GLOBALLY SMOOTH EXPRESSIONS FOR SHOCK PRESSURE DECAY IN IMPACTS Thomas Ruedas[1,2], [1]Institute for Planetology, University of Münster, Germany; [2]Institute of Planetary Research, German Aerospace Center (DLR), Berlin, Germany (t.ruedas@uni-muenster.de)

Introduction: Hypervelocity meteorite impacts on planets subject the target to very high shock pressures. The isobars in the affected region are approximately spherical and centered on a point at a certain depth below the point of impact. The decay of the shock pressure with increasing distance from that center is usually represented in the form [1]

$$\lg p = a + n \lg\left(\frac{r}{R}\right), \tag{1}$$

where p is the shock pressure, r is the distance from the center of the sphere, R is the radius of the impactor, and a and n are fitting constants; a similar relation is also used for the particle velocity. Although its simplicity seems to speak in its favor, this form has some problems: it is not bounded from above, thus necessitating an imposed maximum value that depends on additional assumptions; the slope of the decay is not accurately described by a single n for all r; the transition between domains with different decay parameters introduces kinks into $p(r)$ at poorly defined points.

It is therefore proposed to replace the decay law eq. 1 with other functional forms that are bounded and smooth and can be fully derived from data from numerical experiments. The proposed functions are applied to data of dunite-on-dunite impacts from the literature.

Method: The model function for the pressure decay should be positive everywhere and have a single maximum at $r = 0$, i.e., at the center of the shock pressure sphere. It is convenient to normalize the pressure $p(r)$ with the pressure determined from the impedance match solution p_{IM}, which can be calculated from the material parameters. The following alternative two model functions are considered:

$$\frac{p}{p_{\mathrm{IM}}} = \frac{a}{b + \left(\frac{r}{R}\right)^n} \tag{2a}$$

$$\frac{p}{p_{\mathrm{IM}}} = a \operatorname{arccot}\left[b\left(\frac{r}{R}\right)^n\right]; \tag{2b}$$

these functions will be referred to as the "inverse-r" and the "arccotangent" model, respectively. In the inverse-r model, the fitting parameters a and b are coupled by the condition $p(0)/p_{\mathrm{IM}} = a/b$. If one wishes to enforce the impedance-match solution as the solution at $r = 0$, the constraint $p(0)/p_{\mathrm{IM}} = 1$ results in $a = b$ and $a = 2/\pi$ for the inverse-r and the arccotangent model, respectively.

Results: Numerical experiments have been carried out by various authors for different material combinations, but there are few materials with such a broad range of velocities covered as dunite. [2] have modeled dunite-on-dunite impacts for impact velocities v between 10 and 60 km/s, and this dataset has recently been extended to lower v down to 4 km/s by [3]. Both datasets are of good quality and match quite well in the case $v = 10$ km/s, where they overlap. The fits presented here are based on the data from these experiments, which were taken from the figures in the respective publications.

Table 1: Some fits of eqs. 2 to dunite ($\varrho = 3320\,\mathrm{kg/m^3}$, $C = 6.5$ km/s, $S = 0.9$, [4, 2]) data. The data for $v = 10$ km/s are for the fit to the combined datasets of [2, 3].

v (km/s)	p_{IM} (GPa)	a	b	n	misfit
Inverse-r model (eq. 2a)					
4	55.1	0.797	1.687	1.213	$7.066 \cdot 10^{-4}$
7	112.1	1.355	1.575	1.299	$9.614 \cdot 10^{-5}$
10	182.6	1.311	1.564	0.994	$1.925 \cdot 10^{-3}$
20	514.6	5.224	5.888	2.243	$3.916 \cdot 10^{-3}$
60	3336.6	34.451	31.635	3.366	$8.242 \cdot 10^{-3}$
Arccotangent model (eq. 2b)					
4	55.1	0.28	0.539	1.069	$8.446 \cdot 10^{-4}$
7	112.1	0.388	0.374	1.217	$8.635 \cdot 10^{-5}$
10	182.6	0.543	0.727	0.778	$1.880 \cdot 10^{-3}$
20	514.6	0.566	0.234	1.834	$3.739 \cdot 10^{-3}$
60	3336.6	0.692	0.056	2.804	$8.276 \cdot 10^{-3}$

As can be seen in Figure 1, the fits for both model functions to the data at 4 and 60 km/s are very good, whereas the fit is less tight at 10 km/s. The reason for the less good performance in the latter case is probably that the fit includes two different datasets that have slightly different trends in the crucial overlap region; the individual fits to each dataset match better, but differ visibly from each another, because the data from [2] do not reach far into the far field. Also shown are the fits from [3] for 4 and 10 km/s and the general fit from [2] with the parameters for dunite (with a value of 0.22 replacing the erroneous b value 0.022 from their Table II), both of which have the form of eq. 1; those formulae have been capped with the impedance-match solution, following common practice.

The parameters for the fit are velocity-dependent in the formulae by [2, 3] as well as in the models presented here. The previous workers have therefore constructed fits with the fitting parameters as functions of v. Following their example, the following tentative functions have been derived for the inverse-r model:

$$a(v) = 0.0806^{1.478} \tag{3a}$$

$$b(v) = 0.2658 v^{1.161} \tag{3b}$$

$$n(v) = -0.2034 + 1.9535 \lg v; \tag{3c}$$

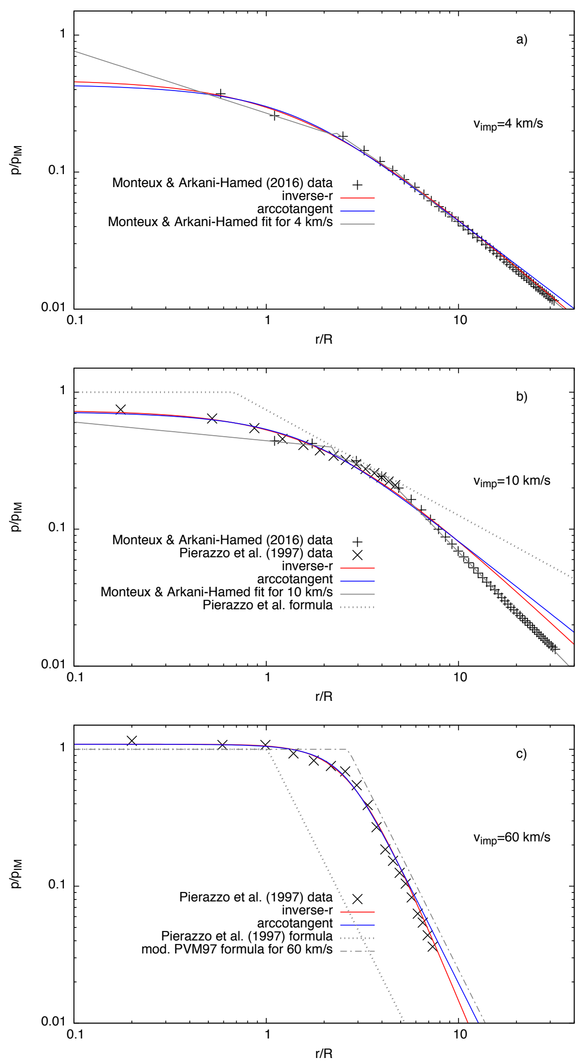

Figure 1: Numerical experiment data for dunite and different fits of $p(r)/p_{\mathrm{IM}}$ for impactor velocities v of 4 (a), 10 (b), and 60 km/s (c); the data points are from [2, 3].

and for the arccotangent model:

$$a(v) = 0.1869 v^{0.333} \quad (4a)$$

$$b(v) = 1.1766 v^{-0.636} \quad (4b)$$

$$n(v) = -0.0408 + 1.5386 \lg v. \quad (4c)$$

However, due to the inhomogeneity and the limitations in sampling of the two available datasets, eqs. 3 and 4 work only well at v substantially higher than 10 km/s. It is expected that a homogeneous dataset covering the entire v range of interest would improve the quality of a general, v-dependent fit substantially. The v-dependent fits by previous workers are apparently not suited for use much beyond their v range of calibration; for instance, the v-dependent exponent n in the general formula from [2] even reverses sign at v less than approximately 5 km/s and probably ceases to be reliable even at v above that limit. Even at 10 km/s, the possibly insufficient coverage of the [2] data of the far field may make their $p(r)$ curve unreliable at large r, as can be seen in Figure 1b, although the fact that the models from [3] included fluidization and damage effects may also play a role. It was also found that there is a significant offset between the data and the p curve of [2] at high v if the transition between the isobaric core region and the decay regime is made at the isobaric core radius determined from their formula. Therefore, their data were refitted to yield the modified formula shown in Figure 1c.

It is hoped that future numerical models of hypervelocity impacts will cover the whole range of v from a few to several tens of km/s for different material combinations with a sufficiently dense sampling of all decay domains, enabling an improved general fit of the data with one of the model functions proposed here with fitting parameters represented in v-dependent functional form. The figure shows that predicted p decays can vary substantially between different parameterizations, and as a consequence, so will shock heating estimates especially for the far field. Inaccurate calculations of p will thus translate into inaccurate thermal models for use in applications of these functions in mantle convection models.

References: [1] T. J. Ahrens, et al. (1977) in *Impact and Explosion Cratering* (Edited by D. J. Roddy, et al.) 639–656 Pergamon Press, Elmsford, N.Y. [2] E. Pierazzo, et al. (1997) *Icarus* 127(2):408 `doi`. [3] J. Monteux, et al. (2016) *Icarus* 264:246 `doi`. [4] H. J. Melosh (1989) *Impact cratering: a geologic process* no. 11 in Oxford Monographs on Geology and Geophysics Oxford University Press.