Conceptualising social housing within the wider housing market: a vacancy chain model

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Abstract

There have been increasing calls for studies of housing systems that are more integrative and pluralistic in nature. Understanding the relationship of social housing systems to the wider housing market remains a key challenge. The mobility of households and the structural configuration of supply are both of importance, demanding methods able to reconcile both. This paper propounds vacancy chain models as offering significant potential in this regard, allowing policy analysis and options appraisal to be built on more dynamic conceptualisations of housing systems. The theoretical basis for vacancy chain models is developed before an account is given of a model developed of the Bradford (UK) social rented sector. The results suggest that social renting is very closely linked to the wider housing market and consequently the impacts of policy and investment may be felt beyond the sector. Observations on the future development of vacancy chain models are offered.

Key words: social housing, vacancy chain model, residential mobility, housing supply, housing market, policy evaluation

Introduction

There has been a rise in interest in the microstructures of housing markets as a way of understanding housing's relationship to the wider economy and conceptualisations of the workings of market and society (Smith & Munro, 2008). This calls, in general, for detailed and multi-layered studies of the internal structure and operation of different parts of the housing system from a range of perspectives, including new microeconomic and cultural-economic conceptualisations of local markets (Kauko, 2003; Watkins, 2008; Wallace, 2008; Gibb, 2009). The focus of most scholars' recent interest appears to have been the private housing market, perhaps understandably given the market's absolute size and its importance to wider social and economic policy. This has arguably left micro-level analysis of social, or non-market, housing somewhat neglected. Despite some important contributions in the 1990s (e.g. Keenan, 1998; Burrows, 1999) this neglect is surprising given high rates of turnover among social housing residents (Pawson & Bramley, 2000) and recent policy concerns with the impact of mobility on neighbourhood sustainability and community cohesion; and the relationships between social housing 'neighbourhoods' and social exclusion. With regard to the latter, Murie & Musterd (2004) helpfully note that future research into disadvantaged neighbourhoods must adopt what they term 'a layered approach' (p. 1457) that seeks to situate local micro factors within an understanding of the neighbourhoods' broader metropolitan and regional contexts and linkages. The imperative for further research is heightened following Oxley et al.'s (2008) conceptual work on competition within social housing and between the social and private sectors. Those authors acknowledge the effect of competition on social housing provision and vice versa, even if it is in some cases at a relatively low level. Although they do not present any empirical estimates of competition, they do point to the widening role of 'choice' in social housing systems, and increased focus on tenure mixing, as indicative of a broader trend.

Together with Kauko's (2003) call for plurality at the structural level, this all confirms that methodologies which are simultaneously alive to the importance of microstructures and their relationships to macrostructures will become increasingly valuable. It is within this context that the prime motivation for this paper arises. Its principal focus is on household mobility within a part of the housing system and between that part and the rest of the system. This paper develops a method that allows a greater understanding of the microstructures of the social housing sector and its links with the wider housing market, and does so in a way that constructs a framework for the assembly and integration of evidence on different parts of the housing system.

The paper draws its inspiration from a rather neglected literature relating to vacancy chain studies. In so doing it supports the recent contentions of a small number of authors (Emmi & Magnusson, 1995b; Nordvik, 2004;

Magnusson Turner, 2008) that vacancy chain models of housing systems can hold significant potential for policy evaluation and options analysis. Recent advances in the availability of micro data mean that realising the potential of vacancy chain models is now a realistic proposition. The paper reports on the development of a vacancy chain model of the social housing system in a large UK city (Bradford). The model simultaneously allows an analysis of detailed intra-sector patterns of mobility while recognising the importance of the flows between social housing and other parts of the housing system. The aim was to develop a conceptually simple model that could be applied to housing systems to understand the interactions between their constituent parts and, more specifically, to model the effects of changing patterns of supply and demand on them. At its broadest level the model can be populated with evidence from a range of perspectives and methods, but can also capitalise on the insights generated by detailed analysis of micro data on household mobility.

The paper begins by restating the case for studies of mobility within housing systems. It sketches out the broad contributions that housing mobility researchers have made, but concludes that, for a variety of reasons, the disaggregation required to draw detailed conclusions is often traded off in the pursuit of more holistic, multi-sector studies. The paper then goes on to develop a simple conceptual framework that focuses attention on the vacant dwelling as a worthwhile object of analysis, it being simultaneously the unit of housing supply and the mechanism that enables all mobility in the system. This is then formalised into a vacancy chain model, which in effect traces the opportunities for, and consequences of, household mobility. An exposition is then given of how the model was applied to the social rented housing sector in Bradford using a detailed data set on household moves over a five-year period.

Bradford was chosen partly because it had what was described as a 'low demand' housing market which was leading to failures in policy targeting. Levels of intra-sector turnover in the city were high. In 2000 there were 26 704 local authority dwellings. Of these, 4 896 were let to new tenants during the previous 12 months representing a turnover rate of 18.3 per cent. Of the city's 9 326 housing association dwellings, 1 658 (33.9 per cent) were let during the previous 12 months (DTLR, 2000). Detailed analysis of how households were moving within and between tenures was, however, not well developed. This raised important policy questions in Bradford, not least among those that were funding new social housing. At the time the research was undertaken there was a concern that housing associations were meeting a different type of housing need than local authority housing, with each tenure having a different 'relationship' to the private market. Although these concerns have since partly been the subject of an emerging literature on supply-side competition between different parts of the housing system (see for example Sinai & Waldfogel, 2005 and Nordvik, 2006), the role of microstructures, particularly within cities or ‘market areas’ has been relatively underplayed.

Housing dynamics and household mobility

Within the fields of housing studies and urban economics there is a great deal of interest in housing market dynamics. This implies that scholars have increasingly acknowledged the centrality of market mechanisms to questions of housing while, at the same time, have sought to understand the links between the constituent components of the market. Studies of various types such as those of neighbourhood dynamics (e.g., Galster, 2003; Kearns & Parkes, 2003), market formation and dynamics (e.g., Smith et al., 2006) and household formation and market-entry (e.g., Ermisch & Di Salvo, 1997) are all united to some extent by their concern with the mobility implied by individual agency within wider systems and structures.

This has inevitably given rise to a whole-systems approach to understanding housing. Whether from the sociological perspectives afforded by the ‘housing pathways’ thesis of Clapham (2005), social policy concerns about spatial and social divisions (e.g. Forrest, 1987; Pawson, 2004), or economic modelling that treats more and more aspects of urban systems endogenously, the goal has increasingly been to integrate knowledge of different housing sectors rather than treat those sectors on their own terms.

This has been a valuable enterprise, even if it has been potentially to the cost of detailed understandings of individual housing sectors. Clarifying the roles played by different housing sectors (often with a focus on social housing) has become very important, and therefore so has attempts to better understand the links between these sectors.

One result of such whole-systems approaches has been the rise of a specific interest in mobility. This has been necessary because understanding mobility does not just illuminate the operation of housing markets and systems; it provides the immediate connection to a host of wider policy imperatives such as those arising from labour markets, education, land use planning and others. Much of the focus on mobility has been through the lens of owner-occupied housing. This has particularly been the case when the relationships between the housing market and economic growth have been under consideration. There are countless motivations for studying mobility in the private housing market: modelling spatial equilibria in the Alonso-Muth-Mills tradition (Glaeser, 2007); determining location choice, either structurally (Wheaton, 1977) or in a more disaggregate sense (e.g. Fernandez et al., 2005); understanding housing's contribution to labour market impedance (Henley, 1998), or its relationship with health (Pevalin et al., 2008) and impacts on social capital and life-chances (De Souza Briggs, 1998).

But mobility within social housing has also been under the spotlight, if sometimes for slightly different reasons such as those arising from policy concerns about housing supply and management. Studies of mobility have helped to answer perhaps three broad groups of questions. First, the internal dynamics of the sector have been of interest because of the social problems associated with turnover, particularly internal population 'churn' within social housing 'estates' (Keenan, 1998; Pawson & Bramley, 2000). Second, net flows of population out of the social rented tenure have periodically called into question the role of social housing and the long term viability of social housing models. This is often associated with the unattractiveness of the tenure itself in comparison to other housing options in the wider housing market and changing demographics (Burrows, 1999) although it should be noted that tenure mobility can also occur 'in situ' through tenure restructuring programmes (ibid.). Finally, the links between particular housing policies and broader social welfare policy can be expounded through the movement patterns of target population segments. The potentially (but unproven) constraining effects of housing allowances on mobility and on job seeking (see Kemp, 2007) are a case in point.

Yet, for all this breadth, studies of mobility that attempt to both integrate different parts of the housing system and examine the minutiae of mobility seem few in number. Many micro level studies, while individually useful, perpetuate a relatively narrow concern with a particular sector or tenure, and are ill-suited to assessing the transmission of phenomena between sectors or tenures. To a point, this is entirely understandable. The rich policy heritage of housing studies has served to effectively filter and funnel academic concerns with housing down a select number of routes, determined by the specific policy concerns of the day. There are also significant pragmatic limitations that have been imposed by data availability, the administrative foundations of which have tended to favour certain sectors over others.

In sum, this results in rather significant practical constraints for the analyst wishing to understand the contingent and external effects of policy. There is a somewhat imperfect analytical infrastructure with which to simultaneously understand something about the transmission of effects throughout the whole housing system, and between and within its constituent parts. Given the centrality of housing to many key debates within studies of population, labour markets and the economy, and the structure and operation of urban systems more generally, it would seem that attempts to develop analytical tools that can be applied across the housing system but that respect the detailed interactions resulting from mobility should receive greater attention. To begin to address this, the paper now sketches out a conceptual housing system, composed of distinct but interrelated parts, within which the mobility of households (both within and between these parts) is of prime analytical importance.

A conceptual housing system

Housing systems within a wide variety of contexts can be understood to be composed of constituent parts. The precise definition of these parts and the extent to which they overlap may vary between different political and economic systems, but the emergence of the submarkets thesis has usefully reminded us of the need to delineate housing systems, either on a spatial basis or otherwise. In McMaster & Watkins' (2006) review of over 25 years of attempts to find evidence for price segmentation in local housing markets, they state that: 'the overwhelming conclusion is that urban housing markets are more appropriately explained as a series of inter-linked submarkets' (p. 913). Elsewhere there is a significant body of evidence that different parts of housing systems interact with each other in different ways. While gated communities are in some senses the apotheosis of self-segregation within housing markets (although for complex economic reasons: see Manzi & Smith-Bowers, 2005), it does not take physical features to demarcate different parts of the market. Regulatory regimes, consumers' preferences and incomes, product differentiation, and urban structure have all been highlighted as having a role to play in explaining housing market segmentation (Watkins, 2001; Kauko, 2005). Indeed, much of the research in this area has ultimately served to reinforce Grigsby's (1963) pioneering conception of submarkets, which relied on the thesis that housing is most aptly seen as a 'service' comprising of differing 'bundles' of attributes. Households implicitly value such constituent attributes, and demand in the market varies according to the attributes of both the households and the housing stock. One of the most important consequences of such an analysis is that, from a theoretical perspective at least, dwellings in the same sub-market are not necessarily spatially contiguous. The submarkets thesis is especially useful because it at once emphasises that the housing system has ‘parts’ to it – which has both analytical and policy value – and that these dynamically interact to form a ‘whole’. Both the parts and the whole are of importance.

Extending the conceptualisation of submarkets to social housing implies some distinct challenges. Clearly, social housing is not allocated within a market framework, although there has been a succession of policy shifts throughout global housing systems that have sought either to directly insert market mechanisms into the internal machinery of social housing (e.g., Fitzpatrick & Pawson, 2007; Dufty, 2007; Van Daalen & Van Der Land, 2008) or have otherwise served to increase the competitive exposure of social housing supply to the wider market through, inter alia, rent restructuring (Tang, 2008), tenure diversification (Munro, 2007) and removing state control from housing management (Gibb & Nygaard, 2006).

This means that while it is clear that social housing systems do not in themselves constitute marketplaces, they nevertheless exist within broader systems dominated by market mechanisms. The functioning of the wider housing market and the policy choices made in respect to it are therefore likely to have a direct impact on social housing systems as a result of inter-tenure mobility. The aggregate patterns of such mobility are tempting objects of analysis but rather miss the point. It is the individual choices, constraints and movements of households into, within, and out of social housing – each time with reference to opportunities both within social housing and also beyond it – that matter. The analytical and policy boundaries that we impose post hoc matter less. To lose the micro-focus on households and housing opportunities would therefore be to lose the analytical power of the market, for each and every decision is made with reference to a set of opportunities and constraints within the market. These decisions in turn govern mobility. For the majority of households, such mobility might be a within-tenure move that conforms to our policy/analytical frameworks of convenience, but for a significant minority it will not.

The final element of the conceptual housing system sketched here is that of structural change to the supply side of the system. The overall stock of housing opportunities does not remain stable over time. Opportunities for future mobility are, in the main, created and absorbed through the normal process of past mobility. The movement of one household creates an opportunity for the movement of another household. This dynamic is, in each case, fundamentally structured by the availability or otherwise of other housing opportunities. Building new housing units brings about an addition to these opportunities; conversely, processes like demolition or the de-conversion of flats into single dwellings serve to reduce the overall number of housing opportunities. The impacts of housing and planning policies are in actuality transmitted through an intricate web of mobility, where ‘new’ and ‘existing’ housing coexists and competes within a range of tenures. These effects can be detected in studies of the impact of structural housing market change on turnover (e.g., Dieleman et al., 2000). The impact of new social housing construction, therefore, will not only meet identified existing needs within existing social housing populations but will have an impact on the wider housing market as a result of the interactions between sectors previously discussed. Understanding these complex transmission effects will allow a more sophisticated set of housing and planning policy evaluation tools to be created.

Developing a vacancy chain model

To summarise the discussion up to this point, it has been argued that there is a need for housing market models that can account for micro level dynamics between households and properties and that is conceptually relevant across different parts of the housing system (e.g. different tenures). If a model that had these properties could be deployed, the way that the social housing system is embedded within, and contingent on, conditions in the wider housing market might be better understood. Furthermore, by understanding the transmission of housing opportunities throughout the market, policymakers might be able to better understand the potential impacts of investment or policy change.

The recognition that the behaviour of agents at highly disaggregate levels can lead to systematic, observable patterns at more aggregate scales has long been recognised, as have the potential implications for policy (Meen and Andrew, 2004). Approaches that employ Cellular Automata and Agent Based Modelling (Kennedy et al., 2007; Batty, 2009) hold some promise but are relatively unrefined in the way they provide a framework for the examination of different parts of the housing system.

This section of the paper considers the potential contribution that can be made by vacancy chain models. The aim is not to develop a maximally endogenous model of the wider housing market but to propound vacancy chain models as providing a useful dynamic framework within which to understand the transmission of effects, through mobility, within and between parts of the housing system. To this framework can be added evidence derived from multiple methods and data sources.

Advantages of vacancy chain models

Vacancy chain models are a class of models particularly suited to a housing systems analytical framework because they are able to reconcile both micro- and macro-structural aspects of mobility. They have the potential to recreate complex patterns of micro-level interactions while analysing their impacts at more aggregate or abstract levels (Chase, 1991; Nordvik 2004). Vacancy chain models were originally used to analyse employment patterns, but soon found applications in housing research (Lansing et al., 1969; White, 1971). A critical spur for the development of such models has been that they can permit insights into the impacts of policy interventions within housing systems, for example to guide resource allocation (Magnusson Turner, 2008).

Social housing providers and regulators use a variety of measures for the purposes of monitoring performance and planning investment. These performance management techniques typically rely on 'snapshots' of vacancy and turnover statistics. These static measures provide a poor basis for future forecasting and underplay the embeddedness of the sector within the wider housing market. This is particularly problematic for the question of investment: how can we be sure that new housing supply meets the needs of those for whom policy intends?

Social housing has historically been a scarce resource, and the selection of indicators and measures has tended to reflect this. The length of waiting lists, for example, continues to be regarded as an important measure of demand. In a context of full stock utilisation, turnover continues to be used as an indicator of neighbourhood stability. Yet it is very difficult to use these measures meaningfully to model effective potential demand where the local context is one where occupancy rates are low or turnover is high because the looseness of the market at the small area level permits a relatively footloose use of the stock across different tenures (Keenan, 1998). The failures of investment planning using 'static' conceptualisations of demand were powerfully demonstrated in the 1990s in many British post-industrial cities, including Bradford. During the crisis of 'low demand' for housing, scrutiny turned to the sustainability of supply side investments in social housing when local housing market studies were showing that low cost home ownership was sometimes cheaper than social renting for working families (Nevin et al., 2001). Continued investment in social housing was missing the policy target to the point that new units were being demolished after only a few years, while a maintenance backlog on existing units continued to accrue.

In short, static performance indicator data is highly context specific and may not be a good guide for future investment. The data themselves provide few clues because they are typically analysed in a tenure vacuum and without reference to the housing market beyond. Turnover within social housing is an important process, but is not entirely an internal one. More dynamic conceptual and analytic models are needed if the future impacts of policy interventions are to be more reliably estimated. As Watkins (2008), in a wide-ranging review of the contribution of microeconomics to understanding housing markets, put it:

> ... the failure to develop a stylised model of the structure and operation of local market systems continues to act as a significant constraint on our ability to understand the linkages between market performance and the efficacy of policy interventions. (p. 168)

The importance of 'filtering'

Yet, serious attempts at 'stylistically' modelling the dynamic effects of intervention have a (perhaps controversial) pedigree. The most significant example is probably the concept of filtering (Kristof, 1965; Altshuler, 1969), which is in itself closely related to the fundamental processes of vacancy chains. Filtering, as both a process and a policy goal, assumes that as new housing units come on-stream in higher value sub-markets (say, 'executive' housing), the units vacated by those who move into the new homes will become occupied by households previously living in lower sub-markets. Houses, then, can move down through a hierarchy of sub-markets over time, perhaps eventually becoming obsolete and subsequently abandoned, rehabilitated or demolished. Policymakers have sometimes used this mechanism to advocate supply-side housing subsidies. Critics like Lowry (1960) have noted that filtering can also be used to specifically justify subsidising higher- rather than lower-value dwellings. Galster (1997) specifically doubts the effectiveness of such subsidies in benefiting the lowest income households. Despite Baer and Williamson's (1988) warning to keep the filtering process and result conceptually separate, it is probably the case that the ability of the filtering concept (despite the challenge of empirical evidence) to support blanket policies that promote supply side intervention at the higher end of the market (and thus downplay the importance of social housing provision) has led to its fall from analytic fashion – and, with it, the allied technique of vacancy chain modelling.

Model description

Tracing the occurrence of housing vacancies is the logical opposite to tracing the movements of households. By analysing housing vacancies and their tendency to move between parts of the housing system (hereafter, 'sectors') in the opposite direction to households, a general abstraction of movement patterns can be built up from micro-level evidence of individual household moves and market transactions. A focus on vacancies allows the supply-side effects of mobility to be more clearly seen. Essentially, a vacancy chain model is centred on an empirical estimation of the probability of moves occurring between system sectors and then using these probabilities to iterate the model forward to estimate who gets what stock (and hence, where). The central computational device is a matrix, **P** describing origin-destination probabilities. The model is linked to a stock-flow accounting framework which is used to track the creation, absorption, and transfer of vacancies which arise from a range of demographic and supply-side processes (after Fielder & Smith, 1996). A 'vacancy event' occurs whenever a house becomes vacant or becomes occupied. Sometimes this results in a 'vacancy transfer'; i.e., one property becomes empty as its household moves to occupy another. Sometimes there may be a net loss of vacancies (through various 'vacancy absorption' events), while at other times there may be a net gain in vacancies (through 'vacancy creation' events). This accounting framework is depicted diagrammatically in Figure 1.

[Figure 1 around here]

Data collection

Vacancy chain models typically require very detailed individual transaction records to accurately reconstitute movement patterns. Early vacancy chain models (e.g., Lansing et al., 1969; Murie et al. 1976) relied on survey methods to identify some chain starting points and use these as the basis for follow-up questionnaires to trace the linked series of subsequent transactions. More recently Emmi & Magnusson (1995a; 1995b; 1997) and Magnusson Turner (2008) have developed vacancy chain models from linked census data sets. Both methods of data collection have limitations arising from the censoring of data. In the case of survey data, the dataset is relatively 'thin' and it is not easy to determine how much it captures of the entirety of a (local) housing system at that point in time. Conversely, using census data in place of survey data results in the loss of detail on moves occurring between census dates.

The widespread adoption of sophisticated database systems by social housing providers, however, increasingly permits easier access to comprehensive and complete datasets on residential mobility. It is possible to combine data from multiple social landlords to recreate more or less complete transactional ledgers for local social housing systems. This paper is based on such a dataset which was compiled by the author from administrative datasets in Bradford.

While the task of data assembly was onerous, it is likely to become easier in the near future as governments and the private sector pursue projects aimed at improving micro-level and small-area data, and the integration of administrative databases. Of particular promise in England, for example, are the inclusion of council housing in the Tenant Services Authority's 'Continuous Recording' system; the National Register of Social Housing project (ODPM, 2004); the adoption of common housing registers; and linked electoral roll data produced by private sector data integrators.

In operationalising the vacancy chain model, three broad types of data were collected. These were:

1. Information on the social housing stock, including its physical and location attributes;

2. Information on households within social housing, and their characteristics;

3. Data on supply, demand and outcomes. This was formulated as a flow of supply (vacancies arising in the stock of housing), a flow of demand (unallocated households arising in the pool of households, for example through exits from housing, in-migration, or newly-created households), and information on outcomes (the allocation of households to dwellings).

Some stock and household characteristics were additionally derived on the basis of location using a Geographic Information System (GIS).

Table 1 shows the basic variables that were collected for the stock, households, and mobility events. Data were collected from seven of the nine largest social landlords in Bradford and standardised. In all, information was collected on some 50 128 dwellings, 39 452 households, and 11 773 mobility events involving the creation, absorption, or transfer of a vacancy over a period of five financial years (1996/7 to 2000/1). These data represented just less than 85 per cent of the city's social housing stock during that time. The data were cleaned and stored in a relational database that linked creation, absorption and transfer events using a system of unique household and stock reference numbers, and permitted flexible querying on the basis of both households and stock.

[Table 1 around here]

From the database, the variables could be used to segment the social housing system on the basis of stock characteristics, household characteristics, or a combination of both. However, the initial model reported in this paper is very modest in scope and restricts itself to three stock characteristics: property size, property type and whether the landlord is a local authority or housing association. Table 2 summarises the sectors used in the model.

[Table 2 around here]

Formal model specification

This section of the paper provides a formal specification of the vacancy chain model together with a concurrent demonstration using data collected for the social housing system in Bradford.

To maintain conceptual simplicity, only variables relating to crude housing type, size and tenure characteristics were used. It would be possible, and indeed desirable, to develop a model in which formally derived sub-markets, derived on the basis of the interaction of supply (vacant stock) and demand (household) characteristics, were used for segmentation. These possibilities are returned to at the end of the paper. Emmi & Magnusson (1997) provide an explicit treatment of the importance of segmentation on the operation of vacancy chain models, but a fuller discussion is beyond the scope of the present paper.

To begin, counts of vacancy events are entered into a supermatrix **N**, which is composed of submatrices $\mathbf{N}^T$, $\mathbf{N}^A$ and $\mathbf{N}^C$ corresponding to vacancy transfer, absorption and creation events respectively. A fourth submatrix, $\mathbf{N}^0$ is included for conceptual convenience and represents 'null' events which in actuality do not occur. Figure 2 provides a diagrammatic representation of **N** and its constituent parts. It should be noted that, unlike other more formal specifications of vacancy chain models, the number of vacancy creation and absorption events is not intended to balance. This is because the system is not closed and the total number of vacancies will expand (or contract) during the study period in response to house building, demolition and net migration.

[Figure 2 around here]

[Table 3 around here]

The actual data within **N**, as extracted from the project database, are provided in Table 3. Sectors are defined according to tenure (HA = housing association; LA = local authority), dwelling type (FLT = flat or maisonette; HSE = house or bungalow), and number of bedrooms. So, for example, 142 vacancies in 2-bedroom local authority flats (LA/FLT/2) were subsequently transferred to 1-bedroom local authority flats (LA/FLT/1) as households moved to larger properties. Three absorption events PRIVATE, MOVE IN and DEMOLISH have been included, analogous respectively to household moves from the Bradford private sector; household moves from outside Bradford; and demolition of a housing unit. Four creation events PRIVATE, DEATH, NEW BUILD and MOVE OUT are also included. These are analogous respectively to household moves to the Bradford private sector; death of a single-person household; new construction of a household unit; and household moves out of Bradford.

The observed probability of a transition is simply the count of that type of transition divided by the total of all vacancy events originating in the same sector. Therefore, the probability p of a move from sector j to sector k is:

$$p_{jk} = \frac{\mathbf{n}_{jk}^{T}}{\sum \mathbf{n}_{j}^{T} + \sum \mathbf{n}_{j}^{A}} \qquad (1)$$

where $\mathbf{n}^{T}_{jk}$ is the corresponding cell of the transition counts submatrix $\mathbf{N}^{T}$;

$\sum \mathbf{n}^{T}_{j}$ is the sum of all cells in row j of $\mathbf{N}^{T}$; and

$\sum \mathbf{n}^{A}_{j}$ is the sum of all cells in row j of the absorptions submatrix $\mathbf{N}^{A}$.

This can be formally expressed using matrix notation. Following Emmi & Magnusson (1995b) and using the horizontal concatenation operator (:) the row sum vector **n** is,

$$\mathbf{n} = \left[\left(\mathbf{N}^{T}\right) : \left(\mathbf{N}^{A}\right)\right]\mathbf{1} \tag{2}$$

which is the concatenation of the transition matrix and the absorption matrix multiplied by unit column vector $\mathbf{1}$. This results in,

$$\mathbf{n} = \begin{bmatrix} 631 \\ 730 \\ 68 \\ 455 \\ 308 \\ 160 \\ 82 \\ 14 \\ 5087 \\ 4115 \\ 5829 \\ 5823 \\ 2738 \\ 5050 \\ 4323 \\ 3739 \end{bmatrix} \tag{3}$$

which, as can be seen, are the row sums of the original fully-specified table of transition counts. It follows therefore, that a matrix of transition probabilities is simply the counts divided by the row totals:

$$\mathbf{P} = «\mathbf{n}»^{-1}\mathbf{N}^{t} \qquad (4)$$

where $«\mathbf{n}»$ is the vector to diagonal matrix transformation[1] of $\mathbf{n}$.

The probabilities represented in $\mathbf{P}$ can be iterated forward to simulate the transfer of vacancies between states as time progresses. If the probability of a vacancy transferring from sector j to k in one move is $\mathbf{p}_{jk}$, then the probability of it transferring in two moves is $\mathbf{p}^{2}_{jk}$. These probabilities added together for every move give the total probability of a vacancy transferring to sector k after any number of moves. Hence,

$$\mathbf{P}_{t=\infty} = \mathbf{P} + \mathbf{P}^{2} + \mathbf{P}^{3} + \dots \qquad (5)$$

The fully populated matrix of transfer probabilities **P** is shown in Table 4. In normal algebraic notation, this power expansion series can be expressed as

$$\begin{aligned} P_{t=\infty} &= \frac{1}{1-P} - 1 \\ &= (1-P)^{-1} - 1 \end{aligned} \qquad (6)$$

Of course, the actual process must take into account the initial vacancy. Therefore the probability of a vacancy transferring to the original sector after any number of moves is always $1 + P$. So, for transfers from one sector to another, $P_{jk} = (1 - P)^{-1} - 1$, and for transfers between the same sector, $P_{jj} = (1 - P)^{-1}$. This can be represented more simply in matrix algebra using the identity matrix[2], $\mathbf{I}$. If we go back to the original power expansion series, adding $\mathbf{I}$ will add unity (1) to each of the elements on the diagonal. The result is a matrix which predicts how many times a vacancy created in sector j will enter sector k before being absorbed. (Remember that the rows of $\mathbf{P}$ do not sum to 1 because at every stage some vacancies will move to an absorbing state.) Again building from the vacancy chain literature, this matrix, $\mathbf{M}$, also known as the Markov multiplier matrix or fundamental matrix, is calculated thus:

$$\begin{aligned} \mathbf{M} &= \mathbf{I} + \mathbf{P} + \mathbf{P}^2 + \mathbf{P}^3 + \dots \\ &= (\mathbf{I} - \mathbf{P})^{-1} \end{aligned} \tag{7}$$

The cells of $\mathbf{M}$ describe the expected number of times that a vacancy created in sector j will be in state j before being absorbed. The ‘multiplier effects,’ or expected chain lengths for vacancy starting in each sector, $\mathbf{m}$, are given as the row sums of $\mathbf{M}$, i.e., $\mathbf{m} = \mathbf{M1}$. While these multiplier effects are useful diagnostically (e.g. to examine the effects of changes on the model) they are also useful in analysing the effects on mobility of introducing new vacancies, such as through a house-building programme.

Results

Table 5 shows the vector **M1**, representing the expected chain lengths of vacancies by origin in the Bradford social housing system between 1996/7 and 2000/1. It is evident from this model of Bradford's social housing sector that chain lengths are short. This is because of the endogenous nature of the sectors used in the model: it is highly likely that a vacancy will pass quickly to a sector that is treated as exogenous, for example as a result of migration into the city or a move from the private sector housing. This is in itself an important finding because it highlights the limitations of policy options appraisal and evaluation that do not sufficiently account for spatial or sectoral linkages.

[Table 5 around here]

A number of conclusions from the initial model are possible. First of all, the short chain lengths highlight in Bradford's case the significance of mobility between social rented housing and the wider housing market. A significant number of vacancy 'events' imply that a household crosses a tenure or market area boundary and therefore the impacts of policies aimed solely at the social rented sector will very quickly have an impact in other parts of the housing system, including the private housing market. This reinforces the importance of understanding the microstructure of housing systems and adds legitimacy to a vacancy chain approach.

Second, it is clear that vacancies arising in housing association properties (sectors prefixed HA) tended to instigate longer chains than those arising in local authority housing (LA). This is because there is a higher propensity for vacancies to transfer out of local authority properties into the private sector – in other words, local authority housing is, proportionately, meeting more needs that arise outside of social housing, while housing association housing is catering, proportionately, more for transfers within social renting. This has important spatial consequences. As can be seen from Figure 3 there are particular geographies associated with local authority and housing association stock in Bradford. Local authority housing 'estates' tend to be larger and more peripheral, while housing association properties tend to comprise smaller developments in inner-city neighbourhoods. This suggests that the entry points to social renting tend to be in peripheral neighbourhoods which are dominated by local authority housing, while moves within the sector that involve changing to a housing association landlord would imply at the same time a move to a different type of neighbourhood. It would be possible to extend a vacancy chain model to more explicitly account for the spatiality of the sectors; a possibility which is returned to later.

[Figure 3 around here]

A third clear conclusion is that vacancies originating in 'traditional' houses initiate longer chains than those originating in flats. Moreover, the average chain initiated by a vacancy arising in a 2-bedroomed HA house is, at 2.20 moves, substantially longer than those originating in any other sector. This suggests that such properties are ceteris paribus more successful in catering for need arising within the existing social rented stock.

Although it is clear that spatial and sectoral links are extremely important, the precise nature of these links remains the potential subject of further investigation. Furthermore, the policy implications depend both on this and on more subjective interpretations of the results – probably in tandem with other studies. For example, short chain lengths might suggest that properties are successfully meeting need from outside the modelled system – in other words, from beyond Bradford and/or from outside the social rented sector. Alternatively put, investment in new social housing supply that gives rise to short chains has only a limited local impact before it is absorbed or otherwise 'leaks out' of the local social sector. Regardless of the perspective adopted it is clear that questions of investment ought sensibly to be accompanied by an analysis of its likely dynamic impacts in the future, as chains of housing opportunities unfold. A simple vacancy chain model of the type developed in this paper can help to illuminate these dynamics.

Conclusions

This paper has demonstrated the possibilities of constructing a simple vacancy chain model using rich micro-data on housing transitions within the social rented sector. It highlights the positive contribution that vacancy chain models can make to our knowledge of the microstructures of local housing systems and residential mobility and also to the evaluation of housing and planning policies that have a supply-side impact. In particular, it demonstrates the very close links at a micro level between different parts of the housing system and highlights the need for policy evaluation tools to adequately recognise these. In the case of the findings for Bradford, the model suggests that the social housing system cannot be meaningfully described at all as being isolated from the wider housing market.

With this in mind and returning to the need to develop more pluralist conceptualisations of housing systems noted at the outset, there are a number of possible directions for the future development of vacancy chain models. Two are here considered in turn. First, there is a set of potential developments that would allow vacancy chain models to say more about the nature of links between different parts, or sectors, within the housing system; and second, there are some issues related to the way that those sectors are formally defined. Together these constitute an agenda for further research.

Links between sectors

A key attribute of the conceptual housing system outlined earlier is the importance of understanding the dynamics between its constituent parts, however defined. This demands that dynamic micro models such as vacancy chain models begin to look beyond their own rather tightly-drawn boundaries in the same way that more macro models successfully do. Consequently, a more useful model of the local housing system would be one that treats a wider range of sectors endogenously – particularly housing in the private sector – and is thus able to account for mobility between different tenures. However, rather than empirically derive transition propensities for such sectors, it may be possible to synthesise data using secondary sources such as local or national, housing surveys. The emerging body of work on spatial micro simulation (Ballas & Clarke, 2001), and techniques such as cellular automata and agent based modelling (Fernandez et al., 2005; Kennedy et al., 2007; Batty, 2009 forthcoming) might permit further insights by facilitating the production of synthetic data on mobility within and between sectors for which there is a paucity of micro data.

Sector definition

Assuming the possibility of developing the model along the lines just discussed, a key technical challenge remains. Vacancy chain models are, like most dynamic models, particularly susceptible to error propagation. If a modelled sector is a poor analogue of the 'real world', the error that this introduces increases through multiple iterations of the model (Emmi & Magnusson, 1995b; Chase, 1991). For housing models, this places a particular emphasis on the definition of the system sectors, which must be as internally homogenous as possible to accurately establish the probabilities of transitions. Scholars' longstanding interest in submarket definition is thus of immediate relevance to vacancy chain modelling, and the long list of variables set out in Table 1 earlier should be considered as potential candidates for use in a more sophisticated scheme of model sectors. Of particular interest, as the distinction between local authority and housing association properties in this paper suggests, is the question of how to deal with the spatial dimension of submarkets. But there is a balance to be struck. There is an inherent tension between the construction of internally homogenous sectors and the need to keep the numbers of those sectors manageable. A small number of large sectors may fail the homogeneity test, while a large number of small sectors would suffer from errors introduced as a result of small cell counts. At the very least, there appears to be an imperative for further work to explore how the body of work on the microeconomic specification of submarkets can be applied to housing vacancy chain models. This also will require some further thought as to how social housing systems might fit into the submarkets paradigm.

As a final observation, although vacancy chain models offer some undoubted benefits, they could arguably be more 'user friendly' in their construction and use. Although specialist mathematics software capable of manipulating matrices is ideal, all operations can be carried out using everyday spreadsheet packages like Microsoft Excel (albeit requiring advanced IT skills). Collecting, standardising and integrating datasets from landlords' administrative systems can be time consuming, but as this paper shows, is possible. Initiatives aimed at harmonising administrative data flows will make the construction of vacancy chain models easier in the future. Finally, the further development and use of vacancy chain model should include a focus on permitting the more intuitive use of the model results. Using vacancies as the analytical object in place of households is potentially counterintuitive in studies of residential mobility. Yet vacancies are housing opportunities and are, as such, at the heart of the operation of housing systems. Understanding how vacancies arise and are propagated can lead to valuable insights to the impact of supply side policies. The diagnostic outputs of vacancy chain models (such as chain lengths) have an abstract quality and the language arguably lacks direct relevance to policymakers. Introducing a more formal temporal dimension to vacancy chain models would not be without difficulty but might enhance their capacity to offer more specific interpretations of use to policymakers.

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Table 1. Variables and variable groups

Variable	Example values
Household characteristics	
Size	Number of persons
Household type	e.g., number of dependents
Ethnicity	Census categories
Age of head of household	Years
Dwelling characteristics	
Landlord type	Local authority, housing association
Type	House, bungalow, flat, maisonette, bed-sit
Age	Pre 1919, 1919–44, 1945–1964, 1965–1979, 1980+
Construction type	Traditional, non-traditional, pre-fabricated
Size	Number of bedrooms
Neighbourhood name	Name of neighbourhood used by social housing managers
Local school performance	School ‘league tables’ – spatial relation using GIS
Proximity to public transport stop	Metres – spatial relation using GIS
Neighbourhood deprivation	Index of local deprivation – spatial relation using GIS

Table 2. Model sectors

Sector	Description
HA/FLT/1	1 bedroom Housing Association flat or maisonette
HA/FLT/2	2 bedroom Housing Association flat or maisonette
HA/FLT/3	3 bedroom Housing Association flat or maisonette
HA/FLT/4	4+ bedroom Housing Association flat or maisonette
HA/HSE/1	1 bedroom Housing Association house or bungalow
HA/HSE/2	2 bedroom Housing Association house or bungalow
HA/HSE/3	3 bedroom Housing Association house or bungalow
HA/HSE/4	4+ bedroom Housing Association house or bungalow
LA/FLT/1	1 bedroom Local Authority flat or maisonette
LA/FLT/2	2 bedroom Local Authority flat or maisonette
LA/FLT/3	3 bedroom Local Authority flat or maisonette
LA/FLT/4	4+ bedroom Local Authority flat or maisonette
LA/HSE/1	1 bedroom Local Authority house or bungalow
LA/HSE/2	2 bedroom Local Authority house or bungalow
LA/HSE/3	3 bedroom Local Authority house or bungalow
LA/HSE/4	4+ bedroom Local Authority house or bungalow

Table 3. Vacancy events and components of supermatrix **N**

		Vacancy transfer events (destination sector) ($\mathbf{N}^T$)																Vacancy absorption events ($\mathbf{N}^A$)		
		HA/FLT/1	HA/FLT/2	HA/FLT/3	HA/FLT/4	HA/HSE/1	HA/HSE/2	HA/HSE/3	HA/HSE/4	LA/FLT/1	LA/FLT/2	LA/FLT/3	LA/FLT/4	LA/HSE/1	LA/HSE/2	LA/HSE/3	LA/HSE/4	PRIVATE	MOVE IN	DEMOLISH
Vacancy transfer events (origin sector) ($\mathbf{N}^T$)	HA/FLT/1	87	22	3	5	7	2	1	0	5	2	4	4	13	5	7	6	437	21	0
	HA/FLT/2	28	239	12	5	13	7	1	1	2	9	2	0	1	14	15	4	366	11	0
	HA/FLT/3	2	9	7	1	0	3	0	0	0	0	0	0	0	0	0	0	46	0	0
	HA/FLT/4	5	8	0	74	0	1	0	1	6	2	21	3	0	6	5	15	394	14	0
	HA/HSE/1	8	13	0	0	55	20	6	0	1	0	0	0	0	1	0	0	200	4	0
	HA/HSE/2	3	15	1	0	17	54	8	0	0	0	0	0	0	0	0	0	60	2	0
	HA/HSE/3	1	4	0	0	4	8	12	2	0	0	0	0	0	0	0	0	48	3	0
	HA/HSE/4	1	1	0	1	2	2	0	0	0	0	0	0	0	0	0	0	5	2	0
	LA/FLT/1	2	1	0	0	0	1	0	0	483	158	66	65	47	67	41	29	3995	116	16
	LA/FLT/2	0	1	0	0	0	0	0	0	142	277	92	45	27	98	50	28	3052	104	199
	LA/FLT/3	1	2	0	0	0	0	0	0	60	99	558	173	11	49	94	50	4182	116	434
	LA/FLT/4	0	0	0	0	0	0	0	0	69	46	196	563	12	36	57	139	3945	154	606

	LA/HSE/1	1	1	0	0	2	1	0	0	69	52	25	14	190	90	43	24	2177	48	1
	LA/HSE/2	4	6	0	1	2	2	0	0	85	161	95	36	97	378	107	53	3660	126	237
	LA/HSE/3	2	3	0	2	1	2	1	0	34	88	207	105	49	136	290	84	3214	89	16
	LA/HSE/4	1	3	2	0	1	0	0	0	25	39	64	257	31	79	77	345	2600	74	141
Vacancy creation events (**N**^{C})	PRIVATE	187	115	16	58	106	40	8	7	1153	843	1141	1252	555	1061	886	859			
	DEATH	44	60	11	20	3	7	0	0	188	126	137	216	43	109	100	0			
	NEW BUILD	20	11	23	10	15	44	16	19	0	0	0	4	0	1	0	0			
	MOVE OUT	13	17	0	6	10	4	1	0	86	61	91	145	35	58	69	75			

Table 4. Transition probability matrix, **P**.

		$\mathbf{P}^T$																$\mathbf{P}^A$			$\mathbf{p}^A$	$\mathbf{p}$
		Vacancy transfer destination state																Vacancy absorption state				
		HA/FLT/1	HA/FLT/2	HA/FLT/3	HA/FLT/4	HA/HSE/1	HA/HSE/2	HA/HSE/3	HA/HSE/4	LA/FLT/1	LA/FLT/2	LA/FLT/3	LA/FLT/4	LA/HSE/1	LA/HSE/2	LA/HSE/3	LA/HSE/4	PRIVATE SEC	MOVE IN	DEMOLISH	ALL	Row sum
$\mathbf{P}^T$ Vacancy transfer origin state	HA/FLT/1	.138	.035	.005	.008	.011	.003	.002	.000	.008	.003	.006	.006	.021	.008	.011	.010	.693	.033	.000	.726	**1.000**
	HA/FLT/2	.038	.327	.016	.007	.018	.010	.001	.001	.003	.012	.003	.000	.001	.019	.021	.005	.501	.015	.000	.516	**1.000**
	HA/FLT/3	.029	.132	.103	.015	.000	.044	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.676	.000	.000	.676	**1.000**
	HA/FLT/4	.011	.018	.000	.163	.000	.002	.000	.002	.013	.004	.046	.007	.000	.013	.011	.033	.646	.031	.000	.677	**1.000**
	HA/HSE/1	.026	.042	.000	.000	.179	.065	.019	.000	.003	.000	.000	.000	.000	.003	.000	.000	.649	.013	.000	.662	**1.000**
	HA/HSE/2	.019	.094	.006	.000	.106	.338	.050	.000	.000	.000	.000	.000	.000	.000	.000	.000	.375	.013	.000	.388	**1.000**
	HA/HSE/3	.012	.049	.000	.000	.049	.098	.146	.024	.000	.000	.000	.000	.000	.000	.000	.000	.585	.037	.000	.622	**1.000**
	HA/HSE/4	.071	.071	.000	.071	.143	.143	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.357	.143	.000	.500	**1.000**
	LA/FLT/1	.000	.000	.000	.000	.000	.000	.000	.000	.095	.031	.013	.013	.009	.013	.008	.006	.785	.023	.003	.811	**1.000**
	LA/FLT/2	.000	.000	.000	.000	.000	.000	.000	.000	.035	.067	.022	.011	.007	.024	.012	.007	.742	.025	.048	.815	**1.000**
	LA/FLT/3	.000	.000	.000	.000	.000	.000	.000	.000	.010	.017	.096	.030	.002	.008	.016	.009	.717	.020	.074	.812	**1.000**
	LA/FLT/4	.000	.000	.000	.000	.000	.000	.000	.000	.012	.008	.034	.097	.002	.006	.010	.024	.677	.026	.104	.808	**1.000**
	LA/HSE/1	.000	.000	.000	.000	.001	.000	.000	.000	.025	.019	.009	.005	.069	.033	.016	.009	.795	.018	.000	.813	**1.000**
	LA/HSE/2	.001	.001	.000	.000	.000	.000	.000	.000	.017	.032	.019	.007	.019	.075	.021	.010	.725	.025	.047	.797	**1.000**
	LA/HSE/3	.000	.001	.000	.000	.000	.000	.000	.000	.008	.020	.048	.024	.011	.031	.067	.019	.743	.021	.004	.768	**1.000**
	LA/HSE/4	.000	.001	.001	.000	.000	.000	.000	.000	.007	.010	.017	.069	.008	.021	.021	.092	.695	.020	.038	.753	**1.000**
$\mathbf{P}^C$ Vacancy creation state	PRIVATE	.023	.014	.002	.007	.013	.005	.001	.001	.139	.102	.138	.151	.067	.128	.107	.104					**1.000**
	DEATH	.041	.056	.010	.019	.003	.007	.000	.000	.177	.118	.129	.203	.040	.102	.094	.000					**1.000**
	NEW BUILD	.123	.067	.141	.061	.092	.270	.098	.117	.000	.000	.000	.025	.000	.006	.000	.000					**1.000**
	MOVE OUT	.019	.025	.000	.009	.015	.006	.001	.000	.128	.091	.136	.216	.052	.086	.103	.112					**1.000**
$\mathbf{p}^C$	ALL	.026	.020	.005	.009	.013	.009	.002	.003	.140	.101	.134	.159	.062	.121	.104	.092					**1.000**

Table 5. Expected chain lengths, **M1**.

Vacancy origin state	**Chain length**
HA/FLT/1	1.39
HA/FLT/2	1.82
HA/FLT/3	1.56
HA/FLT/4	1.45
HA/HSE/1	1.58
HA/HSE/2	2.20
HA/HSE/3	1.69
HA/HSE/4	1.87
LA/FLT/1	1.23
LA/FLT/2	1.23
LA/FLT/3	1.23
LA/FLT/4	1.24
LA/HSE/1	1.23
LA/HSE/2	1.25
LA/HSE/3	1.29
LA/HSE/4	1.32

Figure 1. Vacancy stock-flow accounting framework.

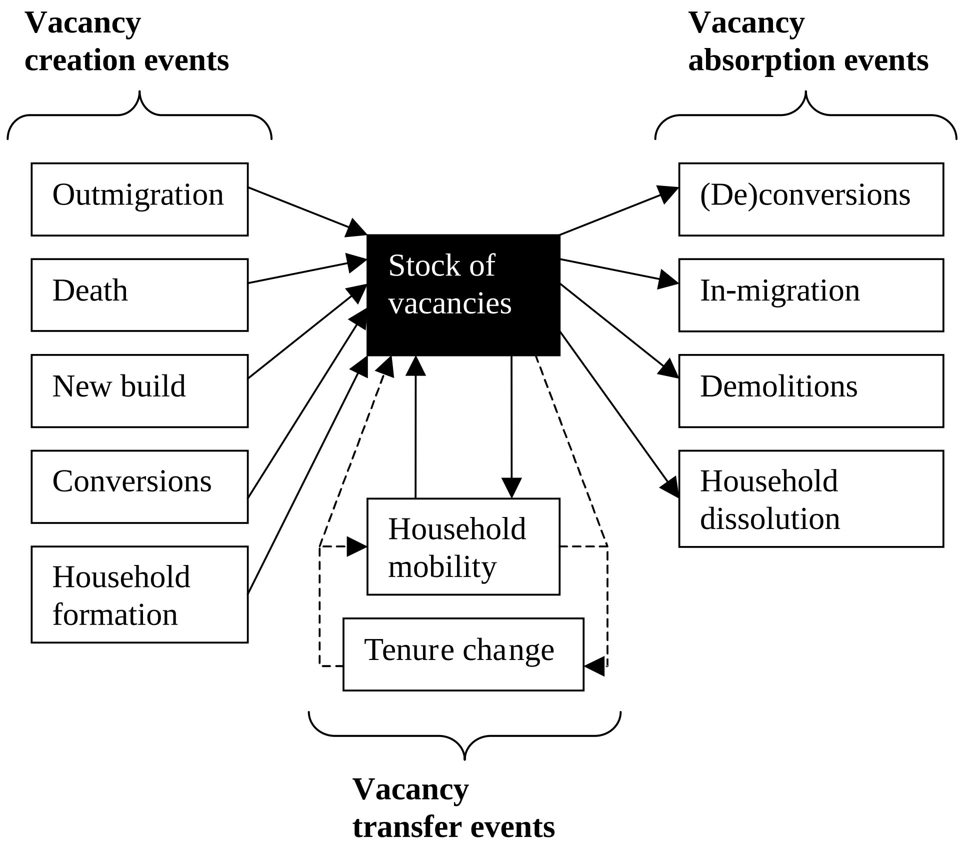
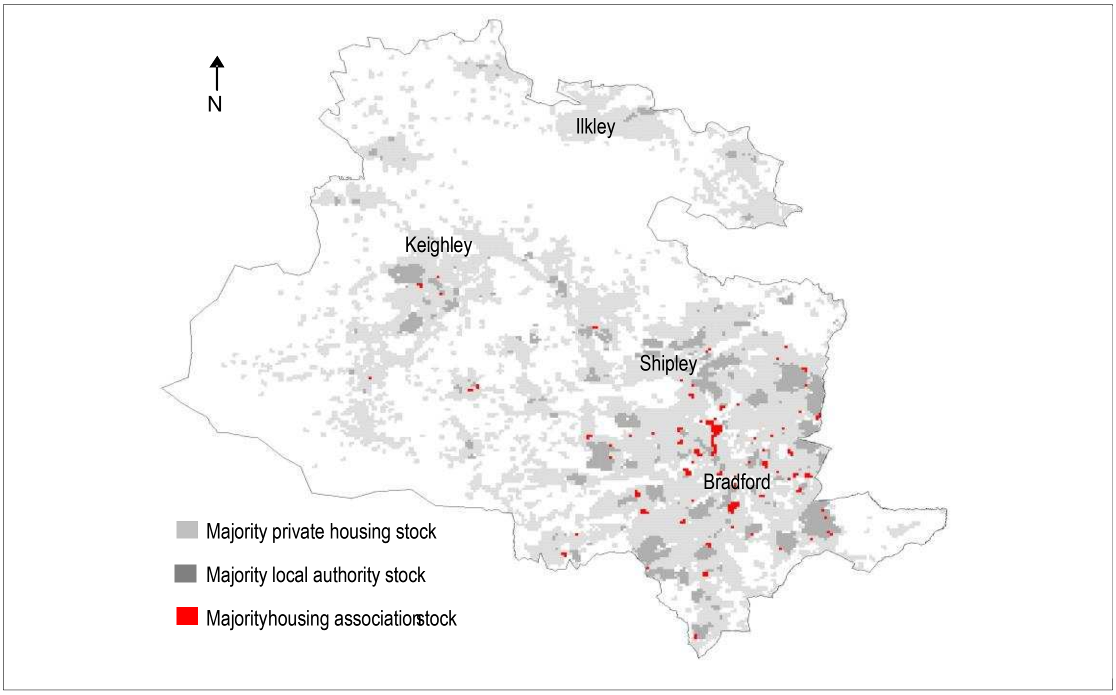


Figure 2. Matrix and vector components of the fully-specified supermatrix, **N**.

	Destination housing market sectors 1,…, r	Vacancy absorbing states s,…, q	Sum
Origin housing market sectors 1,…, r	Vacancy transfers $\mathbf{N}^{T}$	Vacancy absorptions $\mathbf{N}^{A}$	**n**
Vacancy creation states s,…, r	Vacancy creations $\mathbf{N}^{C}$	Null $\mathbf{N}^{0}$	
Sum	$\mathbf{n}'$		

Source: adapted from Emmi and Magnusson (1995b).

Figure 3. Residential areas and social housing in Bradford metropolitan district.



Notes

[1] It may be helpful to point out that in order to scale the individual elements of a matrix by the corresponding element of a vector, it is first necessary to transform that vector into a diagonal matrix. This is denoted by enclosing the vector to be transformed in double chevrons, thus: if

$$\mathbf{N} = \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \mathbf{n}_3 \end{bmatrix}$$

then

$$\begin{aligned} «\mathbf{N}» &= \mathbf{N}(\mathbf{I}) \\ &= \begin{bmatrix} \mathbf{n}_1 & 0 & 0 \\ 0 & \mathbf{n}_2 & 0 \\ 0 & 0 & \mathbf{n}_3 \end{bmatrix}. \end{aligned}$$

[2] The identity matrix, $\mathbf{I}$ is simply a square matrix of the appropriate dimensions where the diagonal cells are all 1, and the off-diagonal cells are all zero. Hence,

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}.$$