

Multiple superconducting ring ratchets for ultrasensitive detection of non-equilibrium noise.

V.L. Gurtovoi^{1,2}, M. Exarchos³, V.N. Antonov^{2,3}, A.V. Nikulov¹, and V.A. Tulin¹

¹*Institute of Microelectronics Technology and High Purity Materials,*

Russian Academy of Sciences, 142432 Chernogolovka, Moscow District, Russia.

²*Moscow Institute of Physics and Technology, 29 Institutskiy per., 141700 Dolgoprudny, Moscow Region, Russia.*

³*Physics Department, Royal Holloway University of London, Egham, Surrey TW20 0EX, UK*

Magnetic quantum periodicity in the dc voltage is observed when asymmetric rings are switched between superconducting and normal state by a noise or ac current. We use this effect for detection of a non-equilibrium noise in a system of 667 asymmetric aluminum rings of $1 \mu\text{m}$ in diameter connected in series. We are able to detect the noise at the level close to the equilibrium. one can be detected with the help of such system with enough great number of asymmetric rings.

1. INTRODUCTION

The superconducting loop can be considered as an artificial atom as its spectrum of permitted states of the mobile charge carriers is discrete due to the flux quantization [1–3]. It is similar to the quantization postulated by Bohr's as far back as 1913 in order to describe stable electron orbits of atom. The transition from normal to superconducting state of a mesoscopic or macroscopic loop is the transition from continuous to discrete spectrum of permitted states of the mobile charge carriers. This transition is observed when whole ring [4] or its segment [5, 6] is switched to the superconducting state. The artificial atoms therefore can provide the experimental opportunities to study quantization phenomena.

The quantization is a consequence of the requirement of a single-value of the complex wave function, $\Psi = |\Psi|e^{i\varphi}$, at any point of the ring when all ring segments are in superconducting state, $\oint_l dl \nabla \varphi = n2\pi$. In the presence of a magnetic vector potential A this results in appearance of the persistent current in the loop

$$I_p = \frac{sq}{m2\pi r} \oint_l dl \Psi^* (-i\hbar \nabla - qA) \Psi = \frac{n\Phi_0 - \Phi}{L_k} \quad (1)$$

,where $\Phi = \oint_l dl A$ and $\Phi_0 = 2\pi\hbar/q$ have a usual meaning. $L_k = ml/sq^2|\Psi|^2$ is the kinetic inductance of the ring with the length $l = 2\pi r$, the cross section s , the density of superconducting pairs $|\Psi|^2$, m is the electron mass and the pair charge $q = 2e$.

The persistent current appears when all ring segment are switched to the superconducting state and should decay when at least one segment is switched to the resistive state [5, 6]. Numerous measurements [9–11] testify to the predominant probability $P_n \propto 1 - \exp[-E_n/k_B T]$ of the permitted state n with the minimal value of the kinetic energy

$$E_n = \frac{1}{2m} \int_V dV \Psi^* (-i\hbar \nabla - qA)^2 \Psi = \frac{(n\Phi_0 - \Phi)^2}{2L_k} \quad (2)$$

The measurements of the critical current [10, 11] testify that the persistent current should have the same direction, with the predominant probability

$P_n(\Phi)/P_{n+1}(\Phi) \approx \exp 40 \approx 10^{17}$ at a given magnetic flux $\Phi \approx (n + 0.25)\Phi_0$ and the temperature $T \approx 1 \text{ K}$. This happens because of a large kinetic energy: $(n\Phi_0 - \Phi)^2/2L_k = (I_p\Phi_0/2)(n - \Phi/\Phi_0)^2 \approx 80k_B(n - \Phi/\Phi_0)^2 \text{ K}$ at a typical value $I_p \approx 1 \mu\text{A}$ [12]. The voltage V_{dc} across the loop appears only when the loop or the loop segment is switching between the superconducting and the normal states due to the ratchet effect [5, 6]. This voltage should oscillate with magnetic field likewise the average value of the persistent current $\bar{I}_p = (\bar{n}\Phi_0 - \Phi)/L_k$, where $\bar{n} = \sum_n n P_n(\Phi)$ and $P_n(\Phi)$ is the probability of the switching in superconducting state with the quantum number n at magnetic flux inside the ring Φ . Such an oscillations between the superconducting and normal states induced by the ac current were observed in an asymmetric rings [10, 13–17]. In this work we use this effect for ultra-sensitive detection of the non-equilibrium noise in a system of a large number of the aluminium rings connected in series.

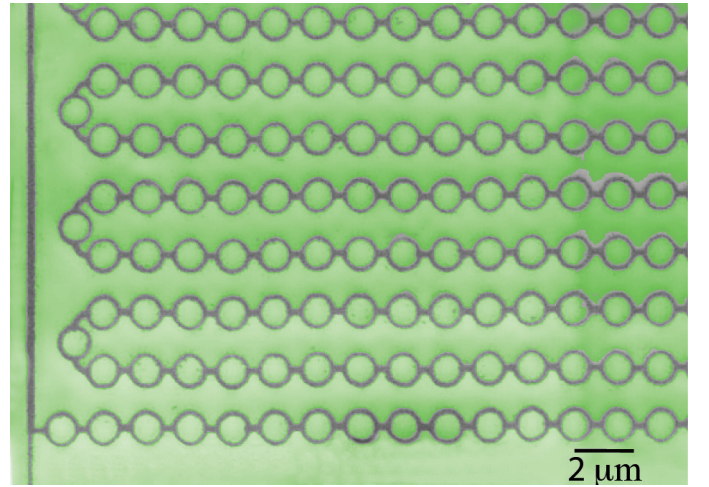


FIG. 1: A fragment of structure consisting of 667 asymmetric rings with diameter of $1 \mu\text{m}$.

2. EXPERIMENTAL DETAILS

We study a chain of 667 asymmetric $1 \mu\text{m}$ diameter rings made of 30 nm thick aluminium film, see Fig. 1. The rings are composed of two arms of 100 and 125 nm wide. The chain has a normal state resistance of $R_n = 5400 \Omega$. It became superconducting at $T_c \approx 1.320 \text{ K}$ with the transition width of $\Delta T_c \approx 0.009 \text{ K}$, Fig.2. The temperature dependence of the critical current measured without magnetic field is described by the theoretical relation $I_c = I_c(T=0)(1 - T/T_c)^{3/2}$, where $I_c(T=0) \approx 520 \mu\text{A}$, Fig.2. The critical current density $j_c(T=0) \approx 10^7 \text{ A/cm}^2$ equals approximately the de-pairing current density [10]. Measurements were carried out by applying the bias current across the chain, dc to 200 kHz, using the ultra low distortion generator with differential output. The rectified voltage at the chain was probed by an instrumentation amplifier in a frequency band from 0 to 30 Hz. Noise level of the amplification system was 20 nVpp for $f_b = 0$ to 1 Hz. It should be noted that rectification effects do not depend on frequency of the bias current at least up to 1 MHz. The magnetic field was applied perpendicular to the ring's plane. When scanning the magnetic field we use a low sweeping rate in order to be within the time resolution of the measurement system of 30 ms.

The magnetic field dependences of the critical currents $I_{c+}(B_{sol})$ and $I_{c-}(B_{sol})$ were taken from a repeating, 10 Hz, current-voltage characteristics in a slowly varying magnetic field B_{sol} . Once the output of amplifier exceeded the threshold voltage, determined by the noise of the measuring system, the critical current is probed. The instrumental delay between the current measurement and the voltage threshold conditioning was

$30 \mu\text{s}$. This procedure allowed us to measure sequentially critical currents in the positive I_{c+} and negative I_{c-} directions. Contrarily the Little-Parks oscillations $R(B) = V(B)/I$ were recorded at a constant bias current $I = 0.1 \div 2.0 \mu\text{A}$. The field dependences of rectified voltage $V_{dc}(B)$ were measured using sinusoidal bias current $I(t) = I_0 \sin(2\pi ft)$ with the amplitude I_0 up to $50 \mu\text{A}$ and frequency $f = 0.5 \div 5 \text{ kHz}$. Because of incomplete screening, the minima of the $R(B_{sol})$ and zero rectified voltage $V_{dc}(B_{sol})$ were shifted by $-B_{res} \approx -0.15 \text{ G}$. When shifted by B_{res} the $R(B)$ had minima (maxima) at $B_{sol} + B_{res} = n\Phi_0/S$ ($B_{sol} + B_{res} = (n + 0.5)\Phi_0/S$) correspondingly. As one should expect $V_{dc}(B)$ also intersects zero at these values. We confirmed also that $I_{c+}(B_{sol} + B_{res}) = I_{c-}(-B_{sol} - B_{res})$ in all our experiments. This is result of the fundamental principle: the simultaneous reversal of the total external field \mathbf{B} and the current I is equivalent to the rotation through 180° , $I_{c+}(B) = I_{c-}(-B)$. Therefore $B_{sol} + B_{res}$ was indeed the total external field.

3. CHARACTERISATION OF THE CHAIN OF RINGS

We observe the Little-Parks resistance oscillations at $T > T_c$, which testify the existence of the persistent current just above the superconducting transition, see Fig. 3 [18]. The resistance increases from $R/R_n \approx 0.22$ at $B = 0$ to $R/R_n \approx 0.90$ at $B \approx \Phi_0/2S$ corresponding to the decrease of the critical temperature by $\Delta T_c \approx -0.01 \text{ K}$. After repeating the experiment with the different bias current in a range from 1 nA to 500 nA, we found that this shift corresponds to $I \approx 300 \text{ nA}$. Con-

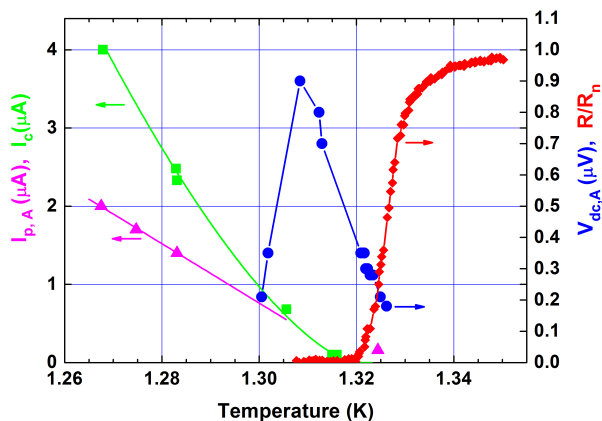


FIG. 2: The superconducting resistive transition R/R_n , the temperature dependence of the critical current I_c (squares, the line is the theoretical dependence), the amplitude of the persistent current $I_{p,A}$ (triangles, the straight line is the theoretical dependence) and the dc voltage $V_{dc,A}$ induced by an arbitrary noise (circles).

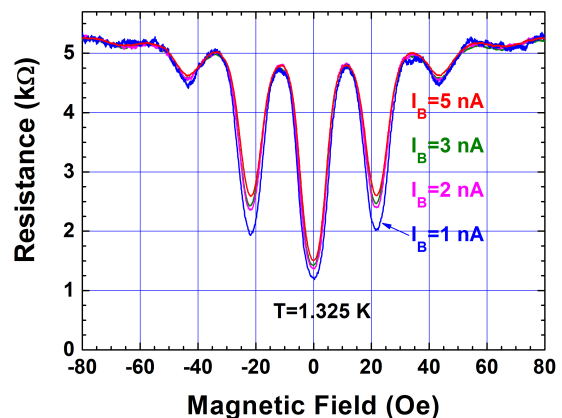


FIG. 3: The Little-Parks oscillations of the 667 rings taken at different values of the bias current and the temperature of the resistive transition: 1) $I = 1 \text{ nA}$, $T \approx 1.3244 \text{ K}$; 2) $I = 2 \text{ nA}$, $T \approx 1.3253 \text{ K}$; 3) $I = 3 \text{ nA}$, $T \approx 1.3254 \text{ K}$; 4) $I = 5 \text{ nA}$, $T \approx 1.3257 \text{ K}$. The period of oscillations $B_0 = \Phi_0/S \approx 22 \text{ Oe}$ corresponds to the ring's area $S = 0.94 \mu\text{m}^2$.

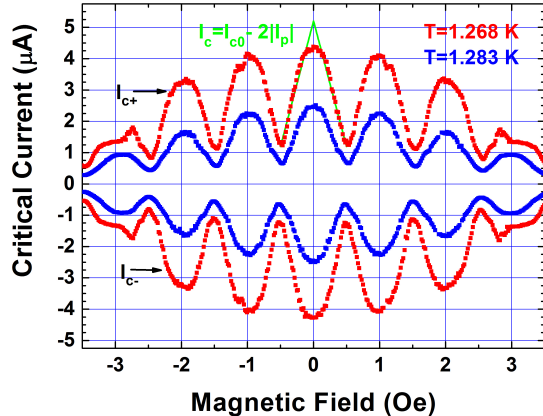


FIG. 4: Magnetic dependence of the critical current of 667 rings measured at the temperature $T = 1.2675 K$ in the opposite directions $I_{c+}(\Phi/\Phi_0)$ (line) and $I_{c-}(\Phi/\Phi_0)$ (criss-crosses). The experimental dependence $-I_{c-}(\Phi/\Phi_0)$ is compared with (3). The values $I_{c0} = 5.2 \mu A$ and $I_{p,A} = 2 \mu A$ were used for the theoretical dependence.

sequently the amplitude of the persistent current oscillations $I_{p,A} \approx 150 nA$ at $T = 1.325 K$ ($> T_c$) largely exceeds the bias current $I = 1 \div 5 nA$ used in the experiments of Fig.3.

The persistent current causes also the oscillations of the critical current in the superconducting state at $T < T_c$, Fig.4. Magnetic dependence of the critical current of a symmetric ring is given by [10]:

$$I_c = I_{c0} - 2|I_p| = I_{c0} - 2I_{p,A}2|n - \frac{\Phi}{\Phi_0}| \quad (3)$$

This relation nicely describes the critical current I_{c+} , I_{c-} of our chain of rings, Fig.4. The discrepancy between theoretical and experimental values near $\Phi = 0$ may be explained by the influence of the narrow wire segment between the rings, the width of which is smaller than the total width of two ring-halves. The fitting of the experimental curve with (3) allows to find the amplitude of the persistent current oscillation $I_{p,A}(T)$ at different temperatures. We found that temperature evolution of the critical current is described by $I_{p,A} = I_{p,A}(T=0)(1 - T/T_c)$, with $I_{p,A}(T=0) \approx 50 \mu A$, Fig.2. The amplitude should be equal zero at $T > T_c = 1.320 K$. However, as we have shown above, the Little-Parks oscillations indicate at a non-zero value of $I_{p,A}$ due to thermal fluctuations.

From the theory one can expect that the critical current of symmetric rings measured with the opposite bias should be equal to each other $I_{c+} = I_{c-}$. In the case of asymmetric ring with different cross section of the ring-halves this does not hold and the anisotropy of the critical current $I_{c,an} = I_{c+} - I_{c-}$ should be seen. We have observed the anisotropy when dealing with the individual rings [10]. It appeared when the magnetic field was applied having a maximum of as non-zero $I_{c,an}$ close to the

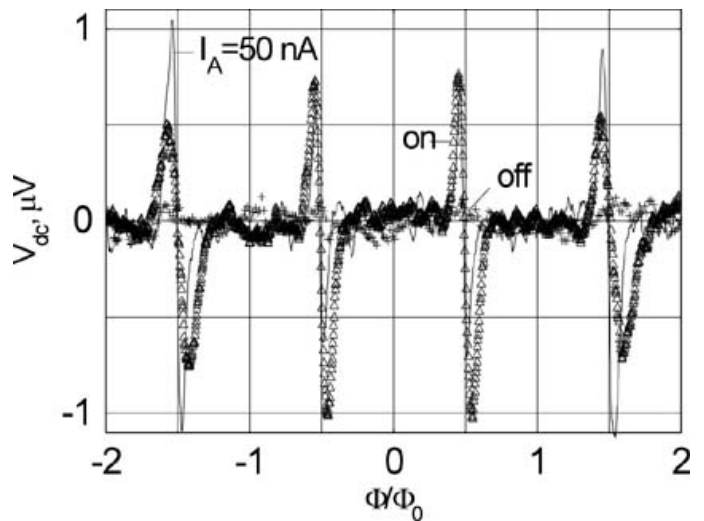


FIG. 5: V_{dc} taken at $T = 1.307 K$ when thermometer electronics were turned off (criss-crosses) and on (triangles), and when the dc voltage is induced by controllable ac bias current with the amplitude $I_A = 50 nA$.

quarter of the flux $\Phi_0/4$ through the arc of the ring. In the current experiment with the chain of the rings we notice only a slight anisotropy of the critical current close to $\Phi = (n + 0.5)\Phi_0$, Fig.4. Close to these points there were also maxima of the rectification voltage $V_{dc}(\Phi/\Phi_0)$, Fig. 5. The latter can be used for sensitive detection of the weak electromagnetic signals and noise, which is discussed in the next section.

4. DETECTION OF WEAK ELECTROMAGNETIC SIGNALS

Detection and analysis of the electromagnetic noise is very important for quantum circuits, like quantum bits, superconducting quantum interference device to mention a few. Fig.6 demonstrates response of the chain of rings to weak electromagnetic signal. The signal is generated by electronics used for temperature measurements. The amplitude of the V_{dc} near $\Phi \approx (n + 0.5)\Phi_0$ does not exceed $0.1 \mu V$ when the electronics was turned off. It increases up to $1 \mu V$ when the thermometer electronics had been turned on. We apply calibrated ac bias current to the chain of rings, and found that $I_A \approx 50 nA$ induces approximately the same value of V_{dc} .

The asymmetric superconducting ring is a ratchet because of the predominate probability of one of the directions of the persistent current. The V_{dc} appears when the persistent current exists, and the rings are switched between superconducting and normal state. V_{dc} decreases at higher temperatures, $T > 1.308 K$, when the amplitude of the persistent current $I_{p,A}$ decreases, Fig. 2. The limiting factor at the lower temperatures is the increase of the critical current I_c . Despite the amplitude of the persistent current is higher below $1.3K$, the weak ac bias

current I_A can not switch the rings to the normal state unless it exceeds the I_c . Therefore the rectification voltage V_{dc} appeared only in a narrow temperature range, Fig. 2. This region expands with the increase of I_A . The minimum detected amplitude was $I_A < 10$ nA. A larger number of asymmetric superconducting rings should be used in order to increase sensitivity.

The V_{dc} increases with the number of rings N in similar way as the voltage of series of any dc power sources [13]. Consequently the amplitude of the detectable noise decreases with the N [17]. We can estimate the amplitude of the minimal detectable noise current as $\delta V_{dc}/R_n \text{Eff}_{Re}$, where $R_n = NR_{n,1}$ is the resistance of chain of rings, $\text{Eff}_{Re} = V_{dc,A}/R_n I_A$ is the rectification efficiency and δV_{dc} is the voltage noise level of the measurement system, ≈ 0.2 μV . The rectification efficiency of asymmetric aluminium rings is abnormally high $\text{Eff}_{Re} \approx 0.33$ in the superconducting state at $T < 0.98T_c$ because of the hysteresis of the current - voltage characteristics in this temperature region [10]. A noise with the amplitude down to $I_A \approx 0.1$ nA could therefore be detected. The rectification efficiency however decreases at $T \rightarrow T_c$, so we are able to detect a noise with the amplitude $I_A > 10$ nA in our experiment.

In order to boost the rectification efficiency, which in fact is a sum of contribution of individual rings, $\text{Eff}_{Re} = V_{dc,A}/R_n I_A = \sum_{i=1}^{i=N} V_{dc,i}(B,T)/R_n I_A$, one need to ensure a maximum of the dc voltage $V_i(B,T)$ induced by the I_A by each ring i at a particular B and a temperature T . Therefore all rings should have the same area $S = \pi r^2$, close to each other the critical temperature T_c . It is a technologically challenging to ensure the homogeneous system of rings with the same T_c . The resistive transition of a homogeneous system has a finite width because of thermal fluctuations [19], giving a limit

of ≈ 0.006 K. The width $\Delta T_c(0.1 \div 0.9R_n) \approx 0.009$ K of our system was near the ideal one, therefore it was close to the ideal homogeneity.

The other way to improve efficiency is to look at $\text{Eff}_{Re,1} = V_{dc,i}/R_{n,1} I_A$ of the individual ring. Here $R_{n,1} \approx 5400/667 \Omega \approx 8 \Omega$ is the resistance of individual ring. The dc voltage up to $V_{dc,i} \approx V_{dc,A}/667 \approx 1.5$ nV is induced by the bias current $I_A \approx 50$ nA on each ring so that $\text{Eff}_{Re,1} \approx 0.004$. The $V_{dc,A}$ depends on the amplitude of the persistent current $I_{p,A}$ rather than I_A . The amplitude I_A should exceed the critical current (3) in order to have non zero $V_{dc}(\Phi/\Phi_0)$. Therefore the value $I_{p,A}$ should be large while the critical current $I_c = I_{c0} - 2I_{p,A}2|n - \Phi/\Phi_0|$ should be small. According to the theory the critical and persistent currents are $I_{c0} = 2sn_s q \hbar / m \sqrt{3} \xi(T)$ and $I_{p,A} = 0.5 \Phi_0 / L_k = sn_s q 0.5 \hbar / m r$ [19]. They are equal to each other when $1 - T/T_c \approx (\sqrt{3} \xi(0)/4r)^2$ as $\xi(T) = \xi(0)(1 - T/T_c)^{-1/2}$. Therefore in order to increase the rectification efficiency and the temperature range one needs to use rings with a smaller radius r . Thus for the rings the diameter $2r \approx 0.3$ μm one will get he temperature region $T/T_c \approx 0.9$ for signal of 50 nA.

5. CONCLUSION

We have demonstrated rectification of the ac signal close to the T_c of the sequential chain of large number of asymmetric aluminium rings. Because of homogeneity of the rings we are able to sum up a small V_{dc} contributions of the individual rings to noticeable voltage of μV range when exposed to the signal of a few nA. Thus such a system can be used for the sensitive detection of noise in the quantum circuits.

-
- [1] O. Astafiev, A. M. Zagoskin, A. A. Abdumalikov Jr., Yu. A. Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura, J. S. Tsai, Resonance Fluorescence of a Single Artificial Atom. *Science* **327** 840 (2010)
- [2] J. Q. You and F. Nori, Atomic physics and quantum optics using superconducting circuits. *Nature* **474**, 589 (2011)
- [3] F. Lecocq, I. M. Pop, I. Matei, E. Dumur, A. K. Feofanov, C. Naud, W. Guichard, and O. Buisson, Coherent Frequency Conversion in a Superconducting Artificial Atom with Two Internal Degrees of Freedom. *Phys. Rev. Lett.* **108**, 107001 (2012)
- [4] V. L. Gurtovoi and A. V. Nikulov, Comment on Vortices induced in a superconducting loop by asymmetric kinetic inductance and their detection in transport measurements. *Phys. Rev. B* **90**, 056501 (2014)
- [5] A.V. Nikulov and I.N. Zhilyaev, The Little-Parks Effect in an Inhomogeneous Superconducting Ring. *J. Low Temp.Phys.* **112**, 227 (1998).
- [6] A.V. Nikulov, The Meissner effect puzzle and the quantum force in superconductor. *Phys. Lett. A* **376**, 3392 (2012); arXiv:1211.0002 (2012)
- [7] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-Relativistic Theory*. Volume 3, Third Edition, Elsevier Science, Oxford, 1977.
- [8] R.P. Feynman, R.B. Leighton, and M. Sands *The Feynman Lectures on Physics*, Addison-Wesley Publishing Company, Reading, Massachusetts, 1963.
- [9] N.C. Koshnick, H. Bluhm, M. E. Huber, K.A. Moler, Fluctuation Superconductivity in Mesoscopic Aluminium Rings. *Science* **318**, 1440 (2007).
- [10] V. L. Gurtovoi, S. V. Dubonos, A. V. Nikulov, N. N. Osipov, and V. A. Tulin, Dependence of the magnitude and direction of the persistent current on the magnetic flux in superconducting rings. *Zh.Eksp.Teor.Fiz.* **132**, 1320 (2007) (*JETP* **105**, 1157 (2007)); arXiv: 0903.3539 (2009)
- [11] Gurtovoi V.L., Dubonos S.V., Karpi S.V., Nikulov A.V., Tulin V.A.: Contradiction between the Results of Observations of Resistance and Critical Current Quantum Oscillations in Asymmetric Superconducting Rings. *Zh.Eksp.Teor.Fiz.* **132**, 297 (2007) (*JETP* **105**,

- 262 (2007)); arXiv: 1011.6149 (2010).
- [12] V.L. Gurtovoi, S.V. Dubonos, A.V. Nikulov, V.A. Tulin, Two permitted states of superconducting ring observed at measurements of its critical current. Proceedings of the 34 Meeting on the Low Temperature Physics, v.2, p.273-274 (2006); arXiv: 1102.5681 (2011)
- [13] S.V. Dubonos, V.I. Kuznetsov, I. N. Zhilyaev, A. V. Nikulov, A. A. Firsov, Observation of the External-ac-Current-Induced dc Voltage Proportional to the Steady Current in Superconducting Loops. *Pisma Zh.Eksp.Teor.Fiz.* **77**, 439 (2003) (*JETP Lett.* **77**, 371 (2003)); arXiv: cond-mat/0303538 (2003).
- [14] S.V. Dubonos, V.I. Kuznetsov, and A.V. Nikulov, Segment of an Inhomogeneous Mesoscopic Loop as a DC Power Source. in Proceedings of 10th International Symposium "NANOSTRUCTURES: Physics and Technology" St Petersburg: Ioffe Institute, p. 350 (2002); arXiv: physics/0105059 (2001)
- [15] A. A. Burlakov, V.L. Gurtovoi, S.V. Dubonos, A.V. Nikulov, and V.A. Tulin, Little-Parks Effect in a System of Asymmetric Superconducting Rings. *Pisma Zh.Eksp.Teor.Fiz.* **86**, 589 (2007) (*JETP Lett.* **86**, 517 (2007)); arXiv: 0805.1223 (2008).
- [16] V.L. Gurtovoi, A.I. Ilin, A.V. Nikulov, V.A. Tulin, Weak dissipation does not result in disappearance of persistent current. *Fizika Nizkikh Temperatur*, **36**, 1209 (2010); arXiv: 1004.3914 (2010)
- [17] A.A. Burlakov, V.L. Gurtovoi, A.I. Ilin, A.V. Nikulov, V.A. Tulin, A possibility of persistent voltage observation in a system of asymmetric superconducting rings. *Phys. Lett. A* **376**, 2325 (2012); arXiv: 1207.0791 (2012).
- [18] W.A. Little and R.D. Parks, Observation of Quantum Periodicity in the Transition Temperature of a Superconducting Cylinder. *Phys.Rev.Lett.* **9**, 9 (1962)
- [19] M. Tinkham, *Introduction to Superconductivity*. McGraw-Hill Book Company (1975).
- [20] A. Th. A. M. de Waele, W. H. Kraan, R. de Bruyn Ouboter, and K. W. Taconis, *Physica* **37**, 114 (1967).
- [21] A. C. Bleszynski-Jayich, W. E. Shanks, B. Peaudecerf, E. Ginossar, F. von Oppen, L. Glazman, and J. G. E. Harris, Persistent Currents in Normal Metal Ring. *Science* **326**, 272 (2009).
- [22] H. Bluhm, N.C. Koshnick, J.A. Bert, M.E. Huber, and K.A. Moler, Persistent Currents in Normal Metal Rings. *Phys. Rev. Lett.* **102**, 136802 (2009).