Condition-Based Maintenance of Multi-Component Systems with Degradation State-Rate Interactions

Nipat Rasmekomen, Ajith Kumar Parlikad

Institute for Manufacturing, Cambridge University Engineering Department

(Accepted for publication in Reliability Engineering & Systems Safety)

Abstract: This paper presents an approach to optimise condition-based maintenance (CBM) of multi-component systems where the state of certain components could affect the rate of degradation of other components, i.e., state-rate degradation interactions. We present a real example of an industrial cold box in a petrochemical plant, where data collected on fouling of its tubes show that the extent of fouling of one tube affects the rate of fouling of other tubes due to overloading. A regression model is used to characterise the state-rate degradation interactions for this example. Further, we optimise the condition-based maintenance policy for this system using simulated annealing. The outcomes of the case study demonstrate that modelling degradation interactions between components in the system can have significant positive impact on CBM policy of the system. The paper therefore tackles a problem that has not been addressed in the literature, paving way for further developments in this important area of research with practical applications.

Keywords**:** Maintenance, Degradation, Interactions, Multi-Component Systems, Regression, Simulated Annealing, Optimisation

1 INTRODUCTION

This paper presents an approach to model the degradation interactions that exist between states and rates of degradation of different components in a multi-component system and to optimise the condition-based maintenance policy, i.e., their inspection timings and maintenance/replacement thresholds. The term “multi-component system” is most widely used in academic literature to refer to both a complex system consisting of more than one asset (i.e. multi-asset system) or an engineering asset consisting of more than one component (multi-component asset). Traditional research on maintenance optimisation considered a complex engineering system as a collection of individual (and independent) components, and maintenance models for such systems were devised with this independence in mind (Cho & Parlar, 1991). Research development in this area is often based on an extension of a single component (Barata, et al., 2001) into a maintenance model for independent multi-component systems (Barata, et al., 2002). However, considering the complexities involved in complex engineering systems and the need to extract more value from maintenance activities, it is no longer sensible to treat each component in such systems as an independent individual component.

There are usually dependencies between the components in a complex system and these dependencies lead to further complications in understanding the behaviour of the system. Of particular interest is a class of dependency that is commonly known as ‘stochastic dependence’, where failure or degradation of some components in the system could affect the failure or degradation of other components in the system (Nicolai & Dekker, 2008). This type of dependency is evident in different industries, especially in mechanical systems (Sun, et al., 2009). Literature in this area predominantly focus on the interactions which are triggered by failure of a component (failure interactions). Examples of such studies can be found in (Lai, 2007) and (Zhang et al., 2011).

While failure interactions exhibit stochastic dependencies upon a complete failure of a component in the system, there is another type of stochastic dependence which is not necessarily triggered by a component failure. Such dependencies are also sometimes triggered by degradation of a component. Degradation processes of components in the system could actually be influenced by degradation of a component in a degraded state without having to completely fail. Such incidents are defined here as degradation interactions. There are few papers that study degradation interactions. Straub (2009) have applied Dynamic Bayesian Networks to characterise degradation interactions. The complexity of the algorithm would however make it computationally difficult for modelling the development of multi-component system degradation over time due to the size of the networks required. Meanwhile, Bian and Gebraeel (2013) also did not account for the continuous degradation of the performance (states) of the system.

Extensive review of the literature has revealed that research on degradation interactions has so far mainly been limited to reliability modelling of multi-component systems where the states of the components in a system are dependent. The degradation states of dependent components are characterised by a joint probability distribution, enabling the estimation of the reliability of the system. Multivariate normal distribution (Wang & Coit, 2004), Bayesian Networks (Hu, et al., 2012) or copula functions (Wang & Pham, 2012) have been used to represent such probability distributions. However, the interaction between states of degradation does not wholly represent the definition of the term ‘degradation interactions’. This is because degradation process of a component is not only represented by its states of degradation but also its rate of degradation. It is evident from the review that there is a gap in the literature where degradation interactions involving degradation rates of the components are not addressed.

In this paper, we use a real industrial example to motivate the need for further research in this area and addresses this challenge by developing an approach to improve predictability of the condition of the system through explicit consideration (and modelling) of interactions between states and rates of degradation between components in the system, thereby improving the maintenance policy for each component in the system. The optimal maintenance policy strikes the balance between maintenance investments and the benefits of improved operating conditions of the system.

The structure of this paper is as follows. To start with, a description of the system is given in Section 2. A short summary of the modelling approach is outlined in Section 3. Section 4 and 5 describes the degradation models and the maintenance optimisation model respectively and discusses the results of the solution. Finally, suggested future work and a summary are then concluded in Section 6.

2 SYSTEM DESCRIPTION

The system under consideration is part of an industrial cold box unit in a petrochemical plant. We consider two components which are gas tubes, feeding excess-heated gas into the cold box unit. The excess heat would then be used by the cold box unit to heat up other 'cold' gas to be ready for further processes in the plant. The performance of the system depends on the amount of excess heat in which the cold box can obtain from the two gas tubes. More excess-heated gas delivered into the cold box unit via the gas tubes would lead to more energy savings.

As the tubes are feeding the excess-heated gas into the cold box unit, fouling would occur within the tubes and consequently reducing the amount of heat which can be delivered into the cold box. Pressures in the tube are measured to act as surrogates to the degradation states of a component at a certain time. Lower pressure would indicate less amount of fouling and more effective heat transfer into the cold box. As fouling occur during operation, the pressure would be increased on the tube and hence result in decreasing performance of the system.

Degradation interactions between the two components can occur as when one tube is already subject to a high fouling state, the excess-heated gas would then be forced to go through the other tube and hence overloading that other tube which then leads to accelerated fouling as a result. This scenario calls for the use of the degradation model for the components with degradation interactions as described earlier.

Current maintenance policies of the system include a daily condition monitoring and the system is allowed to operate until the pressure of a gas tube exceeds the safety threshold level. The failed tube is then maintained, allowing the pressure level to reduce and increase the performance of the system. However, this may not be the optimal maintenance policy for the tubes because of the loss due to performance degradation of the system. By moving the maintenance threshold earlier than the safety threshold, the system would be reducing the loss due to performance degradation albeit with increased maintenance costs. This suggests that investments in more maintenance of the tubes may allow the system to reduce its average cost in the long term.

It should be noted here that, due to confidentiality issues, the numerical figures used in this paper is masked (e.g., currency has been changed to GBP) and scaled to protect the identity of the source and to help visualise the results. The next section presents an approach to model the interaction between the state of one tube and the rate of degradation of the second tube. Following this, in section 5 we use simulated annealing to optimise the condition-based maintenance policy. In both sections, we first provide a general model followed by its application to the cold box example.

3 MODELLING APPROACH

Figure 1 demonstrates the structure of the modelling approach used in this paper, along with the techniques used for each phase of the approach. This approach consists of three main parts, namely, Independent Degradation Model, Interactive Degradation Rate Generic Path (IDRGP) Model and CBM Optimisation Model.

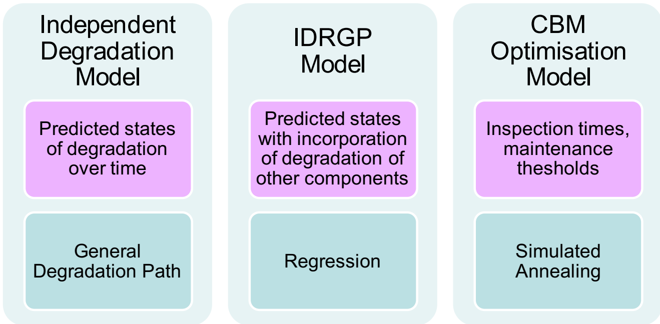

Figure 1: Modelling Approach

The Independent Degradation Model is developed using a General Degradation Path (GDP) model and is used to understand the underlying independent degradation of components in the system. This would form a basis for improved degradation predictions in the IDRGP Model where degradation interactions are included. CBM Optimisation Model is then used to optimise a maintenance policy based on the degradation model.

4 DEGRADATION MODEL

First, we consider a generic system consisting of *N* components. Each component is periodically inspected to reveal the degradation state of the component. The following assumptions are made in the development of this model:

- The components are subject only to gradual degradation and not to sudden failures.
- Only one condition or performance indicator is used to represent the degradation states of each component.
- All components are non-repairable components whose degradation states monotonically increase with its age until its replacement.
- Replacement will bring the component back to its brand new state.
- A single inspection will reveal the states of all components in the system simultaneously.
- The Degradation rate of Component #N is dependent on the states of Component #1 to #N-1.
- Components #1 to #N-1 degrade independently.

Historical degradation data is used to build the degradation model. Each data point is denoted by $(T_{k(i)}, S_{k(i)})$ for i = 1,2,3,…,n. The degradation rate of Component #k at age T_k, $S'_k(T_k)$ is defined as: $S'_k(T_k) = \Delta S_k/\Delta T_k$ where $\Delta S_k = S_k - S_{k,0}$ and $\Delta T_k = T_k - T_{k,0}$.

The degradation data for the cold box tubes was collected in three different scenarios. First, the individual degradation processes were recorded. Then, the effect of the states of Tube#1 (S_1) on the degradation rates of Tube#2 (S'_2), were recorded using a valve in the cold box to shift gas load in the direction from Tube#1 to Tube#2 only. Similarly, the final scenario collected the degradation data to see the effects of S_2 on S'_1.

The historical degradation data for the two tubes under consideration in the cold box example is shown in Figure 2. Since both of the gas tubes are regularly monitored, the historical degradation states (S_1 and S_2) are recorded daily i.e. $\Delta T = 1$. The sudden changes in the degradation states of the components result from the effects of replacement actions. After each replacement of a tube, the degradation state of the tube is reduced to zero.

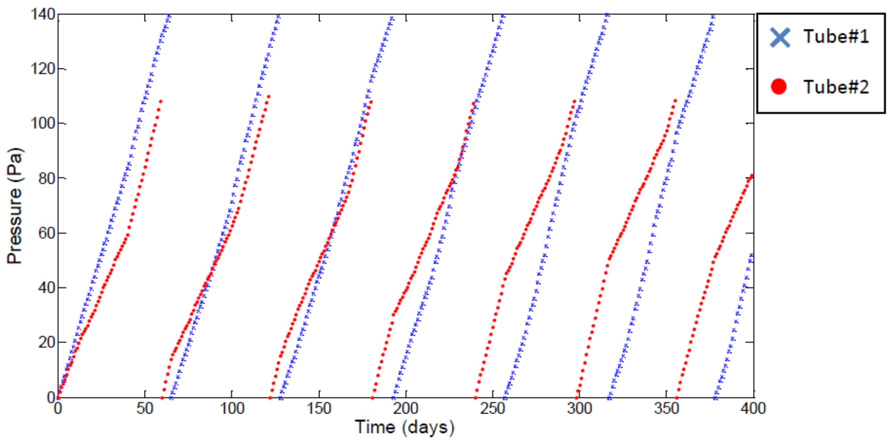

Figure 2

The rest of this section is dedicated to finding an expression for the state and rate of degradation of Component #k (k = 1,2,3,…,N) at age T_k, given by $S_k(T_k)$ and $S'_k(T_k)$, based on such data.

In section 4.1 we present a model to represent the independent degradation processes of the components in the system. Following this, in section 4.2, degradation interactions are introduced to the model.

4.1 Independent Degradation Model

In order to determine an expression for $S'_k(T_k)$ as well as $S_k(T_k)$, an expression for ΔS_k is required. Let ΔS_k be a function f of both ΔT_k and T_k, i.e., the amount of degradation can depend not only on time, but also on the age of the component. For instance, if a component has an increasing degradation rate,

$\Delta S_{k(i)}$ measured at an earlier working age would be smaller than at a later working age even if $\Delta T_{k(i)}$ remained the same. Hence,

$$\Delta S_{k(i)} = f(\Delta T_{k(i)}, T_{k(i-1)}) \tag{1}$$

In order to estimate the function f, we can use Multiple Linear Regression, which is a technique that aims to estimate a functional relationship between multiple independent variables and a dependent variable (Golberg, 2004). Traditionally, GDP models would have defined S_k as a function of T_k. This adaptation from is key to explicitly model degradation rates of the component.

Now, let us apply this to our cold box example. The first step to modelling the independent degradation process of the gas tubes is to define ΔS_k as a function of ΔT and T_k for k = 1,2. In the cold box system, both tubes are regularly inspected at a constant $\Delta T = 1$. $S_k{'}$ as a function of T_k is shown in Figure 3 and Figure 4.

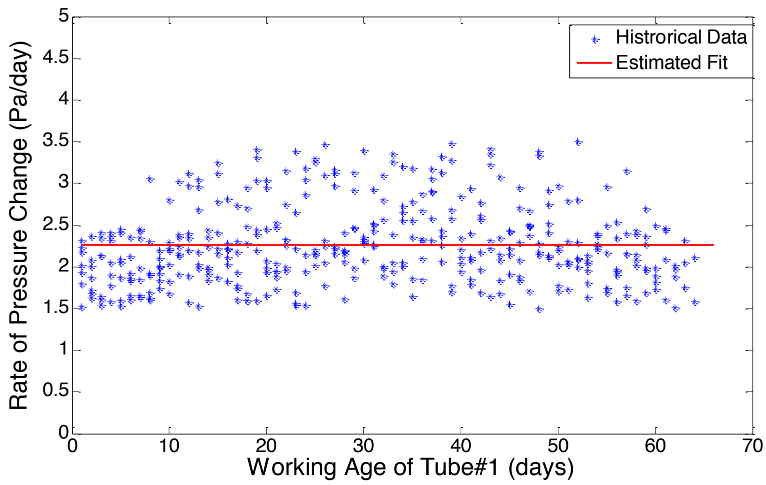

Figure 3: Estimated Rate of Pressure Change from the historical data on Tube 1 ($S_1{'}$)

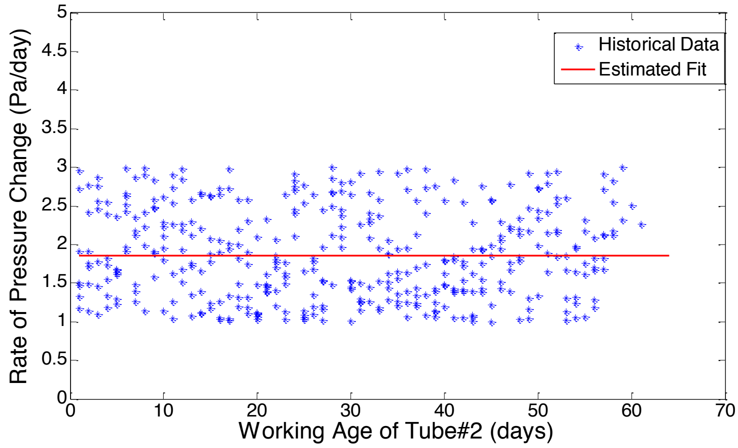

Figure 4: Estimated Rate of Pressure Change from the historical data on Tube 2 ($S_2{}'$)

Using multiple linear regression, the independent degradation model of the components can be found as follows:

$$S_1' \sim \mathcal{N}(2.26, 0.22) \tag{2}$$

$$S_1(T_1) = 2.26T_1 \tag{3}$$

$$S_2' \sim \mathcal{N}(1.86, 0.34) \tag{4}$$

$$S_2(T_2) = 1.86T_2 \tag{5}$$

4.2 Interactive degradation model

Degradation interactions are now introduced to model the degradation process of components in the system case where the degradation rate of Component #N is influenced by the states of $N-1$ components i.e. $S_N'(T_N)$ is also a function of $S_{l,0}$ for l = 1,2,$\cdots$, $N-1$.

Since $S_N'(T_N) = \Delta S_N / \Delta T_N$ and ΔT_N is an independent variable, S_l could only affect $S_N'(T_N)$ through ΔS_N. Hence,

$$\Delta S_N = g(\Delta T_N, T_N, S_{1,0}, S_{2,0}, \ldots S_{N-1,0}) \tag{6}$$

Again, regression can be used to estimate the function *g* in a similar approach to that shown in the previous section.

Now, for the cold box example, based on the historical data with $\Delta T = 1$, we have $S_k' = \Delta S_k$. From equations (2)-(5), the model for each of the two tubes can be defined as;

$$S_1' = \Delta S_1 = 2.26 + f_1(S_2) \tag{7}$$

$$S_2' = \Delta S_2 = 1.86 + f_2(S_1) \tag{8}$$

Gaussian Process Regression (GPR) is used for estimating the functions here because of its ability to allow the observed data to influence the shape of the regression function. This feature is particularly helpful in this case since the types of relationships between the parameters were not explicit. For further details on GPR techniques, please refer to (Ebden, 2008). This would then lead to the expressions for the states of the tubes as a function of its working age as;

$$S_k(T_k) = \Delta S_k + S_{k,0} \; for \; k = 1,2 \tag{9}$$

Figure 5 and Figure 6 show the estimated functions for $f_1(S_2)$ and $f_2(S_1)$ respectively.

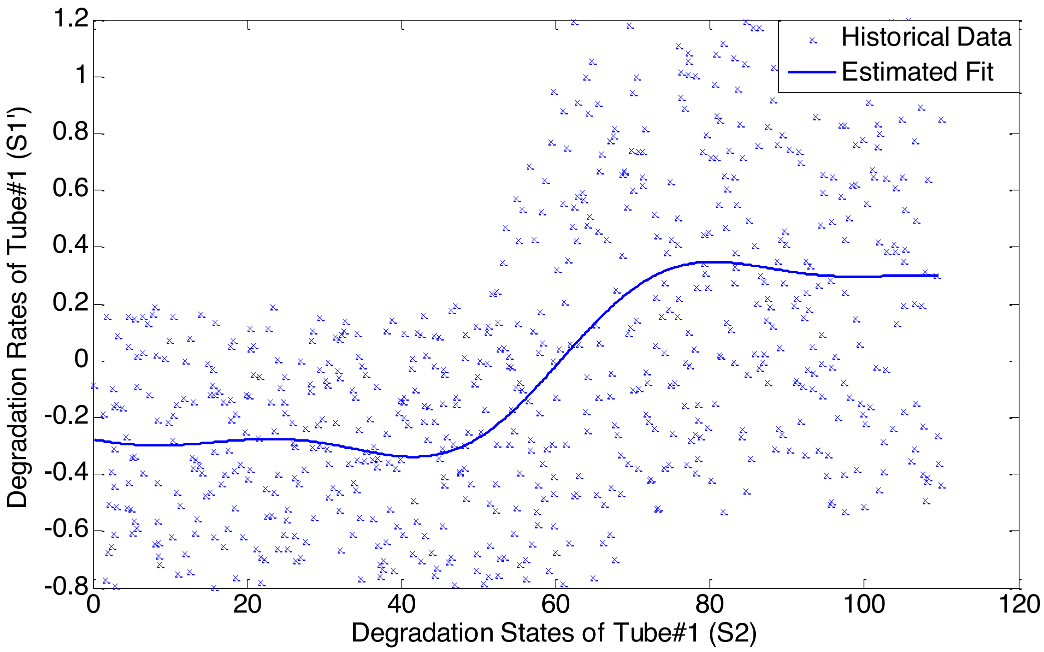

Figure 5

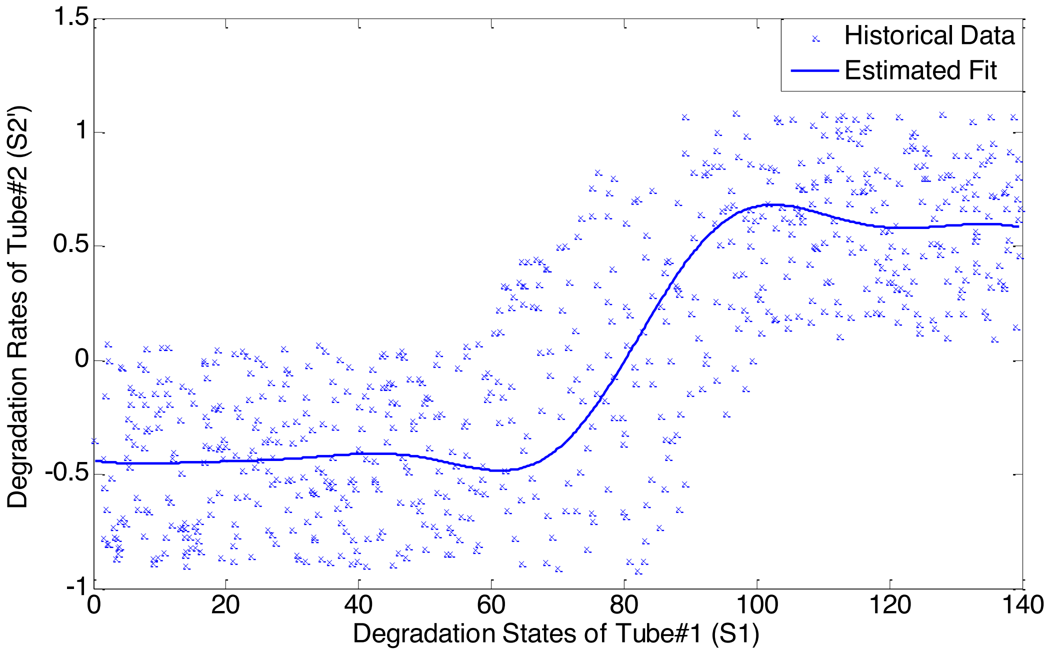

Figure 6

The next section presents the CBM Optimisation Model based on the degradation models developed in this section.

5 CBM OPTIMISATION MODEL

The performance of the system at a certain time is a function of the condition of the components. The goal of this optimisation is to minimise the average cost of the system over time, which accounts for the performance of the system and maintenance costs, by finding an optimal maintenance threshold for each component. An adaptation of the Simulated Annealing Algorithm is implemented to search for an optimal CBM policy for the system. This section provides the necessary steps to achieving this.

5.1 Problem Formulation

The reliability of the system ($R(T)$) acts as a trigger for an inspection action. Once this reliability level fall below an inspection threshold value (r), an inspection action is performed on all components in the system simultaneously.

$R(T)$ is defined as:

$$R(T) \equiv P(S_1(T_1) \leq W_1, S_2(T_2) \leq W_2, \dots, S_M(T_M) \leq W_M) \tag{10}$$

Upon inspection of each component, possible scenarios which trigger replacement actions on the components are listed as follows. First, in cases where the state of a component is found to be in excess of its forced replacement threshold **W**, the component is correctively replaced and hence the state of the component and its working age are set to 0. If the state of the component is still found to be below the forced replacement threshold but in excess of the preventive replacement threshold ($\boldsymbol{D_{pr}}$), where $D_{pr}^{k} \leq W_k$ for k = 1,…,M, the component is preventively replaced. Finally, if the component is found in a state which is not in excess of any of these thresholds, the component is then allowed to operate in the system until the next inspection.

5.2 Objective Function

The average cost function is used as an objective function for the optimisation. Two main factors contributing to this cost function are maintenance costs and the loss due to performance degradation of the system.

The average cost function of the system over the length of a finite-time planning horizon τ ($B(\tau, \boldsymbol{D})$) is defined as:

$$B(\tau, \boldsymbol{D}) = \frac{G(\tau, \boldsymbol{D}) + H(\tau, \boldsymbol{D})}{\tau} \tag{11}$$

where

$G(\tau, \boldsymbol{D})$ is an accumulated cost of maintenance of the system from time 0 to τ, given by:

$G(\tau, \boldsymbol{D}) = n_{in} * c_{in} + \boldsymbol{n_{cr}} \cdot \boldsymbol{c_{cr}} + \boldsymbol{n_{pr}} \cdot \boldsymbol{c_{pr}}$ where

n_{in} is the expected total number of inspection actions performed on the system up to time τ.

$\boldsymbol{n_{cr}} = [n_{cr}^{1} \quad n_{cr}^{2} \quad \cdots \quad n_{cr}^{M}]$, when n_{cr}^{k} (k=1,2,…,M) is the expected number of corrective replacement actions performed on Component #k up to time τ.

$\boldsymbol{n_{pr}} = [n_{pr}^{1} \quad n_{pr}^{2} \quad \cdots \quad n_{pr}^{M}]$, when n_{pr}^{k} (k=1,2,…,M) is the expected number of preventive replacement actions performed on Component #k up to time τ.

$H(\tau, \boldsymbol{D})$ is an accumulated loss due to performance degradation of the system from time 0 to τ, given by:

$$H(\tau, \boldsymbol{D}) = \sum_{T=0}^{\tau} h(S_1(T), S_2(T), \dots, S_M(T)) \tag{12}$$

where $h(\cdot)$ is the performance of the system as a function of the state of the different components.

5.3 Decision variables

The decision variables for the CBM Optimisation Model are as follows:

- Inspection Threshold (*r*) - The inspection threshold is defined as a reliability level below which inspection actions are required to be performed on all components in the system. This level is decided based on the cost of inspection (c_{in}) and the risk of leaving the components operating in poor conditions and crossing the forced replacement threshold. A higher inspection threshold leads to more frequent inspection, incurring more inspection costs. A lower inspection threshold leads to less inspection costs but more risk of leaving the components operating in poor conditions and crossing the forced replacement threshold.
- Preventive Replacement Thresholds ($\boldsymbol{D_{pr}}$) - The preventive replacement threshold of a component is defined as the component's degradation state beyond which the component is preventively replaced. These thresholds indicate whether a preventive replacement action is required upon realising the degradation state of a component through an inspection. A Preventive Replacement Threshold is decided based on the cost of preventive replacement and risk of leaving the components operating in poor conditions and crossing the forced replacement threshold. Higher preventive replacement thresholds lead to less preventive replacement costs but more risk of leaving the components operating in poor conditions and crossing the forced replacement threshold. Lower preventive replacement thresholds lead to more preventive replacement actions, incurring more preventive replacement costs, but the component will be operating in better conditions.

$\boldsymbol{D} = \begin{bmatrix} D_{pr}^1 & D_{pr}^2 & \cdots & D_{pr}^M & r \end{bmatrix}$ is then the CBM policy for the system. This policy then determines when inspection and replacement actions are performed.

The aim of this model is to find an optimal maintenance policy $\boldsymbol{D}^*$ which minimise the expected average cost function of the system over the planning horizon from time 0 to τ.

$$\boldsymbol{D}^* = arg\,\min_{\boldsymbol{D}} B(\tau, \boldsymbol{D})$$

5.4 Maintenance Optimisation using Simulated Annealing

As $B(\tau, \boldsymbol{D})$ is a nondifferentiable/nonsmooth function of **D**, limited techniques are applicable to finding an optimal maintenance policy $\boldsymbol{D}^*$ which minimise the average cost function $B(\tau, \boldsymbol{D})$. A complete search would guarantee optimality for the maintenance policy and provide an absolute minimum average cost of the system. However, the amount of execution time it takes for the complete search algorithm to perform on multi-component systems is usually too long. In practice, there are usually limits on the amount of feasible execution time required of an optimisation algorithm. A compromise can be found in heuristic search algorithms when limited execution time is allowed for optimising larger systems. An optimised maintenance plan through these techniques is an approximation of the solution to the optimisation problem. Although heuristic search algorithms do not necessarily provide an absolute minimum average cost of the system, execution time of these techniques are usually much shorter compared to a complete search algorithm.

A heuristic search technique called Simulated Annealing algorithm is used to find an optimal maintenance plan for the multi-component system in this paper. Simulated Annealing is often used to address combinatorial problems due to the presence of discrete variables (Saraiva, Pereira, Mendes, & Sousa, 2011). It draws an analogy between the cooling process of a solid material and the solving of an optimisation problem (Glover & Greenberg, 1989). A flow chart (Satoh & Nara, 1991) is shown in Figure 7 to summarise the Simulated Annealing Algorithm, which is applied to the context of CBM optimisation in this paper.

The simulated annealing algorithm starts with an initial guess policy for the problem at a high initial temperature. The initial average cost is then determined at this temperature. A cooling schedule is used to determine the next temperature in the process. The next trial policy is then created via a perturbation function. This perturbation shifts the trial policy from the previous policy, depending on the current temperature. At high temperature, this shift is more significant and, as the temperature reduces, smaller shifts are applied. The average cost is then calculated for this new trial policy. If this new average cost is lower than the previous average cost, the new policy is accepted. Otherwise, the probability that the new policy is accepted is $e^{\frac{PreviousCost-NewCost}{Temperature}}$. Once the criterion on the number of accepted policies is satisfied, the temperature is reduced further and the process is repeated until it reaches the final temperature where the algorithm is terminated. The simulated annealing algorithm for this problem is shown in Appendix A.

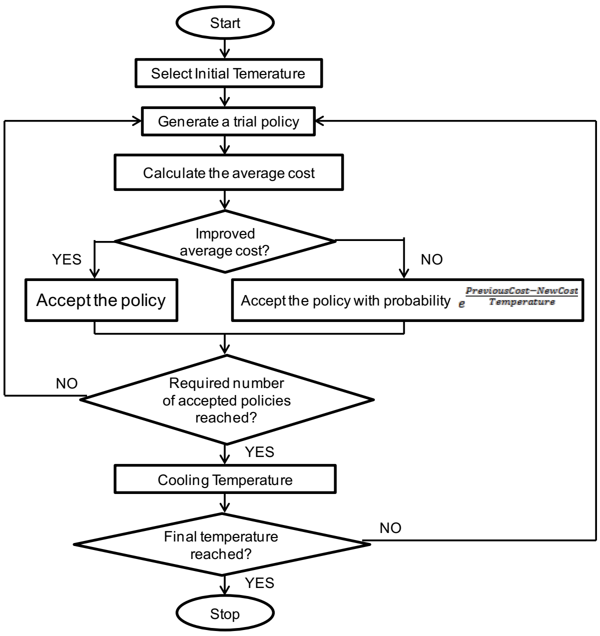

Figure 7

The Perturbation function is a function used to diversify the trial maintenance policy based on the previous trial. The Perturbation function also depends on the temperature as a high temperature causes stronger shifts between the trial maintenance policies. As the temperature decreases, the shifts between the trial maintenance policies are reduced as the average cost function or the energy level settles. The initial and final temperature is chosen to reflect this accordingly. A wide range between the initial and final temperature leads to a more complete search, albeit with a longer execution time compared to a smaller range between the initial and final temperatures.

The Perturbation function used in this research is tailored according to the need to reflect the boundaries of the search space (i.e., maintenance thresholds) and the suitable execution time. This adaptation of the Perturbation function is key to applying the Simulated Annealing Algorithm to optimise CBM policy of multi-component systems. The algorithm used for the perturbation function is shown in Appendix B.

A linear or exponential cooling schedule can be used depending on the requirements. A linear cooling schedule distributes the range of temperatures among the iterations equally. This is particularly useful when there are a lot of local minimum values around the initial guess. By going through more iterations at higher temperatures, there is a better chance of finding the area where the absolute minimum value is located before moving the refined search at lower temperature to that area. However, it should also be noted that, by using a linear cooling schedule, the quality of the final solution may be affected since the refined search at lower temperatures would have gone through less cooling iterations (Ledesma, Aviña, & Sanchez, 2008). In contrast to the linear cooling schedule, an exponential cooling schedule presents sharp decreases in temperature in the early iterations where the temperatures are high. This way the algorithm can search quickly through large sections of the search space and go through more iterations at lower temperatures for a refined search. The choice of cooling schedule is dependent on the nature of the problem and the knowledge of the energy function (average cost function). Alternative cooling schedules have also received the attention of researchers in this area (Nourani & Andresen, 1998).

Finally, the initial guess (policy) is chosen by using an optimal policy made up from individually optimised decision variables, which are determined by a complete search for each optimal value individually, where each component is assumed to be subject to independent degradation. The guess policy may also be decided based on engineering knowledge of the system. Similar to setting the initial and final temperatures, the number of accepted policies required at each temperature of the Simulated Annealing Algorithm can be adjusted to suit the amount of affordable execution time. Large numbers of required accepted policies could lead to an improvement in the final result as the search is more complete, albeit with a longer execution time. A small number of required accepted policies reduces the execution time but may jeopardise the quality of the final result.

The algorithm that calculates the average cost as well as the inspection and replacement times is shown in Appendix C. Note that the calculation of n_{pr}^{k} is included in this algorithm. Essentially, these parameters are incremented if on inspection, the state is found to be worse than the preventive maintenance threshold. Similarly, n_{cr}^{k} is incremented if the state is found to be worse than the safety threshold.

5.5 CBM Optimisation for the cold box example

For the cold box example, the objective function of the optimisation is the average cost over time of the system at different maintenance thresholds D_1 and D_2. The key decision variables for this model are optimal maintenance thresholds for each gas tube.

The average cost function of the system is defined by the sum of the accumulated maintenance costs and the accumulated loss due to performance degradation of the system over a 5-year planning horizon. The accumulated maintenance cost is given by:

$$G(\tau, \boldsymbol{D}) \quad = n_{pr}^1 \cdot c_{pr}^1 + n_{pr}^2 \cdot c_{pr}^2$$

Where,

τ = the length of the planning horizon = 5 years = 1826 days

$\boldsymbol{D} = (D_1, D_2)$ = the maintenance threshold for Tube#1 and Tube#2

n_{pr}^1 = the expected number maintenance actions performed on Tube#1

n_{pr}^2 = the expected number maintenance actions performed on Tube#2

c_{pr}^1 = the cost of a maintenance action performed on Tube#1 = £1645

c_{pr}^2 = the cost of a maintenance action performed on Tube#2 = £1837

The loss due to performance degradation of the system at a certain time is measured by the loss of energy which is affected by the reduced heat transfer through the tubes. This loss is calculated via a function of the state of the two components. The loss function of the system is presented in Figure 8 based on the states of the components.

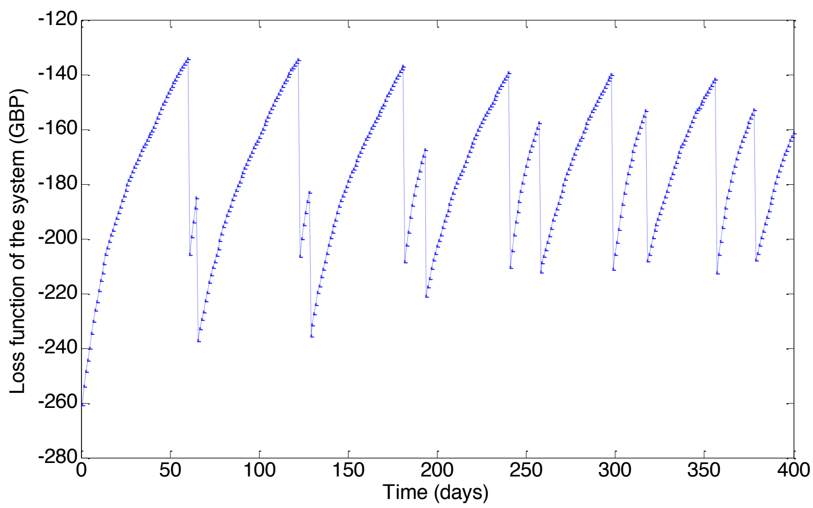

Figure 8

The accumulated loss due to performance degradation of the system over the planning horizon ($H(\tau, \boldsymbol{D})$) is then given by:

$H(\tau, \boldsymbol{D}) = \sum_{T=0}^{\tau} h(S_1(T), S_2(T))$

The input parameters required for optimising the maintenance policy using the Simulated Annealing Algorithm are summarised in Table 1.

Table 1: Input parameters

Parameters	**Value**
τ	1826 days
W_1	140 Pa
W_2	110 Pa
c_{pr}^1	£1645
c_{pr}^2	£1837
c_{cr}^1	£1645
c_{cr}^2	£1837
InitialTemp	1
FinalTemp	0.0001
InitialGuess	(140,110)
Required_Accepted	20
Cool(Temp)	0.9*Temp

The average cost function of the system, $B(\tau, \boldsymbol{D})$, calculated at different iterations of $\boldsymbol{D}$, is shown in Figure 9. Different colours in the figure represent the different level of cost. A search for the optimal maintenance policy for the system, which minimises the average cost function of the system, is found at $\boldsymbol{D}^*$ = (D_1^*,D_2^*) = (127,105). With these thresholds, the expected average cost function of the system would be -£124/day. This represents an improvement of £10/day compared to when using only safety pressure level thresholds for maintenance of the tubes at -£114/day.

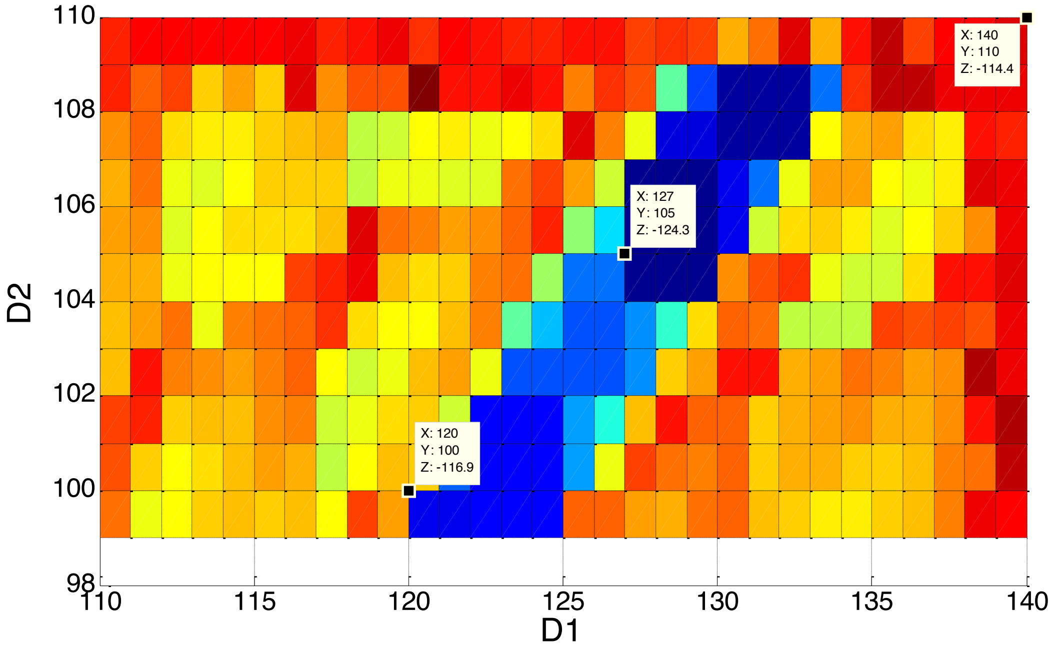

Figure 9: Average cost function of the system at different iterations of D

Simulated Annealing Algorithm is used to search for the optimal maintenance policy for the system, which minimises the average cost function of the system. At **D*** = (D_1*,D_2*) = (127,105), the expected average cost function of the system is minimised at -£124/day. This represents an improvement from £10/day compared to when using only safety pressure level thresholds for maintenance of the tubes at -£114/day.

5.6 Discussion

By using independent degradation to optimise maintenance of the cold-box unit system, the influence of the component's degradation states on the other component's degradation rates is neglected. With the rest of the parameters remain the same, the search for the optimal maintenance policy for the system would suggest a maintenance policy for the system as (120,100).

In the case that this maintenance policy is implemented for this system, the average cost of the system would be -£117/day instead (see Figure 9). Although this already represents an improvement of £3/day from the current maintenance policy of using the safety thresholds as maintenance thresholds, it is still not able to match the level of improvements provided by the CBM policy derived in Section 6.4 at £10/day when the maintenance policy of (127,105) is implemented. This means that by modelling degradation interactions, the company have more than tripled the improvements on the average cost of the cold box unit system compared to using independent individual degradation models.

6 CONCLUSIONS

This paper has proposed an approach to modelling degradation interactions and maintenance optimisation for multi-component systems where degradation rate of a component is affected by states of other components in the system. An industrial case study is also provided to support that degradation interactions can affect maintenance of the systems. By introducing degradation interactions in the model, improved degradation predictions of components would lead to improved CBM policy for the system. It should also be noted here that it is possible for the model to handle a repair action, instead of a replacement action, through the variables ξ_c and ξ_p in the cost calculation algorithm. For perfect replacement, both variables are reset to 0. For corrective repair and/or preventive repair, appropriate positive values are assigned to ξ_c and ξ_p respectively to reflect the state to which the components will return to due to imperfect repair. Suggested lines of future work can be done on addressing assumptions such as imperfect or non-instantaneous inspection and maintenance actions.

7 Appendix A

```
Simulated Annealing Algorithm
Input: τ, W, c_in, c_cr, c_pr, h(S_1(T_1), S_2(T_2), ..., S_M(T_M)) and Degradation Model Parameters
Perturbation Function (Perturb), Cooling Schedule (Cool), InitialTemp, FinalTemp, InitialGuess,
Required_Accepted
Output: D*
begin
    Accepted ← 0
    Finished ← 0
    Temp ← InitialTemp
    D ← InitialGuess
    Execute Algorithm 1
    Oldcost ← B(τ, D)
    while Finished = 0 do
        Currentpolicy ← D
        if Accepted ≥ Required_Accepted do
            if Temp < FinalTemp do
                Finished ← 1
                BREAK
            else
                Temp ← Cool(Temp)
                Accepted ← 1
            end
        end
        D ← Perturb (Currentpolicy,Temp)
        Execute Algorithm 1
        Newcost ← B(τ, D)
        if Oldcost < Newccost do
            if δ ≤ e^((Oldcost−Newcost)/Temp) do
                Oldcost ← Newcost
                Accepted ← Accepted + 1
            else
                D ← Currentpolicy
            end
        else
            Oldcost ← Newcost
            Accepted ← Accepted + 1
        end
    end
    D* ← D
end
```

8 Appendix B

Perturbation Function

Input: Currentpolicy, Temp, $\boldsymbol{W}$

Output: Newpolicy

begin

Select ← γ (γ is a random integer of a uniform distribution on the interval [1, M+1])

Boundaries ← True

SWITCH

CASE 1 do
- while Boundaries do
  - $D_{pr}^{1} \leftarrow D_{pr}^{1} + \theta$
  - (θ is a random number of a normal distribution with zero mean and standard deviation $\frac{W_1}{6}$Temp)
  - if $0 < D_{pr}^{1} < W_1$ do
    - Boundaries ← False
  - end
- end

CASE 2 do
- while Boundaries do
  - $D_{pr}^{2} \leftarrow D_{pr}^{2} + \theta$
  - (θ is a random number of a normal distribution with zero mean and standard deviation $\frac{W_2}{6}$Temp)
  - if $0 < D_{pr}^{2} < W_2$ do
    - Boundaries ← False
  - end
- end

⋮

CASE M do
- while Boundaries do
  - $D_{pr}^{M} \leftarrow D_{pr}^{M} + \theta$
  - (θ is a random number of a normal distribution with zero mean and standard deviation $\frac{W_M}{6}$Temp)
  - if $0 < D_{pr}^{M} < W_M$ do
    - Boundaries ← False
  - end
- end

CASE M+1 do
- while Boundaries do
  - $r \leftarrow r + \theta$
  - (θ is a random number of a normal distribution with zero mean and standard deviation Temp)
  - if $0 < r < 1$ do
    - Boundaries ← False
  - end
- end

end

end

9 Appendix C

Algorithm 1: Average Cost Calculation Algorithm

Input: $\boldsymbol{D}$

Output: $B(\tau, \boldsymbol{D}), \boldsymbol{T_{in}}, \boldsymbol{T_{cr}^1}, \dots, \boldsymbol{T_{cr}^M}, \boldsymbol{T_{pr}^1}, \dots, \boldsymbol{T_{pr}^M}$

begin

Initialise $T \leftarrow 1$

while $T \leq \tau$ do

Calculate $R(T)$

Calculate $\tilde{S}_1(T_1), \dots, \tilde{S}_M(T_M)$

Calculate $h(\tilde{S}_1(T_1), \dots, \tilde{S}_M(T_M))$

$H(T, \boldsymbol{D}) \leftarrow H(T-1, \boldsymbol{D}) + h(\tilde{S}_1(T_1), \dots, \tilde{S}_M(T_M))$

if $R(T) < r$ do

$\boldsymbol{T_{in}} \leftarrow [\boldsymbol{T_{in}} \quad T]$

$n_{in} \leftarrow n_{in} + 1$

if $(\tilde{S}_1(T_1) > W_1) \cup \cdots \cup (\tilde{S}_M(T_M) > W_M)$ do

for each $k \in \{1,2,3, \dots, M\}$ do

if $\tilde{S}_k(T_k) > W_k$ do

$\boldsymbol{T_{cr}^k} \leftarrow [\boldsymbol{T_{cr}^k} \quad T]$

$n_{cr}^k \leftarrow n_{cr}^k + 1$

$T_k \leftarrow \xi_c$

$\tilde{S}_k(T_k) \leftarrow \tilde{S}_k(\xi_c)$

end

end

else if $(\tilde{S}_1(T_1) > D_{pr}^1) \cup \cdots \cup (\tilde{S}_M(T_M) > D_{pr}^M)$ do

for each $k \in \{1,2,3, \dots, M\}$ do

if $\tilde{S}_k(T_k) > D_{pr}^k$ do

$\boldsymbol{T_{pr}^k} \leftarrow [\boldsymbol{T_{pr}^k} \quad T]$

$n_{pr}^k \leftarrow n_{pr}^k + 1$

$T_k \leftarrow \xi_p$

$\tilde{S}_k(T_k) \leftarrow \tilde{S}_k(\xi_p)$

end

end

end

end

$T \leftarrow T + 1, T_1 \leftarrow T_1 + 1, \dots, T_M \leftarrow T_M + 1$

end

Calculate $B(\tau, \boldsymbol{D})$ using (5.1)

end

10 REFERENCES

Barata, J., Guedes Soares, C., Marseguerra, M. and Zio, E. (2001). Monte Carlo simulation of deteriorating systems. s.l., s.n., pp. 879-886.

Barata, J., Guesdes Soares, C., Marseguerra, M. & Zio, E. (2002). Simulation modelling of repairable multi-component deteriorating systems for 'on condition' maintenance optimisation. Reliability Engineering and System Safety, 76, pp. 255-264.

Bian, L. and Gebraeel, N. (2013). Stochastic Modeling and Real-Time Prognostics for Multi-Component Systems with Degradation-Rate-Interactions. IIE Transactions, [online] Available at: <http://www.tandfonline.com/ doi/pdf/10.1080/0740817X.2013.812269>.

Chatterjee, S. & Hadi, A. S. (2013). Regression analysis by example. s.l.:John Wiley & Sons.

Cho, D. and Parlar, M. (1991). A survey of maintenance models for multi-unit systems. European Journal of Operations Research, 51, pp. 1-23.

Ebden, M. (2008). Gaussian Processes for Regression: A Quick Introduction. [online] Available at: <http://www.robots.ox.ac.uk/~mebden/reports/GPtutorial.pdf>

Glover, F. and Greenberg, H. J. (1989). New approaches for heuristic search: A bilateral linkage with artificial intelligence. European Journal of Operational Research, 39, pp. 119-130.

Golberg, M. A. (2004). Introduction to Regression Analysis. s.l.:WIT Press.

Greene, W. (2010). The Least Squares Estimator. [Online] Available at: people.stern.nyu.edu/wgreene/MathStat/ GreeneChapter4.pdf

Hu, J., Zhang, L. and Liang, W. (2012). Opportunistic predictive maintenance for complex multi-component systems based on DBN-HAZOP model. Process Safety and Environmental Protection, 90(5), p. 376–388.

Lai, M.T. (2007). Periodical replacement model for a multi-unit system subject to failure rate interaction. Quality & Quantity, 41, 401-411.

Montgomery, D. C., Peck, E. A. & Vining, G. G. (2012). Introduction to Linear Regression Analysis. s.l.:John Wiley & Sons.

Nicolai, R.P., Dekker, R. (2008). Optimal Maintenance of Multi-component Systems: A Review. *Complex System Maintenance Handbook,* 263-286. Springer, London.

Saraiva, J. T., Pereira, M. L., Mendes, V. T. and Sousa, J. C. (2011). A Simulated Annealing based approach to solve the generator maintenance scheduling problem. Electric Power Systems Research, p. 1283–1291.

Satoh, T. and Nara, K. (1991). Maintenance Scheduling by Using Simulated Annealing Method. IEEE Transactions on Power Systems, 6(2), pp. 850-857.

Straub, D. (2009). Stochastic Modeling of Deterioration Processes through Dynamic Bayesian Networks. Journal of Engineering Mechanics, 135, 1089-1099.

Sun, Y., Ma, L. and Mathew, J. (2009). Failure analysis of engineering systems with preventive maintenance and failure interations. Computers & Industrial Engineering, 57, pp. 539-549.

Wang, P. and Coit, D. W. (2004). Reliability Prediction based on Degradation Modeling for Systems with Multiple Degradation Measures. s.l., s.n., pp. 302-307.

Wang, Y. and Pham, H. (2012). Modeling the Dependent Competing Risks With Multiple Degradation Processes and Random Shock Using Time-Varying Copulas. IEEE Transactions on Reliability, 61, pp. 13-22.

Yang, K. and Xue, J. (1996). Continuous State Reliability analysis. Annual Reliability and Maintainability Symposium, Las Vegas, 22-25 January 1996, Proceedings, 251-257.

Zhang , Y., Zheng, R., Shen, G., and Chen, B. (2011). Reliability Analysis for CNC Machine Tool Based on Failure Interaction. International Conference on Intelligent Computing and Information Science, Chongqing, China, 8-9 January 2011, Proceedings Part I, 489-496. Springer, Berlin.