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PAPER

Guiding of elastic waves in a two-dimensional graded phononic crystal plate

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Abstract

The guiding of elastic waves in a two-dimensional graded phononic crystal plate is investigated. This effect is induced by the resonance coupling of attachments and matrix in a silicon pillar-substrate system and the resonance frequencies of guided surface modes can be tuned by tailoring the geometry and material properties of the pillars. The resonance frequencies increase with radius and Young's modulus, and decrease with height and density of the pillars, which provides several possibilities for the guiding of elastic waves. These devices show the capability of spatially selecting different frequencies into designed channels, thus acting as a phononic multi-channel filter.

1. Introduction

Graded phononic crystal (GPnC) exhibits a gradual variation of constitutive parameters of inclusions along one direction, which can be tailored by filling factors, inclusion geometry, and material properties [1–4]. Based on former studies, 2D GPnCs, which are able to focus propagating waves, have been employed to implement acoustic lenses, beamwidth compressors, and bending waveguides [5–10]. For PnCs with pillars as attachments, resonance effects are very important in determining the properties of the system and its possible applications [11–15]. By placing several locally resonant layers of different resonance frequencies on a PnC, a broadband sound shield which is induced by the combination of different local resonant bandgaps can be achieved [16]. As a kind of acoustic wavelength division demultiplexer, an acoustic rainbow trapping device can distribute different frequencies of a broadband incident temporal acoustic signal into different channels [17]. Based on a graded index inhomogeneity design, the focusing and collimation performance of structure-embedded acoustic lenses has been proposed [18–20]. Surface acoustic waves have been explored in-depth, which can be applied in material characterization, photon or phonon modulation, acoustic sensors and optomechanics [21, 22]. Thus, tailoring the propagation of surface modes is significant for these multitudinous applications. In this work, we study the guiding of elastic waves on the surface of a GPnC plate. The resonance effect in a pillar-type GPnC plate is thoroughly investigated by analysing various influence factors. Our finding provides a guide for the realization of acoustic filter preparation at specific frequencies. The induced elastic waves propagate along multi-resonance channels, some of the frequencies components are bound in the resonators. Hence, this device can separate frequencies spatially.

2. Methods

Coupled mode theory describes the interaction between few fields in a periodically perturbed structure; the fields in the unperturbed condition are considered, while the periodic modulation perturbation couples these modes [23, 24]. The coupled mode in PnC generally means the coupling between inclusions/resonators and matrix, which can result in a bandgap in the dispersion relations and Fano-like line shape in transmission spectra. The coupling can be modulated by the characteristics of different systems. The resonance can be

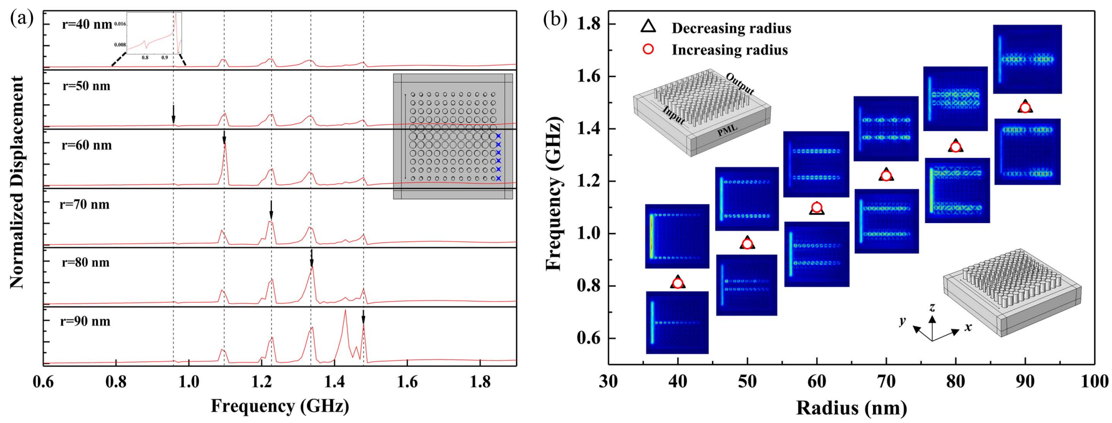

Figure 1. (a) Normalized displacements of different sampling points in GPnC plate with decreasing radius from the middle to the edge. The arrows indicate the largest displacements at one frequency among all sampling points. The inset shows the geometry of the model and the sampling points (marked as blue crosses). (b) The displacement peak values of pillars with decreasing and increasing radius from the middle to the edge respectively. The grey insets are the 3D geometry of models with decreasing radius (left upper) and increasing radius (right down). The coloured insets close to the data show stress fields of the interface, the upper and lower ones correspond to the decreasing and increasing radius, respectively.

enhanced and the coupling is strengthened by introducing erection of pillars on the solid regions in phononic structure [25]. Here we explore other ways to tune the resonance coupling in solid PnCs.

Simulations provide an effective method to investigate the resonance frequency of periodic nanostructure which is inconveniently obtained by theoretical analysis. The modelling software Comsol, based on the finite element method, is employed to study the phononic properties of the designed phononic nanostructure. Pillars on a surface are resonators storing mechanical energy which act as multi-resonance channels. A 2D pillar-type GPnC plate is used to analyse the guiding of elastic waves on the surface in the attachment-matrix system, where the GPnC is realized by varying the size and height of attachments along one direction gradually. Perfectly matched layers (PMLs) are used all around the matrix to avoid backscattering from the boundaries.

3. Results and discussion

Figure 1 shows the relationship between frequencies and displacements with decreasing and increasing radii of attachments in a silicon pillar-type PnC plate. The model of the 2D GPnC plate is comprised of 11 rows of silicon pillars in the x direction and 11 columns in the y direction on a silicon substrate. The lattice constant is 200 nm and the radius of the pillar is set as $r(y) = r_0 + n(y)^{*}\Delta$, where n is the ordinal number of pillars, Δ is the variation. The radii of pillars change gradually from 40 to 90 nm with a step size of $\Delta = 10$ nm. The thickness of the substrate in this model is 600 nm with 200 nm PML. We define the radii of pillars decreasing from the middle to the edge as decreasing radius, vice versa as increasing radius. The parameters of silicon are set as Young's module $E = 131$ GPa, Poisson ratio $\sigma = 0.27$, density $\rho = 2330$ kg m^{-3} [26]. The excitation line source is set as a unit force Fx with fixed frequency which is located in front of the phononic structure. The frequency is then swept in order to obtain the frequency dependent propagating in the structure. The output signals are taken from the sampling points in the rear of the phononic structure marked as blue crosses. The silicon matrix is surrounded by PMLs and a fixed boundary condition is used for the bottom. To study the guided waves in the solid, the surface of the substrate, which is also the interface of attachments and matrix, is selected as a cut plane to show the result intuitively.

Figure 1(a) shows the displacements of six different sampling points for output signals for the GPnC plate with decreasing radius. The arrows indicate the largest displacements at one frequency among all sampling points. The grey inset shows the geometry of the model and the sampling points (behind different sizes' pillars). Each sampling point exhibits six local displacement peaks and the peaks are located almost at the same frequencies indicated by dash lines. The frequencies of the displacement peaks are around 0.81 GHz, 0.96 GHz, 1.09 GHz, 1.22 GHz, 1.33 GHz, and 1.48 GHz, respectively. The first two local displacement peaks are relatively small, hence the zoom-in figure is shown as an inset. The origin of the large displacement at 1.43 GHz is not yet fully understood. However, we believe that this mode is not caused by periodicity truncation of the structure as PMLs are applied around the matrix [27], but a mode which is achieved due to the close proximity of the 80 and 90 nm pillars. The displacement peak shifts to higher frequencies with increasing radius. This demonstrates that a specific radius corresponds to a preferred guided frequency. Here, it is interesting to compare crystals with increasing and decreasing pillar radii. The displacement peak values with decreasing and increasing radius are

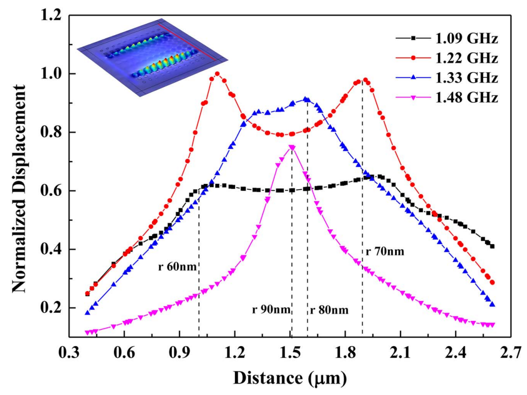

Figure 2. Displacement distributions of the line profile at the output with decreasing radius. The dash lines indicate the locations of pillars. The inset is the displacement field of the interface at 1.09 GHz, the red line is the line profile of sampling.

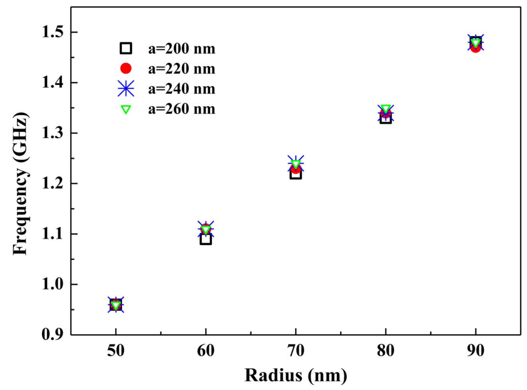

Figure 3. The displacement peak values of pillars with decreasing radius with different lattice constants.

shown in figure 1(b). The grey insets are the 3D geometry of models. The coloured insets which are around the data show stress fields of the interface, the upper and lower ones correspond to the decreasing and increasing radius, respectively. As shown in the coloured insets, the guiding of elastic waves can be observed unambiguously along different channels. The displacement peaks with the decreasing and increasing radius are almost located at the same frequencies, which shows the scattering mechanism is not the dominant factor in phonon transport here. Therefore, the key point of controlled guiding is to design the resonance characteristics of the pillars.

Figure 2 shows the displacement distributions at the output line along the y direction for the decreasing radius. The frequencies here correspond to the local displacement peaks in figure 1. At different frequencies, the large displacements are observed at different locations which are related to the radii of the pillars. For example, in the displacement distribution at 1.22 GHz, the large displacements appear at 1.1 and 1.9 μm, which is the location of pillars r with 70 nm. This observation corroborates our conclusion that the radii of pillars determine the channels of propagation of elastic waves. The inset shows the displacement field at 1.09 GHz, which demonstrates the strong guiding. We deduce that the guiding is induced by the resonance effect in the system and the guided frequency and propagation channel is determined by the geometry of attachments. To further explore the strong guiding effect, we compare results for different lattice constants.

As shown in figure 3, the displacement peak values of pillars with decreasing radius change little with increasing the lattice constant. The results show that the displacement peak is determined by the resonance effect produced by pillars and not influenced by scattering. The guiding of the elastic waves in the GPnC plate is caused by the interaction of vibrations of pillars with the travelling wave in the substrate. Hence, the displacement peak corresponds to the resonance frequency induced by the coupling of pillars and substrate. In 2D infinite GPnC and hole-type GPnC plates, the focus length can be adjusted by modulation of the lattice constant [2, 7, 28, 29]. However, due to the resonance effect induced by pillars here, the elastic waves are guided instead of the focused. The coupling between resonators and matrix can be changed by modulating the quality factor, including

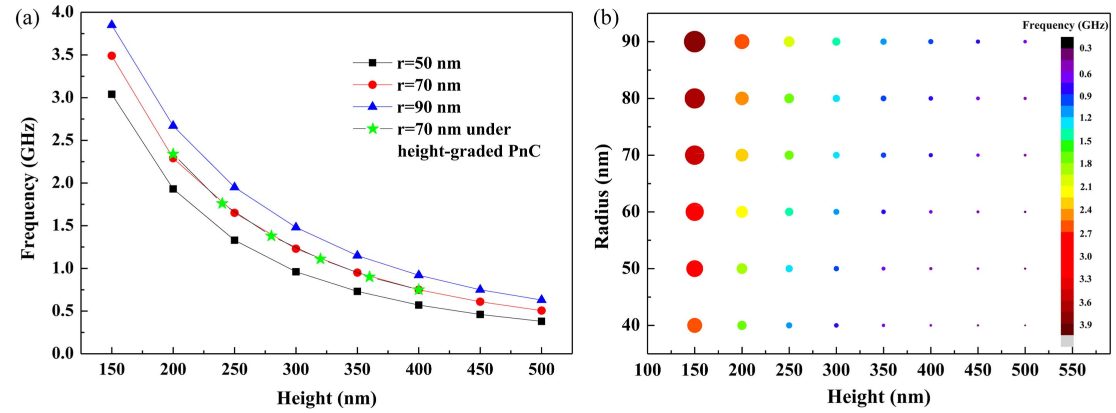

Figure 4. (a) Comparison of resonance frequencies with different height of pillars in GPnC plate with decreasing radius (the star-marked points are taken from GPnC plate with increasing height from the middle to the edge which is introduced below); (b) the relationship between resonance frequencies and geometry parameters (radius and height) of pillars, the size and colour of circles indicate the values of frequencies.

geometry and material properties. The influence of the geometry of the resonators on the guiding of elastic waves is analysed in detail in the following.

Here we focus on the influence of the height of the pillars on the resonance frequency. The influence of the thickness of the substrate is discussed below. As figure 4(a) shows, with increasing height of the pillars, the resonance frequencies decrease gradually and the gradient of frequencies becomes smaller. With increasing the height of pillars the difference values of resonance frequencies between different sizes pillars decrease, e.g., the difference value of frequencies between pillars with $r = 50$ nm and $r = 90$ nm is 0.81 GHz at 150 nm height condition while the difference value is 0.25 GHz at 500 nm height condition. Since this phenomenon is determined by the coupling of attachments and substrate, it is not easy to obtain a general theoretical formula to get the relationship of resonance frequency and parameter of attachments, thus the simulation is essential to explore it. The relationship between resonance frequencies and geometry parameters of pillars is shown in figure 4(b), the size and colour of circles indicate the values of resonance frequencies. From this bubble figure, the change trend of frequencies can be observed unambiguously, which increases with the radius and decreases with the height of pillar. Based on our simulation results, with pillars of height $h = 50$ nm the system demonstrates the characteristics of focusing of elastic waves which is similar to acoustic lens; with $h = 100$ nm the system shows the hybrid effect of focusing and guiding of elastic waves; when the height reaches to 150 nm or higher, the guiding appears unambiguously; if the height exceeds than 600 nm, the guiding of lower resonance frequencies which propagate along the channels with slim pillars are hard to be observed. Therefore, the strong guiding of elastic waves in GPnC plate can also be achieved with the appropriate setting of the height of pillars. It can be explained well by the coupling effect between attachments and matrix. By modulating the coupling of pillars and substrate one can realize a continuous change from localized resonance to Bragg scattering. When the height of pillar is smaller than the appropriate range of height (150 nm here), the coupling between pillars and substrate is so strong that the local resonance weakens and the scattering effect enhances, thus the guiding turns into the acoustic focusing effect; when the height of pillar is larger than 600 nm, the resonance frequency of pillar is small and most energy will be stored in the pillars, so that the coupling of pillars and substrate is too weak to produce the guiding distinctly.

The dispersion relation and the corresponding transmission of the GPnC with decreasing radius (the model in figure 1) are shown in figure 5, which provides another viewpoint to explore the guiding phenomenon. The dispersion relation is calculated by using the supercell which contains one column of pillars along the y direction and applies periodic boundary condition in the x direction. The transmission spectrum is the energy transmission spectrum which is obtained by the sum of the square of displacement components. Large bandgaps which forbid the propagation of elastic waves can be observed from figure 5(a). The local resonance modes and propagation modes are both shown in the dispersion relation as flat or flat-like bands and non-flat bands, respectively. Corresponding to the transmission peaks at 0.81 and 0.96 GHz, the bands are relatively flat which show local resonance modes, that cause a disadvantage for wave guiding. However, the resonant frequencies at 1.09 , 1.22, 1.33, and 1.48 GHz show good transmission conditions which demonstrate strong guiding. The transmission spectrum agrees with the dispersion relation and the transmission peaks correspond to the resonance frequencies. Therefore, the guiding in the GPnC plate can be used as a multi-channel frequency filter which exhibits multi-resonance channels. When the excitation frequency is at one resonance frequency, the elastic waves mainly propagates along these specific channels and distribute most energy in the surface and

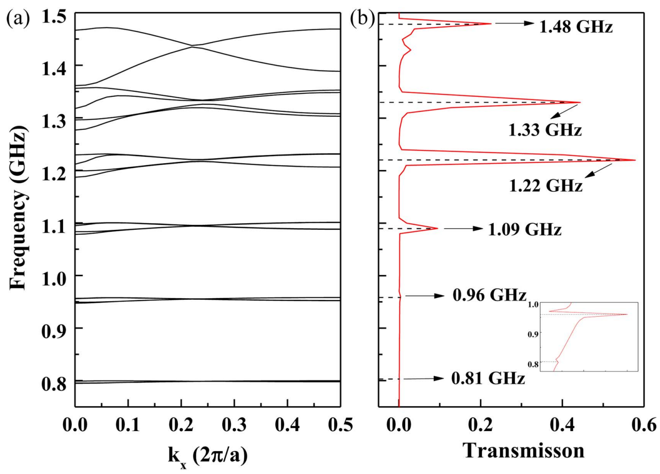

Figure 5. (a) Dispersion relation and (b) transmission of the GPnC plate with decreasing radius in figure 1.

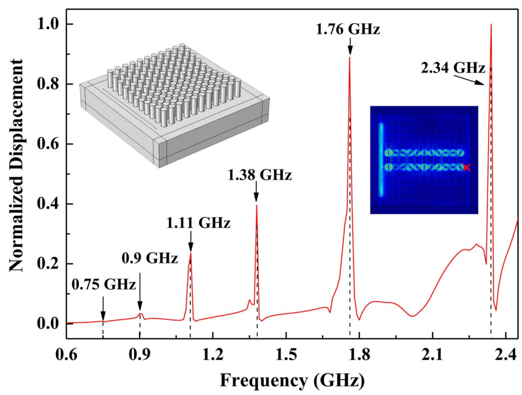

Figure 6. Normalized displacements of GPnC plate with increasing height from the middle to the edge. The radii of pillars are 70 nm, height of pillars are from 200 to 400 nm with variation $\Delta = 40$ nm, and the other settings are same to GPnC plate with decreasing radius in figure 1. The grey inset is the 3D geometry of GPnC plate with increasing height, the coloured inset is the stress field at 1.76 GHz and the red cross shows the sampling point.

subsurface of the substrate and the pillars in the channels. In the following, we will introduce another approach where instead of the gradient radii of pillars, the pillar heights change gradually.

The numerical simulation is performed for a GPnC plate with increasing height of pillars from the middle to the edge and all the other conditions are the same with GPnC plate with decreasing radius. Figure 6 shows six local displacement peaks corresponding to resonance frequencies, which is in close similarity to the results obtained on radius graded PnC, and thus also demonstrates the guiding in the GPnC plate with increasing height. The values of resonance frequencies here are plotted in figure 4(a), which agrees well with the change in the trend of frequency with the height of pillars. This result strengthens the conclusion that the guiding is induced by resonance coupling of attachments and matrix and provides another way to control the propagation of elastic waves and drop different frequencies into different channels.

Besides the above discussions about the influence of the geometry of pillars on guiding, further influence factors are also explored. The simulation results show that resonance frequencies increase with Young's modulus and decrease with density. The shapes of cross-sections of pillars in *z* plane also influence resonance frequencies due to the discrepancy of boundary conditions. Since the most energy of guided wave exists on the surface and the area adjacent to the surface of substrate and resonators at resonance frequencies, the thickness of the substrate which is outside of subsurface range has no influence on the guiding of elastic waves in GPnC plate.

4. Conclusion

The guiding of elastic waves, which are induced by the resonant coupling of attachments and matrix, is shown in 2D pillar-type GPnC plates. The resonance frequency is influenced by geometry and material properties of the resonators, which provides a new way to control phonon transport. For the GPnC plate with gradient-radius, the strong guiding of elastic waves can be achieved with the appropriate setting of the pillar height with a certain pillar radius. The guiding of elastic waves are not limited to the gradient-radius condition, but also apply in gradient-height, gradient-Young's modulus and gradient-density conditions. The increase of radius and Young's modulus of pillars will increase the resonance frequencies while the increase of height and density will decrease them, which offers several ways to guide the propagation of elastic waves. The GPnC plate can be designed to separate the different frequencies component of elastic waves into the different channels, which provides the possibility to design wavelength division demultiplexers or multi-channel frequency filters.

Acknowledgments

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