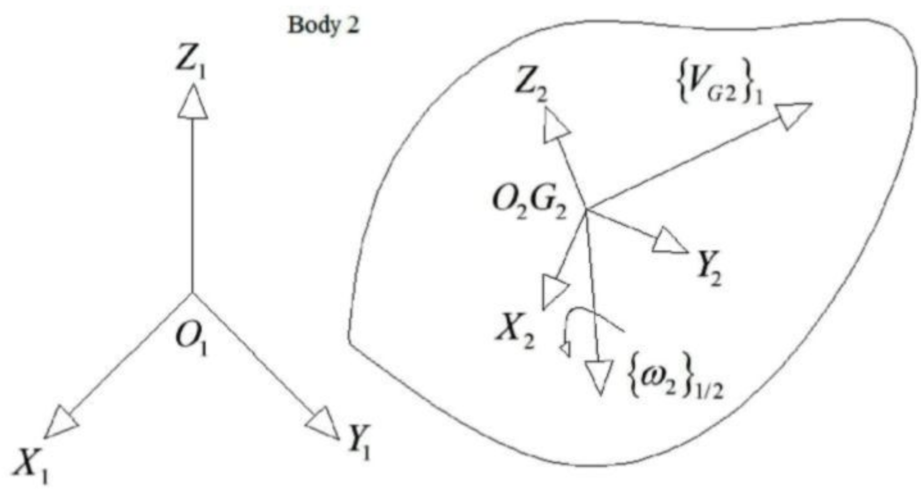


Figure 4.5: Rigid body motions

It can be shown that in setting up the equations of motion for any rigid body, the translational equations for all bodies in a system may conveniently be referred to a single fixed inertial frame O_1. The rotational equations, however, are better referred to a body centered frame, in this case O_2. A considerable simplification in these equations will result if the frame O_2 is selected; such that its axes are the principal axes of the body $\left(I_1 = I_{xx}, I_2 = I_{yy}, I_3 = I_{zz}\right)$ and the products of inertia are zero. The equations that result are known as Euler's equations of motion:

$$M_x = I_1\alpha_x + (I_3 - I_2)\omega_y\omega_z \tag{4.45}$$

$$M_y = I_2\alpha_y + (I_1 - I_3)\omega_x\omega_z \tag{4.46}$$

$$M_z = I_3\alpha_z + (I_2 - I_1)\omega_x\omega_y \tag{4.47}$$

4.5 Case study of McPherson suspension system

The following study is intended to demonstrate the application of the vector theory. The following calculations are typical of the processes carried out using MBS software. The methods do not represent exactly the internal machinations of their procedure. The lower arm used here is a part of McPherson suspension system. The calculations will include a series of analyses as follow:

- Velocity analysis
- Acceleration analysis
- Dynamic force analysis

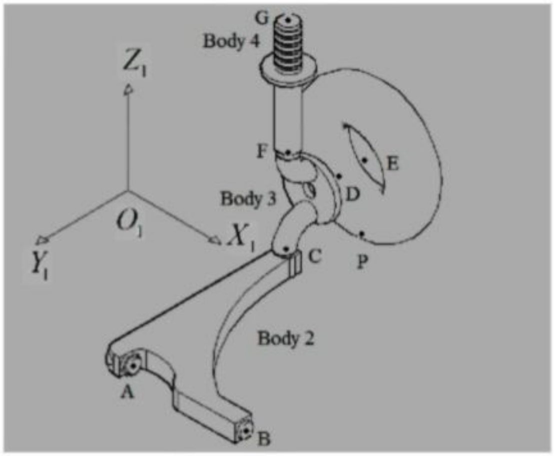
The dimension of lower arm of suspension system is depicted on Figure 4.7

Figure 4.6: Dimension of real lower arm of suspension system

The geometric data required to analyze the lower arm is defined in Figure 4.8. Note that in this example, the x-axis is orientated towards the front of the vehicle rather than pointing to the rear. This is therefore the front left suspension system on the vehicle.

4.5.1 Velocity analysis

The starting point for this analysis is to establish a boundary condition, or road input, at the tire contact patch P as the vehicle negotiates the road hump, shown in Figure 4.9, with a forward speed of 20 m/s.



Point	X(mm)	Y(mm)	Z(mm)
A	0	0	0
B	0	-293	0
C	300	15	10
D	320	10	90
E	398	10	90
F	316	22	200
G	322	25	670
P	398	10	-175

Figure 4.7: McPherson suspension system example geometry data

The analysis is simplified by ignoring the compliance in the tire; the profile of the road hump is taken as a sine function. In other words, sinusoidal method is selected for determining the profile of the road. This method is a combination of all sinusoidal waves with different frequencies and angular phases. It must be mentioned that the reason of using this method is due to optimization method. It is used the same method of determining road profile for doing optimization.

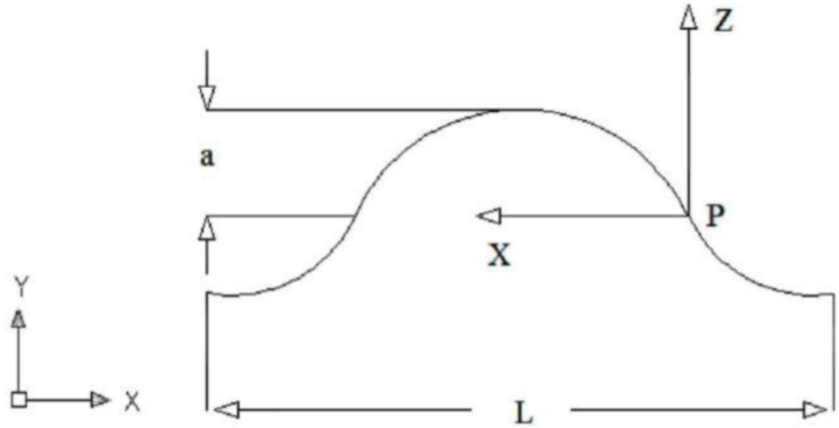


Figure 4.8: Road input definition for input analysis

The local x–z axis taken to reference the geometry of the road hump is located at a point where the vertical velocity V_{p_z} of the contact point P reaches a maximum with a corresponding vertical acceleration A_{p_z} equals to zero. The profile of the road hump can be defined using the following relation.

$$y_q(z) = a\sin\left(\frac{2\pi x}{L}\right) \tag{4.48}$$

Due to requirement to have a numerical analysis in lower arm, it is important to assign 400 mm to L and 14 mm to a. As well as, the total height of the bumps is 30 mm; hence, the following relation is obtained.

$$y_q(z) = 0.03\sin(22.2x + \varphi_n) \tag{4.49}$$

the vertical velocity of point p can be expressed as follows:

$$\frac{dz}{dt} = \frac{dz}{dx}\frac{dx}{dt} \tag{4.50}$$

Where

$$\frac{dx}{dt} = 10000\,mm/s \text{ and } \frac{dz}{dx} = 0.67\cos(22.2x + \varphi_n) \tag{4.51}$$

Giving

$$\frac{dz}{dt} = 0.67\cos(22.2x + \varphi_n)10000 \tag{4.52}$$

The maximum value of $\frac{dz}{dt}$ $\cos(22.2x + \varphi_n) = 1$ occurs when x giving

$$\left(\frac{dz}{dt}\right)_{\max} = 6700 \, {}^{mm}\!/\!{}_{s} \tag{4.53}$$

The acceleration $\frac{d^2z}{dx^2}$ is given by the following relation:

$$\frac{d^2z}{dx^2} = -14.9\sin(22.2x + \varphi_n) \tag{4.54}$$

It has maximum value for, $\cos x$ will be resulted value of zero; therefore, the velocity and acceleration which will be imposed to suspension system are as follows: $V_{p_z} = 6700 \, {}^{mm}\!/\!{}_{s}$ $A_{p_z} = 0 \, mm/s^2$

It is necessary to identify the unknowns that define the problem and the same number of equations as unknowns leading to a solution. The angular velocities of the rigid bodies representing suspension components can be used to find the translational velocities at points within the system. Thus this analysis can proceed, if it is possible to develop 10 equations to solve the 9 unknowns:

$$f\omega_2, \omega x_3, \omega y_3, \omega z_3, \omega x_4, \omega y_4, \omega z_4, V_{p_x}, V_{p_y}$$

In order to get equations for determine the unknowns; the triangle law of vector can be practical. Therefore, as a starting point we will develop a set of three equations using the following relation.

$$\{V_{CF}\}_1 = \{V_C\}_1 - \{V_F\}_1 \tag{4.55}$$

In this form, the equation (4.55) does not introduce any of the 9 unknowns listed above. It will therefore be necessary to initially define $\{V_{CF}\}_1$, $\{V_C\}_1$ and $\{V_F\}_1$ in terms of angular velocity.

$$\{V_C\}_1 = \{V_{CB}\} = \{\omega_2\}_1 \times \{R_{CB}\} \tag{4.56}$$

$$\{\omega_2\}_1 = f\omega_2 \{R_{AB}\}_1 = f\omega_2 \begin{bmatrix} 0 \\ 293 \\ 0 \end{bmatrix} rad/s \tag{4.57}$$

$$\begin{bmatrix} V_{Cx} \\ V_{Cy} \\ V_{Cz} \end{bmatrix} = f\omega_2 \begin{bmatrix} -345 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 300 \\ 308 \\ 10 \end{bmatrix} = \begin{bmatrix} 2930 f\omega_2 \\ 0 \\ -87900 f\omega_2 \end{bmatrix} mm/s \tag{4.58}$$

Determine an expression for $\{V_F\}_1$ at the point F:

$$\{V_F\}_1 = \{V_{FG}\}_1 = \{\omega_4\}_1 \times \{R_{FG}\} \tag{4.59}$$

$$\begin{bmatrix} V_{Fx} \\ V_{Fy} \\ V_{Fz} \end{bmatrix} = \begin{bmatrix} \omega x_4 \\ \omega y_4 \\ \omega z_4 \end{bmatrix} \times \begin{bmatrix} -6 \\ -3 \\ -470 \end{bmatrix} = \begin{bmatrix} -470\omega y_4 + 3\omega z_4 \\ 470\omega x_4 - 6\omega z_4 \\ -3\omega x_4 + 6\omega y_4 \end{bmatrix} mm/s \tag{4.60}$$

For V_{CF} we also have:

$$\{V_{CF}\}_1 = \{\omega_3\}_1 \times \{R_{CF}\}_1 \tag{4.61}$$

$$\begin{bmatrix} V_{CFx} \\ V_{CFy} \\ V_{CFz} \end{bmatrix} = \begin{bmatrix} \omega x_3 \\ \omega y_3 \\ \omega z_3 \end{bmatrix} \times \begin{bmatrix} -16 \\ -7 \\ -190 \end{bmatrix} = \begin{bmatrix} -190\omega y_3 + 7\omega z_3 \\ 190\omega x_3 - 16\omega z_3 \\ -7\omega x_3 + 16\omega y_3 \end{bmatrix} mm/s$$
(4.62)

It is possible to apply the triangle law of vector addition to equate the expression for the $\{V_{CF}\}_1$ in equation (4.55) with $\{V_C\}_1$ in equation (4.58) and $\{V_F\}_1$ in equation (4.60):

$$\{V_{CF}\}_1 = \{V_C\}_1 - \{V_F\}_1 \tag{4.63}$$

$$\begin{bmatrix} -190\omega y_3 + 7\omega z_3 \\ 190\omega x_3 - 16\omega z_3 \\ -7\omega x_3 + 16\omega y_3 \end{bmatrix} = \begin{bmatrix} 2930 f \omega_2 \\ 0 \\ -87900 f \omega_2 \end{bmatrix} - \begin{bmatrix} -470\omega y_4 + 3\omega z_4 \\ 470\omega x_4 - 6\omega z_4 \\ -3\omega x_4 + 6\omega y_4 \end{bmatrix} \tag{4.64}$$

Rearranging (4.64) yields the first three equations required to solve the analysis:

Equation 1: $$190\omega y_3 - 7\omega z_3 + 470\omega y_4 - 3\omega z_4 + 2930 f \omega_2 = 0 \tag{4.65}$$

Equation 2: $$190\omega x_3 - 16\omega z_3 + 470\omega x_4 - 6\omega z_4 = 0 \tag{4.66}$$

Equation 3: $$7\omega x_3 - 16\omega y_3 + 3\omega x_4 - 6\omega y_4 - 87900 f \omega_2 = 0 \tag{4.67}$$

We can now proceed to set up the next set of three equations working from point P to pint C and using the triangle law of vector addition:

$$\{V_{PC}\}_1 = \{V_P\}_1 - \{V_C\}_1 \tag{4.68}$$

Now we determine an expression for the velocity $\{V_{PC}\}_1$:

$$\{V_{CP}\}_1 = \{\omega_3\}_1 \times \{R_{CP}\}_1 \tag{4.69}$$

$$\begin{bmatrix} V_{CPx} \\ V_{CPy} \\ V_{CPz} \end{bmatrix} = \begin{bmatrix} \omega x_3 \\ \omega y_3 \\ \omega z_3 \end{bmatrix} \times \begin{bmatrix} -98 \\ 5 \\ 185 \end{bmatrix} = \begin{bmatrix} 185\omega y_3 - 5\omega z_3 \\ -185\omega x_3 - 98\omega z_3 \\ 5\omega x_3 + 98\omega y_3 \end{bmatrix} mm/s \tag{4.70}$$

For the point P, we determined the amount of its velocity on Z direction, therefore, the vector of the point P will be:

$$\{V_P\}_1 = \begin{bmatrix} V_{P_x} \\ V_{P_y} \\ 6700 \end{bmatrix} mm/s \tag{4.71}$$

The expression for $\{V_C\}_1$ can be found in equation (4.58). Hence, it is possible to apply the triangle law of vector addition to equate the expression for $\{V_{PC}\}_1$ in equation (4.68) with $\{V_C\}_1$ in equation (4.58) and $\{V_P\}_1$ in equation (4.71):

$$\{V_{PC}\}_1 = \{V_P\}_1 - \{V_C\}_1 \tag{4.72}$$

$$\begin{bmatrix} 375\omega y_3 - 12\omega z_3 \\ -375\omega x_3 - 82\omega z_3 \\ 12\omega x_3 + 82\omega y_3 \end{bmatrix} = \begin{bmatrix} -470\omega y_4 + 6\omega z_4 \\ 470\omega x_4 - 3\omega z_4 \\ -6\omega x_4 + 3\omega y_4 \end{bmatrix} - \begin{bmatrix} V_{P_x} \\ V_{P_y} \\ 14889 \end{bmatrix} \tag{4.73}$$

Rearranging (4.73) yields the next set of three equations required to solve the analysis:

Equation 4: $-185\omega y_3 + 5\omega z_3 + 2930 f\omega_2 - V_{P_x} = 0$ (4.74)

Equation 5: $-185\omega x_3 - 98\omega z_3 + V_{P_y} = 0$ (4.75)

Equation 6: $87900 f\omega_2 + 5\omega x_3 + 98\omega y_3 = -14889$ (4.76)

Finally, we can proceed to set up the last set of three equations working from point P to pint F and working from B to F by using the triangle law of vector addition:

$$\{V_{PF}\}_1 = \{V_P\}_1 - \{V_F\}_1 \tag{4.77}$$

$$\{V_{FB}\}_1 = \{V_F\}_1 - \{V_B\}_1 \tag{4.78}$$

The amount of $\{V_G\}_1$ is equal to zero, therefore we will have:

$$\{V_{FG}\}_1 = \{V_F\}_1 \tag{4.79}$$

$$\{V_F\}_1 = \omega_4 \times \{R_{FG}\}_1 \tag{4.80}$$

$$\begin{bmatrix} V_{Fx} \\ V_{Fy} \\ V_{Fz} \end{bmatrix} = \begin{bmatrix} \omega x_4 \\ \omega y_4 \\ \omega z_4 \end{bmatrix} \times \begin{bmatrix} -6 \\ -3 \\ -470 \end{bmatrix} = \begin{bmatrix} -470\omega y_4 + 3\omega z_4 \\ 470\omega x_4 - 6\omega z_4 \\ -3\omega x_4 + 6\omega y_4 \end{bmatrix} mm/s \tag{4.81}$$

The velocity of $\{V_{PF}\}_1$ can be determined as:

$$\{V_{PF}\}_1 = \{\omega_4\}_1 \times \{R_{PF}\}_1 \tag{4.82}$$

$$\begin{bmatrix} V_{PFx} \\ V_{PFy} \\ V_{PFz} \end{bmatrix} = \begin{bmatrix} \omega x_3 \\ \omega y_3 \\ \omega z_3 \end{bmatrix} \times \begin{bmatrix} 82 \\ -12 \\ -375 \end{bmatrix} = \begin{bmatrix} -375\omega y_3 + 12\omega z_3 \\ 375\omega x_3 + 82\omega z_3 \\ -12\omega x_3 - 82\omega y_3 \end{bmatrix} mm/s \tag{4.83}$$

An expression for $\{V_P\}_1$ is already found in equation (4.71) and an expression for $\{V_F\}_1$ in equation (4.81). Hence, it is possible to apply the triangle law of vector addition to equate the expression for $\{V_{PF}\}_1$ in equation (4.84) with $\{V_P\}_1$ and $\{V_F\}_1$:

$$\{V_{PF}\}_1 = \{V_P\}_1 - \{V_F\}_1 \tag{4.84}$$

$$\begin{bmatrix} 375\omega y_3 - 12\omega z_3 \\ -375\omega x_3 - 82\omega z_3 \\ 12\omega x_3 + 82\omega y_3 \end{bmatrix} = \begin{bmatrix} -470\omega y_4 + 6\omega z_4 \\ 470\omega x_4 - 3\omega z_4 \\ -6\omega x_4 + 3\omega y_4 \end{bmatrix} - \begin{bmatrix} V_{P_x} \\ V_{P_y} \\ 14889 \end{bmatrix} \tag{4.85}$$

Rearranging (4.85) yields the final set of three equations required to solve the analysis.

Equation 7: $375\omega y_3 - 12\omega z_3 + 470\omega y_4 - 6\omega z_4 + V_{P_x} = 0$ (4.86)

Equation 8: $-375\omega x_3 - 82\omega z_3 - 470\omega x_4 + 3\omega z_4 + V_{P_y} = 0$ (4.87)

Equation 9: $12\omega x_3 + 82\omega y_3 + 6\omega x_4 - 3\omega y_4 = -14889$ (4.88)

The 9 equations can be set up in matrix form ready for solution. The solution of 9 by 9 will require access to a mathematical or spreadsheet program that offers the capability to invert the matrix.

$$\begin{bmatrix} 2930 & 0 & 190 & -7 & 0 & 470 & -6 & 0 & 0 \\ 0 & 190 & 0 & -16 & 470 & 0 & -3 & 0 & 0 \\ -87900 & 7 & -16 & 0 & 6 & -3 & 0 & 0 & 0 \\ 2930 & 0 & -185 & 5 & 0 & 0 & 0 & -1 & 0 \\ 0 & -185 & 0 & -98 & 0 & 0 & 0 & 0 & 1 \\ 87900 & 5 & 98 & 0 & 0 & 0 & 0 & 0 & 0 \\ 58600 & 0 & 0 & 0 & 0 & -375 & 12 & -1 & 0 \\ 0 & 0 & 0 & 0 & 375 & 0 & 82 & 0 & -1 \\ 92588 & 0 & 0 & 0 & 12 & 82 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f\omega_2 \\ \omega x_3 \\ \omega y_3 \\ \omega z_3 \\ \omega x_4 \\ \omega y_4 \\ \omega z_4 \\ V_{Px} \\ V_{Py} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -14889 \\ 0 \\ 0 \\ -14889 \end{bmatrix} \tag{4.89}$$

Solving equation (4.89) yields the following answers for the 9 unknowns in the Table 4.1. The dimension of $f\omega_2$ is $rad/_{mm.s}$ and the other dimensions are $rad/_s$.

Table 4.1: Velocity analysis of suspension system

$f\omega_2$	ωx_3	ωy_3	ωz_3	ωx_4	ωy_4	ωz_4	V_{P_x}	V_{P_y}
0.28	4076	-607	-21300	-2406	-140	-5257	5810	-1333413

It is now possible to use the scale factor found, $f\omega_2$ to calculate the angular velocity vectors $\{\omega_2\}_1$:

$$\{\omega_2\}_1 = f\omega_2 \{R_{AB}\}_1 = 0.28 \begin{bmatrix} 0 \\ 293 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 82 \\ 0 \end{bmatrix} rad/_s \tag{4.90}$$

In summary, the angular velocity vectors for the rigid bodies are as follows:

$$\{\omega_2\}_1^T = [0\ 82\ \ 0]\ rad/_s$$

$$\{\omega_3\}_1^T = [4076\ -607\ \ 21300]\ rad/_s$$

$$\{\omega_4\}_1^T = [-2406\ -140\ -5257\ ]\ rad/_s$$

Finally, by determining the angular velocity of the different parts of suspension system, it is possible to proceed to calculate the translational velocities at all moving points in the system.

4.5.2 Acceleration analysis

Before proceeding with acceleration analysis, it is necessary to identify the unknowns that define the problem. The angular accelerations and angular velocities of the rigid bodies representing suspension components can be used to find the translational accelerations at points within the system. An example is shown for the lower arm of suspension system in Figure 4.10.

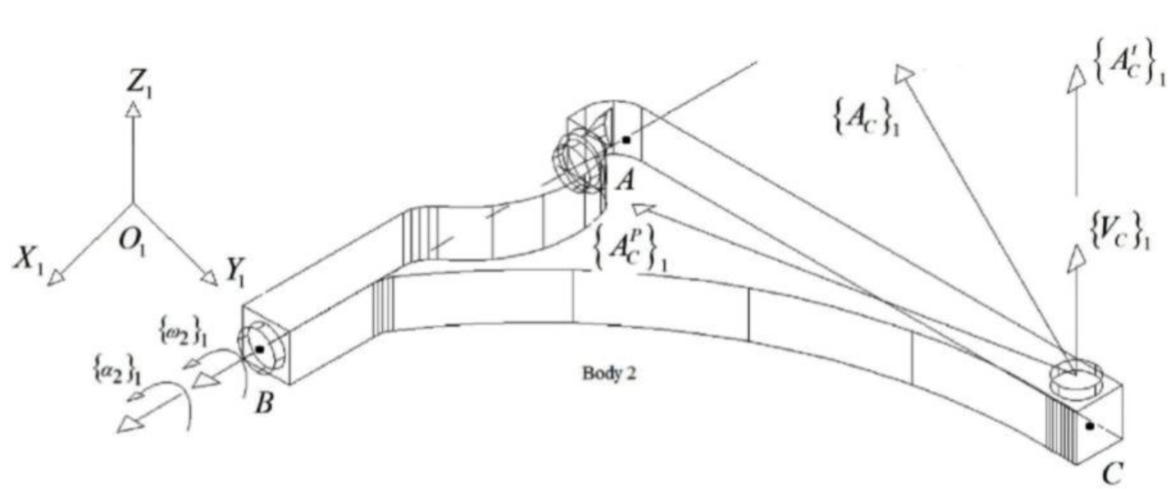


Figure 4.9: Angular and translational acceleration and velocity for the lower McPherson arm

The vector $\{\alpha_2\}_1$ for the angular acceleration of body 2 will act along the axis of rotation through AB. The components of this vector would adopt signs consistent with a positive rotation about this axis as shown in Figure 4.10. The acceleration vector $\{\alpha_2\}_1$ could, for example, be represented as follows:

$$\{\alpha_2\}_1 = f\alpha_2 \{R_{AB}\}_1 \tag{4.91}$$

Also, it follows that since point A is considered fixed with an acceleration $\{A_A\}_1$ equals to zero; absolute acceleration $\{A_C\}_1$ of point C can be found from a consideration of the triangle law of vector addition giving the following relation.

$$\{A_C\}_1 = \{A_{CA}\}_1 \tag{4.92}$$

It follows that the translational acceleration $\{A_C\}_1$ of point C can be found from the following relation.

$$\{A_C\}_1 = \{A_C^P\}_1 + \{A_C^t\}_1 \tag{4.92}$$

Where the centripetal acceleration $\{A_C^P\}_1$ is given by:

$$\{A_C^P\}_1 = \{\omega_2\}_1 \times \{\{\omega_2\}_1 \times \{R_{CA}\}_1\} = \{\omega_2\}_1 \times \{V_C\}_1 \tag{4.93}$$

and the transverse acceleration $\{A_C^t\}_1$ is given by:

$$\{A_C^t\}_1 = \{\alpha_2\}_1 \times \{R_{CB}\}_1 \tag{4.94}$$

The velocity analysis, clearly the same approach, can be taken for the whole bodies of the McPherson suspension system. Thus, this analysis can proceed if it is possible to develop 9 equations to solve the following 9 unknowns.

$$f\alpha_2, \alpha x_3, \alpha y_3, \alpha z_3, \alpha x_4, \alpha y_4, \alpha z_4, A_{Px}, A_{Py}$$

Working through the problem, it will be seen that the same strategy used in the velocity analysis can be developed using the triangle law of vector addition to generate sets of equations. As a starting point, a set of three equations will be developed using the following relation.

$$\{A_{FC}\}_1 = \{A_F\}_1 - \{A_C\}_1 \tag{4.95}$$

For the acceleration of point F, $\{A_F\}_1 = \{A_{FG}\}_1$ is obtained, therefore:

$$\{A_F\}_1 = \{A_{FG}\}_1 = \{\omega_4\}_1 \times \{V_{FG}\}_1 + \{\alpha_4\}_1 \times \{R_{FG}\}_1 \tag{4.96}$$

$$\begin{bmatrix} A_{Fx} \\ A_{Fy} \\ A_{Fz} \end{bmatrix} = \begin{bmatrix} -2406 \\ -140 \\ -5257 \end{bmatrix} \begin{bmatrix} -2406 \\ -140 \\ -5257 \end{bmatrix} \begin{bmatrix} -6 \\ -3 \\ -470 \end{bmatrix} + \begin{bmatrix} \alpha x_4 \\ \alpha y_4 \\ \alpha z_4 \end{bmatrix} \begin{bmatrix} -6 \\ -3 \\ -470 \end{bmatrix} mm/s^2 \tag{4.97}$$

$$\begin{bmatrix} A_{Fx} \\ A_{Fy} \\ A_{Fz} \end{bmatrix} = \begin{bmatrix} -5.8E9 - 470\alpha y_4 + 3\alpha z_4 \\ -2.5E8 + 470\alpha x_4 - 6\alpha z_4 \\ 2.7E9 - 3\alpha x_4 + 6\alpha y_4 \end{bmatrix} mm/s^2 \tag{4.98}$$

Determining an expression for the acceleration $\{A_C\}_1$ at point C using values for $\{\omega_2\}_1$ and $\{V_C\}_1$ found from the earlier velocity analysis gives the following relations.

$$\{A_C\}_1 = \{\omega_2\}_1 \times \{V_C\}_1 + \{\alpha_2\}_1 \times \{R_{CB}\}_1 \tag{4.99}$$

$$\{\alpha_2\}_1 = f\alpha_2 \{R_{AB}\}_1 = f\alpha_2 \begin{bmatrix} 0 \\ 293 \\ 0 \end{bmatrix} rad/s^2 \tag{4.100}$$

$$\begin{bmatrix} A_{Cx} \\ A_{Cy} \\ A_{Cz} \end{bmatrix} = \begin{bmatrix} 0 \\ 82 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 82 \\ 0 \end{bmatrix} \begin{bmatrix} 300 \\ 308 \\ 10 \end{bmatrix} + f\alpha_2 \begin{bmatrix} 0 \\ 293 \\ 0 \end{bmatrix} \begin{bmatrix} 300 \\ 308 \\ 10 \end{bmatrix} mm/s^2 \tag{4.101}$$

$$\begin{bmatrix} A_{Cx} \\ A_{Cy} \\ A_{Cz} \end{bmatrix} = \begin{bmatrix} 2930 f\alpha_2 - 2E6 \\ 0 \\ -87900 f\alpha_2 - 67240 \end{bmatrix} mm/s^2 \tag{4.102}$$

Determining an expression for the relative acceleration $\{A_{FC}\}_1$ of point F related to point C using values for $\{\omega_3\}_1$ and $\{V_{FC}\}_1$ found from the earlier velocity analysis gives the following relations.

$$\{A_{FC}\}_1 = \{\omega_3\}_1 \times \{V_{FC}\}_1 + \{\alpha_3\}_1 \times \{R_{FC}\}_1 \tag{4.103}$$

$$\begin{bmatrix} A_{FCx} \\ A_{FCy} \\ A_{FCz} \end{bmatrix} = \begin{bmatrix} 4076 \\ -607 \\ -21300 \end{bmatrix} \begin{bmatrix} 4076 \\ -607 \\ -21300 \end{bmatrix} \begin{bmatrix} 16 \\ 7 \\ 190 \end{bmatrix} + \begin{bmatrix} \alpha x_3 \\ \alpha y_3 \\ \alpha z_3 \end{bmatrix} \begin{bmatrix} 16 \\ 7 \\ 190 \end{bmatrix} mm/s^2 \quad (4.104)$$

$$\begin{bmatrix} A_{FCx} \\ A_{FCy} \\ A_{FCz} \end{bmatrix} = \begin{bmatrix} -2.4E10 + 190\alpha y_3 - 7\alpha z_3 \\ -8.8E8 - 190\alpha x_3 + 16\alpha z_3 \\ -4.5E9 + 7\alpha x_3 - 16\alpha y_3 \end{bmatrix} mm/s^2 \quad (4.105)$$

It is possible to apply the triangle law of vector addition to equate the expression for $\{A_{FC}\}_1$ in equation (4.105) with $\{A_F\}_1$ in equation (4.98) and $\{A_C\}_1$ in equation (4.102):

$$\{A_{FC}\}_1 = \{A_F\}_1 - \{A_C\}_1 \quad (4.106)$$

$$\begin{bmatrix} -2.4E10 + 190\alpha y_3 - 7\alpha z_3 \\ -8.8E8 - 190\alpha x_3 + 16\alpha z_3 \\ -4.5E9 + 7\alpha x_3 - 16\alpha y_3 \end{bmatrix} = \begin{bmatrix} -5.8E9 - 470\alpha y_4 + 3\alpha z_4 \\ -2.5E8 + 470\alpha x_4 - 6\alpha z_4 \\ 2.7E9 - 3\alpha x_4 + 6\alpha y_4 \end{bmatrix} - \begin{bmatrix} 2930 f\alpha_2 - 2E6 \\ 0 \\ -87900 f\alpha_2 - 67240 \end{bmatrix} \quad (4.107)$$

Rearranging (4.107) yields the first three equations required to solve the analysis:

Equation 1: $2930 f\alpha_2 + 190\alpha y_3 - 7\alpha z_3 + 470\alpha y_4 - 3\alpha z_4 = 1.8E10$ (4.108)

Equation 2: $-190\alpha x_3 + 16\alpha z_3 - 470\alpha x_4 + 6\alpha z_4 = 6.3E8$ (4.109)

Equation 3: $-87900 f\alpha_2 + 7\alpha x_3 - 16\alpha y_3 + 3\alpha x_4 - 6\alpha y_4 = 7.2E9$ (4.110)

We can now proceed to set up the next set of three equations working from point P to point C and using the triangle law of vector addition:

$$\{A_{PC}\}_1 = \{A_P\}_1 - \{A_C\}_1 \quad (4.111)$$

Determining an expression for the acceleration $\{A_{PC}\}_1$ by using values for $\{\omega_3\}_1$ and $\{V_{PC}\}_1$ found from the earlier velocity analysis gives the following relations.

$$\{A_{PC}\}_1 = \{\omega_3\}_1 \times \{V_{PC}\}_1 + \{\alpha_3\}_1 \times \{R_{PC}\}_1 \tag{4.112}$$

$$\begin{bmatrix} A_{PCx} \\ A_{PCy} \\ A_{PCz} \end{bmatrix} = \begin{bmatrix} 4076 \\ -607 \\ -21300 \end{bmatrix} \begin{bmatrix} 4076 \\ -607 \\ -21300 \end{bmatrix} \begin{bmatrix} 98 \\ -5 \\ -185 \end{bmatrix} + \begin{bmatrix} \alpha x_3 \\ \alpha y_3 \\ \alpha z_3 \end{bmatrix} \begin{bmatrix} 98 \\ -5 \\ -185 \end{bmatrix} mm/s^2 \tag{4.113}$$

$$\begin{bmatrix} A_{PCx} \\ A_{PCy} \\ A_{PCz} \end{bmatrix} = \begin{bmatrix} -2.8E10 - 185\alpha y_3 + 5\alpha z_3 \\ -2.8E8 + 185\alpha x_3 + 98\alpha z_3 \\ 5.4E9 - 5\alpha x_3 - 98\alpha y_3 \end{bmatrix} mm/s^2 \tag{4.114}$$

For the point P, it is determined the amount of its acceleration on Z direction, therefore, the vector of the point P will be as follows:

$$\{A_P\}_1 = \begin{bmatrix} A_{P_x} \\ A_{P_y} \\ 0 \end{bmatrix} mm/s^2 \tag{4.115}$$

An expression for $\{A_C\}_1$ in equation (4.101) is already obtained. Hence, it is possible to apply the triangle law of vector addition to equate the expression for $\{A_{PC}\}_1$ in equation (4.113) with $\{A_C\}_1$ in equation (4.101) and $\{A_P\}_1$ in equation (4.115).

$$\begin{bmatrix} A_{PCx} \\ A_{PCy} \\ A_{PCz} \end{bmatrix} = \begin{bmatrix} A_{P_x} \\ A_{P_y} \\ A_{P_z} \end{bmatrix} - \begin{bmatrix} A_{Cx} \\ A_{Cy} \\ A_{Cz} \end{bmatrix} \tag{4.116}$$

$$\begin{bmatrix} -2.8E10 - 185\alpha y_3 + 5\alpha z_3 \\ -2.8E8 + 185\alpha x_3 + 98\alpha z_3 \\ 5.4E9 - 5\alpha x_3 - 98\alpha y_3 \end{bmatrix} = \begin{bmatrix} A_{P_x} \\ A_{P_y} \\ 0 \end{bmatrix} - \begin{bmatrix} 2930 f\alpha_2 - 2E6 \\ 0 \\ -87900 f\alpha_2 - 67240 \end{bmatrix} mm/s^2 \tag{4.117}$$

Rearranging (4.118) yields the next set of three equations required to solve the analysis:

Equation 4: $185\alpha x_3 + 98\alpha z_3 - A_{P_y} = 2.8E8$ (4.118)

Equation 5: $185\alpha x_3 + 98\alpha z_3 - A_{P_y} = 2.8E8$ (4.119)

Equation 6: $-87900 f\alpha_2 - 5\alpha x_3 - 98\alpha y_3 = -5.4E9$ (4.120)

Finally, it is possible to proceed to set up the last set of three equations working from point P to point F and working from B to F by using the triangle law of vector addition:

$$\{A_{PF}\}_1 = \{A_P\}_1 - \{A_F\}_1 \tag{4.121}$$

$$\{A_{FB}\}_1 = \{A_F\}_1 - \{A_B\}_1 \tag{4.122}$$

The amount of $\{A_B\}_1$ is equal to zero, therefore the following relations will be obtained.

$$\{A_{FB}\}_1 = \{A_F\}_1 \tag{4.123}$$

$$\{A_F\}_1 = f\omega_2 \{R_{AB}\}_1 \times \{V_{FB}\}_1 + f\alpha_2 \{R_{AB}\}_1 \times \{R_{FB}\}_1 \tag{4.124}$$

$$\begin{bmatrix} A_{Fx} \\ A_{Fy} \\ A_{Fz} \end{bmatrix} = \begin{bmatrix} 0 \\ 82 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 82 \\ 0 \end{bmatrix} \begin{bmatrix} 316 \\ 315 \\ 200 \end{bmatrix} + \begin{bmatrix} 0 \\ 293 f\alpha_2 \\ 0 \end{bmatrix} \begin{bmatrix} 316 \\ 315 \\ 200 \end{bmatrix} mm/s^2 \tag{4.125}$$

$$\begin{bmatrix} A_{Fx} \\ A_{Fy} \\ A_{Fz} \end{bmatrix} = \begin{bmatrix} 58600 f\alpha_2 - 2.1E6 \\ 0 \\ -92588 f\alpha_2 - 1.3E6 \end{bmatrix} mm/s^2 \tag{4.126}$$

As well as the acceleration of $\{A_{PF}\}_1$ can be determined as follows:

$$\{A_{PF}\}_1 = \{\omega_4\}_1 \times \{V_{PF}\}_1 + \{\alpha_4\}_1 \times \{R_{PF}\}_1 \tag{4.127}$$

$$\begin{bmatrix} A_{PFx} \\ A_{PFy} \\ A_{PFz} \end{bmatrix} = \begin{bmatrix} -2406 \\ -140 \\ -5257 \end{bmatrix} \begin{bmatrix} -2406 \\ -140 \\ -5257 \end{bmatrix} \begin{bmatrix} 82 \\ -12 \\ -375 \end{bmatrix} + \begin{bmatrix} \alpha x_4 \\ \alpha y_4 \\ \alpha z_4 \end{bmatrix} \begin{bmatrix} 82 \\ -12 \\ -375 \end{bmatrix} mm/s^2 \tag{4.128}$$

$$\begin{bmatrix} A_{PFx} \\ A_{PFy} \\ A_{PFz} \end{bmatrix} = \begin{bmatrix} -7E9 - 375\alpha y_4 - 12\alpha z_4 \\ 1.5E9 + 375\alpha x_4 + 82\alpha z_4 \\ 3.2E9 - 12\alpha x_4 - 82\alpha y_4 \end{bmatrix} mm/s^2 \tag{4.129}$$

An expression for $\{A_P\}_1$ in equation (4.115) and an expression for $\{A_F\}_1$ in equation (4.125) are already obtained. Hence, it is possible to apply the triangle law of vector addition to equate the expression for $\{A_{PF}\}_1$ in equation (4.128) with $\{A_P\}_1$ and $\{A_F\}_1$:

$$\begin{bmatrix} A_{PFx} \\ A_{PFy} \\ A_{PFz} \end{bmatrix} = \begin{bmatrix} A_{P_x} \\ A_{P_y} \\ A_{P_z} \end{bmatrix} - \begin{bmatrix} A_{Fx} \\ A_{Fy} \\ A_{Fz} \end{bmatrix} \tag{4.130}$$

$$\begin{bmatrix} -7E9 - 375\alpha y_4 + 12\alpha z_4 \\ 1.5E9 + 375\alpha x_4 + 82\alpha z_4 \\ 3.2E9 - 12\alpha x_4 - 82\alpha y_4 \end{bmatrix} = \begin{bmatrix} A_{P_x} \\ A_{P_y} \\ 0 \end{bmatrix} - \begin{bmatrix} 58600 f\alpha_2 - 2.1E6 \\ 0 \\ -92588 f\alpha_2 - 1.3E6 \end{bmatrix} mm/s^2 \tag{4.131}$$

Rearranging (4.131) yields the final set of three equations required to solve the analysis.

Equation 7: $58600 f\alpha_2 - 375\alpha y_4 + 12\alpha z_4 - A_{P_x} = 7E9$ (4.132)

Equation 8: $375\alpha x_4 + 82\alpha z_4 - A_{P_y} = -1.5E9$ (4.133)

Equation 9: $-92588 f\alpha_2 - 12\alpha x_4 - 82\alpha y_4 = -3.2E9$ (4.134)

The 9 equations can now be set up in matrix form ready for solution as follows:

$$\begin{bmatrix} 2930 & 0 & 190 & -7 & 0 & 470 & -3 & 0 & 0 \\ 0 & -190 & 0 & 16 & -470 & 0 & 6 & 0 & 0 \\ -87900 & 7 & -16 & 0 & 3 & -6 & 0 & 0 & 0 \\ 2930 & 0 & -185 & 5 & 0 & 0 & 0 & -1 & 0 \\ 0 & 185 & 0 & 98 & 0 & 0 & 0 & 0 & -1 \\ -87900 & -5 & -98 & 0 & 0 & 0 & 0 & 0 & 0 \\ 58600 & 0 & 0 & 0 & 0 & -375 & 12 & -1 & 0 \\ 0 & 0 & 0 & 0 & 375 & 0 & 82 & 0 & -1 \\ -92588 & 0 & 0 & 0 & -12 & -82 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f\alpha_2 \\ \alpha x_3 \\ \alpha y_3 \\ \alpha z_3 \\ \alpha x_4 \\ \alpha y_4 \\ \alpha z_4 \\ A_{Px} \\ A_{Py} \end{bmatrix} = \begin{bmatrix} 1.8E10 \\ 6.3E8 \\ 7.2E9 \\ 2.8E10 \\ 2.8E8 \\ -5.4E9 \\ 7E9 \\ -1.5E9 \\ -3.2E9 \end{bmatrix} \qquad (4.135)$$

Solving equation (4.135) yields the following answers for the 9 unknowns in the Table 4.2. The dimension of $f\alpha_2$ is ${rad}/{mm.s^2}$ and the other dimensions are ${rad}/{s^2}$.

Table 4.2: Acceleration analysis of suspension system

$f\alpha_2$	αx_3	αy_3	αz_3	αx_4	αy_4	αz_4	A_{Px}	A_{Py}
-9.7E5	-1E10	1.4E9	5E10	5.9E9	2.7E8	9.6E9	-4.8E10	3E12

In summary, the angular acceleration vectors for the rigid bodies are as follows:

$$\{\alpha_2\}_1^T = [0 \; -2.8E8 \;\; 0] \; {rad}/{s^2}$$

$$\{\alpha_3\}_1^T = [-1E10 \; 1.4E9 \;\; 5E10] \; {rad}/{s^2}$$

$$\{\alpha_4\}_1^T = [5.9E9 \;\; 2.7E8 \;\; 9.6E9] \; {rad}/{s^2}$$

Now by determining the angular acceleration of the every bodies of suspension system, it is possible to get the acceleration of each point of the McPherson suspension system.

4.5.3 Dynamic force analysis

For the suspension considered here, a theoretical solution could be formulated on the basis of the static forces of the suspension system. However, for brevity a full theoretical solution will not be performed here. The six equations of motion for lower arm of suspension system will be set up using the velocities and accelerations found earlier. Respectively, it is possible to set up the equations of motions for the other bodies. The lower arm of suspension system, body 2, can be considered in isolation as illustrated with the free-body diagram shown in Figure 4.11.

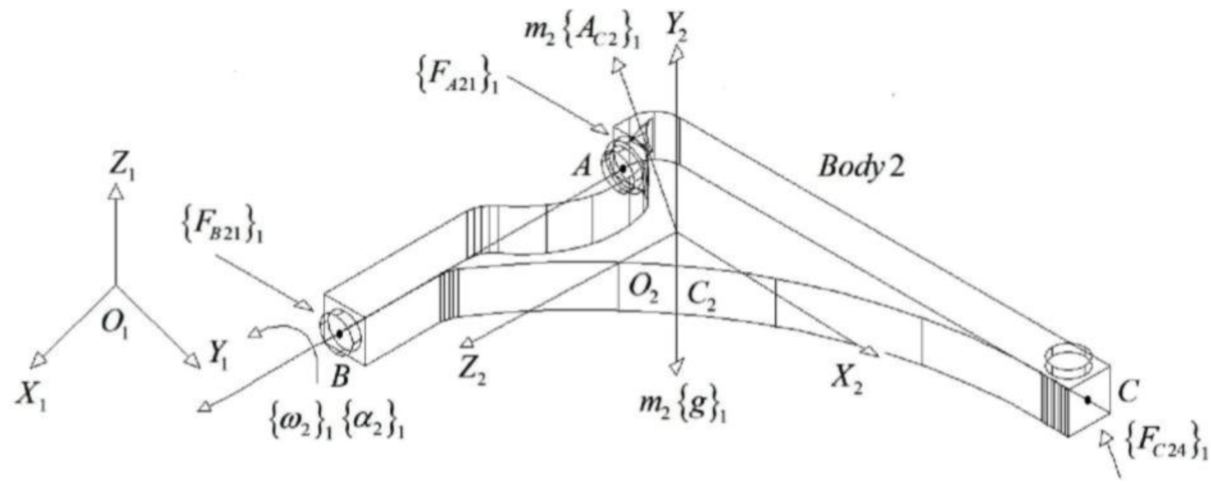


Figure 4.10: Free body diagrams for suspension lower arm Body 2

For the dynamic analysis, all of the physical properties of the lower arm of suspension system, namely mass, mass moments of inertia, center of mass location and orientation of the body principal axis system, have been determined by designing software, solid works. For a dynamic analysis, the mass center locations of all moving bodies are required in order to set up the equations of motion. For this specific example for lower arm of suspension system, (body 2) the position of the mass centre C_2 relative to the inertial reference frame O_1 is defined by the position vector $\{R_{C2O1}\}_1$. It must be mentioned that the inertial reference frame has been considered on A and assumed to be as follows:

$$\{R_{C2O1}\}_1^T = [0.089 \; -0.082 \; -0.00013]\; m \tag{4.136}$$

The mass of lower arm of suspension system; m_2 has been defined to be 1.2 kg. The mass moments of inertia for lower arm, measured about the principal axes of the body O_2 and output coordinate system, have been declared in (4.37) which is inertia matrix.

$$[I_2]_{2/2} = \begin{bmatrix} 0.007366 & 0.004957 & -0.000016 \\ 0.004957 & 0.010405 & -1.83E-6 \\ -16E-6 & -1.83E-6 & 0.017642 \end{bmatrix} \text{kg m}^2 \tag{4.137}$$

By considering the velocity and acceleration of lower arm of suspension system that have been determined before, for the acceleration of mass center, it is found the following relation:

$$\{A_{C2}\}_1 = \{A_{C2A}\}_1 = \{\omega_2\}_1 \times \{V_{C2}\}_1 + \{\alpha_2\}_1 \times \{R_{C2A}\}_1 \tag{4.138}$$

Where

$$\{V_{C2}\}_1 = \{V_{C2A}\}_1 = \{\omega_2\}_1 \times \{R_{C2A}\}_1 \tag{4.139}$$

$$\begin{bmatrix} V_{C2x} \\ V_{C2y} \\ V_{C2z} \end{bmatrix} = \begin{bmatrix} 0 \\ 82 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0.089 \\ -0.082 \\ -0.00013 \end{bmatrix} = \begin{bmatrix} -0.011 \\ 0 \\ -7.3 \end{bmatrix} m/s \tag{4.140}$$

Therefore, the following relation is obtained.

$$\begin{bmatrix} A_{C2x} \\ A_{C2y} \\ A_{C2z} \end{bmatrix} = \begin{bmatrix} 0 \\ 82 \\ 0 \end{bmatrix} \times \begin{bmatrix} -0.011 \\ 0 \\ -7.3 \end{bmatrix} + \begin{bmatrix} 0 \\ -2.8E8 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0.089 \\ -0.082 \\ -0.00013 \end{bmatrix} = \begin{bmatrix} 36E3 \\ 0 \\ 2.5E7 \end{bmatrix} m/s^2 \qquad (4.141)$$

It is required to define a vector $\{g\}_1$ for gravitational acceleration which for the reference frame O_1 used here would be as follows:

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -9.81 \end{bmatrix} m/s^2 \qquad (4.142)$$

For lower arm, by sum of forces and by applying Newton`s, second law gives the following relations.

$$\sum \{F_2\}_1 = m_2 \{A_{C2}\}_1 \qquad (4.143)$$

$$\{F_{B21}\}_1 + \{F_{A21}\}_1 + \{F_{C23}\}_1 + m_2 \{g\}_1 = m_2 \{A_{C2}\}_1 \qquad (4.144)$$

$$\begin{bmatrix} F_{B21x} \\ F_{B21y} \\ F_{B21z} \end{bmatrix} + \begin{bmatrix} F_{A21x} \\ F_{A21y} \\ F_{A21z} \end{bmatrix} + \begin{bmatrix} F_{C23x} \\ F_{C23y} \\ F_{C23z} \end{bmatrix} + 1.2 \begin{bmatrix} 0 \\ 0 \\ -9.81 \end{bmatrix} = 1.2 \begin{bmatrix} 36E3 \\ 0 \\ 2.5E7 \end{bmatrix} N \qquad (4.145)$$

The summation of forces in equation (4.145) leads to the first three equations:

Equation 1: $F_{B21x} + F_{A21x} + F_{C23x} = 4.3E4$ (4.146)

Equation 2: $F_{B21y} + F_{A21y} + F_{C23y} = 0$ (4.147)

Equation 3: $F_{B21z} + F_{A21z} + F_{C23z} = 3E7$ (4.148)

For the rotational equation, it is convenient to refer the vectors to the reference frame O_2 fixed in and rotating with lower arm. The rotational equations of motion for lower arm may be written as Euler`s equations of motion in vector form as follow:

$$\sum\{M_{C2}\}_{1/2} = [I_2]_{2/2}\{\alpha_2\}_{1/2} + [\omega_2]_{1/2}[I_2]_{2/2}\{\omega_2\}_{1/2} \tag{4.149}$$

In order to eliminate the inertial force $m_2\{A_2\}_1$ which is acting through the mass center, it should sum moments which are acting on mass center of lower arm. In order to carry out the moment balance, it is required to establish new relative position vectors $\{R_{BC2}\}_{1/2}$, $\{R_{AC2}\}_{1/2}$ and $\{R_{CC2}\}_{1/2}$.

Therefore

$$\{R_{BC2}\}_{1/2}^T = [-0.089\ -0.211\ 0.00013]\ m$$

$$\{R_{AC2}\}_{1/2}^T = [-0.089\ 0.082\ 0.00013]\ m$$

$$\{R_{CC2}\}_{1/2}^T = [0.211\ 0.097\ 0.01]\ m$$

By determining the moment balance of the constraint forces acting at A, B and C:

$$\sum\{M_{C2}\}_{1/2} = \{R_{BC2}\}_{1/2} \times \{F_{B21}\}_{1/2} + \{R_{AC2}\}_{1/2} \times \{F_{A21}\}_{1/2} + \{R_{CC2}\}_{1/2} \times \{F_{C21}\}_{1/2} \tag{4.150}$$

$$\sum\{M_{C2}\}_{1/2} = \begin{bmatrix} -0.089 \\ -0.211 \\ 0.00013 \end{bmatrix} \begin{bmatrix} F_{B21x} \\ F_{B21y} \\ F_{B21z} \end{bmatrix} + \begin{bmatrix} -0.089 \\ 0.082 \\ 0.00013 \end{bmatrix} \begin{bmatrix} F_{A21x} \\ F_{A21y} \\ F_{A21z} \end{bmatrix} + \begin{bmatrix} 0.211 \\ 0.097 \\ 0.01 \end{bmatrix} \begin{bmatrix} F_{C21x} \\ F_{C21y} \\ F_{C21z} \end{bmatrix} Nm \tag{4.151}$$

Considering the rotational intentional terms, the following relations are obtained.

$$\sum\{M_{C2}\}_{1/2} = [I_2]_{2/2}\{\alpha_2\}_{1/2} + [\omega_2]_{1/2}[I_2]_{2/2}\{\omega_2\}_{1/2} \tag{4.152}$$

$$\sum\{M_{C2}\}_{1/2}=\begin{bmatrix} I_{XX} & I_{XY} & I_{XZ} \\ I_{YX} & I_{YY} & I_{YZ} \\ I_{ZX} & I_{ZY} & I_{ZZ} \end{bmatrix}\begin{bmatrix} \alpha_{2x2} \\ \alpha_{2y2} \\ \alpha_{2z2} \end{bmatrix}+\begin{bmatrix} \omega_{2x2} \\ \omega_{2y2} \\ \omega_{2z2} \end{bmatrix}\begin{bmatrix} I_{XX} & I_{XY} & I_{XZ} \\ I_{YX} & I_{YY} & I_{YZ} \\ I_{ZX} & I_{ZY} & I_{ZZ} \end{bmatrix}\begin{bmatrix} \omega_{2x2} \\ \omega_{2y2} \\ \omega_{2z2} \end{bmatrix}\rightarrow \qquad (4.153)$$

$$\begin{bmatrix} 0.007366 & 0.004957 & -0.000016 \\ 0.004957 & 0.010405 & -1.83E-6 \\ -16E-6 & -1.83E-6 & 0.017642 \end{bmatrix}\begin{bmatrix} 0 \\ -2.8E8 \\ 0 \end{bmatrix}+$$

$$\begin{bmatrix} 0 \\ 82 \\ 0 \end{bmatrix}\begin{bmatrix} 0.007366 & 0.004957 & -0.000016 \\ 0.004957 & 0.010405 & -1.83E-6 \\ -16E-6 & -1.83E-6 & 0.017642 \end{bmatrix}\begin{bmatrix} 0 \\ 82 \\ 0 \end{bmatrix}$$

Equating (4.96) with (4.98) yields the rotational equations of motion for lower arm.

Equation4:

$$0.082F_{A21z}-0.00013F_{A21y}-0.00013F_{B21y}-0.211F_{B21z}-0.01F_{C21y}+0.097F_{C21z}=-1.4E6 \qquad (4.154)$$

Equation5:

$$0.00013F_{A21x}+0.089F_{A21z}+0.00013F_{B21x}+0.089F_{B21z}+0.01F_{C21x}-0.211F_{C21z}=-2.9E6 \qquad (4.155)$$

Equation 6:

$$-0.082F_{A21x}-0.089F_{A21y}+0.211F_{B21x}-0.089F_{B21y}-0.097F_{C21x}+0.211F_{C21y}=500 \qquad (4.156)$$

In order to determine these 9 forces, it should to make 3 other equations to have a set of 9 equations with 9 variables. Hence, it is considered that the point A fixed to the chassis of the vehicle, and calculates the angular momentum of this point to generate the last 3 equations. Therefore, we will have the following relation.

$$\sum\{M_A\}_{1/2}=\{R_{BA}\}_{1/2}\times\{F_{B21}\}_{1/2}+\{R_{CA}\}_{1/2}\times\{F_{C21}\}_{1/2}+\dots$$
$$[I_2]_1\{\alpha_2\}_{1/2}+[\omega_2]_{1/2}[I_2]_1\{\omega_2\}_{1/2} \tag{4.157}$$

The angular velocity and angular acceleration on point A is zero, as mentioned before, because this point is fixed in the body. Therefore, left side of the equality is zero; hence the following relation is obtained.

$$\{R_{BA}\}_{1/2}\times\{F_{B21}\}_{1/2}+\{R_{CA}\}_{1/2}\times\{F_{C21}\}_{1/2}+[I_2]_1\{\alpha_2\}_{1/2}+[\omega_2]_{1/2}[I_2]_1\{\omega_2\}_{1/2}=0 \tag{4.158}$$

It must be mentioned that in order to impose the effect of mass center on point A, which is declared in equation (4.158), it must transfer the momentum from mass center to point A. Hence, it should to calculate the I_2 according to point A.

$$[I_2]_1=\begin{bmatrix}0.015538 & -0.003908 & -0.000029\\ -0.003909 & 0.020024 & -0.000011\\ -0.000029 & 0.000011 & 0.035433\end{bmatrix}\text{kg m}^2 \tag{4.159}$$

Therefore

$$\begin{bmatrix}0\\-0.293\\0\end{bmatrix}\begin{bmatrix}F_{B21x}\\F_{B21y}\\F_{B21z}\end{bmatrix}+\begin{bmatrix}0.3\\0.015\\0.01\end{bmatrix}\begin{bmatrix}F_{C21x}\\F_{C21y}\\F_{C21z}\end{bmatrix}+\begin{bmatrix}0.015538 & -0.003908 & -0.000029\\ -0.003909 & 0.020024 & -0.000011\\ -0.000029 & 0.000011 & 0.035433\end{bmatrix}\begin{bmatrix}0\\9.7E5\\0\end{bmatrix}+\dots$$

$$\dots+\begin{bmatrix}0\\42.7\\0\end{bmatrix}\begin{bmatrix}0.015538 & -0.003908 & -0.000029\\ -0.003909 & 0.020024 & -0.000011\\ -0.000029 & 0.000011 & 0.035433\end{bmatrix}\begin{bmatrix}0\\42.7\\0\end{bmatrix}=0 \tag{4.160}$$

Finally, by solving (4.160) yields

Equation 7: $\quad -0.293F_{B21z}+0.015F_{C21z}-0.01F_{C21y}=3791 \qquad (4.161)$

Equation 8: $0.01F_{C21x} - 0.3F_{C21z} = -19423$ (4.162)

Equation 9: $0.293F_{B21x} + 0.3F_{C21y} - 0.015F_{C21x} = -17.8$ (4.163)

The 9 equations can now be set up in matrix form ready for solution:

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & -0.00013 & 0.082 & 0 & -0.00013 & -0.211 & 0 & -0.01 & 0.097 \\ 0.00013 & 0 & 0.089 & 0.00013 & 0 & 0.089 & 0.01 & 0 & -0.211 \\ -0.082 & -0.089 & 0 & 0.211 & -0.089 & 0 & -0.097 & 0.211 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.293 & 0 & -0.01 & 0.015 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0 & -0.3 \\ 0 & 0 & 0 & 0.293 & 0 & 0 & -0.015 & 0.3 & 0 \end{bmatrix} \begin{bmatrix} F_{A21x} \\ F_{A21y} \\ F_{A21z} \\ F_{B21x} \\ F_{B21y} \\ F_{B21z} \\ F_{C23x} \\ F_{C23y} \\ F_{C23z} \end{bmatrix} = \begin{bmatrix} 4.3E4 \\ 0 \\ 3E7 \\ -1.4E6 \\ -2.9E6 \\ 500 \\ 3791 \\ -19423 \\ -17.8 \end{bmatrix}$$

(4.164)

Solving equation (4.164) yields the following answers for the 9 unknowns in the Table 4.3.

Table 4.3: Dynamic force analysis of lower arm (N)

F_{A21x}	F_{A21y}	F_{A21z}	F_{B21x}	F_{B21y}	F_{B21z}	F_{C23x}	F_{C23y}	F_{C23z}
-1.9E7	43	-6.3E5	1.9E7	1.9E7	6.5E5	29655	-1.9E7	9885

Now by determining dynamic forces of lower arm of McPherson suspension system, ABAQUS for stress analysis of lower arm during its operation can be used. In dynamic mode, there are two important points for proceeding in dynamic analysis of lower arm of suspension system. The first point is related to static load condition of lower arm, according to vehicle's weight and its passengers, there is 5000 N static force in point c. The second point is related to the area where forces are imposed to lower arm. The static and dynamic forces are imposing to point c, which is an area with a radius equal to 20 mm. It must be mentioned that for dynamic forces, the important force on this point is one of them which is oriented along z axis. Because of location of lower arm in operation, this force has a significant impact. Therefore, the following pressure values are imposing to lower arm during its operation:

$$P_{Dynamic} = 7488636.4 \; {}^{N}\!/_{m^2} \qquad\qquad P_{Static} = 6585360.7 \; {}^{N}\!/_{m^2}$$

By using these forces in finite element software, such as ABAQUS, the stress condition of lower arm during its operation can be determined. In general, the Procedure that has been used for determining dynamic forces on lower arm is based on vector analysis. Vector analysis has been done by determining a desired road profile for our calculation. By assigning proper characters for vehicle, such as speed, it can consider the velocity and acceleration that imposed to the suspension system during car's movement. In order to determine the velocity and acceleration of lower arm, the velocity and acceleration of whole suspension body have been determined. Finally, by using lower arms' velocity and acceleration and using second Newton's law and rotational momentum, it is possible to determine the dynamic forces of lower arm in different orientations.

4.6 Conclusion

The first requirement for doing stress analysis is determining load condition of specific mechanical part. Hence, the condition also is the same for lower arm of suspension system. During operation of suspension system, it might be affected by imposed force by the road; consequently, different parts of the road also will be in the same situation as lower arm. In order to determine force condition of lower arm of suspension system dynamic analysis, as an analytical method for determining dynamic condition of moving parts, can be used.

Dynamic analysis has comprised three main parts; these parts can be divided to velocity analysis, acceleration analysis and dynamic force analysis. First step in determining force condition of lower arm is determining velocity and acceleration of it. Hence, there is a hand in hand relationship between velocity and acceleration of lower arm with other components of suspension system. Therefore, in first calculation, all velocity and acceleration of all components of suspension system must be determined.

By using the determined velocity and the acceleration of lower arm, which are derived by using governing equation o f motion, it is possible to write force equation of lower arm of suspension system. These equations are derived by using angular rotation of lower arm of suspension system. The important point here is choosing, the amount of velocity and acceleration of vehicle and the road profile. Because they crucially affect the other parts velocity and acceleration and consequently load force condition. Now, it is possible to use these values for stress analysis of lower arm of suspension system.

CHAPTER 5

VIBRATION AND STRESS ANALYSIS

5.1 Introduction

The study of this part aims to apply vibration and stress analysis on lower arm of suspension system which is a common part in suspension system. In this study the lower arm of McPherson suspension system has been studied. Hence, finite element method is selected for doing vibration and stress analysis. These analyses are applied by using ABAQUS, which is renowned software in finite element analysis. Moreover, some codes have been developed in MATLAB for determining frequency response of a part of our vibration model which is contained mass of lower arm suspension system.

5.2 Vibration analysis

For vibration analysis of lower arm, it is going to compare vibration of whole suspension system with optimized values of design variables and non-optimized values of design variables. The point here is, when a vibration system assigned to whole suspension system, mass of lower arm is considered in unsprung mass. Therefore, by analysis the vibration of unsprung mass, it will not be unavoidable to determine the vibration condition of lower arm during its operation.

Moreover, by using ABAQUS, it is possible to determine the natural frequency of lower arm; it is also determined by using optimized values of design variables of vibration model and by determining the natural frequency of vibration model. It is possible to compare these two frequencies to each other in order to survey condition for applying resonance and beating.

5.1.1 Natural frequency of vibration model

The natural frequency equations of quarter model of vehicle have been introduced in this sub-section. Natural frequencies of vehicle models can be found by using the un-damped and free vibration of motion equations. The normal solution of the equation of motion is given by the following equation.

$$\{x(t)\} = \{X\} e^{i\omega t} \tag{5.1}$$

Substituting equation (5.1) into equation (3.11) without damping force terms, the following generalized Eigen value problem for the natural frequencies ω and their corresponding mode shape vectors $\{X\}$ can be written as follows:

$$\left([K] - \omega^2 [M]\right)\{X\} = 0 \tag{5.2}$$

The natural frequencies are obtained from the following equation.

$$\det\left([K] - \omega^2 [M]\right) = 0 \tag{5.3}$$

Consequently, the corresponding Eigen values, mode shapes, $\{X_i\}$ for natural frequencies ω_i are found by equation (5.3). From equations (3.13), (3.14), and (5.3) following equation can be obtained.

$$\det\begin{bmatrix} k_s - \omega^2 M_s & -k_s & -ak_s & bk_s \\ -k_s & k_s + k_t - \omega^2 M_u & ak_s & -bk_s \\ -ak_s & ak_s & a^2 k_s - \omega^2 I_y & -abk_s \\ bk_s & -bk_s & -abk_s & b^2 k_s - \omega^2 I_x \end{bmatrix} = 0 \tag{5.4}$$

By determining the above determinant, the following equations can be obtained.

$$A_1 = k_s \times k_t \times \omega^4 \times I_y \times I_x \tag{5.5}$$

$$A_2 = k_s \times \omega^6 \times M_u \times I_y \times I_x \tag{5.6}$$

$$A_3 = \omega^6 \times M_s \times k_s \times I_y \times I_x \tag{5.7}$$

$$A_4 = \omega^4 \times M_s \times k_t \times a^2 \times k_s \times I_x \tag{5.8}$$

$$A_5 = \omega^4 \times M_s \times k_t \times I_y \times b^2 \times k_s \tag{5.9}$$

$$A_6 = \omega^6 \times M_s \times k_t \times I_y \times I_x \tag{5.10}$$

$$A_7 = \omega^6 \times M_s \times M_u \times a^2 \times k_s \times I_x \quad (5.11)$$

$$A_8 = \omega^6 \times M_s \times M_u \times I_y \times b^2 \times k_s \quad (5.12)$$

$$A_9 = \omega^8 \times M_s \times M_u \times I_y \times I_x \quad (5.13)$$

The natural frequency of the quarter-car model will be determined by solving the following equation; its variable has been specified as ω.

$$A_1 - A_2 - A_3 + A_4 + A_5 - A_6 - A_7 - A_8 + A_9 = 0 \quad (5.14)$$

By solving equation (5.14) according to the optimized conditions which have determined in last section, it is possible to compare natural frequency of quarter vibration model of vehicle with natural frequency of lower arm of suspension system. Hence, for imposed force frequency $\omega = 1$, the following relation will be obtained.

$$\begin{aligned} &125425762E6\ \omega^4 - 14755972E4\ \omega^6 - 3688993E5\ \omega^6 + 35411E10\ \omega^4 + 752675E8\ \omega^4 \\ &- 313564405E4\ \omega^6 - 4166E8\ \omega^6 - 8855E7\ \omega^6 + 3688993E3\ \omega^8 = 0 \end{aligned} \quad (5.15)$$

By solving this equation, natural frequency of quarter vibration model of vehicle can be determined. In order to make a comparison between different natural frequencies of vibration model, it must be mentioned that values of natural frequency are determined for sixth of road frequencies. Table 5.1 depicts these values:

Table 5.1: Natural frequency of quarter vibration model of vehicle (Hz)

Road frequency	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8
$\omega=1$	0	0	0	0	78.9	12.1	-78.9	-12.1
$\omega=2$	0	0	0	0	69	12.1	-69	-12.1
$\omega=3$	0	0	0	0	63.5	12.1	-63.5	-12.1
$\omega=4$	0	0	0	0	77.4	12.1	-77.4	-12.1
$\omega=5$	0	0	0	0	77.9	12.1	-77.9	-12.1
$\omega=6$	0	0	0	0	76.6	11.3	-76.6	-11.3

5.1.2 Natural frequency of lower arm

By determining the natural frequency of the lower arm by ABAQUS, it is possible to get practical points in redesigning lower arm in order to have better operation of it. The frequency of the road forces, according to the weight of the vehicle and its speed and path of the road, can be changed in specific ranges. In other words, the road frequency during vehicle moving will be changed between 0 to 20 Hz. Hence, by assumption the frequency of the road forces, it is possible to use it for redesigning lower arm in a range of frequencies that will not cause either resonance or beating. These two phenomena lead to increase the imposed force by the road; consequently, it will cause noise and vibration in the chassis. Table 5.2 shows both six natural frequency and Eigen values of real model of lower arm of suspension system according to the real condition of its operation.

Table 5.2: Natural frequency and Eigen value of lower arm

Mode	Natural Frequency (Hz)	Eigen Value
1	192.8	1.5E6
3	976.74	3.6E7
3	1243.8	3.8E7
4	1693.9	6.1E7
5	2424.7	1.1E8
6	2552.7	2.3E8

The ride quality normally associated with the vehicle's response to bumps which is a factor of the relatively low frequency bounces and rebound movements of the suspension system. Following a bump, the lower arm of suspension system of a vehicle will experience a series of oscillations that will cycle according to the natural frequency of it. Ride is perceived as most comfortable when the natural frequency is in the range of about 1 Hz to 1.5 Hz. When the frequency approaches 2 Hz, occupants perceive the ride as harsh.

In order to have a better understanding over our work, the vibration condition of vibration model must be determined by comparing values in optimized condition with non-optimized one. Moreover, the sprung and un-sprung masses have a significant effect on the values of natural frequency. Hence, it must be surveyed. It should be known that the amplitude affects human sensitivity to frequency; there are some frequencies that are especially uncomfortable. For example, a frequency in the range of 30 to 50 CPM will produce motion sickness.

It is obvious that the lower arm will not have any destructive resonance vibration condition in suspension system, because, there is not any compatibility between natural frequencies of lower arm and natural frequencies of vibration model of suspension system. In contrast, occurrence of beating is possible for lower arm. Frequencies of vibration model of suspension system are determined for first sixth road frequency, because, the optimized values after 6 hertz are approximately in the same ranges. Hence, the values for natural frequency for more than 6 hertz will be in the same range of 6 hertz.

By determining the natural frequency of suspension system for non-optimized values in some cases for design values, it is possible to have some natural frequencies avoiding beating for lower arm. In other way, due to the specific frequency, the values of design variables are not as same as optimized values. Therefore, there is not minimum road force on lower arm and consequently the amount of stress on lower arm of suspension system will increase.

There must be a balance between stress condition of lower arm and its vibration problem of beating. It is possible to optimize imposed force on lower arm of suspension system in order to have minimum amount of stress on lower arm. However, it can be possible to have maximum amount of vibration on lower arm. Making a balance for stress and vibration is really important, because the effect of destructive vibration can cause increasing stress. However, in non destructive vibration also, there is some destructive effect on lower arm. For instance, fatigue is a phenomenon which is unavoidable for lower arm of suspension system when operates in a vibration condition for a long period of time.

Another important point about the natural frequency of the suspension system components is deflection. The static deflection rate of the suspension is not the same as the spring rate. Springs are located inboard of the wheels where they are normally subjected to the mechanical advantage of the suspension linkages. Static deflection is related to the distance of sprung mass (essentially the body) that moves downward in response to weight. The mode shapes of the lower arm are depicted on Figure 5.1. Vibration modes of lower arm of suspension system are determined for first six vibration mode.

During operation of suspension system, its lower arm can be vibrated in several modes, these vibration modes depends on the forces which are imposed to the lower arm and its boundary condition. It is possible for lower arm to vibrate in any of these six modes depend on its condition, especially on fourth first modes.

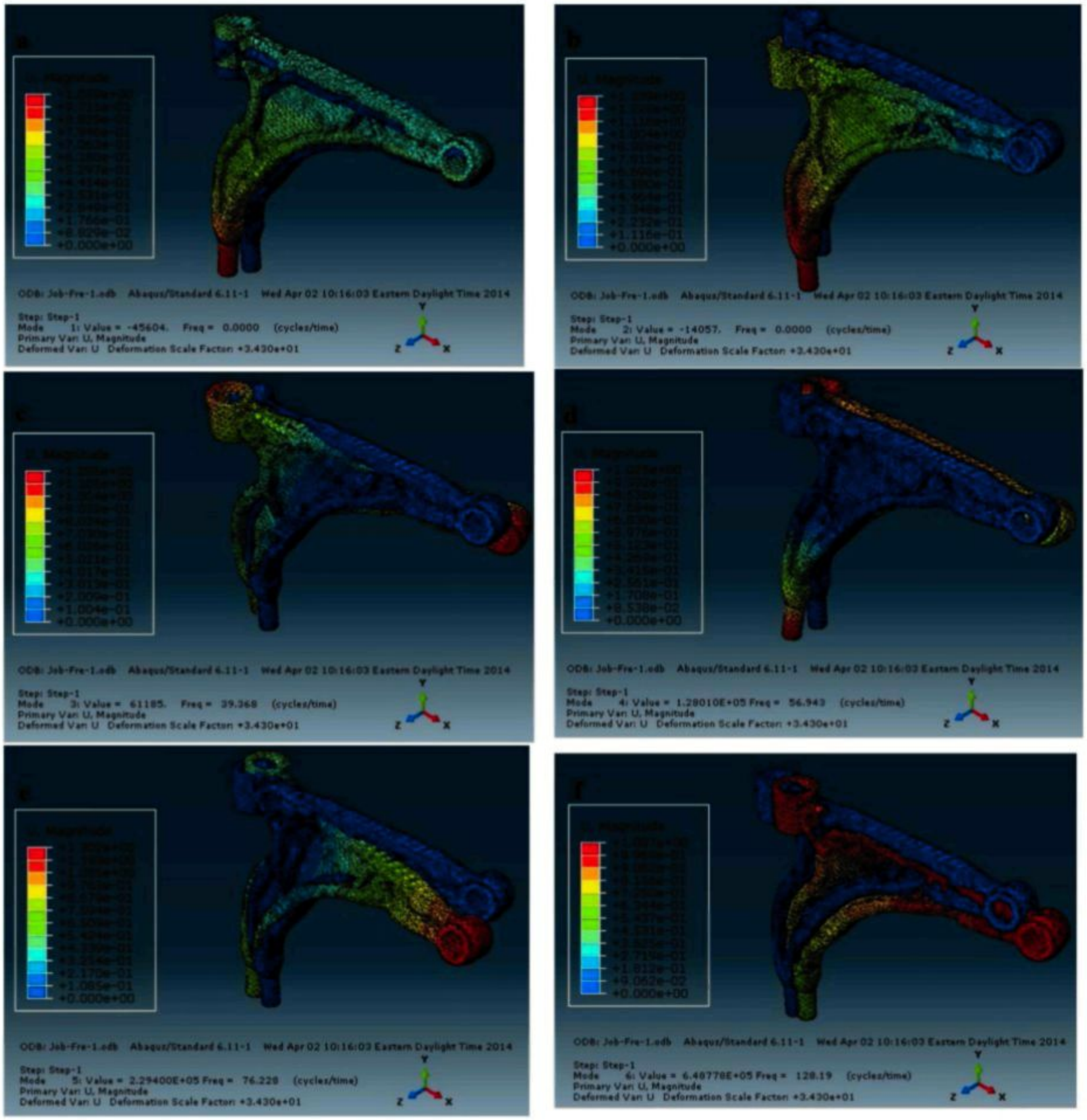


Figure 5.1: Mode shapes of lower arm of McPherson suspension system a) mode shape 1, b) mode shape 2 c) mode shape 3, d) mode shape 4, e) mode shape 5 and f) mode shape 6

5.1.3 Unsprung mass vibration

According to these optimized values, it is possible now to determine frequency response of quarter vibration model of suspension system. This response will be helpful in determining vibration analysis of suspension system after optimization. Figure 5.2 depicts time domain and frequency domain of acceleration of unsprung mass for optimized value of unsprung mass and other different values.

In order to determine frequency response of unsprung mass, it should to determine the frequency response of our vibration model; it should to determine transfer function which is used for determining acceleration response of unsprung mass. The reason for considering acceleration of unsprung mass is related to mass of lower arm of suspension system. In assigning a vibration model to vehicle and suspension system, the mass of suspension system is considered in unsprung mass. Hence, in order to determine vibration condition of lower arm of suspension systems during its operation, it is possible to determine vibration condition of unsprung mass.

Matlab is used for developing a code for determining transfer function of acceleration of unsprung mass. For vibration analysis, FFT analysis is used. A fast Fourier transform (FFT) is an algorithm to compute the discrete Fourier transform (DFT) and it's inverse. Fourier analysis converts time (or space) to frequency and vice versa; an FFT rapidly computes such transformations by factorizing the DFT matrix into a product of sparse (mostly zero) factors. Hence, time domain diagram of transfer function is determined and using FFT. It is possible to determine the specific frequency that the unsprung mass responded in that.

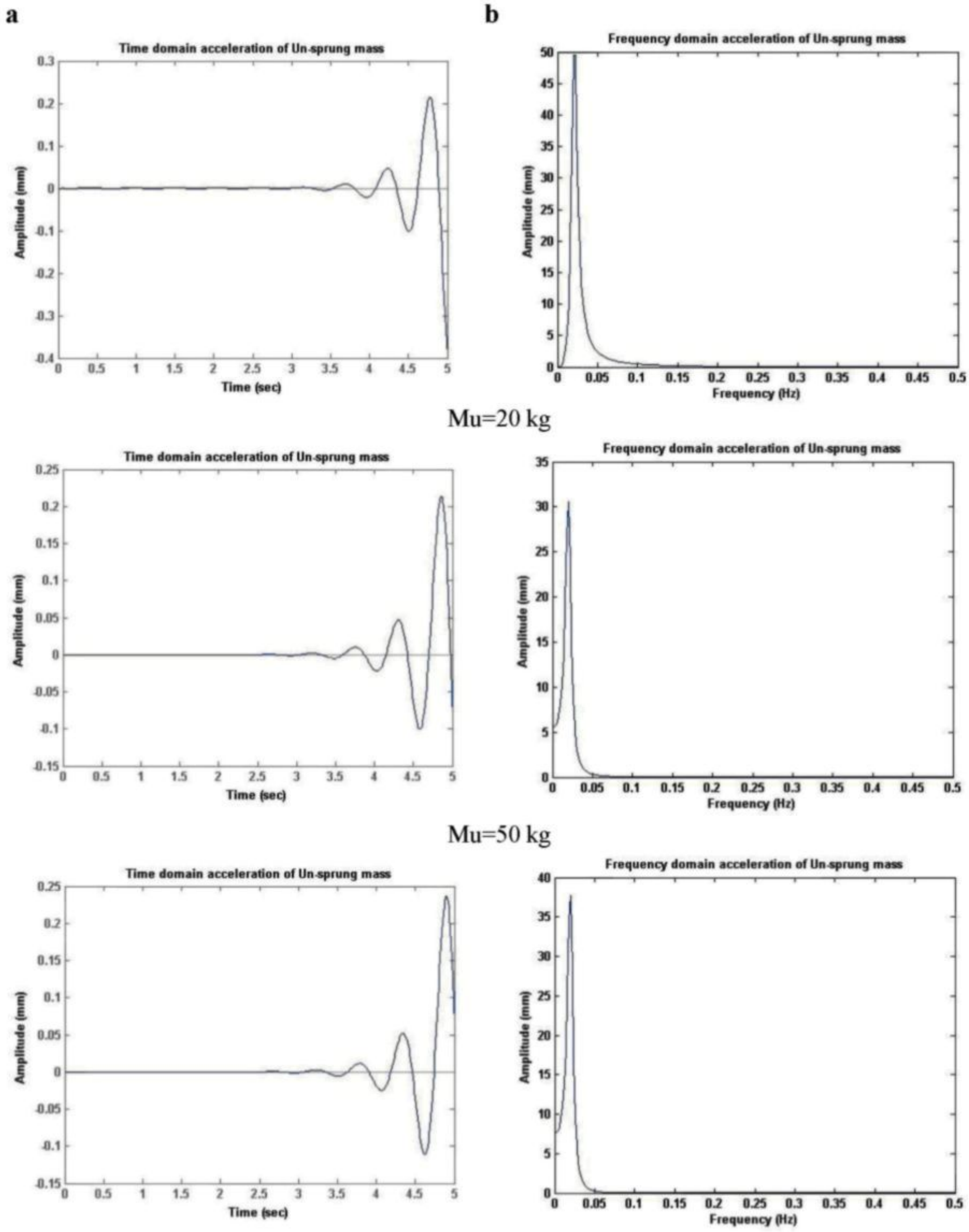


Figure 5.2: Time (column a) and frequency (column b) domain of acceleration of un-sprung mass

In order to have a better mastery over behavior of acceleration of unsprung mass, frequency response of unsprung mass according to different values of optimized values rather than non optimized values must be considered. The reason for considering unsprung mass has been related to lower arm of suspension system. For vibration modeling of a suspension system, the mass of suspension system and its components are considered as unsprung mass. Figure 5.3 depicts frequency response amplitude of acceleration of unsprung mass according to both optimized and non optimized values of design variable.

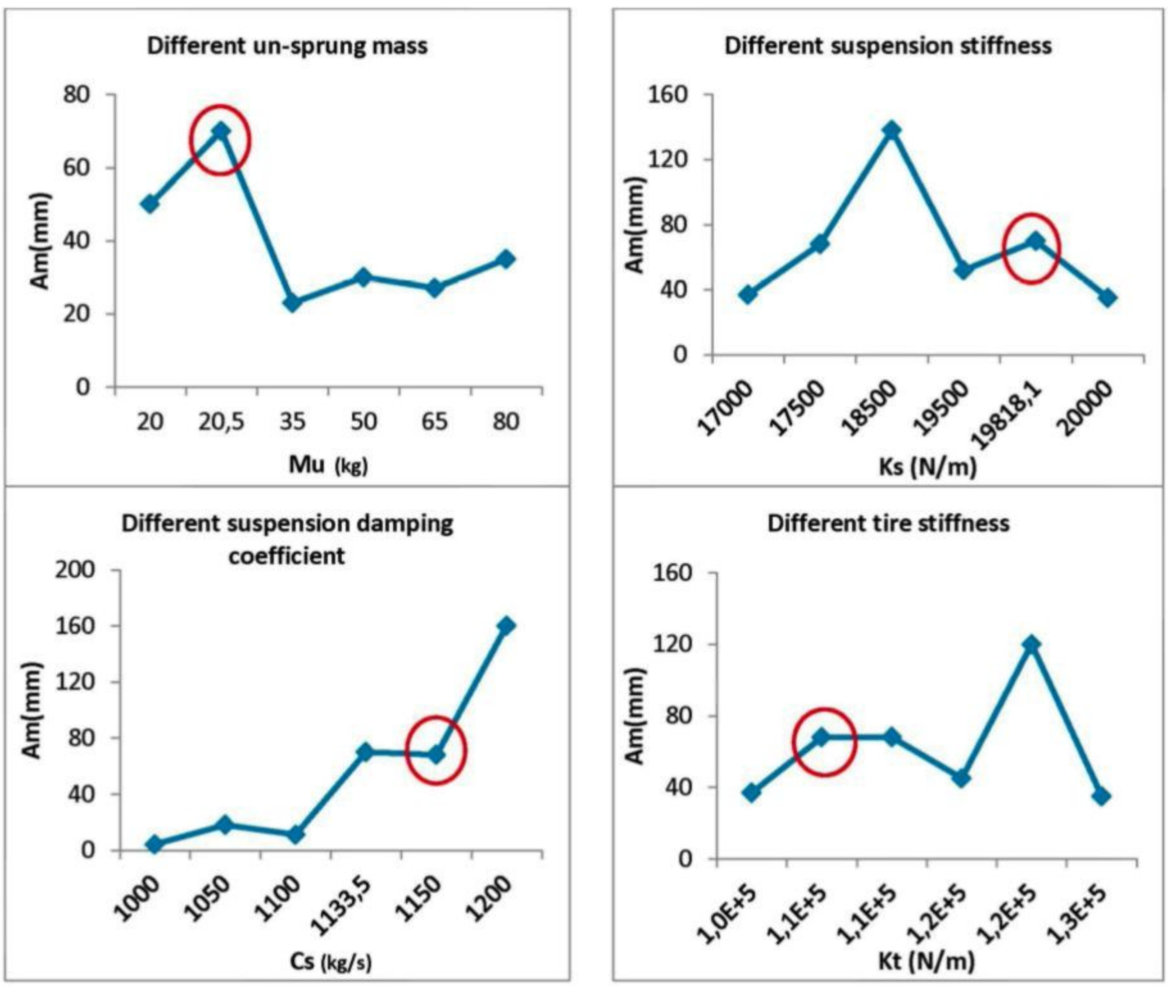


Figure 5.3: Behavior of acceleration of un-sprung mass according to different shape variables

As it obvious in the Figure 5.3, all the optimized values have been depicted by red circle. In unsprung mass, by increasing values of the mass, the vibration reduces and the optimized mass has maximum vibration amplitude. As mentioned before, this study focuses on unsprung mass, because it has contained lower arm mass. Therefore in optimized values of unsprung mass, low vibration resistance is obtained. The same condition is occurred in tire stiffness; there is almost maximum vibration in optimized value of tire stiffness. The same condition is also existed for suspension damping coefficient; the optimized value of damping coefficient has almost maximum vibration, however, the optimized value for damping coefficient take upper level of its range. In figure of suspension stiffness, by increasing values of suspension stiffness, vibration is almost increased. Moreover, the optimized value of suspension stiffness is located on upper of suspension stiffness range.

5.3 Stress analysis

Regarding to the analytical analysis applied to determine velocity and acceleration of lower arm of suspension system, it is possible to determine load condition of lower arm in attached to the chassis and to the other linkages of suspension system. By simulating the load and boundary conditions of lower arm in ABAQUS and by assigning proper material and physical properties such as density, young module, and poison ration, it is possible to do stress analysis in dynamic condition.

5.1.4 Material properties

CAE is considered as an important tool for automotive production. Chassis mounted components, such as suspension system and all its components, are subjected to random excitations from the road during its life cycle. During components design, the component should sustain the random vibration loads for the life cycle of the vehicle. These random vibration loads are also responsible for the fatigue failure of the component.

The material used in producing this type of lower arm is completely different rather than regular materials used for producing different parts of the suspension systems. The base material used in producing this arm is aluminum which is light material. Due to the natural properties of aluminum, distribution of stress through it will be distinguished rather than steel. Table 5.4 depicts the properties of lower arm and its ingredient material which is aluminum A357. It must be mentioned that the properties of lower arm is determined by designing software, Solid-works.

Table 5.3: Properties of lower arm and its ingredient material (Al357)	
Density kg/m^3	2670
Mass kg	1.197
Volume mm^3	448416.3
Surface area mm^2	101621.005
Coefficient of Poisson	0.33
Modulus of Elasticity N/m^2	72.2E9
Elongation Mpa	11.3±1.4%
Elastic Resistance Mpa	298.1±6.9

The load and boundary conditions considered in analysis of lower arm of suspension system has been related to the operation and position of this part in suspension system. In dynamic condition, it must to tolerate the forces created by the weight of the car with passengers, dynamic force imposed to lower arm by its linkage to the tire and acceleration of its mass due to movement. However, the spring and damper used in other part have some duty to reduce the forces; hence, this part will be imposed by pressure load and body acceleration.

5.1.5 Force and boundary condition

According to the boundary condition of lower arm in real condition, there are two fixed regions which have been attached to the vehicles chassis. Figure 5.4 depicts this boundary condition, the orange regions present positions that lower arm has been attached to the vehicle's chassis. The boundary condition has not any kind of rotation or displacement in any orientation.

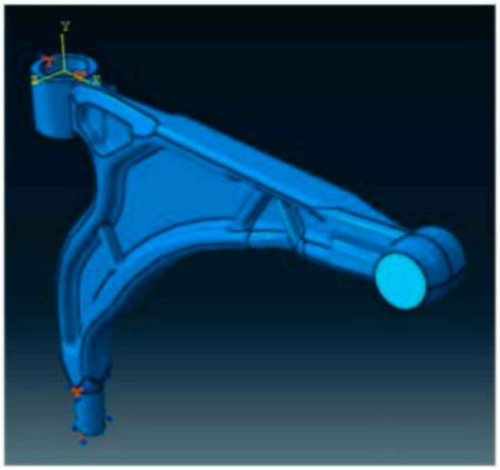
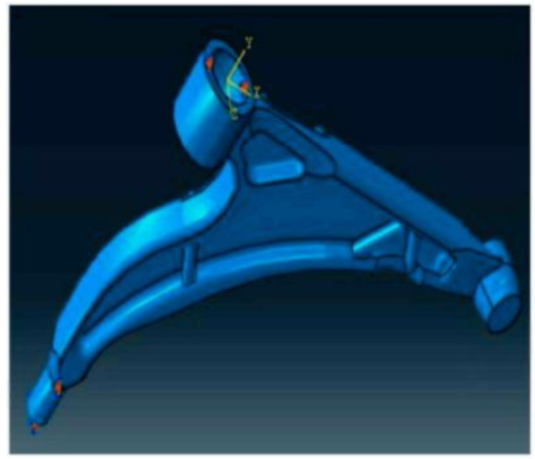
 

Figure 5.4: Boundary condition of lower arm of suspension system

The dynamic load condition of lower arm of suspension system has been determined analytically. Hence, it has been possible to impose them to lower arm, in order to make a real condition in analysis. Figure 5.5 depicts load condition of lower arm in dynamic condition, which has contained acceleration force in figure (a), static force in figure (b), dynamic force in figure (c) and combination of forces (d).

The body force in lower arm is due to mass acceleration during operation of suspension system; this acceleration has been made by angular acceleration. The static force has been created by weight of vehicle and its passengers; the value of static force is F= 5000 N and its equal pressure according to related area is P= 6585360.73 N/m2. The dynamic force has been made by reaction of other parts of suspension system and its value is F=5686 N and pressure is P= 7488636.4 N/m2.

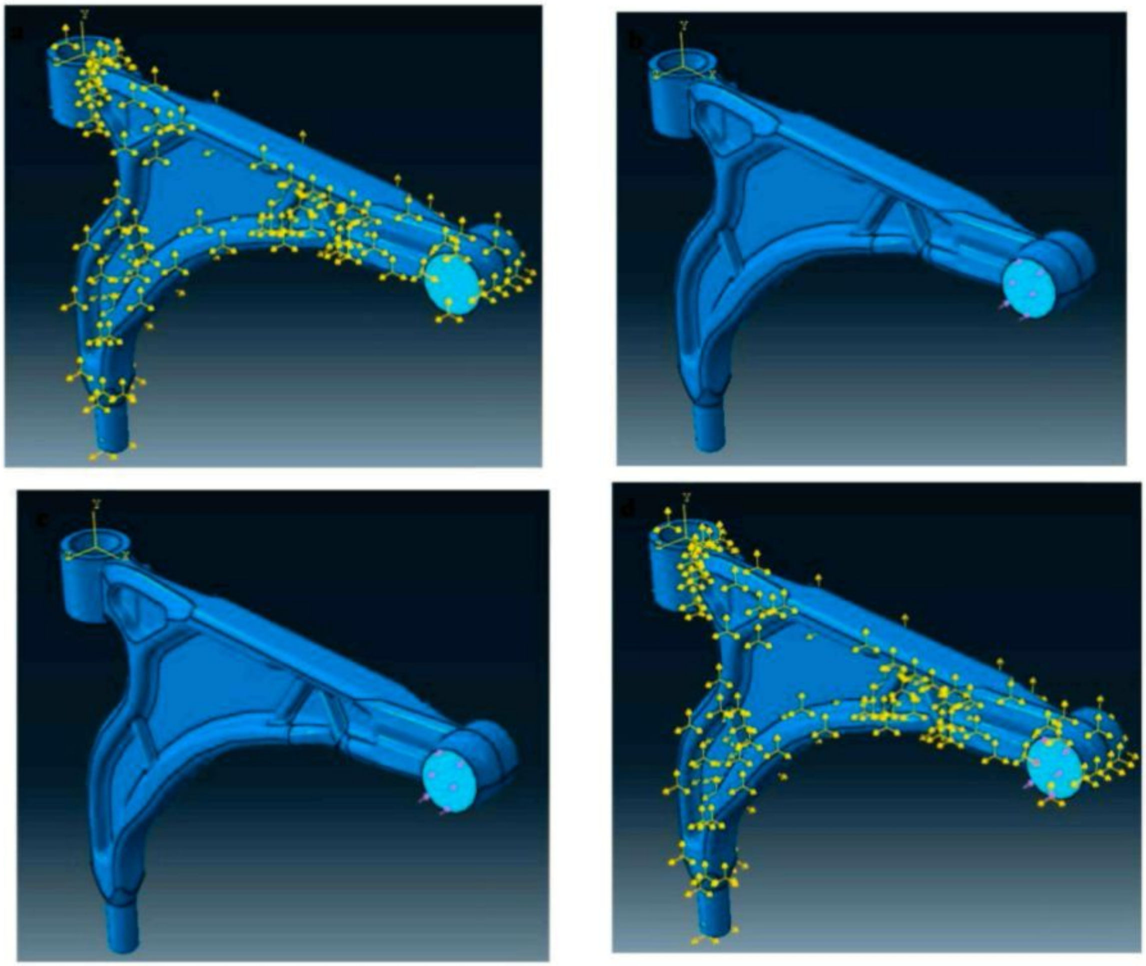


Figure 5.5: Force condition of lower arm of suspension system a) acceleration force, b) static force, c) dynamic force, d) combination of forces

5.1.6 Mesh properties

A finite element analysis of any physical problem requires a mesh of finite elements to be generated. The generation of finite element meshes is a fundamental step in analysis using any finite element software such as ABAQUS, as well as, all finite element methods. The results accuracy of the finite element analysis measured on the exact solution of the mathematical model depends highly on the use of an appropriate mesh. Hence, the element mesh type, which is used in ABAQUS in our analysis, is tetrahedral. Figure 5.6 depicts two sizes of meshes. Figure (a) shows bigger mesh and figure (b) shows smaller one.

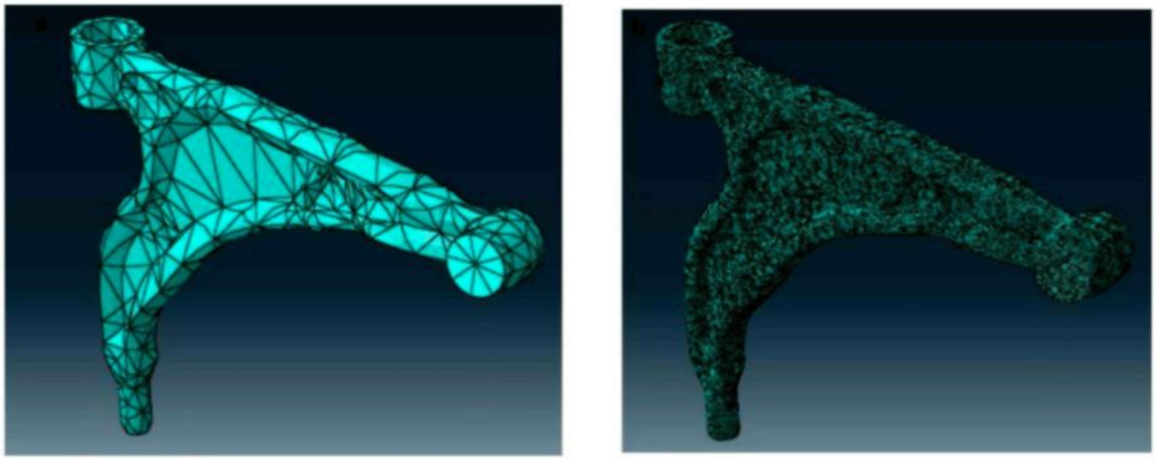


Figure 5.6: Mesh condition of lower arm of suspension system a) bigger mesh b) smaller mesh

The mesh size affects the value of created increments in its problem. In other words by reducing mesh size, number of nodes and faces in our part will be increased; required time for solving the problem will be increased respectively. Main problem in increasing size of the mesh has been related to geometry problem. Lower arm contained a lot of layers and sub-parts; in other words, it has a complicated geometry. Hence, by selecting big size in meshing, disordering is occurred in this part. Therefore, the part will not be meshed for analysis.

5.1.7 Stress condition

In order to survey the results of the analysis on lower arm of suspension system, two paths have been considered for surveying of distribution of stress and displacement in the lower suspension arm. The paths have been selected between the loaded area and fixed area. Figure 5.7 depicts the paths on lower arm.

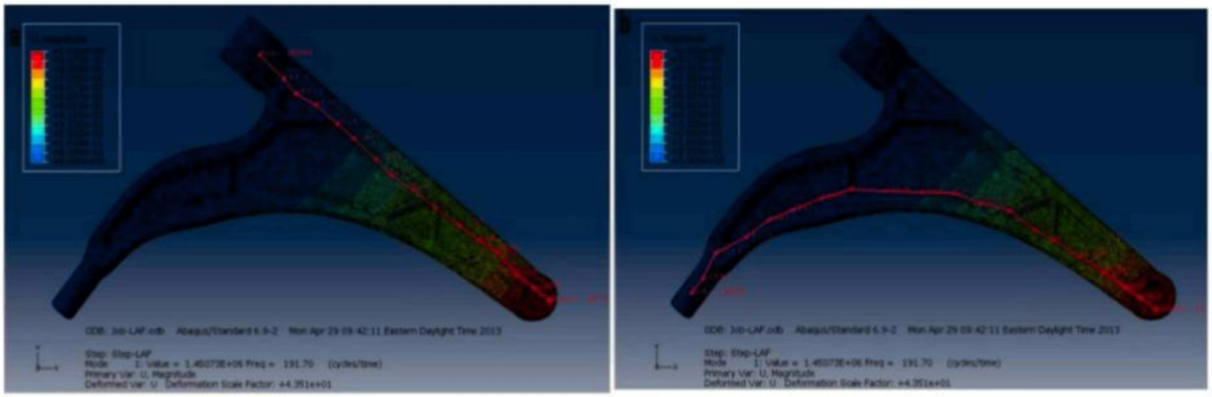


Figure 5.7: Lower arm paths for analysis, a and b are respectively path number 1 and path number 2

By running the program in order to impose the loads to the lower arm, distribution of stress and displacement can be detective. Figure 5.8 shows the maximum principal stress in the lower arm. Low stress regions have been determined by blue color and respectively, high stress places have been determined by red color. Generally, at each counter of stress, it is obvious that the amount of highest stress has been occurred in some regions which are close to the fixed point and equals to 4.7 Mpa.

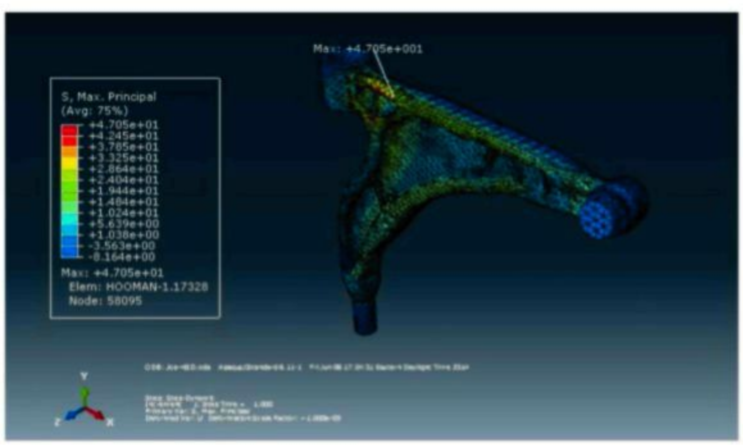


Figure 5.8: Maximum principal stress diagram of lower arm of suspension system

By considering the stress on the first and second path, the distribution of stress in the first path is more than the distribution of stress in the second path. Figure 5.9 depicts the stress on the path one and path two. The path 1, which is the start of imposing load until the end of the path 1, stress has been changed from 0 to 20 MPa. In contrast, the distribution of stress in the path 2 has been changed from 0 to 12 MPa. Hence, stress distribution on path 1 is much more than distribution of stress on path 2. Moreover, data shown in Figure 5.8 strongly indorse data shown in Figure 5.9. This can be related to the stress which is occured in the length of path 1.

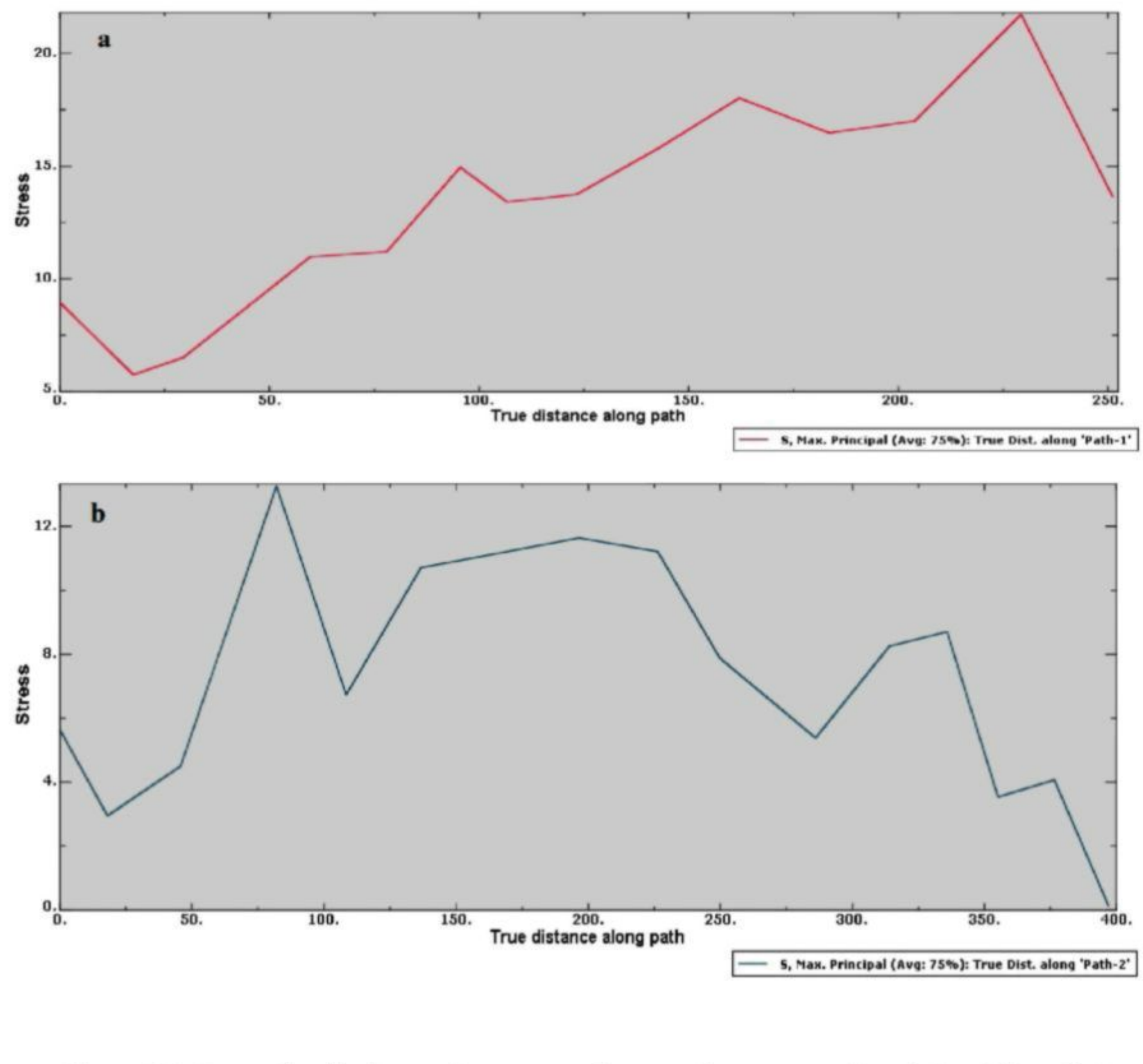


Figure 5.9: Stress distribution on lower arm of suspension system, a) path 1 and b) path 2

During imposing load on lower arm of suspension system, there will be deformation on the lower arm. Figure 5.10 depicts the deformation of lower arm due to pressure load with a deformation shape factor of 3.136E6. It is obvious that the regions of lower arm, which are close to place of imposed load, have highest amount of displacement. Respectively, the regions which are occurred in the fixed regions have lower displacement. According to the Figure 5.10, the highest amount of displacement equals to 1.229E-12 at the place of imposed load and the lowest displacement is occurred on mounted place with amount of zero.

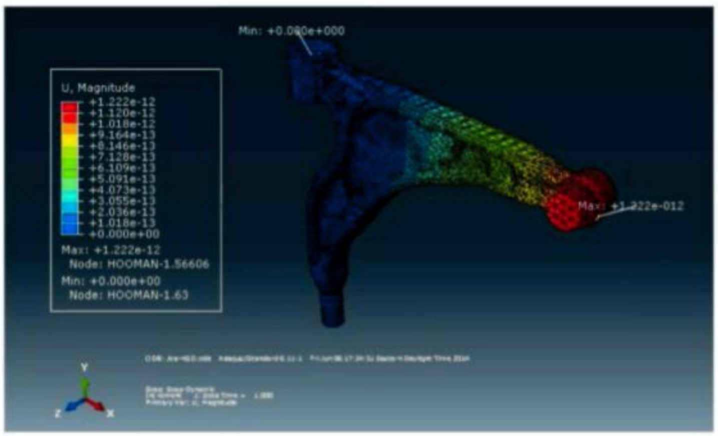


Figure 5.10: Displacement of lower arm of McPherson suspension system

5.4 Conclusion

Stress and vibration analysis have been studied at this part of research. According to the objectives of the thesis, this is the important part of analysis. Regarding to vibration analysis, it has been applied for lower arm of suspension system, sprung mass of vibration model of vehicle and its suspension system, as well as, for unsprung mass of vibration model of suspension system. Regarding to stress analysis using finite element method, stress condition of lower arm during operation of suspension system has been surveyed.

In order to apply vibration analysis, MATLAB and ABAQUS have been used; it is used to determine frequency response of unsprung mass of vibration model, natural frequency of sprung mass of vibration model and natural frequency of lower arm of suspension system. For vibration analysis, natural frequency of lower arm of suspension system has been compared by natural frequency of sprung mass of suspension system. This comparison is studied to determine different conditions of resonance and beating which are destructive for lower arm of suspension system. This is related to increase stress in lower arm of suspension system which is destructive for the system.

Stress analysis has been applied by using ABAQUS, as strong finite element software for stress analysis. One of the useful ways for stress analysis is finite element method; it is really useful and precise, especially by using software. By modeling lower arm of suspension system in modeling software, such as SOLIDWORKS, it is possible to import it to ABAQUS for stress analysis. Different steps must be done for this analysis in ABAQUS, such as determining the properties of lower arm, step analysis for type of problem, force and boundary condition, and mesh condition. It must be mentioned that for using ABAQUS, dimension of imported model is really important; it means that according to the dimension, which can be either meter or millimetre, the other properties such as density and young modulus must be changed.

It has been determined that, it is possible to the lower arm of suspension system to get beat during its operation by effect of sprung mass. It means that natural frequency of lower arm of suspension system and natural frequency of sprung mass of vibration model of suspension system can have compatibility on each other, this is destructive. Moreover, for stress condition of lower arm, distribution of stress is intensive near the boundary condition and in stretched side of lower arm.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

The general analysis applied on suspension system and especially its lower arm has been studied focusing in stress and vibration. Road profile has important role in vibration and stress condition of suspension system, because its irregularities impose directly to the vehicle's tire. Hence, power spectrum density of road quality affected the calculations in two ways. First, in dynamic analysis of suspension system by its effect on both velocity and acceleration; second, in optimization, by its role in determining the objective function for optimizing of imposed force to the vehicle's tire by the road.

In order to determine the objective function for optimizing imposed force to the vehicle's tire by the road, a proper vibration model must be assigned to the vehicle and its suspension system. The type of this model is very important in yielding objective function. In other words, the numbers of degrees of freedom in vibration system are important. In order to have a vibration model close to the real condition of vehicle and its suspension system, a 3d quarter model of vehicle and its suspension system have been considered.

According to the goal of the optimization, a proper method for determining an objective function must be selected. Genetic algorithm has been used for optimization in this study; as the numbers of desired variables that must be optimized are more than 2 variables and desired variables are related to each other. Hence, the method must be able to determine values of variable with convinced approximate, moreover, behavior of variables is relatively changed.

In order to have optimization by minimizing of road force, optimization toolbar of Matlab has been used. Applying genetic algorithm has different stages with specific procedure, where all of these stages are collected in optimization Matlab toolbar. Before using optimization toolbar, objective function of road force must get discrete. Different codes are developed in Matlab in order to prepare an entrance data for optimization toolbar.

According to the optimized values of design variables which are used for minimizing road force, it is possible to survey vibration condition of lower arm of suspension system. Surveying has been done by determining a transfer function for accelerating of unsprung mass, as well as, by using optimized values of design variables. The FFT of transfer function is calculated in order to determine the respond frequency and its amplitude. Comparing the determined values can be helpful for getting mastery over vibration operation of lower arm of suspension system according to optimized and non optimized values of design variables of suspension system.

Moreover, according to optimized values of design variables of suspension system, it is possible to determine natural frequency of vibration model of vehicle and its suspension system. These different natural frequencies are determined according to different frequencies of the road forces. Furthermore, by using ABAQUS as finite element software, it is possible to determine natural frequencies of lower arm of suspension system. By comparing these two different frequencies, it is possible to survey vibration behavior of lower arm of suspension system during its operation according to natural frequency of vibration model of vehicle and suspension system.

For stress analysis, in first step, load condition of lower arm of suspension system must be determined. Hence, a typical model of McPherson suspension system has been selected for analysis. Relating to the road profile considered for analysis and velocity of vehicle, it is possible to obtain both velocity and acceleration governing equations. These equations are used for whole components of McPherson suspension system in order to determine their velocity and acceleration. These determined values are used to determine dynamic force condition of lower arm of suspension system during its operation. By using dynamic forces which are governing on lower arm of suspension system, in ABAQUS, it is possible to determine stress condition of lower arm during its operation.

6.2 Contribution and conclusion

In general, two different aspects of lower arm of suspension system have been studied in this work which is stress and vibration condition. In order to determine these aspects, specific methods have been used for getting these goals. At first, it must be mentioned that, this study is planned based on modeling and simulation. Hence, the main important part in this study has been done by using analytical software, namely ABAQUS, SOLIDWORKS and MATLAB.

Regarding to vibration analysis, different aspects of science concepts have been used; optimization for minimizing is the important approach applied by using genetic algorithm. In order to determine frequency response of our vibration model, Pseudo excitation is used. Finally, for determining vibration behavior of acceleration of lower arm during its operation, Fast Fourier Transform is used to determine responded frequency of acceleration of lower arm. Most of the work done in this part has been developed by writing codes and programming.

Regarding to stress analysis, vector and dynamic analysis are used. For using finite element method in order to determine stress condition of lower arm, force condition of lower arm must be determined. Analytical analysis has the most contribution in this part in order to determine load condition of lower arm. Hence, the vibration and stress condition of lower arm have studied separately; however, in some ways the influence of each parameter on each of other parameters have been considered.

The most important variable in optimizing is unsprung mass, because, studying lower arm of suspension system is the aim of this study; moreover, the mass of lower arm has been contented in unsprung mass. According to the optimization results in each frequency condition of road force, the unsprung mass has got minimum values, in order to reduce dynamic forces of the road. However, in 2 frequencies of road (2 and 3 Hertz), the amount of optimized values of unsprung mass have not minimum values; they have got lower values in variation range of unsprung mass. Hence, in order to have minimum value of road force on lower arm of suspension system and consequently lower stress on it, the amount of mass in lower arm must be as minimum value.

In the other side, by applying vibration analysis on suspension system, it is clarified that by reducing the unsprung mass, the vibration residence will be decreased. It means that at minimum unsprung mass, there is maximum amplitude in vibration. In first optimized value of road frequency for design variables for suspension system, in order to have minimum amount of road force, the amount of unsprung mass must be minimized. Therefore, increasing in vibration is unavoidable. There is reverse relation between vibration of lower arm and its stress condition according to mass factor.

For damping coefficient of suspension system, the optimized value for first road frequency has almost maximum amount of value in its limits. As well as, the amount of vibration at this optimized point is more than that the other values of damping coefficient for non optimized values. Therefore, in order to have minimum values of road force, it should to increase amount of damping coefficient. However, by increasing the damping coefficient, vibration will increase.

Regarding to suspension stiffness in order to reduce the amount of road force on lower arm, it should to have maximum amount of suspension stiffness. In other words, the optimized values for suspension stiffness according to its range limits will be close to upper limits of suspension stiffness. The amount of vibration at this point approximately is as equal as other non optimized values.

Finally, in order to have minimum amount of stress on lower arm of suspension system, the tire stiffness value must be minimum for first frequency road force. In other words, for decreasing road force, tire stiffness has to be lower value in its range. It also has approximately maximum vibration rather than other non optimized values. It must be mentioned that, in order to reduce the road force to have minimum stress on lower arm, it should to be optimized design variables of suspension system. By considering these optimized values, it is obvious that the road force is decreased on lower arm. However, the amount of vibration rather than non optimized values is increased. Hence, in order to have an optimum suspension system character, it should to consider effect of vibration and stress at the same time.

Natural frequency of lower arm and vibration model of suspension system, have been determined, hence, in lower frequency of lower arm, there are more capability of beating. Where, the first natural frequency of lower arm can be a multiple of first and second natural frequency of vibration model of vehicle and its suspension system. Therefore, during operation of suspension system, in some road frequencies, it is possible for lower arm to start to beat.

By applying finite element analysis in dynamic situation, it is obvious that the boundary condition of the lower arm crucially has been affected by both dynamic and static forces. These forces are imposing to the lower arm, as well as, affecting on acceleration of lower arm; these forces are considerable. The maximum value of stress is occurred in fixed region closed to the region of imposed force. If it is considered the nearest fixed region and force region as two ends of a beam, the results are compatible according to general laws of strength of material about stress.

6.3 Recommendation for future work

There are recommendations for future work that can be applied for advancing the following studies. The determined parameters of suspension system that cause reduction in imposed force by the road to the suspension system, as well as, the optimized design values of suspension system, have been surveyed. It is possible to offer specific advanced works based on these facts by doing experimental tests and some advancing simulation.

It is possible to determine vibration behavior of acceleration of unsprung mass in all frequencies of road force. By computing the FFT diagram of unsprung mass in different frequencies, it is possible to determine an exact idea about vibration behavior of lower arm according to optimized design values of suspension system. Hence, advancing vibration analysis to a variety range of frequencies can be useful in understanding vibration behavior of lower arm of suspension system.

In order to reduce stress on lower arm of suspension system, it should to optimize force of the road, because, it has an important role and effect on internal forces of other members of suspension system. Hence, specific design variables of suspension systems can be determined; these variables have an important role in designing a suspension system. However, the effect of these optimized values is not determined in lower arm of suspension system. Therefore, it will be important to follow the effect of reducing road force on lower arm.

Load condition of lower arm of suspension system can be determined according to the different values of design variables of suspension system for optimized and non optimized values in different road frequencies. By determining the load condition of lower arm of suspension system according to optimized values of suspension system, it is possible to use these forces for finite element analysis, in order to determine stress condition of lower arm. This kind of analysis will give a visual point of view about the stress condition of lower arm in different force conditions. Moreover, comparing the force condition with non optimized values variables of suspension system also will be very useful.

A part of calculation and simulation is done by MATLAB for optimization analysis and stress analysis is done by ABAQUS. It is possible to synchronize these software programs together. It directly uses optimization values for calculating forces and imposes them directly to lower arm in ABAQUS. From this factor, it is possible to use outgo data of MATLAB in order to do some design optimization in lower arm of suspension system for redesigning it for reducing its mass in order to have less stress on it.

It should to validate natural frequency of lower arm by experimental analysis, in order to make a comparison between finite element results and experimental results. It may be possible to do the same thing on dynamic force of lower arm. Hence, by imposing desired values of road force, it is possible to calculate the acceleration of mass center of lower arm, as well as, force reaction of its boundary condition.

References

[1] Ebrahimi, F., 2011, ADVANCES IN VIBRATION Edited by Farzad Ebrahimi, Janeza Trdine 9, 51000 Rijeka, Croatia.

[2] Blekhman, I. I., 2000, Vibrational Mechanics, World Scientific publishing Co. Pte. Ltd.

[3] Krodkiewski, J. M., 2008, "Mechanical vibration," The University of Melourne.

[4] Sharifi, M., and Shahriari, B., 2012, "Pareto Optimization of Vehicle Suspension Vibration for a Nonlinear Half- car Model Using a Multi-objective Genetic Algorithm," **1**(8), pp. 17–22.

[5] Submitted, T., 2010, "RANDOM VIBRATION OF A ROAD VEHICLE," (July).

[6] Loading, F. E., Numerical, D., Is, I., and Slow, V., "Dynamic Analysis by Numerical Integration," pp. 1–12.

[7] Blüthner, R., Hinz, B., Menzel, G., Schust, M., and Seidel, H., 2006, "On the significance of body mass and vibration magnitude for acceleration transmission of vibration through seats with horizontal suspensions," J. Sound Vib., **298**(3), pp. 627–637.

[8] Salih, W. M., 2011, "A novel design of lower vehicle arm based on optimization technique," **6**(4), pp. 768–776.

[9] Hemin, M. M., and Alokaidi, S. H., 2011, "Optimization of Lower Arm Vehicle Using Response Surface Methodology," **1**, pp. 150–164.

[10] Qassem, W., 1996, "Model prediction of vibration effects on human subject seated on various cushions.," Med. Eng. Phys., **18**(5), pp. 350–8.

[11] Analysis, S., Truck, O. N., Husaini, M., and Abd, B. I. N., 2009, "A report submitted in fulfillment of the requirements for the award of the Bachelor of Mechanical Engineering with Automotive Engineering," (November).

[12] Saoudi, A., Bouazara, M., and Marceau, D., 2011, "Fatigue Failure Study of the Lower Suspension Vehicle Arm Using a Multiaxial Criterion of the Strain Energy Density," Strojniški Vestn. – J. Mech. Eng., **57**(04), pp. 345–356.

[13] Bogsjö, K., 2007, "Evaluation of stochastic models of parallel road tracks," Probabilistic Eng. Mech., **22**(4), pp. 362–370.

[14] Gobbi, M., and Mastinu, G., 2001, "Analytical Description and Optimization of the Dynamic Behaviour of Passively Suspended Road Vehicles," J. Sound Vib., **245**(3), pp. 457–481.

[15] Hong Hao, Member, ASCE, and T. C. A., 1998, "Analytical modeling of traffic-induced ground vibrations.pdf," J. Eng. Mech.

[16] Qu, J., and Shih, A. J., 2003, "Analytical Surface Roughness Parameters of a Theoretical Profile Consisting of Elliptical Arcs," Mach. Sci. Technol., **7**(2), pp. 281–294.

[17] Dziuk, G., Elliott, C. M., and Kingdom, U., 2007, "SURFACE FINITE ELEMENTS FOR PARABOLIC," **25**(4), pp. 385–407.

[18] Larson, M. G., and Bengzon, F., 2010, "The Finite Element Method: Theory, Implementation, and Practice."

[19] David Murray, 2011, "Frequency Response and Continuous-time Fourier Series Recall course objectives."

[20] Fonseca, C. M., and Fleming, P. J., 1993, "Genetic Algorithms for Multiobjective Optimization :," (July).

[21] Segla, S., 2007, "Optimization and comparison of passive , active , and semi-active vehicle suspension systems Technical University of Kosice," **2**, pp. 0–5.

[22] Shaopu, Y., Shaohua, L. I., and Yongjie, L. U., 2009, "Dynamics of vehicle-pavement coupled system based on a revised flexible roller contact tire model," **52**(3), pp. 721–730.

[23] Engineering, C., and Engineering, M., 2002, "ANALYSES OF DYNAMIC RESPONSE OF VEHICLE AND TRACK COUPLING SYSTEM WITH RANDOM IRREGULARITY OF TRACK VERTICAL PROFILE," **258**, pp. 147–165.

[24] Baillargeon, B. P., Vel, S. S., and Koplik, J. S., "Utilizing ABAQUS to Analyze the Active Vibration Suppression of Structural Systems," pp. 81–94.

[25] Car, L. T., and Miata, M., 2004, "Suspension Systems," pp. 171–187.

[26] Roy, S., Liu, Z., Systems, M., and Systems, S., 2006, "ROAD VEHICLE SUSPENSION AND PERFORMANCE EVALUATION USING A TWO-DIMENSIONAL VEHICLE MODEL," pp. 1–26.

[27] Solomon, U., and Padmanabhan, C., 2011, "Semi-active hydro-gas suspension system for a tracked vehicle," J. Terramechanics, **48**(3), pp. 225–239.

[28] No, A., and Shirgaon, G., 1999, "OPTIMUM DESIGN OF A PASSIVE SUSPENSION SYSTEM OF A VEHICLE SUBJECTED TO ACTUAL RANDOM ROAD EXCITATIONS," **219**, pp. 193–205.

[29] Rychlik, I., 2012, "Modelling of road profiles using roughness indicators."

[30] Mú, P., 2012, "Longitudinal Road Profile Spectrum Approximation by Split Straight Lines," **1**(February), pp. 243–251.

[31] Loizos, a., and Plati, C., 2008, "An alternative approach to pavement roughness evaluation," Int. J. Pavement Eng., **9**(1), pp. 69–78.

[32] Sun, L., 2003, "Simulation of pavement roughness and IRI based on power spectral density," Math. Comput. Simul., **61**(2), pp. 77–88.

[33] Patil, M. K., and Palanichamy, M. S., 1988, "A mathematical model of tractor-occupant system with a new seat suspension for minimization of vibration response," Appl. Math. Model., **12**(1), pp. 63–71.

[34] Xia, H., Cao, Y. M., and De Roeck, G., 2010, "Theoretical modeling and characteristic analysis of moving-train induced ground vibrations," J. Sound Vib., **329**(7), pp. 819–832.

[35] Verros, G., 2005, "Design Optimization of Quarter-car Models with Passive and Semi-active Suspensions under Random Road Excitation," J. Vib. Control, **11**(5), pp. 581–606.

[36] Agharkakli, A., Sabet, G. S., and Barouz, A., 2012, "Simulation and Analysis of Passive and Active Suspension System Using Quarter Car Model for Different Road Profile," **3**, pp. 636–644.

[37] Litak, G., Borowiec, M., Friswell, M. I., and Szabelski, K., 2008, "Chaotic vibration of a quarter-car model excited by the road surface profile," Commun. Nonlinear Sci. Numer. Simul., **13**(7), pp. 1373–1383.

[38] MATLAB, 2011, "Frequency Response (FR) Principle Transmissibility Applications to automobile suspension design and."

[39] Lin, J. ., Zhang, Y. ., Li, Q. ., and Williams, F. ., 2004, "Seismic spatial effects for long-span bridges, using the pseudo excitation method," Eng. Struct., **26**(9), pp. 1207–1216.

[40] Lin, J. H., Zhang, Y. H., and Zhao, Y., 2011, "Pseudo Excitation Method and Some Recent Developments," Procedia Eng., **14**, pp. 2453–2458.

[41] Guo, X.-L., and Li, D.-S., 2004, "Experiment study of structural random loading identification by the inverse pseudo excitation method," Struct. Eng. Mech., **18**(6), pp. 791–806.

[42] Verros, G., 2005, "Design Optimization of Quarter-car Models with Passive and Semi-active Suspensions under Random Road Excitation," J. Vib. Control, **11**(5), pp. 581–606.

[43] Li, C., Liang, M., and Yu, Q., 2011, "Multi-objective Optimization of Vehicle Passive Suspension with a Two-Terminal Mass Using Chebyshev Goal Programming," pp. 633–638.

[44] Gon, P. C., and Ambr, J. A. C., 2004, "Optimization of Vehicle Suspension Systems for Improved Comfort of Road Vehicles Using Flexible Multibody Dynamics," pp. 113–131.

[45] Sun, L., Cai, X., and Yang, J., 2007, "Genetic algorithm-based optimum vehicle suspension design using minimum dynamic pavement load as a design criterion," J. Sound Vib., **301**(1-2), pp. 18–27.

[46] Malhotra, R., Singh, N., and Singh, Y., 2011, "Genetic Algorithms : Concepts , Design for Optimization of Process Controllers," **4**(2), pp. 39–54.

[47] Wang, K., Inman, D. J., Leo, D. J., Robertshaw, H. H., and Singh, M. P., 2004, "Vibration Analysis of Cracked Composite Bending-torsion Beams for Damage Diagnosis Vibration Analysis of Cracked Composite Bending-torsion Beams for Damage Diagnosis."

[48] Karuppaiah, N., Sujatha, C., and Ramamurti, V., "NUMERICAL ANALYSIS OF VIBRATION RESPONSE IN A LIGHT PASSENGER VEHICLE," pp. 295–304.

[49] Guo, L.-X., and Zhang, L.-P., 2010, "Vehicle Vibration Analysis in Changeable Speeds Solved by Pseudoexcitation Method," Math. Probl. Eng., **2010**, pp. 1–14.

[50] Brebbia, C. a., and Nardini, D., 1983, "Dynamic analysis in solid mechanics by an alternative boundary element procedure," Int. J. Soil Dyn. Earthq. Eng., **2**(4), pp. 228–233.

[51] Abdullah, S., Kadhim, N. A., Ariffin, A. K., and Hosseini, M., 2007, "Dynamic Analysis of an Automobile Lower Suspension Arm Using Experiment and Numerical Technique."

[52] Ross, C. T. F., 1998, "Finite element analysis," pp. 185–251.

[53] Berggren, M., 2002, "Introduction to the Finite Element Method.pdf."

[54] Suli, E., 2012, "Finite Element Methods for Partial Differential Equations."