Excited Heavy Mesons From Lattice QCD

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Abstract. I discuss some recent investigations of excited mesons using first-principles lattice QCD calculations. Over the last few years we have made significant advances in studying near-threshold states, resonances and related scattering phenomena. I illustrate this progress by presenting results from scattering channels involving charm mesons, including DK scattering relevant for the enigmatic $D_s(2317)$ and $D\bar{K}$ scattering. I also comment on applications to other channels and future prospects.

INTRODUCTION

The last decade has witnessed a renewed interest in hadron spectroscopy driven by a wealth of data from experiments. A number of 'puzzling' observations have been made and, as we have heard at this conference, these are the subject of active discussions and ongoing experimental investigations. Particular examples include the various "X,Y,Z's" in charmonium, charged charmonium-like structures and the charm-strange $D_{s0}(2317)$. There has been a lot of speculation as to the nature of these structures and various QCD-inspired approaches have been used in attempts to describe them and their properties. Possible interpretations include tetraquarks, molecular states of hadrons, hadro-charmonia and hybrid mesons where the gluonic field is excited. States with exotic quantum numbers, i.e. those that cannot arise from solely a quark-antiquark pair, are particularly interesting because they are a smoking gun for physics beyond a simple quark model. For example, exotic spin (J), parity (P), charge-conjugation (C) combinations (e.g. $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$) or exotic flavour states (e.g. charmonium and bottomonium-like states with non-zero charge).

Lattice QCD provides a method to perform ab-initio calculations in the non-perturbative regime of QCD: four-dimensional space-time is discretised on a finite four-dimensional hypercubic lattice and the calculation of quantities in the path integral formulation then becomes an ordinary (but very large) integration problem. If a Euclidean (imaginary-time) space-time metric is used, the integrals can be evaluated effectively using importance-sampling Monte Carlo methods. The masses and other properties of hadrons are then extracted from analyses of correlation functions involving interpolating operators built from quark and gluon fields. Calculations of the low-lying spectrum of hadrons have long been benchmarks of lattice methods but only in the last few years has there been significant progress in using lattice QCD to study excited hadrons and, even more recently, in investigating unstable and near-threshold states.

In these proceedings I briefly summarise some of the Hadron Spectrum Collaboration's recent work in the charm sector; other Hadron Spectrum Collaboration results are discussed elsewhere [1]. A more general review of recent lattice results for hadron spectroscopy can be found in [2]. I begin with a calculation of the spectrum of excited charmonia before discussing DK and $D\bar{K}$ scattering and then concluding with an outlook.

EXCITED CHARMONIA

In recent years, through the development of a combination of novel techniques we have made significant progress in using lattice QCD to compute spectra of excited light isovector [5, 6] and isoscalar [7, 8] mesons, charmonia[3] and open-charm mesons[9]. Figure 1 shows the spectra of excited charmonia from a calculation with dynamical strange and degenerate up and down quarks corresponding to $m_\pi \approx 400$ MeV. The lattice is anisotropic with a temporal lattice spacing, a_t, finer than the spatial lattice spacing, $a_s \approx 0.12$ fm, and $\xi \equiv a_s/a_t \approx 3.5$; the spatial extent is $L \approx 2.9$ fm

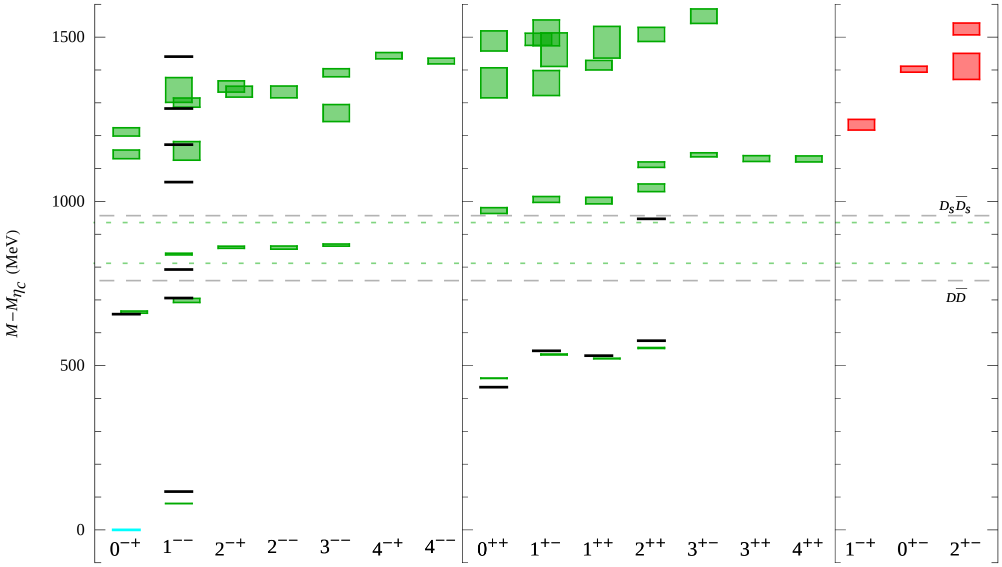

FIGURE 1. Charmonium spectrum labelled by J^{PC} from Ref. [3]. Red and green boxes are from a lattice QCD calculation with $m_\pi \approx 400$ MeV and black lines are experimental values from the PDG [4]. The calculated (experimental) masses are shown with the calculated (experimental) η_c mass subtracted. The vertical size of the boxes represents the one sigma statistical uncertainty on either side of the mean. Dashed lines indicate the lowest non-interacting $D\bar{D}$ and $D_s\bar{D}_s$ levels using calculated D and D_s masses (fine green dashing) and using the experimental masses (coarse grey dashing).

($L/a_s = 24$). Only connected contributions to the charmonium correlators are included; full details of the calculations and results from another lattice volume are given in Ref. [3].

The computed spectrum in Figure 1 includes many states with exotic J^{PC} quantum numbers and these are shown in the right panel; by considering the overlaps of states with interpolating operators [6, 10] we identify them as hybrid mesons. In the non-exotic channels the majority of the states can be understood in terms of quark-model $^{2S+1}L_J$ multiplets (L is the orbital angular momentum and S the total quark spin) but there are also some states that do not fit into this pattern; again, by considering operator-state overlaps, we identify these as non-exotic hybrids. In Ref. [3] we highlight the states identified as hybrids and show that the pattern can be interpreted as a colour-octet quark-antiquark pair coupled to a 1^{+-} chromomagnetic gluonic excitation. The lightest gluonic excitation appears at an energy scale $\sim 1.2 - 1.3$ GeV above the lightest conventional meson. This pattern and energy scale are consistent with what was found in the baryon, light meson and open-charm meson sectors.

As discussed in Ref. [3], we do not see any clear evidence for multi-hadron states in our extracted spectra. The study only considered correlation functions containing fermion-bilinear interpolating operators; to reliably study multi-hadron states we need to enlarge the basis of operators to include those with more fermion fields, something I will return to in the next section. We note that states above threshold can have large hadronic widths and a conservative approach is to only consider the above mass values accurate up to the hadronic width [6, 11].

DK SCATTERING

The vast majority of mesons are unstable and decay strongly to two or more lighter hadrons. Moreover, many of the unexplained structures that have been observed in experiments are near or above strong-decay thresholds. It is therefore essential to study resonances, near-threshold states and related phenomena within QCD. However, in the Euclidean formulation of lattice QCD direct access to dynamical properties is lost. The Lüscher method [12] and its extensions allow, in principle, at least in certain cases, indirect access to infinite-volume scattering observables from the discrete spectrum of multi-hadron states in a finite volume. The Hadron Spectrum Collaboration has developed techniques to compute excited multi-hadron spectra with a high statistical precision using carefully constructed multi-

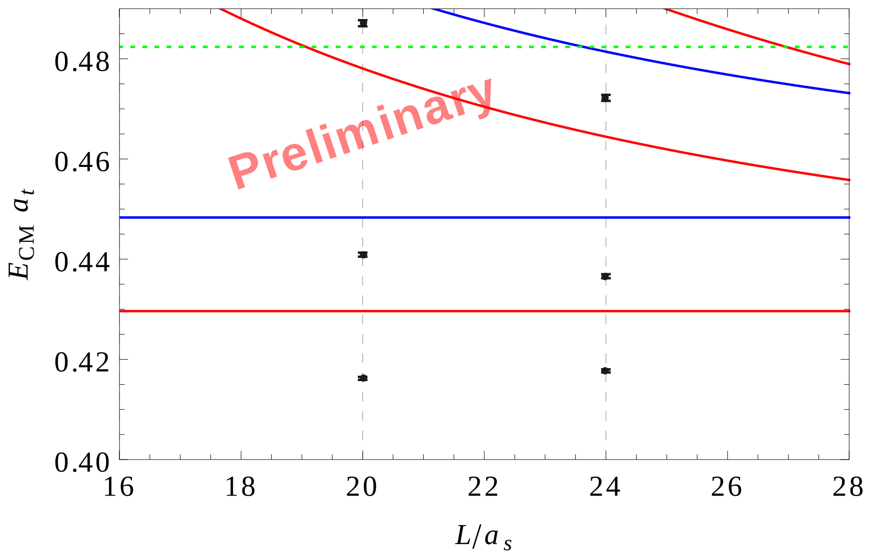

FIGURE 2. Preliminary charm-strange finite-volume spectrum (strangeness = 1, charm = 1, isospin = 0) as a function of the spatial lattice extent L with overall zero momentum and scalar quantum numbers (A_1^+ irrep of the octahedral group). Black points show the CM-frame energies (with statistical uncertainties) computed on two different lattice volumes. Lines/curves are non-interacting energies: red are DK, blue are $D_s\eta$ and the dotted-green line is the $D_s\pi\pi$ threshold.

hadron interpolating operators [13] and then determine scattering amplitudes using extensions of the Lüscher method. Our work in the light-meson sector [13, 11, 14, 15, 16] and an overview of the methodology have been discussed elsewhere at this conference [1]; here I will briefly summarise some preliminary results in the charm sector.

To investigate charm-strange mesons in the energy region where elastic isospin-0 DK scattering is the only relevant channel, i.e. below $D_s\eta$ threshold, we have computed the finite-volume spectra in a number of relevant quantum-number channels. This is relevant for, amongst other things, the enigmatic $D_{s0}(2317)$, a charm-strange meson with $J^P = 0^+$ that appears just below DK threshold and is narrow [17], while in quark-potential models it was generally expected to be broad and above threshold. The lattice setup is as above, $m_\pi \approx 400$ MeV and the computations were performed on two volumes with spatial extents $L/a_s = 20$ and 24.

Reliably determining the finite-volume energies necessitates the computation of correlation functions containing 'two-meson' DK operators as well as fermion-bilinear operators. Each energy level extracted constrains the scattering matrix, in the case of elastic scattering parameterized by a phase shift $\delta(E_{\mathrm{CM}})$, at that centre-of-mass (CM) frame energy E_{CM}. In order to more fully constrain the energy-dependence of the phase shift, we consider, as well as systems at zero momentum, systems with an overall non-zero momentum in the frame of the lattice; these give additional E_{CM} values when boosted to the CM frame.

Figure 2 shows an example of a computed spectrum with zero overall momentum and scalar quantum numbers (A_1^+ irrep of the octahedral group) relevant for isospin-0 DK scattering in S-wave. There appears to be a significant interaction below $D_s\eta$ threshold and the 'extra' level in this energy region may signal the presence of a near-threshold bound state or resonance. These features are also seen in spectra with non-zero overall momentum.

To make these statements more quantitative, we use extensions of the Lüscher method to determine the S-wave scattering phase shift δ from the computed spectra. A number of different parameterizations of the scattering matrix (effective range, Breit-Wigner and various K-matrix parameterizations) are considered and for each the best-fit parameters are determined by fitting the energy levels given by the parameterization to the computed energy levels [11, 14, 15]. Up to 35 energy levels with various relevant quantum numbers and overall momenta are considered on the two volumes. An example of the energies given by such a parameterization is shown in Figure 3 where there is seen to be good agreement with the computed energies.

With a parameterization of the scattering matrix in hand we can determine where its poles are – these correspond to resonances, bound states, etc. For a range of different parameterizations that are able to describe the data well, we find a pole on the real axis of the physical sheet below threshold with $a_t m \approx 0.420$ ($a_t \Delta m \approx 0.010$ below threshold) corresponding to a bound state with a mass ≈ 2380 MeV (≈ 55 MeV below threshold). This can be compared to the

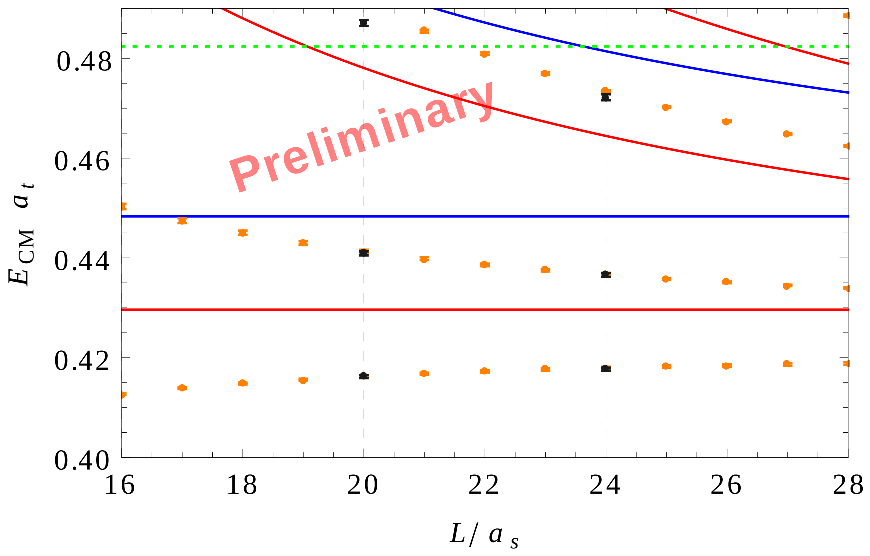

FIGURE 3. As Figure 2 with the addition of the finite-volume energy levels given by an example parameterization of the scattering matrix (orange points).

experimentally-observed $D_{s0}(2317)$ which appears $\approx 40 - 50$ MeV below threshold. Note that in this discussion of the preliminary results, we have ignored various systematic effects including the possible mixing with higher partial waves that can occur because of the reduced symmetry of the finite cubic volume used compared to the symmetry of an infinite volume.

$D\bar{K}$ SCATTERING

We now turn to the flavour-exotic isospin-1 $D\bar{K}$ channel (strangeness = -1, charm = 1, isospin = 1), interesting because any resonance or bound-state here could not be solely a quark-antiquark pair and so would have to be something more exotic (e.g. a molecular meson or a tetraquark). The calculation proceeds in a similar way to that described above, except now there are only 'two-meson' $D\bar{K}$ operators because no fermion-bilinear operators have the required flavour.

Figure 4 shows an example of a computed spectrum with zero overall momentum and scalar quantum numbers (A_1^+ irrep of the octahedral group) relevant for isospin-1 $D\bar{K}$ scattering in S-wave. In contrast to the DK channel discussed above, here the energies are shifted by a small amount above the non-interacting energies suggesting a weak repulsive interaction. This is confirmed by the phase shift extracted using the Lüscher method shown in Figure 5. Note that in these preliminary results, potential mixing with higher partial waves and coupling to other channels above inelastic thresholds have been ignored.

OUTLOOK

In recent years we have made significant advances in studying excited, unstable and near-threshold hadrons in lattice QCD. By robustly extracting a large number of energy levels in many different quantum-number channels, we have been able to map out the energy dependence of the phase shift, or more generally the scattering amplitudes, in unprecedented detail. This has enabled us to reliably determine the presence of resonances, bound-states and other phenomena and to compute their properties. In these proceedings I have briefly summarised some of our recent work in the charm sector including preliminary results on DK and $D\bar{K}$ scattering. Future work will address other scattering channels, for example those relevant for the "X,Y,Z's" and charged charmonium-like structures. Through these investigations and computing other properties of hadrons, for example their coupling to photons, the prospects for understanding the various puzzling structures within ab-initio QCD calculations look promising.

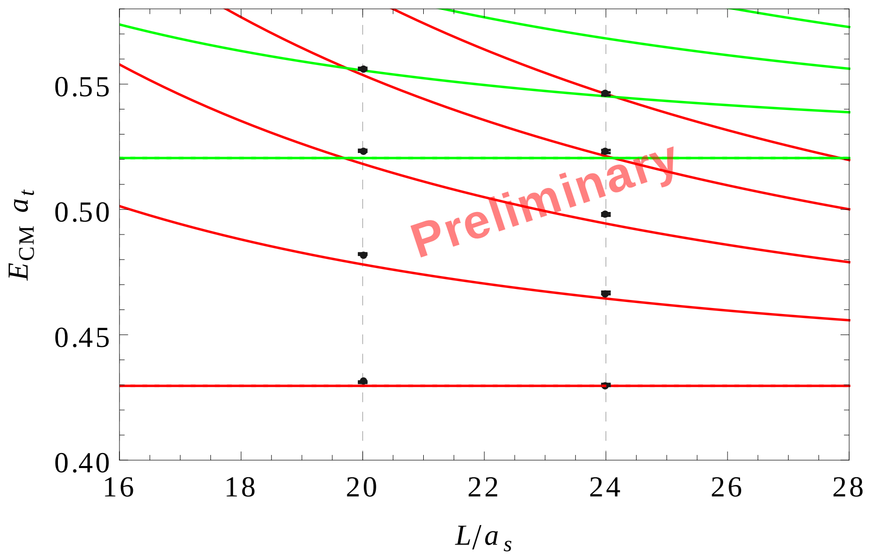

FIGURE 4. Preliminary $D\bar{K}$ finite-volume spectrum (strangeness = -1, charm = 1, isospin = 1) as a function of the spatial lattice extent L with overall zero momentum and scalar quantum numbers (A_1^+ irrep of the octahedral group). Black points show the CM-frame energies (with statistical uncertainties) computed on two different lattice volumes. Lines/curves are non-interacting energies: red are $D\bar{K}$ and green are $D^*\bar{K}^*$.

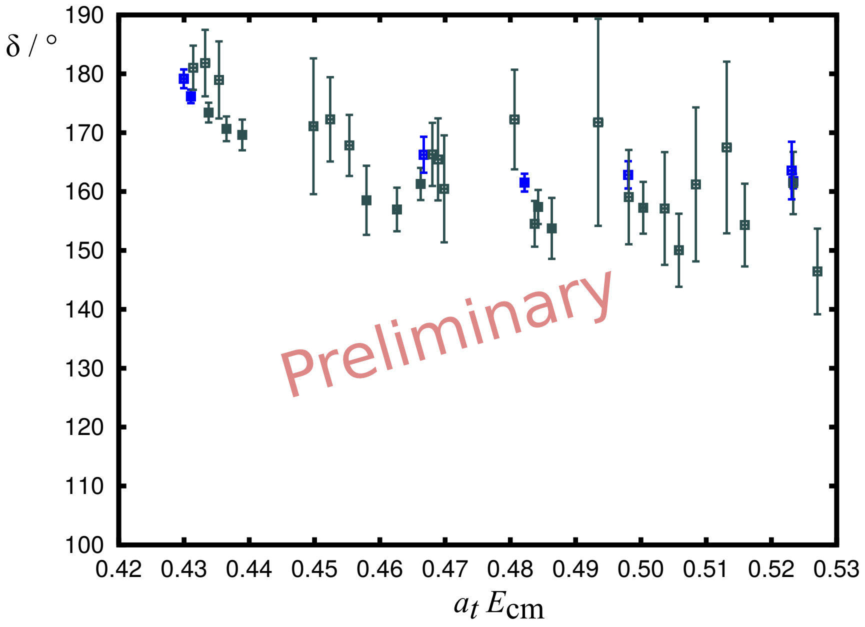

FIGURE 5. Preliminary S-wave scattering phase shift, δ, for $D\bar{K}$ isospin-1 scattering as a function of the CM-frame energy (statistical uncertainties on δ are shown). Blue points were determined for systems at rest relative to the lattice whereas grey points are from systems with overall non-zero momentum. In these preliminary results potential mixing with higher partial waves and coupling to other channels above inelastic thresholds have been ignored.

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