Formation Splitting and Merging

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Abstract. This paper presents an approach to swarm split and rejoin maneuvers of a system of multi-robots formations. A post split formation is split into low-degree sub-swarms when the swarm encounters an obstacle. The sub-swarms reestablish links with other sub-swarms and converge into its pre-split formation after avoiding collisions with the obstacles. The leader-follower control strategy is used for maintaining formation shape in the sub-swarms. A set of artificial potential field functions is proposed for avoiding inter-robot, inter-formation and obstacle collisions and attraction to their designated targets. The Direct Method of Lyapunov is then used to establish stability of the given system. The effectiveness of the proposed nonlinear acceleration control laws is demonstrated through a computer simulation.

Keywords: Formation, Lyapunov; Nonholonomic mobile robots, Low-degree.

1 Introduction

Recent years have seen considerable attention focussed on the problem of coordinated motion and cooperative control of multi-robot systems. This interest arises from complex systems such as ecology, social sciences, evolutionary biological sciences, control theory in science and engineering systems [1, 2]. These systems have individual characteristics in the way they interact with other agents and also reveal certain group phenomena which can give useful ideas for developing control theory for robotic systems. Often observed, biological systems such as groups of ants, fish, birds and bacteria reveal some amazing cooperative behaviors in their motion [1], such as reaching a target or moving in a formation to name a few. Researchers use computer simulations and animations to generate generic simulated flocking creatures called *boids* to study problems in ecology in the context of animal aggregation and social cohesion in animal groups [3]. The pioneering work on this area was done by Reynolds [4].

Now consider a cohesive swarm system in some initial configuration. When the swarm encounters an obstacle, splitting may be useful to maneuver the swarm around the obstacle. In many instances, it is more desirable for some agents to move to one side of the obstacle while the others move to the other side and later merge. The swarm system splits into clustered subswarms, moves around the obstacle and later re-establishes into its pre-split formation. In this paper, we consider creating two or more low-degree post-split sub-formations from some pre-split formation. This is achieved by breaking the link between different post-split sub-formations. However, the links between agents in the post-split sub-formations are still maintained. As such the agents in the same sub-formations maintain their formation structure while avoiding an obstacle. A leader-following based formation scheme is used to maintain the geometric formation shape in each sub-formation. The post-split sub-formations later re-establish links with other sub-swarms and converge into its pre-split formation once collision avoidance with the obstacles is achieved. The control strategy formulates a low degree formation which allows for slight distortions in the sub-formations. These distortions would normally appear if the group encounters an obstacle. Based on artificial potential fields, the Direct Method of Lyapunov is then used to derive continuous acceleration-based controllers which render our system stable.

The remainder of this paper is structured as follows: in Section 2, the robot model is defined; in Section 3, the artificial potential field functions are defined under the influence of kinodynamic constraints; in Section 4, the Lyapunov function is constructed, while in Section 5, stability analysis of the robotic system is carried out; in Section 6, we demonstrate the effectiveness of the proposed controllers via a computer simulation; and finally, Section 7 concludes the paper.

2 Vehicle Model

We will consider h, $h \in \mathbb{N}$ formations with n, $n \in \mathbb{N}$, car-like mobile robots, where the ith agent in the hth formation is denoted by $\mathcal{A}_{hi}$. We denote the hth formation as $\mathcal{A}_h$. Without loss of generalization, we let $\mathcal{A}_{h1}$ represent the leader in the hth formation structure while the others in $\mathcal{A}_h$ take the role of followers. Let (x_{hi}, y_{hi}) represents the Cartesian coordinates and gives the reference point of $\mathcal{A}_{hi}$, which is the midpoint of the centers of the front and rear axles. Moreover, θ_{hi} gives the orientation of $\mathcal{A}_{hi}$ with respect to the z_1-axis of the z_1z_2-plane while ϕ_{hi} gives the steering angle with respect to its longitudinal axis of $\mathcal{A}_{hi}$. L_{hi} represents the distance between the centers of the front and rear axles and l_{hi} is the length of each axle of $\mathcal{A}_{hi}$.

Next, to ensure that each robot safely steers past an obstacle we construct circular regions that protect the robot. Given the *clearance parameters* $\epsilon_1 > 0$ and $\epsilon_2 > 0$, we enclose the each vehicle by a protective circular region centered at (x_{hi}, y_{hi}) with radius $r_{hi} = \frac{1}{2}\sqrt{(L_{hi} + 2\epsilon_1)^2 + (l_{hi} + 2\epsilon_2)^2}$ for $h = 1, \ldots, M$, and $i = 1, \ldots, n$.

These generate the *nonholonomic constraints* on the system. The kinodynamic model of the system, adopted from [6] is

$$\left.\begin{array}{l} \dot{x}_{hi} = v_{hi}\cos\theta_{hi} - \frac{L_{hi}}{2}\omega_{hi}\sin\theta_{hi}, \quad \dot{y}_{hi} = v_{hi}\sin\theta_{hi} + \frac{L_{hi}}{2}\omega_{hi}\cos\theta_{hi}, \\ \dot{\theta}_{hi} = \frac{v_{hi}}{L_{hi}}\tan\phi_{hi} := \omega_{hi}, \quad \dot{v}_{hi} := \sigma_{hi1}, \qquad \dot{\omega}_{hi} := \sigma_{hi2}, \end{array}\right\} \quad (1)$$

for $h = 1, \ldots, M$, and $i = 2, \ldots, n$. Here, v_{hi} and ω_{hi} are, respectively, the instantaneous translational and rotational velocities of $\mathcal{A}_{hi}$, while σ_{hi1} and σ_{hi2} are the instantaneous translational and rotational accelerations of $\mathcal{A}_{hi}$. Without any loss of generality, we assume that $\phi_{hi} = \theta_{hi}$.

2.1 Leader-Follower Based Formation Scheme

To desire a substantial degree of rigidness in our formation of $\mathcal{A}_h$, we assign a Cartesian coordinate system $(X_h - Y_h)$ fixed on the leaders body of the $\mathcal{A}_h$, as shown in Fig. 1 adopted from [5], based on the concept of an instantaneous co-rotating frame of reference.

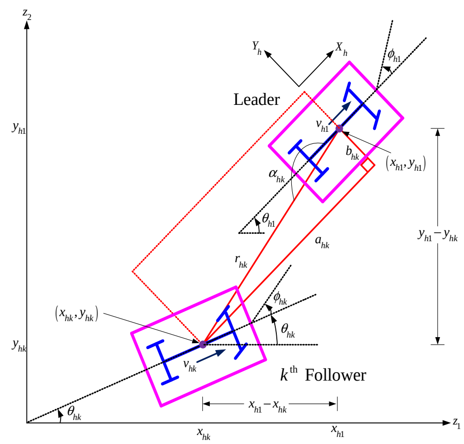

Fig. 1. The proposed scheme utilizing a rotation of axes with axis fixed at the leader.

We consider the position of the kth follower in $\mathcal{A}_h$ by considering the relative distances of $\mathcal{A}_{hk}$ in $\mathcal{A}_h$, from its leader, $\mathcal{A}_{h1}$ along the given X_h and Y_h directions. Thus, we have:

$$\begin{aligned} A_{hk} &= -(x_{h1} - x_{hk}) \cos\theta_{h1} - (y_{h1} - y_{hk}) \sin\theta_{h1}, \\ B_{hk} &= (x_{h1} - x_{hk}) \sin\theta_{h1} - (y_{h1} - y_{hk}) \cos\theta_{h1}, \end{aligned} \tag{2}$$

for $h = 1, \ldots, M$, and $k = 2, \ldots, n$ and A_{hk} and B_{hk} are the relative positions with respect to the X_h-Y_h coordinate system of the kth followers in A_h.

2.2 Split/Rejoin Maneuvers

One of the leaders of the sub-formations takes the role of the supreme leader of the entire flock of sub-formations. The rest of the leaders then have a dual role. They become followers to the supreme leader and also take the responsibility to lead its group. To establish an attractive force between the leaders the follower robots are required to follow the supreme leader via a concept known as mobile ghost targets (a_h, b_h) positioned relative to the supreme leader's position. The supreme leader will move towards its defined target with center (p_{111}, p_{112}), while the ghost targets move relative to the leader's position and the follower robots move towards their designated ghost targets.

3 Artificial Potential Field Function

This section formulates collision free trajectories of the robot system under kinodynamic constraints in a given workspace.

3.1 Attractive Potential Field Functions

Attraction to Target For the establishment and advancement of the group of M formations having n mobile robots, a target is assigned to the supreme leader. For the attraction of the supreme leader, $\mathcal{A}_{11}$ to its designated target, we consider an attractive potential function

$$V_{11}(\mathbf{x}) = \frac{1}{2}\left[(x_{11} - p_{111})^2 + (y_{11} - p_{112})^2 + v_{11}^2 + \omega_{11}^2\right]. \qquad (3)$$

The leader, $\mathcal{A}_{11}$ will move towards its defined target with center (p_{111}, p_{112}), while the other leader robots, $\mathcal{A}_{h1}$ for $h = 2, \ldots, M$ follow the ghost target relative to the supreme leader. For this we consider

$$V_{h1}(\mathbf{x}) = \frac{1}{2}\left[(x_{h1} - x_{11} + a_h)^2 + (y_{h1} - y_{11} + b_h)^2 + v_{h1}^2 + \omega_{h1}^2\right]. \qquad (4)$$

The leaders, $\mathcal{A}_{h1}$ for $h = 2, \ldots, M$ will be maintaining a desired relative position to supreme leader.

For $\mathcal{A}_{hi}$ for $i = 2, \ldots, n$ to maintain its desired relative position with respect to the leader, $\mathcal{A}_{h1}$, we utilize the following potential function for $h = 1, \ldots, M$ and $i = 2, \ldots, n$

$$V_{hi}(\mathbf{x}) = \frac{1}{2}\left[(A_{hi} - a_{hi})^2 + (B_{hi} - b_{hi})^2 + +v_{hi}^2 + \omega_{hi}^2\right]. \qquad (5)$$

Auxiliary Function To guarantee the convergence of the supreme leader to its designated target, we define

$$G_{11}(\mathbf{x}) = \frac{1}{2}\left[(x_{11} - p_{111})^2 + (y_{11} - p_{112})^2 + (\theta_{11} - p_{113})^2\right], \qquad (6)$$

where p_{13} is the prescribed final orientation of the supreme leader robot. To guarantee the convergence of the leader mobile robots, $\mathcal{A}_{h1}$ to their designated ghost targets, we design an auxiliary function for $h = 2, \ldots, M$ as

$$G_{h1}(\mathbf{x}) = \frac{1}{2}\left[[(x_{h1} - x_{11} + a_h)^2 + (y_{h1} - y_{11} + b_h)^2 + (\theta_{h1} - p_{h13})^2\right], \qquad (7)$$

where p_{h3} is the prescribed final orientation of the leader robot, $\mathcal{A}_{h1}$ and

$$G_{hi}(\mathbf{x}) = \frac{1}{2}\left[(A_{hi} - a_{hi})^2 + (B_{hi} - b_{hi})^2 + (\theta_{hi} - p_{hi3})^2\right], \tag{8}$$

for $i = 2, \ldots, n$ and $h = 1, \ldots, M$. These auxiliary functions are then multiplied to the repulsive potential field functions to be designed in the following subsections.

3.2 Repulsive Potential Field Functions

We desire the leader, $\mathcal{A}_{h1}$ and its followers, $\mathcal{A}_{hk}$ avoid all fixed and moving obstacles intersecting their paths.

Fixed Obstacles Let us fix $q \in \mathbb{N}$ solid obstacles within the boundaries of the workspace. We assume that the lth obstacle is a circular disk with center (o_{l1}, o_{l2}) and radius ro_l. For $\mathcal{A}_{hi}$ to avoid the lth obstacle, we consider

$$FO_{hil}(\mathbf{x}) = \frac{1}{2}\left[(x_{hi} - o_{l1})^2 + (y_{hi} - o_{l2})^2 - (ro_l + r_{hi})^2\right], \tag{9}$$

as an avoidance function, where $h = 1, \ldots, M$, $i = 1, \ldots, n$, and $l = 1, \ldots, q$.

Moving Obstacles To generate feasible trajectories, we consider moving obstacles of which the system has prior knowledge. Here, each mobile robot, has to be treated as a moving obstacle for all other mobile robots in the workspace.

Minimum Inter-Robot Distance We desire to maintain a minimum inter robot separation distance between the robots. This prevents $\mathcal{A}_{hi}$ from getting very close to (or colliding with) $\mathcal{A}_{hj}$ [6], especially during the re-establishment of the prescribed formation when the system is distorted. We can consider the following obstacle avoidance function

$$MO_{hij}(\mathbf{x}) = \frac{1}{2}\left[(x_{hi} - x_{hj})^2 + (y_{hi} - y_{hj})^2 - (r_{hi} + r_{hj})^2\right], \tag{10}$$

for $h = 1, \ldots, M$, and $i, j = 1, \ldots, n$, with $j \neq i$.

Inter Formation Avoidance We also desire for each formation structure in the system to avoid any other formation structure in the workspace. For ith body of $\mathcal{A}_h$ to evade the uth body of $\mathcal{A}_m$, we adopt

$$DO_{himu}(\mathbf{x}) = \frac{1}{2}\left[(x_{hi} - x_{mu})^2 + (y_{hi} - y_{mu})^2 - (r_{hi} + r_{mu})^2\right], \tag{11}$$

for $i, u = 1, \ldots, n$ and $h, m = 1, \ldots, M$ with $m \neq h$.

Dynamic Constraints Practically, the steering angles of the mobile robots are limited due to mechanical singularities while the translational speed is restricted due to safety reasons. Subsequently, we have $|v_{hi}| < v_{\max}$, where $v_{\max}$ is the *maximal achievable speed* of the $\mathcal{A}_{hi}$ and $|\omega_{hi}| < \frac{v_{\max}}{|\rho_{\min}|}$, where $\rho_{\min} := \frac{L_{hi}}{\tan(\phi_{\max})}$. This condition arises due to the boundness of the steering angle ϕ_{hi}. That is, $|\phi_{hi}| \leq \phi_{\max} < \pi/2$, where $\phi_{\max}$ is the *maximal steering angle*. Hence, we consider the following avoidance functions:

$$U_{hi1}(\mathbf{x}) = \frac{1}{2}\left(v_{\max} - v_{hi}\right)\left(v_{\max} + v_{hi}\right), \tag{12}$$

$$U_{hi2}(\mathbf{x}) = \frac{1}{2}\left(\frac{v_{\max}}{|\rho_{\min}|} - \omega_{hi}\right)\left(\frac{v_{\max}}{|\rho_{\min}|} + \omega_{hi}\right), \tag{13}$$

for $h = 1, \ldots, M$ and $i = 1, \ldots, n$.

4 Acceleration Controllers

The nonlinear acceleration control laws for system (1), will be designed using LbCS as proposed in [6].

4.1 Lyapunov Function

We now construct the total potentials, that is, a Lyapunov function for system (1).

$$L_{(1)}(\mathbf{x}) = \sum_{h=1}^{M}\sum_{i=1}^{n}\left[V_{hi}(\mathbf{x}) + G_{hi}(\mathbf{x})Z(\mathbf{x})\right], \tag{14}$$

where

$$Z(\mathbf{x}) = \sum_{l=1}^{q}\frac{\alpha_{hil}}{FO_{hil}(\mathbf{x})} + \sum_{s=1}^{2}\frac{\beta_{his}}{U_{his}(\mathbf{x})} + \sum_{\substack{j=1\\ j\neq i}}^{n}\frac{\eta_{hij}}{MO_{hij}(\mathbf{x})} + \sum_{u=1}^{n}\sum_{\substack{m=1\\ m\neq h}}^{M}\frac{\gamma_{himu}}{DO_{himu}(\mathbf{x})}.$$

Utilizing the attractive and repulsive potential field functions, continuous time-invariant acceleration control laws, $\sigma_{hi1}, \sigma_{hi2}$, can be generated, that intrinsically guarantees stability,in the sense of Lyapunov, of system (1) as well.

5 Stability Analysis

Theorem 1. *Let* $(p_{h11}, p_{h12}) = (a_h, b_h)$ *for* $h = 2, \ldots, M$ *be the position of the target of the leader in* $\mathcal{A}_h$*, and* p_{hi3} *for* $i = 1, \ldots, n$*, be the desired final* *orientations of the robots in each $\mathcal{A}_h$. Given a_{hi} and b_{hi} in $\mathcal{A}_h$, let p_{hi1} and p_{hi2} satisfy*

$$a_{hi} = -(p_{h11} - p_{hi1})\cos\theta_{h1} - (p_{h12} - p_{hi2})\sin\theta_{h1},$$
$$b_{hi} = (p_{h11} - p_{hi1})\sin\theta_{h1} - (p_{h12} - p_{hi2})\cos\theta_{h1},$$

for $i = 2, \ldots, n$ and $h = 1, \ldots, M$.

Given $\mathbf{x}_{hi}^* := (p_{hi1}, p_{hi2}, p_{hi3}, 0, 0) \in \mathbb{R}^5$, if $\mathbf{x}_e := (\mathbf{x}_{11}^*, \mathbf{x}_{12}^*, \ldots, \mathbf{x}_{hn}^*) \in \mathbb{R}^{5\times n\times M}$ is an equilibrium point for (1), then $\mathbf{x}_e \in D(L_{(1)}(\mathbf{x}))$ is a stable equilibrium point of system (1).

Proof. One can easily verify the following, for $i = 1, \ldots, n$ and $h = 1, \ldots, M$:

1. $L_{(1)}(\mathbf{x})$ is defined, continuous and positive over the domain $D(L_{(1)}(\mathbf{x})) = \{\mathbf{x} \in \mathbb{R}^{5\times M\times n} : FO_{hil}(\mathbf{x}) > 0, l = 1, \ldots, q; DO_{himu}(\mathbf{x}) > 0, m = 1, \ldots, M, u = 1, \ldots, n, m \neq h;\ MO_{hij}(\mathbf{x}) > 0, j = 1, \ldots, n, j \neq i;\ U_{his}(\mathbf{x}) > 0, s = 1, 2\}$;
2. $L_{(1)}(\mathbf{x}^*) = 0$;
3. $L_{(1)}(\mathbf{x}) > 0\ \ \forall \mathbf{x} \in D(L_{(1)}(\mathbf{x}))/\mathbf{x}_e$.

Next, consider the time derivative of the candidate Lyapunov function along a particular trajectory of system (1), we obtain the following semi-negative definite function

$$\dot{L}_{(1)}(\mathbf{x}) = -\sum_{i=1}^{n} \left(\delta_{hi1} v_{hi}^2 + \delta_{hi2}\omega_{hi}^2\right) \leq 0,$$

for $h = 1, \ldots, M$ and $i = 1, \ldots, n$, where $\delta_{hi1} > 0$, and $\delta_{hi2} > 0$ are constants commonly known as convergence parameters.

Thus, $\dot{L}_{(1)}(\mathbf{x}) \leq 0\ \forall \mathbf{x} \in D(L_{(1)}(\mathbf{x}))$ and $\dot{L}_{(1)}(\mathbf{x}_e) = 0$. Finally, it can be easily verified that $L_{(1)}(\mathbf{x}) \in C^1\left(D(L_{(1)}(\mathbf{x}))\right)$, which makes up the fifth and final criterion of a Lyapunov function. □

Remark 1. This result is in no contradiction with Brockett's Theorem [7] as we have not proven asymptotic stability.

6 Simulation Results

In this section, we illustrate the effectiveness of the proposed continuous time-invariant controllers within the framework of the Lyapunov-based control scheme by simulating a virtual scenario.

We consider the motion of a pair of 4 cars in a split/rejoin formation in a two dimensional space with static obstacles in its path. Each follower robot in each formation structure is assigned a unique position relative to its leader as seen in Fig. 2. This is achieved by assigning appropriate values to (a_{hk}, b_{hk}) to obtain a geometric formation structure. While a leaders $\mathcal{A}_{11}$ and $\mathcal{A}_{21}$ move towards its intended target, the followers, $\mathcal{A}_{hk}$ for $h = 1, 2$ and $k = 2, \ldots, 4$ in each sub-formation is observed to maintain a low-degree formation. The sub swarms

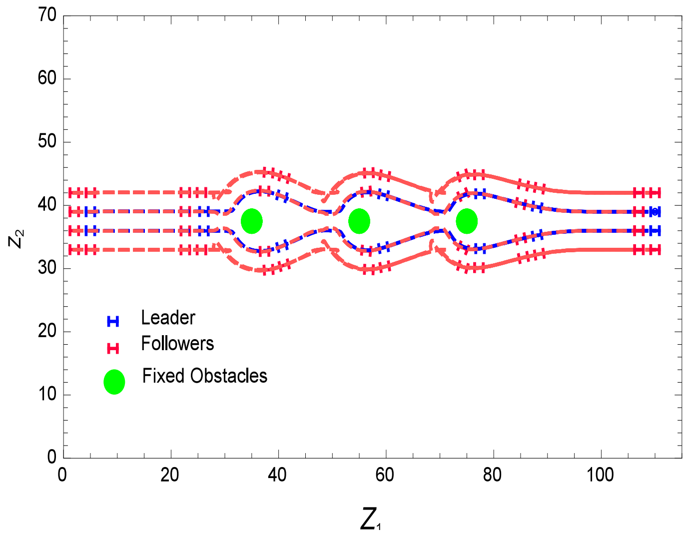

Fig. 2. The proposed scheme showing the split/rejoin formation of the robots.

split from their initial formation when the swarm encounters an obstacle. The sub-formations reestablish links with other sub-swarms and converge into its pre-split formation after avoiding collisions with the obstacles. The leader-follower control strategy is used for maintaining formation shape in the sub-swarms. Assuming that the appropriate units have been accounted for, Table 1 provides the corresponding initial and final configurations of the two robots and other necessary parameters required to simulate the scenario.

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7 Conclusion

In this paper we have proposed a leader follower scheme for the coordination of multi robot systems in an environment with obstacles. A split/rejoin maneuver is observed between sub-swarms. After avoiding collisions with the obstacles, the sub-swarms reestablishes its links with the other sub-swarms and converge into its pre-split formation. The leader-follower control strategy is used for maintaining formation shape in the sub-swarms. An advantage of the proposed scheme is that we can have multiple formations structures having different geometric shapes. The approach also considers inter-robot and inter-formation collision avoidance. The effectiveness of the proposed control laws were demonstrated via a computer simulation.

Table 1. Numerical values of initial and final states, constraints and parameters.

Initial Configuration	
Rectangular positions of leaders	$(x_{11}, y_{11}) = (5, 39), (x_{21}, y_{21}) = (5, 36)$
Desired relative distances of followers	$a_{12} = a_{22} = b_{14} = b_{24} = 0, b_{12} = b_{13} = -3$ $a_{13} = a_{14} = a_{23} = a_{24} = b_{22} = b_{23} = 3$
Translational velocity	$v_{hi} = 0.5$ for $i = 1, \ldots, 4$, $h = 1, 2$
Rotational velocities	$\omega_{hi} = 0$, for $i = 1, \ldots, 4$, $h = 1, 2$
Angular positions	$\theta_{hi} = 0$, for $i = 1, \ldots, 4$, $h = 1, 2$
Constraints and Parameters	
Dimensions of robots	$L_{hi} = 1.6$, $l_{hi} = 1.2$ for $i = 1, \ldots, 4$, $h = 1, 2$
$\mathcal{A}_1$ leader's target :	$(p_{111}, p_{112}) = (110, 39)$, $rt_{11} = 0.5$,
Fixed Obstacles	$(o_{11}, o_{12}) = (35, 37.5), (o_{21}, o_{22}) = (55, 37.5)$, $(o_{31}, o_{32}) = (75, 37.5)$
Max. translational velocity	$v_{\max} = 5$
Max. steering angle	$\phi_{\max} = \pi/2$
Clearance parameters	$\epsilon_1 = 0.1$, $\epsilon_2 = 0.05$
Control and Convergence Parameters	
Collision avoidance	$\alpha_{hil} = 0.1$, for $i = 1, \ldots, 4$, $h = 1, 2$, $l = 1, \ldots, 3$ $\eta_{hij} = 0.01$ for $h = 1, 2$, $i, j = 1, \ldots, 4$, $j \neq i$, $\gamma_{himu} = 0.01$, for $h, m = 1, 2$, $i, j = 1, \ldots, 4$, $h \neq m$
Dynamics constraints	$\beta_{his} = 1$, for $i = 1, \ldots, 4$, $h, s = 1, 2$,
Convergence	$\delta_{h11} = 12000$, $\delta_{h12} = 12000$, for $h = 1, 2$ $\delta_{hij} = 50$, for $i = 1, \ldots, 4$, $h, j = 1, 2$

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