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**NETWORK GAME THEORY MODELS OF SERVICES
AND QUALITY COMPETITION WITH APPLICATIONS
TO FUTURE INTERNET ARCHITECTURES AND
SUPPLY CHAINS**

A Dissertation Presented

by

SARA SABERI

Submitted to the Graduate School of the
University of Massachusetts Amherst in partial fulfillment
of the requirements for the degree of

DOCTOR OF PHILOSOPHY

September 2016

Management

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Approved as to style and content by:

Anna Nagurney, Chair

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Management

To my beloved family.

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ABSTRACT

NETWORK GAME THEORY MODELS OF SERVICES AND QUALITY COMPETITION WITH APPLICATIONS TO FUTURE INTERNET ARCHITECTURES AND SUPPLY CHAINS

SEPTEMBER 2016

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The Internet has transformed the way in which we conduct business and perform economic and financial transactions. One key challenge of the Internet is the inefficiency of the mechanisms by which technology is deployed and the business and economic models surrounding these processes (Wolf et al. (2014)). Equilibrium models for the Internet generally assume basic economic relationships. However, in new paradigms

for the Internet and in supply chain networks, price is not the only factor; quality of service (QoS) is also of increasing importance.

Supply chains networks, which give us the means to manufacture products and deliver them to points of demand across the globe, are also under many pressures to offer differentiated products and services (Nagurney (2014)). It is well-known today that success is determined by how well the entire supply chain performs, rather than the performance of its individual entities.

This dissertation contributes to the analysis, design, and management of the future Internet and supply chain networks with a focus on price and quality competition in service-oriented networks.

Specifically, I focus on economic models for the Internet of the future by developing both a basic and a general network economic game theory model of a quality-based service-oriented Internet to study competition among service providers. To study and analyze the underlying dynamics of the various economic decision-makers, subsequently, I develop a dynamic network economic model of a service-oriented Internet with price and quality competition using projected dynamical systems theory. Then, to assess the prices for various contract durations at the demand markets, I consider a game theory model of a service-oriented Internet in which the network providers compete in usage service rates, quality levels, and duration-based contracts. Finally, I construct a model that captures the competition among manufacturers and freight service providers in a supply chain network. This model is the first one in the literature that handles both price and quality competition with multiple modes of shipment from both equilibrium and dynamic perspectives.

For each model, I derive the governing equilibrium conditions and provide the equivalent variational inequality formulations. In order to illustrate the modeling

framework and the algorithm, I present computed solutions to several numerical examples for each model as well as sensitivity analysis results.

This dissertation is heavily based on the following papers: Saberi, Nagurney, and Wolf (2014), Nagurney et al. (2014a), Nagurney et al. (2015b), and Nagurney et al. (2015a) as well as additional results and conclusions.

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	v
ABSTRACT	vii
LIST OF TABLES	xiv
LIST OF FIGURES	xv
CHAPTER	
1. INTRODUCTION AND RESEARCH MOTIVATION	1
1.1 Next Generation Internet	6
1.1.1 ChoiceNet	8
1.2 Supply Chains and Freight Services	10
1.3 Quality of Service	13
1.3.1 QoS in the Internet	13
1.3.2 QoS in the Supply Chain	15
1.4 Literature Review	17
1.4.1 Price and Quality Competition Among Service Providers	18

1.4.2	Time-Based Competition Between Internet Service Providers	23
1.5	Dissertation Overview	26
2.	METHODOLOGIES	30
2.1	Variational Inequality Theory	31
2.2	Projected Dynamical Systems	37
2.3	The Relationships between Variational Inequalities and Game Theory	41
2.4	Algorithm - The Euler Method	43
3.	A SERVICE-ORIENTED INTERNET NETWORK ECONOMIC MODEL WITH PRICE AND QUALITY COMPETITION IN BOTH CONTENT AND NETWORK PROVISION	46
3.1	The Basic Model	47
3.1.1	The Analysis of Two-Sided Pricing in the Basic Model	51
3.2	The Network Economic Game Theory Model of Price and Quality Competition in a Service-Oriented Internet	56
3.3	Explicit Formulae for the Euler Method Applied to Variational Inequality (3.45) with $F(X)$ Defined by (3.46) – (3.49)	63
3.4	Numerical Examples and Sensitivity Analysis	64
3.5	Summary and Conclusions	86
4.	A DYNAMIC SERVICE-ORIENTED INTERNET NETWORK ECONOMIC MODEL WITH PRICE AND QUALITY COMPETITION	88
4.1	The Dynamic Network Economic Model of a Service-Oriented Internet with Price and Quality Competition	89
4.1.1	Modeling of Quality in a Service-Oriented Internet	90

4.1.2	The Behavior of the Content Providers and Their Price and Quality Dynamics	92
4.1.3	The Behavior of the Network Providers and Their Price and Quality Dynamics	94
4.1.4	The Projected Dynamical System	96
4.1.5	Stationary/Equilibrium Point	98
4.1.6	Stability Under Monotonicity	100
4.2	The Algorithm and Numerical Examples	106
4.2.1	Explicit Formulae for the Euler Method Applied to the Service-oriented Internet with Price and Quality Competition	107
4.3	Summary and Conclusions	116
5.	A DIFFERENTIATED SERVICE-ORIENTED INTERNET NETWORK ECONOMIC MODEL WITH DURATION-BASED CONTRACTS	120
5.1	The Competitive Duration-Based Differentiated Service-Oriented Internet Game Theory Model	121
5.1.1	Variational Inequality Formulation	125
5.2	Explicit Formulae for the Euler Method Applied to the Internet Network Model with Contract Durations	129
5.3	Numerical Examples	130
5.4	Summary	140
6.	A SUPPLY CHAIN NETWORK MODEL WITH COMPETITION IN PRICE AND QUALITY BETWEEN MULTIPLE MANUFACTURERS AND FREIGHT SERVICE PROVIDERS WITH MULTIPLE MODES OF SHIPMENT	142

6.1	The Supply Chain Network Model with Price and Quality Competition	145
6.1.1	The Firms' Behavior	146
6.1.2	The Freight Service Providers' Behavior	148
6.1.3	The Nash Equilibrium Conditions and Variational Inequality Formulation	150
6.2	The Dynamics	155
6.3	Explicit Formulae for the Euler Method Applied to the Multitiered Supply Chain Network Problem	159
6.4	Numerical Examples	160
6.5	Summary and Conclusions	183
7.	CONCLUSIONS AND FUTURE RESEARCH	186
7.1	Conclusions	186
7.2	Future Research	190
	BIBLIOGRAPHY	192

LIST OF TABLES

Table	Page
3.1 Demand Functions for Example 3.4	80
4.1 Notation for the Dynamic Network Economic Model of a Service-Oriented Internet with Price and Quality Competition	91
6.1 Example Features	162

LIST OF FIGURES

Figure	Page
1.1 ChoiceNet Principle	9
3.1 Network Topology for the Basic Model's Content Flow	48
3.2 The Network Structure of the Basic Model's Financial Payment Flows	48
3.3 The Network Structure of the Multi-Provider Model's Content Flows	57
3.4 Graphic of the Multi-Provider Model with a Focus on Payments	57
3.5 Network Topology of Content Flows for Example 3.1	65
3.6 Network Topology of Content Flows for Example 3.2	67
3.7 Network Topology of Content Flows for Example 3.3	70
3.8 Prices and Quality Levels of Content Providers for Example 3.3	73
3.9 Prices and Quality Levels of Network Providers for Example 3.3	74
3.10 Effect of p_{t_1} Value on Utilities, Prices, and Quality in Example 3.1	75
3.11 Effect of p_{t_1} and p_{t_2} Values on Utilities, Prices, and Quality in Example 3.2 with $p_{t_1} = p_{t_2}$	76

3.12	Effect of p_{t_1} and p_{t_2} Values on Total Utility in Example 3.3	77
3.13	Network Topology of Content Flows for Example 3.4	78
4.1	The Network Structure of the Model's Content Flows	90
4.2	The Network Structure of the Model's Financial Payment Flows	96
4.3	Network Topology for Example 4.1	104
4.4	Network Topology for Example 4.2	109
4.5	Network Topology for Example 4.3	111
4.6	Network Topology for Example 4.4	113
4.7	Prices of Content Provider 1 and Network Provider 1 for Example 4.4	116
4.8	Prices of Content Provider 2 and Network Provider 2 for Example 4.4	117
4.9	Quality Levels of Content Providers and Network Providers for Example 4.4	118
5.1	The Bipartite Structure of the Competition Among the Network Providers	121
5.2	Network Topology for Example 5.1	131
5.3	Network Topology for Example 5.2	133
5.4	Effect of Increasing p_0 on Demand Market 2's Contracts	135
5.5	Effect of Increasing p_0 on Network Providers' Profits	136

5.6	Network Topology for Example 5.3	136
5.7	Effect of Increasing Maximum Quality Level on Network Providers’ Profits - Same for Both Providers	138
5.8	Effect of Increasing Maximum Quality Level on Network Providers’ Profits - Different for Each Provider	139
6.1	The Supply Chain Network Structure of the Game Theory Model	147
6.2	The Supply Chain Network Topology for Example 6.1	162
6.3	Prices and Quality Levels for the Product and Freight of Example 6.1	164
6.4	The Supply Chain Network Topology for Example 6.2	166
6.5	Prices and Quality Levels for Products and Modes 1 and 2 of Example 6.2	168
6.6	The Supply Chain Network Topology for Example 6.3 and Variant	169
6.7	The Supply Chain Network Topology for Example 6.4 and Variant	173
6.8	The Supply Chain Network Topology for Example 6.5 and Variant	178

CHAPTER 1

INTRODUCTION AND RESEARCH MOTIVATION

Using certain protocols, rules, and policies, networks provide the infrastructure for connectivity and operations for service providers in our societies (Nagurney, Dong, and Zhang (2002)). For instance, communication networks facilitate the spreading of information at speeds never before imagined. Transportation networks give us the means for mobility, shipment, and delivery of goods as fast as overnight or within the day (see Sheffi (1985), Nagurney (2006b), and United States Patent (2015)).

Logistical and supply chain networks enable firms to look at the overall movement of products from start to end, allowing organizations to see the value in creating partnerships and in working together to ensure the best possible service provided to the end-customer. In other words, supply chains are networks of suppliers, manufacturers, transportation service providers, storage facility managers, retailers, and consumers at the demand markets (Nagurney (2006a)). Supply chains are the backbones of our globalized network economy and provide the infrastructure for the production, storage, and distribution of goods and associated services as varied as food products, pharmaceuticals, vehicles, computers and other high tech equipment, building materials, furniture, clothing, toys, and even electricity (Nagurney (1999)).

In addition to these positive roles, today's networks have downsides such as their large-scale nature and complexity and increasing congestion, especially in, but not limited to, transportation and telecommunications networks. Also, alternative

behaviors of the networks' users, which can lead to paradoxical phenomena (cf. Braess, Nagurney, and Wakkolbinger (2005)), as well as interactions between the networks themselves are some realities of today's networks. The decisions made by the entities in the networks, in turn, may affect not only themselves but others as well, in terms of profits and costs, the timeliness of deliveries, and the quality of services (Nagurney (1999, 2006a)).

In this competitive global market, to maintain an edge, every business needs to achieve optimum levels of efficiency (Forker, Mendez, and Hershauer (1997)). Creating a variety of products and taking into consideration quality of products (Millen and Maggard (1997)) have been recognized as one of best ways to maintain a competitive edge in different supply chain networks. In fact, quality is the business of doing business (Murthy (2001)).

Communication networks, in particular the Internet, represent the essential infrastructure for business, government, and personal communication. They provide the backbone for numerous economic transactions and social interactions and have transformed manufacturing, transportation, and finance. Almost 40% of the world population has an Internet connection today, in comparison with 1995, when the percentage was less than 1% (Internet Live Stats (2016)). There are now approximately 3.4 billion Internet users out of a global population of 7.3 billion (Miniwatts Marketing Group (2016)). However, emerging technologies and applications have pushed the capabilities required of the Internet beyond what the current infrastructure can provide. To address these limitations, the networking research community has taken up the task of designing new architecture for the future Internet, accompanied by proper economic pricing mechanisms in order to make them manageable (see Wolf et al. (2012)).

The future Internet needs to live up to the diversified requirements of next-generation applications and new users' requirements comprising mobility, security,

and flexibility. Zhang et al. (2010) point out that economic relationships are far more mysterious than the underlying technology, as the business relationships that give rise to observed connections are mostly hidden from view. Our knowledge drops even further when we face services offered over a new paradigm that have the ability to create new functionalities that let users choose winners and losers. While there has been dramatic success in infrastructure research, resulting in a high bandwidth Internet backbone supporting simple end-to-end connections, there has been less success in terms of service-oriented Internet pricing research (Faizullah and Marsic (2005)). In fact, economic complexity in designing the next generation Internet (NGI) is advancing the role of pricing models including quality competition (see Jain, Durresi, and Paul (2011) and Wolf et al. (2012)).

Pricing models have been structured to consider quality and quantity to satisfy new requirements of applications and demand markets. However, pricing based on quality and the amount of usage, and, as is now typical, contracts of one to two years duration, may result in network congestion since network resource utilization may change over time, unless there are network upgrades. Furthermore, consumers may desire more flexibility and more choices, depending upon their location, and the type of viewing or other experience desired. Hence, it is expected that contract duration will become an important feature in the pricing of network services with shorter duration contracts garnering greater interest (Hwang and Weiss (2000)).

In general, the success of the entire supply chain is determined by the performance of all entities in the chain, rather than that of individual one. Quality has become one of the most essential factors in the success of supply chains of various products from food and agro-based products to other perishable products such as blood (Nagurney and Masoumi (2012)), pharmaceuticals (Masoumi, Yu, and Nagurney (2012)), medical nuclear supply chains (Nagurney and Nagurney (2012)), durable

manufactured products, including automobiles (see Shank and Govindarajan (1994)) to high tech products, such as microprocessors (see Goettler and Gordon (2011) and Goettler and Gordon (2014)) and services associated with the Internet (see Kruse (2010) and Nagurney et al. (2013a)). Quality and price have also been identified empirically as critical factors in transport mode selection for product/goods delivery (cf. Floden, Barthel, and Sorkina (2010) and Saxin, Lammgard, and Floden (2005)).

Poor freight service quality can lead to damaged and perished goods as Wang and Mozur (2014) noted for China's biggest electronic commerce shopping day known as Singles' Day as the biggest shopping day of the year in China with steep discount. The result was major delivery problems and costs associated with shipping in China exceeding even shipments from China to the US because of logistical challenges. Hence, the interplay between product quality and price and that of freight service quality and price, with respect to consumer demands, as well as specific product requirements, are not well-known.

On the other hand, the impact of traffic congestion, including wasted time, frustration, and losses in productivity is not insignificant and its effects on energy consumption and environmental emissions is immense. Congestion is also highly relevant in cities in terms of freight distribution and last mile deliveries. The US is experiencing a freight capacity crisis that threatens the strength and productivity of the US economy. According to the American Road & Transportation Builders Association (see Jeanneret (2006)), nearly 75% of US freight is carried in the US on highways, and bottlenecks are causing truckers 243 million hours of delay annually with an estimated associated cost of \$8 billion.

In this dissertation, I contribute to the modelling, analysis, and design of communication and supply chain networks with a focus on quality of service and price competition between decision-makers in the Internet and in freight shipment networks.

Specifically, after providing an introduction and research motivation as well as the foundational methodologies in Chapters 1 and 2, I construct a generalized network framework in Chapter 3 to focus on quality of service for all service providers in a future Internet multi-tier network. All providers with different functionalities and services are competing to set their prices and quality of service to maximize their profits. Then, in Chapter 4, I propose a dynamic adjustment process, which models how different service providers in the future Internet network adjust their prices, along with how the service providers define the quality of their services to satisfy the heterogeneous demands of consumers/demand markets for the Internet services.

In Chapter 5, I subsequently address the issue of contract duration in existing markets associated with the Internet that requires customers be locked-in for extended periods of time. Such inflexibility is detrimental to the users and may also impede innovation in Internet services. A general question that arises is how will the quality and the duration of Internet network contracts affect the pricing? For this issue, I formulate a model that captures the flexibility of contract duration as well as quality for Internet services. Then, I focus on another logistic network – that of the freight transportation network in Chapter 6. In that setting, I develop a supply chain network design model with multiple manufacturers and freight service providers competing on price and quality, while multiple modes of shipment for each freight service provider are considered.

This chapter is organized as follows: Section 1.1 includes an overview of the next generation Internet and the ChoiceNet project as a new network architecture for NGI. In Section 1.2 and Section 1.3, I describe freight services in supply chain and quality of service in the Internet and the supply chain. An appropriate literature review is provided in Section 1.4, and, finally, in Section 1.5, I present a dissertation overview.

1.1. Next Generation Internet

Without a doubt, the Internet has changed the world. It has developed from a small communication network among a few scientists to the most important medium for information exchange and the dominant communication environment for business, educational, entertainment, and social interactions. The Internet now is much more than it was ever envisaged to be (Paul, Pan, and Jain (2011)). It has become the backbone of modern society, rather than simply a communication system. Today, services, which were not even envisioned early in the Internet age, such as cloud computing and video streaming, are becoming mainstream (Wolf et al. (2012)).

In spite of its good –not perfect– functionality, there are recognized problems that cannot be patched within the constraints of the current architecture of the Internet (Donnet, Iannone, and Bonaventure (2008)). Few of the most relevant problems for which the present Internet architecture has failed to supply a satisfactory solution have been discussed in Jain (2006).

As our reliance on a highly dependable and secure information technology infrastructure continues to increase, it is no longer clear that the emerging and future needs of our society can be met by the current Internet infrastructure (Trossen (2009)). In addition, recent trends in technology and network use have pushed the capabilities required of the Internet beyond what can be provided by the currently deployed infrastructure (Labovitz et al. (2010)). Research initiatives, therefore, have been launched to study the design and development principles of the next generation Internet.

If history is a guide, the potential of the future Internet will be primarily driven by innovative services and applications (Man-Sze (2009)). The Directorate for Computer and Information Science and Engineering (CISE) has formulated a program to stimulate innovative and creative research to explore, design, and evaluate trustworthy Future

Internet Architectures (FIA). In the United State of America, the National Science Foundation (NSF) funded four projects as a part of this program in 2010 summer¹ and the fifth one (ChoiceNet) in 2011. The FIA projects include “Named Data Networking”² (focusing on security of content in the future Internet), “MobilityFirst”³ (working on robustness and trustworthy networks), “NEBULA”⁴ (developing new trustworthy data to support cloud computing model for network services), “eXpressive Internet Architecture”⁵ (enabling flexible context-dependent mechanisms for establishing trust between the communicating principals), and “ChoiceNet”⁶ (developing a new architecture to enable sustained innovation in the core of the network, using economic principles).

In Europe, the Future Internet Research and Experimentation (FIRE)⁷ is addressing this need by creating a multidisciplinary research environment for investigating and experimentally validating highly innovative and revolutionary ideas for new networking and service paradigms. Typical E.U. projects include ECRYPT II⁸ (working on future encryption technologies), INTERSECTION⁹ (focusing on the vulnerabilities at the

¹NSF FIA, 2014, NSF Future Internet Architecture Project, <http://www.nets-fia.net>.

²Named Data Networking (NDN) Project, <http://www.named-data.net>.

³Mobility First Future Internet Architecture Project, <http://mobilityfirst.winlab.rutgers.edu>.

⁴NEBULA: Future Internet Architecture, <http://nebula.cis.upenn.edu>.

⁵eXpressive Internet Architecture, <http://www.cs.cmu.edu/~xia>.

⁶Choicenet: Network Innovation through Choice, <https://code.renci.org/gf/project/choicenet>, and choicenet.info.

⁷Future Internet Research & Experimentation, <http://cordis.europa.eu/fp7/ict/fire>.

⁸European Network of Excellence in Cryptology II, European Union 7th Framework Program, <http://www.ecrypt.eu.org>.

⁹Infrastructure for heTERogeneous, Resilient, SEcure, Complex, Tightly Inter-Operating Networks, European Union 7th Framework Program, <http://www.intersection-project.eu>.

interaction point of different service providers), AWISSENET¹⁰ (concentrating on security and error resilience on wireless ad-hoc networks and sensor networks), and SWIFT¹¹ (focusing on future cross-layer identity management framework).

One of the critical concerns in designing the future Internet architecture is how to integrate new technologies into an ecosystem that involves users, service providers, and developers in such a way that new ideas can be deployed and used in practice in a sustainable fashion. To answer this question, investigators from the University of Massachusetts Amherst and three other institutes have worked on the architectural design of an economy plane for the Internet in the form of ChoiceNet project.

1.1.1 ChoiceNet

Competition is necessary for promoting the long-term economic viability of networks. It provides incentives for continual innovation and investment in network operators' and service providers' facilities. Therefore, "design for competition" must be an important principle for any future Internet architecture (Chuang (2011)). This is the main ambition of the ChoiceNet project. ChoiceNet is an FIA project with four institutions, including the University of Massachusetts Amherst as the lead, the University of Kentucky, North Carolina State University, and the University of North Carolina/RENCI.

The key element of the ChoiceNet project is "choices" that can drive innovations necessary for future networks. Designing for choice is similar to designing for competition (Clark et al. (2005)) which enables entities to select among a range

¹⁰Ad-hoc personal area network & Wireless Sensor SEcure NETwork, European Union 7th Framework Program, <http://www.awissenet.eu>.

¹¹Secure Widespread Identities for Federated Telecommunications, European Union 7th Framework Program, <http://www.ist-swift.org>.

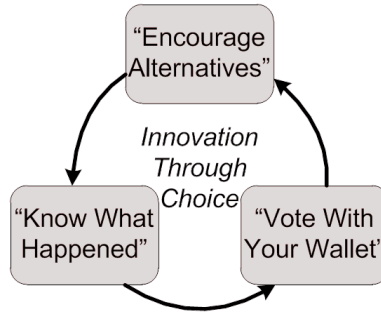


Figure 1.1. ChoiceNet Principle

of alternative services that may differ in functionality, performance, and cost. By creating the “economy plane”, ChoiceNet aims to create an environment to allow more innovation within the Internet architecture. As a matter of fact, any global-scale distributed communications infrastructure, such as the future Internet, requires significant capital investments; therefore, appropriate incentives must exist for the network owners to have a sustainable investment in new facilities and services (Chuang (2011)).

Compared to other studies focusing on economic aspects of networking, ChoiceNet integrates the economic interactions between networking technologies and the economic effects in the network architecture itself by offering three key principles (Figure 1.1). Other studies (see McKnight and Bailey (1998)) look at networking technology and economic effects as separate issues (e.g., analysis of economic behavior based on the given Internet structure and technologies (Semret et al. (2000))). To that end, the first principle of an economy plane in ChoiceNet is to “encourage alternatives” by creating different types of services and alternative services of the same type. Once alternatives are available, users can evaluate their choices as they “know what happened”. Using this information, users can “vote with their wallet” and choose services that are most suitable for them or continue using a particular service. For complete information about ChoiceNet see Wolf et al. (2012).

As part of my doctoral dissertation, I have developed economic game theory models and pricing for the next generation Internet under the ChoiceNet project.

1.2. Supply Chains and Freight Services

The efficiency of a supply chain has the power to make or break a business. If the systems are fine-tuned, this translates into a business gaining a competitive advantage in the marketplace. As global supply chains become more complex year by year companies must adapt to these changes and alter their strategies accordingly. In addition, optimizing supply chain productivity requires a commitment to quality from every entity and the use of process excellence techniques (Nagurney, Li, and Nagurney (2013)) to maximize efficiency and minimize costs (DeBenedetti (2015)).

The shipping environment in the supply chain can be a complex system in which decision-makers do not have full visibility into the different parties that exist and the roles that they serve (Nagurney (2014) and Liu and Nagurney (2012)). Nagurney (2004) stated that transportation networks and their efficient management have been studied since ancient times. For instance, Romans imposed controls over chariot traffic during different times of day in order to deal with congestion (see Banister and Button (1993)).

Since transportation and logistics involve activities associated with the movement of products and information to, from, and between the members of the supply chains, transportation network equilibrium is applied to formulate general supply chains (Nagurney (2006b), Nagurney, Liu, and Woolley (2007), and Nagurney (2007)), power supply chains (Wu et al. (2006) and Nagurney et al. (2007)), and financial networks with intermediation (Liu and Nagurney (2007)).

More recently, efficient transportation has been recognized as an essential determining factor in providing consistent service to beneficiaries and reducing cost substantially. Many decision-makers struggle with a seemingly simple question of the best way to get their specific product to their desired destinations, as it depends on a variety of shipment criteria and carrier capabilities (Coyle et al. (2012)).

When a consignee pays for the freight, customers may select their carrier (Samir (2014)). Specifically, for online shopping, the retailers can increase the sale and customer satisfaction when they expand the flexibility of shipping options - including shipping rates, shipping modes, and shipping speeds (UPS (2014)).

For instance, in pharmaceutical and food supply chains, professionals struggle with the shipment of temperature-sensitive products so that they will remain in the appropriate temperature range and arrive at the suitable time (Yu and Nagurney (2013)). Additional concerns of security affect nearly every carrier with a product in a supply chain (Nagurney and Nagurney (2012), Nagurney and Masoumi (2012), and Masoumi, Yu, and Nagurney (2012)), but are of special concern to distributors with a high value product that criminals find an easy market for, such as over the counter medicine, apparel, jewelry, non-alcoholic beverages, and other industries plagued by their popularity as criminal targets (Coughlin (2012)).

The growth of intercontinental multi-channel distribution, containerization, direct to business, and direct to customer shipping has led to fierce competition among freight service providers who are subjected to pricing pressures and increased expectations to handle more complex services (Hakim (2014) and DHL (2014)). To maintain their competitive edge, freight service providers are increasingly focusing on positioning themselves as more than just a commodity business. These providers may offer flexibility to meet customer needs of safety and/or traceability and, furthermore, differentiate themselves from the rest of the competition; thereby, migrating towards

being more value-oriented than cost-oriented (Bowman (2014) and Glave, Joerss, and Saxon (2014)).

As consumers' demands have become more diversified and personalized, mass production has taken a backseat to customized production and faster delivery times has put transport systems under pressure. Since the 1980s, an express industry has been developed to satisfy the resulting need for small, frequent-batch and door-to-door transportation. The online retailer Amazon.com recently submitted a patent (United States patent (2013)) for anticipatory and speculative shipping. In this patent, based on advanced forecasts of customer behavior (e.g. previous purchases, behavior during homepage visits, and demographics), they actually ship the products before the customer orders it! The product is shipped towards a region where a purchase is expected and is redirected during transport when the order is placed, thus, allowing almost instant deliveries (Bensinger (2014)).

In the era of global trade, issues surrounding competition, opportunities, investment, and outsourcing have induced transport and logistics companies to look for different services to grow and improve their competitive advantage. In general, a good transport system is defined according to the number of "rights" of supply. This involves getting the products at the right time, in the right condition, and via a cost effective manner accompanied with desired level of service quality (Nagurney and Li (2014a)). Transport owners that cannot offer the desired level of quality are forced to leave the market, as was the case when the intermodal company CargoNet withdrew from the Swedish rail market, claiming unreliable infrastructure as one of the main reasons (Floden and Woxenius (2013)). In fact, quality of service is driving logistics performance in both developed and emerging economies. Clearly, quality in freight service is gaining in importance (Kormnyos and Tànczos (2007), Achou (2010), Deflorio, Perboli, and

Tadei (2010), and Hao and Lin (2013)) and carriers must take quality into account in shipping the products to be able to survive in the current market.

The next section focuses on quality of service in communication and transportation networks.

1.3. Quality of Service

In today's world of fierce competition, there are severe pressures on any organization to find new ways of creating and delivering value to customers through supply chain management and carriers are no exception to this (Kannan, Bose, and Kannan (2012)). Carriers can transport either data through information networks such as in the case of network service providers in the Internet or ship goods via different modes of transport in a supply chain.

1.3.1 QoS in the Internet

During the dramatic development of the Internet within the past several years, electronic commerce has been penetrating all aspects of the business world and leading to the appearance of a new business economy (Stahl, Dai, and Whinston (2001)). With greater involvement of the Internet in the commercial world, it has been realized that in spite of its attractive capability and versatility, it still has some notable limitations to meet the requirements of all business activities, such as security and quality of service (Gibbens and Kelly (1999)).

The Internet was historically built on the simple concept of Best Effort (BE). Best Effort means that there is only one class of service to which all traffic belongs while there is no delivery confirmation and no guarantee for timely delivery and there is a possibility of traffic loss. The diversification of users' demand after the

commercialization of the Internet forced Internet providers to offer different levels of services (Shin, Weiss, and Correa (2004)). In addition, critical real-time and business-oriented applications require improved levels of services, or quality of service from the network.

The relationship between Best Effort and QoS is similar to that of regular mail and priority mail for which users pay a higher price than regular mail. However, guaranteeing QoS in the Internet is not that easy. Initially, QoS was introduced in telecommunications to measure how well a particular service performs (Hardy (2001)).

In the Internet, quality of service for various traffic types is the ability to provide different priorities of content transmission. It can guarantee a certain level of performance of a data flow and its reliability. The main categories of data traffic in the Internet are real-time traffic such as voice services, interactive data and streaming traffic such as web browsing, and delay-tolerant traffic such as e-mail and file transfer. Depending on the type of data streaming, various services make different demands on the network (Davies, Hardt, Kelly (2004)).

The efficiency of each service type is measured by several parameters including bounds on the packet delay, delay variation, loss rate (Fulp et al. (1998)), and jitter (Shin, Weiss, and Correa (2004)) either qualitatively (relative) or quantitatively (absolute). QoS is assured by reserving resources, primarily bandwidth and sometimes buffer space (Zhao, Olshefski, and Schulzrinne (2001)).

By requiring QoS to be geared to the end user's expectations, we may expect new legislation and rules to create a new reliable and yet more expensive service-oriented Internet, which is different from the Best Effort type of network that we have known (Altman et al. (2012)). In a service-oriented architecture (SOA), business processes can be realized by comprising various services, which autonomously provide a more

complex functionality (cf. Krafzig, Banke, and Slama (2004)) with more intricate economic relationships between providers.

The next generation Internet is expected to be service-oriented with each provider offering specific services. In the Internet of services with comparable functionalities, but varying quality levels, services are available at different costs in the service marketplace, so that users can decide which services from which service provider to select (Wolf et al. (2012)). NGI, typically, includes multi-tier service providers. For example, a content service provider is a website that handles the distribution of online content such as blogs, videos, music or files, whereas a network service provider is an entity that provides network access or long-haul network transport. These offer equal or rather similar services at different QoS levels and different costs. This gives users the opportunity to select those services which meet their anticipations and QoS requirements best (Schuller et al. (2010)).

Quality of service is not limited to Internet and telecommunication services. The next section focuses on quality of service in the supply chain.

1.3.2 QoS in the Supply Chain

In today's world of fierce competition, the service sector has become important in the economies of countries all over the world and services will continue to be a dominant force in the world economy in the future as well. All businesses today are part of supply chains. Supply chains are like the circulatory system that encompasses all flows of product services, information, and finance between entities. As a natural progression to this thought, every entity in the supply chain focuses on contributing quality, value, and satisfaction to the immediate customer which may result in greater profit for all entities (Kamakoty and Sohani (2013)). That is why service quality has been a major area of research for almost three decades in any organization and is

not limited to the Internet and communication services (Kannan, Bose, and Kannan (2012)).

In any supply chain, the ultimate success of a firm will depend on its managerial ability to deliver high quality services which results in customers' satisfaction. Quality may mean the quality of tangibles/intangibles, the quality of logistics, the quality of the processes, the quality of peripheral services, the quality of the service provider/service user, and the operational/technical quality. The relationship of service quality with improved supply chain performance is widely accepted (Mentzer, Flint, and Kent (1999), Mentzer, Flint, and Hult (2001), and Perry and Sohal (1999)). Meanwhile, quality of service in logistics is recognized as a critical factor in gaining a competitive advantage (Seth, Deshmukh, and Vrat (2006a-b)) and a key for subsistence and success (Kannan, Bose, and Kannan (2012)).

In food and agricultural supply chains, food products are required by European Council-regulation to be traceable from producer to end customer in order to withdraw perished food (Folinas, Manikas, and Manos (2006)). To enable food traceability, Ringsberg and Mirzabeiki (2013) suggested use of advanced technologies such as RFID-tagging to provide the required high quality transport. It is stated that there is a growing expectation that quality assurance will dominate the process of production and distribution in food chains in the future (Trienekens and Zuurbier (2008)). Therefore, a diagnostic model for quality control in agro supply chain logistics has been developed based on real-time product quality information (Van der Vorst, van Kooten, and Luning (2011)). The notion of quality controlled logistics hypothesizes that the flow of goods can be controlled and supply chain designs can be altered in real-time or pro-actively if the quality of the product can be predicted at each echelon of a supply chain.

The quality of service in a logistic supply chain can be defined as reliability, adhering to emission standards, cargo handling competency, or quality of in-house infrastructure and vehicles. Quality in DHL logistics, is defined as having no errors in shipments, low product damage, on-time orders, high productivity, excellent alignment with customer requirements, and full regulatory compliance (DHL (2015)).

In this dissertation, for freight service providers, I define and quantify quality as the quality conformance level, that is, the degree to which a specific service conforms to a design or specification (Nagurney and Li (2014b), Gilmore (1974), and Juran and Gryna (1988)). Hence, quality may vary from a 0% service level to a 100% service level (see, e.g., Juran and Gryna (1988), Campanella (1990), Feigenbaum (1983), and Shank and Govindarajan (1994)). When the quality of a freight service is at 0% level, the shipment has no specific quality, while a 100% service level demonstrates that the shipment is at perfect possible quality.

While there is a plethora of rich and continuous research literature available on how to measure quality of service in different supply chains (e.g., Mentzer, Flint, and Kent (1999), Mentzer, Flint, and Hult (2001), Perry and Sohal (1999), Seth, Deshmukh, and Vrat (2006a-b), and Gupta and Singh (2012)), the area of quality competition in the logistic component of a supply chain network is still in its infancy.

1.4. Literature Review

In this section, I present a review of the existing models in which service providers are competing to set quantity, price, and quality of their services in a service-oriented network. I emphasize in this section the fact that existing models are either designed for a monopoly service provider or one level of providers. Also, they missed the fact that each provider in a network provides a quality of service which will affect the consumers' demand. To the best of my knowledge, there is no study that considers

the effect of a flexible contract and quality of service for all providers on the pricing strategy in a network.

In fact, the existing models are not general and strong enough to demonstrate the complexity of the competitive relationships between service providers in the service-oriented networks such as the Internet and supply chains. Therefore, the construction of general game theory models for these networks that consider an oligopoly of service providers, quality of service for all providers in a multi-tier network, and heterogeneous demand markets is of importance and relevant.

1.4.1 Price and Quality Competition Among Service Providers

Pricing for any corporation has to be in line with its strategic goals (Farm and McCarthy (1999)). Generally, pricing policy defines how a company sets the prices of its products and services. The pricing model in an oligopoly market goes back to the “Bertrand” equilibrium model (Bertrand (1883)) in which firms set their prices first and then their customers choose quantities at the prices set. For example, this pricing model has been used in power supply chains to price electricity (Hobbs (1986), Rudkevich, Duckworth, and Rosen (1998), Brennan and Melanieb (1998), Hobbs, Metzler, and Pang (2000), Gan and Shen (2004), Lise et al. (2006), and Soleymani, Ranjbar, and Shirani (2008)).

In contrast to the Bertrand competition model, companies may independently compete on the quantity of their products or services a la “Cournot” competition (Cournot (1838)) and consumers pay for their amounts of orders. Cournot competition is applied extensively in different supply chains of products to set the right amounts of quantity in the network, including but not limited to food supply chains (Nagurney et al. (2013b) and Yu and Nagurney (2013)), pharmaceutical supply chains (Masoumi,

Yu, and Nagurney (2012)) and medical nuclear supply chains (Nagurney and Nagurney (2012) and Nagurney, Nagurney, and Li (2015)).

Since the 1990s, quality has emerged as one of the major competitive issues and higher quality goods have led to increasing returns (Balachander and Srinivasan (1994)). For instance, Japanese firms made dramatic gains in market share in industries such as automobiles, semiconductors, and consumer electronics because of the superior quality and reliability of their products, while American manufacturers lost their markets since their products were perceived by consumers as offering poorer quality than equivalently priced foreign products (Banker, Khosla, and Sinha (1998)). Applying this to the Internet network, the timing and bandwidth requirements of multimedia demanded new ways of dealing with data in communication systems (Hutchison, Mauthe, and Yeadon (1997)) and, therefore, it urged the incorporation of quality into Internet services and its effect on pricing of services. Kelly, Maulloo, and Tan (1998) addressed the issue of fair pricing within a large-scale broadband network and how available bandwidth should be shared between competing streams of traffic with an optimization framework.

Early pricing approaches for the Internet include Paris metro pricing (Odlyzko (1999)), responsive pricing (MacKie-Mason, Murphy, and Murphy (1997)), smart-market pricing (MacKie-Mason and Varian (1995)), two-tier market pricing (Semret et al. (2000)), and edge-pricing (Shenker et al. (1996)). Gibbens, Mason, and Steinberg (2000) assessed Paris metro pricing in a network with competition between subnetworks who provide multiple services in the presence of congestion. For pricing of Internet services, a number of researchers assume a monopoly of service provider. For instance, Dasilva, Petr, and Akar (2000) discussed static pricing policies for a service provider in multi-service networks. The service provider could offer the incentives for each user to choose the service that best matches her needs. See Gibbens (2000) for a survey on

controlling and pricing in communication networks and effective and fair allocation of scarce resources in the Internet.

In addition, equilibrium models for Internet networks generally assume basic economic relationships and consider price as the only factor that affects demand (cf. Kausar, Briscoe, and Crowcroft (1999), Laffont et al. (2003), Zhang et al. (2010), Altman, Hanawal, and Sundaresan (2010), and Musacchio, Schwartz, and Walrand (2011)). For instance, Zhang et al. (2010) proposed a two-stage Stackelberg game with Cournot and Bertrand competition. The price of a service offered by a content provider (CP) is determined as a function of the user's demand and the network access price. The network providers NPs charge CPs by maximizing their profit as a function of market share and the CPs' marginal cost. Laffont et al. (2003) modeled Bertrand competition with two network providers, multi-content providers, and heterogeneous users. A new pricing mechanism "off-net cost pricing principle" was proposed to find the optimum price to charge users and content providers. They analyzed the impact of an access charge on welfare and profit. The outcomes showed that the access charge determines the allocation of communication costs and affects the level of traffic. Economides and Tag (2012) also investigated what price network providers should charge users and content providers in order to maximize profits. Their analysis showed that the NP and the users are better-off while the CPs and the social surplus are always worse-off under network freedom (a non-neutral network).

However, in new paradigms for the Internet, price is not the only factor, and quality of service, as the ability to provide different priorities to applications, users, or data flows, is rising to the fore, due, in part, to increasingly demanding consumers. Considering quality as a delay cost function, Mieghem and Mieghem (2002) presented a quantity-quality-based framework which integrates technology and economics to derive mutually consistent pricing and scheduling of differentiated services for a heterogeneous

market of users. By combining the robustness and fairness of generalized processor sharing approach with the optimality and incentive-compatible pricing of generalized μ rule ($Gc\mu$), they proposed a new scheduling rule, called “ $Gc\mu$ -PS”. Cao et al. (2002) modeled a leader-follower cooperative game between one Internet service provider and its users to price the services considering QoS. They concluded that the solution to a leader-follower cooperative game is not Pareto optimal.

Other researchers focused on price competition in an oligopoly market of service providers. He and Walrand (2003) proposed a generic model for pricing Internet services in a multi-provider network. The results demonstrated that a noncooperative game can be unfair and discourage future upgrades of the network. A revenue-sharing policy, on the other hand, would be more efficient and encourages service providers to collaborate without cheating. Hermalin and Katz (2007) modeled the simultaneous choice of network providers for charging households and content providers, when the NP is able to offer several levels vs. one level of service quality. They concluded that restricting the network provider to supplying one level of quality has more negative outcomes. To control traffic in a congested network and analyze the bounds on the efficiency of oligopoly equilibria, Ozdaglar (2008) studied price competition between some service providers who own the routes in a network and set prices to maximize their profits. In a two-sided market framework, Njoroge et al. (2009) developed a game theoretic model to analyze the competition between two interconnected Internet service providers that compete in quality and prices for both heterogenous content providers and consumers. Musacchio, Schwartz, and Walrand (2011) investigated a two-sided market where content providers and network providers invest jointly in the network infrastructure and share the revenue. Users’ demand is determined as a function of the product of CPs’ and NPs’ investment (can be assumed as their quality) and decreases exponentially if the price goes up.

The pricing models are not limited to communication networks. Various studies have been instrumental in including competition and decision-making at each stage of a supply chain and, later, integrating them to form a unified structure. Some of the pioneers in the study of quality competition are: Akerlof (1970), Spence (1975), Sheshinski (1976), and Mussa and Rosen (1978), who discussed firms' decisions on price and quality in a quality differentiated monopoly market with heterogeneous customers. Dixit (1979) and Gal-Or (1983) initiated the study of quantity and quality competition in an oligopolistic market with multiple firms, where several symmetric cases of oligopolistic equilibria were considered. Brekke, Siciliani, and Straume (2010) investigated the relationship between competition and quality via a spatial price-quality competition model. Others who have contributed to the topic of quality competition include: Ronnen (1991), Banker, Khosla, and Sinha (1998), Johnson and Myatt (2003), and Acharyya (2005).

Yamada et al. (2011), building on the work of Nagurney, Dong, and Zhang (2002), focused on constructing a supply chain-transport supernetwork equilibrium model based on the behavior of freight carriers. The model accounts for the interaction between freight carriers and transport network users and endogenously determines the transportation costs generated in the supply chain networks. The study primarily focuses on road transportation. Xia and Ma (2012) developed a multimodal and multiproduct transportation network model that includes spatial aspects in order to offer a methodology to forecast the transportation demand and the freight flows in the transportation network while maximizing profit.

In addition, Hasan (2009) implemented an international freight simultaneous transportation equilibrium model developed by the UN economic and social commission for Western Asia. The multimodal multicommodity model makes cost and flow predictions; thereby, facilitating path/link redistribution between origin-destination pairs to

minimize costs. Yamada et al. (2009) proposed a model for strategic transport planning, particularly in freight terminal development and interregional freight transport network design. The modeling is undertaken within the framework of bilevel programming, where a multimodal multiclass user traffic assignment technique is incorporated within the lower-level problem, and the upper-level problem determines the best combination of actions such that the freight-related benefit-cost ratio is maximized. Holgun-Veras et al. (2011) conducted an experimental economic investigation (in the US and the UK) of shipper-carrier interactions on choice of mode and shipment size in freight transport. The theoretical and empirical evidence from this study concluded that freight mode choice can be best understood as the outcome of shipper-carrier interactions and to a large extent is due to shipment sizes.

1.4.2 Time-Based Competition Between Internet Service Providers

Initially, the Internet was government-funded and, thus, free to the users. Later, two pricing models were developed: one, where a flat fee was charged, and the second, where a basic charge covered a certain time and quantity of data with additional time/data charged incrementally. More than a decade ago, it was realized that such pricing models may not be applicable in a rapidly changing Internet (Faizullah and Marsic (2000)). For example, in the US, Comcast differentiates its monthly charge for business users based upon the desired download and upload speeds¹². Mediacom Cable not only differentiates among speeds, but adds a limit to the total quantity of data transfer¹³. In Canada, Rogers also offers similar pricing schemes¹⁴. For an overview of earlier Internet pricing models see Stiller, Reichl, and Leinen (2001).

¹²<http://www.comcast.com>

¹³<http://mediacom.com>

¹⁴<http://Rogers.com>

There exist several early mathematical models in which duration and quality of services are included in the pricing of Internet services. For example, Wang, Peha, and Sirbu (1997) examined the optimal pricing problem for guaranteed, integrated services in a network with capacity limitation. Demand elasticity for the service and the opportunity cost of providing that service are used to determine the optimal price for each service. Their model is a time-varying price schedule instead of a single price, since price is a function of time of day and demand for network services usually changes with time of day.

For a broadband multiservice network, Kelly (1997) addressed the issue of charging and examined the relationship between various fairness criteria and smart market approaches for dynamic pricing. He determined how a user chooses the charge per unit time that the user is willing to pay and then the optimality of the system is achieved when the user's choice of charge and network choice of allocated rates are in equilibrium. Courcoubetisaib and Siris (1999) investigated a framework for managing and pricing differentiated services that offers some level of performance guarantees which is called service level agreements (SLA). The framework defines and prices the amount of resources used by a specific SLA and, therefore, a manager can decide the number of such contracts that can be offered simultaneously based on the available bandwidth for the network. In this way, the pricing method can provide users with an incentive to select traffic contracts that reflect their actual needs and maximize the sum of utilities of users (social welfare) while maintaining a certain level of SLA. The proposed framework is quite general and can be used with a variety of mechanisms for implementing differentiated services.

Some scholars proposed two part pricing. For instance, Ferrari and Delgrossi (1998) derived a pricing formula from some charging policies. In order to satisfy these policies, the charging formula contains two parts: reservation charges and

usage charge. The reservation charges are the prices per unit time assigned to the buffer space, computing capacity, and schedulability for each type of service while the usage charge depends on real-time vs. non-real-time communications. In the same fashion, Jormakka, Grgic, and Siris (2001) proposed a charging mechanism for network connectivity services which includes a subscription component and a usage component. The subscription component of a charging mechanism is a one-time site connection fee, which is paid once when the user is connected to the provider and is related to the cost of equipment and labor necessary for connecting. The usage component, on the other hand, is associated with resource reservation and consumption in the backbone. It might depend on measures of resource usage such as the duration (time), the volume transferred, and the class of quality. This mechanism can describe a wide range of pricing schemes that are applied in the current telecommunications market for quality guaranteed services and Best Effort services.

To address dynamic bandwidth management, Hwang, Kim, and Weiss (2002) formulated an economic model as an optimization problem for dynamically provisioned differentiated service networks. This optimization problem is based on economic edge-pricing theory. Applying the price data of bandwidth commodity markets, they proposed a way to capture the opportunity cost as a part of edge-pricing for differentiated services. The price and cost of services are calculated in a dynamic profit maximization problem. Another type of edge-pricing algorithm which is based on the effective bandwidth concept is an ex-post charging mechanism, developed by Bailey, Gamvros, and Raghavan (2007), to prevent frequent congestion in the network. Effective bandwidth is defined by a scalar that summarizes resource usage on a communications link and presents the capacity of the outgoing link. It is called “ex-post” since the charging algorithm is determined in advance while the actual charge is calculated thereafter. As they mentioned, this pricing mechanism can penalize customers with high utilization and/or bursty traffic and charges customers with

higher prices that demand better quality (determined by lower probability of packet loss) for their traffic.

1.5. Dissertation Overview

This dissertation consists of seven chapters. An overview of the motivation for, contributions, and background literature to the research that I have conducted is provided in Chapter 1. Chapter 2 provides a thorough review of the methodologies, foundational models and theories, including variational inequality theory and Projected Dynamical Systems (PDS) theory, that this research is based on.

In Chapter 3 of the dissertation, according to the discussion of Section 1.3, I consider an Internet with a service-oriented architecture, in which content and network providers interact and compete in prices and quality of services. There are two models, a basic model and a general one. The methodology is inspired, for the first model, by Altman, Legout, and Xu (2011) and, for the second model, by El Azouzi, Altman, and Wynter (2003). Altman, Legout, and Xu (2011) studied the effect of side payments, while taking into account the different levels of quality offered by a network provider in the Internet with one content provider, one network provider, and one demand market. With a basic model, I complete Altman, Legout, and Xu (2011)'s model by including the quality of both providers into the demand function and assuming a production cost function for the content provider. El Azouzi, Altman, and Wynter (2003) modeled an oligopoly market of content providers and one network provider in a bi-criteria Nash equilibrium competition between content providers. Their model restricts the network to one network provider and quality for only the content providers' service. Therefore, it cannot reveal the competition among the network providers for users. In contrast, my model overcomes these limitations by including multiple providers, multiple users (demand markets), and demands as a function of the prices and quality levels of all

providers. On top of that, this model presents a general framework for modeling alternative cost functions and demand functions associated with the services and the demand markets. This chapter is based on Saberi, Nagurney, and Wolf (2014).

I develop a projected dynamical system model of a service-oriented Internet in Chapter 4. Such dynamical systems were introduced by Dupuis and Nagurney (1993) and have been used in a variety of applications from transportation, spatial economic and oligopolistic market problems (see Nagurney and Zhang (1996), Nagurney (1999), and the references therein) to supply chain network problems (cf. Nagurney (2006a), Nagurney, Cruz, and Toyasaki (2008), and Cruz (2008)) and finance (see Nagurney (2008)). In addition, PDSs have been applied in population games by Sandholm (2010) and in neuroscience by Girard et al. (2008). More recently, PDSs have been utilized to capture the dynamics of oligopolistic competition with the inclusion of quality (see Nagurney and Li (2014a)) and to model the dynamics of a service-oriented Internet with only quality associated with content provision by Nagurney et al. (2013a), and also to capture that associated with network provision by Nagurney and Wolf (2014). Here, for the first time, I model the dynamics of both price and quality competition of both content providers and of network providers. This work is an attempt to complete both the latter models in terms of price setting while considering quality of service for both content and network provision. The continuous-time dynamic model that I propose describes the evolution of prices charged by the content providers and the network providers, as well as their quality levels of content and network transport provision, respectively. I provide qualitative results, including stability analysis, and also present a discrete-time algorithm for iterative computation and tracking of the prices and quality levels until a stationary point, equivalent to an equilibrium state is achieved. This work extends and completes the static Internet network economic model of Saberi, Nagurney, and Wolf (2014) by describing the underlying dynamic

behavior, accompanied by qualitative analysis, and with the provision of additional numerical examples. This chapter is based on Nagurney et al. (2014a).

In Chapter 5 of the dissertation, I formulate a competitive oligopoly market of Internet network providers, motivated by ChoiceNet (cf. Wolf et al. (2012) and Wolf et al. (2014)), although not limited to it, and the economic relationships among them. The entities are able to offer different network services and to create contracts for their users according to the users' desires and needs. The model developed in this chapter is straightforward enough to understand for both users and network providers and creates an opportunity to control the total charge for a communication by a modification of the parameters. The users/demand markets select contracts based on three main criteria: the amount of usage contracted per period of time (the usage rate) during the contract duration, the quality level of service, and the contract duration. Here I consider a reserved usage amount per unit of time. The earlier work on the network economic game theoretical modeling of future Internet architectures focused on introducing quality, with an emphasis on service provision, which is maintained through network transport/provision in Nagurney et al. (2013a), and also on capturing the behavior of both content providers and network providers, with the latter competing on price and quality in Nagurney and Wolf (2014). In Nagurney et al. (2014a), the dynamics are associated with content and network provider competition where consumers respond to the prices and the quality of both content provision and network provision. Here, in contrast, my goal is to extend the game theoretical modeling of competitive network providers and services by including not only quality of service but also contract durations as strategic variables, in addition to the reserved usage rates. This chapter is based on Nagurney et al. (2015b).

In Chapter 6, I focus on the development of game theory models in both equilibrium and dynamic settings for a supply chain network with multiple manufacturers and

multiple freight service providers handling freight transportation. The decision-makers including manufacturers and freight service providers at each echelon are competing in both prices and quality. Quality of the product is traced along the supply chain with consumers differentiating among the products offered by manufacturers. Also, quality of freight service shipment is accounted for in the model. Heretofore, the integration of price and quality competitive behavior with both manufacturers and freight service providers has not been examined in a rigorous theoretical and computationally tractable framework. This framework is inspired, in part, by the work of Nagurney et al. (2013a) and Saberi, Nagurney, and Wolf (2014). The former studied a network economic game theory model of a service-oriented Internet with choices and quality competition. In addition, Saberi, Nagurney, and Wolf (2014) proposed a network economic game theory model of service-oriented Internet architectures with price and quality competition between content and network providers. Here, I allow for multiple modes of transportation and each freight service provider can have a different number of mode options. I consider a mode in a general way in that it can correspond to intermodal transportation. This chapter is based on Nagurney et al. (2015a).

Finally, Chapter 7 summarizes the obtained results and presents the conclusions. Suggestions and directions for future research are also presented.

CHAPTER 2

METHODOLOGIES

This chapter provides an overview of some of the fundamental theories and methodologies that are utilized in this dissertation. I first recall variational inequality theory, which is utilized throughout this dissertation as the basic methodology, and is applied in Chapters 3 to 6. Variational inequality theory is a powerful methodology that can be applied to numerous problems to solve network economic equilibrium models and is applied in this dissertation to analyze the equilibria of price and quality competition in the Internet and in supply chain networks.

After the review of variational inequality theory, I present projected dynamical systems theory, which is used in Chapters 4 and 6, to analyze the associated dynamics.

Following that, I discuss the relationships between variational inequalities and game theory as well as the qualitative properties of the variational inequality model of Nash equilibrium. Finally, I recall the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993) and employed in this dissertation to solve variational inequalities and projected dynamical systems. It is a computational algorithm and provides discrete-time realizations of the continuous-time adjustment processes associated with projected dynamical systems.

Further details and proofs of theorems concerning variational inequalities and projected dynamical systems can be found in Nagurney (1999), Dupuis and Nagurney (1993), and Nagurney and Zhang (1996).

2.1. Variational Inequality Theory

In this section, I briefly overview the theory of variational inequalities, which is used throughout this dissertation to solve finite-dimensional network equilibrium problems. Then, I present qualitative results, specifically concerning the existence and uniqueness of solutions. All definitions and theorems are taken from Nagurney (1999) except where noted. Variational inequality theory was first defined over an infinite-dimensional space by Hartman and Stampacchia (1966). Then, finite-dimensional theory was advanced when Dafermos (1981) recognized that traffic network equilibrium conditions, as stated by Smith (1979), had a structure of a variational inequality. For further discussion and proofs see Nagurney (1999). We assume here that all vectors, except where noted, are column vectors.

Definition 2.1

The finite-dimensional variational inequality problem, $VI(F, \mathcal{K})$, is to determine a vector $X^ \in \mathcal{K} \subset \mathcal{R}^n$, such that*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \quad (2.1)$$

where F is a given continuous function from \mathcal{K} to \mathcal{R}^n and \mathcal{K} is a given closed convex set. Note that $\langle \cdot, \cdot \rangle$ is an inner product in n -dimensional Euclidean space, such that

$$\langle F(X^*), X - X^* \rangle = \sum_{i=1}^n F_i(X^*) \times (X_i - X_i^*). \quad (2.2)$$

The variational inequality problem is a general formulation that encompasses a set of mathematical problems, including nonlinear equations, optimization problems, complementarity problems and fixed point problems (see Nagurney (1999)). Optimization problems, including constrained and unconstrained, can be formulated as variational

inequality problems. The following is a brief discussion of the relationship between the variational inequality problem and the optimization problem. All the proofs of the following variational inequality theorems can be found in Nagurney (1999) (see also Kinderlehrer and Stampacchia (1980)).

Proposition 2.1

Let X^ be a solution to the optimization problem:*

$$\text{Minimize } f(X) \tag{2.3}$$

subject to:

$$X \in \mathcal{K},$$

where f is continuously differentiable and \mathcal{K} is closed and convex. Then X^ is a solution of the variational inequality problem:*

$$\langle \nabla f(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{2.4}$$

where $\nabla f(X)$ is the gradient vector of f with respect to X , that is

$$\nabla f(X) = \begin{pmatrix} \frac{\partial f(X)}{\partial X_1} \\ \frac{\partial f(X)}{\partial X_2} \\ \dots \\ \frac{\partial f(X)}{\partial X_n} \end{pmatrix}. \tag{2.5}$$

Proposition 2.2

If $f(X)$ is a convex function and X^* is a solution to $\text{VI}(\nabla f, \mathcal{K})$, then X^* is a solution to the optimization problem (2.3). In the case that the feasible set $\mathcal{K} = \mathcal{R}^n$, then the unconstrained optimization problem is also a variational inequality problem.

In the case where a certain symmetry conditions holds, the variational inequality problem can be reformulated as an optimization problem. I now present the definitions of positive semidefinite, positive-definite and strongly positive-definite.

Definition 2.2

An $n \times n$ matrix $M(X)$, whose elements $m_{ij}(X); i, j = 1, \dots, n$, are functions defined on the set $\mathcal{S} \subset \mathcal{R}^n$, is said to be positive-semidefinite on \mathcal{S} if

$$v^T M(X)v \geq 0, \quad \forall v \in \mathcal{R}^n, X \in \mathcal{S}. \quad (2.6)$$

It is said to be positive-definite on \mathcal{S} if

$$v^T M(X)v > 0, \quad \forall v \neq 0, v \in \mathcal{R}^n, X \in \mathcal{S}. \quad (2.7)$$

It is said to be strongly positive-definite on \mathcal{S} if

$$v^T M(X)v \geq \alpha \|v\|^2, \text{ for some } \alpha > 0, \quad \forall v \in \mathcal{R}^n, X \in \mathcal{S}. \quad (2.8)$$

Theorem 2.1

Assume that $F(X)$ is continuously differentiable on \mathcal{K} and that the Jacobian matrix

$$\nabla F(X) = \begin{bmatrix} \frac{\partial F_1}{\partial X_1} & \cdots & \frac{\partial F_1}{\partial X_n} \\ \vdots & \dots & \vdots \\ \frac{\partial F_n}{\partial X_1} & \cdots & \frac{\partial F_n}{\partial X_n} \end{bmatrix} \quad (2.9)$$

is symmetric and positive-semidefinite. Then, there is a real-valued convex function $f : \mathcal{K} \mapsto \mathcal{R}^1$ satisfying

$$\nabla f(X) = F(X) \quad (2.10)$$

with X^* the solution of $\text{VI}(F, \mathcal{K})$ also being the solution of the mathematical programming problem:

$$\text{Minimize } f(X)$$

subject to:

$$X \in \mathcal{K},$$

where $f(X) = \int F(X)^T dx$, and \int is a line integral.

The variational inequality problem can be reformulated as a convex optimization problem when the Jacobian matrix of $F(X)$ is symmetric and positive semidefinite. Historically, many equilibrium problems were reformulated as optimization problems, under precisely such an assumption of symmetry. The assumption, however, in terms of applications was restrictive and precluded the more realistic modeling of multiple commodities, multiple modes and/or classes in competition.

However, the variational inequality is the more general problem formulation that can also handle a function $F(X)$ with an asymmetric Jacobian (see Nagurney (1999)). This fact allows VIs to be utilized to study a broad range of equilibrium problems.

Existence of a solution to a variational inequality problem follows from continuity of the function F entering the variational inequality, provided that the feasible set \mathcal{K} is compact, as stated in Theorem 2.2. I now provide qualitative properties, specifically, the conditions for existence and uniqueness of a solution.

Theorem 2.2

If \mathcal{K} is a compact convex set and $F(X)$ is continuous on \mathcal{K} , then the variational inequality problem admits at least one solution X^ .*

In the case that the feasible set \mathcal{K} is unbounded, we have

Theorem 2.3

VI(F, \mathcal{K}) admits a solution if and only if there exists an $\mathcal{R} > 0$ and a solution of VI(F, \mathcal{S}), X_R^ , such that $\|X_R^*\| < \mathcal{R}$, where $\mathcal{S} = \{X : \|X\| \leq \mathcal{R}\}$.*

Given certain monotonicity conditions, the qualitative properties of existence and uniqueness can be obtained easily. Next, I utilize certain monotonicity conditions to discuss the qualitative properties of existence and uniqueness. I recall some basic definitions.

Definition 2.3 (Monotonicity)

$F(X)$ is monotone on \mathcal{K} if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \quad (2.11)$$

Definition 2.4 (Strict Monotonicity)

$F(X)$ is strictly monotone on \mathcal{K} if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, X^1 \neq X^2. \quad (2.12)$$

Definition 2.5 (Strong Monotonicity)

$F(X)$ is strongly monotone on \mathcal{K} if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq \alpha \|X^1 - X^2\|^2, \quad \forall X^1, X^2 \in \mathcal{K}, \quad (2.13)$$

where

$$\alpha > 0.$$

Definition 2.6 (Lipschitz Continuity)

$F(X)$ is Lipschitz continuous on \mathcal{K} if there exists an $L > 0$, such that

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \leq L \|X^1 - X^2\|^2, \quad \forall X^1, X^2 \in \mathcal{K}. \quad (2.14)$$

L is called the Lipschitz constant.

Theorem 2.4 (Uniqueness Under Strict Monotonicity)

Suppose that $F(X)$ is strictly monotone on \mathcal{K} . Then the solution to the VI(F, \mathcal{K}) problem is unique, if one exists.

Theorem 2.5 (Uniqueness Under Strong Monotonicity)

Suppose that $F(X)$ is strongly monotone on \mathcal{K} . Then there exists precisely one solution X^ to $\text{VI}(F, \mathcal{K})$.*

From the above theorems, in the case of an unbounded feasible set \mathcal{K} , strong monotonicity of the function F guarantees both existence and uniqueness because existence follows from the fact that strong monotonicity implies coercivity whereas uniqueness follows from the fact that strong monotonicity implies strict monotonicity. On the other hand, if the feasible set \mathcal{K} is compact, then continuity of F guarantees the existence, and the strict monotonicity condition is sufficient to provide uniqueness.

2.2. Projected Dynamical Systems

In this section, I present the definition of a projected dynamical system (cf. Dupuis and Nagurney (1993), Nagurney and Zhang (1996)), and then discuss the relationship between projected dynamical systems and variational inequality problems. I also recall some fundamental qualitative properties of the solution to the ordinary differential equation that defines such a projected dynamical system, as well as stability analysis. All the definitions and theorems can be found in Nagurney and Zhang (1996) and Nagurney and Siokos (1997), except where noted.

Definition 2.7 (Vector Projection)

Given $X \in \mathcal{K}$ and $v \in R^n$, define the projection of the vector v at X (with respect to \mathcal{K}) by

$$\Pi_{\mathcal{K}}(X, v) = \lim_{\delta \rightarrow 0} \frac{(P_{\mathcal{K}}(X + \delta v) - X)}{\delta} \quad (2.15)$$

with $P_{\mathcal{K}}$ denoting the projection map:

$$P_{\mathcal{K}}(X) = \operatorname{argmin}_{X' \in \mathcal{K}} \|X' - X\|, \quad (2.16)$$

where $\| \cdot \| = \langle x, x \rangle$.

Projected dynamical systems are different from classical dynamical systems in that the right-hand side, which is a projection operator, is discontinuous, due to the imposed constraints of each specific application. One can easily notice that if X lies in the interior of the feasible set \mathcal{K} , then the projection in the direction v is simply v . The class of ordinary differential equations that are of concern in this dissertation take on the following form:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0 \in \mathcal{K}, \quad (2.17)$$

where \dot{X} denotes the rate of change of vector X , \mathcal{K} is closed convex set, corresponding to the constraint set in a particular application, and $F(X)$ is a vector field defined on \mathcal{K} . I refer to the ordinary differential equation in (2.17) as ODE(F, \mathcal{K}).

The classical dynamical system, in contrast to (2.17), is of the form:

$$\dot{X} = -F(X), \quad X(0) = X_0 \in \mathcal{K}. \quad (2.18)$$

Definition 2.8 (The Projected Dynamical Systems)

Define the projected dynamical system (referred to as PDS(F, \mathcal{K})) $X_0(t) : \mathcal{K} \times R \mapsto \mathcal{K}$ as the family of solutions to the Initial Value Problem (IVP) (2.18) for all $X_0 \in \mathcal{K}$.

Definition 2.9 (An Equilibrium Point)

The vector $X^ \in \mathcal{K}$ is a stationary point or an equilibrium point of the projected dynamical system PDS(F, \mathcal{K}) if*

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)). \quad (2.19)$$

In other words, X^* is a stationary point or an equilibrium point if, once the projected dynamical system is at X^* , it will remain at X^* for all future times. Definition 2.9 establishes that X^* is an equilibrium point of the projected dynamical system $\text{PDS}(F, \mathcal{K})$ if the vector field F vanishes at X^* . Since in the case that X^* lies on the boundary of \mathcal{K} , one may have $F(X^*) \neq 0$. The contrary is only true when X^* is an interior point of the constraint set \mathcal{K} . Note that for classical dynamical systems, the necessary and sufficient condition for an equilibrium point is that the vector field vanish at that point, that is, $-F(X^*) = 0$.

Theorem 2.6 (see Dupuis and Nagurney (1993)), establishes the equivalence between the set of equilibria of a projected dynamical system and the set of solutions of a variational inequality problem.

Theorem 2.6

Assume that \mathcal{K} is a convex polyhedron. Then the equilibrium points of the $\text{PDS}(F, \mathcal{K})$ coincide with the solutions of $\text{VI}(F, \mathcal{K})$. Therefore, $X^ \in \mathcal{K}$ satisfies*

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)) \tag{2.20}$$

also satisfies

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{2.21}$$

Based on the following fundamental assumption which is implied by Lipschitz continuity (Definition 2.6), I now address the issues of existence and uniqueness of an equilibrium pattern through the theory of projected dynamical systems.

Assumption 2.1 (Linear Growth Condition)

There exists a $B < \infty$ such that the vector field $-F : R^n \mapsto R^n$ satisfies the linear growth condition $\|F(X)\| \leq B(1 + \|X\|)$, $X \in \mathcal{K}$, and also

$$\langle -F(X) + F(y), X - y \rangle \leq B \|X - y\|^2, \quad \forall X, y \in \mathcal{K}. \quad (2.22)$$

Theorem 2.7 (Existence, Uniqueness, and Continuous Dependence)

Assume Assumption 2.1. Then

(i) For any $X_0 \in \mathcal{K}$, there exists a unique solution $X_0(t)$ to the initial value problem;

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0; \quad (2.23)$$

(ii) If $X_n \rightarrow X_0$ as $n \rightarrow \infty$, then $X_n(t)$ converges to $X_0(t)$ uniformly on every compact set of $[0, \infty)$.

The second statement of the Theorem 2.7 is sometimes called the continuous dependence of the solution path to the ODE(F, \mathcal{K}) on the initial value. As a result, whenever Assumption 2.1 holds, the PDS(F, \mathcal{K}) is well-defined and inhabits \mathcal{K} , and is, therefore, a sufficient condition for the fundamental properties of projected dynamical systems as stated in Theorem 2.7.

I now turn to addressing the stability of the system (see Nagurney and Zhang (1996)).

Definition 2.10 (Stability of the System)

The system defined by equation (2.23) is stable if, for every X_0 and every equilibrium point X^* , the Euclidean distance $\|X^* - X_0(t)\|$ is a monotonically non increasing function of time t .

The equilibrium point X^* is unstable, if the system defined by equation (2.23) is not stable.

2.3. The Relationships between Variational Inequalities and Game Theory

I briefly recall some of the relationships between variational inequalities and game theory in this section. All the definitions and theorems can be found in Nagurney (1999), except where noted.

The seminal work by Nash (1950, 1951) formally developed the theory of the noncooperative game, which consists of multiple players, each of whom acts in his/her own interest. In particular, consider a game with m players, each player i having a strategy vector $X_i = \{X_{i1}, \dots, X_{in}\}$ selected from a closed, convex set $\mathcal{K}^i \subset R^n$. Each player i seeks to maximize his/her own utility function, $U_i: \mathcal{K} \mapsto R$, where $\mathcal{K} = \mathcal{K}^1 \times \mathcal{K}^2 \times \dots \times \mathcal{K}^m \subset R^{mn}$. The utility of player i , U_i , depends not only on his/her own strategy vector, X_i , but also on the strategy vectors of all the other players, $(X_1, \dots, X_{i-1}, X_{i+1}, \dots, X_m)$. An equilibrium is achieved if no one can increase his/her utility by unilaterally altering the value of its strategy vector. I first recall the formal definition of the Nash equilibrium.

Definition 2.11 (Nash Equilibrium)

A Nash equilibrium is a strategy vector

$$X^* = (X_1^*, \dots, X_m^*) \in \mathcal{K}, \quad (2.24)$$

such that

$$U_i(X_i^*, \hat{X}_i^*) \geq U_i(X_i, \hat{X}_i^*), \quad \forall X_i \in \mathcal{K}^i, \forall i, \quad (2.25)$$

where $\hat{X}_i^* = (X_1^*, \dots, X_{i-1}^*, X_{i+1}^*, \dots, X_m^*)$.

In other words, under Nash equilibrium, no unilateral deviation in strategy by any single player makes her better off.

It has been shown (cf. Hartman and Stampacchia (1966) and Gabay and Moulin (1980)) that the Nash equilibrium problem with continuously differentiable and concave utility functions can be formulated as a variational inequality problem defined on \mathcal{K} .

Theorem 2.8 (Variational Inequality Formulation of Nash Equilibrium)

Under the assumption that each utility function U_i is continuously differentiable and concave, X^* is a Nash equilibrium if and only if $X^* \in \mathcal{K}$ is a solution of the variational inequality

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad X \in \mathcal{K}, \quad (2.26)$$

where $F(X) \equiv (-\nabla_{X_1} U_1(X), \dots, -\nabla_{X_m} U_m(X))^T$, is a column vector and where $\nabla_{X_i} U_i(X) = (\frac{\partial U_i(X)}{\partial X_{i1}}, \dots, \frac{\partial U_i(X)}{\partial X_{in}})$.

I now introduce conditions for existence and uniqueness of a Nash equilibrium. Rosen (1965) demonstrated existence under the assumptions that \mathcal{K} is compact and each U_i is continuously differentiable.

Theorem 2.9 (Existence Under Compactness and Continuous Differentiability)

Suppose that the feasible set \mathcal{K} is compact and each U_i is continuously differentiable. Then existence of a Nash equilibrium is guaranteed.

Karamardian (1969a-b) proved existence and uniqueness of a Nash equilibrium under the strong monotonicity condition.

Theorem 2.10 (Existence and Uniqueness Under Strong Monotonicity)

Assume that $F(X)$, as given in Theorem 2.8, is strongly monotone on \mathcal{K} . Then there exists precisely one Nash equilibrium X^ .*

Additionally, based on Theorem 2.5, uniqueness of a Nash equilibrium can be guaranteed under the assumptions that $F(X)$ is strictly monotone and an equilibrium exists.

Theorem 2.11 (Uniqueness Under Strict Monotonicity)

Suppose that $F(X)$, as given in Theorem 2.8, is strictly monotone on \mathcal{K} . Then the Nash equilibrium, X^ , is unique, if it exists.*

2.4. Algorithm - The Euler Method

In this section, I consider the computation of a stationary point of (2.18). The algorithm that is proposed is the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). It has been applied to-date to solve a plethora of dynamic network models (see, e.g., Nagurney and Zhang (1996) and Nagurney and Dong (2002)). The algorithm not only provides a discretization of the continuous time trajectory defined by (2.18) but also yields a stationary, that is, an

equilibrium point that satisfies variational inequality (2.1). The Euler method will be used throughout this dissertation for the computation of equilibria in the case of dynamic models. In the case of this method, we have:

$$F_\tau(X) = F(X) \quad \forall \tau \in T, \quad \text{and} \quad X \in \mathcal{K}, \quad (2.27)$$

where, at iteration τ , the algorithm solves the strictly convex quadratic programming problem:

$$X^{\tau+1} = \min_{X \in \mathcal{K}} \frac{1}{2} \langle X, X \rangle - \langle X^\tau - a_\tau F(X^\tau), X \rangle, \quad (2.28)$$

where τ denotes an iteration counter.

The procedure of the Euler method at iteration τ takes the form:

$$X^{\tau+1} = P_{\mathcal{K}}(X^\tau - \alpha_\tau F(X^\tau)), \quad (2.29)$$

where F is the function in (2.1), and $P_{\mathcal{K}}$ denotes the operator of projection (see Nagurney (1999)) onto the closed convex set \mathcal{K} , defined by

$$P_{\mathcal{K}}(X) = \operatorname{argmin}_{X' \in \mathcal{K}} \|X' - X\|. \quad (2.30)$$

I now provide the complete statement of this algorithm.

Step 0: Initialization

Set $X^0 \in \mathcal{K}$.

Let $\tau = 1$ and set the sequence $\{\alpha_\tau\}$ so that $\sum_{\tau=1}^{\infty} \alpha_\tau = \infty$, $\alpha_\tau > 0$ for all τ , and $\alpha_\tau \rightarrow 0$ as $\tau \rightarrow \infty$.

Step 1: Computation

Compute $X^\tau \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\langle X^\tau + \alpha_\tau F(X^{\tau-1}) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \quad (2.31)$$

Step 2: Convergence Verification

If $|X^\tau - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau = \tau + 1$, and go to Step 1.

Convergence conditions for this method can be found in Dupuis and Nagurney (1993) and have been studied in a variety of network-based problems in Nagurney and Zhang (1996) and the references therein. This concludes Chapter 2 of this dissertation.

CHAPTER 3

A SERVICE-ORIENTED INTERNET NETWORK ECONOMIC MODEL WITH PRICE AND QUALITY COMPETITION IN BOTH CONTENT AND NETWORK PROVISION

In this chapter, I develop game theory models in equilibrium settings for multiple service providers in a next generation Internet architecture. There are two types of service providers, content and network service providers, who are competing with each other to set the price and quality level of their services. They share the same demand market, so that, not only they have price and quality competition with peer providers but also with other types of providers. The users in demand markets have different preferences and would like to pay differently. The contributions to the literature are:

- I include quality levels, in addition to prices, for both network and content providers, as they engage in competition for users at the demand markets.
- Consumers have more choices in that they can select network and content providers.
- I handle heterogeneity in the providers' cost functions and in the users' demands and do not limit myself to linear demand functions.
- I provide a natural underlying set of adjustment processes until the equilibrium, or equivalently, the stationary point, is achieved.

- The theoretical framework is supported by a rigorous algorithm that is well-suited for implementation.
- I perform sensitivity analysis in order to investigate the impact of the transfer prices on the providers' prices, quality levels, and their utilities, which reflect their profits.

This chapter is based on Saberi, Nagurney, and Wolf (2014) and is organized as follows. In Section 3.1, a basic model of a service-oriented Internet and its analysis are presented. A game theory model of service providers (CPs and NPs) is then constructed and analyzed in Section 3.2 to show the competitive behavior of content and network providers in prices and quality of services and their interactions with the users at the demand markets. This model extends the work of Nagurney et al. (2013a) and Nagurney and Wolf (2013) in that quality of both content and of network provision is captured. In addition, I allow for side payments and utilize direct demand functions (rather than their inverses). I demonstrate that the Nash equilibrium conditions are equivalent to the solution of variational inequality problems. The closed form expressions yielded by the Euler method (Dupuis and Nagurney (1993) and Nagurney and Zhang (1996)) is described in Section 3.3 for the price and quality of each provider. The algorithm is then applied to compute solutions to several examples in Section 3.4, accompanied by sensitivity analysis, in order to provide insights into the network economics. I summarize and present my conclusions in Section 3.5.

3.1. The Basic Model

In this section, a basic model is presented for illustration purposes. Figure 3.1 shows the structure of the content flows and Figure 3.2 depicts the structure of the financial payments in a basic (preliminary) model of a quality-based service-oriented

Internet, which consists of a single content provider, CP_1 , a single network provider, NP_1 , and one demand market (user) u_1 . For simplicity, a user refers to a market of users.

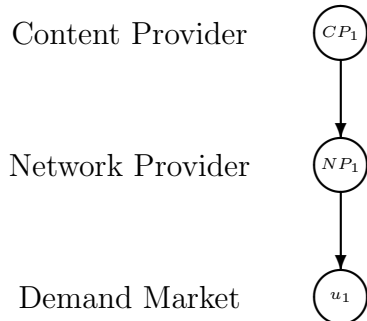


Figure 3.1. Network Topology for the Basic Model's Content Flow

The network provider and the content provider determine the equilibrium price and quality for their services offered to the user. According to Figure 3.2, the network provider charges the user a price p_{s_1} for transferring a unit of content while maintaining the quality at q_{s_1} . The user is also charged by the content provider a price p_{c_1} for each content of quality q_{c_1} that he receives through the network provider.

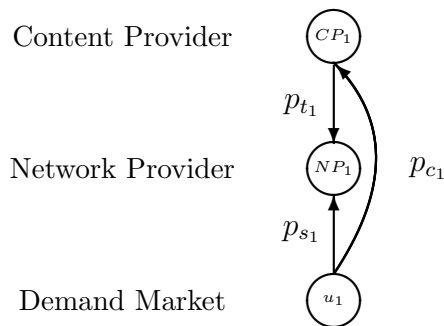


Figure 3.2. The Network Structure of the Basic Model's Financial Payment Flows

I consider a usage base price, rather than a flat rate price, for both network and content provision since I am modelling a service-oriented Internet in which all providers offer different services at various prices and quality. The user signals his preferences

via a demand function d_{111} (3.1), for the content produced by CP_1 and transferred by NP_1 , which depends on the price and the quality of both network and content provision, as follows:

$$d_{111} = d_0 - \alpha p_{s_1} - \beta p_{c_1} + \gamma q_{s_1} + \delta q_{c_1}. \quad (3.1)$$

The α , β , γ , and δ are all ≥ 0 . d_0 is the demand at zero usage based on the price and the best effort service delivery (i.e., $q_{s_1} = q_{c_1} = 0$). Based on this demand function, the user will request more service as the price goes down or the quality increases in network and content provision. The α and β reflect the sensitivity of the user to the network and content provider's prices, respectively. I consider different price sensitivity for content and network provider charges according to the assumption that there is an intrinsic value in the network besides the services offered by the content providers; otherwise, α and β would be equal. The γ and δ illustrate the effect of the quality of service of the network and the content providers on the user's demand. In this simple, illustrative service-oriented Internet model, the network provider also charges the content provider a transfer price p_{t_1} per unit of content transfers for the right to access end users. By charging a transfer price p_{t_1} I have a two-sided market. I also assume that the demand function is monotonically decreasing in price but increasing in quality.

The quality of the network, q_{s_1} , can be defined by various metrics such as latency, jitter, or bandwidth. Latency is a measure of the delay that the traffic experiences as it traverses a network and jitter is defined as the variation in that delay. Bandwidth is measured as the amount of data that can pass through a point in a network over time (see Smith and Garcia-Luna-Aceves (2008)). Here, I define the quality as the "expected delay", which is computed by the Kleinrock function (see Altman, Legout, and Xu (2011)) as the reciprocal of the square root of delay:

$$q_{s_1} = \frac{1}{\sqrt{Delay}} = \sqrt{b(d_{111}, q_{s_1}) - d_{111}}, \quad (3.2)$$

where $b(d_{111}, q_{s_1})$ is the total bandwidth of the network and is a function of demand and quality, that is:

$$b(d_{111}, q_{s_1}) = d_{111} + q_{s_1}^2. \quad (3.3)$$

Therefore, the greater the demand at higher quality, the larger the amount of bandwidth used. The network provider incurs a cost of transferring the demand while supporting q_{s_1} for data shipment, denoted by CS_1 . I assume a convex, continuous, and differentiable transfer function for NP_1 :

$$CS_1 = CS_1(d_{111}, q_{s_1}) = R(d_{111} + q_{s_1}^2), \quad (3.4)$$

where R is the unit cost of bandwidth. The quality of content provided can be specified for a specific domain of content, e.g., video streaming. In this case, quality is defined as the quality of videos produced by the content provider and CP_1 's production cost, CC_1 , is a convex and continuous function of quality of service:

$$CC_1 = CC_1(q_{c_1}) = Kq_{c_1}^2. \quad (3.5)$$

This model is different from the model of Altman, Legout, and Xu (2011) since I introduce quality and a cost function for content provision. Based on the network structure, the user demand would be equal to the content provider's supply and the network provider's shipments. I assume that there is competition between the noncooperatively competing CP_1 and NP_1 and I seek to determine the Nash equilibrium price and quality that maximize their respective utilities. The network provider's income in a two-sided market would be the summation of the revenue of transferring services from the content provider to the user and providing Internet access for users.

Let \mathcal{S}_{CP} denote the price and quality strategies of CP_1 where $\mathcal{S}_{CP} \equiv \{(p_{c_1}, q_{c_1}) \mid p_{c_1} \geq 0 \text{ and } q_{c_1} \geq 0\}$. The utility of the content provider, U_{CP_1} , which corresponds to his profits, is the difference between his revenue and his cost, and is given by:

$$U_{CP_1} = U_{CP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) = (p_{c_1} - p_{t_1})d_{111} - CC_1 = (p_{c_1} - p_{t_1})d_{111} - Kq_{c_1}^2. \quad (3.6)$$

Let \mathcal{S}_{NP} denote the price and quality strategies of NP_1 where $\mathcal{S}_{NP} \equiv \{(p_{s_1}, q_{s_1}) \mid p_{s_1} \geq 0 \text{ and } q_{s_1} \geq 0\}$. The utility of the network provider, U_{NP_1} , represents his profits and also is the difference between his revenue and his cost:

$$U_{NP_1} = U_{NP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) = (p_{s_1} + p_{t_1})d_{111} - CS_1 = (p_{s_1} + p_{t_1} - R)d_{111} - Rq_{s_1}^2. \quad (3.7)$$

Here, since the basic model builds on the model of Altman, Kegout, and Xu (2011), and to enable the subsequent analytics in Section 3.1.1, I assume that the demand function is linear as in (3.1). In Section 3.2, I relax this assumption in this general model.

3.1.1 The Analysis of Two-Sided Pricing in the Basic Model

In this game, the two noncooperative agents, CP_1 and NP_1 , seek to maximize their individual utilities with respect to their prices and quality. CP_1 maximizes his utility with respect to p_{c_1} and q_{c_1} :

$$\text{Maximize } U_{CP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) = (p_{c_1} - p_{t_1})d_{111} - Kq_{c_1}^2. \quad (3.8)$$

NP_1 also maximizes his utility but with respect to p_{s_1} , and q_{s_1} :

$$\text{Maximize } U_{NP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) = (p_{s_1} + p_{t_1} - R)d_{111} - Rq_{s_1}^2, \quad (3.9)$$

with all the prices and the quality levels being nonnegative.

Although the network provider needs to determine the transfer price, p_{t_1} , to charge the content provider, he cannot maximize his utility with respect to p_{t_1} simultaneously with p_{s_1} . Note that the utilities are linear functions of p_{t_1} (with the same derivatives with respect to p_{t_1} but different sign), so that if p_{t_1} is under the control of one of the providers, it would simply be set at an extreme value and, subsequently, lead to zero demand and zero income (see Kesidis (2012) and Altman, Legout, and Xu (2011)). As a result, I need to fix the p_{t_1} and maximize both U_{NP_1} and U_{CP_1} regarding the 4-tuple $(p_{s_1}, q_{s_1}, p_{c_1}, \text{ and } q_{c_1})$. However, a subsequent and important question would be how large the side payment should be and whether NP_1 can get any benefit by charging CP_1 . To overcome this issue, after optimizing the utility of CP_1 and NP_1 , I check whether NP_1 's profit is strictly increasing in p_{t_1} at $p_{t_1} = 0$ and under what conditions.

Definition 3.1: Nash Equilibrium in Prices and Quality

A price and quality level pattern $(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*) \in \mathcal{S}_{CP} \times \mathcal{S}_{NP}$ is said to constitute a Nash equilibrium if:

$$U_{CP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*) = \max_{(p_{c_1}, q_{c_1}) \in \mathcal{S}_{CP}} U_{CP_1}(p_{c_1}, q_{c_1}, p_{s_1}^*, q_{s_1}^*), \quad (3.10)$$

$$U_{NP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*) = \max_{(p_{s_1}, q_{s_1}) \in \mathcal{S}_{NP}} U_{NP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}, q_{s_1}). \quad (3.11)$$

Theorem 3.1: Variational Inequality Formulations of Nash Equilibrium in Prices and Quality

Assume that the content provider's profit function, $U_{CP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1})$, is concave with respect to the variables (p_{c_1}, q_{c_1}) and is continuous and continuously differentiable.

Assume, also, that for the network provider's profit function, $U_{NP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1})$, is concave with respect to the variables (p_{s_1}, q_{s_1}) and is continuous and continuously differentiable.

Then $(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*) \in \mathcal{S}_{CP} \times \mathcal{S}_{NP}$ is a Nash equilibrium according to Definition 3.1 if and only if it satisfies the variational inequality problem:

$$\begin{aligned} & -\frac{\partial U_{CP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*)}{\partial p_{c_1}} \times (p_{c_1} - p_{c_1}^*) - \frac{\partial U_{CP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*)}{\partial q_{c_1}} \times (q_{c_1} - q_{c_1}^*) \\ & -\frac{\partial U_{NP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*)}{\partial p_{s_1}} \times (p_{s_1} - p_{s_1}^*) - \frac{\partial U_{NP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*)}{\partial q_{s_1}} \times (q_{s_1} - q_{s_1}^*) \geq 0, \\ & \forall (p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) \in \mathcal{S}_{CP} \times \mathcal{S}_{NP}, \end{aligned} \quad (3.12)$$

or, equivalently, the variational inequality problem:

$$\begin{aligned} & (-d_{111} + \beta(p_{c_1}^* - p_{t_1})) \times (p_{c_1} - p_{c_1}^*) + (2Kq_{c_1}^* + \delta(p_{t_1} - p_{c_1}^*)) \times (q_{c_1} - q_{c_1}^*) \\ & + (-d_{111} + \alpha(p_{s_1}^* + p_{t_1} - R)) \times (p_{s_1} - p_{s_1}^*) + (2Rq_{s_1}^* + \gamma(R - p_{s_1}^* - p_{t_1})) \times (q_{s_1} - q_{s_1}^*) \geq 0, \\ & \forall (p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) \in \mathcal{S}_{CP} \times \mathcal{S}_{NP}, \end{aligned} \quad (3.13)$$

where d_{111} in (3.13) is evaluated at $(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*)$.

Proof: (3.12) follows directly from Gabay and Moulin (1980) and Dafermos and Nagurney (1987). In order to obtain (3.13) from (3.12), I note that:

$$-\frac{\partial U_{CP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*)}{\partial p_{c_1}} = -d_{111} + \beta(p_{c_1}^* - p_{t_1}), \quad (3.14)$$

$$-\frac{\partial U_{CP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*)}{\partial q_{c_1}} = 2Kq_{c_1}^* + \delta(p_{t_1} - p_{c_1}^*). \quad (3.15)$$

Similarly, I note that

$$-\frac{\partial U_{NP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{c_1}^*)}{\partial p_{s_1}} = -d_{111} + \alpha(p_{s_1}^* + p_{t_1} - R), \quad (3.16)$$

$$-\frac{\partial U_{NP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*)}{\partial q_{s_1}} = 2Rq_{s_1}^* + \gamma(R - p_{s_1}^* - p_{t_1}). \quad (3.17)$$

Making the substitutions for the marginal utilities in (3.12) given by (3.14) – (3.17) yields variational inequality (3.13). \square

Theorem 3.2: Uniqueness of the Nash Equilibrium Satisfying Variational Inequality (3.12)

The Nash equilibrium $(p_{c_1}^, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*) \in \mathcal{S}_{CP} \times \mathcal{S}_{NP}$ satisfying variational inequality (3.12) is unique, if the function F is strictly monotone over the feasible set $\mathcal{S}_{CP} \times \mathcal{S}_{NP}$, under the imposed assumptions (see Definition 2.4) with the function F consisting of minus the marginal utility functions of the providers w.r.t their price and quality variables.*

I now provide some insights as to under what conditions F for the simple model will be strictly monotone. I note that Jacobian of F , since $F = -\nabla U(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1})$, in view of the demand function, the revenue functions, and the cost functions, is given by:

$$\nabla F = \begin{pmatrix} -\frac{\partial^2 U_{CP_1}}{\partial p_{c_1}^2} & -\frac{\partial^2 U_{CP_1}}{\partial q_{c_1} \partial p_{c_1}} & -\frac{\partial^2 U_{CP_1}}{\partial p_{s_1} \partial p_{c_1}} & -\frac{\partial^2 U_{CP_1}}{\partial q_{s_1} \partial p_{c_1}} \\ -\frac{\partial^2 U_{CP_1}}{\partial p_{c_1} \partial q_{c_1}} & -\frac{\partial^2 U_{CP_1}}{\partial q_{c_1}^2} & -\frac{\partial^2 U_{CP_1}}{\partial p_{s_1} \partial q_{c_1}} & -\frac{\partial^2 U_{CP_1}}{\partial q_{s_1} \partial q_{c_1}} \\ -\frac{\partial^2 U_{NP_1}}{\partial p_{c_1} \partial p_{s_1}} & -\frac{\partial^2 U_{NP_1}}{\partial q_{c_1} \partial p_{s_1}} & -\frac{\partial^2 U_{NP_1}}{\partial p_{s_1}^2} & -\frac{\partial^2 U_{NP_1}}{\partial q_{s_1} \partial p_{s_1}} \\ -\frac{\partial^2 U_{NP_1}}{\partial p_{c_1} \partial q_{s_1}} & -\frac{\partial^2 U_{NP_1}}{\partial q_{c_1} \partial q_{s_1}} & -\frac{\partial^2 U_{NP_1}}{\partial p_{s_1} \partial q_{s_1}} & -\frac{\partial^2 U_{NP_1}}{\partial q_{s_1}^2} \end{pmatrix} = \begin{pmatrix} 2\beta & -\delta & \alpha & -\gamma \\ -\delta & 2K & 0 & 0 \\ \beta & -\delta & 2\alpha & -\gamma \\ 0 & 0 & -\gamma & 2R \end{pmatrix}. \quad (3.18)$$

I know that if ∇F is positive-definite, then F is strictly monotone for this model and the solution to variational inequality (3.12) is unique. Of course, if the Jacobian is strictly diagonally dominant then it will be positive-definite.

Theorem 3.3

The network provider, NP_1 , will benefit from charging the content provider, CP_1 , if $4\alpha R > \gamma^2$ and the user is more sensitive to the price that NP_1 charges him than the price that CP_1 charges him. In other words, if $4\alpha R - \gamma^2 > 0$, and $\alpha > \beta$, then NP_1 would set a positive p_{t_1} to increase his profit.

Proof: According to the Nash equilibrium, the best response of NP_1 and CP_1 can be found when the derivatives $\frac{\partial U_{NP_1}}{\partial p_{s_1}}$, $\frac{\partial U_{NP_1}}{\partial q_{s_1}}$, $\frac{\partial U_{CP_1}}{\partial p_{c_1}}$, and $\frac{\partial U_{CP_1}}{\partial q_{c_1}}$ are all zero, under the assumption that the associated variables are all positive. Then, I will have:

$$p_{s_1} = \frac{d_0 - \beta p_{c_1} + \gamma q_{s_1} + \delta q_{c_1} - \alpha(p_{t_1} - R)}{2\alpha}, \quad (3.19)$$

$$q_{s_1} = \frac{\gamma(p_{s_1} + p_{t_1} - R)}{2R}, \quad (3.20)$$

$$p_{c_1} = \frac{d_0 - \alpha p_{s_1} + \gamma q_{s_1} + \delta q_{c_1} + \beta p_{t_1}}{2\beta}, \quad (3.21)$$

$$q_{c_1} = \frac{\delta(p_{c_1} - p_{t_1})}{2K}. \quad (3.22)$$

By substituting (3.22) into (3.21) and then substituting the resultant equation and (3.20) into (3.19), at the Nash equilibrium, the following expressions are obtained:

$$p_{s_1}^* = \text{Max}\left\{0, \frac{2RK\beta[d_0 - R\alpha - (\beta - \alpha)p_{t_1}]}{\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)} + R - p_{t_1}\right\}, \quad (3.23)$$

$$q_{s_1}^* = \text{Max}\left\{0, \frac{K\gamma\beta[d_0 - R\alpha - (\beta - \alpha)p_{t_1}]}{\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)}\right\}, \quad (3.24)$$

$$p_{c_1}^* = \text{Max}\left\{0, \frac{2RK\alpha[d_0 - R\alpha - (\beta - \alpha)p_{t_1}]}{\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)} + p_{t_1}\right\}, \quad (3.25)$$

$$q_{c_1}^* = \text{Max}\left\{0, \frac{R\delta\alpha[d_0 - R\alpha - (\beta - \alpha)p_{t_1}]}{\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)}\right\}, \quad (3.26)$$

$$d_{111} = \text{Max}\left\{0, \frac{2RK\alpha\beta[d_0 - R\alpha - (\beta - \alpha)p_{t_1}]}{\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)}\right\}. \quad (3.27)$$

Hence, the utilities of the network and content providers are:

$$U_{NP_1} = \frac{RK^2\beta^2(4R\alpha - \gamma^2)[d_0 - R\alpha - (\beta - \alpha)p_{t_1}]^2}{[\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)]^2}, \quad (3.28)$$

$$U_{CP_1} = \frac{KR^2\alpha^2(4K\beta - \delta^2)[d_0 - R\alpha - (\beta - \alpha)p_{t_1}]^2}{[\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)]^2}. \quad (3.29)$$

I now have the utility functions based on p_{t_1} . To determine whether NP_1 should charge CP_1 or not, I obtain the derivative of U_{NP_1} w.r.t p_{t_1} and check if it is increasing when $p_{t_1} = 0$.

$$\frac{\partial U_{NP_1}}{\partial p_{t_1}} = (\alpha - \beta)[d_0 - R\alpha - (\beta - \alpha)p_{t_1}] \frac{2RK^2\beta^2(4\alpha R - \gamma^2)}{[\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)]^2}. \quad (3.30)$$

When $p_{t_1} = 0$, $\frac{\partial U_{NP_1}}{\partial p_{t_1}}$ would be:

$$(\alpha - \beta)[d_0 - R\alpha] \frac{2RK^2\beta^2(4\alpha R - \gamma^2)}{[\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)]^2}. \quad (3.31)$$

With the assumption of a large d_0 , $\frac{\partial U_{NP_1}}{\partial p_{t_1}}$ is positive if $4\alpha R - \gamma^2 > 0$ and $\alpha > \beta$. \square

3.2. The Network Economic Game Theory Model of Price and Quality Competition in a Service-Oriented Internet

In this section, I develop a network economic game theory model for a multi-provider service-oriented network with heterogeneous markets of users. The network structure of the problem, which depicts the direction of the content flows, is given in Figure 3.3. See Figure 3.4 for a graphic depiction of the financial payments

in this general model. I assume m content providers, a typical one denoted by CP_i ; $\{i = 1, \dots, m\}$, n network providers, denoted by NP_j ; $\{j = 1, \dots, n\}$, and o markets of users, denoted by u_k ; $\{k = 1, \dots, o\}$. These providers compete under the Nash concept of noncooperative behavior to set their prices and quality levels so as to maximize their utilities, which are in the form of profits.

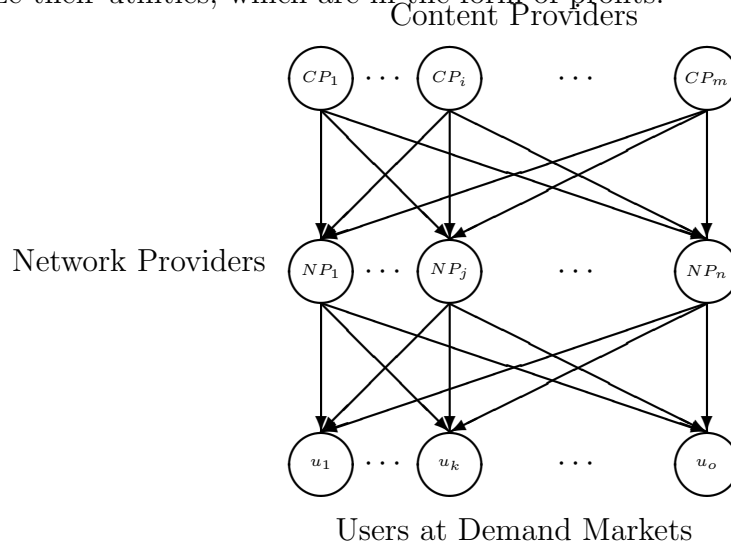


Figure 3.3. The Network Structure of the Multi-Provider Model's Content Flows

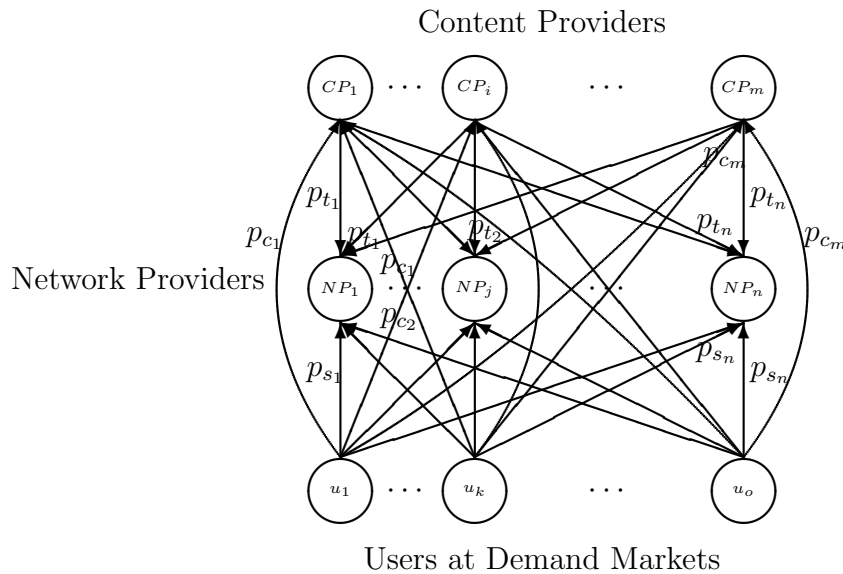


Figure 3.4. Graphic of the Multi-Provider Model with a Focus on Payments

To receive a unit of content service from CP_i with quality q_{c_i} , which is transmitted by NP_j with quality q_{s_j} , a user pays p_{c_i} and p_{s_j} to the CP_i and NP_j , respectively. The content providers also pay the network providers for transferring their content to the users. Each network provider NP_j has a fixed transmission fee p_{t_j} that he charges the CPs per unit of content. I group the p_{t_j} , p_{s_j} , q_{s_j} , p_{c_i} , and q_{c_i} for $i = 1, \dots, m$; $j = 1, \dots, n$, into vectors p_t , p_s , q_s , p_c , and q_c , respectively.

The users are heterogeneous in their demands and signal their preferences through a demand function d_{ijk} for the content produced by content provider i and transmitted by NP_j to demand market k :

$$d_{ijk} = d_{ijk}(p_c, q_c, p_s, q_s), \quad \forall i, j, k. \quad (3.32)$$

In this game theory model, the demand d_{ijk} does not only depend on the price and quality of CP_i and NP_j , but also on the prices and quality levels of the other content and network providers as a result of competition among the providers. Moreover, unlike the specialized, illustrative model in Section 3.1, the demand functions above need not be linear, as in (3.1), and in the work of Altman, Legout, and Xu (2011) and El Azouzi, Altman, and Wynter (2003).

Herein, if p_{s_j} and p_{c_i} (q_{s_j} , and q_{c_i}) decrease (increase), d_{ijk} naturally goes up, but it decreases if the price (quality) of the other providers decreases (increases). I now describe the behavior of the content providers.

Each content provider CP_i produces distinct (but substitutable) content of specific quality q_{c_i} , and sells at a unit price of p_{c_i} . The total supply of CP_i , SCP_i , is given by:

$$SCP_i = \sum_{j=1}^n \sum_{k=1}^o d_{ijk}, \quad i = 1, \dots, m. \quad (3.33)$$

Each CP_i has a production cost, CC_i , which is a function of his supply and his quality of service:

$$CC_i = CC_i(SCP_i, q_{c_i}), \quad i = 1, \dots, m. \quad (3.34)$$

I assume that the production cost functions are convex, continuous, and continuously differentiable functions.

Also, I assume that the content providers are profit-maximizers, where the profit or utility of CP_i ; $i = 1, \dots, m$, which is the difference between his total revenue and his total cost, is given by the expression:

$$U_{CP_i} = U_{CP_i}(p_c, q_c, p_s, q_s) = \sum_{j=1}^n (p_{c_i} - p_{t_j}) \sum_{k=1}^o d_{ijk} - CC_i. \quad (3.35)$$

Let \mathcal{K}_i^1 denote the feasible set corresponding to CP_i , where $\mathcal{K}_i^1 \equiv \{(p_{c_i}, q_{c_i}) \mid p_{c_i} \geq 0 \text{ and } q_{c_i} \geq 0\}$.

I now describe the behavior of the network providers.

A network provider NP_j ; $j = 1, \dots, n$, is distinguishable by means of his quality q_{s_j} , the fee p_{t_j} that he charges each content provider to transfer one unit of content to the users, and the fee p_{s_j} that he charges users to transfer them one unit of content. By charging p_{t_j} , I have a two-sided market. Here, as in Section 3.1, the p_{t_j} s are assumed to be an exogenous parameter in this multi-provider model. I assume that all content providers are connected to all network providers and, subsequently, to all users. The total amount of content of services transported by NP_j , TNP_j , is given by:

$$TNP_j = \sum_{i=1}^m \sum_{k=1}^o d_{ijk}, \quad j = 1, \dots, n. \quad (3.36)$$

NP_j incurs the cost, CS_j , of maintaining his network based on the offered quality and the total traffic passing through his bandwidth:

$$CS_j = CS_j(TNP_j, q_{s_j}), \quad j = 1, \dots, n. \quad (3.37)$$

Similar cost functions were used in Altman, Legout, and Xu (2011), where it was noted that the (transport) network provider has to cover the costs of operating the backbone, the last mile, upgrades, etc. I also assume that these cost functions are convex, continuous, and continuously differentiable functions. The utility of NP_j ; $j = 1, \dots, n$ is defined as the difference between his income and his cost, that is:

$$U_{NP_j} = U_{NP_j}(p_c, q_c, p_s, q_s) = (p_{s_j} + p_{t_j})TNP_j - CS_j. \quad (3.38)$$

Let \mathcal{K}_j^2 denote the feasible set corresponding to NP_j , where $\mathcal{K}_j^2 \equiv \{(p_{s_j}, q_{s_j}) \mid p_{s_j} \geq 0 \text{ and } q_{s_j} \geq 0\}$.

I now consider the Nash equilibrium that captures the providers' behavior.

Definition 3.2: Nash Equilibrium in Price and Quality

A price and quality level pattern $(p_c^*, q_c^*, p_s^*, q_s^*) \in \mathcal{K}^3 \equiv \prod_{i=1}^m \mathcal{K}_i^1 \times \prod_{j=1}^n \mathcal{K}_j^2$, is said to constitute a Nash equilibrium if for each content provider CP_i ; $i = 1, \dots, m$:

$$U_{CP_i}(p_{c_i}^*, \hat{p}_{c_i}^*, q_{c_i}^*, \hat{q}_{c_i}^*, p_s^*, q_s^*) \geq U_{CP_i}(p_{c_i}, \hat{p}_{c_i}^*, q_{c_i}, \hat{q}_{c_i}^*, p_s^*, q_s^*), \quad \forall (p_{c_i}, q_{c_i}) \in \mathcal{K}_i^1, \quad (3.39)$$

where

$$\hat{p}_{c_i}^* \equiv (p_{c_1}^*, \dots, p_{c_{i-1}}^*, p_{c_{i+1}}^*, \dots, p_{c_m}^*) \text{ and } \hat{q}_{c_i}^* \equiv (q_{c_1}^*, \dots, q_{c_{i-1}}^*, q_{c_{i+1}}^*, \dots, q_{c_m}^*), \quad (3.40)$$

and if for each network provider NP_j ; $j = 1, \dots, n$:

$$U_{NP_j}(p_c^*, q_c^*, p_{s_j}^*, \hat{p}_{s_j}^*, q_{s_j}^*, \hat{q}_{s_j}^*) \geq U_{NP_j}(p_{s_j}, p_c^*, q_c^*, \hat{p}_{s_j}^*, q_{s_j}, \hat{q}_{s_j}^*), \quad \forall (p_{s_j}, q_{s_j}) \in \mathcal{K}_j^2, \quad (3.41)$$

where

$$\hat{p}_{s_j}^* \equiv (p_{s_1}^*, \dots, p_{s_{j-1}}^*, p_{s_{j+1}}^*, \dots, p_{s_n}^*) \text{ and } \hat{q}_{s_j}^* \equiv (q_{s_1}^*, \dots, q_{s_{j-1}}^*, q_{s_{j+1}}^*, \dots, q_{s_n}^*). \quad (3.42)$$

According to (3.39) and (3.41), a Nash equilibrium is established if no provider can unilaterally improve upon his profits by selecting an alternative vector of quality levels and prices.

Theorem 3.4: Variational Inequality Formulations of Nash Equilibrium for the Service-Oriented Internet

Assume that the provider utility functions are concave, continuous, and continuously differentiable. Then $(p_c^*, q_c^*, p_s^*, q_s^*) \in \mathcal{K}^3$ is a Nash equilibrium according to Definition 3.2 if and only if it satisfies the variational inequality:

$$\begin{aligned} & - \sum_{i=1}^m \frac{\partial U_{CP_i}(p_c^*, q_c^*, p_s^*, q_s^*)}{\partial p_{c_i}} \times (p_{c_i} - p_{c_i}^*) - \sum_{i=1}^m \frac{\partial U_{CP_i}(p_c^*, q_c^*, p_s^*, q_s^*)}{\partial q_{c_i}} \times (q_{c_i} - q_{c_i}^*) \\ & - \sum_{j=1}^n \frac{\partial U_{NP_j}(p_c^*, q_c^*, p_s^*, q_s^*)}{\partial p_{s_j}} \times (p_{s_j} - p_{s_j}^*) - \sum_{j=1}^n \frac{\partial U_{NP_j}(p_c^*, q_c^*, p_s^*, q_s^*)}{\partial q_{s_j}} \times (q_{s_j} - q_{s_j}^*) \geq 0, \\ & \forall (p_c, q_c, p_s, q_s) \in \mathcal{K}^3, \end{aligned} \quad (3.43)$$

or, equivalently,

$$\begin{aligned} & \sum_{i=1}^m \left[- \sum_{j=1}^n \sum_{k=1}^o d_{ijk} - \sum_{j=1}^n \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial p_{c_i}} \times (p_{c_i}^* - p_{t_j}) + \frac{\partial CC_i(SCP_i, q_{c_i}^*)}{\partial SCP_i} \cdot \frac{\partial SCP_i}{\partial p_{c_i}} \right] \times (p_{c_i} - p_{c_i}^*) \\ & + \sum_{i=1}^m \left[- \sum_{j=1}^n \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial q_{c_i}} \times (p_{c_i}^* - p_{t_j}) + \frac{\partial CC_i(SCP_i, q_{c_i}^*)}{\partial q_{c_i}} \right] \times (q_{c_i} - q_{c_i}^*) \end{aligned}$$

$$\begin{aligned}
& + \sum_{j=1}^n \left[- \sum_{i=1}^m \sum_{k=1}^o d_{ijk} - \sum_{i=1}^m \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial p_{s_j}} \times (p_{s_j}^* + p_{t_j}) + \frac{\partial CS_j(TNP_j, q_{s_j}^*)}{\partial TNP_j} \cdot \frac{\partial TNP_j}{\partial p_{s_j}} \right] \times (p_{s_j} - p_{s_j}^*) \\
& + \sum_{j=1}^n \left[- \sum_{i=1}^m \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial q_{s_j}} \times (p_{s_j}^* + p_{t_j}) + \frac{\partial CS_j(TNP_j, q_{s_j}^*)}{\partial q_{s_j}} \right] \times (q_{s_j} - q_{s_j}^*) \geq 0, \\
& \forall (p_c, q_c, p_s, q_s) \in \mathcal{K}^3. \tag{3.44}
\end{aligned}$$

Variational inequality (3.44) can be put into standard form (cf. (2.1)): determine $X^* \in \mathcal{K}^3$ such that:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{3.45}$$

where $F(X)$ is a continuous function such that $F(X) : X \mapsto \mathcal{K} \subset R^N$, and \mathcal{K} is a closed and convex set. The term $\langle \cdot, \cdot \rangle$ denotes the inner product in N -dimensional Euclidean space. I define $X \equiv (p_c, q_c, p_s, q_s)$, and $F(X) \equiv (F_{p_c}, F_{q_c}, F_{p_s}, F_{q_s})$. The specific components of F are given by: for $i = 1, \dots, m; j = 1, \dots, n$:

$$F_{p_{c_i}} = \frac{\partial CC_i(SCP_i, q_{c_i})}{\partial SCP_i} \cdot \frac{\partial SCP_i}{\partial p_{c_i}} - \sum_{j=1}^n \sum_{k=1}^o d_{ijk} - \sum_{j=1}^n \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial p_{c_i}} \times (p_{c_i} - p_{t_j}), \tag{3.46}$$

$$F_{q_{c_i}} = \frac{\partial CC_i(SCP_i, q_{c_i})}{\partial q_{c_i}} - \sum_{j=1}^n \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial q_{c_i}} \times (p_{c_i} - p_{t_j}), \tag{3.47}$$

$$F_{p_{s_j}} = \frac{\partial CS_j(TNP_j, q_{s_j})}{\partial TNP_j} \cdot \frac{\partial TNP_j}{\partial p_{s_j}} - \sum_{i=1}^m \sum_{k=1}^o d_{ijk} - \sum_{i=1}^m \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial p_{s_j}} \times (p_{s_j} + p_{t_j}), \tag{3.48}$$

$$F_{q_{s_j}} = \frac{\partial CS_j(TNP_j, q_{s_j})}{\partial q_{s_j}} - \sum_{i=1}^m \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial q_{s_j}} \times (p_{s_j} + p_{t_j}), \tag{3.49}$$

where $\mathcal{K} = \mathcal{K}^3$ and $N = 2m + 2n$.

3.3. Explicit Formulae for the Euler Method Applied to Variational Inequality (3.45) with $F(X)$ Defined by (3.46) – (3.49)

For computational purposes, the Euler method (cf. Section 2.4), which is a discrete-time algorithm serving as an approximation to the continuous-time trajectories, is utilized.

The elegance of this procedure for the computation of solutions to this network economic model of the service-oriented Internet can be seen in the following explicit formulae. Indeed, variational inequality (3.44) yields the following closed form expressions, at each iteration, for the price and quality levels of each content and network provider $i = 1, \dots, m$; $j = 1, \dots, n$:

$$p_{c_i}^{\tau+1} = \max \left\{ 0, p_{c_i}^{\tau} + a_{\tau} \left(\sum_{j=1}^n \sum_{k=1}^o d_{ijk} + \sum_{j=1}^n \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial p_{c_i}} \times (p_{c_i}^{\tau} - p_{t_j}) - \frac{\partial CC_i(SCP_i, q_{c_i}^{\tau})}{\partial SCP_i} \cdot \frac{\partial SCP_i}{\partial p_{c_i}} \right) \right\}, \quad (3.50)$$

$$q_{c_i}^{\tau+1} = \max \left\{ 0, q_{c_i}^{\tau} + a_{\tau} \left(\sum_{j=1}^n \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial q_{c_i}} \times (p_{c_i}^{\tau} - p_{t_j}) - \frac{\partial CC_i(SCP_i, q_{c_i}^{\tau})}{\partial q_{c_i}} \right) \right\}, \quad (3.51)$$

$$p_{s_j}^{\tau+1} = \max \left\{ 0, p_{s_j}^{\tau} + a_{\tau} \left(\sum_{i=1}^m \sum_{k=1}^o d_{ijk} + \sum_{i=1}^m \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial p_{s_j}} \times (p_{s_j}^{\tau} + p_{t_j}) - \frac{\partial CS_j(TNP_j, q_{s_j}^{\tau})}{\partial TNP_j} \cdot \frac{\partial TNP_j}{\partial p_{s_j}} \right) \right\}, \quad (3.52)$$

$$q_{s_j}^{\tau+1} = \max \left\{ 0, q_{s_j}^\tau + a_\tau \left(\sum_{i=1}^m \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial q_{s_j}} \times (p_{s_j}^\tau + p_{t_j}^\tau) - \frac{\partial CS_j(TNP_j, q_{s_j}^\tau)}{\partial q_{s_j}} \right) \right\}. \quad (3.53)$$

Notice that all the functions to the left of the equal signs in (3.50)-(3.53) are evaluated at their respective variables computed at the τ -th iteration.

I now provide the convergence result. The proof is direct from Theorem 5.8 in Nagurney and Zhang (1996).

Theorem 3.5: Convergence

In this service-oriented Internet model, assume that $F(X) = -\nabla U(p_c, q_c, p_s, q_s)$ is strongly monotone. Also, assume that F is uniformly Lipschitz continuous. Then, there exists a unique equilibrium price and quality pattern $(p_c^, q_c^*, p_s^*, q_s^*) \in \mathcal{K}^3$ and any sequence generated by the Euler method as given by (3.50)-(3.53), where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \rightarrow 0$, as $\tau \rightarrow \infty$ converges to $(p_c^*, q_c^*, p_s^*, q_s^*)$.*

3.4. Numerical Examples and Sensitivity Analysis

I implemented the Euler method (cf. (3.50) - (3.53)) to compute solutions to service-oriented Internet network problems using Matlab programming. For the computations, I utilized a DELL XPS Series laptop with an Intel Core Duo processor with 3 GB RAM. The Euler method was considered to have converged if, at a given iteration, the absolute value of the difference of each price and each quality level differed from its respective value at the preceding iteration by no more than $\epsilon = 10^{-6}$. The sequence $\{a_\tau\}$ was: $.1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \dots)$. I initialized the algorithm by setting $p_{c_i}^0 = q_{c_i}^0 = p_{s_j}^0 = q_{s_j}^0 = 0.00, \forall i, j$.

Example 3.1

In this example, I have two content providers, CP_1 and CP_2 , one network provider, NP_1 , and one market of users, u_1 (see Figure 3.5).

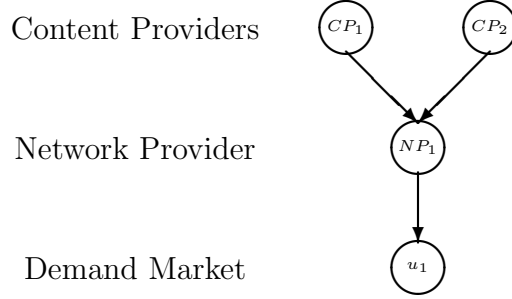


Figure 3.5. Network Topology of Content Flows for Example 3.1

The demand functions are as below:

$$d_{111} = 100 - 2.8p_{s_1} - 2.1p_{c_1} + 1.3p_{c_2} + 1.62q_{s_1} + 1.63q_{c_1} - .42q_{c_2},$$

$$d_{211} = 112 - 2.8p_{s_1} + 1.3p_{c_1} - 2.7p_{c_2} + 1.62q_{s_1} - .42q_{c_1} + 1.58q_{c_2}.$$

The cost functions of the content providers, CP_1 and CP_2 , are:

$$CC_1 = 1.7q_{c_1}^2, \quad CC_2 = 2.4q_{c_2}^2.$$

The cost function of the network provider, NP_1 , is:

$$CS_1 = 2.2(d_{111} + d_{211} + q_{s_1}^2).$$

The utilities of the content providers are:

$$U_{CP_1} = (p_{c_1} - p_{t_1})d_{111} - CC_1, \quad U_{CP_2} = (p_{c_2} - p_{t_1})d_{211} - CC_2.$$

The utility of the network provider is:

$$U_{NP_1} = (p_{s_1} + p_{t_1})(d_{111} + d_{211}) - CS_1.$$

Here, $p_{t_1} = 33$.

The Jacobian of $F(X) = -\nabla U(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1})$, denoted by $J(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1})$, is

$$J(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1}) = \begin{pmatrix} 4.2 & -1.63 & -1.3 & .42 & 2.8 & -1.62 \\ -1.63 & 3.4 & 0 & 0 & 0 & 0 \\ -1.3 & .42 & 4.5 & -1.58 & 2.8 & -1.62 \\ 0 & 0 & -1.58 & 4.8 & 0 & 0 \\ .8 & -1.21 & 1.4 & -1.16 & 11.2 & -3.24 \\ 0 & 0 & 0 & 0 & -3.24 & 4.4 \end{pmatrix}.$$

Since the symmetric part of $J(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1})$, $(J + J^T)/2$, has only positive eigenvalues, which are: 1.54, 2.80, 3.11, 4.65, 6.89, and 13.51, the $F(X)$ in Example 3.1 is strongly monotone since $\nabla F(X)$, as above, is positive-definite. Thus, according to Theorem 3.5, there exists a unique equilibrium, which, according to Theorem 3.7 in Nagurney and Zhang (1996) is also globally exponentially stable for the utility gradient process.

The Euler method required 1922 iterations and 12.79 CPU seconds for convergence. The computed equilibrium solution is:

$$p_{c_1}^* = 75.68, \quad p_{c_2}^* = 63.62, \quad p_{s_1}^* = 0,$$

$$q_{c_1}^* = 20.46, \quad q_{c_2}^* = 10.08, \quad q_{s_1}^* = 22.68,$$

with incurred demands of:

$$d_{111} = 89.64, \quad d_{211} = 82.68.$$

The utility of NP_1 is 4175.73, that of CP_1 is 3114.25, and that of CP_2 is 2288.16. It is interesting that the network provider NP_1 is better off by not charging the user, that is, $p_{s_1}^* = 0$, and only charges the CPs for transferring the content to the user. Meanwhile, the users' demand for services offered by CP_1 is higher ($d_{111} > d_{211}$) in comparison with that of CP_2 , since CP_1 provides content services at a higher quality ($q_{c_1}^* > q_{c_2}^*$).

Example 3.2

The network topology of Example 3.2 is given in Figure 3.6. I have one content provider, CP_1 , two network providers, NP_1 and NP_2 , and one market of users, u_1 .

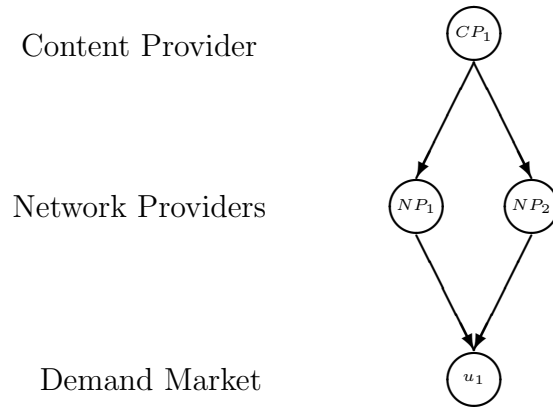


Figure 3.6. Network Topology of Content Flows for Example 3.2

The demand functions are:

$$d_{111} = 100 - 1.8p_{s_1} + .5p_{s_2} - 1.83p_{c_1} + 1.59q_{s_1} - .6q_{s_2} + 1.24q_{c_1},$$

$$d_{121} = 100 + .5p_{s_1} - 1.5p_{s_2} - 1.83p_{c_1} - .6q_{s_1} + 1.84q_{s_2} + 1.24q_{c_1}.$$

The network providers' cost functions are:

$$CS_1 = 1.7(d_{111} + q_{s_1}^2), \quad CS_2 = 1.8(d_{121} + q_{s_2}^2).$$

The cost function of CP_1 is:

$$CC_1 = 1.84(d_{111} + d_{121} + q_{c_1}^2).$$

The utility function of CP_1 is:

$$U_{CP_1} = (p_{c_1} - p_{t_1})d_{111} + (p_{c_1} - p_{t_2})d_{121} - CC_1.$$

The utility functions of the network providers are:

$$U_{NP_1} = (p_{s_1} + p_{t_1})d_{111} - CS_1, \quad U_{NP_2} = (p_{s_2} + p_{t_2})d_{121} - CS_2.$$

I set $p_{t_1} = p_{t_2} = 0$.

The Jacobian of $-\nabla U(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, denoted by $J(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, is

$$J(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2}) = \begin{pmatrix} 7.32 & -2.48 & 1.3 & -.99 & 1 & -1.24 \\ -2.48 & 3.68 & 0 & 0 & 0 & 0 \\ 1.83 & -1.24 & 3.6 & -1.59 & -1 & .6 \\ 0 & 0 & -1.59 & 3.4 & 0 & 0 \\ 1.83 & -1.24 & -.5 & .6 & 3 & -1.84 \\ 0 & 0 & 0 & 0 & -1.84 & 3.6 \end{pmatrix}.$$

Since the symmetric part of $J(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, $(J + J^T)/2$, has only positive eigenvalues, which are: 9.44, 5.78, 3.5, 2.57, 1.4, and 1.87, I know that the $F(X)$

in Example 3.2 is strongly monotone. Hence, I can conclude that the equilibrium solution is unique.

The equilibrium solution was achieved after 2931 iterations of the Euler method and 18.58 seconds of CPU time:

$$p_{c_1}^* = 29.19, \quad p_{s_1}^* = 27.66, \quad p_{s_2}^* = 37.38,$$

$$q_{c_1}^* = 18.43, \quad q_{s_1}^* = 12.14, \quad q_{s_2}^* = 18.18,$$

with incurred demands of:

$$d_{111} = 46.72, \quad d_{121} = 53.37.$$

The utilities of NP_1 and NP_2 are 962.58, and 1303.77, respectively, and the utility of CP_1 is 2112.75. Note that NP_2 offers his services at a higher quality, but at a higher price than NP_1 .

Example 3.3

The network topology of this example is depicted in Figure 3.7. I have two content providers, two network providers, and three markets of users.

The demand functions are:

$$d_{111} = 112 - 2.1p_{s_1} + .6p_{s_2} - 1.85p_{c_1} + .5p_{c_2} + .64q_{s_1} - .04q_{s_2} + .76q_{c_1} - .4q_{c_2},$$

$$d_{112} = 100 - 2.2p_{s_1} + .6p_{s_2} - 2.3p_{c_1} + .5p_{c_2} + .7q_{s_1} - .4q_{s_2} + .61q_{c_1} - .4q_{c_2},$$

$$d_{113} = 95 - .2p_{s_1} + .6p_{s_2} - 2.2p_{c_1} + .5p_{c_2} + .1q_{s_1} - .4q_{s_2} + .66q_{c_1} - .4q_{c_2},$$

$$d_{121} = 112 + .6p_{s_1} - .2p_{s_2} - 1.85p_{c_1} + .5p_{c_2} - .4q_{s_1} + .1q_{s_2} + .76q_{c_1} - .4q_{c_2},$$

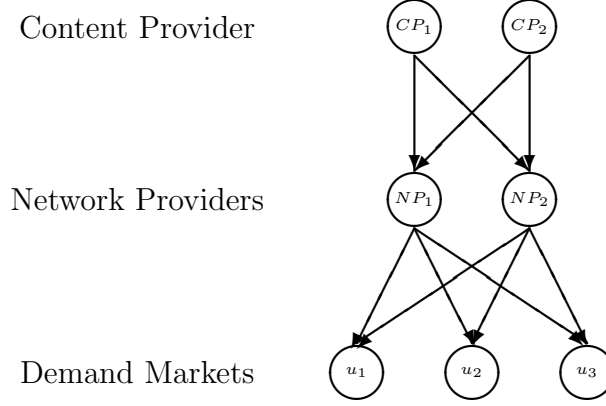


Figure 3.7. Network Topology of Content Flows for Example 3.3

$$\begin{aligned}
d_{122} &= 100 + .6p_{s_1} - 2p_{s_2} - 2.3p_{c_1} + .5p_{c_2} - .4q_{s_1} + .9q_{s_2} + .61q_{c_1} - .4q_{c_2}, \\
d_{123} &= 95 + .06p_{s_1} - 2.3p_{s_2} - 2.2p_{c_1} + .5p_{c_2} - .04q_{s_1} + .68q_{s_2} + .66q_{c_1} - .4q_{c_2}, \\
d_{211} &= 99 - 2.1p_{s_1} + .06p_{s_2} + .5p_{c_1} - 1.85p_{c_2} + .64q_{s_1} - .04q_{s_2} - .4q_{c_1} + .76q_{c_2}, \\
d_{212} &= 110 - 2.2p_{s_1} + .6p_{s_2} + .5p_{c_1} - 2.3p_{c_2} + .7q_{s_1} - .4q_{s_2} - .4q_{c_1} + .61q_{c_2}, \\
d_{213} &= 115 - .2p_{s_1} + .6p_{s_2} + .5p_{c_1} - 2.2p_{c_2} + .1q_{s_1} - .4q_{s_2} - .4q_{c_1} + .66q_{c_2}, \\
d_{221} &= 99 + .6p_{s_1} - .2p_{s_2} + .5p_{c_1} - 1.85p_{c_2} - .4q_{s_1} + .1q_{s_2} - .4q_{c_1} + .76q_{c_2}, \\
d_{222} &= 110 + .6p_{s_1} - 2p_{s_2} + .5p_{c_1} - 2.3p_{c_2} - .4q_{s_1} + .9q_{s_2} - .4q_{c_1} + .61q_{c_2}, \\
d_{223} &= 115 + .06p_{s_1} - 2.3p_{s_2} + .5p_{c_1} - 2.2p_{c_2} - .04q_{s_1} + .68q_{s_2} - .4q_{c_1} + .66q_{c_2}.
\end{aligned}$$

The network providers' cost functions are:

$$CS_1 = 1.2(d_{111} + d_{112} + d_{113} + d_{211} + d_{212} + d_{213} + q_{s_1}^2),$$

$$CS_2 = 3.2(d_{121} + d_{122} + d_{123} + d_{221} + d_{222} + d_{223} + q_{s_2}^2).$$

The cost functions of the content providers are:

$$CC_1 = 2.7q_{c_1}^2, \quad CC_2 = 3.1q_{c_2}^2.$$

The utility functions of the content providers are:

$$U_{CP_1} = (p_{c_1} - p_{t_1})(d_{111} + d_{112} + d_{113}) + (p_{c_1} - p_{t_2})(d_{121} + d_{122} + d_{123}) - CC_1,$$

$$U_{CP_2} = (p_{c_2} - p_{t_1})(d_{211} + d_{212} + d_{213}) + (p_{c_2} - p_{t_2})(d_{221} + d_{222} + d_{223}) - CC_2.$$

The utility functions of the network providers are:

$$U_{NP_1} = (p_{s_1} + p_{t_1})(d_{111} + d_{112} + d_{113} + d_{211} + d_{212} + d_{213}) - CS_1,$$

$$U_{NP_2} = (p_{s_2} + p_{t_2})(d_{121} + d_{122} + d_{123} + d_{221} + d_{222} + d_{223}) - CS_2.$$

I set $p_{t_1} = 23$ and $p_{t_2} = 21$.

The Jacobian of $-\nabla U(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, denoted by $J(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, is

$$J = \begin{pmatrix} 25.4 & -4.06 & -3 & 2.4 & 3.24 & -.6 & 3.24 & -.84 \\ -4.06 & 5.4 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & 2.4 & 25.4 & -4.06 & 3.24 & -.6 & 3.24 & -.84 \\ 0 & 0 & -4.06 & 6.2 & 0 & 0 & 0 & 0 \\ 4.85 & -.83 & 4.85 & -.83 & 18 & -2.88 & -2.52 & 1.68 \\ 0 & 0 & 0 & 0 & -2.88 & 2.4 & 0 & 0 \\ 4.85 & -.83 & 4.85 & -.83 & -2.52 & 1.68 & 18 & -3.36 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3.36 & 6.4 \end{pmatrix}.$$

The symmetric part of $J(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, $(J + J^T)/2$, has only positive eigenvalues, which are: 1.85, 4.46, 5.42, 5.48, 10.71, 21.47, 28.25, and 29.56. Hence, the $F(X)$ in Example 3.3 is also strongly monotone and I know that the equilibrium solution is unique.

The equilibrium solution below is achieved after 1758 iterations and 19.95 CPU seconds:

$$\begin{aligned}
 p_{c_1}^* &= 40.57, & p_{c_2}^* &= 41.49, & p_{s_1}^* &= 8.76, & p_{s_2}^* &= 5.35, \\
 q_{c_1}^* &= 13.96, & q_{c_2}^* &= 12.76, & q_{s_1}^* &= 36.67, & q_{s_2}^* &= 12.15,
 \end{aligned}$$

with incurred demands of:

$$\begin{aligned}
 d_{111} &= 68.11, & d_{112} &= 35.60, & d_{113} &= 30.87, \\
 d_{211} &= 51.55, & d_{212} &= 41.80, & d_{213} &= 47.10, \\
 d_{121} &= 53.93, & d_{122} &= 21.68, & d_{123} &= 25.62, \\
 d_{221} &= 37.37, & d_{222} &= 27.89, & d_{223} &= 41.86.
 \end{aligned}$$

In this example, NP_1 has a lower cost of bandwidth in comparison with that of NP_2 . This can be related to the technology. NP_1 may be using advanced technology and, therefore, incurs a lower cost. Hence, NP_1 can set up his services at a higher quality ($q_{s_1} > q_{s_2}$) and absorbs a higher percentage of the total demand ($TNP_1 > TNP_2$).

Please refer to Figures 3.8 and 3.9 to view the trajectories of the prices and the quality levels generated by the Euler method at iterations 0, 40, 80, ..., 1720, and 1758.

As mentioned in Section 3.1.1, the transfer prices are not variables in this model. However, the value of these prices: p_{t_j} ; $j = 1, \dots, n$, may impact the equilibrium values of the price and quality variables and the incurred utilities of the entities in the model. In order to make the impact of their values clearer, I provide sensitivity analysis results. For Example 3.1, with a single network provider, NP_1 , I varied the value of p_{t_1} from 0 to 40 to determine the effect on NP_1 's utility, price, and quality

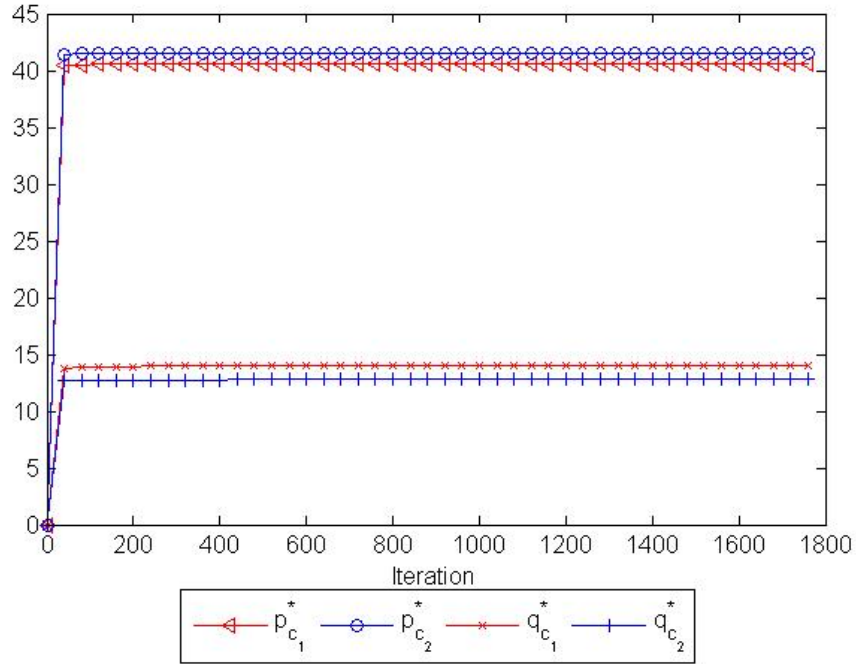


Figure 3.8. Prices and Quality Levels of Content Providers for Example 3.3

level, and on the two content providers', CP_1 and CP_2 , utilities, prices, and quality levels. The results are reported in Figure 3.10.

For Example 3.1, by increasing the value of p_{t_1} , I found that the utility of both CPs and that of NP_1 increases. Also, the prices charged by the CPs increase while the price charged by NP_1 decreases as the value of p_{t_1} increases. On the other hand, the quality of all providers does not change considerably (cf. Figure 3.10).

It is interesting that, when $p_{t_1} \geq 33$, the price charged by the network provider, NP_1 , $p_{s_1}^* = 0$, and the utilities of both content providers remain essentially unchanged. Therefore, in this case, the best value of p_{t_1} for all entities would be 33. Hence, in this example, all providers benefit with a positive p_{t_1} .

For Examples 3.2 and 3.3, in which I have two network providers, two kinds of sensitivity analyzes were performed. The results for the first sensitivity analysis are

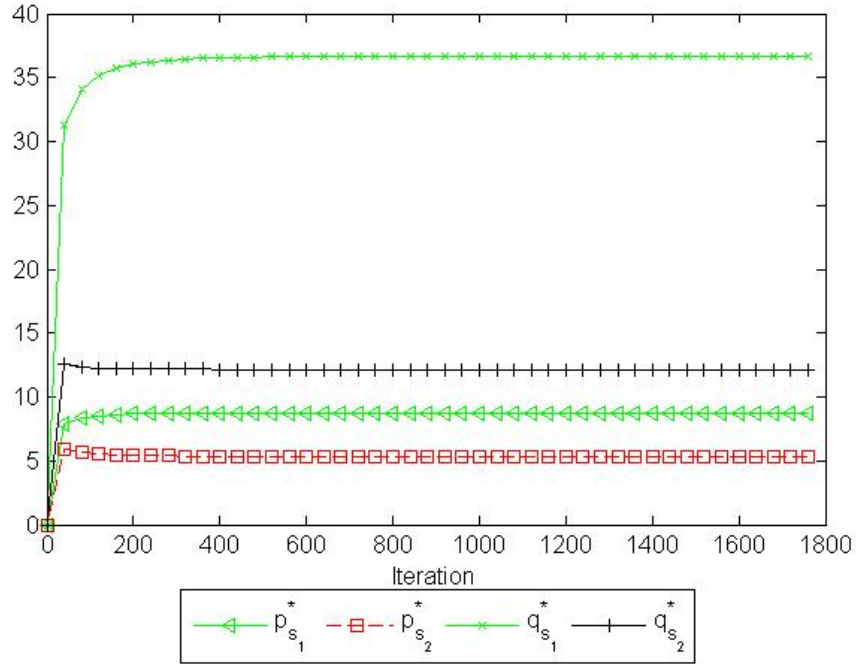


Figure 3.9. Prices and Quality Levels of Network Providers for Example 3.3

reported in Figure 3.11. For the first sensitivity analysis, the value of both p_{t_1} and p_{t_2} increase simultaneously from 0 to 40. As can be seen from the results in Figure 3.11, the utilities of all providers decrease with increasing values of the p_{t_j} s.

For the second sensitivity analysis in this set, I let $p_{t_1} + p_{t_2} = 40$, so that p_{t_1} starts at 40 and decreases to 0 while p_{t_2} starts at 0 and increases to 40. This transfer pricing scheme illustrates the case where the two network providers charge the content providers differently. The results are reported in Figure 3.12. I determine that the total utility of providers computed as the sum of the NPs' and the CPs' utilities, which correspond to their profits, is maximized when both network providers charge equally (cf. Figure 3.12).

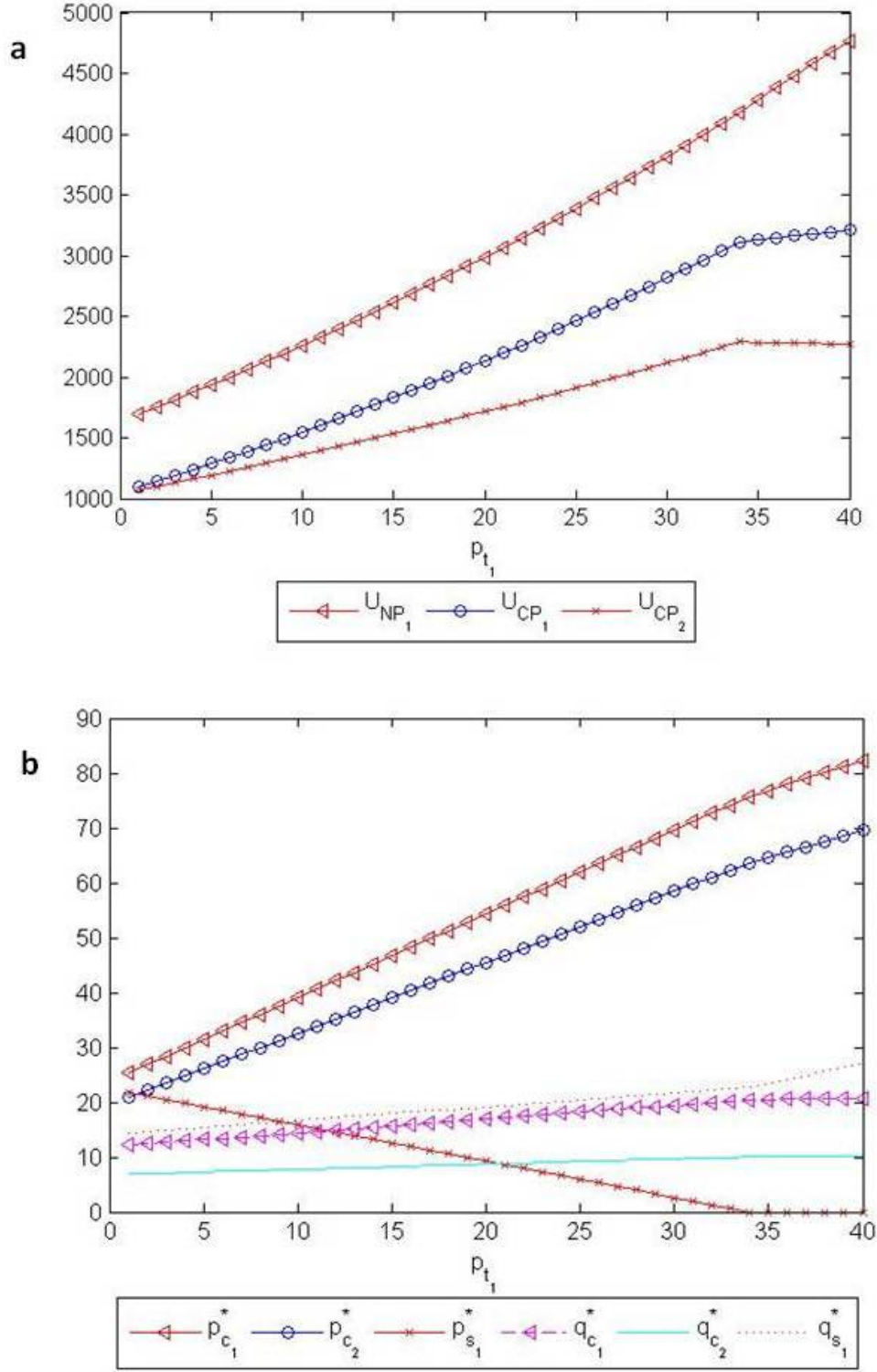


Figure 3.10. Effect of p_{t_1} Value on Utilities, Prices, and Quality in Example 3.1

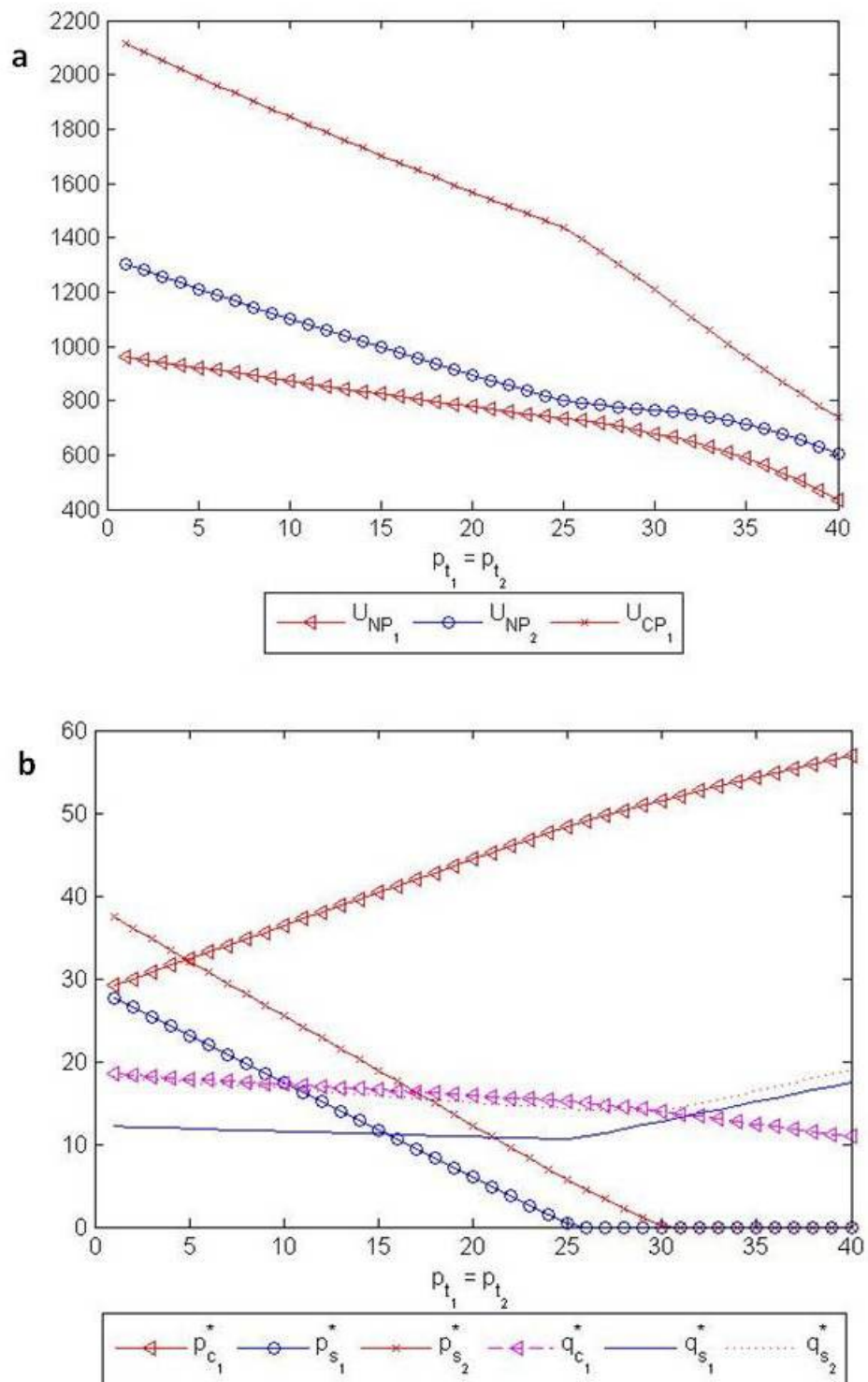


Figure 3.11. Effect of p_{t_1} and p_{t_2} Values on Utilities, Prices, and Quality in Example 3.2 with $p_{t_1} = p_{t_2}$

By examining other values for the sum of p_{t_1} and p_{t_2} , with $n = 30$, $n = 50$, and $n = 60$, I reach the conclusion, computationally, that for a pricing scheme of $p_{t_1} + p_{t_2} = n$ the optimal total utility of all providers is obtained when $p_{t_1} = p_{t_2} = n/2$ for n as above.

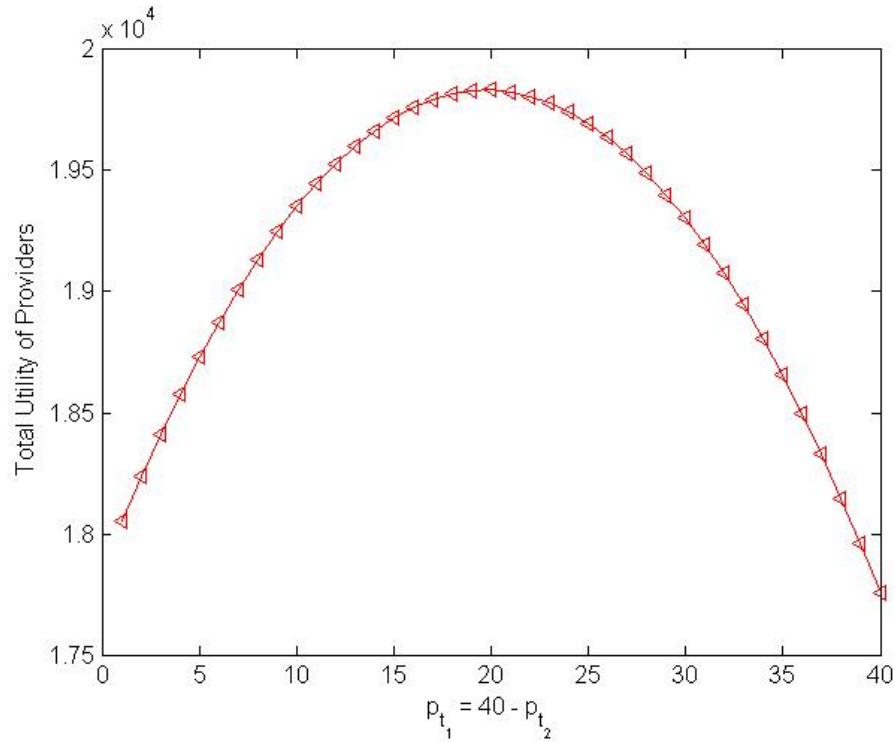


Figure 3.12. Effect of p_{t_1} and p_{t_2} Values on Total Utility in Example 3.3

Interesting results have been observed by performing sensitivity analysis. First, in a market with a monopolistic network provider all providers can increase their utility with a positive value of p_{t_1} . When I have multiple network providers, all providers achieve a higher utility by not charging content providers. On the other hand, if the network providers are allowed to charge content providers (lack of neutrality regulations), the social welfare or summation of all providers' utilities would be maximized if the network providers charge equally. I obtained such conclusions based on the results for Examples 3.2 and 3.3.

Nevertheless, as mentioned in Musacchio and Kim (2009), Njoroge et al. (2010), Altman, Caron, and Kesidis (2010), Musacchio, Schwartz, and Walrand (2009), and Economides and Tag (2012), the overall effect of implementing network neutrality regulations (e.g., having the $p_{t,j}$ s be zero) may still be both positive and negative depending on the parameter values and the model structure. This further emphasizes the importance of a computational framework to investigate the impacts of different values of transfer prices and their impacts, along with any other sensitivity analysis that may be desired.

Example 3.4

In this example, there are 4 content providers, 3 network providers, and 5 markets of user (Figure 13). Here, there are $4 \times 3 \times 5 = 60$ demand functions and 7 profit functions for the providers.

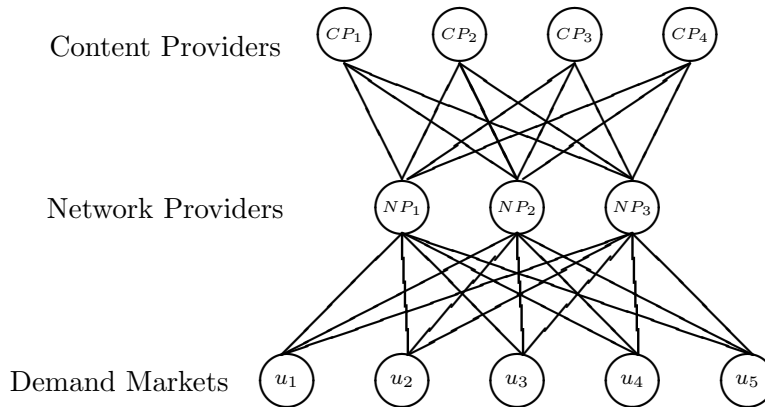


Figure 3.13. Network Topology of Content Flows for Example 3.4

The demand functions for demand market k for content from content provider i that is transferred by network provider j has the following form:

$$\begin{aligned}
d_{ijk} &= d_{ik}^0 - \beta_{ik} p_{c_i} + \sum_{f=1, f \neq i}^m (\beta'_{fk} p_{c_f}) \\
&= -\alpha_{jk} p_{s_j} + \sum_{l=1, l \neq j}^n (\alpha'_{lk} p_{s_l}) \\
&= +\delta_{ik} q_{c_i} - \sum_{f=1, f \neq i}^m (\delta'_{fk} q_{c_f}) \\
&= +\gamma_{jk} q_{s_j} - \sum_{l=1, l \neq j}^n (\gamma'_{lk} q_{s_l}), \quad \forall i, j, k.
\end{aligned}$$

The parameters for the demand functions are given in Table 3.1.

The cost function for network provider j has the following form:

$$CS_j = \sigma_j \left(\sum_{i=1}^m \sum_{k=1}^o d_{ijk} + q_{s_j}^2 \right), \quad \forall j,$$

where $\sigma_1 = 1.2$, $\sigma_2 = 3.2$, and $\sigma_3 = 2.5$.

Also, the cost function for content provider i is given by:

$$CC_i = \kappa_i (q_{c_i}^2), \quad \forall i,$$

where $\kappa_1 = 2.7$, $\kappa_2 = 3.1$, $\kappa_3 = 2.9$, and $\kappa_4 = 3.2$.

The utility of each provider is the difference of its revenue and cost. The transfer price for network providers are:

$$p_{t_1} = 10, \quad p_{t_2} = 14, \quad p_{t_3} = 13.$$

Table 3.1: Demand Functions for Example 3.4

d_{ijk}	d_{ik}^0	p_{c1}	p_{c2}	p_{c3}	p_{c4}	p_{s1}	p_{s2}	p_{s3}	q_{c1}	q_{c2}	q_{c3}	q_{c4}	q_{s1}	q_{s2}	q_{s3}
d_{111}	112	-1.85	0.3	0.3	0.3	-2.1	0.4	0.4	0.76	-0.3	-0.3	-0.3	0.64	-0.4	-0.4
d_{211}	99	0.3	-2	0.3	0.3	-2.1	0.4	0.4	-0.3	0.56	-0.3	-0.3	0.64	-0.4	-0.4
d_{311}	100	0.3	0.3	-2.3	0.3	-2.1	0.4	0.4	-0.3	-0.3	0.61	-0.3	0.64	-0.4	-0.4
d_{411}	97	0.3	0.3	0.3	-1.99	-2.1	0.4	0.4	-0.3	-0.3	-0.3	0.71	0.64	-0.4	-0.4
d_{121}	112	1.85	0.3	0.3	0.3	0.4	-1.92	0.4	0.76	-0.3	-0.3	-0.3	-0.4	0.1	-0.4
d_{221}	99	0.3	-2	0.3	0.3	0.4	-1.92	0.4	-0.3	0.56	-0.3	-0.3	-0.4	0.1	-0.4
d_{321}	100	0.3	0.3	-2.3	0.3	0.4	-1.92	0.4	-0.3	-0.3	0.61	-0.3	-0.4	0.1	-0.4
d_{421}	97	0.3	0.3	0.3	-1.99	0.4	-1.92	0.4	-0.3	-0.3	-0.3	0.71	-0.4	0.1	-0.4
d_{131}	112	-1.85	0.3	0.3	0.3	0.4	0.4	-1.8	0.76	-0.3	-0.3	-0.3	-0.4	-0.4	0.5
d_{231}	99	0.3	-2	0.3	0.3	0.4	0.4	-1.8	-0.3	0.56	-0.3	-0.3	-0.4	-0.4	0.5
d_{331}	100	0.3	0.3	-2.3	0.3	0.4	0.4	-1.8	-0.3	-0.3	0.61	-0.3	-0.4	-0.4	0.5
d_{431}	97	0.3	0.3	0.3	-1.99	0.4	0.4	-1.8	-0.3	-0.3	-0.3	0.71	-0.4	-0.4	0.5
d_{112}	98	-1.85	0.3	0.3	0.3	-1.9	0.4	0.4	0.59	-0.3	-0.3	-0.3	0.49	-0.4	-0.4
d_{212}	100	0.3	-2.2	0.3	0.3	-1.9	0.4	0.4	-0.3	0.73	-0.3	-0.3	0.49	-0.4	-0.4
d_{312}	89	0.3	0.3	-1.89	0.3	-1.9	0.4	0.4	-0.3	-0.3	0.61	-0.3	0.49	-0.4	-0.4
d_{412}	110	0.3	0.3	0.3	-2.14	-1.9	0.4	0.4	-0.3	-0.3	-0.3	0.71	0.49	-0.4	-0.4
d_{122}	98	-1.85	0.3	0.3	0.3	0.4	-2.2	0.4	0.59	-0.3	-0.3	-0.3	-0.4	0.7	-0.4
d_{222}	100	0.3	-2.2	0.3	0.3	0.4	-2.2	0.4	-0.3	0.73	-0.3	-0.3	-0.4	0.7	-0.4
d_{322}	89	0.3	0.3	-1.89	0.3	0.4	-2.2	0.4	-0.3	-0.3	0.61	-0.3	-0.4	0.7	-0.4
d_{422}	110	0.3	0.3	0.3	-2.14	0.4	-2.2	0.4	-0.3	-0.3	-0.3	0.71	-0.4	0.7	-0.4
d_{132}	98	-1.85	0.3	0.3	0.3	0.4	0.4	-2	0.59	-0.3	-0.3	-0.3	-0.4	-0.4	0.65
d_{232}	100	0.3	-2.2	0.3	0.3	0.4	0.4	-2	-0.3	0.73	-0.3	-0.3	-0.4	-0.4	0.65
d_{332}	89	0.3	0.3	-1.89	0.3	0.4	0.4	-2	-0.3	-0.3	0.61	-0.3	-0.4	-0.4	0.65
d_{432}	110	0.3	0.3	0.3	-2.14	0.4	0.4	-2	-0.3	-0.3	-0.3	0.71	-0.4	-0.4	0.65
d_{113}	102	-2	0.3	0.3	0.3	-2	0.4	0.4	0.66	-0.3	-0.3	-0.3	0.3	-0.4	-0.4
d_{213}	95	0.3	-1.97	0.3	0.3	-2	0.4	0.4	-0.3	0.46	-0.3	-0.3	0.3	-0.4	-0.4
d_{313}	115	0.3	0.3	-2.2	0.3	-2	0.4	0.4	-0.3	-0.3	0.59	-0.3	0.3	-0.4	-0.4

Continued

Table 3.1

d_{ijk}	d_{ik}^0	p_{c1}	p_{c2}	p_{c3}	p_{c4}	p_{s1}	p_{s2}	p_{s3}	q_{c1}	q_{c2}	q_{c3}	q_{c4}	q_{s1}	q_{s2}	q_{s3}
d_{413}	97	0.3	0.3	0.3	-2.05	-2	0.4	0.4	-0.3	-0.3	-0.3	0.6	0.3	-0.4	-0.4
d_{123}	102	-2	0.3	0.3	0.3	0.4	-1.95	0.4	0.66	-0.3	-0.3	-0.3	-0.4	0.1	-0.4
d_{223}	95	0.3	-1.97	0.3	0.3	0.4	-1.95	0.4	-0.3	0.46	-0.3	-0.3	-0.4	0.1	-0.4
d_{323}	115	0.3	0.3	-2.2	0.3	0.4	-1.95	0.4	-0.3	-0.3	0.59	-0.3	-0.4	0.1	-0.4
d_{423}	97	0.3	0.3	0.3	-2.05	0.4	-1.95	0.4	-0.3	0.3	-0.3	0.6	-0.4	0.1	-0.4
d_{133}	102	-2	0.3	0.3	0.3	0.4	0.4	-2.24	0.66	-0.3	-0.3	-0.3	-0.4	-0.4	0.68
d_{233}	95	0.3	-1.97	0.3	0.3	0.4	0.4	-2.24	-0.3	0.46	-0.3	-0.3	-0.4	-0.4	0.68
d_{333}	115	0.3	0.3	-2.2	0.3	0.4	0.4	-2.24	-0.3	-0.3	0.59	-0.3	-0.4	-0.4	0.68
d_{433}	97	0.3	0.3	0.3	-2.05	0.4	0.4	-2.24	-0.3	-0.3	-0.3	0.6	-0.4	-0.4	0.68
d_{114}	97	-1.9	0.3	0.3	0.3	-1.9	0.4	0.4	0.49	-0.3	-0.3	-0.3	0.59	-0.4	-0.4
d_{214}	101	0.3	-2.2	0.3	0.3	-1.9	0.4	0.4	-0.3	0.7	-0.3	-0.3	0.59	-0.4	-0.4
d_{314}	90	0.3	0.3	-2	0.3	-1.9	0.4	0.4	-0.3	-0.3	0.65	-0.3	0.59	-0.4	-0.4
d_{414}	109	0.3	0.3	0.3	-1.87	-1.9	0.4	0.4	-0.3	-0.3	-0.3	0.7	0.59	-0.4	-0.4
d_{124}	97	-1.9	0.3	0.3	0.3	0.4	-2.14	0.4	0.49	-0.3	-0.3	-0.3	-0.4	0.6	-0.4
d_{224}	101	0.3	-2.2	0.3	0.3	0.4	-2.14	0.4	-0.3	0.7	-0.3	-0.3	-0.4	0.6	-0.4
d_{324}	90	0.3	0.3	-2	0.3	0.4	-2.14	0.4	-0.3	-0.3	0.65	-0.3	-0.4	0.6	-0.4
d_{424}	109	0.3	0.3	0.3	-1.87	0.4	-2.14	0.4	-0.3	-0.3	-0.3	0.7	-0.4	0.6	-0.4
d_{134}	97	-1.9	0.3	0.3	0.3	0.4	0.4	-1.89	0.49	-0.3	-0.3	-0.3	-0.4	-0.4	0.8
d_{234}	101	0.3	-2.2	0.3	0.3	0.4	0.4	-1.89	-0.3	0.7	-0.3	-0.3	-0.4	-0.4	0.8
d_{334}	90	0.3	0.3	-2	0.3	0.4	0.4	-1.89	-0.3	-0.3	0.65	-0.3	-0.4	-0.4	0.8
d_{434}	109	0.3	0.3	0.3	-1.87	0.4	0.4	-1.89	-0.3	-0.3	-0.3	0.7	-0.4	-0.4	0.8
d_{115}	101	-2	0.3	0.3	0.3	-1.85	0.4	0.4	0.65	-0.3	-0.3	-0.3	0.4	-0.4	-0.4
d_{215}	96	0.3	-1.95	0.3	0.3	-1.85	0.4	0.4	-0.3	0.48	-0.3	-0.3	0.4	-0.4	-0.4
d_{315}	114	0.3	0.3	-2.24	0.3	-1.85	0.4	0.4	-0.3	-0.3	0.56	-0.3	0.4	-0.4	-0.4
d_{415}	98	0.3	0.3	0.3	-1.86	-1.85	0.4	0.4	-0.3	-0.3	-0.3	0.65	0.4	-0.4	-0.4
d_{125}	101	-2	0.3	0.3	0.3	0.4	-2.31	0.4	0.65	-0.3	-0.3	-0.3	-0.4	0.2	-0.4
d_{225}	96	0.3	-1.95	0.3	0.3	0.4	-2.31	0.4	-0.3	0.48	-0.3	-0.3	-0.4	0.2	-0.4
d_{325}	114	0.3	0.3	-2.24	0.3	0.4	-2.31	0.4	-0.3	-0.3	0.56	-0.3	-0.4	0.2	-0.4
d_{425}	98	0.3	0.3	0.3	-1.86	0.4	-2.31	0.4	-0.3	-0.3	-0.3	0.65	-0.4	0.2	-0.4

Continued

Table 3.1

d_{ijk}	d_{ik}^0	p_{c1}	p_{c2}	p_{c3}	p_{c4}	p_{s1}	p_{s2}	p_{s3}	q_{c1}	q_{c2}	q_{c3}	q_{c4}	q_{s1}	q_{s2}	q_{s3}
d_{135}	101	-2	0.3	0.3	0.3	0.4	0.4	-1.95	0.65	-0.3	-0.3	-0.3	-0.4	-0.4	0.58
d_{235}	96	0.3	-1.95	0.3	0.3	0.4	0.4	-1.95	-0.3	0.48	-0.3	-0.3	-0.4	-0.4	0.58
d_{335}	114	0.3	0.3	-2.24	0.3	0.4	0.4	-1.95	-0.3	-0.3	0.56	-0.3	-0.4	-0.4	0.58
d_{435}	98	0.3	0.3	0.3	-1.86	0.4	0.4	-1.95	-0.3	-0.3	-0.3	0.65	-0.4	-0.4	0.58

The utility functions are:

$$\begin{aligned}
U_{NP_1} = & 6.12p_{c_1} + 6.984p_{c_2} + 7.356p_{c_3} + 6.492p_{c_4} + 46.8p_{s_1} - 9.6p_{s_2} \\
& - 9.6p_{s_3} + 1.62q_{c_1} + 1.884q_{c_2} + 1.776q_{c_3} + 1.356q_{c_4} - 11.616q_{s_1} + 9.6q_{s_2} \\
& + 9.6q_{s_3} - 1.2q_{s_1}^2 - (p_{s_1} + 10) \times (5.1p_{c_1} + 5.82p_{c_2} + 6.13p_{c_3} + 5.41p_{c_4} \\
& + 39p_{s_1} - 8p_{s_2} - 8p_{s_3} + 1.35q_{c_1} + 1.57q_{c_2} + 1.48q_{c_3} + 1.13q_{c_4} - 9.68q_{s_1} \\
& + 8q_{s_2} + 8q_{s_3} - 2020) - 2424,
\end{aligned}$$

$$\begin{aligned}
U_{NP_2} = & 16.32p_{c_1} + 18.624p_{c_2} + 19.616p_{c_3} + 17.312p_{c_4} - 25.6p_{s_1} + 134.656p_{s_2} \\
& - 25.6p_{s_3} + 4.32q_{c_1} + 5.0244q_{c_2} + 4.736q_{c_3} + 3.616q_{c_4} + 25.6q_{s_1} - 21.76q_{s_2} \\
& + 25.6q_{s_3} - 3.2q_{s_2}^2 - (p_{s_2} + 14) \times (5.1p_{c_1} + 5.82p_{c_2} + 6.13p_{c_3} + 5.41p_{c_4} \\
& - 8p_{s_1} + 42.08p_{s_2} - 8p_{s_3} + 1.35q_{c_1} + 1.57q_{c_2} + 1.48q_{c_3} + 1.13q_{c_4} + 8q_{s_1} \\
& - 6.8q_{s_2} + 8q_{s_3} - 2020) - 6464,
\end{aligned}$$

$$\begin{aligned}
U_{NP_3} = & 12.75p_{c_1} + 14.55p_{c_2} + 15.325p_{c_3} + 13.525p_{c_4} - 20p_{s_1} - 20p_{s_2} \\
& + 98.8p_{s_3} + 3.375q_{c_1} + 3.925q_{c_2} + 3.7q_{c_3} + 2.825q_{c_4} + 20q_{s_1} + 20q_{s_2} \\
& - 32.1q_{s_3} - 2.5q_{s_3}^2 - (p_{s_3} + 13) \times (5.1p_{c_1} + 5.82p_{c_2} + 6.13p_{c_3} + 5.41p_{c_4} \\
& - 8p_{s_1} - 8p_{s_2} + 39.52p_{s_3} + 1.35q_{c_1} + 1.57q_{c_2} + 1.48q_{c_3} + 1.13q_{c_4} + 8q_{s_1} \\
& + 8q_{s_2} - 12.84q_{s_3} - 2020) - 5050,
\end{aligned}$$

$$\begin{aligned}
U_{CP_1} = & (p_{c_1} - 10) * (1.5p_{c_2} - 9.6p_{c_1} + 1.5p_{c_3} + 1.5p_{c_4} - 9.75p_{s_1} + 2p_{s_2} + 2p_{s_3} \\
& + 3.15q_{c_1} - 1.5q_{c_2} - 1.5q_{c_3} - 1.5q_{c_4} + 2.42q_{s_1} - 2q_{s_2} - 2q_{s_3} + 510) - 2.7q_{c_1}^2 \\
& + (p_{c_1} - 14) \times (1.5p_{c_2} - 9.6p_{c_1} + 1.5p_{c_3} + 1.5p_{c_4} + 2p_{s_1} - 10.52p_{s_2} + 2p_{s_3} \\
& + 3.15q_{c_1} - 1.5q_{c_2} - 1.5q_{c_3} - 1.5q_{c_4} - 2q_{s_1} + 1.7q_{s_2} - 2q_{s_3} + 510) \\
& + (p_{c_1} - 13) \times (1.5p_{c_2} - 9.6p_{c_1} + 1.5p_{c_3} + 1.5p_{c_4} + 2p_{s_1} + 2p_{s_2} - 9.88p_{s_3} + 3.15q_{c_1} \\
& - 1.5q_{c_2} - 1.5q_{c_3} - 1.5q_{c_4} - 2q_{s_1} - 2q_{s_2} + 3.21q_{s_3} + 510),
\end{aligned}$$

$$\begin{aligned}
U_{CP_2} = & (p_{c_2} - 10) \times (1.5p_{c_1} - 10.32p_{c_2} + 1.5p_{c_3} + 1.5p_{c_4} - 9.75p_{s_1} + 2p_{s_2} + 2p_{s_3} \\
& - 1.5q_{c_1} + 2.93q_{c_2} - 1.5q_{c_3} - 1.5q_{c_4} + 2.42q_{s_1} - 2q_{s_2} - 2q_{s_3} + 491) - 3.1q_{c_2}^2 \\
& + (p_{c_2} - 14) \times (1.5p_{c_1} - 10.32p_{c_2} + 1.5p_{c_3} + 1.5p_{c_4} + 2p_{s_1} - 10.52p_{s_2} + 2p_{s_3} \\
& - 1.5q_{c_1} + 2.93q_{c_2} - 1.5q_{c_3} - 1.5q_{c_4} - 2q_{s_1} + 1.7q_{s_2} - 2q_{s_3} + 491) \\
& + (p_{c_2} - 13) \times (1.5p_{c_1} - 10.32p_{c_2} + 1.5p_{c_3} + 1.5p_{c_4} + 2p_{s_1} + 2p_{s_2} - 9.88p_{s_3} \\
& - 1.5q_{c_1} + 2.93q_{c_2} - 1.5q_{c_3} - 1.5q_{c_4} - 2q_{s_1} - 2q_{s_2} + 3.21q_{s_3} + 491),
\end{aligned}$$

$$\begin{aligned}
U_{CP_3} = & (p_{c_3} - 10) \times (1.5p_{c_1} + 1.5p_{c_2} - 10.63p_{c_3} + 1.5p_{c_4} - 9.75p_{s_1} + 2p_{s_2} + 2p_{s_3} \\
& - 1.5q_{c_1} - 1.5q_{c_2} + 3.02q_{c_3} - 1.5q_{c_4} + 2.42q_{s_1} - 2q_{s_2} - 2q_{s_3} + 508) - 2.9q_{c_3}^2 \\
& + (p_{c_3} - 14) \times (1.5p_{c_1} + 1.5p_{c_2} - 10.63p_{c_3} + 1.5p_{c_4} + 2p_{s_1} - 10.52p_{s_2} + 2p_{s_3} - 1.5q_{c_1} \\
& - 1.5q_{c_2} + 3.02q_{c_3} - 1.5q_{c_4} - 2q_{s_1} + 1.7q_{s_2} - 2q_{s_3} + 508) \\
& + (p_{c_3} - 13) \times (1.5p_{c_1} + 1.5p_{c_2} - 10.63p_{c_3} + 1.5p_{c_4} + 2p_{s_1} + 2p_{s_2} - 9.88p_{s_3} \\
& - 1.5q_{c_1} - 1.5q_{c_2} + 3.02q_{c_3} - 1.5q_{c_4} - 2q_{s_1} - 2q_{s_2} + 3.21q_{s_3} + 508),
\end{aligned}$$

$$\begin{aligned}
U_{CP_4} = & (p_{c_4} - 10) \times (1.5p_{c_1} + 1.5p_{c_2} + 1.5p_{c_3} - 9.91p_{c_4} - 9.75p_{s_1} + 2p_{s_2} + 2p_{s_3} \\
& - 1.5q_{c_1} - 1.5q_{c_2} - 1.5q_{c_3} + 3.37 * q_{c_4} + 2.42q_{s_1} - 2q_{s_2} - 2q_{s_3} + 511) - 3.2q_{c_4}^2 \\
& + (p_{c_4} - 14) \times (1.5p_{c_1} + 1.5p_{c_2} + 1.5p_{c_3} - 9.91p_{c_4} + 2p_{s_1} - 10.52p_{s_2} + 2p_{s_3} - 1.5q_{c_1} \\
& - 1.5q_{c_2} - 1.5q_{c_3} + 3.37q_{c_4} - 2q_{s_1} + 1.7q_{s_2} - 2q_{s_3} + 511) \\
& + (p_{c_4} - 13) \times (1.5p_{c_1} + 1.5p_{c_2} + 1.5p_{c_3} - 9.91p_{c_4} + 2p_{s_1} + 2p_{s_2} - 9.88p_{s_3} - 1.5q_{c_1} \\
& - 1.5q_{c_2} - 1.5q_{c_3} + 3.37q_{c_4} - 2q_{s_1} - 2q_{s_2} + 3.21 * q_{s_3} + 511).
\end{aligned}$$

The Euler method required 9046 iterations and 212.56 CPU seconds for convergence.

The equilibrium result is:

$$p_{c_1}^* = 32.27, \quad p_{c_2}^* = 26.37, \quad p_{c_3}^* = 27.35, \quad p_{c_4}^* = 30.51,$$

$$p_{s_1}^* = 21.77, \quad p_{s_2}^* = 0, \quad p_{s_3}^* = 5.45,$$

$$q_{c_1}^* = 34.89, \quad q_{c_2}^* = 19.90, \quad q_{c_3}^* = 23.46, \quad q_{c_4}^* = 28.71,$$

$$q_{s_1}^* = 123.32, \quad q_{s_2}^* = 11.48, \quad q_{s_3}^* = 40.95.$$

The utilities of network providers are:

$$U_{NP_1} = 18209.15, \quad U_{NP_2} = 1796.99, \quad U_{NP_3} = 5856.37.$$

The content providers' utilities are:

$$U_{CP_1} = 8666.85, \quad U_{CP_2} = 5376.46, \quad U_{CP_3} = 6101.34, \quad U_{CP_4} = 7686.85.$$

According to the result¹, NP_1 transfers almost 60 percent of total demand for all demand markets and CP_1 has the largest supply (around 30%)² among the content providers.

3.5. Summary and Conclusions

In this chapter, I developed a modeling and computational framework for a service-oriented Internet using game theory and variational inequality theory. First, I modeled a simple, illustrative Internet with a single content provider and a single network provider and analyzed the effect of the price that the network provider charges the content provider for data transmission. User's demand is a function of price and quality of both providers and goes up (down) as the price (quality) of the providers decreases. The analysis showed that the network provider benefits from charging the content provider if the user is more sensitive towards the network provider's fee.

I then modeled a market of multiple providers. The providers (content and network providers) are assumed to compete in an oligopolistic manner using quality and price of offered services to users as strategic variables. All providers are noncooperative and are assumed to be utility maximizers with their utilities consisting of profits. The users, in turn, reflect their preferences for the services produced by a content provider and transported by a network provider through the demand functions, which are functions of price and quality of not only that network and content provider, but also of the other providers. I also provided the equilibrium model's equivalent variational

¹ $TNP_1 = 1192.41$, $TNP_2 = 205.40$, and $TNP_3 = 630.16$

² $SCP_1 = 574.22$, $SCP_2 = 434.53$, $SCP_3 = 478.91$, and $SCP_4 = 540.30$

inequality formulation with nice features for computational purposes. I used the Euler method to solve numerical examples in order to illustrate the proposed model.

Next chapter, I focus on the dynamics of this model.

CHAPTER 4

A DYNAMIC SERVICE-ORIENTED INTERNET NETWORK ECONOMIC MODEL WITH PRICE AND QUALITY COMPETITION

This chapter completes the general model in Chapter 3. The dynamic adjustment process for the evolution of the price and quality of content service providers and network service providers is captured in this chapter by developing a projected dynamical systems model for a service-oriented Internet. In an oligopoly market, the providers are competing to maximize their profits while satisfying the demands of heterogeneous users for the Internet services at different quality levels and prices.

This chapter is based on Nagurney et al. (2014a) and is organized as follows. In Section 4.1, I develop the model and describe the content providers' and the network providers' decision-making behaviors, and formulate the dynamics of the prices and the quality levels of the content and the network providers as a projected dynamical system (cf. Dupuis and Nagurney (1993), Zhang and Nagurney (1995), Nagurney and Zhang (1996), and Nagurney (2006a)). I establish that the set of stationary points of the projected dynamical system coincides with the set of solutions to the derived variational inequality problem in Section 3.2. The associated stability results are also provided. In Section 4.2, I present the algorithm to track the trajectories of the prices and quality levels over time until the equilibrium values are attained. I then apply the discrete-time algorithm to several numerical examples to further illustrate the

model. I summarize the results and present my conclusions in Section 4.3, along with suggestions for future research.

4.1. The Dynamic Network Economic Model of a Service-Oriented Internet with Price and Quality Competition

In this section, I develop the dynamic network economic model of a service-oriented Internet with price and quality competition. Unlike earlier models that focused on dynamics (cf. Nagurney et al. (2013a) and Nagurney and Wolf (2013)), the new model allows for distinct quality levels associated with content provision and with transport network service provision. Moreover, I utilize direct demand functions, rather than inverse demand (price) functions, to capture the demand for content and network provision. Users (consumers) at the demand markets provide feedback to the content providers and the network providers in terms of the prices that they charge and their quality levels through the demands. Here, the demands are for the combination of content and network provision.

The network structure of the problem, which depicts the direction of the content flows, is given in Figure 4.1. Specifically, I assume m content providers, with a typical content provider denoted by CP_i ; n network providers, which provide the transport of the content to the consumers at the demand markets, with a typical network provider denoted by NP_j , and o demand markets of users, with a typical demand market denoted by u_k .

The notation for the model is given in Table 4.1. I first discuss what is meant by quality in the context of our model and describe specific functional forms, which are then utilized in the numerical examples. I then describe the behavior of the content providers and, subsequently, that of the network providers. I construct the projected

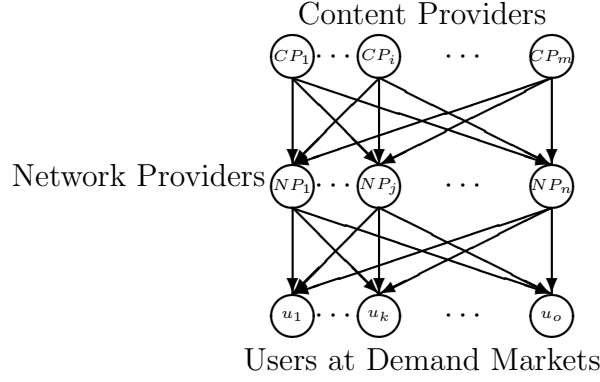


Figure 4.1. The Network Structure of the Model's Content Flows

dynamical system which formulates, in a unified manner, the dynamics of the content provider prices and quality levels and those of the network providers.

4.1.1 Modeling of Quality in a Service-Oriented Internet

The quality of content provided can be specified for a specific domain of content, e.g., video streaming. In this case, quality is defined as the quality of videos produced by the content provider CP_i and the production cost CC_i is a convex and continuous function of quality of service as well as demand. Here I assume that the demand is equal to the supply, so that $CC_i = CC_i(SCP_i, q_{c_i})$.

A possible functional form for CC_i is given by $K(SCP_i^2 + q_{c_i}^2)$. Of course, a special case of this functional form would be $Kq_{c_i}^2$, which would mean that the production cost of CP_i depends only on the quality of his product content.

The quality of the network transport service associated with NP_j , q_{s_j} , in turn, can be defined by various metrics such as the latency, jitter, or bandwidth. In this framework, as in Chapter 3, Section 3.1, I define the quality as the “expected delay,” which is computed by the Kleinrock function (see Altman, Legout, and Xu (2011)) as the reciprocal of the square root of delay: $q_{s_j} = \frac{1}{\sqrt{Delay}} = \sqrt{b(d, q_{s_j}) - D}$, where $b(d, q_{s_j})$ is the total bandwidth of the network and is a function of demand d and

Table 4.1. Notation for the Dynamic Network Economic Model of a Service-Oriented Internet with Price and Quality Competition

Notation	Definition
p_{c_i}	the price CP_i ; $i = 1, \dots, m$, charges the users for a unit of his content. The p_{c_i} ; $i = 1, \dots, m$, are grouped into the vector $p_c \in R_+^m$.
p_{s_j}	the price NP_j ; $j = 1, \dots, n$, charges the users for a unit of content transmitted by NP_j , with the p_{s_j} ; $j = 1, \dots, n$, grouped into the vector $p_s \in R_+^n$.
p_{t_j}	the fixed transmission fee that NP_j ; $j = 1, \dots, n$, charges the content providers for transmitting a unit of content.
q_{c_i}	the quality of CP_i 's content. The q_{c_i} ; $i = 1, \dots, m$, are grouped into the vector $q_c \in R_+^m$.
q_{s_j}	the quality of NP_j 's transmission service. The q_{s_j} ; $j = 1, \dots, n$, are grouped into the vector $q_s \in R_+^n$.
$d_{ijk}(p_c, q_c, p_s, q_s)$	the demand for CP_i 's content; $i = 1, \dots, m$, transmitted by NP_j ; $j = 1, \dots, n$, at demand market u_k ; $k = 1, \dots, o$. The demand function d_{ijk} is monotonically decreasing (increasing) in p_{c_i} and p_{s_j} (q_{c_i} and q_{s_j}), and monotonically increasing (decreasing) in the other prices (quality levels).
SCP_i	the total supply of content of CP_i ; $i = 1, \dots, m$.
TNP_j	the total amount of content transmitted by NP_j ; $j = 1, \dots, n$.
$CC_i(SCP_i, q_{c_i})$	the total cost of CP_i ; $i = 1, \dots, m$, to produce the content.
$CS_j(TNP_j, q_{s_j})$	the total cost of NP_j ; $j = 1, \dots, n$, to maintain its network based on the total traffic passed through and its quality level.

quality, that is: $b(d, q_{s_j}) = d + q_{s_j}^2$. Hence, the greater the demand at higher quality, the larger the amount of bandwidth used. The network provider incurs a cost of transferring the demand while supporting q_{s_j} for data transport, denoted by CS_j . I assume a convex, continuous, and differentiable transfer function for NP_j of the following general form: $CS_j(TNP_j, q_{s_j}) = R(TNP_j + q_{s_j}^2)$, where R is the unit cost of bandwidth.

4.1.2 The Behavior of the Content Providers and Their Price and Quality Dynamics

Each CP_i produces distinct (but substitutable) content of specific quality q_{c_i} , and sells at a unit price p_{c_i} . The total supply of CP_i , SCP_i , is given by:

$$SCP_i = \sum_{j=1}^n \sum_{k=1}^o d_{ijk}, \quad i = 1, \dots, m. \quad (4.1)$$

I assume that the content providers are profit-maximizers, where the profit or utility of CP_i , U_{CP_i} ; $i = 1, \dots, m$, which is the difference between his total revenue and his total cost, is given by the expression:

$$U_{CP_i}(p_c, q_c, p_s, q_s) = \sum_{j=1}^n (p_{c_i} - p_{t_j}) \sum_{k=1}^o d_{ijk} - CC_i(SCP_i, q_{c_i}). \quad (4.2)$$

Let \mathcal{K}_i^1 denote the feasible set corresponding to CP_i , where $\mathcal{K}_i^1 \equiv \{(p_{c_i}, q_{c_i}) \mid p_{c_i} \geq 0, \text{ and } q_{c_i} \geq 0\}$. Hence, the price charged by each CP_i and his quality level must be nonnegative. I assume that the utility functions in (4.2) for all i are continuous, continuously differentiable, and concave.

I now propose a dynamic adjustment process for the evolution of the content providers' prices and quality levels. In this framework, the rate of change of the price

charged by CP_i ; $i = 1, \dots, m$, is in proportion to $\frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial p_{c_i}}$, as long as the price p_{c_i} is positive. Namely, when $p_{c_i} > 0$,

$$\dot{p}_{c_i} = \frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial p_{c_i}}, \quad (4.3)$$

where \dot{p}_{c_i} denotes the rate of change of p_{c_i} . However, when $p_{c_i} = 0$, the nonnegativity condition on the price forces the price p_{c_i} to remain zero when $\frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial p_{c_i}} \leq 0$. Hence, in this case, I am only guaranteed of having possible increases in the price. Namely, when $p_{c_i} = 0$,

$$\dot{p}_{c_i} = \max\left\{0, \frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial p_{c_i}}\right\}. \quad (4.4)$$

Note that (4.4) is economically meaningful since when the marginal utility (profit) with respect to the price charged by CP_i is positive then we can expect the price that he charges for the content to increase; similarly, if the marginal utility (profit) with respect to the price that he charges is negative, then we can expect the price that he charges for the content to decrease. The max operator in (4.4) guarantees that the price will not take on a negative value, since it must satisfy the nonnegativity constraint.

I may write (4.3) and (4.4) concisely for each CP_i ; $i = 1, \dots, m$, as:

$$\dot{p}_{c_i} = \begin{cases} \frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial p_{c_i}}, & \text{if } p_{c_i} > 0 \\ \max\left\{0, \frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial p_{c_i}}\right\}, & \text{if } p_{c_i} = 0. \end{cases} \quad (4.5)$$

As for CP_i 's quality level, when $q_{c_i} > 0$, then

$$\dot{q}_{c_i} = \frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial q_{c_i}}, \quad (4.6)$$

where \dot{q}_{c_i} denotes the rate of change of q_{c_i} ; otherwise:

$$\dot{q}_{c_i} = \max\left\{0, \frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial q_{c_i}}\right\}, \quad (4.7)$$

since q_{c_i} must be nonnegative.

Combining (4.6) and (4.7), I may write, for each CP_i ; $i = 1, \dots, m$:

$$\dot{q}_{c_i} = \begin{cases} \frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial q_{c_i}}, & \text{if } q_{c_i} > 0 \\ \max\left\{0, \frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial q_{c_i}}\right\}, & \text{if } q_{c_i} = 0. \end{cases} \quad (4.8)$$

The system (4.8) is also economically meaningful, since we can expect the quality level associated with CP_i 's content to increase (decrease) if the associated marginal utility (profit) is positive (negative). In addition, I am guaranteed that the quality of CP_i 's content is never negative.

4.1.3 The Behavior of the Network Providers and Their Price and Quality Dynamics

Each NP_j ; $j = 1, \dots, n$, selects his quality q_{s_j} and the price p_{t_j} that he charges each content provider to transfer one unit of content to the users, and the price p_{s_j} that he charges users to transfer them one unit of content. Theoretically, every content provider is connected to every network provider and, subsequently, to all users, as depicted in Figure 4.1. However, solution of the model will determine which links have positive flows on them in terms of content. The total amount of content of services transported by NP_j , TNP_j , is given by:

$$TNP_j = \sum_{i=1}^m \sum_{k=1}^o d_{ijk}, \quad j = 1, \dots, n. \quad (4.9)$$

The utility of NP_j ; $j = 1, \dots, n$, U_{NP_j} , corresponds to his profit and is the difference between his income and his cost, that is:

$$U_{NP_j}(p_c, q_c, p_s, q_s) = (p_{s_j} + p_{t_j})TNP_j - CS_j(TNP_j, q_{s_j}). \quad (4.10)$$

Let \mathcal{K}_j^2 denote the feasible set corresponding to network provider j , where $\mathcal{K}_j^2 \equiv \{(p_{s_j}, q_{s_j}) \mid p_{s_j} \geq 0, \text{ and } q_{s_j} \geq 0\}$. Hence, NP_j 's price and quality must both be nonnegative. The utility functions in (4.10) for all j are assumed to be continuous, continuously differentiable, and concave.

Although the network provider needs to determine the price to charge the content provider, p_{t_j} , he cannot maximize his utility with respect to p_{t_j} simultaneously with p_{s_j} . Note that the providers' utilities are linear functions of p_{t_j} , so that if p_{t_j} is under the control of one of the providers, it would simply be set at an extreme value and, subsequently, lead to zero demand and zero income. Therefore, p_{t_j} is assumed to be an exogenous parameter in this model.

I now describe the dynamics. Using similar arguments to those in Section 4.1.2, we have that the rate of change of the price for NP_j , \dot{p}_{s_j} ; $j = 1, \dots, n$, can be expressed as:

$$\dot{p}_{s_j} = \begin{cases} \frac{\partial U_{NP_j}(p_c, q_c, p_s, q_s)}{\partial p_{s_j}}, & \text{if } p_{s_j} > 0 \\ \max\{0, \frac{\partial U_{NP_j}(p_c, q_c, p_s, q_s)}{\partial p_{s_j}}\}, & \text{if } p_{s_j} = 0. \end{cases} \quad (4.11)$$

Analogously, for the quality level of NP_j ; $j = 1, \dots, n$, I may write:

$$\dot{q}_{s_j} = \begin{cases} \frac{\partial U_{NP_j}(p_c, q_c, p_s, q_s)}{\partial q_{s_j}}, & \text{if } q_{s_j} > 0 \\ \max\{0, \frac{\partial U_{NP_j}(p_c, q_c, p_s, q_s)}{\partial q_{s_j}}\}, & \text{if } q_{s_j} = 0. \end{cases} \quad (4.12)$$

Before proceeding to the construction of the projected dynamical systems model, I depict the financial payment flows associated with this dynamic network economic

model in Figure 4.2. The directions of the arrows reflect the direction of the financial payments. The prices charged, in turn, would have the opposite direction to the associated financial payment.

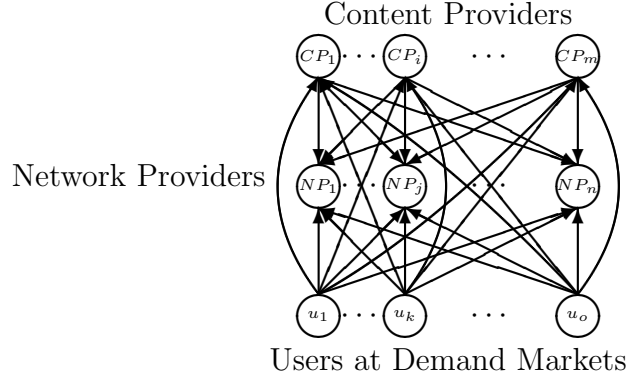


Figure 4.2. The Network Structure of the Model's Financial Payment Flows

4.1.4 The Projected Dynamical System

Consider now the dynamic network economic model in which the content provider prices evolve according to (4.5) and their quality levels evolve according to (4.8). Similarly, the quality levels of the network providers evolve according to (4.12) and the prices that they charge according to (4.11). Let X denote the $(2m + 2n)$ -dimensional vector consisting of the vectors: (p_c, q_c, p_s, q_s) . I also define the feasible set $\mathcal{K} \equiv \prod_{i=1}^m K_i^1 \times \prod_{j=1}^n K_j^2$. Finally, I define the $(2m + 2n)$ -dimensional vector $F(X)$ with components:

$$\begin{aligned} & -\frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial p_{c_i}}, -\frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial q_{c_i}}; \quad i = 1, \dots, m; \\ & -\frac{\partial U_{NP_j}(p_c, q_c, p_s, q_s)}{\partial p_{s_j}}, -\frac{\partial U_{NP_j}(p_c, q_c, p_s, q_s)}{\partial q_{s_j}}; \quad j = 1, \dots, n. \end{aligned} \quad (4.13)$$

All vectors are assumed to be column vectors.

Then, the dynamic model described above can be rewritten as the projected dynamical system (cf. (2.17)) defined by the following initial value problem:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X^0, \quad (4.14)$$

where $\Pi_{\mathcal{K}}$ is the projection operator of $-F(X)$ onto \mathcal{K} and X^0 is the initial point $(p_c^0, q_c^0, p_s^0, q_s^0)$ corresponding to the initial price and quality levels of the content and the network providers. Specifically, according to Definition 2.7, $\Pi_{\mathcal{K}}$ is the projection, with respect to \mathcal{K} , with \mathcal{K} being a convex polyhedron, of the vector $-F(X)$ at X , defined as:

$$\Pi_{\mathcal{K}}(X, -F(X)) = \lim_{\delta \rightarrow 0} \frac{P_{\mathcal{K}}(X - \delta F(X)) - X}{\delta}, \quad (4.15)$$

with $P_{\mathcal{K}}$ being the projection map:

$$P_{\mathcal{K}}(X) = \operatorname{argmin}_{z \in \mathcal{K}} \|X - z\|, \quad (4.16)$$

and where $\|\cdot\| = \langle x, x \rangle$. In this model, the projection operator takes on a nice explicit form because the feasible set \mathcal{K} is the nonnegative orthant.

The trajectory associated with (4.14) provides the dynamic evolution of the prices charged and the quality levels of both the content providers and the network providers and the dynamic interactions among the content and the network providers and the users at the demand markets through the demand functions.

As emphasized in Definition 2.7, the dynamical system (4.14) is non-classical in that the right-hand side is discontinuous in order to guarantee that the constraints, that is, the nonnegativity assumption on all the prices and quality levels, are satisfied. Dupuis and Nagurney (1993) introduced such dynamical systems and they have been used, to-date, in numerous competitive applications. Here, for the first time, I model

the dynamics of both price and quality competition of both content and network providers.

4.1.5 Stationary/Equilibrium Point

I now present the relationship between the stationary points of the projected dynamical system (4.14) and the solutions, commonly referred to as equilibria (Theorem 2.6), of the associated variational inequality problem: determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \quad (4.17)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in n -dimensional Euclidean space, F is a continuous function from \mathcal{K} to R^n , and \mathcal{K} is closed and convex set. According to Theorem 2.6, (cf. Dupuis and Nagurney (1993)), the stationary points of the projected dynamical system (4.14) coincide with the solution of variational inequality (4.17). Hence, I can immediately write down the variational inequality governing the equilibrium state (stationary point) associated with the above dynamic network economic model, in which no content provider nor any network provider has any incentive to alter his pricing and quality level strategies, as given below.

Corollary 4.1

$(p_c^*, q_c^*, p_s^*, q_s^*) \in \mathcal{K}$ is a stationary point of the projected dynamical system (4.14) if and only if it satisfies the variational inequality:

$$\begin{aligned} & - \sum_{i=1}^m \frac{\partial U_{CP_i}(p_c^*, q_c^*, p_s^*, q_s^*)}{\partial p_{c_i}} \times (p_{c_i} - p_{c_i}^*) - \sum_{i=1}^m \frac{\partial U_{CP_i}(p_c^*, q_c^*, p_s^*, q_s^*)}{\partial q_{c_i}} \times (q_{c_i} - q_{c_i}^*) \\ & - \sum_{j=1}^n \frac{\partial U_{NP_j}(p_c^*, q_c^*, p_s^*, q_s^*)}{\partial p_{s_j}} \times (p_{s_j} - p_{s_j}^*) - \sum_{j=1}^n \frac{\partial U_{NP_j}(p_c^*, q_c^*, p_s^*, q_s^*)}{\partial q_{s_j}} \times (q_{s_j} - q_{s_j}^*) \geq 0, \end{aligned}$$

$$\forall(p_c, q_c, p_s, q_s) \in \mathcal{K}, \quad (4.18)$$

or, equivalently,

$$\begin{aligned}
& \sum_{i=1}^m \left[- \sum_{j=1}^n \sum_{k=1}^o d_{ijk} - \sum_{j=1}^n \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial p_{c_i}} \times (p_{c_i}^* - p_{t_j}) \right. \\
& \quad \left. + \frac{\partial CC_i(SCP_i, q_{c_i}^*)}{\partial SCP_i} \cdot \frac{\partial SCP_i}{\partial p_{c_i}} \right] \times (p_{c_i} - p_{c_i}^*) \\
& + \sum_{i=1}^M \left[- \sum_{j=1}^n \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial q_{c_i}} \times (p_{c_i}^* - p_{t_j}) + \frac{\partial CC_i(SCP_i, q_{c_i}^*)}{\partial q_{c_i}} \right] \times (q_{c_i} - q_{c_i}^*) \\
& \quad + \sum_{j=1}^n \left[- \sum_{i=1}^m \sum_{k=1}^o d_{ijk} - \sum_{i=1}^m \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial p_{s_j}} \times (p_{s_j}^* + p_{t_j}) \right. \\
& \quad \left. + \frac{\partial CS_j(TNP_j, q_{s_j}^*)}{\partial TNP_j} \cdot \frac{\partial TNP_j}{\partial p_{s_j}} \right] \times (p_{s_j} - p_{s_j}^*) \\
& + \sum_{j=1}^n \left[- \sum_{i=1}^m \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial q_{s_j}} \times (p_{s_j}^* + p_{t_j}) + \frac{\partial CS_j(TNP_j, q_{s_j}^*)}{\partial q_{s_j}} \right] \times (q_{s_j} - q_{s_j}^*) \geq 0, \\
& \quad \forall(p_c, q_c, p_s, q_s) \in \mathcal{K}. \quad (4.19)
\end{aligned}$$

Variational inequalities (4.18) and (4.19) are precisely the ones obtained in (3.43) and (3.44) for the static counterpart of this dynamic network economic model in which the content providers compete in price and quality until the Nash equilibrium is achieved whereby no content provider can improve upon his profits by altering his price and/or quality level. Similarly, the network providers also compete in price and quality until no network provider can improve upon his profits by altering his strategies and, hence, a Nash equilibrium is also achieved.

Recall that a content price pattern and quality level pattern (p_c^*, q_c^*) is said to constitute a Nash equilibrium if for each content provider CP_i ; $i = 1, \dots, m$:

$$U_{CP_i}(p_{c_i}^*, \hat{p}_{c_i}^*, q_{c_i}^*, \hat{q}_{c_i}^*, p_s^*, q_s^*) \geq U_{CP_i}(p_{c_i}, \hat{p}_{c_i}^*, q_{c_i}, \hat{q}_{c_i}^*, p_s^*, q_s^*), \quad \forall (p_{c_i}, q_{c_i}) \in \mathcal{K}_i^1, \quad (4.20)$$

where $\hat{p}_{c_i}^* \equiv (p_{c_1}^*, \dots, p_{c_{i-1}}^*, p_{c_{i+1}}^*, \dots, p_{c_m}^*)$ and $\hat{q}_{c_i}^* \equiv (q_{c_1}^*, \dots, q_{c_{i-1}}^*, q_{c_{i+1}}^*, \dots, q_{c_m}^*)$.

Similarly, a network price pattern and quality level pattern (p_s^*, q_s^*) is said to constitute a Nash equilibrium if for each network provider NP_j ; $j = 1, \dots, n$:

$$U_{NP_j}(p_c^*, q_c^*, p_{s_j}^*, \hat{p}_{s_j}^*, q_{s_j}^*, \hat{q}_{s_j}^*) \geq U_{NP_j}(p_c^*, q_c^*, p_{s_j}, \hat{p}_{s_j}^*, q_{s_j}, \hat{q}_{s_j}^*), \quad \forall (p_{s_j}, q_{s_j}) \in \mathcal{K}_j^2, \quad (4.21)$$

where $\hat{p}_{s_j}^* \equiv (p_{s_1}^*, \dots, p_{s_{j-1}}^*, p_{s_{j+1}}^*, \dots, p_{s_n}^*)$ and $\hat{q}_{s_j}^* \equiv (q_{s_1}^*, \dots, q_{s_{j-1}}^*, q_{s_{j+1}}^*, \dots, q_{s_n}^*)$.

4.1.6 Stability Under Monotonicity

I now investigate whether, and under what conditions, the dynamic, continuous-time adjustment process defined by (4.14) approaches a stationary point/equilibrium. Recall that Lipschitz continuity of $F(X)$ (Definition 2.6) guarantees the existence of a unique solution to (4.14). In other words, $X^0(t)$ solves the initial value problem (IVP)

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X^0, \quad (4.22)$$

with $X^0(0) = X^0$.

I propose the following definitions of stability for the adjustment process, which are adaptations of those introduced in Zhang and Nagurney (1995) (see also Definition 2.10). I use $B(X, r)$ to denote the open ball with radius r and center X .

I now present some fundamental definitions, for completeness, and some basic qualitative results.

Definition 4.1

An equilibrium price and quality pattern X^* is stable, if for any $\epsilon > 0$, there exists a $\delta > 0$, such that for all initial $X \in B(X^*, \delta)$ and all $t \geq 0$

$$X(t) \in B(X^*, \epsilon). \quad (4.23)$$

The equilibrium point X^* is unstable, if it is not stable.

Definition 4.2

An equilibrium price and quality pattern X^* is asymptotically stable, if it is stable and there exists a $\delta > 0$ such that for all initial prices and qualities $X \in B(X^*, \delta)$

$$\lim_{t \rightarrow \infty} X(t) \longrightarrow X^*. \quad (4.24)$$

Definition 4.3

An equilibrium price and quality pattern X^* is globally exponentially stable, if there exist constants $b > 0$ and $\mu > 0$ such that

$$\|X^0(t) - X^*\| \leq b\|X^0 - X^*\|e^{-\mu t}, \quad \forall t \geq 0, \forall X^0 \in \mathcal{K}. \quad (4.25)$$

Definition 4.4

An equilibrium price and quality pattern X^* is a global monotone attractor, if the Euclidean distance $\|X(t) - X^*\|$ is nonincreasing in t for all $X \in \mathcal{K}$.

Definition 4.5

An equilibrium X^* is a strictly global monotone attractor, if $\|X(t) - X^*\|$ is monotonically decreasing to zero in t for all $X \in \mathcal{K}$.

I now investigate the stability of the dynamic adjustment process under various monotonicity conditions.

Recall (Definition 2.3, Definition 2.4, and Definition 2.5) that $F(X)$ is *monotone* if

$$\langle F(X) - F(X^*), X - X^* \rangle \geq 0, \quad \forall X, X^* \in \mathcal{K}. \quad (4.26)$$

$F(X)$ is *strictly monotone* if

$$\langle F(X) - F(X^*), X - X^* \rangle > 0, \quad \forall X, X^* \in \mathcal{K}, X \neq X^*. \quad (4.27)$$

$F(X)$ is *strongly monotone*, if there is an $\eta > 0$, such that

$$\langle F(X) - F(X^*), X - X^* \rangle \geq \eta \|X - X^*\|^2, \quad \forall X, X^* \in \mathcal{K}. \quad (4.28)$$

The monotonicity of a function F is closely related to the positive-definiteness of its Jacobian ∇F (cf. Nagurney (1999)). Specifically, if ∇F is positive-semidefinite, then F is monotone; if ∇F is positive-definite, then F is strictly monotone; and, if ∇F is strongly positive-definite, in the sense that the symmetric part of ∇F , $(\nabla F^T + \nabla F)/2$, has only positive eigenvalues, then F is strongly monotone.

In the context of this network economic model, where $F(X)$ is the vector of negative marginal utilities, I note that if the utility functions are twice differentiable and the Jacobian of the negative marginal utility functions (or, equivalently, the negative of

the Hessian matrix of the utility functions) for the model is positive-definite, then the corresponding $F(X)$ is strictly monotone.

I now present an existence and uniqueness result, the proof of which follows from the basic theory of variational inequalities (cf. Nagurney (1999)).

Theorem 4.1

Suppose that F is strongly monotone. Then there exists a unique solution to variational inequality (4.18), equivalently, to variational inequality (4.19).

I summarize in the following theorem the stability properties of the utility gradient process, under various monotonicity conditions on the marginal utilities.

Theorem 4.2

(i). *If $F(X)$ is monotone, then every stationary point of (4.14), provided its existence, is a global monotone attractor for the utility gradient process.*

(ii). *If $F(X)$ is strictly monotone, then there exists at most one stationary point /equilibrium of (4.14). Furthermore, given existence, the unique equilibrium is a strictly global monotone attractor for the utility gradient process.*

(iii). *If $F(X)$ is strongly monotone, then the stationary point/equilibrium of (4.14), which is guaranteed to exist, is also globally exponentially stable for the utility gradient process.*

Proof: The stability assertions follow from Theorems 3.5, 3.6, and 3.7 in Nagurney and Zhang (1996), respectively. The uniqueness in (ii) is a classical variational inequality result, whereas existence and uniqueness as in (iii) follows from Theorem 4.1. \square

Example 4.1

I present Example 4.1 in order to illustrate some of the above concepts and results. The network consists of a single content provider, CP_1 , a single network provider, NP_1 , and users at a single demand market, u_1 , as depicted in Figure 4.3.

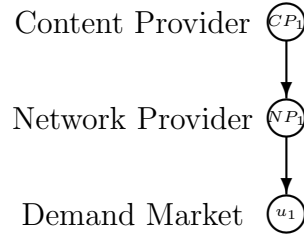


Figure 4.3. Network Topology for Example 4.1

The data are as follows. The price p_{t_1} is 10. The demand function is:

$$d_{111} = 100 - .5p_{s_1} - .8p_{c_1} + .6q_{s_1} + .5q_{c_1}.$$

The cost functions of CP_1 and NP_1 are, respectively:

$$CC_1 = 2(d_{111}^2 + q_{c_1}^2), \quad CS_1 = 2.2(d_{111} + q_{s_1}^2),$$

and their utility/profit functions are, respectively:

$$U_{CP_1} = (p_{c_1} - p_{t_1})d_{111} - 2(d_{111}^2 + q_{c_1}^2),$$

$$U_{NP_1} = (p_{s_1} + p_{t_1})d_{111} - 2.2(d_{111} + q_{s_1}^2).$$

Hence, I have that:

$$\begin{aligned}
F_{p_{c_1}} &= - \frac{\partial U_{CP_1}}{\partial p_{c_1}} \\
&= - \frac{\partial [(p_{c_1} - 10)d_{111} - 2(d_{111}^2 + q_{c_1}^2)]}{\partial p_{c_1}} \\
&= - [d_{111} + (p_{c_1} - 10) \times \frac{\partial d_{111}}{\partial p_{c_1}} - 4d_{111} \times \frac{\partial d_{111}}{\partial p_{c_1}}] \\
&= - [d_{111} + (p_{c_1} - 10) \times (-.8) - 4d_{111} \times (-.8)] \\
&= - 428 + 2.1p_{s_1} + 4.16p_{c_1} - 2.52q_{s_1} - 2.1q_{c_1};
\end{aligned}$$

$$\begin{aligned}
F_{q_{c_1}} &= - \frac{\partial U_{CP_1}}{\partial q_{c_1}} \\
&= - \frac{\partial [(p_{c_1} - 10)d_{111} - 2(d_{111}^2 + q_{c_1}^2)]}{\partial q_{c_1}} \\
&= - [(p_{c_1} - 10) \times \frac{\partial d_{111}}{\partial q_{c_1}} - (4d_{111} \times \frac{\partial d_{111}}{\partial q_{c_1}} + 4q_{c_1})] \\
&= - [(p_{c_1} - 10).5 - 4d_{111} \times .5 - 4q_{c_1}] \\
&= 205 - p_{s_1} - 2.1p_{c_1} + 1.2q_{s_1} + 5q_{c_1};
\end{aligned}$$

$$\begin{aligned}
F_{p_{s_1}} &= - \frac{\partial U_{NP_1}}{\partial p_{s_1}} \\
&= - \frac{\partial [(p_{s_1} + 10)d_{111} - 2.2(d_{111} + q_{s_1}^2)]}{\partial p_{s_1}} \\
&= - [d_{111} + (p_{s_1} + 10) \times \frac{\partial d_{111}}{\partial p_{s_1}} - 2.2 \times \frac{\partial d_{111}}{\partial p_{s_1}}] \\
&= - [d_{111} + (p_{s_1} + 10) \times (-.5) - 2.2 \times (-.5)] \\
&= - 96.1 + p_{s_1} + .8p_{c_1} - .6q_{s_1} - .5q_{c_1};
\end{aligned}$$

$$\begin{aligned}
F_{q_{s_1}} &= - \frac{\partial U_{NP_1}}{\partial q_{s_1}} \\
&= - \frac{\partial [(p_{s_1} + 10)d_{111} - 2.2(d_{111} + q_{s_1}^2)]}{\partial q_{s_1}} \\
&= - [(p_{s_1} + 10) \times \frac{\partial d_{111}}{\partial q_{s_1}} - (2.2 \times \frac{\partial d_{111}}{\partial q_{s_1}} + 4.4q_{s_1})] \\
&= - [(p_{s_1} + 10) \times .6 - 2.2 \times .6 - 4.4q_{s_1}] \\
&= - 4.68 - .6p_{s_1} + 4.4q_{s_1}.
\end{aligned}$$

The Jacobian matrix of $-\nabla U(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1})$, denoted by $J(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1})$, is

$$J(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) = \begin{pmatrix} 4.16 & -2.1 & 2.1 & -2.52 \\ -2.1 & 5 & -1 & 1.2 \\ .8 & -.5 & 1 & -.6 \\ 0 & -.6 & 0 & 4.4 \end{pmatrix}.$$

Since the symmetric part of $J(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1})$, $(J^T + J)/2$, has only positive eigenvalues, which are: .43, 2.40, 4.03, and 7.70, the $F(X)$ in Example 4.1 (cf. (4.13)) is strongly monotone. Thus, according to Theorem 4.2, there exists a unique equilibrium, which is also globally exponentially stable for the utility gradient process. In the next section, I compute the equilibrium solution to this and other numerical examples.

4.2. The Algorithm and Numerical Examples

Note that, for computation purposes, I need to identify a discrete-time adjustment process or algorithm which will track the continuous-time process (4.14) until a stationary point is achieved (equivalently, an equilibrium point). In this section, I

use the Euler method (cf. Section 2.4), for computational procedure. Specifically, iteration τ of the Euler method is given by:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})). \quad (4.29)$$

4.2.1 Explicit Formulae for the Euler Method Applied to the Service-oriented Internet with Price and Quality Competition

The elegance of the Euler method for the computation of solutions to this network economic model of a service-oriented Internet can be seen in the following explicit formulae. Indeed, variational inequality problem (4.19) yields the following closed form expressions for the price and the quality of each content and network provider $i = 1, \dots, m; j = 1, \dots, n$:

$$p_{c_i}^{\tau+1} = \max \left\{ 0, p_{c_i}^{\tau} + a_{\tau} \left(\sum_{j=1}^n \sum_{k=1}^o d_{ijk} + \sum_{j=1}^n \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial p_{c_i}} \times (p_{c_i}^{\tau} - p_{t_j}) \right. \right. \\ \left. \left. - \frac{\partial CC_i(SCP_i, q_{c_i}^{\tau})}{\partial SCP_i} \cdot \frac{\partial SCP_i}{\partial p_{c_i}} \right) \right\}, \quad (4.30)$$

$$q_{c_i}^{\tau+1} = \max \left\{ 0, q_{c_i}^{\tau} + a_{\tau} \left(\sum_{j=1}^n \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial q_{c_i}} \times (p_{c_i}^{\tau} - p_{t_j}) - \frac{\partial CC_i(SCP_i, q_{c_i}^{\tau})}{\partial q_{c_i}} \right) \right\}, \quad (4.31)$$

$$p_{s_j}^{\tau+1} = \max \left\{ 0, p_{s_j}^{\tau} + a_{\tau} \left(\sum_{i=1}^m \sum_{k=1}^o d_{ijk} + \sum_{i=1}^m \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial p_{s_j}} \times (p_{s_j}^{\tau} + p_{t_j}) \right. \right. \\ \left. \left. - \frac{\partial CS_j(TNP_j, q_{s_j}^{\tau})}{\partial TNP_j} \cdot \frac{\partial TNP_j}{\partial p_{s_j}} \right) \right\}, \quad (4.32)$$

$$q_{s_j}^{\tau+1} = \max \left\{ 0, q_{s_j}^{\tau} + a_{\tau} \left(\sum_{i=1}^m \sum_{k=1}^o \frac{\partial d_{ijk}}{\partial q_{s_j}} \times (p_{s_j}^{\tau} + p_{t_j}) - \frac{\partial CS_j(TNP_j, q_{s_j}^{\tau})}{\partial q_{s_j}} \right) \right\}. \quad (4.33)$$

Note that, all the functions to the right of the equal signs in (4.30)-(4.33) are evaluated at their respective variables computed at the τ -th iteration.

I now provide the convergence result. The proof is direct from Theorem 5.8 in Nagurney and Zhang (1996).

Theorem 4.3: Convergence

In the service-oriented Internet network economic problem, assume that $F(X) = -\nabla U(p_c, q_c, p_s, q_s)$ is strongly monotone. Also, assume that F is uniformly Lipschitz continuous. Then, there exists a unique equilibrium price and quality pattern $(p_c^, q_c^*, p_s^*, q_s^*) \in \mathcal{K}$ and any sequence generated by the Euler method as given by (4.30)-(4.33), where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^\infty a_\tau = \infty$, $a_\tau > 0$, $a_\tau \rightarrow 0$, as $\tau \rightarrow \infty$ converges to $(p_c^*, q_c^*, p_s^*, q_s^*)$ satisfying (4.19).*

I implemented the Euler method (cf. (4.30) - (4.33)) to compute solutions to service-oriented Internet network economic problems in Matlab. The Euler method was deemed to have converged if, at a given iteration, the absolute value of the difference of each price and each quality level differed from its respective value at the preceding iteration by no more than $\epsilon = 10^{-6}$. The sequence $\{a_\tau\}$ used was: $.1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \dots)$. I initialized the algorithm by setting $p_{c_i}^0 = q_{c_i}^0 = p_{s_j}^0 = q_{s_j}^0 = 0, \quad \forall i, j$.

Example 4.1 Revisited

I first applied the Euler method to compute the equilibrium prices and quality levels for Example 4.1. The Euler method required 136 iterations for convergence to the computed equilibrium:

$$p_{c_1}^* = 94.50, \quad q_{c_1}^* = 2.51 \quad p_{s_1}^* = 24.40, \quad q_{s_1}^* = 4.38,$$

with an incurred demand of $d_{111} = 16.10$. The utility/profit of CP_1 is 829.32 and that of NP_1 is 475.70.

If I change p_{t_1} to 0, then the new equilibrium is:

$$p_{c_1}^* = 35.39, \quad q_{c_1}^* = 2.59, \quad p_{s_1}^* = 87.14, \quad q_{s_1}^* = 4.52,$$

with an incurred demand of $d_{111} = 16.08$. The utility/profit of CP_1 is now 882.01 and that of NP_1 is 505.92.

Hence, in this example, NP_1 would be better off in terms of his profit, if he does not charge CP_1 , that is, $p_{t_1} = 0$ since the users are more sensitive to the content provider's price.

Example 4.2

In Example 4.2, there are 2 content providers, CP_1 and CP_2 , a single network provider, NP_1 , and users at a single demand market, u_1 , as depicted in Figure 4.4.

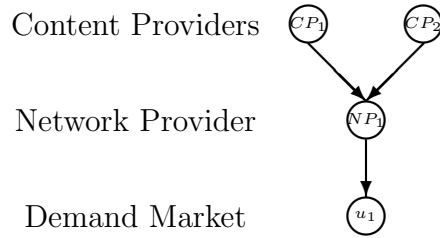


Figure 4.4. Network Topology for Example 4.2

The data are as follows. The demand functions are:

$$d_{111} = 100 - 1.6p_{c_1} + .65p_{c_2} - 1.35p_{s_1} + 1.2q_{c_1} - .42q_{c_2} + 1.54q_{s_1},$$

$$d_{211} = 112 + .65p_{c_1} - 1.5p_{c_2} - 1.35p_{s_1} - .42q_{c_1} + 1.3q_{c_2} + 1.54q_{s_1}.$$

The cost functions of the content providers are:

$$CC_1 = 1.7q_{c_1}^2, \quad CC_2 = 2.4q_{c_2}^2$$

and their utilities/profit functions are:

$$U_{CP_1} = (p_{c_1} - p_{t_1})d_{111} - CC_1, \quad U_{CP_2} = (p_{c_2} - p_{t_1})d_{211} - CC_2.$$

The cost function of the network provider is:

$$CS_1 = 2.1(d_{111} + d_{211} + q_{s_1}^2),$$

and its utility/profit function is:

$$U_{NP_1} = (p_{s_1} + p_{t_1})(d_{111} + d_{211}) - CS_1.$$

p_{t_1} is assumed to be 10.

The Jacobian matrix of $-\nabla U(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1})$, denoted by $J(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1})$, is

$$J(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1}) = \begin{pmatrix} 3.2 & -1.2 & -.65 & .42 & 1.35 & -1.54 \\ -1.2 & 3.4 & 0 & 0 & 0 & 0 \\ -.65 & .42 & 3 & -1.3 & 1.35 & -1.54 \\ 0 & 0 & -1.3 & 4.8 & 0 & 0 \\ .95 & -.78 & .85 & -.88 & 5.4 & -3.08 \\ 0 & 0 & 0 & 0 & -3.08 & 4.2 \end{pmatrix}.$$

Since the symmetric part of $J(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1})$, $(J^T + J)/2$, has only positive eigenvalues, which are 1.52, 1.61, 2.37, 4.22, 5.61, and 8.67, the $F(X)$ in Example 4.2 is strongly monotone. Thus, according to Theorem 4.2, there exists a unique equilibrium, which is also globally exponentially stable for the utility gradient process.

The Euler method converged in 2341 iterations to the following solution:

$$p_{c_1}^* = 51.45, \quad p_{c_2}^* = 56.75, \quad p_{s_1}^* = 42.64,$$

$$q_{c_1}^* = 14.63, \quad q_{c_2}^* = 12.66, \quad q_{s_1}^* = 37.06,$$

with incurred demands of:

$$d_{111} = 66.32, \quad d_{211} = 70.13.$$

The utility/profit of CP_1 is 2385.21 and of CP_2 : 2894.58. The utility/profit of NP_1 is 4011.92.

Example 4.3

In Example 4.3, there is a single content provider, CP_1 , two network providers, NP_1 and NP_2 , and a single demand market, u_1 , as depicted in Figure 4.5.

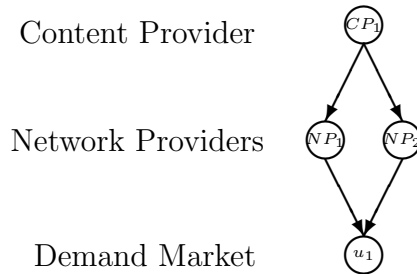


Figure 4.5. Network Topology for Example 4.3

The demand functions are:

$$d_{111} = 100 - 1.7p_{c_1} - 1.5p_{s_1} + .8p_{s_2} + 1.76q_{c_1} + 1.84q_{s_1} - .6q_{s_2},$$

$$d_{121} = 100 - 1.7p_{c_1} + .8p_{s_1} - 1.8p_{s_2} + 1.76q_{c_1} - .6q_{s_1} + 1.59q_{s_2}.$$

The cost function of CP_1 is:

$$CC_1 = 1.5(d_{111} + d_{121} + q_{c_1}^2),$$

and its utility/profit function is:

$$U_{CP_1} = (p_{c_1} - p_{t_1})d_{111} + (p_{c_1} - p_{t_2})d_{121} - CC_1.$$

The network providers' cost functions are:

$$CS_1 = 1.8(d_{111} + q_{s_1}^2), \quad CS_2 = 1.7(d_{121} + q_{s_2}^2),$$

with their utility/profit functions given by:

$$U_{NP_1} = (p_{s_1} + p_{t_1})d_{111} - CS_1, \quad U_{NP_2} = (p_{s_2} + p_{t_2})d_{121} - CS_2.$$

I set $p_{t_1} = 10$ and $p_{t_2} = 7$.

The Jacobian matrix of $-\nabla U(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, denoted by $J(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, is

$$J(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2}) = \begin{pmatrix} 6.8 & -3.52 & .7 & -1.24 & 1 & -.99 \\ -3.52 & 3 & 0 & 0 & 0 & 0 \\ 1.7 & -1.76 & 3 & -1.84 & -0.8 & .6 \\ 0 & 0 & -1.84 & 3.6 & 0 & 0 \\ 1.7 & -1.76 & -.8 & .6 & 3.6 & -1.59 \\ 0 & 0 & 0 & 0 & -1.59 & 3.4 \end{pmatrix}.$$

The symmetric part of $J(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, $(J^T + J)/2$, has only positive eigenvalues, which are .66, 1.32, 1.84, 3.96, 5.85, and 9.77. Hence, the $F(X)$ in Example 4.3 is also strongly monotone and I know from Theorem 4.2, that there exists a unique equilibrium, which is also globally exponentially stable for the utility gradient process.

The Euler method required 120 iterations for convergence. The computed equilibrium solution is:

$$p_{c_1}^* = 64.90, \quad p_{s_1}^* = 57.98, \quad p_{s_2}^* = 43.24,$$

$$q_{c_1}^* = 64.41, \quad q_{s_1}^* = 33.82, \quad q_{s_2}^* = 22.70,$$

with incurred demands of:

$$d_{111} = 99.28, \quad d_{121} = 87.38.$$

The utility/profit of CP_1 is 4006.15. The utilities/profits of NP_1 and NP_2 are 4511.38, and 3366.23, respectively.

Example 4.4

In Example 4.4, there are two content providers, CP_1 and CP_2 , two network providers, NP_1 and NP_2 , and two markets of users, u_1 and u_2 , as depicted in Figure 4.6.

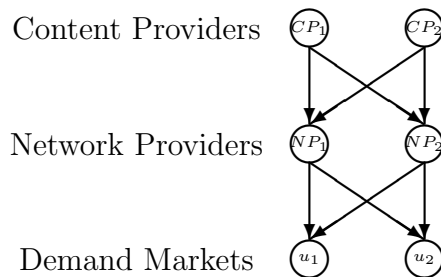


Figure 4.6. Network Topology for Example 4.4

The demand functions are:

$$d_{111} = 100 - 2.1p_{c_1} + .5p_{c_2} - 2.3p_{s_1} + .6p_{s_2} + .63q_{c_1} - .4q_{c_2} + .62q_{s_1} - .4q_{s_2},$$

$$d_{112} = 112 - 2.2p_{c_1} + .5p_{c_2} - 2.4p_{s_1} + .6p_{s_2} + .75q_{c_1} - .4q_{c_2} + .56q_{s_1} - .4q_{s_2},$$

$$d_{121} = 100 - 2.1p_{c_1} + .5p_{c_2} + .6p_{s_1} - 2.2p_{s_2} + .63q_{c_1} - .4q_{c_2} - .4q_{s_1} + .59q_{s_2},$$

$$d_{122} = 112 - 2.2p_{c_1} + .5p_{c_2} + .6p_{s_1} - 2.1p_{s_2} + .75q_{c_1} - .4q_{c_2} - .4q_{s_1} + .68q_{s_2},$$

$$d_{211} = 110 + .5p_{c_1} - 2.3p_{c_2} - 2.3p_{s_1} + .6p_{s_2} - .4q_{c_1} + .76q_{c_2} + .62q_{s_1} - .4q_{s_2},$$

$$d_{212} = 104 + .5p_{c_1} - 2.05p_{c_2} - 2.4p_{s_1} + .6p_{s_2} - .4q_{c_1} + .61q_{c_2} + .56q_{s_1} - .4q_{s_2},$$

$$d_{221} = 110 + .5p_{c_1} - 2.3p_{c_2} + .6p_{s_1} - 2.2p_{s_2} - .4q_{c_1} + .76q_{c_2} - .4q_{s_1} + .59q_{s_2},$$

$$d_{222} = 104 + .5p_{c_1} - 2.05p_{c_2} + .6p_{s_1} - 2.1p_{s_2} - .4q_{c_1} + .61q_{c_2} - .4q_{s_1} + .68q_{s_2}.$$

The cost functions of the content providers are:

$$CC_1 = 3.7(q_{c_1}^2), \quad CC_2 = 5.1(q_{c_2}^2),$$

and their profit functions are, respectively:

$$U_{CP_1} = (p_{c_1} - p_{t_1})(d_{111} + d_{112}) + (p_{c_1} - p_{t_2})(d_{121} + d_{122}) - CC_1,$$

$$U_{CP_2} = (p_{c_2} - p_{t_1})(d_{211} + d_{212}) + (p_{c_2} - p_{t_2})(d_{221} + d_{222}) - CC_2.$$

The network providers' cost functions are:

$$CS_1 = 4.1(d_{111} + d_{112} + d_{211} + d_{212} + q_{s_1}^2), \quad CS_2 = 3.9(d_{121} + d_{122} + d_{221} + d_{222} + q_{s_2}^2),$$

and their profit functions are:

$$U_{NP_1} = (p_{s_1} + p_{t_1})(d_{111} + d_{112} + d_{211} + d_{212}) - CS_1,$$

$$U_{NP_2} = (p_{s_2} + p_{t_2})(d_{121} + d_{122} + d_{221} + d_{222}) - CS_2.$$

I set $p_{t_1} = 23$, and $p_{t_2} = 22$.

The Jacobian matrix of $-\nabla U(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, denoted by

$J(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, is

$$J = \begin{pmatrix} 17.2 & -2.76 & -2 & 1.6 & 3.5 & -0.38 & 3.1 & -0.47 \\ -2.76 & 7.4 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 1.6 & 17.4 & -2.74 & 3.5 & -0.38 & 3.1 & -0.47 \\ 0 & 0 & -2.74 & 10.2 & 0 & 0 & 0 & 0 \\ 3.3 & -0.58 & 3.35 & -0.57 & 18.8 & -2.36 & -2.4 & 1.6 \\ 0 & 0 & 0 & 0 & -2.36 & 8.2 & 0 & 0 \\ 3.3 & -0.58 & 3.35 & -0.57 & -2.4 & 1.6 & 17.2 & -2.54 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2.54 & 7.8 \end{pmatrix}.$$

The symmetric part of $J(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, $(J^T + J)/2$, has only positive eigenvalues, which are 6.54, 7.01, 7.57, 8.76, 10.24, 20.39, 20.94, and 22.75. Hence, the $F(X)$ in Example 4.4 is also strongly monotone and I know that the equilibrium solution is unique. The Euler method required 189 iterations for convergence, yielding:

$$p_{c_1}^* = 41.52, \quad p_{c_2}^* = 40.93, \quad p_{s_1}^* = 0.0, \quad p_{s_2}^* = 0.58,$$

$$q_{c_1}^* = 7.09, \quad q_{c_2}^* = 4.95, \quad q_{s_1}^* = 5.44, \quad q_{s_2}^* = 6.08,$$

with incurred demands of:

$$d_{111} = 37.04, \quad d_{112} = 45.42, \quad d_{121} = 35.91, \quad d_{122} = 45.21,$$

$$d_{211} = 38.83, \quad d_{212} = 42.00, \quad d_{221} = 37.70, \quad d_{222} = 41.79.$$

The profits of the content providers are, respectively, 2924.52 and 2828.79, and that of the network providers: 2964.97 and 2855.11.

Please refer to Figures 4.7, 4.8, and 4.9 to view the trajectories of the prices and the quality levels generated by the Euler method at iterations 0, 10, 20, ..., 180, 189.

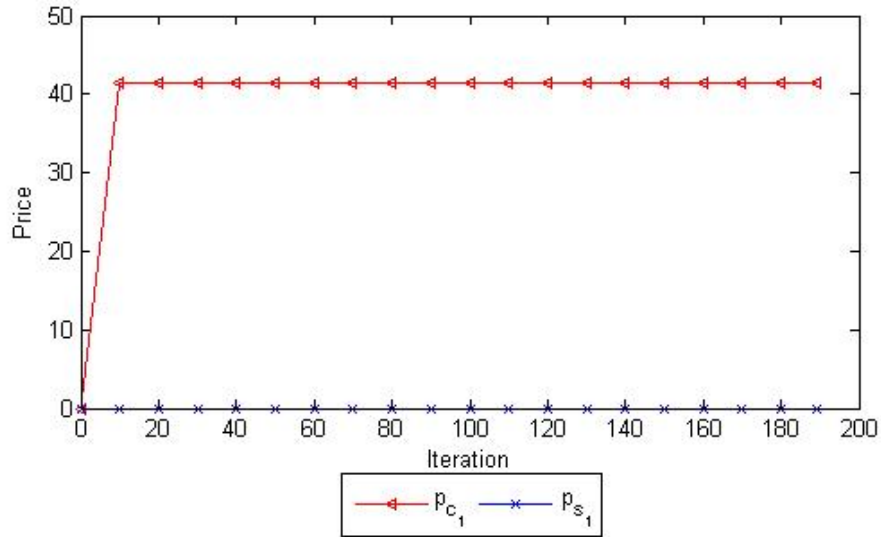


Figure 4.7. Prices of Content Provider 1 and Network Provider 1 for Example 4.4

4.3. Summary and Conclusions

In this chapter, I developed a new dynamic network economic model of a service-oriented Internet. The model handles price and quality competition among the content providers, who provide Internet services, and among the network providers, who

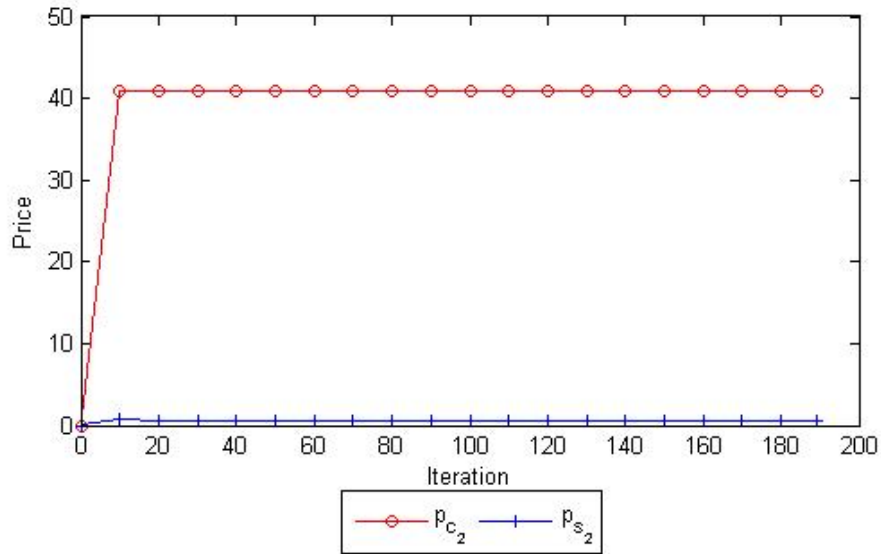


Figure 4.8. Prices of Content Provider 2 and Network Provider 2 for Example 4.4

transport the Internet services. Consumers' direct demand functions that depend on the prices and the quality levels of both content and network providers, are utilized rather than their inverses, which allows for prices as strategic variables. The framework yields insights into the evolutionary processes of quality selection and the pricing of Internet services.

Specifically, the projected dynamical systems model that I constructed provides a continuous-time adjustment process of the content providers' and the network providers' prices and quality levels, and guarantees that prices and quality levels remain nonnegative, as required by the constraints. The set of equilibrium/stationary points coincides with the set of solutions to the associated variational inequality problem. Qualitative properties, including stability analysis results, are also provided.

I proposed the Euler method, which provides a discretization of the continuous-time adjustment process and yields closed form expressions for the prices and the quality levels at each iteration step. This algorithm also tracks the values of the prices and

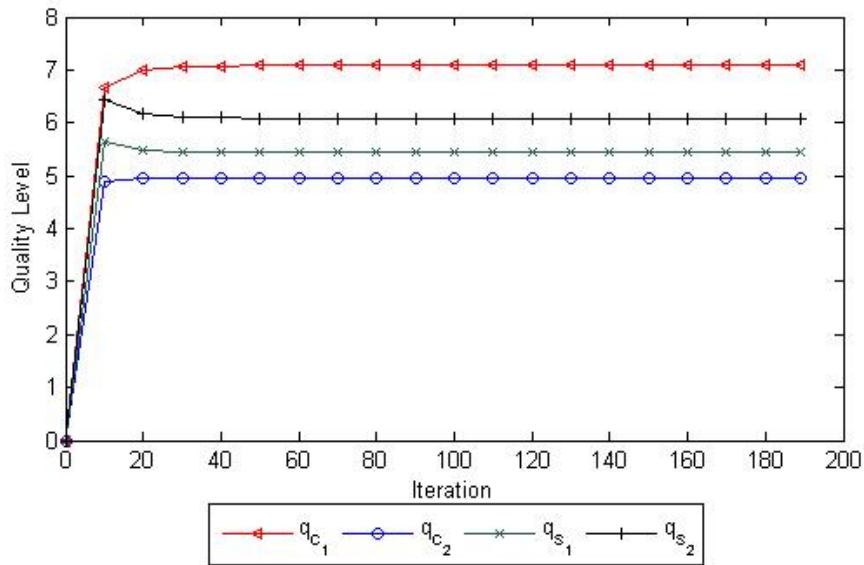


Figure 4.9. Quality Levels of Content Providers and Network Providers for Example 4.4

quality levels over time until the equilibrium point is achieved. Convergence results were also given. The generality and practicality of this model and the computational procedure are illustrated through several numerical examples.

The NGI, as an exciting new area of research, is full of additional questions for investigation, some of which are identified below.

- The price mechanisms used in my model are usage-based with bandwidth-based pricing for the content and network providers. What would be the equilibrium outcomes if a flat-rate or a two-part tariff pricing mechanism would be applied instead? Would such pricing mechanisms increase the users' demand?
- Since long-term contracts lock in consumers, and have low flexibility, it would be interesting to consider short-term contracts, which might enable users to select among the service offerings from different providers, in a more dynamic manner. How would the pricing dynamics change in an NGI with short-term contracts?

- In this model, content providers and network providers have no restrictions on their services, with the exception that the prices that they charge and their service quality levels must be nonnegative. However, providers in the future Internet might be faced with some additional restrictions, that is, constraints. For example, what would be the dynamics and the equilibrium prices and quality levels for a content provider with a production capacity limitation? To what extent would the equilibrium price and quality level of a network provider with capacity restrictions for data transmission change in comparison with the case with no such limitations? Presently, I handled capacity limitations through the nonlinearity of the underlying cost functions, which can capture “congestion”.
- I might wish to consider an upper bound or a non-zero lower bound for the quality level of a content or network provider’s services. A non-zero, but positive, lower bound on the quality level, for example, might occur due to an imposed governmental regulation.
- Empirical studies could be used to validate this model and to yield a parameterization of the model that matches a practical NGI scenario.

I believe that the framework constructed in this chapter can serve as the foundation to address the above issues in future research.

According to the suggestion for future work, I focus in the next chapter on the pricing model in a service-oriented Internet which offers flexible contracts to users, so that they can select the services according to their preferred level of quality of service, price, and contract duration.

CHAPTER 5

A DIFFERENTIATED SERVICE-ORIENTED INTERNET NETWORK ECONOMIC MODEL WITH DURATION-BASED CONTRACTS

In contemplating the current Internet limitations, the ChoiceNet project as a new architecture for the Internet of the future aims to provide more options and flexibility for all entities in the market by offering more choices to support and encourage innovation. It is not unreasonable to expect having different levels of quality of service and flexible contract duration for connectivity in the future Internet.

In this dissertation, I considered quality of service for content and network providers in their pricing competition and analyzed this game theory model in both static (in Chapter 3) and dynamic versions (in Chapter 4) to describe the dynamic adjustment process for setting equilibrium prices and quality levels.

In this chapter, I develop and analyze a pricing model for a service-oriented Internet in which contract duration is not fixed anymore and can change according to customer preferences. In this model, network providers compete in usage service rates, quality levels, and duration-based contracts. Also, the governing equilibrium conditions of the model which yield the service usage, quality levels and durations are formulated as a variational inequality problem.

This chapter is based on Nagurney et al. (2015b) and is organized as follows. In Section 5.1, I develop the competitive duration-based contract pricing model for a service-oriented Internet network with differentiated services and derive the variational

inequality formulation. I also provide some qualitative properties of the equilibrium pattern. In Section 5.2, I present the computational scheme, which has nice features for ease of implementation, and compute solutions to a series of numerical examples in Section 5.3. Then, I summarize the results and present my conclusions in Section 5.4.

5.1. The Competitive Duration-Based Differentiated Service-Oriented Internet Game Theory Model

The focus of this game theory model is on duration-based contracts associated with network provision. I assume m network providers, with a typical provider denoted by i ; $i = 1, 2, \dots, m$ and n users/demand markets, with a typical one denoted by j ; $j = 1, 2, \dots, n$, as shown in Figure 5.1. A demand market may correspond to an individual, a household, and/or a business. The users reveal their preferences for the network providers' services through the demand price functions, which depend on the service usage rates, the quality of services, and the contract durations. I further detail the model's underlying functions and their generality below.

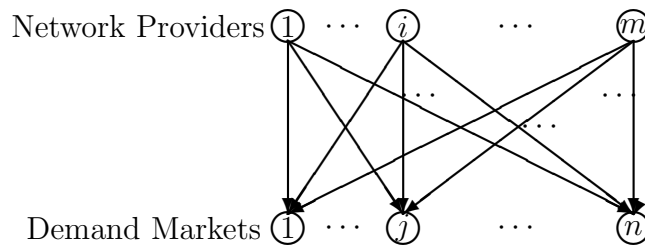


Figure 5.1. The Bipartite Structure of the Competition Among the Network Providers

Let p_{ij} denote the price for transmission of bit units of data (can be individual ones, Megabits, etc.) from network provider i to demand market j , for the selected number of bit units per unit time (the reserved usage rate), at the quality level and the contract duration. Let d_{ij} represent the number of bit units per unit time contracted

for between i and j , corresponding to the reserved usage rate, and let q_{ij} denote the contracted quality of service, which ranges between 0 and 100, with 100 denoting perfect quality. T_{ij} represents the duration of the contract between network provider i and demand market j in units of time. Henceforth, I simply refer to usage rate with the understanding that I mean a reserved usage rate. Indeed, although the consumer may not use up all of his usage rate over the contract duration, the network provider still needs to plan as if the user will in order to provide the desired quality of service and to manage the network resources accordingly. In Section 5.2, I provide specific units in the context of the numerical examples for the prices and decision variables. Here, I consider a general framework that can be adapted to any currency, time unit, etc., as needed.

Each network provider i ; $i = 1, \dots, m$, is faced, due to technological limitations, with a maximum usage rate to a particular demand market j , \bar{d}_{ij} , in terms of the number of megabits per time unit, and may also impose a nonnegative minimum, \underline{d}_{ij} , so that

$$\underline{d}_{ij} \leq d_{ij} \leq \bar{d}_{ij}, \quad \forall i, j. \quad (5.1)$$

Also, due to technological limitations, network provider i may have a maximum level of quality \bar{q}_{ij} that he can offer, where $\bar{q}_{ij} \leq 100$. Hence,

$$0 \leq q_{ij} \leq \bar{q}_{ij}, \quad \forall i, j. \quad (5.2)$$

A single parameter with quality, subject to a bound, as above, was also used in El Azouzi, Altman, and Wynter (2003). Finally, the contract durations for a given network provider and demand market pair may also be bounded, with \bar{T}_{ij} denoting the upper bound and \underline{T}_{ij} the nonnegative lower bound, so that

$$\underline{T}_{ij} \leq T_{ij} \leq \bar{T}_{ij}, \quad \forall i, j. \quad (5.3)$$

For example, some service providers may decide to have a positive lower bound for the contract duration for ease of management. I group the usage rates for service into the vector $d \in R_+^{mn}$, the quality levels into the vector $q \in R_+^{mn}$, and the contract durations into the vector $T \in R_+^{mn}$.

The price of i 's service provision to j , p_{ij} , is a function of the reserved usage rates, the quality levels, and the durations of the contracts, as follows

$$p_{ij} = p_{ij}(d, q, T), \quad \forall i, j. \quad (5.4)$$

Note that, according to (5.4), the price of transmission between (i, j) depends not only on the usage per unit time in terms of the number of bit units, the quality of transmission between (i, j) , and the contract duration, but also on the values of these variables associated with other network providers and with other demand markets. This functional form also captures that users should be aware of the services offered by the network providers at other demand markets. Indeed, I argue for transparency in ChoiceNet, so that users can make informed decisions.

I assume that the demand price function for each pair (i, j) is monotonically decreasing in its reserved service usage rate and in the duration of the contract between (i, j) but increasing in terms of service quality between the pair.

Each network provider incurs a cost for delivering the service at a specific quality and usage rate and maintaining the quality within the contract duration. I assume that the cost is a convex function (MacKie-Mason and Varian (1995) and Roughgarden (2005)) of the usage rates, the quality levels, and the durations of the contracts. The cost c_{ij} incurred by network provider i for serving j is of the form:

$$c_{ij} = c_{ij}(d, q, T), \quad \forall i, j. \quad (5.5)$$

The demand price functions (5.4) and the cost functions (5.5) are assumed to be continuous and continuously differentiable. The generality of the expressions in (5.4) and (5.5) allows for modeling and application flexibility. Moreover, the cost functions in (5.5) reveal that the cost on a “link” as depicted in Figure 5.1 can depend not only on the usage on that link but also on those on the other links. Since there may be competition for network resources, such cost functions can capture competition, albeit at a high level, among the network providers during transmission.

The strategic variables of network provider i are its service usage rates, the quality levels, and the contract durations $\{d_i, q_i, \text{ and } T_i\}$, where $d_i = (d_{i1}, \dots, d_{in})$, $q_i = (q_{i1}, \dots, q_{in})$, and $T_i = (T_{i1}, \dots, T_{in})$.

The utility or profit of network provider i is the difference between his revenue and his total cost and is given by the expression:

$$U_i = \sum_{j=1}^n p_{ij} T_{ij} d_{ij} - \sum_{j=1}^n c_{ij}, \quad \forall i. \quad (5.6)$$

In (5.6), the first term after the equal sign is the total revenue and the second term is the total cost of network provider i .

Let K^i denote the feasible set corresponding to network provider i , where $K^i \equiv \{(d_i, q_i, T_i) \mid (5.1), (5.2), \text{ and } (5.3) \text{ hold}\}$ and define $K \equiv \prod_{i=1}^m K^i$. The network providers compete in a noncooperative manner in the sense of Nash (1950, 1951), each one seeking to maximize his profit. I wish to determine the vectors of the equilibrium service usage rates, quality levels, and contract durations (d^*, q^*, T^*) , according to the definition below.

Definition 5.1: The Differentiated Service-Oriented Internet Network Equilibrium with Contract Durations

A service usage rate, quality, and contract duration pattern $(d^*, q^*, T^*) \in K$ is an equilibrium if, for each network provider i ; $i = 1, \dots, m$:

$$U_i(d_i^*, q_i^*, T_i^*, \hat{d}_i^*, \hat{q}_i^*, \hat{T}_i^*) \geq U_i(d_i, q_i, T_i, \hat{d}_i^*, \hat{q}_i^*, \hat{T}_i^*), \quad \forall (d_i, q_i, T_i) \in K^i, \quad (5.7)$$

where

$$\hat{d}_i^* = (d_1^*, \dots, d_{i-1}^*, d_{i+1}^*, \dots, d_m^*),$$

$$\hat{q}_i^* = (q_1^*, \dots, q_{i-1}^*, q_{i+1}^*, \dots, q_m^*),$$

and

$$\hat{T}_i^* = (T_1^*, \dots, T_{i-1}^*, T_{i+1}^*, \dots, T_m^*). \quad (5.8)$$

According to (5.7), an equilibrium is established if no network provider can unilaterally improve his profit by selecting an alternative vector of reserved service usage rates, quality levels, and durations of contracts, given the decisions of the other network providers.

5.1.1 Variational Inequality Formulation

Variational inequalities have been used to formulate a spectrum of problems arising in engineering, operations research, management sciences, transportation science, economics, and finance (cf. Nagurney (1999), Nagurney (2006a), Nagurney and Qiang (2009), and references therein). I now present a variational inequality formulation of the service-oriented Internet network equilibrium.

Theorem 5.1: Variational Inequality Formulation

Assume that the profit function $U_i(d, q, T)$ is concave with respect to the variables $\{d_{i1}, \dots, d_{in}\}$, $\{q_{i1}, \dots, q_{in}\}$, and $\{T_{i1}, \dots, T_{in}\}$ and is continuous and continuously differentiable for each network provider $i; = 1, \dots, m$. Then, $(d^*, q^*, T^*) \in K$ is an Internet network equilibrium service usage rate, quality, and contract duration pattern according to Definition 5.1 if and only if it satisfies the variational inequality:

$$\begin{aligned} & - \sum_{i=1}^m \sum_{j=1}^n \frac{\partial U_i(d^*, q^*, T^*)}{\partial d_{ij}} \times (d_{ij} - d_{ij}^*) - \sum_{i=1}^m \sum_{j=1}^n \frac{\partial U_i(d^*, q^*, T^*)}{\partial q_{ij}} \times (q_{ij} - q_{ij}^*) \\ & - \sum_{i=1}^m \sum_{j=1}^n \frac{\partial U_i(d^*, q^*, T^*)}{\partial T_{ij}} \times (T_{ij} - T_{ij}^*) \geq 0, \quad \forall (d, q, T) \in K, \end{aligned} \quad (5.9)$$

or, equivalently, $(d^*, q^*, T^*) \in K$ is an equilibrium service usage rate, quality, and contract duration pattern if and only if it satisfies the variational inequality:

$$\begin{aligned} & \sum_{i=1}^m \sum_{j=1}^n \left[\sum_{l=1}^n \frac{\partial c_{il}(d^*, q^*, T^*)}{\partial d_{ij}} - p_{ij}(d^*, q^*, T^*) \times T_{ij}^* - \sum_{l=1}^n \frac{\partial p_{il}(d^*, q^*, T^*)}{\partial d_{ij}} \times d_{il}^* \times T_{il}^* \right] \times (d_{ij} - d_{ij}^*) \\ & + \sum_{i=1}^m \sum_{j=1}^n \left[\sum_{l=1}^n \frac{\partial c_{il}(d^*, q^*, T^*)}{\partial q_{ij}} - \sum_{l=1}^n \frac{\partial p_{il}(d^*, q^*, T^*)}{\partial q_{ij}} \times d_{il}^* \times T_{il}^* \right] \times (q_{ij} - q_{ij}^*) \\ & + \sum_{i=1}^m \sum_{j=1}^n \left[\sum_{l=1}^n \frac{\partial c_{il}(d^*, q^*, T^*)}{\partial T_{ij}} - p_{ij}(d^*, q^*, T^*) \times d_{ij}^* - \sum_{l=1}^n \frac{\partial p_{il}(d^*, q^*, T^*)}{\partial T_{ij}} \times d_{il}^* \times T_{il}^* \right] \\ & \quad \times (T_{ij} - T_{ij}^*) \geq 0, \quad \forall (d, q, T) \in K. \end{aligned} \quad (5.10)$$

Proof: (5.9) follows directly from Gabay and Moulin (1980) and Dafermos and Nagurney (1987). In order to obtain variational inequality (5.10) from variational inequality (5.9), I note that:

$$-\frac{\partial U_i}{\partial d_{ij}} = \left[\sum_{l=1}^n \frac{\partial c_{il}}{\partial d_{ij}} - p_{ij} \times T_{ij} - \sum_{l=1}^n \frac{\partial p_{il}}{\partial d_{ij}} \times d_{il} \times T_{il} \right]; \quad i = 1, \dots, m; j = 1, \dots, n, \quad (5.11)$$

$$-\frac{\partial U_i}{\partial q_{ij}} = \left[\sum_{l=1}^n \frac{\partial c_{il}}{\partial q_{ij}} - \sum_{l=1}^n \frac{\partial p_{il}}{\partial q_{ij}} \times d_{il} \times T_{il} \right]; \quad i = 1, \dots, m; j = 1, \dots, n, \quad (5.12)$$

and

$$-\frac{\partial U_i}{\partial T_{ij}} = \left[\sum_{l=1}^n \frac{\partial c_{il}}{\partial T_{ij}} - p_{ij} \times d_{ij} - \sum_{l=1}^n \frac{\partial p_{il}}{\partial T_{ij}} \times d_{il} \times T_{il} \right]; \quad i = 1, \dots, m; j = 1, \dots, n. \quad (5.13)$$

Multiplying the right-most expression in (5.11) by $(d_{ij} - d_{ij}^*)$ and summing the resultant over all i and all j ; multiplying the right-most expression in (5.12) by $(q_{ij} - q_{ij}^*)$ and summing the resultant over all i and j , and, similarly, multiplying the right-most expression in (5.13) by $(T_{ij} - T_{ij}^*)$ and summing the resultant over all i and j yields (5.10). The conclusion follows. \square

I now put variational inequality (5.10) into standard form (cf. (2.1)), that is: Determine $X^* \in \mathcal{K} \subset R^N$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \quad (5.14)$$

where F is a given continuous function from \mathcal{K} to R^N , and \mathcal{K} is a closed and convex set.

I define the mn -dimensional vectors $X \equiv (d, q, T)$ and $F(X) \equiv (F^1(X), F^2(X), F^3(X))$ with the (i, j) -th component, F_{ij}^1 , of $F^1(X)$ given by

$$F_{ij}^1(X) \equiv -\frac{\partial U_i}{\partial d_{ij}}, \quad (5.15)$$

the (i, j) -th component, F_{ij}^2 , of $F^2(X)$ given by

$$F_{ij}^2(X) \equiv -\frac{\partial U_i}{\partial q_{ij}}, \quad (5.16)$$

and the (i, j) -th component, F_{ij}^3 , of $F^3(X)$ given by

$$F_{ij}^3(X) \equiv -\frac{\partial U_i}{\partial T_{ij}}, \quad (5.17)$$

and with the feasible set $\mathcal{K} \equiv K$. Then, clearly, variational inequality (5.10) can be put into standard form (5.14).

The next theorem is immediate from the standard theory of variational inequalities (Theorem 2.2) since the feasible set \mathcal{K} in this model is compact and the function F that enters variational inequality (5.14) for this model under the imposed assumptions is continuous.

Theorem 5.2: Existence

A solution X^ to variational inequality (5.14) is guaranteed to exist.*

Theorem 5.3: Uniqueness

If $F(X)$ is strictly monotone, that is:

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, X^1 \neq X^2, \quad (5.18)$$

then the solution to variational inequality (5.14) is unique.

Proof: Follows from the standard theory of variational inequalities.

$F(X)$ is strictly monotone if $\nabla F(X)$ is positive-definite over the feasible set \mathcal{K} .

5.2. Explicit Formulae for the Euler Method Applied to the Internet Network Model with Contract Durations

The feasible set underlying variational inequality (5.10) consists of box-type constraints, a feature that I exploit for computational purposes. Specifically, for the computation of the equilibrium pattern, I apply the Euler method (cf Section 2.4), which has been used to compute solutions to numerous network equilibrium problems (see, e.g., Nagurney and Zhang (1996), Cruz (2008), Nagurney et al. (2014a), and Toyasaki, Daniele, and Wakolbinger (2014)).

In particular, the Euler method yields, at each iteration, explicit formulae for the solution of the variational inequality problem (5.10) for the service usage rates, quality levels, and contract durations, respectively:

$$d_{ij}^{\tau+1} = \max \left\{ \underline{d}_{ij}, \min \{ \bar{d}_{ij}, d_{ij}^{\tau} - a_{\tau} F_{ij}^1(X^{\tau}) \} \right\}, \quad (5.19)$$

$$q_{ij}^{\tau+1} = \max \left\{ 0, \min \{ \bar{q}_{ij}, q_{ij}^{\tau} - a_{\tau} F_{ij}^2(X^{\tau}) \} \right\}, \quad (5.20)$$

$$T_{ij}^{\tau+1} = \max \left\{ \underline{T}_{ij}, \min \{ \bar{T}_{ij}, T_{ij}^{\tau} - a_{\tau} F_{ij}^3(X^{\tau}) \} \right\}. \quad (5.21)$$

Note that all the functions to the right of the equal signs in (5.19) - (5.21) are evaluated at their respective variables computed at the τ -th iteration. This algorithm can also be interpreted as a discrete-time adjustment process. I now provide the convergence result. The proof is direct from Theorem 5.8 in Nagurney and Zhang (1996).

Theorem 5.4: Convergence

In the differentiated service-oriented Internet network game theory model with contract durations, if $F(X) = -\nabla U(d, q, T)$ is strictly monotone at an equilibrium pattern and

F is uniformly Lipschitz continuous, then there exists a unique equilibrium service usage rate, quality, and contract duration pattern $(d^, q^*, T^*) \in K$ and any sequence generated by the Euler method as given by (5.19) - (5.21), where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \rightarrow 0$, as $\tau \rightarrow \infty$ converges to (d^*, q^*, T^*) .*

5.3. Numerical Examples

In this section, I present numerical examples which were solved via the Euler method (cf. (5.19) - (5.21)). I implemented the Euler method in Matlab on a VAIO S Series laptop with an Intel Core i7 processor and 12 GB RAM. The algorithm was considered to have converged if, at a given iteration, the absolute value of the difference of each variable differed from its respective value at the preceding iteration by no more than $\epsilon = 10^{-4}$. The sequence $\{a_\tau\}$ was: $(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \dots)$. I initialized the algorithm for all the examples by setting $d_{ij}^0 = \underline{d}_{ij}$; $q_{ij}^0 = \underline{q}_{ij}$; $T_{ij}^0 = \underline{T}_{ij}$, $\forall i, j$.

The examples begin with a simple network of two network providers and a single demand market (user), which I then extend to two network providers and two demand markets, and, finally, to two network providers and three demand markets. In the numerical examples, the contract durations, T_{ij} s, are in hours, the reserved service usage rates, d_{ij} s, are in Megabits/second, and, to simplify the presentation, the prices p_{ij} are in cents/Megabit multiplied by 10^{-5} . I use linear demand functions (see Altman, Legout, and Xu (2011), El Azouzi, Altman, and Wynter (2003), and Zhang et al. (2010)). The data were selected to be consistent with current advertized pricing of ISPs such as COMCAST¹.

¹<http://www.comcast.com>.

Example 5.1

The topology of the first example is given in Figure 5.2.

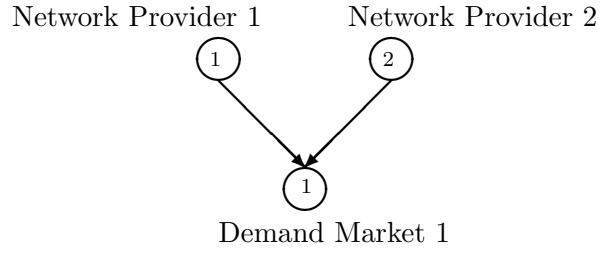


Figure 5.2. Network Topology for Example 5.1

The price functions at Demand Market 1 are:

$$p_{11} = 12 - .167 d_{11} - .0334 d_{21} + .032 q_{11} - .0064 q_{21} - .182 T_{11} - .0546 T_{21},$$

$$p_{21} = 12 - .0334 d_{11} - .167 d_{21} - .0064 q_{11} + .032 q_{21} - .0546 T_{11} - .182 T_{21}.$$

These functions reflect that Demand Market 1 is more sensitive to the contract duration than to the service usage rate. The network providers likely use different technologies for their services; therefore, their cost functions are distinct. The cost functions for Network Providers 1 and 2 are, respectively:

$$c_{11} = (.0049 q_{11}^2 + .001715 q_{11} + .029 d_{11})T_{11}, \quad c_{21} = (.0037 q_{21}^2 + .053 d_{21}^2)T_{21}.$$

The utility functions of the network providers are:

$$U_1 = p_{11}d_{11}T_{11} - c_{11}, \quad U_2 = p_{21}d_{21}T_{21} - c_{21}.$$

Network Provider 1 can offer services at a higher minimum service usage rate but at a lower minimum contract duration in comparison to Network Provider 2, where:

$$\begin{aligned} 23 \leq d_{11} \leq 250, & \quad 0 \leq q_{11} \leq 100, & \quad 8 \leq T_{11} \leq 40, \\ 15 \leq d_{21} \leq 200, & \quad 0 \leq q_{21} \leq 100, & \quad 11 \leq T_{21} \leq 40. \end{aligned}$$

Applying the Euler algorithm, the equilibrium solution and the incurred prices at equilibrium are, after 2,957 iterations:

$$d_{11}^* = 28.28, \quad d_{21}^* = 20.97,$$

$$T_{11}^* = 17.83, \quad T_{21}^* = 17.39,$$

$$q_{11}^* = 92.17, \quad q_{21}^* = 90.63,$$

$$p_{11} = 4.75, \quad p_{21} = 5.73.$$

The Jacobian matrix of $F(X) = -\nabla U(d, q, T)$, denoted by $J(d_{11}, d_{21}, T_{11}, T_{21}, q_{11}, q_{21})$, for this problem evaluated at $X^* = (d_{11}^*, d_{21}^*, T_{11}^*, T_{21}^*, q_{11}^*, q_{21}^*)$ is:

$$J = \begin{bmatrix} 5.96 & 0.59 & 3.25 & 0.97 & -0.575 & 0.115 \\ 0.58 & 7.655 & 0.95 & 3.16 & 0.11 & -0.56 \\ 3.25 & 0.94 & 10.29 & 1.54 & 0 & 0.18 \\ 0.70 & 3.16 & 1.15 & 7.63 & 0.13 & 0 \\ -0.57 & 0 & 0 & 0 & 0.17 & 0 \\ 0 & -0.56 & 0 & 0 & 0 & 0.13 \end{bmatrix}.$$

The eigenvalues of $\frac{1}{2}(J + J^T)$ are: 0.08, 0.11, 4.28, 4.49, 9.45, and 13.43, which are all positive. Therefore, both the uniqueness of the equilibrium solution and the conditions for convergence of the algorithm are guaranteed.

Hence, the contract period for Network Provider 1 at Demand Market 1 is 17.83 hours and that for Network Provider 2 is 17.39 hours. The revenue in cents for Network Provider 1 for the contract is $p_{11}d_{11}T_{11} \times 10^{-5} \times 3600$ seconds/hour = 86.26 cents. Network Provider 1 faces a cost of $c_{11} \times 10^{-5} \times 3600$ seconds/hour=27.37 cents for this contract and earns a profit of 58.91 cents. Note that this is the profit for a single user for the specific contract. The quality provided by Network Provider 1 of its service is higher than that provided by Network Provider 2. Network Provider 2, on the other hand, earns 75.15 cents in revenue, has 33.61 cents in cost, which results in a profit of 41.54 cents.

In this example, if the contract duration was 1 month, the revenue of a network provider per user would be approximately \$35, which is consistent with today’s Internet pricing from service providers such as COMCAST.

Example 5.2 and Sensitivity Analysis in Price

This example has the identical data to that of Example 5.1 except that Demand Market 2 is added as in Figure 5.3.

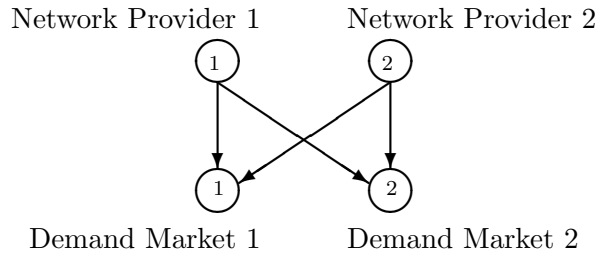


Figure 5.3. Network Topology for Example 5.2

The price functions for Demand Market 1 are as in Example 5.1. Demand Market 2 is less sensitive to the contract duration, the quality, and the service usage rate than Demand Market 1. The price functions for Demand Market 2 are:

$$p_{12} = 6 - .063 d_{12} - .0126 d_{22} + .026 q_{12} - .0052 q_{22} - .117 T_{12} - .0351 T_{22},$$

$$p_{22} = 6 - .0126 d_{12} - .063 d_{22} - .0052 q_{12} + .026 q_{22} - .0351 T_{12} - .117 T_{22}.$$

The cost functions for the network providers are:

$$c_{1j} = (.0049 q_{1j}^2 + .001715 q_{1j} + .029 d_{1j})T_{1j}, \quad j = 1, 2$$

$$c_{2j} = (.0037 q_{2j}^2 + .053 d_{2j}^2)T_{2j}, \quad j = 1, 2.$$

The bounds on the variables are:

$$23 \leq d_{1j} \leq 250, \quad 0 \leq q_{1j} \leq 100, \quad 8 \leq T_{1j} \leq 40, \quad j = 1, 2,$$

$$15 \leq d_{2j} \leq 200, \quad 0 \leq q_{2j} \leq 100, \quad 11 \leq T_{2j} \leq 40, \quad j = 1, 2.$$

The utilities of Network Providers 1 and 2 are, respectively:

$$U_1 = p_{11}d_{11}T_{11} + p_{12}d_{12}T_{12} - (c_{11} + c_{12}), \quad U_2 = p_{21}d_{21}T_{21} + p_{22}d_{22}T_{22} - (c_{21} + c_{22}).$$

The Jacobian of $F(X)$ is also positive-definite for this example.

The computed equilibrium, after 6,244 iterations, is:

$$d_{11}^* = 28.28, \quad d_{12}^* = 45.39, \quad d_{21}^* = 20.98, \quad d_{22}^* = 20.71,$$

$$T_{11}^* = 17.83, \quad T_{12}^* = 15.18, \quad T_{21}^* = 17.39, \quad T_{22}^* = 12.47,$$

$$q_{11}^* = 92.16, \quad q_{12}^* = 100.00, \quad q_{21}^* = 90.72, \quad q_{22}^* = 72.64.$$

At equilibrium, the prices of network services are:

$$p_{11} = 4.75, \quad p_{12} = 2.89, \quad p_{21} = 5.73, \quad p_{22} = 3.50.$$

Following the methodology used for Example 5.1, it follows that the revenue of Network Provider 1 is now 157.87 cents and his cost is 54.93 cents. Therefore, Network Provider 1 earns 102.94 cents for providing the services to the two demand markets. Network Provider 2's profit is now 55.12 cents at a revenue of 107.75 cents and a cost of 52.63 cents.

In order to understand the impact of changes in price functions, I denoted the constant term in the price functions of Demand Market 2 as p_0 and allowed p_0 to vary from 6 (its initial value) in both p_{12} and p_{22} to 16 in increments of 2. The results are reported in Figure 5.4. We see that not only the prices that Demand Market 2 is charged, but also the service usage rates and the durations of the contracts for this demand market increase. These changes lead to higher profits for not only Network Provider 2 but also interestingly for Network Provider 1 (cf. Figure 5.5).

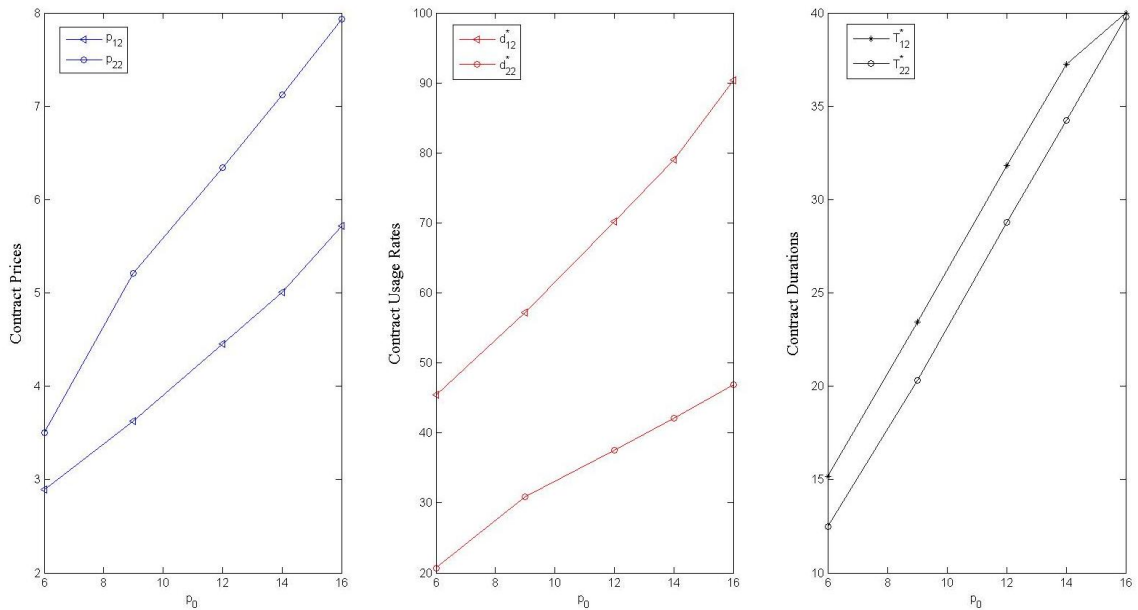


Figure 5.4. Effect of Increasing p_0 on Demand Market 2's Contracts

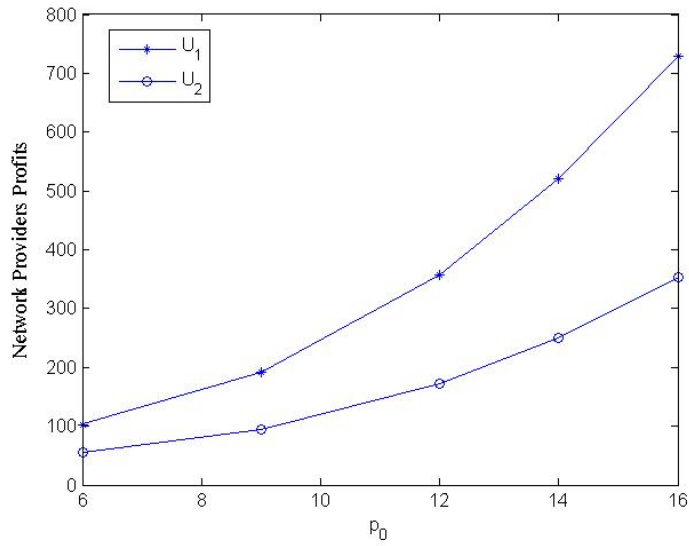


Figure 5.5. Effect of Increasing p_0 on Network Providers' Profits

Example 5.3 and Sensitivity Analysis in Quality Upper Bounds

Example 5.3 extends Example 5.2 to include a third demand market as shown in Figure 5.6.

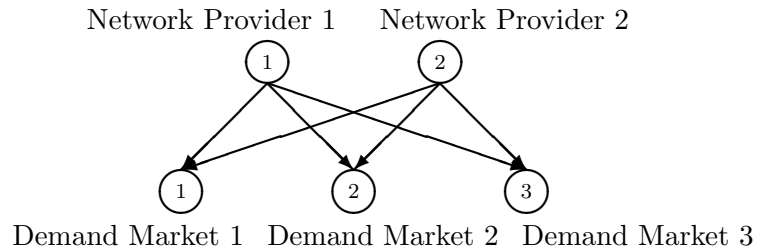


Figure 5.6. Network Topology for Example 5.3

The price and demand functions for Demand Markets 1 and 2 are as in Example 5.2. The price functions for Demand Market 3 are:

$$p_{13} = 9 - .115 d_{13} - .023 d_{23} + .028 q_{13} - .0056 q_{23} - .211 T_{13} - .0633 T_{23},$$

$$p_{23} = 9 - .023 d_{13} - .115 d_{23} - .0056 q_{13} + .028 q_{23} - .0633 T_{13} - .211 T_{23}.$$

The cost functions for Demand Market 3 are:

$$c_{1j} = (.0049 q_{1j}^2 + .001715 q_{1j} + .029 d_{1j})T_{1j}, \quad j = 3,$$

$$c_{2j} = (.0037 q_{2j}^2 + .053 d_{2j}^2)T_{2j}, \quad j = 3,$$

with those for Demand Markets 1 and 2 as in Example 5.2.

The bounds on the variables are:

$$23 \leq d_{1j} \leq 250, \quad 0 \leq q_{1j} \leq 100, \quad 8 \leq T_{1j} \leq 40, \quad j = 1, 2, 3,$$

$$15 \leq d_{2j} \leq 200, \quad 0 \leq q_{2j} \leq 100, \quad 11 \leq T_{2j} \leq 40, \quad j = 1, 2, 3.$$

The utility functions of Network Providers 1 and 2 are:

$$U_1 = p_{11}d_{11}T_{11} + p_{12}d_{12}T_{12} + p_{13}d_{13}T_{13} - (c_{11} + c_{12} + c_{13}),$$

$$U_2 = p_{21}d_{21}T_{21} + p_{22}d_{22}T_{22} + p_{23}d_{23}T_{23} - (c_{21} + c_{22} + c_{23}).$$

The Jacobian of $F(X)$ for this example is also positive-definite. The new equilibrium solution, computer after 8,681 iterations, is:

$$d_{11}^* = 31.48, \quad d_{12}^* = 45.39, \quad d_{13}^* = 30.16, \quad d_{21}^* = 23.55, \quad d_{22}^* = 20.71, \quad d_{23}^* = 19.87,$$

$$T_{11}^* = 20.31, \quad T_{12}^* = 15.18, \quad T_{13}^* = 13.49, \quad T_{21}^* = 19.84, \quad T_{22}^* = 12.47, \quad T_{23}^* = 13.00,$$

$$q_{11}^* = 100.00, \quad q_{12}^* = 100.00, \quad q_{13}^* = 76.77, \quad q_{21}^* = 100.00, \quad q_{22}^* = 72.64, \quad q_{23}^* = 67.11.$$

The equilibrium prices are:

$$p_{11} = 5.29, \quad p_{12} = 2.89, \quad p_{13} = 3.77, \quad p_{21} = 6.43, \quad p_{22} = 3.50, \quad p_{23} = 4.57.$$

Following the methodology used for Examples 5.1 and 5.2, I determine that Network Provider 1 earns a profit of 169.81 cents and Network Provider 2 a profit of 99.21 cents. The total cost of Network Provider 1 is now 78.71 cents and that of Network Provider 2 is 83.71 cents.

In order to investigate the effects of the maximum quality level of the network providers on their profits, I conducted a numerical sensitivity analysis. Quality disruptions may occur for various reasons, including natural, man-made, or technological issues. I used Example 5.3 as a baseline but varied the quality upper bounds from 10 through 100 in increments of 10 with both providers having the same quality upper bound. The profits/utilities of the providers are displayed in Figure 5.7.

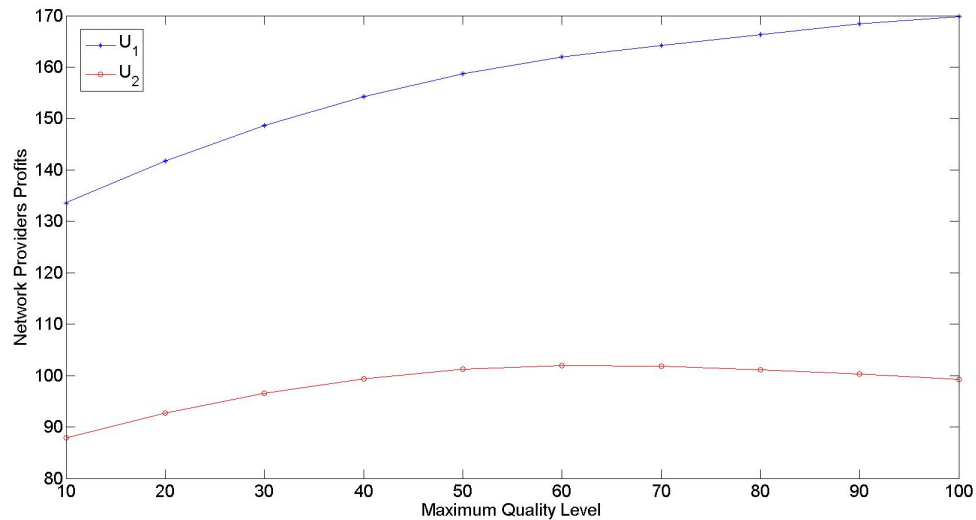


Figure 5.7. Effect of Increasing Maximum Quality Level on Network Providers' Profits - Same for Both Providers

The results show that the profit of Network Provider 1 increases as the maximum quality level increases, while the profit of Network Provider 2 is less sensitive to changes in the maximum quality level. Also, Figure 5.7 reveals that, when both providers have similar maximum levels of quality, Network Provider 1's utility/profit is highest when

the maximum quality level for both providers is at 100, while the highest utility/profit for Network Provider 2 is obtained when the maximum quality level for both is at 60.

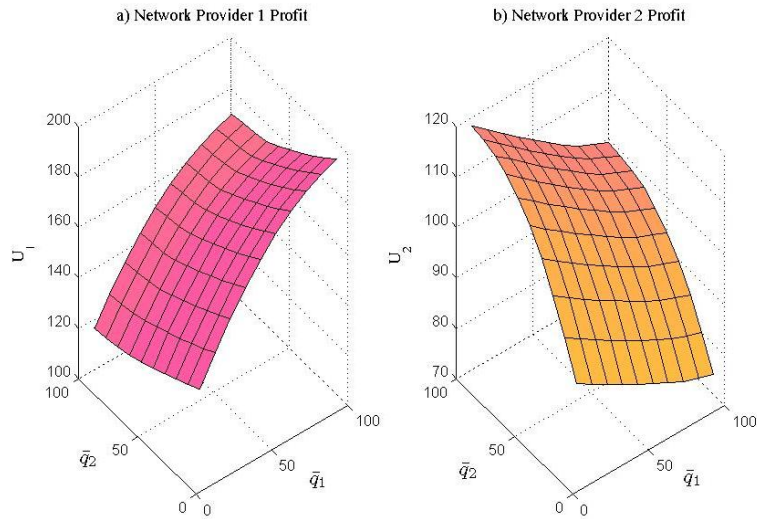


Figure 5.8. Effect of Increasing Maximum Quality Level on Network Providers' Profits - Different for Each Provider

Additional sensitivity analysis results are given in Figure 5.8 to investigate the impact on profits of distinct quality level upper bounds for the providers. The results in Figure 5.8 reveal that each network provider is better off at a higher maximum quality level while the maximum quality level of the other provider is fixed. Also, each provider benefits by increasing his maximum quality level bound while the maximum quality level of the other provider is lower. For each provider, the lowest utility/profit occurs when that provider has the lowest level of maximum quality (10) whereas the other provider has the highest maximum quality level (100).

These examples illustrate the importance of computations in gaining insights that may not be obvious otherwise because of the number of decision-makers, demand markets, and the complexity of interactions among them.

Note that if $F(X)$ is monotone, a property that would be satisfied for utility functions as in Theorem 5.1, in which case $\nabla F(X)$ is positive-semidefinite, then an

algorithm such as the extragradient method of Korpelevich (1977) could be utilized, with $F(X)$ also being Lipschitz continuous.

5.4. Summary

In this chapter, I developed a game theory model for a differentiated service-oriented Internet with duration-based contracts and quality competition. The Internet service providers are competing in an oligopoly market in terms of quality, duration, and price to the users in the demand markets.

The theoretical formalism was established using variational inequalities, which provides us with tools for both qualitative analysis and computational schemes. In order to show the applicability of the model, three examples are given. The first one shows the simplest market and the last one extends two previous ones with more decision-makers.

Numerical examples, supplemented with sensitivity analysis, demonstrated the efficacy of both the model and algorithmic framework. The results show that the model can capture the current pricing that we have in the Internet service market. The outcomes reveal that the idea of more flexibility in the next generation Internet, (e.g. ChoiceNet) is possible for future providers considering technological constraints in terms of contract duration, quality of service and even quantity of services.

This model can be used as the base for future studies focusing on shorter contracts for a service-oriented Internet with a more complicated architecture, resource limitations, and bandwidth sharing.

As I explained in Chapter 1, the Internet as a communication network, which transfers data between entities, is quite similar to a supply chain with freight service providers, which carry goods from one point to another point. Therefore, it would

be interesting to analyze the price and quality competition between freight service providers with different modes of shipment. This is done in the next chapter.

CHAPTER 6

A SUPPLY CHAIN NETWORK MODEL WITH COMPETITION IN PRICE AND QUALITY BETWEEN MULTIPLE MANUFACTURERS AND FREIGHT SERVICE PROVIDERS WITH MULTIPLE MODES OF SHIPMENT

In this chapter, a supply chain with freight service providers is considered. Supply chain networks and the Internet have many features in common. They both are involved in delivering products (in terms of goods or communication data) and quality of service should also be taken into account since quality is perceived as the most important element which leads to company success.

I develop a game theory model in both equilibrium and dynamic settings in an oligopoly market of manufacturers and freight service providers. The new static and dynamic models in this chapter also build on the work of Nagurney, Dong, and Zhang (2002), which introduced supply chain network equilibrium models but here the competition is in price and quality and not in quantities. See, also, the dynamic multilevel financial/informational/logistical framework of Nagurney, Dong, and Zhang (2002), the supernetwork model with freight carriers in Yamada et al. (2011), and the maritime chain model with carriers, ports, and shippers of Talley and Ng (2013). For a plethora of supply chain network equilibrium models, along with the underlying dynamics, see the book by Nagurney (2006a). For an overview of projected dynamical systems, which is the methodology that I utilize to describe the underlying competitive

dynamics and the evolution of prices and quality, see Chapter 2, Section 2.2. My contributions to existing knowledge and literature are:

- I model explicit competition among manufacturing firms and freight service providers (carriers) in terms of prices and quality of the products that the firms offer and the prices and quality of the freight services provided. This multi-faceted inclusion of competition from price and quality dimensions leads to results that not just quantify quality at the product and service ends, but also helps to assess the trade-offs between quality and costs at each echelon of the supply chain that ultimately influences the demand. A model that considers oligopolistic competition among manufacturers and freight service providers under price and quality with multiple modes of transportation and non-separable demand and cost functions is attempted for the first time in this chapter.
- The analysis for freight service providers contains price and quality evaluations for multiple modes of transportation. The transportation costs, resultantly, differ by mode, leading to a pertinent evaluation of quality vs. costs for the freight service providers and the modes of transportation they offer to the customers. In my frame of reference, modes could also imply intermodal transport of products.
- I handle heterogeneity in the providers' cost functions and in the consumers' demands and do not limit myself to specific functional forms. Utility of each manufacturing firm considers price and quality for not just his own products, but that of other manufacturing firms as well. Similarly, the utility of each freight service provider includes the implications of other providers' prices and quality for various modes in addition to his own. Also, I impose bounds on the prices and quality levels with positive minimum quality levels corresponding to minimum quality standards, relevant for policy-making.

- I provide qualitative properties of the equilibrium price and quality pattern and also present the underlying dynamics associated with the evolution of the prices and quality levels over time until the equilibrium is achieved.
- The theoretical framework is supported by a rigorous algorithm that is well-suited for implementation.
- The computational scheme is applied to a spectrum of numerical examples in order to illustrate the generality of the framework.

This chapter is based on Nagurney et al. (2015a). The structure of Chapter 6 is as follows. Section 6.1 presents the multitiered supply chain network game theory model with manufacturers and freight service providers. I capture the firms' behavior that accounts for the prices and quality levels of the products at the demand markets. In parallel, I model freight service providers' behavior that deals with the prices and quality levels of their services for various modes. The freight service providers compete in terms of price and quality that differ by mode. A variational inequality formulation is derived, which unifies the firms' and freight service providers' behaviors. An existence result for a solution to the unified variational inequality formulation (cf. Nagurney (1999)) is also given. A projected dynamical systems model is, subsequently, constructed in Section 6.2 to capture the underlying dynamics of the competitive behavior. In Section 6.3, I present an algorithm for solving the proposed variational inequality formulation, accompanied by convergence results. At each iteration, the algorithm yields closed form expressions for the prices and qualities of the firms and freight service providers. It also serves as a time-discretization of the continuous time adjustment processes in prices and quality levels. Section 6.4 illustrates the model and the computational algorithm through several numerical examples in order to gain managerial insights. In Section 6.5, I summarize the results and present the conclusions.

6.1. The Supply Chain Network Model with Price and Quality Competition

In the supply chain network, there are N manufacturing firms involved in the production of substitutable products that are transported by O freight service providers or carriers to Q demand markets. I denote a typical manufacturing firm by F_i ; $i = 1, \dots, N$, a typical freight service provider by C_j ; $j = 1, \dots, O$, and a typical demand market by k ; $k = 1, \dots, Q$. Each freight service provider C_j ; $j = 1, \dots, O$ has M_j possible modes of transport/shipment, associated with which is also a distinct quality. The modes of shipment may include rail, air, truck, sea, or even bicycles for last mile deliveries, etc.

Moreover, for the sake of modeling flexibility and generality, a *mode* in this framework may represent a composition of modes as in the case of intermodal transportation. The freight service providers are responsible for picking up the products at the manufacturers and delivering them to consumers at the demand markets. Note that each freight service provider may have a different number of modes available to him based on vehicle ownership and access, contracts, prior relationships, geographical issues, etc. The supply chain network representation of this game theory model is depicted in Figure 6.1. The manufacturing firms compete with one another as do the freight service providers.

Firm F_i manufactures a product of quality q_i at the price p_i . As in Nagurney and Li (2014b), I define and quantify quality as the quality conformance level, that is, the degree to which a specific product conforms to a design or specification (Gilmore (1974) and Juran and Gryna (1988)). I group the prices of all firms' products into the vector $p_F \in R_+^N$, and their quality levels into the vector $q_F \in R_+^N$.

The quality and price associated with freight service provider C_j retrieving the product from firm F_i and delivering it to demand market k via mode m are denoted, respectively, by q_{ijk}^m and p_{ijk}^m ; $i = 1, \dots, N$; $j = 1, \dots, O$; $k = 1, \dots, Q$; $m = 1, \dots, M_j$. Quality with respect to freight in this model corresponds to level of service as emphasized by Mancera, Bruckmann, and Weidmann (2013). I group these quality levels and prices into the vectors $q_C \in R_+^{NOQ \sum_{j=1}^O M_j}$ and $p_C \in R_+^{NOQ \sum_{j=1}^O M_j}$.

The consumers at demand market k ; $k = 1, \dots, Q$, reveal their preferences for firm F_i 's product transported by freight service provider C_j via mode m through a demand function d_{ijk}^m . The demand d_{ijk}^m depends not only on the price and quality of firm F_i 's product, but also, in general, on the prices and quality levels of all other substitutable products as well as on the prices and quality levels associated with transportation:

$$d_{ijk}^m = d_{ijk}^m(p_F, q_F, p_C, q_C), \quad i = 1, \dots, N; j = 1, \dots, O; k = 1, \dots, Q; m = 1, \dots, M_j. \quad (6.1)$$

The generality of the demand functions allows for the modeling of competition on the demand side for the products and freight service provision. I expect that the demand d_{ijk}^m will increase (decrease) as the price (quality) of firm F_i 's product or the shipment price (quality) of freight service provider C_j decreases. I group the demands into the $NOQ \sum_{j=1}^O M_j$ -dimensional vector $d(p_F, q_F, p_C, q_C)$.

6.1.1 The Firms' Behavior

The supply of firm F_i 's product, s_i , is equal to the demand, that is,

$$s_i(p_F, q_F, p_C, q_C) = \sum_{k=1}^Q \sum_{j=1}^O \sum_{m=1}^{M_j} d_{ijk}^m(p_F, q_F, p_C, q_C), \quad i = 1, \dots, N, \quad (6.2)$$

since I expect the markets to clear.

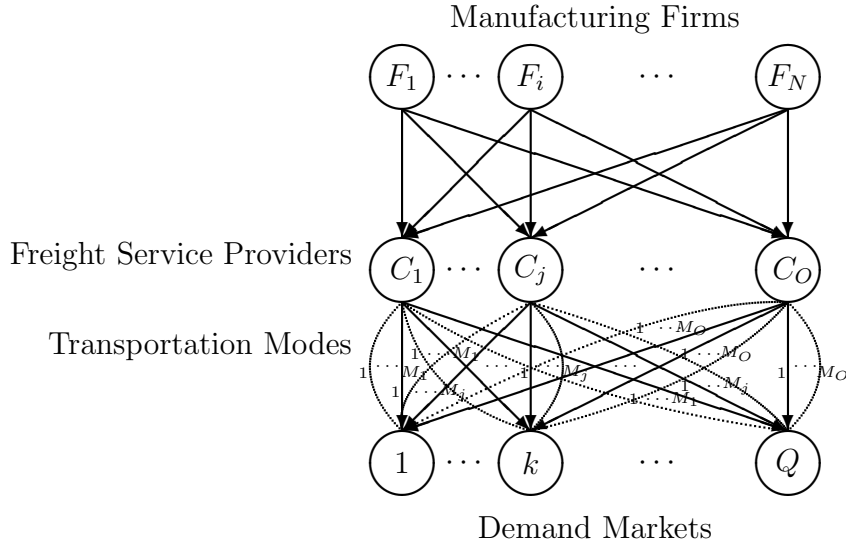


Figure 6.1. The Supply Chain Network Structure of the Game Theory Model

The production cost of firm F_i , PC_i , depends, in general, upon the entire production (supply) pattern, as well as on the product quality levels, that is:

$$PC_i = PC_i(s_F(p_F, q_F, p_C, q_C), q_F), \quad i = 1, \dots, N, \quad (6.3)$$

where $s_F(p_F, q_F, p_C, q_C) \in R_+^N$ is the vector of all the supplies of the products. The generality of the production cost functions allows us to capture competition for resources in manufacturing, whether natural, human, and/or capital.

The utility of firm F_i , U_{F_i} ; $i = 1, \dots, N$, represents his profit, and is the difference between the firm's revenue and the production cost:

$$U_{F_i}(p_F, q_F, p_C, q_C) = p_i \left[\sum_{k=1}^Q \sum_{j=1}^O \sum_{m=1}^{M_j} d_{ijk}^m(p_F, q_F, p_C, q_C) \right] - PC_i(s_F(p_F, q_F, p_C, q_C), q_F). \quad (6.4)$$

Each firm F_i is faced with a nonnegative lower bound \underline{q}_i on the quality of his product as well as an upper bound \bar{q}_i , so that

$$\underline{q}_i \leq q_i \leq \bar{q}_i, \quad i = 1, \dots, N. \quad (6.5)$$

Typically, $\bar{q}_i = 100$ corresponds to perfect quality conformance as discussed in Nagurney and Li (2015). If that is not achievable by a firm, then the upper bound would be set to a lower value. Also, a positive lower bound \underline{q}_i corresponds to a minimum quality standard as discussed in Nagurney and Li (2014b).

In addition, each firm F_i is faced with an upper bound on the price that he charges for his product, that is,

$$0 \leq p_i \leq \bar{p}_i, \quad i = 1, \dots, N. \quad (6.6)$$

The price that firm F_i charges and his quality level correspond to his strategic variables in the competitive game. Let K_i^1 denote the feasible set corresponding to F_i , where $K_i^1 \equiv \{(p_{F_i}, q_{F_i}) \mid (6.5) \text{ and } (6.6) \text{ hold}\}$. I define: $K^1 \equiv \prod_{i=1}^N K_i^1$ and assume that all the above functions are continuous and continuously differentiable. The manufacturers compete in a noncooperative manner which I formalize in Section 6.1.3.

6.1.2 The Freight Service Providers' Behavior

Recall that freight service provider C_j transports a product from firm F_i to demand market k via mode m at a quality level q_{ijk}^m at a unit price of p_{ijk}^m . I group the quality levels of freight service provider C_j into the vector $q_{C_j} \in R_+^{NQM_j}$ and his prices into the vector $p_{C_j} \in R_+^{NQM_j}$. These are his strategic variables.

I denote the transportation cost between firm F_i and demand market k via mode m of freight service provider C_j by TC_{ijk}^m and assume that:

$$TC_{ijk}^m = TC_{ijk}^m(d(p_F, q_F, p_C, q_C), q_C), \quad i = 1, \dots, N; j = 1, \dots, O; k = 1, \dots, Q; m = 1, \dots, M_j, \quad (6.7)$$

that is, the transportation cost may depend, in general, on the vector of demands and the vector of quality levels of all freight service providers. In the transportation costs,

I also include handling costs associated with, for example, loading and unloading and, perhaps, also, storage of the products over a period of time.

The utility or profit function of freight service provider C_j , U_{C_j} , is the difference between his revenue and his transportation costs:

$$U_{C_j}(p_F, q_F, p_C, q_C) = \sum_{i=1}^N \sum_{k=1}^O \sum_{m=1}^{M_j} [p_{ijk}^m d_{ijk}^m(p_F, q_F, p_C, q_C)] - \sum_{i=1}^N \sum_{k=1}^Q \sum_{m=1}^{M_j} TC_{ijk}^m(d(p_F, q_F, p_C, q_C), q_C). \quad (6.8)$$

Each C_j ; $j = 1, \dots, O$, is faced with a lower and upper bound on the quality of transport shipment \underline{q}_{ijk}^m , \bar{q}_{ijk}^m , respectively, and an upper bound for price, \bar{p}_{ijk}^m , between i and k so that

$$\underline{q}_{ijk}^m \leq q_{ijk}^m \leq \bar{q}_{ijk}^m, \quad i = 1, \dots, N; k = 1, \dots, Q; m = 1, \dots, M_j, \quad (6.9)$$

$$0 \leq p_{ijk}^m \leq \bar{p}_{ijk}^m, \quad i = 1, \dots, N; k = 1, \dots, Q; m = 1, \dots, M_j. \quad (6.10)$$

The freight service provider lower bounds are assumed to be nonnegative as in the case of product quality with a positive value corresponding to a minimum quality standard.

Let K_j^2 denote the feasible set corresponding to C_j , where $K_j^2 \equiv \{(p_{C_j}, q_{C_j}) \mid (6.9) \text{ and } (6.10) \text{ hold}\}$. I then define $K^2 \equiv \prod_{j=1}^O K_j^2$. I assume that all the above functions associated with the freight service providers are continuous and continuously differentiable. The freight service providers also compete in a noncooperative manner, as per below.

6.1.3 The Nash Equilibrium Conditions and Variational Inequality Formulation

I now present the Nash equilibrium definition that captures the decision-makers' competitive behavior in this model.

Definition 6.1: Nash Equilibrium in Prices and Quality Levels

A price and quality level pattern $(p_F^*, q_F^*, p_C^*, q_C^*) \in K^3 \equiv \prod_{i=1}^N K_i^1 \times \prod_{j=1}^O K_j^2$, is said to constitute a Nash equilibrium if for each firm $F_i; i = 1, \dots, N$:

$$U_{F_i}(p_i^*, \hat{p}_i^*, q_i^*, \hat{q}_i^*, p_C^*, q_C^*) \geq U_{F_i}(p_i, \hat{p}_i^*, q_i, \hat{q}_i^*, p_C^*, q_C^*), \quad \forall (p_i, q_i) \in K_i^1, \quad (6.11)$$

where

$$\hat{p}_i^* \equiv (p_1^*, \dots, p_{i-1}^*, p_{i+1}^*, \dots, p_N^*) \text{ and } \hat{q}_i^* \equiv (q_1^*, \dots, q_{i-1}^*, q_{i+1}^*, \dots, q_N^*), \quad (6.12)$$

and if for each freight service provider $C_j; j = 1, \dots, O$:

$$U_{C_j}(p_F^*, q_F^*, p_{C_j}^*, \hat{p}_{C_j}^*, q_{C_j}^*, \hat{q}_{C_j}^*) \geq U_{C_j}(p_F^*, q_F^*, p_{C_j}, \hat{p}_{C_j}^*, q_{C_j}, \hat{q}_{C_j}^*), \quad \forall (p_{C_j}, q_{C_j}) \in K_j^2, \quad (6.13)$$

where

$$\hat{p}_{C_j}^* \equiv (p_{C_1}^*, \dots, p_{C_{j-1}}^*, p_{C_{j+1}}^*, \dots, p_{C_O}^*) \text{ and } \hat{q}_{C_j}^* \equiv (q_{C_1}^*, \dots, q_{C_{j-1}}^*, q_{C_{j+1}}^*, \dots, q_{C_O}^*). \quad (6.14)$$

According to (6.11) and (6.13), a Nash equilibrium is established if no decision-maker, whether a manufacturing firm or freight service provider, can unilaterally improve upon his profits by selecting an alternative vector of prices and quality levels.

I assume that the above utility functions are concave. Under previously imposed assumptions on the production cost, transportation cost, and demand functions, I know that the utility functions are continuous and continuously differentiable. I now derive the variational inequality formulation of the governing equilibrium conditions.

Theorem 6.1: Variational Inequality Formulations of Nash Equilibrium in Prices and Quality

Assume that the manufacturing firms' and freight service providers' utility functions are concave, continuous, and continuously differentiable. Then $(p_F^, q_F^*, p_C^*, q_C^*) \in \mathcal{K}^3$ is a Nash equilibrium according to Definition 6.1 if and only if it satisfies the variational inequality:*

$$\begin{aligned}
& - \sum_{i=1}^N \frac{\partial U_{F_i}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial p_i} \times (p_i - p_i^*) - \sum_{i=1}^N \frac{\partial U_{F_i}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial q_i} \times (q_i - q_i^*) \\
& - \sum_{j=1}^O \sum_{i=1}^N \sum_{k=1}^Q \sum_{m=1}^{M_j} \frac{\partial U_{C_j}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial p_{ijk}^m} \times (p_{ijk}^m - p_{ijk}^{m*}) \\
& - \sum_{j=1}^O \sum_{i=1}^N \sum_{k=1}^Q \sum_{m=1}^{M_j} \frac{\partial U_{C_j}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial q_{ijk}^m} \times (q_{ijk}^m - q_{ijk}^{m*}) \geq 0, \quad \forall (p_F, q_F, p_C, q_C) \in \mathcal{K}^3,
\end{aligned} \tag{6.15}$$

or, equivalently,

$$\begin{aligned}
& \sum_{i=1}^N \left[\sum_{l=1}^N \frac{\partial PC_i(s_F(p_F^*, q_F^*, p_C^*, q_C^*), q_F^*)}{\partial s_l} \times \frac{\partial s_l(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial p_i} \right. \\
& \left. - \sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} d_{ijk}^m(p_F^*, q_F^*, p_C^*, q_C^*) - p_i^* \sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} \frac{\partial d_{ijk}^m(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial p_i} \right] \times (p_i - p_i^*) \\
& + \sum_{i=1}^N \left[\sum_{l=1}^N \frac{\partial PC_i(s_F(p_F^*, q_F^*, p_C^*, q_C^*), q_F^*)}{\partial s_l} \times \frac{\partial s_l(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial q_i} \right.
\end{aligned}$$

$$\begin{aligned}
& + \left. \frac{\partial PC_i(s_F^*, q_F^*)}{\partial q_i} - p_i^* \sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} \frac{\partial d_{ijk}^m(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial q_i} \right] \times (q_i - q_i^*) \\
& + \sum_{j=1}^O \sum_{i=1}^N \sum_{k=1}^Q \sum_{m=1}^{M_j} \left[\sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \left[\sum_{r=1}^N \sum_{v=1}^O \sum_{w=1}^Q \sum_{z=1}^{M_v} \right. \right. \\
& \left. \left. \frac{\partial TC_{ljs}^t(d(p_F^*, q_F^*, p_C^*, q_C^*), q_C^*)}{\partial d_{rvw}^z} \times \frac{\partial d_{rvw}^z(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial p_{ijk}^m} \right] \right. \\
& \left. - d_{ijk}^m(p_F^*, q_F^*, p_C^*, q_C^*) - \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial d_{ljs}^t(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial p_{ijk}^m} \times p_{ljs}^{t*} \right] \times (p_{ijk}^m - p_{ijk}^{m*}) \\
& + \sum_{j=1}^O \sum_{i=1}^N \sum_{k=1}^Q \sum_{m=1}^{M_j} \left[\sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \left[\sum_{r=1}^N \sum_{v=1}^O \sum_{w=1}^Q \sum_{z=1}^{M_v} \right. \right. \\
& \left. \left. \frac{\partial TC_{ljs}^t(d(p_F^*, q_F^*, p_C^*, q_C^*), q_C^*)}{\partial d_{rvw}^z} \times \frac{\partial d_{rvw}^z(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial q_{ijk}^m} \right] \right. \\
& \left. + \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial TC_{ljs}^t(d^*, q_C^*)}{\partial q_{ijk}^m} - \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial d_{ljs}^t(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial q_{ijk}^m} \times p_{ljs}^{t*} \right] \times (q_{ijk}^m - q_{ijk}^{m*}) \geq 0, \\
& \qquad \qquad \qquad \forall (p_F, q_F, p_C, q_C) \in \mathcal{K}^3, \quad (6.16)
\end{aligned}$$

where $s_F^* \equiv s_F(p_F^*, q_F^*, p_C^*, q_C^*)$ and $d^* \equiv d(p_F^*, q_F^*, p_C^*, q_C^*)$.

Proof: (6.15) follows from Gabay and Moulin (1980) and Dafermos and Nagurney (1987). In order to obtain (6.16) from (6.15), for each i I have:

$$-\frac{\partial U_{F_i}}{\partial p_i} = \sum_{l=1}^N \frac{\partial PC_i}{\partial s_l} \times \frac{\partial s_l}{\partial p_i} - \sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} d_{ijk}^m - p_i \sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} \frac{\partial d_{ijk}^m}{\partial p_i}, \quad (6.17)$$

$$-\frac{\partial U_{F_i}}{\partial q_i} = \sum_{l=1}^N \frac{\partial PC_i}{\partial s_l} \times \frac{\partial s_l}{\partial q_i} + \frac{\partial PC_i}{\partial q_i} - p_i \sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} \frac{\partial d_{ijk}^m}{\partial q_i}, \quad (6.18)$$

and, for each i, j, k and m , I have:

$$-\frac{\partial U_{C_j}}{\partial p_{ijk}^m} = \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \left[\sum_{r=1}^N \sum_{v=1}^O \sum_{w=1}^Q \sum_{z=1}^{M_v} \frac{\partial TC_{ljs}^t}{\partial d_{rvw}^z} \times \frac{\partial d_{rvw}^z}{\partial p_{ijk}^m} \right] - d_{ijk}^m - \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial d_{ljs}^t}{\partial p_{ijk}^m} \times p_{ljs}^t, \quad (6.19)$$

$$-\frac{\partial U_{C_j}}{\partial q_{ijk}^m} = \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \left[\sum_{r=1}^N \sum_{v=1}^O \sum_{w=1}^Q \sum_{z=1}^{M_v} \frac{\partial TC_{ljs}^t}{\partial d_{rvw}^z} \times \frac{\partial d_{rvw}^z}{\partial q_{ijk}^m} \right] + \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial TC_{ljs}^t}{\partial q_{ijk}^m} - \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial d_{ljs}^t}{\partial q_{ijk}^m} \times p_{ljs}^t. \quad (6.20)$$

Multiplying the right-most expression in (6.17) by $(p_i - p_i^*)$ and summing the result over all i and similarly, multiplying the right-most expression in (6.18) by $(q_i - q_i^*)$, and summing the result over all i , yields, respectively,

$$\sum_{i=1}^N \left[\sum_{l=1}^N \frac{\partial PC_i}{\partial s_l} \times \frac{\partial s_l}{\partial p_i} - \sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} d_{ijk}^m - p_i \sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} \frac{\partial d_{ijk}^m}{\partial p_i} \right] \times (p_i - p_i^*), \quad (6.21)$$

$$\sum_{i=1}^N \left[\sum_{l=1}^N \frac{\partial PC_i}{\partial s_l} \times \frac{\partial s_l}{\partial q_i} + \frac{\partial PC_i}{\partial q_i} - p_i \sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} \frac{\partial d_{ijk}^m}{\partial q_i} \right] \times (q_i - q_i^*), \quad (6.22)$$

Also, multiplying the right-most expression in (6.19) by $(p_{ijk}^m - p_{ijk}^{m*})$ and summing over all i, j, k , and m and similarly, multiplying the right-most expression in (6.20) by $(q_{ijk}^m - q_{ijk}^{m*})$ and summing over all i, j, k , and m yields, respectively:

$$\sum_{j=1}^O \sum_{i=1}^N \sum_{k=1}^Q \sum_{m=1}^{M_j} \left[\sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \left[\sum_{r=1}^N \sum_{v=1}^O \sum_{w=1}^Q \sum_{z=1}^{M_v} \frac{\partial TC_{ljs}^t}{\partial d_{rvw}^z} \times \frac{\partial d_{rvw}^z}{\partial p_{ijk}^m} \right] - d_{ijk}^m - \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial d_{ljs}^t}{\partial p_{ijk}^m} \times p_{ljs}^t \right] \times (p_{ijk}^m - p_{ijk}^{m*}), \quad (6.23)$$

$$\sum_{j=1}^O \sum_{i=1}^N \sum_{k=1}^Q \sum_{m=1}^{M_j} \left[\sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \left[\sum_{r=1}^N \sum_{v=1}^O \sum_{w=1}^Q \sum_{z=1}^{M_v} \frac{\partial TC_{ljs}^t}{\partial d_{rvw}^z} \times \frac{\partial d_{rvw}^z}{\partial q_{ijk}^m} \right] + \frac{\partial TC_{ljs}^t}{\partial q_{ijk}^m} - \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial d_{ljs}^t}{\partial q_{ijk}^m} \times p_{ljs}^t \right] \times (q_{ijk}^m - q_{ijk}^{m*}),$$

$$+ \left[\sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial T C_{ljs}^t}{\partial q_{ijk}^m} - \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial d_{ljs}^t}{\partial q_{ijk}^m} \times p_{ljs}^t \right] \times (q_{ijk}^m - q_{ijk}^{m*}), \quad (6.24)$$

Finally, summing (6.21), (6.22), (6.23), and (6.24), yields variational inequality (6.16). \square

I now put the above Nash equilibrium problem into standard variational inequality form (cf. (2.1)).

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \quad (6.25)$$

I set $\mathcal{K} \equiv \mathcal{K}^3$, which is a closed and convex set, and $n = 2N + 2(NOQ \sum_{j=1}^O M_j)$. I define the vector $X \equiv (p_F, q_F, p_C, q_C)$ and $F(X) \equiv (F_{p_F}, F_{q_F}, F_{p_C}, F_{q_C})$ with the i -th component of F_{p_F} and F_{q_F} given, respectively, by:

$$F_{p_i} = -\frac{\partial U_{F_i}}{\partial p_i}, \quad (6.26)$$

$$F_{q_i} = -\frac{\partial U_{F_i}}{\partial q_i}, \quad (6.27)$$

and the (i, j, k, m) -th component of F_{p_C} and F_{q_C} , respectively, given by:

$$F_{p_{ijk}^m} = -\frac{\partial U_{C_j}}{\partial p_{ijk}^m}, \quad (6.28)$$

$$F_{q_{ijk}^m} = -\frac{\partial U_{C_j}}{\partial q_{ijk}^m}. \quad (6.29)$$

Then, clearly, variational inequality (6.16) can be put into standard form (6.25).

Existence of a solution to variational inequality (6.15), equivalently, variational inequality (6.16), is guaranteed since the feasible set \mathcal{K} is compact and the function $F(X)$ (cf. (6.25)) in this model is continuous, under the assumptions made on the underlying functions (see Theorem 2.7).

6.2. The Dynamics

I now propose dynamic adjustment processes for the evolution of the firms' product prices and quality levels and those of the freight service providers (carriers). Each manufacturing firm adjusts the prices and quality of his products in a direction that maximizes his utility while maintaining the price and quality bounds. Also, each freight service provider adjusts his prices and quality levels in order to maximize his utility while keeping the prices and quality levels within their minimum and maximum levels. This kind of behavior, as I show below, yields a *projected dynamical system*. I also demonstrate that the stationary point of the projected dynamical system coincides with the solution of the variational inequality governing the Nash equilibrium of the supply chain network model introduced in Section 6.1. Hence, the adjustment processes provide a reasonable economic and behavioral description of the underlying competitive interactions.

For a current price and quality level pattern at time t , $X(t) = (p_F(t), q_F(t), p_C(t), q_C(t))$, $-F_{p_i}(X(t)) = \frac{\partial U_{F_i}(p_F(t), q_F(t), p_C(t), q_C(t))}{\partial p_i}$, given by (6.26), is the marginal utility (profit) of firm F_i with respect to the price that he charges for his product, $-F_{q_i}(X(t)) = \frac{\partial U_{F_i}(p_F(t), q_F(t), p_C(t), q_C(t))}{\partial q_i}$, defined in (6.27), is the marginal utility of firm F_i with respect to the quality of his product, and $-F_{p_{ijk}^m}(X(t)) = \frac{\partial U_{C_j}(p_F(t), q_F(t), p_C(t), q_C(t))}{\partial p_{ijk}^m}$, given by (6.28), and $-F_{q_{ijk}^m}(X(t)) = \frac{\partial U_{C_j}(p_F(t), q_F(t), p_C(t), q_C(t))}{\partial q_{ijk}^m}$, defined in (6.29), are, respectively, the marginal utility of freight service provider C_j with respect to price and with respect to quality of shipment, from manufacturing firm F_i to demand market k by mode m . In this framework, the rate of change of the price that firm F_i charges is in proportion to $-F_{p_i}(X)$, as long as the price p_i is positive and less than \bar{p}_i . Namely, when $0 < p_i < \bar{p}_i$, then

$$\dot{p}_i = \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i}, \quad (6.30)$$

where \dot{p}_i denotes the rate of change of p_i . However, when $\frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i} \leq 0$ or $\frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i} \geq \bar{p}_i$, constraint (6.6) forces the price to remain zero or equal to \bar{p}_i , hence

$$\dot{p}_i = \max \left\{ 0, \min \left\{ \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i}, \bar{p}_i \right\} \right\}. \quad (6.31)$$

I may write (6.30) and (6.31) concisely as:

$$\dot{p}_i = \begin{cases} \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i}, & \text{if } 0 < p_i < \bar{p}_i \\ \max \left\{ 0, \min \left\{ \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i}, \bar{p}_i \right\} \right\}, & \text{if } p_i = 0 \text{ or } p_i = \bar{p}_i. \end{cases} \quad (6.32)$$

The rate of change of the product quality of firm F_i , in turn, is in proportion to $-F_{q_i}(X)$, if $\underline{q}_i < q_i < \bar{q}_i$, so that

$$\dot{q}_i = \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i}, \quad (6.33)$$

where \dot{q}_i denotes the rate of change of q_i . However, when $\frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i} \leq \underline{q}_i$ or $\frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i} \geq \bar{q}_i$, constraint (6.5) forces the quality level to remain at least \underline{q}_i or at most \bar{q}_i , respectively. Therefore,

$$\dot{q}_i = \max \left\{ \underline{q}_i, \min \left\{ \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i}, \bar{q}_i \right\} \right\}. \quad (6.34)$$

Combining (6.33) and (6.34), I may write:

$$\dot{q}_i = \begin{cases} \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i}, & \text{if } \underline{q}_i < q_i < \bar{q}_i \\ \max \left\{ \underline{q}_i, \min \left\{ \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i}, \bar{q}_i \right\} \right\}, & \text{if } q_i = \underline{q}_i \text{ or } q_i = \bar{q}_i. \end{cases} \quad (6.35)$$

The rate of change of price p_{ijk}^m , in turn, that freight service provider C_j charges demand market k to ship the product from firm F_i via mode m , is in proportion to

$-F_{p_{ijk}^m}$, as long as $0 < p_{ijk}^m < \bar{p}_{ijk}^m$, so that

$$\dot{p}_{ijk}^m = \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial p_{ijk}^m}, \quad (6.36)$$

where \dot{p}_{ijk}^m is the rate of change of p_{ijk}^m . Otherwise, constraint (6.10) forces the price to be zero or at most equal to \bar{p}_{ijk}^m . Thus,

$$\dot{p}_{ijk}^m = \max \left\{ 0, \min \left\{ \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial p_{ijk}^m}, \bar{p}_{ijk}^m \right\} \right\}. \quad (6.37)$$

I can write (6.36) and (6.37) compactly as:

$$\dot{p}_{ijk}^m = \begin{cases} \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial p_{ijk}^m}, & \text{if } 0 < p_{ijk}^m < \bar{p}_{ijk}^m \\ \max \left\{ 0, \min \left\{ \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial p_{ijk}^m}, \bar{p}_{ijk}^m \right\} \right\}, & \text{if } p_{ijk}^m = 0 \text{ or } p_{ijk}^m = \bar{p}_{ijk}^m. \end{cases} \quad (6.38)$$

Finally, the rate of change of q_{ijk}^m , which is given by \dot{q}_{ijk}^m , is in proportion to $-F_{q_{ijk}^m}$, while the quality of mode m of freight service provider C_j for shipping the product from firm F_i to demand market k , q_{ijk}^m , is more than his lower bound and less than his upper bound. In other words, when $\underline{q}_{ijk}^m < q_{ijk}^m < \bar{q}_{ijk}^m$, I have

$$\dot{q}_{ijk}^m = \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial q_{ijk}^m}, \quad (6.39)$$

otherwise:

$$\dot{q}_{ijk}^m = \max \left\{ \underline{q}_{ijk}^m, \min \left\{ \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial q_{ijk}^m}, \bar{q}_{ijk}^m \right\} \right\}. \quad (6.40)$$

Combining (6.39) and (6.40), the quality level q_{ijk}^m evolves according to

$$\dot{q}_{ijk}^m = \begin{cases} \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial q_{ijk}^m}, & \text{if } \underline{q}_{ijk}^m < q_{ijk}^m < \bar{q}_{ijk}^m \\ \max \left\{ \underline{q}_{ijk}^m, \min \left\{ \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial q_{ijk}^m}, \bar{q}_{ijk}^m \right\} \right\}, & \text{if } q_{ijk}^m = \underline{q}_{ijk}^m \text{ or } q_{ijk}^m = \bar{q}_{ijk}^m. \end{cases} \quad (6.41)$$

Applying (6.32) and (6.35) to all manufacturing firms F_i ; $i = 1, \dots, N$, and applying (6.38) and (6.41) to all modes $m = 1, \dots, M_j$ of freight service providers C_j ; $j = 1, \dots, O$ used in shipping the product from firm F_i ; $i = 1, \dots, N$ to all demand markets k ; $k = 1, \dots, Q$, and combining the resultants, yields the pertinent ordinary differential equation (see (2.17)) for the adjustment processes of the prices and quality levels of firms and freight service providers, in vector form:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X^0. \quad (6.42)$$

Note that X^0 is the initial point $(p_F^0, q_F^0, p_C^0, q_C^0)$ corresponding to the initial price and quality levels of the manufacturing firms and freight service providers and $F(X)$ is the vector of minus the marginal utilities of the manufacturing firms and the freight service providers with respect to their strategic variables of prices and quality levels, with the individual components of $F(X)$ given by (6.26) through (6.29).

The trajectory provides the dynamic evolution of the prices charged and the quality levels of the manufacturing firms' products and carriers' freight services and the dynamic interactions among them. I note that ODE (6.42) ensures that the prices and quality levels of all firms and carriers are always within their lower and upper bounds.

The following theorem from Dupuis and Nagurney (1993) and Theorem 2.6 holds true in this framework since the feasible set \mathcal{K} is convex.

Theorem 6.2

X^ solves the variational inequality problem (6.25) (equivalently, (6.15) and (6.16)), if and only if it is a stationary point of the ODE (6.42), that is,*

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)). \quad (6.43)$$

This theorem demonstrates that the necessary and sufficient condition for a product and freight service price and quality level pattern $X^* = (p_F^*, q_F^*, p_C^*, q_C^*)$ to be a Nash equilibrium, according to Definition 6.1, is that $X^* = (p_F^*, q_F^*, p_C^*, q_C^*)$ is a stationary point of the adjustment processes defined by ODE (6.42), that is, X^* is the point at which $\dot{X} = 0$.

6.3. Explicit Formulae for the Euler Method Applied to the Multitiered Supply Chain Network Problem

The explicit formulae yielded by the Euler method (see Section 2.4) are the following closed form expressions for all firms' product price $p_i; i = 1, \dots, N$ and product quality $q_i; i = 1, \dots, N$, respectively:

$$p_i^{\tau+1} = \max \left\{ 0, \min \left\{ \bar{p}_i, p_i^\tau + a_\tau \left[\sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} d_{ijk}^m(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau) \right. \right. \right. \\ \left. \left. \left. + p_i^\tau \sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} \frac{\partial d_{ijk}^m(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial p_i} \right. \right. \right. \\ \left. \left. \left. - \sum_{l=1}^N \frac{\partial PC_i(s_F(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau), q_F^\tau)}{\partial s_l} \times \frac{\partial s_l(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial p_i} \right] \right\} \right\}, \quad (6.44)$$

$$q_i^{\tau+1} = \max \left\{ \underline{q}_i, \min \left\{ \bar{q}_i, q_i^\tau + a_\tau \left[p_i^\tau \sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} \frac{\partial d_{ijk}^m(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial q_i} \right. \right. \right. \\ \left. \left. \left. - \sum_{l=1}^N \frac{\partial PC_i(s_F(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau), q_F^\tau)}{\partial s_l} \times \frac{\partial s_l(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial q_i} - \frac{\partial PC_i(s_F^\tau, q_F^\tau)}{\partial q_i} \right] \right\} \right\}. \quad (6.45)$$

Also, one can obtain the values for the prices, p_{ijk}^m , and the quality levels, q_{ijk}^m , of the freight service providers: ($i = 1, \dots, N; j = 1, \dots, O; k = 1, \dots, Q; m = 1, \dots, M_j$), according to the following closed form expressions, respectively:

$$p_{ijk}^{m(\tau+1)} = \max \left\{ 0, \min \left\{ \bar{p}_{ijk}^m, p_{ijk}^{m\tau} + a_\tau \left[d_{ijk}^m(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau) \right. \right. \right.$$

$$\begin{aligned}
& + \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial d_{ljs}^t(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial p_{ijk}^m} \times p_{ljs}^{t\tau} \\
& - \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \left(\sum_{r=1}^N \sum_{v=1}^O \sum_{w=1}^Q \sum_{z=1}^{M_v} \frac{\partial TC_{ljs}^t(d(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau), q_C^\tau)}{\partial d_{rvw}^z} \times \frac{\partial d_{rvw}^z(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial p_{ijk}^m} \right) \Big] \Big\} \Big\}, \tag{6.46}
\end{aligned}$$

$$\begin{aligned}
q_{ijk}^{m(\tau+1)} &= \max \left\{ \underline{q}_{ijk}^m, \min \left\{ \bar{q}_{ijk}^m, q_{ijk}^{m\tau} + a_\tau \left[\sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial d_{ljs}^t(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial q_{ijk}^m} \times p_{ljs}^{t\tau} \right. \right. \right. \\
& - \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \left(\sum_{r=1}^N \sum_{v=1}^O \sum_{w=1}^Q \sum_{z=1}^{M_v} \frac{\partial TC_{ljs}^t(d(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau), q_C^\tau)}{\partial d_{rvw}^z} \times \frac{\partial d_{rvw}^z(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial q_{ijk}^m} \right) \\
& \left. \left. \left. - \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial TC_{ljs}^t(d^\tau, q_C^\tau)}{\partial q_{ijk}^m} \right] \right\} \right\}. \tag{6.47}
\end{aligned}$$

Note that all the functions to the left of the equal signs in (6.44) - (6.47) are evaluated at their respective variables computed at the τ -th iteration.

Also, the below convergence result is immediate following Nagurney and Zhang (1996) since the feasible set \mathcal{K} is compact.

Theorem 6.3: Convergence

In this multitiered supply chain network game theory model, assume that

$F(X) = -\nabla U(p_F, q_F, p_C, q_C)$ is strictly monotone. Also, assume that F is uniformly

Lipschitz continuous. Then, there exists a unique equilibrium price and quality pattern

$(p_F^, q_F^*, p_C^*, q_C^*) \in \mathcal{K}$ and any sequence generated by the Euler method as given by (6.44)*

- (6.47), where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \rightarrow 0$, as $\tau \rightarrow \infty$ converges to

$(p_F^, q_F^*, p_C^*, q_C^*)$.*

6.4. Numerical Examples

In this section, I present numerical examples illustrating the multitiered supply chain network game theory framework developed in Sections 6.1 and 6.2. The equilibrium

solutions of the model are computed by applying the Euler method as outlined in Section 6.3. Specifically, I present a spectrum of examples with various combinations of manufacturing firms, freight service providers, and modes. The supply chain network topology for each numerical example is described before the data and solution are presented.

The computations via the Euler method (cf. (6.44) -(6.47)) are carried out using Matlab. The algorithm was implemented on a VAIO S Series laptop with an Intel Core i7 processor and 12 GB RAM. The convergence tolerance is 10^{-6} , so that the algorithm is deemed to have converged when the absolute value of the difference between each successive price and quality level is less than or equal to 10^{-6} . The sequence $\{\alpha_\tau\}$ is set to: $.1\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \dots\}$. I initialize the algorithm by setting the prices and quality levels at their lower bounds. The ranges in which the prices and quality levels vary are noted for each example.

The first two examples are simple examples, for exposition purposes and clarity. The subsequent examples, along with their variants, reveal various aspects of the underlying competition. For the first two examples, I also provide the trajectories of the evolution of the prices and quality. Due to complexity of the model, the number of variables in each example is quite considerable. The examples, number of variables and CPU time for each one are provided in Table 6.1. It demonstrates the breadth of each network considered in the examples.

This framework can be applied to both high value and low value products with appropriate modifications in the underlying functions. For example, valuable goods would require greater quality in freight service provision, but at a higher associated cost. Also, I would expect that their production/manufacturing costs, given the components, to be higher.

Table 6.1. Example Features

Example	No. of Variables	CPU Time
1	4	4.70
2	6	73.34
3	8	186.03
Variant of 3	8	1157.82
4	16	462.31
Variant of 4	16	2675.05
5	28	703.08
Variant of 5	28	7967.54

Example 6.1

In the first example, I have a single manufacturing firm, F_1 , a single freight service provider, C_1 , with one mode of transport, and a single demand market, as depicted in the supply chain network in Figure 6.2.



Figure 6.2. The Supply Chain Network Topology for Example 6.1

The demand function for demand market 1 is:

$$d_{111}^1 = 43 - 1.62p_{111}^1 + 1.6q_{111}^1 - 1.45p_1 + 1.78q_1.$$

The supply of F_1 is:

$$s_1 = d_{111}^1.$$

The production cost of manufacturing firm F_1 is:

$$PC_1 = 1.55(s_1 + 1.15q_1^2).$$

The utility of manufacturing firm F_1 is:

$$U_{F_1} = p_1s_1 - PC_1.$$

The quality and price of the firm are bounded as per the following constraints:

$$0 \leq p_1 \leq 80, \quad 10 \leq q_1 \leq 100.$$

The transportation cost of freight service provider C_1 is:

$$TC_{111}^1 = .5d_{111}^1 + (q_{111}^1)^2.$$

The utility of freight service provider C_1 is:

$$U_{C_1} = p_{111}^1d_{111}^1 - TC_{111}^1,$$

with the following limitations on his price and quality:

$$0 \leq p_{111}^1 \leq 70, \quad 9 \leq q_{111}^1 \leq 100.$$

The Jacobian of $-\nabla U(p_{111}^1, p_1, q_{111}^1, q_1)$, denoted by $J(p_{111}^1, p_1, q_{111}^1, q_1)$, is

$$J = \begin{pmatrix} 3.24 & 1.45 & -1.60 & -1.78 \\ 1.62 & 2.90 & -1.60 & -1.78 \\ -1.60 & 0 & 2.00 & 0 \\ 0 & -1.78 & 0 & 3.57 \end{pmatrix}.$$

The eigenvalues of the symmetric part of J , $(J + J^T)/2$, are all positive and they are: 0.79, 1.14, 3.28, and 6.47. The equilibrium result, after 60 iterations, is:

$$p_{111}^{1*} = 16.63, \quad p_1^* = 19.57, \quad q_{111}^{1*} = 12.90, \quad q_1^* = 10.00.$$

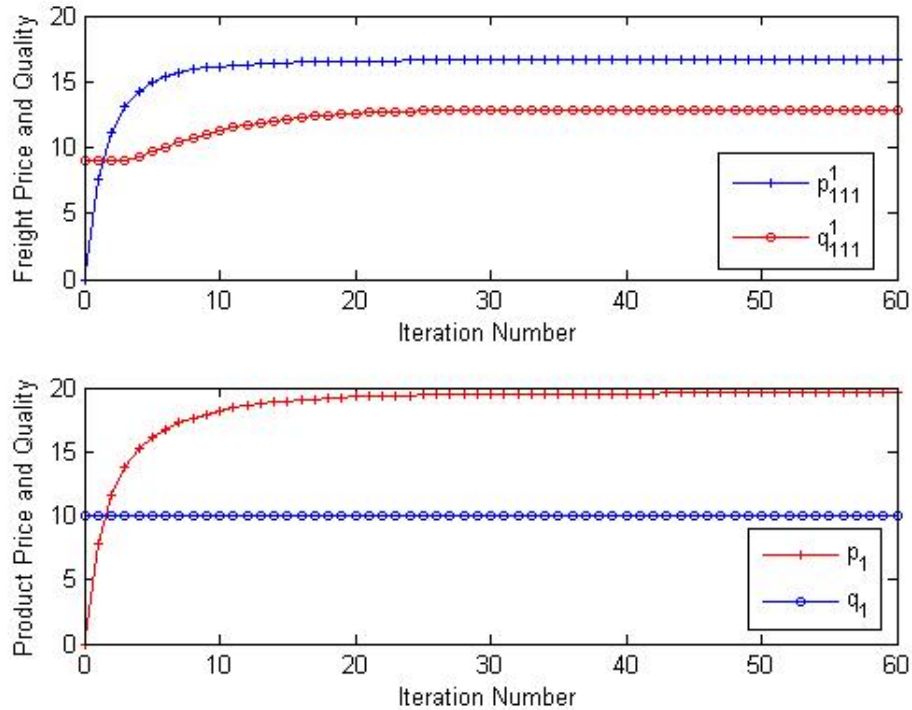


Figure 6.3. Prices and Quality Levels for the Product and Freight of Example 6.1

The iterates displayed in Figure 6.3 provide a discrete-time evolution of the prices and quality levels of the manufacturer and freight service provider as they respond through the time periods to the demands for the product and service. I observe that the prices move much above the quality levels and reach significantly higher values than their points of initiation, while the quality levels do not gain as much. This can be attributed to a lack of competition and enough scope at the demand market for gaining revenues. The manufacturer and freight service provider would try to extract the maximum price out of the market while offering a low quality product and services.

Indeed, in the absence of competition, the manufacturing firm and the freight service provider produce and transport at low quality levels. This explains the low equilibrium values of q_1^* and q_{111}^* . The utility of firm F_1 is 292.60 and that of freight service provider C_1 is 254.95. Also, the demand d_{111}^1 at equilibrium is 26.13. The demand function is assumed so that more weight is given to the quality of the product than of the freight service provision and the price of the freight service provider than the product price. Since there is no competition, the manufacturing firm ends up with a higher utility by selling a low quality product, while the freight service provider gains but not as much as the manufacturer.

Example 6.2

In Example 6.2, I extend Example 6.1 by adding another mode of shipment for freight service provider C_1 . The supply chain network topology is now depicted in Figure 6.4.

The demand functions are:

$$d_{111}^1 = 43 - 1.62p_{111}^1 + 1.6q_{111}^1 - 1.45p_1 + 1.78q_1 + .03p_{111}^2 - .2q_{111}^2,$$

$$d_{111}^2 = 52 - 1.75p_{111}^2 + 1.21q_{111}^2 - 1.45p_1 + 1.78q_1 + .03p_{111}^1 - .2q_{111}^1.$$

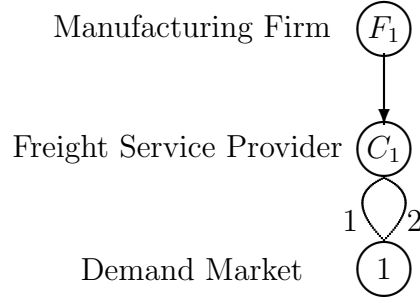


Figure 6.4. The Supply Chain Network Topology for Example 6.2

The contribution of quality of the product is higher in the demand functions than its price. Also, the contribution of price of the freight service provider is higher in the demand functions than the quality he offers. Here, the freight service provider is striving to position himself as a value added service.

The supply of manufacturing firm F_1 is changed to:

$$s_1 = d_{111}^1 + d_{111}^2$$

since there are two modes of shipment available now.

The production cost function of F_1 is the same as Example 6.1. The transportation costs of the freight service provider C_1 for modes 1 and 2 are:

$$TC_{111}^1 = .5d_{111}^1 + (q_{111}^1)^2,$$

$$TC_{111}^2 = .45d_{111}^2 + .54(q_{111}^2)^2 + .0035d_{111}^2q_{111}^2.$$

Note that mode 1's cost remains as in Example 6.1.

The utility of freight service provider C_1 is:

$$U_{C_1} = p_{111}^1d_{111}^1 + p_{111}^2d_{111}^2 - TC_{111}^1 - TC_{111}^2,$$

with the constraints on the price and quality of shipment kept for the first mode as in Example 6.1 and for the added second mode as below:

$$0 \leq p_{111}^2 \leq 70, \quad 9 \leq q_{111}^2 \leq 100.$$

The symmetric part of J , $(J + J^T)/2$, has positive eigenvalues, which guarantees the strict monotonicity of $F(X)$. The equilibrium solution, after 166 iterations, is:

$$p_{111}^{1*} = 21.68, \quad p_{111}^{2*} = 24.16, \quad p_1^* = 27.18,$$

$$q_{111}^{1*} = 14.58, \quad q_{111}^{2*} = 22.43, \quad q_1^* = 25.59.$$

The trajectories in Figure 6.5 provide a discrete-time evolution of the prices and quality levels of the manufacturer and freight service provider. As compared to Figure 6.3, the quality levels, and, therefore, the prices, of both manufacturer and freight service provider increase. This would be a result of the competing modes. I observe that the quality of mode 2 is much better than that of mode 1. Hence, the freight service provider quotes a higher price for mode 2. At the manufacturer's level, I observe a higher price in comparison to the quality level. However, I see the difference between the prices and quality levels to be much less than Figure 6.3 (the trajectories move along more closely in Figure 6.5 than in Figure 6.3 for the manufacturer).

At equilibrium, the utility of manufacturing firm F_1 is 737.29 and that of freight service provider C_1 is 1190.05. The amount shipped via mode 1, d_{111}^1 , is 33.59 and that shipped via mode 2, d_{111}^2 , is 40.73. Interestingly, even though the price offered by service provider C_1 for mode 2 is slightly higher, the quality level of mode 2 is much better than that of mode 1, which increases the demand satisfied by mode 2 as compared to mode 1. Also, the fixed component of the demand function, d_{111}^2 is

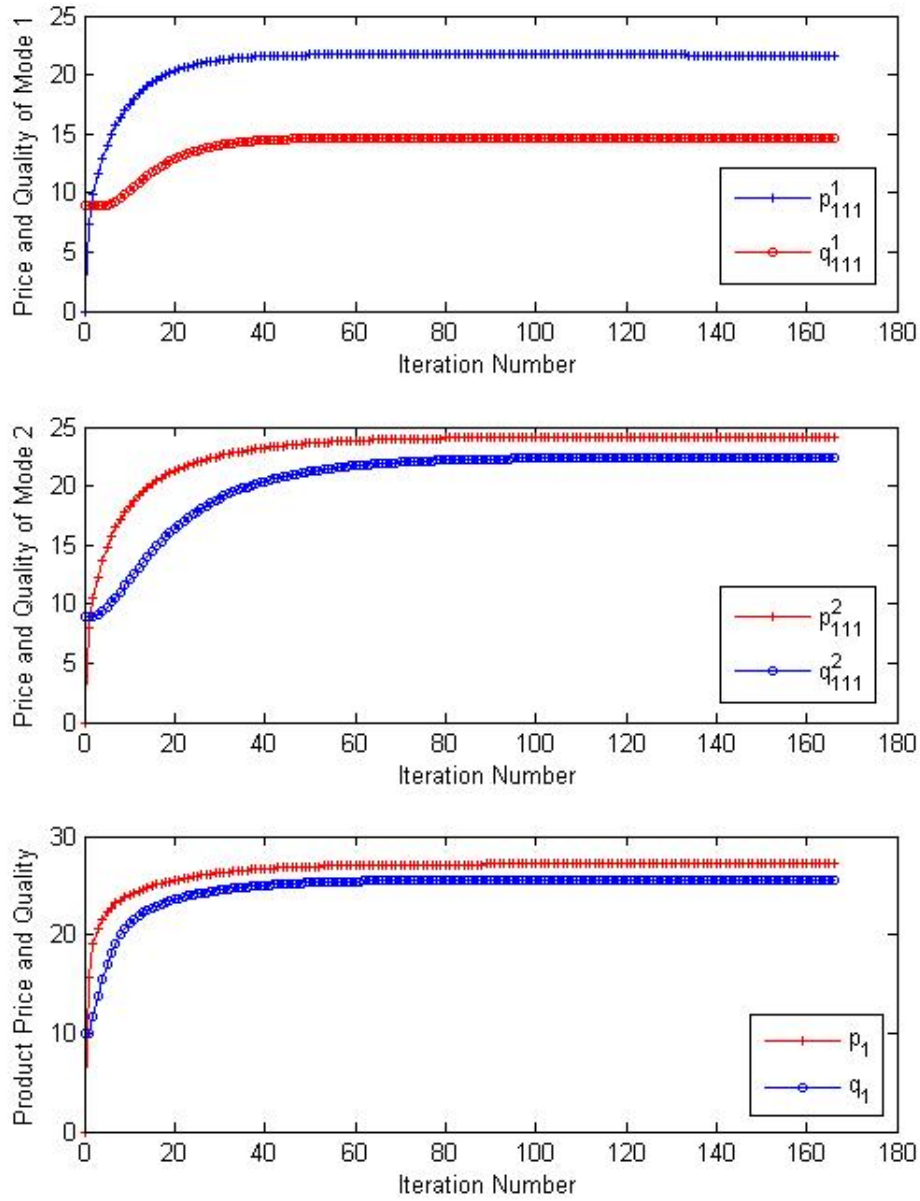


Figure 6.5. Prices and Quality Levels for Products and Modes 1 and 2 of Example 6.2

higher than that of d_{111}^1 . This also contributes to the higher demand shipped by mode 2 to demand market 1.

The differences in the utilities of the manufacturer (737.29) and the freight service provider (1190.05) are explained mainly by the production costs and transportation costs, respectively. It is judicious to assume that the production costs of a manufacturing firm would be higher than the transportation costs incurred by a freight service provider. This difference gets aptly captured in the (comparatively) higher coefficients of the production cost function.

Example 6.3 and Variant

In Example 6.3 and its variant, I extend Example 6.2 by including another freight service provider with one mode of shipment as illustrated in Figure 6.6.

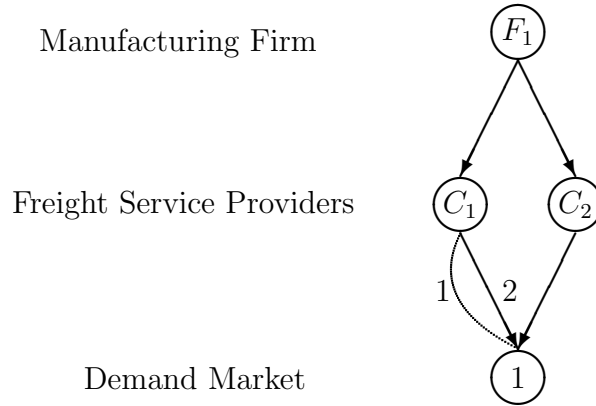


Figure 6.6. The Supply Chain Network Topology for Example 6.3 and Variant

The demand functions are:

$$d_{111}^1 = 43 - 1.62p_{111}^1 + 1.6q_{111}^1 - 1.45p_1 + 1.78q_1 + .03p_{111}^2 - .2q_{111}^2 + .04p_{121}^1 - .1q_{121}^1,$$

$$d_{111}^2 = 52 - 1.75p_{111}^2 + 1.21q_{111}^2 - 1.45p_1 + 1.78q_1 + .03p_{111}^1 - .2q_{111}^1 + .04p_{121}^1 - .1q_{121}^1,$$

$$d_{121}^1 = 47 - 1.79p_{121}^1 + 1.41q_{121}^1 - 1.45p_1 + 1.78q_1 + .03p_{111}^1 - .2q_{111}^1 + .04p_{111}^2 - .1q_{111}^2.$$

The supply of F_1 is:

$$s_1 = d_{111}^1 + d_{111}^2 + d_{121}^1.$$

The production cost of F_1 is the same as in Example 6.2. Therefore, the utility function of F_1 has not changed. The transportation costs of freight service provider C_1 are:

$$TC_{111}^1 = .5d_{111}^1 + (q_{111}^1)^2 + .045d_{121}^1,$$

$$TC_{111}^2 = .45d_{111}^2 + .54(q_{111}^2)^2 + .005d_{111}^2q_{111}^2,$$

and that of freight service provider C_2 is:

$$TC_{121}^1 = .64d_{121}^1 + .76(q_{121}^1)^2.$$

The utility function of C_1 and his price and quality constraints have not changed. The utility of C_2 is:

$$U_{C_2} = p_{121}^1 d_{121}^1 - TC_{121}^1.$$

The maximum and minimum levels of price and quality of C_2 are:

$$0 \leq p_{121}^1 \leq 65, \quad 12 \leq q_{121}^1 \leq 100.$$

The Jacobian of $F(X)$ for this example is also positive-definite. The new equilibrium solution, computed after 218 iterations, is:

$$p_{111}^{1*} = 45.69, \quad p_{111}^{2*} = 45.32, \quad p_{121}^{1*} = 44.82, \quad p_1^* = 53.91,$$

$$q_{111}^{1*} = 31.69, \quad q_{111}^{2*} = 41.32, \quad q_{121}^{1*} = 41.24, \quad q_1^* = 78.43.$$

In addition to the competition between modes captured in Example 6.2, in Example 6.3, I capture the competition among freight service providers. This adds pragmatism and generality. The assumption regarding the demand functions being more inclined

towards the quality of the product manufactured and the prices of the freight service providers remains valid in this instance as well. This supposition induced by the assumed coefficients of the demand and cost functions gets clearly reflected in the equilibrium solution ($p_1^* = 53.91; q_1^* = 78.43$).

At equilibrium, the utility of manufacturing firm F_1 is 961.39 and that of freight service providers C_1 and C_2 are 4753.06 and 2208.92, respectively. Demand market 1 receives amounts of 71.88 and 76.81 via modes 1 and 2 from C_1 , and 79.07 from C_2 . The inclusion of an additional freight service provider helps to increase the total demand as compared to Example 6.2. The increasing demand provides an incentive for manufacturing firm F_1 to increase his quality level and, consequently, his price. This surge in demand also has a positive effect on the utilities of the manufacturing firm and both freight service providers. Higher demand gets satisfied by C_2 since his price is lower and the quality level is at par with the quality provided by C_1 for both modes. Clearly, mode 1 of C_1 carries the lowest amount of the total demand due to the higher price and lower quality combination he offers.

Variant of Example 6.3

I consider a variant of Example 6.3 wherein the demand function is more sensitive to the price of the product manufactured and the quality offered by the freight service providers. Keeping the other data consistent, the demand functions are, hence, modified to the following:

$$d_{111}^1 = 43 - 1.44p_{111}^1 + 1.53q_{111}^1 - 1.82p_1 + 1.21q_1 + .03p_{111}^2 - .2q_{111}^2 + .04p_{121}^1 - .1q_{121}^1,$$

$$d_{111}^2 = 52 - 1.49p_{111}^2 + 1.65q_{111}^2 - 1.82p_1 + 1.21q_1 + .03p_{111}^1 - .2q_{111}^1 + .04p_{121}^1 - .1q_{121}^1,$$

$$d_{121}^1 = 47 - 1.57p_{121}^1 + 1.64q_{121}^1 - 1.82p_1 + 1.21q_1 + .03p_{111}^1 - .2q_{111}^1 + .04p_{111}^2 - .1q_{111}^2.$$

The equilibrium solution, computed after 553 iterations, is:

$$p_{111}^{1*} = 8.71, \quad p_{111}^{2*} = 63.17, \quad p_{121}^{1*} = 16.22, \quad p_1^* = 24.80,$$

$$q_{111}^{1*} = 9.00, \quad q_{111}^{2*} = 93.15, \quad q_{121}^{1*} = 16.92, \quad q_1^* = 23.67.$$

It should be noted that the quality levels offered by the freight service providers take on higher values than their prices as opposed to a vice versa situation in the case of Example 6.3. At equilibrium, the utility of manufacturing firm F_1 is 1952.19 and that of service providers C_1 and C_2 are 1073.86 and 164.99, respectively. The transportation costs increase to ensure high quality transportation. Thus, the utility of the manufacturing firm is higher than the utilities of both freight service providers. This can be explained by the fact that, apart from the price and quality level of the second mode of service provider C_1 , the prices and quality levels of the other mode and the other service provider take on much smaller values than in the equilibrium solution of the previous assumption. Since the emphasis is given to the quality of the freight service provider in the demand functions, the low quality levels result in lower demand. Demand market 1 receives amounts of 9.96 and 92.51 via modes 1 and 2 of freight service provider C_1 , and 24.46 via freight service provider C_2 . The low demand further reduces the utilities.

Example 6.4 and Variant

Example 6.4 and its variant extend the previous numerical examples through the addition of another manufacturing firm, as shown in Figure 6.7. These manufacturers offer substitutable products to the demand markets.

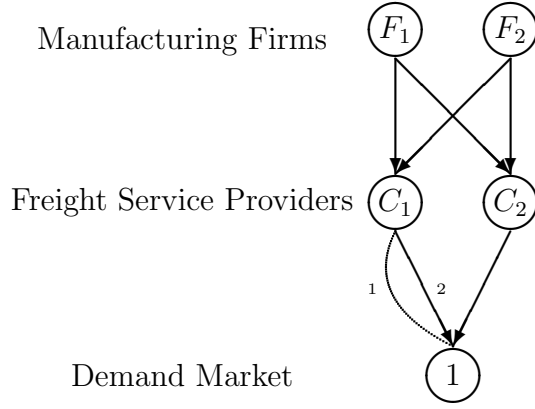


Figure 6.7. The Supply Chain Network Topology for Example 6.4 and Variant

The demand functions for manufacturing firm F_1 are:

$$d_{111}^1 = 43 - 1.62p_{111}^1 + 1.6q_{111}^1 - 1.45p_1 + 1.78q_1 + .08p_2 - .04q_2 + .03p_{111}^2 - .2q_{111}^2 + .04p_{121}^1 - .1q_{121}^1,$$

$$d_{111}^2 = 52 - 1.75p_{111}^2 + 1.21q_{111}^2 - 1.45p_1 + 1.78q_1 + .08p_2 - .04q_2 + .03p_{111}^1 - .2q_{111}^1 + .04p_{121}^1 - .1q_{121}^1,$$

$$d_{121}^1 = 47 - 1.79p_{121}^1 + 1.41q_{121}^1 - 1.45p_1 + 1.78q_1 + .08p_2 - .04q_2 + .03p_{111}^1 - .2q_{111}^1 + .04p_{111}^2 - .1q_{111}^2,$$

and that of manufacturing firm F_2 are:

$$d_{211}^1 = 51 - 1.57p_{211}^1 + 1.26q_{211}^1 - 1.65p_2 + 1.98q_2 + .08p_1 - .04q_1 + .04p_{211}^2 - .1q_{211}^2 + .02p_{221}^1 - .12q_{221}^1,$$

$$d_{211}^2 = 44 - 1.63p_{211}^2 + 1.21q_{211}^2 - 1.65p_2 + 1.98q_2 + .08p_1 - .04q_1 + .04p_{211}^1 - .1q_{211}^1 + .02p_{221}^1 - .12q_{221}^1,$$

$$d_{221}^1 = 56 - 1.46p_{221}^1 + 1.41q_{221}^1 - 1.65p_2 + 1.98q_2 + .08p_1 - .04q_1 + .04p_{211}^1 - .1q_{211}^1 + .02p_{211}^2 - .12q_{211}^2.$$

The supply of F_1 is similar to that in Example 6.3 and that of manufacturing firm F_2 is:

$$s_2 = d_{211}^1 + d_{211}^2 + d_{221}^1.$$

The production cost functions of F_1 and F_2 are:

$$PC_1 = 1.55s_1 + 1.88q_1^2 + .02s_2 + .06q_2,$$

$$PC_2 = 1.47s_2 + 1.94q_2^2 + .041s_1 + .032q_1.$$

Manufacturing firm F_1 has the same utility function and price and quality bounds as in Example 6.3. The utility of manufacturing firm F_2 is:

$$U_{F_2} = p_2 s_2 - PC_2,$$

and the price and quality of his product are constrained in the following manner:

$$0 \leq p_2 \leq 95, \quad 8 \leq q_2 \leq 100.$$

The transportation cost functions of freight service provider C_1 are changed to:

$$TC_{111}^1 = .5d_{111}^1 + (q_{111}^1)^2 + .0045d_{121}^1 + .0045d_{221}^1 + .0045d_{211}^1,$$

$$TC_{111}^2 = .45d_{111}^2 + .54(q_{111}^2)^2 + .0011d_{211}^2,$$

$$TC_{211}^1 = .68d_{211}^1 + .79(q_{211}^1)^2 + .002d_{211}^1 + .002d_{221}^1,$$

$$TC_{211}^2 = .57d_{211}^2 + .74(q_{211}^2)^2 + .005d_{111}^2,$$

and the cost functions of freight service provider C_2 are changed to:

$$TC_{121}^1 = .64d_{121}^1 + .76(q_{121}^1)^2 + .0015d_{221}^1,$$

$$TC_{221}^1 = .59d_{221}^1 + .80(q_{221}^1)^2 + .01d_{121}^1 + .01d_{111}^1 + .01d_{211}^1.$$

The utility of C_1 is:

$$U_{C_1} = p_{111}^1 d_{111}^1 + p_{111}^2 d_{111}^2 + p_{211}^1 d_{211}^1 + p_{211}^2 d_{211}^2 - TC_{111}^1 - TC_{111}^2 - TC_{211}^1 - TC_{211}^2,$$

and that of C_2 is:

$$U_{C_2} = p_{121}^1 d_{121}^1 + p_{221}^1 d_{221}^1 - TC_{121}^1 - TC_{221}^1.$$

The lower and upper bounds of the prices for freight service providers are now:

$$0 \leq p_{i1k}^{M_1} \leq 90, \quad \forall i, k, M_1, \text{ for } M_1 = 2,$$

$$0 \leq p_{i2k}^{M_2} \leq 85, \quad \forall i, k, M_2, \text{ for } M_2 = 1.$$

The equilibrium solution, computed after 231 iterations, is:

$$p_{111}^{1*} = 40.20, \quad p_{111}^{2*} = 40.72, \quad p_{121}^{1*} = 39.79, \quad p_1^* = 48.08,$$

$$p_{211}^{1*} = 51.17, \quad p_{211}^{2*} = 42.88, \quad p_{221}^{1*} = 69.18, \quad p_2^* = 50.89,$$

$$q_{111}^{1*} = 27.73, \quad q_{111}^{2*} = 37.76, \quad q_{121}^{1*} = 36.53, \quad q_1^* = 66.25,$$

$$q_{211}^{1*} = 37.64, \quad q_{211}^{2*} = 29.42, \quad q_{221}^{1*} = 63.97, \quad q_2^* = 75.65.$$

In this example, I consider competition at the manufacturers' level, the freight service providers' level, and between modes of a particular freight service provider. This, further, increases the generality, as well as the complexity, of the problem when compared with Example 6.3. The assumption regarding the demand functions being more inclined towards the quality of the product manufactured and the prices of the freight service providers remains valid in this instance as well. The equilibrium solution ($p_1^* = 48.08$; $q_1^* = 66.25$; $p_2^* = 50.89$; $q_2^* = 75.65$) supports this assumption.

The utilities of manufacturing firms F_1 and F_2 are 1179.39 and 976.85, respectively. Moreover, the utilities of service providers C_1 and C_2 are 8743.66 and 5340.84, respectively. The demand market receives an amount of 132.37 of the product manufactured by F_1 from service provider C_1 and an amount of 70.05 from C_2 . Firm F_2 sends 144.51 units via C_1 and 100.14 units by C_2 .

Due to the added competition at the manufacturers' level, the quality and price of the product manufactured at firm F_1 have declined as compared to Example 6.3.

This was expected since to attain more market share, the prices would be lowered, which would result in a lowering of quality levels. The utility of F_1 is higher than that of F_2 . A product with reduced prices and quality levels would require cheaper prices (and, hence, quality) of the transporters. Resultantly, prices and quality levels of freight service provider C_1 carrying products from F_1 have also been reduced. It is interesting to note that even though the price and quality level of C_2 transporting the product manufactured by F_2 are the highest of all $(p_{221}^{1*}; q_{221}^{1*})$, more demand for F_2 is satisfied by service provider C_2 (100.14) than that of F_1 (70.05). The prices and quality levels of provider C_2 transporting goods of manufacturer F_1 are at par with that of provider C_1 . Clearly, both manufacturers prefer freight service provider C_1 to freight service provider C_2 .

Variant of Example 6.4

I now construct a variant of Example 6.4 wherein the demand function is more sensitive to the price of the product manufactured and the quality offered by the service providers. Keeping the other data consistent, the demand functions are, hence, modified to the following:

$$\begin{aligned}
 d_{111}^1 &= 43 - 1.44p_{111}^1 + 1.53q_{111}^1 - 1.82p_1 + 1.21q_1 + 0.08p_2 - 0.04q_2 + 0.03p_{111}^2 \\
 &\quad - 0.2q_{111}^2 + 0.04p_{121}^1 - 0.1q_{121}^1, \\
 d_{111}^2 &= 52 - 1.49p_{111}^2 + 1.65q_{111}^2 - 1.82p_1 + 1.21q_1 + 0.08p_2 - 0.04q_2 + .03p_{111}^1 \\
 &\quad - 0.2q_{111}^1 + 0.04p_{121}^1 - 0.1q_{121}^1, \\
 d_{121}^1 &= 47 - 1.57p_{121}^1 + 1.64q_{121}^1 - 1.82p_1 + 1.21q_1 + 0.08p_2 - 0.04q_2 + .03p_{111}^1 \\
 &\quad - 0.2q_{111}^1 + 0.04p_{111}^2 - 0.1q_{111}^2,
 \end{aligned}$$

$$\begin{aligned}
d_{211}^1 &= 51 - 1.39p_{211}^1 + 1.66q_{211}^1 - 1.88p_2 + 1.25q_2 + 0.08p_1 - 0.04q_1 + .04p_{211}^2 \\
&\quad - 0.1q_{211}^2 + 0.02p_{221}^1 - 0.12q_{221}^1, \\
d_{211}^2 &= 44 - 1.42p_{211}^2 + 1.58q_{211}^2 - 1.88p_2 + 1.25q_2 + 0.08p_1 - 0.04q_1 + .04p_{211}^1 \\
&\quad - 0.1q_{211}^1 + 0.02p_{221}^1 - 0.12q_{221}^1, \\
d_{221}^1 &= 56 - 1.40p_{221}^1 + 1.63q_{221}^1 - 1.88p_2 + 1.25q_2 + 0.08p_1 - 0.04q_1 + .04p_{211}^1 \\
&\quad - 0.1q_{211}^1 + 0.02p_{211}^2 - 0.12q_{211}^2.
\end{aligned}$$

The equilibrium solution, computed after 568 iterations, is:

$$\begin{aligned}
p_{111}^{1*} &= 8.30, & p_{111}^{2*} &= 64.70, & p_{121}^{1*} &= 15.54, & p_1^* &= 25.02, \\
p_{211}^{1*} &= 28.70, & p_{211}^{2*} &= 18.47, & p_{221}^{1*} &= 36.15, & p_2^* &= 21.38, \\
q_{111}^{1*} &= 9.00, & q_{111}^{2*} &= 96.71, & q_{121}^{1*} &= 16.16, & q_1^* &= 22.71, \\
q_{211}^{1*} &= 28.34, & q_{211}^{2*} &= 17.19, & q_{221}^{1*} &= 38.55, & q_2^* &= 19.24.
\end{aligned}$$

At equilibrium, the utilities of manufacturing firms F_1 and F_2 are 2037.45 and 1511.87, and that of freight service providers C_1 and C_2 are 1729.44 and 737.02. It is important to note that, based on the previous equilibrium solutions, the utilities of the freight service providers were higher than those of the manufacturers. However, based on the variant's solution, the utilities of the freight service providers (focus on quality) are lower than the utilities of the manufacturers (focus on price). This is directly connected to the transportation costs which increase in order to ensure high quality transportation. Demand market 1 receives 104.81 units of F_1 's product from

service provider C_1 and 23.37 units from C_2 . Also, the demand market receives 62.52 units of F_2 's product via C_1 and 49.79 via C_2 .

Example 6.5 and Variant

In this example and its variant, I extend the previous ones by adding another demand market to the supply chain network; see Figure 6.8. The manufacturers and freight service providers compete to serve two demand markets now.

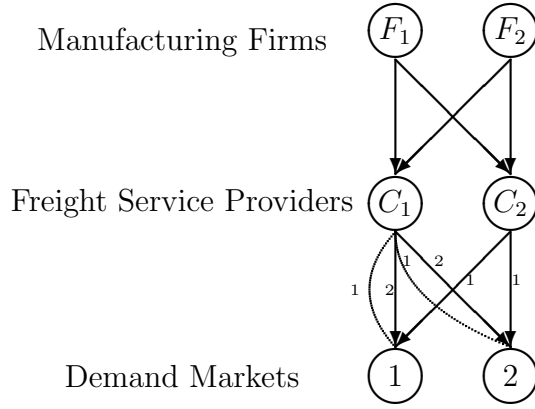


Figure 6.8. The Supply Chain Network Topology for Example 6.5 and Variant

The demand functions at demand market 2 for manufacturing firm F_1 are:

$$\begin{aligned}
 d_{112}^1 &= 50 - 1.63p_{112}^1 + 1.55q_{112}^1 - 1.48p_1 + 1.74q_1 + 0.06p_2 - 0.05q_2 + .05p_{112}^2 \\
 &\quad - 0.23q_{112}^2 + 0.02p_{122}^1 - 0.13q_{122}^1, \\
 d_{112}^2 &= 39 - 1.78p_{112}^2 + 1.21q_{112}^2 - 1.48p_1 + 1.74q_1 + 0.06p_2 - 0.05q_2 + .05p_{112}^1 \\
 &\quad - 0.23q_{112}^1 + 0.02p_{122}^1 - 0.13q_{122}^1, \\
 d_{122}^1 &= 42 - 1.66p_{122}^1 + 1.41q_{122}^1 - 1.48p_1 + 1.74q_1 + 0.06p_2 - 0.05q_2 + .05p_{112}^1 \\
 &\quad - 0.23q_{112}^1 + 0.02p_{112}^2 - 0.13q_{112}^2,
 \end{aligned}$$

and for manufacturing firm F_2 are:

$$\begin{aligned}
d_{212}^1 &= 38 - 1.49p_{212}^1 + 1.34q_{212}^1 - 1.61p_2 + 1.86q_2 + 0.06p_1 - 0.05q_1 + .05p_{212}^2 \\
&\quad - 0.09q_{212}^2 + 0.03p_{222}^1 - 0.08q_{222}^1, \\
d_{212}^2 &= 43 - 1.57p_{212}^2 + 1.26q_{212}^2 - 1.61p_2 + 1.86q_2 + 0.06p_1 - 0.05q_1 + .05p_{212}^1 \\
&\quad - 0.09q_{212}^1 + 0.03p_{222}^1 - 0.08q_{222}^1, \\
d_{222}^1 &= 58 - 1.53p_{222}^1 + 1.31q_{222}^1 - 1.61p_2 + 1.86q_2 + 0.06p_1 - 0.05q_1 + .05p_{212}^1 \\
&\quad - 0.09q_{212}^1 + 0.03p_{212}^2 - 0.08q_{212}^2.
\end{aligned}$$

The supply functions for both manufacturers are changed in the following manner:

$$\begin{aligned}
s_1 &= d_{111}^1 + d_{111}^2 + d_{121}^1 + d_{112}^1 + d_{112}^2 + d_{122}^1, \\
s_2 &= d_{211}^1 + d_{211}^2 + d_{221}^1 + d_{212}^1 + d_{212}^2 + d_{222}^1.
\end{aligned}$$

There is no change to the utility functions of the manufacturing firms. However, the transportation functions of freight service provider C_1 have been changed to:

$$\begin{aligned}
TC_{111}^1 &= .5d_{111}^1 + (q_{111}^1)^2 + .0045d_{121}^1 + .0045d_{221}^1 + .0045d_{211}^1 + .0045d_{112}^1, \\
TC_{111}^2 &= .45d_{111}^2 + .54(q_{111}^2)^2 + .0011d_{211}^2 + .0011d_{212}^2, \\
TC_{211}^1 &= .68d_{211}^1 + .79(q_{211}^1)^2 + .002d_{111}^1 + .002d_{121}^1 + 0.002d_{212}^1, \\
TC_{211}^2 &= .57d_{211}^2 + .74(q_{211}^2)^2 + .005d_{111}^2 + .005d_{212}^2, \\
TC_{112}^1 &= .61d_{112}^1 + .7(q_{112}^1)^2 + .0037d_{111}^1 + .0037d_{122}^1 + 0.0037d_{212}^1, \\
TC_{112}^2 &= .52d_{112}^2 + .58(q_{112}^2)^2 + .0024d_{212}^2,
\end{aligned}$$

$$TC_{212}^1 = .49d_{212}^1 + .59(q_{212}^1)^2 + .0017d_{112}^1 + .0017d_{122}^1,$$

$$TC_{212}^2 = .43d_{212}^2 + .55(q_{212}^2)^2 + .0023d_{112}^2,$$

and that of freight service provider C_2 to:

$$TC_{121}^1 = .64d_{121}^1 + .76(q_{121}^1)^2 + .0015d_{221}^1,$$

$$TC_{221}^1 = .59d_{221}^1 + .80(q_{221}^1)^2 + .014d_{121}^1 + .014d_{111}^1 + .014d_{211}^1,$$

$$TC_{122}^1 = .67d_{122}^1 + .73(q_{122}^1)^2 + .0031d_{222}^1 + .0031d_{212}^1,$$

$$TC_{222}^1 = .45d_{222}^1 + .58(q_{222}^1)^2 + .012d_{122}^1 + .012d_{112}^1 + .012d_{212}^1.$$

With the same constraints on the prices and quality levels, the utilities of freight service providers become:

$$U_{C_1} = p_{111}^1 d_{111}^1 + p_{111}^2 d_{111}^2 + p_{211}^1 d_{211}^1 + p_{211}^2 d_{211}^2 + p_{112}^1 d_{112}^1 + p_{112}^2 d_{112}^2 + p_{212}^1 d_{212}^1 + p_{212}^2 d_{212}^2$$

$$-TC_{111}^1 - TC_{111}^2 - TC_{211}^1 - TC_{211}^2 - TC_{112}^1 - TC_{112}^2 - TC_{212}^1 - TC_{212}^2,$$

$$U_{C_2} = p_{121}^1 d_{121}^1 + p_{221}^1 d_{221}^1 + p_{122}^1 d_{122}^1 + p_{222}^1 d_{222}^1 - TC_{121}^1 - TC_{221}^1 - TC_{122}^1 - TC_{222}^1.$$

The equilibrium solution, after 254 iterations, is:

$$p_{111}^{1*} = 56.79, \quad p_{111}^{2*} = 55.45, \quad p_{112}^{1*} = 72.96, \quad p_{112}^{2*} = 36.93,$$

$$p_{121}^{1*} = 55.19, \quad p_{122}^{1*} = 53.55, \quad p_{211}^{1*} = 62.77, \quad p_{211}^{2*} = 53.28,$$

$$p_{212}^{1*} = 72.94, \quad p_{212}^{2*} = 65.91, \quad p_{221}^{1*} = 76.15, \quad p_{222}^{1*} = 83.73,$$

$$p_1^* = 63.76, \quad p_2^* = 64.90, \quad q_1^* = 100.00, \quad q_2^* = 100.00,$$

$$q_{111}^{1*} = 39.53, \quad q_{111}^{2*} = 51.20, \quad q_{112}^{1*} = 74.61, \quad q_{112}^{2*} = 23.54,$$

$$\begin{aligned}
q_{121}^{1*} &= 50.93, & q_{122}^{1*} &= 51.05, & q_{211}^{1*} &= 46.25, & q_{211}^{2*} &= 36.72, \\
q_{212}^{1*} &= 76.89, & q_{212}^{2*} &= 69.56, & q_{221}^{1*} &= 61.18, & q_{222}^{1*} &= 94.70.
\end{aligned}$$

In this example, I consider competition at the manufacturers' level, the freight service providers' level, and between modes of a particular service provider, wherein all these players are competing to satisfy the demands at two different demand markets. This makes the problem quite complex. The assumption regarding the demand functions being more sensitive to the quality of the product manufactured and the prices of the service providers remains valid in this example as well. The equilibrium solution ($p_1^* = 63.76; q_1^* = 100.00; p_2^* = 64.90; q_2^* = 100.00$) supports this assumption. The price and quality levels have gone up as compared to Example 6.4 since there are two demand markets to be satisfied now as opposed to one.

The utilities of manufacturers F_1 and F_2 have increased to 15244.22 and 19922.55, respectively. Also, the freight service providers C_1 and C_2 are now witnessing higher utilities of 29256.82 and 16905.45, respectively. Since more demand from multiple demand markets has increased the prices and quality levels of products, the utilities have increased. The results indicate that service provider C_1 transports an amount of 279.46 to demand market 1 and an amount of 381.13 to demand market 2. Also, service provider C_2 carries an amount of 207.96 to demand market 1 and 215.20 to demand market 2.

As there is enough demand for products of both manufacturers F_1 and F_2 , the prices of the products are high and the quality levels are at their upper bounds of 100. This happens since the emphasis is on quality rather than price for manufacturers. Resultantly, the overall prices and quality levels of the two freight service providers also go up as compared to Example 6.4.

Variant of Example 6.5

Once again, I consider a variant wherein the demand functions are more sensitive to the price of the product manufactured and the quality offered by the freight service providers. Keeping the other data consistent, the demand functions are, hence, modified to the following:

$$\begin{aligned}
 d_{112}^1 &= 50 - 1.37p_{112}^1 + 1.67q_{112}^1 - 1.91p_1 + 1.33q_1 + .06p_2 - .05q_2 + .05p_{112}^2 - .23q_{112}^2 + .02p_{122}^1 - .13q_{122}^1, \\
 d_{112}^2 &= 39 - 1.41p_{112}^2 + 1.65q_{112}^2 - 1.91p_1 + 1.33q_1 + .06p_2 - .05q_2 + .05p_{112}^1 - .23q_{112}^1 + .02p_{122}^1 - .13q_{122}^1, \\
 d_{122}^1 &= 42 - 1.35p_{122}^1 + 1.70q_{122}^1 - 1.91p_1 + 1.33q_1 + .06p_2 - .05q_2 + .05p_{112}^1 - .23q_{112}^1 + .02p_{122}^2 - .13q_{122}^2, \\
 d_{212}^1 &= 38 - 1.33p_{212}^1 + 1.59q_{212}^1 - 1.87p_2 + 1.29q_2 + .06p_1 - .05q_1 + .05p_{212}^2 - .09q_{212}^2 + .03p_{222}^1 - .08q_{222}^1, \\
 d_{212}^2 &= 43 - 1.36p_{212}^2 + 1.67q_{212}^2 - 1.87p_2 + 1.29q_2 + .06p_1 - .05q_1 + .05p_{212}^1 - .09q_{212}^1 + .03p_{222}^1 - .08q_{222}^1, \\
 d_{222}^1 &= 58 - 1.42p_{222}^1 + 1.68q_{222}^1 - 1.87p_2 + 1.29q_2 + .06p_1 - .05q_1 + .05p_{212}^1 - .09q_{212}^1 + .03p_{212}^2 - .08q_{212}^2.
 \end{aligned}$$

The equilibrium solution, after 769 iterations, is:

$$\begin{aligned}
 p_{111}^{1*} &= 22.05, & p_{111}^{2*} &= 80.01, & p_{112}^{1*} &= 44.02, & p_{112}^{2*} &= 77.79, \\
 p_{121}^{1*} &= 46.56, & p_{122}^{1*} &= 71.98, & p_{211}^{1*} &= 62.01, & p_{211}^{2*} &= 47.77, \\
 p_{212}^{1*} &= 82.80, & p_{212}^{2*} &= 85.62, & p_{221}^{1*} &= 64.72, & p_{222}^{1*} &= 85.00, \\
 p_1^* &= 43.78, & p_2^* &= 52.86, & q_1^* &= 85.79, & q_2^* &= 100.00, \\
 q_{111}^{1*} &= 9.00, & q_{111}^{2*} &= 100.00, & q_{112}^{1*} &= 39.34, & q_{112}^{2*} &= 100.00, \\
 q_{121}^{1*} &= 49.85, & q_{122}^{1*} &= 82.99, & q_{211}^{1*} &= 61.55, & q_{211}^{2*} &= 46.18, \\
 q_{212}^{1*} &= 100.00, & q_{212}^{2*} &= 100.00, & q_{221}^{1*} &= 65.62, & q_{222}^{1*} &= 100.00.
 \end{aligned}$$

The utilities of firms F_1 and F_2 are 6333.31 and 10285.25, respectively. The utilities of freight service providers C_1 and C_2 are 18654.58 and 10277.76, respectively.

As expected, the utilities are increasing from those in Example 6.3 onwards. This particular variant registers the highest. Since the focus of the freight service providers is on quality, there are multiple cases wherein the quality levels of the providers are at their upper bounds. The demand markets have grown which lets the manufacturers and freight service providers increase their prices and quality levels. Higher quality levels, however, ensure that the transportation costs go up which, in turn, reduces the utilities of the freight service providers.

6.5. Summary and Conclusions

In this chapter, I developed a game theory supply chain network model in both static and dynamic versions with multiple manufacturers and freight service providers competing on price and quality. This multi-faceted inclusion of competition in the model assesses the quality conformance level of the product and the level of service of freight service providers along with the prices at which the products and the transportation services were offered. The model handles multiple modes of transportation for delivery of products. The utility of each manufacturer (or service provider) depends on the prices and on the quality levels of the products (or shipment services) he offers as well as those of other competitors.

Variational inequality theory was employed in the formulation of the equilibrium governing the manufacturers' and freight service providers' behaviors with respect to price and quality followed by the rigorous description of the underlying dynamic interactions until a stationary point, equivalently, an equilibrium is achieved. The dynamics were shown to satisfy a projected dynamical system. The computational procedure utilized was the Euler method. The discrete-time algorithm, also serving as an approximation to the continuous-time trajectories, yields an equilibrium price and quality patterns for the manufacturers and the freight service providers.

In order to demonstrate the generality of the framework and the computational scheme, I then provided solutions to a series of numerical examples, beginning with smaller scale examples. In the larger examples, a scenario and its variant were explored while computing and analyzing the solutions for various combinations of manufacturing firms, freight service providers, and modes of transportation. The competition within echelons of the different examples altered the price and quality levels, and, thereby, the utilities, of the entities. I considered a scenario wherein the demand functions were more sensitive to the quality of the product manufactured and the price charged by the freight service providers. The variant took a contrasting position, whereby the demand markets were giving more importance to the price of the product manufactured and the quality levels offered by the freight service providers. These contradictory situations brought about interesting comparisons between the utilities of the manufacturers and the freight service providers and how they changed when the emphasis on price and quality levels changed.

There are many aspects to this proposed framework that are worthy of further discussion and investigation. For instance, additional tiers of supply chain decision-makers could be included. The quality levels might be explicitly modeled for the freight service providers in terms of time-conformance of delivery, reliability of the service, emission standards (to compare the environmental viability of various modes), the quality of in-house transportation infrastructure, and so on. It is interesting to note from the results of this model that in order to capture a higher market share, manufacturers or freight service providers might try to quote a lower price and offer a lower quality level (leading to a lower cost). However, a lower quality product/service might not be able to sustain the market share.

This work fills the gap in the existing literature by capturing quality in transportation as well as production in a multitiered competitive supply chain network, along

with prices as strategic variables. It provides a critical foundation for future research in this area.

CHAPTER 7

CONCLUSIONS AND FUTURE RESEARCH

7.1. Conclusions

The Internet is a network of service providers which provides connections between entities and offers numerous distributed applications and services (Wolf et al. (2012)). Customers' demands are driving the Internet and telecommunication networks towards providing quality-based services. However, providers face many challenges in determining technical and economic solutions to price and bill their services and to establish economic relationships with other providers that are necessary to deliver end-to-end services. Although the underlying technology associated with the existing Internet is rather well-understood, the economics of the associated services have been less studied.

Quality-based demands are not limited to the Internet and communication networks, and supply chain networks are under many pressures to offer differentiated products and services (Li and Nagurney (2015)). Nowadays, customers demand quality products, quality distribution and shipment, quality services, and increasing value. Success in a supply chain network is determined by how well the entire supply chain performs, rather than by the performance of its individual entities (Floden, Barthel, and Sorkina (2010) and Saxin, Lammgard, and Floden (2005)). Logistic and freight companies as the main components of any supply chain need to be strategic in delivering the goods to meet the customers' satisfaction and improve their competitive advantages.

The purpose of this dissertation was to model, analyse, and compute solutions to game theory problems based on communication and supply chain networks with a focus on quality of service and price competition between decision-makers in the Internet and freight shipment networks.

In this dissertation, the methodologies utilized were variational inequality theory, network theory, game theory, optimization theory, and also projected dynamical systems theory to develop oligopolistic models for the service-oriented Internet and transportation networks in supply chains and to study the underlying dynamics.

Specifically, in Chapter 3, I considered a basic and a general game theory model for a two-sided service-oriented Internet network. The basic model focused on a noncooperative game between two profit maximizers, a content service provider and a network service provider. Both providers optimized their profits as the difference between their total revenue and total cost to set the best price and quality of service for a demand market. The network provider's quality of service was defined as the expected delay according to Kleinrock function (Altman, Legout, and Xu (2011)). Therefore, the total cost of transmission depended on the bandwidth usage and it increased as the demand for higher quality intensified. The variational inequality of this game was derived and the fee that the network provider charged the content provider was analyzed. The result showed that if consumers at the demand market are more sensitive to the price that the network provider charges them in comparison with the price that the content provider charges them, then the network provider is better off charging the content provider. The second part of Chapter 3 contained a general model for a service-oriented Internet network, including multitiered network and content providers and heterogeneous demand markets. In this network economic game theory model, all providers were also profit maximizers and competed in a noncooperative fashion to offer the best prices and quality levels to the demand markets. The users

at the demand markets signaled their preferences through demand functions which increased (decreased) as the price (quality) decreased. The equilibrium model's equivalent variational inequality formulation with nice features for computational purposes was provided and the Euler method was applied to solve some numerical examples. The sensitivity analysis of the price, that the network providers charge the content providers, demonstrated that the social welfare or summation of all providers' utilities would be maximized if the network providers charge the content providers equally.

In Chapter 4 of this dissertation, I extended the general model provided in the second part of Chapter 3 to construct a dynamic network economic model of a service-oriented Internet with price and quality competition using projected dynamical systems theory. I captured the dynamics of both price and quality competition of the content providers and the network providers. This continuous-time dynamic model describes the evolution of the prices charged by the content providers and the network providers, in addition to the quality levels of content and network transport provision. The set of equilibrium/stationary points coincided with the set of solutions to the associated variational inequality problem. Also, the conditions, under which the dynamic of the continuous-time adjustment process approached a stationary point, were investigated. Then, the qualitative results, including stability analysis along with a discrete-time algorithm were provided for the iterative computation and tracking of the prices and quality levels until the stationary point was achieved. The Euler method, which provides a discretization of the continuous-time adjustment process and yields closed form expressions for the prices and the quality levels at each iteration step, was proposed to solve numerical examples.

Chapter 5 emphasized time-based pricing for a service-oriented Internet network. A competitive oligopoly market of Internet network providers that captures the economic

relationships, motivated by the ChoiceNet project, was formulated. The network providers offered different network services and created contracts for their users according to the users' desires and needs. Three main criteria including the amount of usage contracted per period of time (the usage rate), the quality level of service, and the contract duration were considered for users' contract selection. Here, I assumed a reserved usage amount per unit of time. Then, the variational inequality formulation of the service-oriented Internet network equilibrium was presented and the existence and uniqueness of the result were discussed. After presenting the explicit formulae and convergence for the equilibrium, the Euler method was used to solve multiple problem sets. The numerical results were consistent with today's Internet pricing from service providers such as COMCAST. The outcome from sensitivity analysis revealed that the network providers can benefit when they provide higher levels of quality to their demand markets.

The similarities and analogies between the service-oriented Internet networks and supply chain networks prompted me to also focus on supply chain networks in Chapter 6 of this dissertation. Chapter 6 developed game theory models in both equilibrium and dynamic settings for a supply chain network with multiple manufacturers and multiple freight service providers handling freight transportation. The manufacturers and freight service providers competed in prices they offer for production and shipment. Quality of the product was traced along the supply chain with consumers at demand markets differentiating among the products offered by the manufacturers. In addition, quality of freight service provision was accounted for with the model with the providers competing on quality as well. In this model, multiple modes of transportation for each freight service provider were allowed. A mode was considered in a general way and can also correspond to intermodal transportation. The qualitative properties of the equilibrium price and quality pattern were provided. Also, the underlying dynamics, associated with the evolution of the prices and quality levels over time, were presented.

The results from numerical examples demonstrated that to capture a higher market share in an oligopoly market, manufacturers or freight service providers might try to offer a lower price and a lower quality level. However, a lower quality level for the product or freight service might not be able to sustain the market share.

7.2. Future Research

While there are certainly several possible and fruitful directions for my future research, here I discuss topics I intend to pursue in the near future.

The model, developed in Chapter 3, provides a basic framework for the analysis of price competition in service-oriented networks. Nowadays, the highly dynamic and competitive business environment makes the decision-making of the networks' entities increasingly complex. To survive and thrive, the providers need to have intelligent and consistent strategies to provide optimal decisions today and benefit from them in the future (Friesz et al. (2008)). I plan to extend the general model in Chapter 3 for analyzing the price and quality levels over the multi-period planning horizon (see Liu and Nagurney (2012)), not only in the Internet network but also in other networks including freight transportation networks, tourism supply chains, and the hospitality industry where quality of service is an emerging matter of fact (Tian et al. (2013)).

The dynamics of the flows in the Internet network and freight transportation network not only change over time (multi-period horizon), but also are time-dependant (Nagurney, Parkes, and Daniele (2007)). Emerging new services and applications in communication and transportation networks fluctuate the customers' demand and result in time-varying demand rate pattern. I plan on expanding the theoretical framework of Chapter 6 to consider an evolutionary variational inequality formulation. In this case, time-dependant demand functions will be considered to study dynamic

problems and represent model adjustment processes and equilibria with lags in the Internet and freight transportation networks.

I am also interested in theoretically and empirically exploring and expanding the research on modeling multitiered networks where multiple decision-makers face conflicting objectives in a competitive economic environment. In the future, we can expect to see more dynamic service offerings, as Internet users move from long-term service provider agreements to more opportunistic service models. Inspired by the work in Nagurney et al. (2013a) and Saberi, Nagurney, and Wolf (2014), I would like to develop innovative modeling frameworks for pricing Internet services which involve uncertainty in the network demand, hierarchical structure of service-providers in the network, and spectrum sharing issues among providers.

As was discussed in Chapter 4, because of various limitations in the current Internet architecture, new network architectures are being explored. For any new network architecture design, there is a critical need to point to the real-world challenges such as path selection across domains or proper economic relationships between entities operating and using the network (Jain, Durrezi, and Paul (2011) and Wolf et al. (2014)) and to deploy innovative solutions for these challenges. Nagurney, Dong, and Zhang (2002) modeled a competitive supply chain network to handle many decision-makers and their independent behaviors. This model captures both the independent behavior of the various decision-makers as well as the effects of their interactions. It can evaluate both price and product flows in this supply chain. To model the competitive behavior of the network providers in the Internet, I plan to extend the model of Nagurney, Dong, and Zhang (2002) to construct a novel oligopoly model with multiple tiers of providers in both the Internet and in supply chain networks with freight transportation and investigate and investigate the pricing strategies among network providers and freight carriers in each such network, respectively.

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