A Cognitive Hierarchy Model of Behavior in Endogenous Timing Games*

Daniel Carvalho and Luís Santos-Pinto†

This version: January 25, 2010

Abstract

This paper applies the cognitive hierarchy model of Camerer, Ho and Chong (2004) to the action commitment game of Hamilton and Slutsky (1990). The model generates the heterogeneity of behavior reported in Huck, Müeller and Normann (2002). The model predicts the spike in the leadership quantity in the first period as well as the spike in the Cournot quantity in the second period. The model predicts delay, a feature that cannot be explained by social preferences. The also model predicts very well the percentage of Stackelberg outcomes, double leadership outcomes, and Stackelberg leaders punished by followers. Notwithstanding, the model produces low first period movement and is unable to generate sufficient percentages of sequential play of Cournot quantities and collusive market outcomes.

JEL Classification Numbers: C72, D43, L13.

Keywords: Endogenous Timing Games; Thinking Steps; Cognitive Hierarchy.

*We are thankful to Vince Crawford, Joel Sobel, Miguel Costa-Gomes, and Nagore Irriberi for many helpful comments. We are also thankful to conference and seminar participants of the Informal Research Workshop at Universidade Nova de Lisboa, Microeconomics Workshop at University of Lausanne, 24th Annual Congress of the European Economic Association in Barcelona, and the 36th EARIE Conference in Ljubljana.

†Daniel Carvalho is a graduate student at Universidade Nova de Lisboa. Luís Santos-Pinto is Assistant Professor of Economics at University of Lausanne. Corresponding author: Luís Santos-Pinto, Faculty of Business and Economics, University of Lausanne, Internef 535, CH-1015, Lausanne, Switzerland. Ph: +41-216923658. E-mail address: LuisPedro.SantosPinto@unil.ch.

1 Introduction

The theoretical literature on endogenous timing tries to identify factors that might lead to the endogenous emergence of sequential or simultaneous play in oligopolistic markets.[1] In Hamilton and Slutsky (1990)'s action commitment game, two firms must decide a quantity to be produced in *one* of two periods before the market clears. If a firm commits to a quantity in the first period, it acts as the leader, but it does not know whether the other firm has chosen to commit early or not. If a firm commits to a quantity in the second period, then it observes the first period production of the rival (or its decision to wait).

Hamilton and Slutsky show that this game has three subgame perfect Nash equilibria: both firms committing in the first period to the simultaneous-move Cournot-Nash equilibrium quantities, and each waiting and the other playing its Stackelberg leader quantity in the first period. They also show that only the Stackelberg equilibria survive elimination of weakly dominated strategies.

Observed behavior in experiments on this canonical model of endogenous timing is at odds with the theory. For example, Huck, Müeller and Normann (2002) test experimentally the predictions of the action commitment game. They find that: (i) Stackelberg outcomes are rare, (ii) simultaneous-move Cournot outcomes are modal, (iii) simultaneous-move outcomes are often played in the second production period (delay), and (iv) behavior is quite heterogeneous—followers punish leaders, collusive outcomes are played, and double Stackelberg leadership is observed.

The questions that the endogenous timing literature tries to address are particularly relevant in terms of new markets, where two or more firms will enter. The experimental evidence suggests that simultaneous-move play may be a better predictor of behavior in markets for new goods than sequential play.[2] It also suggests that there may be substantial heterogeneity in behavior in these markets.

Why do we observe this huge gap between the theoretical predictions and the empirical evidence? One explanation might be that subjects have trouble coordinating their play in

[1] The seminal papers are Saloner (1987), Hamilton and Slutsky (1990), and Robson (1990)

[2] As we have seen the prediction of Stackelberg equilibria rests on equilibrium selection arguments. Simultaneous-move Cournot-Nash equilibria typically exist, however, they do not survive the application of equilibrium refinements.

one of the two Stackelberg equilibria: if both equilibria are exactly the same then it is far from clear which of the two firms is going to assume the leading role in the first period–see Matsumura (2001) on the instability of leader-follower relationships. This implies that playing the Stackelberg leader's quantity is risky by comparison with playing the Cournot-Nash quantity.

It is possible to think of explanations for some aspects of the empirical evidence. However, it is much harder to explain most of the experimental findings. For example, Harsanyi and Selten's (1988) risk-payoff equilibrium selection argument may explain why simultaneous-move outcomes are more frequently played than Stackelberg outcomes. However, it cannot explain the emergence of collusive outcomes or Stackelberg warfare. It is also not clear how this explanation can account for delay in the action commitment game.

Another explanation might be inequity aversion. Santos-Pinto (2008) generalizes Hamilton and Slutsky's (1990) action commitment game by assuming that players are averse to inequality in payoffs. He shows that relatively high levels of inequity aversion rule out asymmetric equilibria, and inequity aversion gives rise to a continuum of simultaneous-move equilibria which include the Cournot-Nash outcome, collusive outcomes as well as Stackelberg warfare. However, inequity aversion is not able to explain delay. Although inequity aversion can cast some light into the heterogeneity in behavior observed, we believe that the discussion can be further enriched with a different focus.

Recent experiments suggest that in strategic settings without clear precedents, people's initial responses often deviate systematically from equilibrium. Moreover, different players seem to employ different levels of reasoning in games. Nagel (1995) was one of the first to provide evidence for this using the p-beauty contest, a dominance solvable game. She found that most people do not follow the Nash equilibrium prediction of behavior; rather, their degree of strategic thinking is limited to a finite number of iterations when eliminating weakly dominated strategies. Other important references on non-equilibrium models of behavior in games are Stahl and Wilson (1995), Costa-Gomes, Crawford and Broseta (2003), and Camerer, Ho and Chong (2004). In these models, a level-k player computes his best response assuming that his rivals employ less thinking steps; the sole exception is that of the level zero players, who do not behave strategically and choose randomly across the decision set.[3]

This paper applies Camerer, Ho and Chong's (2004) cognitive hierarchy model to Hamilton and Slutsky's (1990) action commitment game. We assume that a level zero plays any quantity with equal probability and plays in the first period with 50% probability. Since the action commitment game is a dynamic game and non-equilibrium models of behavior in games are usually applied to static games, we assume that players of levels $k > 1$ use Bayes' rule to update their beliefs about their rivals' level of strategic sophistication. Additionally, given that the cognitive hierarchy model typically delivers small sets of predicted behavior we introduce noise into players' behavior (see Camerer, Palfrey and Rodgers (2007) and Östling, Wang, Chou and Camerer (2008)). Because the set of possible quantity choices is large we assign noise to the quantity choices and assume that players make no mistakes regarding the period of entry. We estimate the model that best fits the data and compare its predictions to observed behavior.

We find that the cognitive hierarchy model is able to explain the heterogeneity in behavior in Huck, Müeller and Normann (2002). The model predicts delay which, as we have seen, cannot be explained by risk-dominance or inequity aversion. The model predicts very well (i) Stackelberg outcomes, (ii) double leadership outcomes, and (iii) Stackelberg leaders punished by followers. The model also predicts the spike in the leadership quantity in the first period as well as the spike in the Cournot quantity in the second period. Notwithstanding, the model does not predict the spike in the Cournot quantity in the first period, overestimates (underestimates) simultaneous play in the second (first) period and underestimates collusive outcomes.

In non-equilibrium models of behavior in games different level zero specifications can sometimes lead to different predictions of behavior. To address this issue we study alternative forms of level zero behavior. First, we consider different probabilities of first period movement. We find that the predictions of the model are essentially the same no matter if the level zero plays in the first period with 50%, 75%, or 99% probability. Second, we consider that a level zero chooses the Cournot quantity in the first period with y per

[3] Ivanov, Levin and Peck (forthcoming) apply a non-equilibrium model of behavior in games to a model of endogenous timing in investment where players decide if they want to invest in a market and, if yes, when they want to carry that action out.

cent probability and sticks to the random behavior with $1-y$ per cent probability, where $y \in \{50, 75, 99\}$. This alternative specification of level zero behavior fits the data better than the benchmark model and can explain the spike in the Cournot quantity in the first period.

The remainder of this paper is organized as follows. Section 2 describes the empirical evidence in Huck, Müeller and Normann (2002). Section 3 shows how the cognitive hierarchy model can be applied to the action commitment game. Section 4 reports the maximum likelihood estimates and discusses the results. Section 5 concludes the paper. The appendix contains the classification of market outcomes, the details of the maximum likelihood estimation, and sensitivity analysis of different level zero specifications.

2 Empirical Evidence

Huck, Müeller and Normann (2002) use a laboratory experiment to test the action commitment model in Hamilton and Slutsky (1990).[4] They assume a linear inverse demand function $p(Q) = \max\{30 - Q, 0\}$, where $Q = q_1 + q_2$, and a cost function $C_i(q_i) = 6q_i$, with $i = 1, 2$. Table 1, taken from Huck, Müeller and Normann (2001), summarizes the quantities, profits, consumer surplus and total welfare for the Cournot and Stackelberg equilibria and for the fully collusive market outcome.[5]

Table 1: Theoretical Predictions for Equilibria and Fully Collusive Outcome

	Cournot	Stackelberg	Collusion
Individual quantities	$q_i^C = 8$	$q^L = 12; q^F = 6$	$(q_i^J = 6)_{sym}$
Total quantities	$Q^C = 16$	$Q^S = 18$	$Q^J = 12$
Profits	$\Pi_i^C = 64$	$\Pi^L = 72; \Pi^F = 36$	$(\Pi_i^J = 72)_{sym}$
Consumers' surplus	$CS^C = 128$	$CS^S = 162$	$CS^J = 72$
Total welfare	$TW^C = 256$	$TW^S = 270$	$TW^J = 216$

[4] Participants in the experiment were students of various backgrounds that were paid according to their results in the game and a participation fee to cover eventual negative profits. Subjects were randomly matched in pairs and were not informed of who their rival was. Every subject had to choose when to enter the market and which quantity to produce.

[5] In this earlier paper, an experiment was performed with the same design except that the timing of the decisions was previously stipulated, either for sequential and simultaneous move. The purpose was to study Stackelberg and Cournot frameworks when subjects where randomnly matched or fixed paired.

Subjects were handed a payoff matrix with discrete quantity values and the respective payoffs their choices would yield considering the quantities that their rival might play and its own profit. The experiment was done with two payoff matrices, one large and one small. The large payoff matrix had quantities ranging from the integers 3 to 15 and the small payoff matrix had only 6, 8 and 12 as possible choices. The play lasted 30 rounds and the subjects were informed, at the end of each round, of the quantity and time of entry their rival had chosen and the respective payoffs.

We focus on the results of the game with the large payoff matrix. In that game the quantities 6, 7, 8, and 9 are weakly dominated by the strategy "enter the market in the second period." The quantities 3, 4, 5, 13, 14, and 15 are strictly dominated. Ruling out weakly dominated strategies the game with the large payoff matrix has three equilibria in pure strategies and one in mixed strategies where one player chooses quantity 10 in the first period with probability 2/5 and decides to wait with probability 3/5.[6]

Table 2 displays the choices made, broken down by quantity and period of play, by subjects who played the game with the large payoff matrix. There are three points worth stressing from the analysis of table 2. The first is that the distribution is highly concentrated in the quantity subset $\{6, 7, 8, 9, 10, 11, 12\}$, that is, very few players choose strictly dominated quantities. In fact, 96.0% of the quantities chosen in the first period and 96.3% of those chosen in the second period lie in that subset. The second is that most players preferred to move in the first period, some 61% of the total, and only 39% decided to wait for the second. The last point is the existence of two spikes in the first period, in quantities 8 and 10, and a third spike in the second period in quantity 8.[7]

[6] In Hamilton and Slutsky (1990), the linearity of the demand and cost functions combined with the continuous action space guarantee that there are no equilibria where players mix a first period choice with the strategy "enter in the second period." With a discrete strategy space there exist various mixed strategy equilibria. Nevertheless, only one of those equilibria is not attained in weakly dominated strategies.

[7] Behavior becomes more cooperative as the number of rounds of play increases. By spliting the sample into two parts, the first encompassing the first fifteen rounds and the second the remaining rounds, we observe that: quantities 6 and 7 were chosen less often in the first part of the sample than in the second part; quantities 9 and 11 were chosen more often in the first part and less often in the second. Throughout, quantities 8, 10 and 12 remain approximatelly constant in both subsets as well as the stricktly dominated quantities.

Table 2: Observed Quantities per Period of Play

Quantity	Period 1	Period 2	Total
3	1	3	4
4	1	2	3
5	3	2	5
6	35	23	58
7	59	61	120
8	142	125	267
9	47	56	103
10	139	40	179
11	53	17	70
12	46	12	58
13	7	2	9
14	4	1	5
15	6	3	9
Total	543	347	890
%	61	39	100

Table 3 , taken from Huck, Müeller and Normann (2002), displays aggregate results for the experiment with the large matrix. First of all, we can see that the players who decided to move in the first period chose a quantity that was, on average, significantly less than the Stackelberg leader's; moreover, those who decided to wait for the second period chose, on average, a quantity much higher than the Stackelberg follower's. Finally, when both firms chose simultaneously in the second period, the average quantity played was very close to the Cournot level.

Table 3: Aggregate Results for the Large Payoff Matrix

	Both firms in period 1	Explicit followers	Both firms in period 2
Average quantity	9.15	8.93	8.40
Standard deviation	1.91	1.75	1.67
Number of observations	543	207	140
%	61%	23%	16%

Table 4 organizes results into market outcomes and displays the percentage of each in terms of the total. Quantity 9, together with 8, is considered admissible as Cournot quantity and quantities 10 and 11, together with 12, as Stackelberg leader's quantities.

Table 4: Observed Market Outcomes

Market outcomes	% cases
Cournot:	
1st period	4.5
Sequential	14.8
2nd period	4.5
Stackelberg:	
Leader 12, follower 6	0.9
Leader 11, follower 7	2.0
Leader 10, follower 7	4.5
First mover punished or rewarded:	
Stackelberg leader punished	11.9
Stackelberg leader rewarded	0.2
Cournot punished	0.9
Cournot rewarded	0.0
Stackelberg and Cournot in 1st period	12.6
Double Stackelberg leadership	6.3
Collusion:	
Collusion successful	6.1
Collusion failed	10.6
Collusion exploited	4.3
Other	16.0

We will briefly go through some of the market outcomes in Table 4. The outcome "Cournot sequential" happens when the first and second movers both choose a Cournot quantity (8 or 9). The outcome "Stackelberg leader punished" happens when the first mover chooses a Stackelberg leader's quantity (12, 11 or 10) and the second mover chooses a quantity greater than his best response to the first mover's. The outcome "Cournot punished" happens when the first mover chooses a Cournot quantity and the second mover chooses a quantity greater than his best response to the first mover's. The outcome "Stackelberg and Cournot in 1st period" happens when both players move in the first period and one of them chooses a Stackelberg leader's quantity while the other chooses a Cournot quantity. The outcome "double Stackelberg leadership" happens when both players play a Stackelberg leader's quantity in the first period. The outcome "collusion successful" happens when both players play a collusive quantity (6 or 7) in either period. The outcome "collusion failed" happens when both players move in the first period, one player chooses a collusive quantity and the other player plays either Stackelberg or Cournot. The outcome "collusion exploited" happens when the first mover chooses a collusive quantity and the second mover chooses a quantity greater than 7. Finally, the market outcome "others" refers to those situations that do not fit into any of the previous cases.[8]

3 A Cognitive Hierarchy in the Action Commitment Model

To explain behavior in games Camerer, Ho and Chong (2004) propose a cognitive hierarchy theory where different players employ different levels of reasoning. Level zero players do not think strategically at all; they randomize equally across all strategies. Players of level $k > 0$ anticipate the decisions of lower-level players and best respond to the mixture of their decisions using normalized frequencies.[9]

Formally, players of level $k > 0$ know the true proportions of lower-level players $f(0)$, $f(1),\ldots, f(k-1)$. Since these proportions do not add to one, they normalize them by dividing by their sum. That is, players with k levels of reasoning have the following beliefs about players with h levels of reasoning:

$$g^{Lk}(h) = \begin{cases} f(h)/\sum_{l=0}^{k-1} f(l), & \forall h < k \\ 0, & \forall h \geq k \end{cases}.$$

This setup leads to an "increasing rational expectations" characteristic, meaning that the deviation between the actual frequencies and the player's beliefs is diminishing in k, the thinking step level.

Camerer, Ho and Chong (2004) discuss the properties that the appropriate distribution of levels should possess: it should be discrete because the thinking steps are integers; it should reflect the fact that, as thinking steps increase, so do the computations that the players carry out. Working memory constraints should make it likely that, the higher is k, the fewer are the players doing one further reasoning level. In other words $f(k)/f(k-1)$

[8] See Appendix A for a complete description of the quantities and periods of play that characterize each market outcome.

[9] Thus, players of level $k > 0$ are assumed to not realize that some players might be thinking at least as 'hard' as they are about the game. This could be due to overconfidence: players believe that their rivals have less insight regarding the game they are playing. It could also be due to the limited capacity that people have to continuously eliminate dominated strategies. However, players of level $k > 0$ are assumed to make an accurate guess about the relative proportions of players using fewer steps than they do.

is decreasing in k. Moreover, the authors assume that the ratio is proportional to $1/k$ and, consequently, the distribution must be the Poisson:

$$f(k|\tau) = \frac{\tau^k e^{-\tau}}{k!},$$

with $k = 0, 1, 2, ...$ and $\tau > 0$. An advantage of this approach is that one only needs to estimate a single parameter, τ, to find out the distribution of players' types in the population as well as quantity and timing predictions.

We now illustrate how a truncated version of the cognitive hierarchy model can be applied to predict behavior in the action commitment game. Let $k \in \{0, 1, 2\}$ and let the best response function of a level-k player, with $k > 0$, to quantity q produced by his rival be given by

$$BR^{Lk}(q) = \arg\max_{q^{Lk}} \left[P\left(q^{Lk} + q\right) - c \right] q^{Lk}.$$

The first step is to determine the best strategy of a level 1 player–$L1$ from now on. The $L1$ player believes that he can only play against a level zero player–$L0$ from now on–and that a $L0$ player picks a random quantity and produces in the first period with probability $x \in (0, 1)$. If the $L1$ player decides to produce in the first period his optimal quantity choice is to produce $BR^{L1}(\bar{q}^{L0})$, where $\bar{q}^{L0}$ denotes the expected output of a $L0$ player. This generates a perceived expected profit of

$$\Pi_1^{L1} = \pi^{L1}\left(BR^{L1}\left(\bar{q}^{L0}\right), \bar{q}^{L0}\right).$$

If, on the contrary, the $L1$ player decides to wait until the second period, one of the following two situations will occur: with probability x the $L0$ player will choose its quantity in the first period and the $L1$ player will be able to best respond to it; with probability $1 - x$ the $L0$ player will only play in the second period and the $L1$ player will have to play $BR^{L1}(\bar{q}^{L0})$. Thus, the perceived expected profit of a $L1$ player who moves in the second period is given by

$$\Pi_2^{L1} = x \int \pi^{L1}\left(BR^{L1}\left(q^{L0}\right), q^{L0}\right) dF(q^{L0}) + (1-x)\pi^{L1}\left(BR^{L1}\left(\bar{q}^{L0}\right), \bar{q}^{L0}\right).$$

By definition of the best response function, we know that

$$\int \pi^{L1}\left(BR^{L1}\left(q^{L0}\right), q^{L0}\right) dF(q^{L0}) \geq \pi^{L1}\left(BR^{L1}\left(\bar{q}^{L0}\right), \bar{q}^{L0}\right)$$

and so $\Pi_2^{L1} \geq \Pi_1^{L1}$. Therefore, the $L1$ player is better off by waiting until the second period to choose his quantity whatever the probability the $L0$ player has of moving in the first period. The intuition behind this result is that since $L0$ players have no strategic interaction with the other types of players, the $L1$ player has nothing to gain if it plays in the first period because it cannot condition the $L0$ players' response. Therefore, waiting for the second period is the optimal choice of a $L1$ player.

The level 2 player–$L2$ from now on–thinks that the population is composed exclusively of $L0$ and $L1$ players. The $L2$ player also knows that a $L0$ player will play a random quantity and will do it in period one with probability $x > 0$ as well as that the best decision of a $L1$ player is to wait for the second period. Therefore, the $L2$ player is faced with a trade-off. If she knew her rival was a $L0$, then she would prefer to move in the second period to be able to best respond to the rival when the rival moves in the first period. If she knew the rival was a $L1$, then she would prefer to move in the first period and reap the benefits of a leadership position. Since the $L2$ only knows the percentage of $L0$'s and $L1$'s, his optimal choice will depend on tau.

If the $L2$ player decides to move in the first period, then his optimal quantity choice, denote it by q_1^{L2}, is the solution to

$$\max_{q^{L2}} g^{L2}(0)\pi^{L2}\left(q^{L2}, \bar{q}^{L0}\right) + g^{L2}(1)\pi^{L2}\left(q^{L2}, BR^{L1}(q^{L2})\right), \tag{1}$$

where $g^{L2}(0)$ is the relative proportion of $L0$ players in the population according to $L2$'s beliefs and, likewise, $g^{L2}(1)$ is that of $L1$ players. These two proportions will be normalized by the $L2$ to sum up to one. We can thus write the $L2$'s perceived expected profit of producing q_1^{L2} in the first period as

$$\Pi_1^{L2} = g^{L2}(0)\,\pi^{L2}\left(q_1^{L2}, \bar{q}^{L0}\right) + [1 - g^{L2}(0)]\pi^{L2}\left(q_1^{L2}, BR^{L1}\left(q_1^{L2}\right)\right).$$

If the $L2$ player decides to postpone his quantity decision to the second period, one of the following two situations will arise. If the $L2$ observes a quantity commitment by the rival, then he believes that he is playing against a $L0$ player and chooses his best response in the second period. In this case the perceived expected profit of the $L2$ is $\int \pi^{L2}\left(BR^{L2}\left(q^{L0}\right), q^{L0}\right) dF(q^{L0})$. If the $L2$ does not observe any quantity commitment in the first period he believes that he is either playing against a $L0$ who moves in the second period or against a $L1$. In this case the optimal quantity choice of the $L2$, denote it by q_2^{L2}, is the solution to

$$\max_{q^{L2}} \mu^{L2}\pi^{L2}\left(q^{L2}, \bar{q}^{L0}\right) + (1-\mu^{L2})\pi^{L2}(q^{L2}, BR^{L1}\left(\bar{q}^{L0}\right)), \tag{2}$$

where μ^{L2} is the (posterior) belief of the $L2$ that the rival is a $L0$ given that there was no movement in the first period. From Bayes' rule we have

$$\mu^{L2} = \frac{(1-x)g^{L2}(0)}{(1-x)g^{L2}(0) + g^{L2}(1)} = \frac{(1-x)g^{L2}(0)}{1-xg^{L2}(0)}.$$

Therefore, the $L2$'s perceived expected profit of producing q_2^{L2} in the second period is

$$\Pi_2^{L2} = xg^{L2}(0)\int \pi^{L2}\left(BR^{L2}\left(q^{L0}\right), q^{L0}\right) dF(q^{L0}) + (1-x)g^{L2}(0)\pi^{L2}\left(q_2^{L2}, \bar{q}^{L0}\right)$$
$$+ [1-g^{L2}(0)]\pi^{L2}(q_2^{L2}, BR^{L1}\left(\bar{q}^{L0}\right)).$$

A $L2$ will choose to produce in the first period if and only if $\Pi_1^{L2} > \Pi_2^{L2}$. This inequality will be satisfied for sufficiently high values of τ, that is, when there are relatively few $L0$ players in the population. If we assume additionally that the $L0$'s behavior is drawn from a uniform distribution over the periods of play and the quantity levels in HMN we have the following predictions in the truncated model (the analysis is approximately correct for $\tau \in (0,1)$):

$L1$: Chooses to wait and (i) if there is commitment by the rival the $L1$ chooses a best response, (ii) if there is no commitment by the rival the $L1$ chooses the Cournot quantity, $q^C = 8$ (the best response to quantity 9, the average quantity produced by the $L0$).

$L2$: If tau is sufficiently high, the $L2$ moves in the first period and produces a Stackelberg leader's quantity q^* where $8 \leq q^* \leq 12$ ($q^* = 10$ in HMN's data). If tau is sufficiently low, the $L2$ chooses to wait and (i) if there is commitment by the rival the $L2$ chooses a best response, (ii) if there is no commitment by the rival the $L2$ updates beliefs about the rival's type and chooses a best response.

We see that the truncated cognitive hierarchy model generates delay and a spike in the Cournot quantity in the second period mostly due to the behavior of $L1$s. The model also generates a spike in a moderate Stackelberg leader's quantity in the first period as long as tau is sufficiently high due to the behavior of $L2$s. However, the model does not generate the spike in the Cournot quantity in the first period.

Further thinking steps are easily added to the model by following the same logic as above. The main predictions will be similar to those of the truncated model with $k \in \{0, 1, 2\}$. The behavior of higher levels will be similar to that of the $L2$. In appendix B behavior is depicted as a function of tau up to $\tau = 4$ and from $L2$ to $L10$.

The assumption that a $L0$ moves in the second period with positive probability, $x < 1$, is critical to the analysis. To see this suppose that a $L0$ never moves in the second period. We know that the optimal choice of a $L1$ is to wait for the second period because he wants to best respond to the $L0$. Now if, for example, a $L1$ is paired against another $L1$, then both players would observe no movement in the first period from the rival. This would be inconsistent with their belief that the population is only composed of $L0$s.

The main difference between Camerer, Ho and Chong (2004) and other non-equilibrium models of behavior in games such as the "n-depth of reasoning" in Nagel (1995) or the "level-k" model in Costa-Gomes and Crawford (2006), is the assumption that a player of level $k > 1$ considers that he is playing against all inferior levels of reasoning instead of the one just immediately below. It turns out that the dynamic nature of the action commitment game rules out the application of a "n-depth of reasoning" or a "level-k" model.

To see this suppose that players think that the whole population is composed of players that are just one thinking step below ($L2$s only acknowledge the existence of $L1$s, $L3$s the existence of $L2$s, and so on). Moreover, suppose that $L1$s and $L3$s wait for the second period and the $L2$s move in the first period. In this case a $L3$ expects to observe a move in the first period because he believes everyone else is an $L2$. However, if the $L3$'s rival turns out to be a $L1$, then the $L3$ does not observe a quantity commitment which is inconsistent with his view of the world. Furthermore, since all other higher level players depend on the actions of the $L3$ player, they cannot be correctly specified within the scope of a "n-depth of reasoning" or a "level-k model." Thus, the cognitive hierarchy model and the assumption that players update beliefs about their rivals' level of strategic sophistication using Bayes' rule are better suited to explain behavior in dynamic games than other non-equilibrium models of behavior in games.

4 Estimation

This section explains how we introduce errors in players' decisions, describes the likelihood function, and reports the estimation results. Technical details of the maximum likelihood estimation can be found in Appendix B.

4.1 Maximum Likelihood Estimation

Cognitive hierarchy models typically produce a rather short set of best responses. In the action commitment game, predicted behavior alternates between moving in the first period with quantity 10 or waiting for the second period and, either best responding to observed quantities in the case of sequential movement, or playing quantity 8 if no movement has been observed.

Following El-Gamal and Grether (1995), Costa-Gomes, Crawford and Broseta (2001), Camerer, Palfrey and Rodgers (2007) and Östling, Wang, Chou and Camerer (2008) we assume that players' best responses are stochastic. More precisely, we assume that players make mistakes when choosing their quantity levels but make correct decisions regarding the period of entry. We think this is a reasonable assumption given the large set of quantity choices (13 possible quantities) and the small set of timing decisions (2 possible periods). Allowing both types of errors would substantially increase the computational burden involved in the estimation procedure.

Additionally, we assume that the probability of playing a quantity close to the predicted quantity by mistake is higher than the probability of playing a distant quantity. To incorporate this behavior, we assume that the mistakes of a level k player, with $k \geq 1$, follow a power function. This function assigns a high probability to quantities adjacent to the predicted one and a small probability to distant quantities.[10] We also assume that errors are independent across types of players.

The estimation goes through all the pairs of decisions. By pair, we mean every possible period and quantity decision: for example, both players moving in the first period, one playing quantity 8 and the other playing quantity 12; or both players moving in the second period and playing quantity 8. This approach captures the interaction of players taking decisions that are conditioned by their rivals' decisions. The information is thus broken down into three possible cases: both players move in the first period; both players move in the second; and one player moves in the first and the other in the second.[11]

The estimation method is maximum likelihood and it is done according to a standard grid search approach. The likelihood function is given by

$$L(\tau,\varepsilon)=\prod_{t_1=1}^{2}\prod_{q_1=3}^{15}\prod_{t_2=1}^{2}\prod_{q_2=3}^{15}\left[f\left(t_1,q_1,t_2,q_2|\tau,\varepsilon\right)\right]^{n_{(t_1,q_1,t_2,q_2)}},$$

where t_i and q_i are the timing and quantity predictions for player i in a given pair, t is the index of timing predictions and q for quantity predictions, $n_{(t_1,q_1,t_2,q_2)}$ is the number of cases that each pair is observed in the data, τ is the parameter of the Poisson distribution, and ε is the error term. Since cognitive hierarchy models are better suited to explain initial responses to games we perform an estimation for the entire sample and another one for the first half of the sample, i.e., for the first fifteen rounds of play.

4.2 Results

In this subsection, we present the most significant results of the maximum likelihood estimation. Interval by interval estimates are displayed in Appendix C. A complete list of all pairs, their percentage in the data and the estimated probability in the model can be found in Appendix D.

[10] The power function was tested against other alternative error specifications, such as the uniform, assigning errors only to the two and four immediate quantities or assuming no errors at all. It performed better than these alternatives.

[11] Crawford and Iriberri (2007) and Harless and Camerer (1995) use a similar estimation procedure.

4.2.1 The Benchmark Model

In our benchmark model we assume that a $L0$ enters in the first period with 50% probability and chooses each quantity with equal probability. Parameter estimates of τ and ε for both the entire sample and the first fifteen rounds, as well as the respective maximum likelihood values, are displayed in Table 5.

Table 5: Maximum likelihood estimates

	τ	ε	ML
Entire set			
Benchmark	2.86	0.65	−2257.49
Alternative	1.74	0.59	−2176.01
First 15 rounds			
Benchmark	3.56	0.65	−1102.71
Alternative	1.74	0.58	−1060.90

The benchmark model's estimate of the average number of thinking steps is 2.86 for the entire set and 3.56 for the first fifteen rounds. These estimates are relatively high by comparison with those in Nagel (1995), specially the one for the first fifteen rounds. Nevertheless, it should be pointed out that higher average values of thinking steps are not uncommon in the literature.[12]

Camerer, Ho an Chong (2004) show that, even though the average estimate for the average thinking steps across a wide range of games is 1.5, there is a relationship between the expected payoff of winning a game and that average value. Subjects tend to think harder when there is the possibility for them to reap a higher reward. The data we use is based on an experiment which yielded, on average, a higher payoff than the one in the typical p-beauty games.[13]

[12] The lowest estimated τ in the seven weeks of the LUPI game in Östling, Wang, Chou and Camerer (2008) is 2.98, the remaining six are above 5 thinking steps and the highest are over 10; in Camerer, Palfrey and Rodgers (2007) there are games for which the predicted τ is also rather high.

[13] Camerer, Ho and Chong (2004) argue that subjects tend do employ a cost-benefit analysis concerning the amount of thinking they do in games. They present evidence that the higher the stakes of a given game, the higher will τ be: they show that subjects tend to think harder in games that yield \$4 than games that yield \$1. Since the LUPI game is based on data from an actual lotto game that existed in Sweden with a prize money of at least €10 000, it makes sense that the game's estimates should be relatively high (even though, of course, the probability of winning the prize is much smaller). In the large matrix experiment we

Table 6: Quality adjustment measures

	Entire set	First half
Upper bound	−1825.90	−896.57
Lower bound		
Benchmark	−2624.53	−1328.21
Alternative	−2773.42	−1402.82
Log-likelihood		
Benchmark	734.07	450.99
	(0.000)	(0.000)
Alternative	1194.82	683.85
	(0.000)	(0.000)

The benchmark model's estimate for the error term is 0.65 for both the entire set and the first fifteen rounds. This estimate indicates that only 35% of the subjects played according to their predicted quantity.[14] The power function assigns roughly a probability of 32% to errors in those quantities immediately adjacent to their predicted quantity and 16% to quantities that were two integers away from the predicted one. Thus, the majority of players' mistakes are, in fact, very close to their predicted behavior.[15]

We now turn towards the quality of the adjustment. Table 6 displays upper and lower bounds to the maximum likelihood value. The upper bound is attained by running the likelihood function with the empirical frequencies of the pairs of play. By definition, this procedure yields the maximum value attainable for the estimation. The lower bound is the maximum likelihood value of a totally random model. This is equivalent to assuming the restriction $\tau = 0$, i.e., all players are $L0$s and therefore the model is totally random. Furthermore, we also present log-likelihood ratios (p-values are in parenthesis). Since we are testing for the significance of τ, we have, under the null hypothesis, that it is not statistically different than 0. This means that what we are testing is whether our model is statistically different from a model where all players are $L0$s. As is evident from the log-likelihood p-values and the comparison with the adjustment's lower bound, the benchmark

used, subjects received the equivalent to $11.44, on average, which, given the reward, places our model's estimates somewhere in the middle of this range.

[14] Error rate estimates of this magnitude are not uncommon in this literature. See, Costa-Gomes, Crawford and Broseta (2003).

[15] The main characteristic of the action commitment game is the wide range of possible actions. When there are more options available and there is noise in players' behavior, the probability of not choosing the optimal option usually increases. The unfamiliar strategic component of an endogenous timing decision also increases the probability of mistakes.

model performs considerably better than a totally random one.

Table 7: Models' aggregate results

	Both players in period 1	Explicit followers	Both players in period 2
Entire set			
Data			
Average quantity	9.15	8.93	8.40
% of observations	61.0%	23.3%	15,7%
Benchmark model			
Average quantity	9.89	7.25	8.10
% of observations	42.2%	24.4%	33.3%
Alternative			
Average quantity	9.45	7.49	8.12
% of observations	42.1%	24.4%	33.6%
First half			
Data			
Average quantity	9.36	8.88	8.32
% of observations	58,9%	22.0%	19,1%
Benchmark			
Average quantity	9.92	7.18	8.07
% of observations	42.5%	24.4%	32.7%
Alternative			
Average quantity	9.44	7.49	8.12
% of observations	42.1%	24.4%	33.6%

The aggregate results of the model are displayed in Table 7. In general, the benchmark model approximates well the average quantity when both players move in the second period. On the contrary, the average quantity predicted when both players move in the first period is higher than in the data whereas the average quantity predicted for explicit followers is lower. The model also predicts a lower percentage of simultaneous play in the first period and a higher percentage of simultaneous play in the second period than those found in the data. The predicted percentage of explicit followers is very close to the data.

Table 8 compares the market outcomes predicted by the cognitive hierarchy model to those observed in the data for the entire sample. Table 9 does the same for the first fifteen rounds of play.

We see from Table 8 that the benchmark cognitive hierarchy model can generate the heterogeneity in behavior observed in the data. The model predicts delay (simultaneous play of Cournot quantities in the second period), a feature that cannot be explained by

Table 8: Market outcomes - entire set

Market outcomes	HMN	CH Model	
		Benchmark	Alternative
Cournot:			
1st period	4.5	1.0	2.5
Sequential	14.8	5.9	9.8
2nd period	4.5	8.2	9.4
Stackelberg:			
Leader 12, follower 6	0.9	1.4	1.1
Leader 11, follower 7	2.0	2.6	2.1
Leader 10, follower 7	4.5	5.4	5.4
First mover punished or rewarded:			
Stackelberg leader punished	11.9	9.3	6.9
Stackelberg leader rewarded	0.2	3.9	2.9
Cournot punished	0.9	1.8	2.6
Cournot rewarded	0.0	2.9	4.1
Stackelberg and Cournot 1st period	12.6	5.0	6.2
Double Stackelberg leadership	6.3	5.9	3.8
Collusion:			
Collusion successful	6.1	2.7	2.2
Collusion failed	10.6	2.0	1.6
Collusion exploited	4.3	2.3	1.9
Other	16.0	39.7	37.6
$\sqrt{\sum(o_i - \hat{o}_i)^2}$		29.1	26.9

equilibrium selection arguments or inequity aversion. As expected, the model underestimates the percentage of first period Cournot quantities. The model predicts very accurately the percentages of Stackelberg outcomes.

Regarding the outcomes where the first mover is either punished or rewarded the model predicts very well the percentage of Stackelberg leaders who get punished by followers but overestimates the percentage of Stackelberg leaders who get rewarded by followers as well as the percentage of first movers who play Cournot and get punished or rewarded by second movers.

The model approximates very well the percentage of double leadership market outcomes but underestimates the percentage of "Stackelberg and Cournot in 1st period" and collusive outcomes, specially "failed collusion." The latter and the difference in the Cournot percentages account for the high percentage of cases under the category "other."

Table 9: Market outcomes - first half

Market outcomes	HMN	CH Model	
		Benchmark	Alternative
Cournot:			
1st period	5.3	1.1	2.5
Sequential	14.2	6.1	9.9
2nd period	6.2	8.3	9.7
Stackelberg:			
Leader 12, follower 6	1.8	1.4	1.1
Leader 11, follower 7	3.6	2.7	2.1
Leader 10, follower 7	5.8	5.6	5.7
First mover punished or rewarded:			
Stackelberg leader punished	10.7	9.4	6.8
Stackelberg leader rewarded	0.0	4.1	2.9
Cournot punished	0.9	1.8	2.5
Cournot rewarded	0.0	2.9	4.0
Stackelberg and Cournot in 1st period	14.7	5.2	6.2
Double Stackelberg leadership	7.1	6.2	3.8
Collusion:			
Collusion successful	4.0	2.7	2.1
Collusion failed	7.6	2.0	1.6
Collusion exploited	2.2	2.2	1.9
Other	16.0	38.2	37.2
$\sqrt{\sum (o_i - \hat{o}_i)^2}$		27.1	25.6

Inspection of Tables 8 and 9 shows us that the cognitive hierarchy model fits the data better in the first fifteen rounds of play than in the entire set. This happens because the model underestimates collusive outcomes and the percentage of collusive outcomes observed in the data for the first fifteen rounds is much smaller than for the entire set. This finding is consistent with the fact that non-equilibrium models of behavior in games are particularly good at predicting initial responses.

Appendix E shows that the predictions of the benchmark model are robust to different probabilities of first period entry by the $L0$.

4.2.2 The Alternative Model

As we have seen the benchmark model is able to account for the spikes in quantity 10 in the first period and in quantity 8 in the second period, but not the spike in quantity 8 in the first period. For this reason, we explore an alternative version of the model where the $L0$ player plays quantity 8 in the first period $y\%$ of the time and sticks to random behavior in the remaining $(1-y)\%$ of the time. The reason we choose this particular specification is to see how far the cognitive hierarchy model can go in explaining the spike in quantity 8 in the first period. Here we report the results for $y = 50\%$.

We see from Table 5 that the alternative model's estimated τ is 1.74 for both the entire set and the first fifteen rounds. The reason for the smaller average number of thinking steps by comparison with the benchmark model is due to the fact that a higher percentage of $L0$ players is needed to account for the spike in the first period Cournot quantity. The alternative model's estimated ε is 0.59 for the entire set and 0.58 for the first fifteen rounds. This lower estimate for ε is consistent with a higher percentage of $L0$ players in the alternative model than in the benchmark model. The alternative model bears a higher likelihood value than the benchmark model. The alternative model, like the benchmark model, also does better than a totally random model (see Table 6).

The alternative model performs better than the benchmark model in predicting aggregate results (see Table 7). The predicted average quantity in the first period is lower and closer to the average quantity in the data, specially when we look at the first fifteen rounds. The predicted average quantity in the explicit followers case is also closer to the data. The alternative model, like the benchmark one, underestimates (overestimates) simultaneous play in the first (second) period.

As far as market outcomes are concerned (see Tables 8 and 9), the alternative model also predicts high heterogeneity in behavior. However, it makes several different predictions by comparison with the benchmark model. The first is that the predicted percentage of sequential play of Cournot quantities is much closer to the data. The alternative model also predicts a higher percentage of “Stackelberg and Cournot in 1st period” outcomes. However, the alternative model does worse in terms of predicting the percentage of first mover punished outcome, second period play of Cournot quantities, and double Stackelberg leadership. Differences between the predictions of the two models in the remaining outcomes are small.

5 Conclusion

This paper shows that a cognitive hierarchy model can explain the heterogeneity of behavior in endogenous timing decisions observed in Huck, Müeller and Normann (2002).[16] The model predicts the first period spike in quantity 10 and the second period spike in quantity 8. The latter is rather important in the sense that it generates the delay in simultaneous move games, one feature of the data that cannot be explained by equilibrium selection arguments or inequity aversion.

The main shortcoming of the benchmark cognitive hierarchy model is that it does not explain the spike in quantity 8 in the first period. We consider an alternative specification of level zero behavior to explain the spike in quantity 8 that the benchmark model does not yield. The alternative model's results reveal a substantial improvement in the percentage of sequential play of Cournot quantities.

Both models, the benchmark and the alternative, are also tested for the entire sample and for the first fifteen rounds of play since cognitive hierarchy models are more suited for initial responses. Focusing on the first fifteen rounds reduces the amount of collusive outcomes in the data and this improves the model's predictions.

[16] An alternative explanation for the heterogeneity in behavior observed in experimental endogenous timing games is the quantal response approach–see McKelvey and Palfrey 1995–where players' beliefs are correctly formed but players do not always choose best responses.

6 References

Camerer, C. F., T.-H. Ho, and J.-K. Chong (2004). "A cognitive hierarchy model of games," *The Quarterly Journal of Economics*, 861-898.

Camerer, C. F., T. R. Palfrey, and B. W. Rodgers (2007). "Heterogeneous quantal response equilibrium and cognitive hierarchies," California Institute of Technology, Division of the Humanities and Social Sciences Working Paper N°1260.

Costa-Gomes, M., V. Crawford (2006). "Cognition and behavior in two person guessing games: an experimental study," *American Economic Review*, Vol. 96, 1737-1768.

Costa-Gomes, M., V. Crawford and B. Broseta (2003). "Experimental studies of strategic sophistication and cognition in normal-form games," *Econometrica*, Vol. 69, 1193-1235.

Crawford, V., N. Iriberri (2007). "Fatal attraction: salience, naiveté, and sophistication in "hide-and-seek" games," *American Economic Review*, Vol. 97(5), 1731-1750.

El-Gamal, M. A., D. M. Grether (1995). "Are people Bayesian: uncovering behavioral strategies," *Journal of the American Statistical Association*, Vol. 97(432), 1137-1145.

Fonseca, M. A., W. Mueller, and H.-T. Normann (2006). "Endogenous timing in duopoly: experimental evidence," *International Journal of Game Theory*, Vol. 34, 443-456.

Hamilton, J., S. Slutsky (1990). "Endogenous timing in duopoly games: Stackelberg or Cournot equilibria," *Games and Economic Behavior*, Vol. 2, 29-46.

Harless, D. W., C. F. Camerer (1995). "An error rate analysis of experimental data testing Nash refinements," *European Economic Review*, Vol. 39, 649-660.

Harsanyi, J., and R. Selten (1988). A General Theory of Equilibrium Selection in Games, Cambridge, MA: MIT Press.

Huck, S., W. Müller, and H.-T. Normann (2001). "Stackelberg beats Cournot – on collusion and efficiency in experimental markets," *Economic Journal*, Vol. 111, 749-765.

Huck, S., W. Müller, and H.-T. Normann (2002). "To commit or not to commit: endogenous timing in experimental duopoly markets," *Games and Economic Behavior*, Vol. 38, 240-264.

Ivanov, A., D. Levin, J. Peck (forthcoming). "Hindsight, foresight and insight: an experimental study of a small-market investment game with common and private values," *American Economic Review*.

Matsumura, T. (2001). "Market instability in a Stackelberg duopoly," *Journal of Economics*, Vol. 75(3), 199-210.

McKelvey, R., T. Palfrey (1995). "Quantal response equilibria in normal form games," *Games and Economic Behavior*, Vol. 10, 6-38.

Nagel, R. (1995). "Unraveling in guessing games: an experimental study," *The American Economic Review*, Vol. 85(5), 1313-1326.

Östling, R., J. T. Y. Wang, E. Chou, and C. F. Camerer (2008). "Strategic thinking and learning in the field and lab: evidence from Poisson LUPI lottery games," SSE/EFI Working Paper Series in Economics and Finance No. 671.

Robson, A. (1990). "Duopoly with endogenous strategic timing: Stackelberg regained," *International Economic Review*, Vol. 31(2), 263-274.

Saloner, G. (1987). "Cournot duopoly with two production periods," *Journal of Economic Theory*, Vol. 42, 183-187.

Santos-Pinto, L. (2008). "Making sense of the experimental evidence on endogenous timing in duopoly markets," *Journal of Economic Behavior and Organization*, Vol. 68, 657-666.

Schwarz, G. (1978). "Estimating the dimension of a model," *The Annals of Statistics*, Vol. 6 (2), 461-464.

Stahl, D. O., P. W. Wilson (1994). "On player's models of other players: theory and experimental evidence," *Games and Economic Behavior*, Vol.10, 218-254.

A Classification of Market Outcomes

Table 10 provides the classification of market outcomes employed. Specifically, the table is composed of four different matrices. The upper left refers to simultaneous move in the first period, the lower right refers to simultaneous move in the second period and the remaining two tables refer to sequential play, for both the cases where each subject is leader or follower. The notation employed is as follows:

Cournot outcomes:

- C_1 - Cournot 1st period
- C_{12} - Sequential play of Cournot quantities
- C_2 - Cournot 2nd period

Stackelberg outcomes:

- S_{12}- Stackelberg leader 12, follower 6
- S_{11} - Stackelberg leader 11, follower 7
- S_{10} - Stackelberg leader 10, follower 7

First mover punished or rewarded:

- SP - Stackelberg leader punished
- SR - Stackelberg leader rewarded
- CP - Cournot punished
- CR - Cournot rewarded

SC - Stackelberg and Cournot in 1st period

DL - Double Stackelberg leadership

Collusive outcomes:

- CS - Collusion successful
- CF - Collusion failed
- CE - Collusion exploited

O - Other market outcomes

Table 10: Market Outcomes Classification

		t=1							t=2						
		6	7	8	9	10	11	12	6	7	8	9	10	11	12
t=1	6	CS	CS	CF	CF	CF	CF	CF	CS	CS	CE	CE	CE	CE	CE
	7	CS	CS	CF	CF	CF	CF	CF	CS	CS	CE	CE	CE	CE	CE
	8	CF	CF	C_1	C_1	SC	SC	SC	CR	CR	C_{12}	C_{12}	CP	CP	CP
	9	CF	CF	C_1	C_1	SC	SC	SC	CR	CR	C_{12}	C_{12}	CP	CP	CP
	10	CF	CF	SC	SC	DL	DL	DL	SR	S_{10}	SP	SP	SP	SP	SP
	11	CF	CF	SC	SC	DL	DL	DL	SR	S_{11}	SP	SP	SP	SP	SP
	12	CF	CF	SC	SC	DL	DL	DL	S_{12}	SP	SP	SP	SP	SP	SP
t=2	6	CS	CS	CR	CR	SR	SR	S_{12}	CS	CS	O	O	O	O	O
	7	CS	CS	CR	CR	S_{10}	S_{11}	SP	CS	CS	O	O	O	O	O
	8	CE	CE	C_{12}	C_{12}	SP	SP	SP	O	O	C_2	C_2	O	O	O
	9	CE	CE	C_{12}	C_{12}	SP	SP	SP	O	O	C_2	C_2	O	O	O
	10	CE	CE	CP	CP	SP	SP	SP	O	O	O	O	O	O	O
	11	CE	CE	CP	CP	SP	SP	SP	O	O	O	O	O	O	O
	12	CE	CE	CP	CP	SP	SP	SP	O	O	O	O	O	O	O

B Maximum Likelihood Estimation

In this appendix we describe in detail the procedures behind the maximum likelihood estimation. Let $f^{Lk}(t)$ denote the probability that a player of level k moves in period t, where $t \in \{1, 2\}$. In the benchmark model $f^{L0}(1) = 1/2$. We assume that players of level $k > 0$ do not make mistakes when choosing their period of entry but that they might make mistakes when choosing their quantity level. Let $f^{Lk}(q|q^{Lk})$ denote the probability that a player of level k plays quantity q where $q \in \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$ given that his predicted quantity is q^{Lk}. We use a power function to model the probability of making mistakes in quantity choices. Let i be the distance of a given quantity level from the predicted one, let N_l be the maximum distance between the predicted quantity and the lower bound of the interval of available quantities, i.e., quantity 3, and let N_u be the maximum distance between the predicted quantity and the upper bound of the interval of available quantities, i.e., quantity 15. We have that

$$\sum_{i=1}^{N_l} x_{q^{Lk}}^{-2^{i-1}} + \sum_{i=1}^{N_u} x_{q^{Lk}}^{-2^{i-1}} = 1.$$

where $x_{q^{Lk}}$ will be different for different predicted quantities, depending on their position within the interval of quantities. For example, if a $L3$ player's predicted behavior is to move in the first period and choose quantity 8, then $f^{L3}(1) = 1$ and

$$f^{L3}(q|8) = \left(\frac{\varepsilon}{16x_8}, \frac{\varepsilon}{8x_8}, \frac{\varepsilon}{4x_8}, \frac{\varepsilon}{2x_8}, \frac{\varepsilon}{x_8}, 1-\varepsilon, \frac{\varepsilon}{x_8}, \frac{\varepsilon}{2x_8}, \frac{\varepsilon}{4x_8}, \frac{\varepsilon}{8x_8}, \frac{\varepsilon}{16x_8}, \frac{\varepsilon}{32x_8}, \frac{\varepsilon}{64x_8} \right),$$

where $x_8 = 3.921876$.

Table 11: Probability of a Playing a Particular Quantity

H	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15
{}	$\frac{\varepsilon}{64x_{10}}$	$\frac{\varepsilon}{32x_{10}}$	$\frac{\varepsilon}{16x_{10}}$	$\frac{\varepsilon}{8x_{10}}$	$\frac{\varepsilon}{4x_{10}}$	$\frac{\varepsilon}{2x_{10}}$	$\frac{\varepsilon}{x_{10}}$	$1-\varepsilon$	$\frac{\varepsilon}{x_{10}}$	$\frac{\varepsilon}{2x_{10}}$	$\frac{\varepsilon}{4x_{10}}$	$\frac{\varepsilon}{8x_{10}}$	$\frac{\varepsilon}{16x_{10}}$
{0}	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{x_8}$	$1-\varepsilon$	$\frac{\varepsilon}{x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{32x_8}$	$\frac{\varepsilon}{64x_8}$
{1,3}	$\frac{\varepsilon}{128x_{11}}$	$\frac{\varepsilon}{64x_{11}}$	$\frac{\varepsilon}{32x_{11}}$	$\frac{\varepsilon}{16x_{11}}$	$\frac{\varepsilon}{8x_{q11}}$	$\frac{\varepsilon}{4x_{11}}$	$\frac{\varepsilon}{2x_{11}}$	$\frac{\varepsilon}{x_{11}}$	$1-\varepsilon$	$\frac{\varepsilon}{x_{11}}$	$\frac{\varepsilon}{2x_{11}}$	$\frac{\varepsilon}{4x_{11}}$	$\frac{\varepsilon}{8x_{11}}$
{1,4}	$\frac{\varepsilon}{64x_{10}}$	$\frac{\varepsilon}{32x_{10}}$	$\frac{\varepsilon}{16x_{10}}$	$\frac{\varepsilon}{8x_{10}}$	$\frac{\varepsilon}{4x_{q10}}$	$\frac{\varepsilon}{2x_{10}}$	$\frac{\varepsilon}{x_{10}}$	$1-\varepsilon$	$\frac{\varepsilon}{x_{10}}$	$\frac{\varepsilon}{2x_{10}}$	$\frac{\varepsilon}{4x_{10}}$	$\frac{\varepsilon}{8x_{10}}$	$\frac{\varepsilon}{16x_{10}}$
{1,5}	$\frac{\varepsilon}{64x_{10}}$	$\frac{\varepsilon}{32x_{10}}$	$\frac{\varepsilon}{16x_{10}}$	$\frac{\varepsilon}{8x_{10}}$	$\frac{\varepsilon}{4x_{q10}}$	$\frac{\varepsilon}{2x_{10}}$	$\frac{\varepsilon}{x_{10}}$	$1-\varepsilon$	$\frac{\varepsilon}{x_{10}}$	$\frac{\varepsilon}{2x_{10}}$	$\frac{\varepsilon}{4x_{10}}$	$\frac{\varepsilon}{8x_{10}}$	$\frac{\varepsilon}{16x_{10}}$
{1,6}	$\frac{\varepsilon}{32x_9}$	$\frac{\varepsilon}{16x_9}$	$\frac{\varepsilon}{8x_9}$	$\frac{\varepsilon}{4x_9}$	$\frac{\varepsilon}{2x_{q9}}$	$\frac{\varepsilon}{x_9}$	$1-\varepsilon$	$\frac{\varepsilon}{x_9}$	$\frac{\varepsilon}{2x_9}$	$\frac{\varepsilon}{4x_9}$	$\frac{\varepsilon}{8x_9}$	$\frac{\varepsilon}{16x_9}$	$\frac{\varepsilon}{32x_9}$
{1,7}	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{x_{q8}}$	$1-\varepsilon$	$\frac{\varepsilon}{x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{32x_8}$	$\frac{\varepsilon}{64x_8}$
{1,8}	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{x_{q8}}$	$1-\varepsilon$	$\frac{\varepsilon}{x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{32x_8}$	$\frac{\varepsilon}{64x_8}$
{1,9}	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{x_{q8}}$	$1-\varepsilon$	$\frac{\varepsilon}{x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{32x_8}$	$\frac{\varepsilon}{64x_8}$
{1,10}	$\frac{\varepsilon}{8x_7}$	$\frac{\varepsilon}{4x_7}$	$\frac{\varepsilon}{2x_7}$	$\frac{\varepsilon}{x_7}$	$1-\varepsilon$	$\frac{\varepsilon}{x_7}$	$\frac{\varepsilon}{2x_7}$	$\frac{\varepsilon}{4x_7}$	$\frac{\varepsilon}{8x_7}$	$\frac{\varepsilon}{16x_7}$	$\frac{\varepsilon}{32x_7}$	$\frac{\varepsilon}{64x_7}$	$\frac{\varepsilon}{128x_7}$
{1,11}	$\frac{\varepsilon}{8x_7}$	$\frac{\varepsilon}{4x_7}$	$\frac{\varepsilon}{2x_7}$	$\frac{\varepsilon}{x_7}$	$1-\varepsilon$	$\frac{\varepsilon}{x_7}$	$\frac{\varepsilon}{2x_7}$	$\frac{\varepsilon}{4x_7}$	$\frac{\varepsilon}{8x_7}$	$\frac{\varepsilon}{16x_7}$	$\frac{\varepsilon}{32x_7}$	$\frac{\varepsilon}{64x_7}$	$\frac{\varepsilon}{128x_7}$
{1,12}	$\frac{\varepsilon}{4x_6}$	$\frac{\varepsilon}{2x_6}$	$\frac{\varepsilon}{x_6}$	$1-\varepsilon$	$\frac{\varepsilon}{x_{q6}}$	$\frac{\varepsilon}{2x_6}$	$\frac{\varepsilon}{4x_6}$	$\frac{\varepsilon}{8x_6}$	$\frac{\varepsilon}{16x_6}$	$\frac{\varepsilon}{32x_6}$	$\frac{\varepsilon}{64x_6}$	$\frac{\varepsilon}{128x_6}$	$\frac{\varepsilon}{256x_6}$
{1,13}	$\frac{\varepsilon}{4x_6}$	$\frac{\varepsilon}{2x_6}$	$\frac{\varepsilon}{x_6}$	$1-\varepsilon$	$\frac{\varepsilon}{x_{q6}}$	$\frac{\varepsilon}{2x_6}$	$\frac{\varepsilon}{4x_6}$	$\frac{\varepsilon}{8x_6}$	$\frac{\varepsilon}{16x_6}$	$\frac{\varepsilon}{32x_6}$	$\frac{\varepsilon}{64x_6}$	$\frac{\varepsilon}{128x_6}$	$\frac{\varepsilon}{256x_6}$
{1,14}	$\frac{\varepsilon}{2x_5}$	$\frac{\varepsilon}{x_5}$	$1-\varepsilon$	$\frac{\varepsilon}{x_5}$	$\frac{\varepsilon}{2x_{q5}}$	$\frac{\varepsilon}{4x_5}$	$\frac{\varepsilon}{8x_5}$	$\frac{\varepsilon}{16x_5}$	$\frac{\varepsilon}{32x_5}$	$\frac{\varepsilon}{64x_5}$	$\frac{\varepsilon}{128x_5}$	$\frac{\varepsilon}{256x_5}$	$\frac{\varepsilon}{512x_5}$
{1,15}	$\frac{\varepsilon}{2x_5}$	$\frac{\varepsilon}{x_5}$	$1-\varepsilon$	$\frac{\varepsilon}{x_5}$	$\frac{\varepsilon}{2x_{q5}}$	$\frac{\varepsilon}{4x_5}$	$\frac{\varepsilon}{8x_5}$	$\frac{\varepsilon}{16x_5}$	$\frac{\varepsilon}{32x_5}$	$\frac{\varepsilon}{64x_5}$	$\frac{\varepsilon}{128x_5}$	$\frac{\varepsilon}{256x_5}$	$\frac{\varepsilon}{512x_5}$

The first row in Table 11 displays the probability of playing each quantity level of a Stackelberg leader with a theoretical predicted quantity of 10. History {} means that we are at the beginning of the game, nothing as occurred yet. Only leaders play with this history. With probability $1-\varepsilon$ the leader plays quantity 10 and makes no mistake; with probability $\frac{\varepsilon}{x_{10}}$ he play either quantity 9 or 11; with probability $\frac{\varepsilon}{2x_{10}}$ he plays either quantity 8 or 12, and so on. The second row in the table displays the probability of playing each quantity level for a player who decides to wait for the second period, has a theoretical predicted quantity of 8, and the first period as elapsed and none of the players has moved–the history is {0}. Therefore, this player plays 8 with probability $1-\varepsilon$, 7 or 9 with probability $\frac{\varepsilon}{x_8}$ each, 6 or 10 with probability $\frac{\varepsilon}{2x_8}$ each, and so on. The remaining rows in the table display the probability of playing each quantity for a player who decides to wait for the second period, and who has observed the rival moving in the first period.

For example, if the history is $\{1, 10\}$ then the first period mover has played 10 and the follower will play his best response of 7 with probability $1-\varepsilon$, 6 or 8 with probability $\frac{\varepsilon}{x7}$ each, 5 or 9 with probability $\frac{\varepsilon}{2x7}$ each, and so on.

The probability of a given pair of timing and quantity choices (t_1, q_1, t_2, q_2) occurring is given by:

$$f(t_1, q_1, t_2, q_2|\tau, \varepsilon) = \sum_{k_1=0}^{K}\sum_{k_2=0}^{K} f^{Lk_1}(t_1) f^{Lk_1}(q_1|q^{LK_1}) f(k_1|\tau) f^{Lk_2}(t_2) f^{Lk_2}(q_2|q^{LK_2}) f(k_2|\tau).$$

Therefore, the likelihood function is

$$L(\tau, \varepsilon) = \prod_{t_1=1}^{2}\prod_{q_1=3}^{15}\prod_{t_2=1}^{2}\prod_{q_2=3}^{15} \left[f(t_1, q_1, t_2, q_2|\tau, \varepsilon)\right]^{n_{(t_1,q_1,t_2,q_2)}},$$

where $n_{(t_1,q_1,t_2,q_2)}$ is the number of cases that the particular pair is observed in the data. The first step in the estimation procedure is the determination of intervals where predicted behavior is constant. These intervals were computed up to $\tau = 4$. There are two reasons for this. The first is that higher levels of τ are unlikely according to the cognitive hierarchy literature. The second reason is that by increasing the interval for τ, it would be necessary to include progressively more thinking steps and this bears a high computational burden since there are no routines or packages for this particular type of estimation. With $\tau = 4$, ten thinking steps are required, which seems reasonable to cover this problem.

Table 12 presents, for the benchmark model and Table 13, for the alternative model, the upper and lower bounds of each interval, as well as the behavior of each type of player. The inclusion of higher thinking steps is done progressively as τ increases. It should be noted that the second period behavior depicted is the one when the history is null. In other words, second period movers best respond to observed first period movements; only when there is no observed movement will they play the quantity indicated in these intervals.

Table 12: Intervals where Predicted Behavior is Constant - Benchmark Model

Interval for τ	$L2$	$L3$	$L4$	$L5$	$L6$	$L7$	$L8$	$L9$	$L10$
[0.0001,0.8351]	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	–	–	–	–	–
[0.8352,0.8444]	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	–	–	–	–	–
[0.8445,0.8580]	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	–	–	–	–	–
[0.8581,0.8980]	I_2Q_8	I_2Q_8	I_1Q_{10}	I_1Q_{10}	–	–	–	–	–
[0.8981,0.9878]	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	–	–	–	–	–
[0.9879,1.0176]	I_2Q_8	I_1Q_{10}	I_2Q_8	I_1Q_{10}	–	–	–	–	–
[1.0177,1.0737]	I_2Q_8	I_1Q_{10}	I_1Q_{10}	I_2Q_8	–	–	–	–	–
[1.0738,1.0905]	I_2Q_8	I_1Q_{10}	I_1Q_{10}	I_1Q_{10}	I_2Q_8	–	–	–	–
[1.0906,1.3012]	I_2Q_8	I_1Q_{10}	I_1Q_{10}	I_1Q_{10}	I_1Q_{10}	–	–	–	–
[1.3013,1.8257]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	–	–	–
[1.8258,1.8550]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	–	–	–
[1.8551,1.9015]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	–	–	–
[1.9016,1.9804]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_1Q_{10}	–	–	–
[1.9805,2.1944]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	–	–	–
[2.1945,2.2481]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	–	–
[2.2482,2.3459]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_1Q_{10}	–	–
[2.3460,2.8563]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	–
[2.8564,3.1886]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8
[3.1887,3.2126]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}
[3.2127,3.2478]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8
[3.2479,3.2899]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_1Q_{10}
[3.2900,3.3826]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8
[3.3827,3.4263]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	I_1Q_{10}
[3.4264,3.4963]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_1Q_{10}	I_2Q_8
[3.4964,3.5604]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_1Q_{10}	I_1Q_{10}
[3.5605,3.8640]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8
[3.8641,4.0000]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}

Table 13: Intervals where Predicted Behavior is Constant - Alternative Model

Interval for τ	$L2$	$L3$	$L4$	$L5$	$L6$	$L7$	$L8$	$L9$	$L10$
[0.0001,0.6859]	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	–	–	–	–	–
[0.6860,0.6906]	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	–	–	–	–	–
[0.6907,0.6974]	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	–	–	–	–	–
[0.6975,0.7227]	I_2Q_8	I_2Q_8	I_1Q_{10}	I_1Q_{10}	–	–	–	–	–
[0.7228,0.7729]	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	–	–	–	–	–
[0.7730,0.7838]	I_2Q_8	I_1Q_{10}	I_2Q_8	I_1Q_{10}	–	–	–	–	–
[0.7839,0.8004]	I_2Q_8	I_1Q_{10}	I_1Q_{10}	I_2Q_8	–	–	–	–	–
[0.8005,0.9839]	I_2Q_8	I_1Q_{10}	I_1Q_{10}	I_1Q_{10}	–	–	–	–	–
[0.9840,1.6199]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	–	–	–
[1.6200,1.6413]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	–	–	–
[1.6414,1.6742]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	–	–	–
[1.6743,1.7422]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_1Q_{10}	–	–	–
[1.7423,1.9156]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	–	–	–
[1.9157,2.0079]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	I_1Q_{10}	–	–	–
[2.0080,2.1871]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}	I_1Q_{10}	I_2Q_8	I_2Q_8	–	–
[2.1872,2.3184]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}	I_1Q_{10}	I_2Q_8	I_1Q_{10}	–	–
[2.3185,2.5563]	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}	I_1Q_{10}	I_1Q_{10}	I_2Q_8	–	–
[2.5564,3.1253]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	–
[3.1254,3.1592]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8
[3.1593,3.2016]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_1Q_{10}
[3.2017,3.2937]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8
[3.2938,3.3352]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	I_1Q_{10}
[3.3353,3.4012]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_1Q_{10}	I_2Q_8
[3.4013,3.4745]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8	I_1Q_{10}	I_1Q_{10}	I_1Q_{10}
[3.4746,3.7894]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_2Q_8
[3.7895,3.9237]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}
[3.9238,4.0000]	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8	I_2Q_8	I_1Q_{10}	I_2Q_8	I_1Q_{10}	I_2Q_8

The estimation was then carried out for each of the mentioned intervals and the maximum was extracted from the set of 27 maxima, one per each interval. The procedure used was a standard grid search: the maximum likelihood expression was calculated for all possible values of tau within each interval and for possible values that ε assumes, i.e., from 0 to 100. The software used was GAUSS. We present the results with two decimal places but we used four decimal places for the parameter τ. The reason for this is the fact that some intervals are rather small and the use of more decimal places is needed for them to be considered.

One question that is frequently posed in the context of cognitive hierarchy models is how would a general distribution of players' types, with each $f(k)$ independent, perform compared to the Poisson distribution. Camerer, Ho and Chong (2004) show that the use of general distributions (they use a seven parameter model) does not introduce significant improvements compared to the Poisson distribution, specially given the simplicity the latter has.

In our case, using a general distribution of players' types would make it impossible to use the interval approach. We would have to have a computer routine that would calculate, for each step, the predictions of the model, then it would have to generate the functions that calculate the probability of each timing and quantity pair and, only then, would be able to calculate the probability and the maximum likelihood. Additionally, the benefits of using a general distribution of players' types would have to be considerable for it to be worthwhile since the model would become much less parsimonious.

C Maximum Likelihood Estimates for Tau

Tables 14 and 15 display the maximum likelihood estimates for tau for each of the intervals for the benchmark and the alternative models, respectively.

Table 14: Maximum Likelihood Estimates per Interval - Benchmark Model

Interval for tau	Entire set			First half		
	tau	epsilon	ML	tau	epsilon	ML
[0.0001,0.8351]	0.1564	0.04	-2505.48	0.1948	0.15	-1259.50
[0.8352,0.8444]	0.8352	0.45	-2689.38	0.8352	0.45	-1337.70
[0.8445,0.8580]	0.8445	0.42	-2655.10	0.8445	0.43	-1320.17
[0.8581,0.8980]	0.8581	0.42	-2650.71	0.8581	0.43	-1317.70
[0.8981,0.9878]	0.9878	0.46	-2536.01	0.9878	0.47	-1256.52
[0.9879,1.0176]	1.0176	0.47	-2526.94	1.0176	0.48	-1251.28
[1.0177,1.0737]	1.0737	0.50	-2490.39	1.0737	0.51	-1231.53
[1.0738,1.0905]	1.0905	0.51	-2479.31	1.0905	0.51	-1225.56
[1.0906,1.3012]	1.3012	0.55	-2434.58	1.3012	0.56	-1199.39
[1.3013,1.8257]	1.5291	0.60	-2317.04	1.6700	0.61	-1137.95
[1.8258,1.8550]	1.8258	0.62	-2321.09	1.8258	0.62	-1137.82
[1.8551,1.9015]	1.8551	0.62	-2315.49	1.8551	0.62	-1134.91
[1.9016,1.9804]	1.9016	0.62	-2314.02	1.9016	0.62	-1133.83
[1.9805,2.1944]	1.9805	0.63	-2296.43	2.1682	0.63	-1124.45
[2.1945,2.2481]	2.1945	0.64	-2293.26	2.2481	0.63	-1121.96
[2.2482,2.3459]	2.2482	0.64	-2291.88	2.2637	0.63	-1123.62
[2.3460,2.8563]	2.8563	0.65	-2275.88	2.8563	0.65	-1111.60
[2.8564,3.1886]	2.8564	0.65	-2257.49	2.8564	0.65	-1103.24
[3.1887,3.2126]	3.1887	0.66	-2268.42	3.1887	0.65	-1107.51
[3.2127,3.2478]	3.2127	0.66	-2267.43	3.2127	0.65	-1107.02
[3.2479,3.2899]	3.2479	0.66	-2267.74	3.2479	0.65	-1107.11
[3.2900,3.3826]	3.2900	0.66	-2264.78	3.2900	0.65	-1105.71
[3.3827,3.4263]	3.3827	0.66	-2266.59	3.3827	0.65	-1106.41
[3.4264,3.4963]	3.4264	0.66	-2265.36	3.4264	0.65	-1105.80
[3.4964,3.5604]	3.4964	0.66	-2265.71	3.4964	0.65	-1105.89
[3.5605,3.8640]	3.5605	0.66	-2258.75	3.5605	0.65	-1102.71
[3.8641,4.0000]	3.8641	0.66	-2264.30	3.8641	0.66	-1104.98

Table 15: Maximum Likelihood Estimates per Interval - Alternative Model

Interval for tau	Entire set			First half		
	tau	epsilon	ML	tau	epsilon	ML
[0.0001,0.6859]	0.2999	0.39	-2525.27	0.3425	0.42	-1259.14
[0.6860,0.6906]	0.6860	0.47	-2594.47	0.6860	0.48	-1284.91
[0.6907,0.6974]	0.6907	0.44	-2536.14	0.6907	0.45	-1268.91
[0.6975,0.7227]	0.6975	0.44	-2558.92	0.6975	0.45	-1266.62
[0.7228,0.7729]	0.7729	0.43	-2429.38	0.7729	0.44	-1198.74
[0.7730,0.7838]	0.7838	0.43	-2423.74	0.7838	0.45	-1195.58
[0.7839,0.8004]	0.8004	0.44	-2400.26	0.8004	0.45	-1183.16
[0.8005,0.9839]	0.9839	0.48	-2352.00	0.9839	0.49	-1154.53
[0.9840,1.6199]	1.3571	0.56	-2179.55	1.4327	0.56	-1064.38
[1.6200,1.6413]	1.6200	0.58	-2185.48	1.6200	0.57	-1065.69
[1.6414,1.6742]	1.6414	0.58	-2183.37	1.6414	0.57	-1064.63
[1.6743,1.7422]	1.6743	0.58	-2183.77	1.6743	0.58	-1064.67
[1.7423,1.9156]	1.7423	0.59	-2176.01	1.7423	0.58	-1060.90
[1.9157,2.0079]	1.9157	0.60	-2176.41	1.9157	0.59	-1062.91
[2.0080,2.1871]	2.0080	0.60	-2179.73	2.0080	0.59	-1062.12
[2.1872,2.3184]	2.1872	0.61	-2185.99	2.1872	0.6	-1064.88
[2.3185,2.5563]	2.3185	0.62	-2188.13	2.3185	0.61	-1065.93
[2.5564,3.1253]	2.5564	0.63	-2181.30	2.5564	0.62	-1063.42
[3.1254,3.1592]	3.1254	0.64	-2215.97	3.1254	0.63	-1079.72
[3.1593,3.2016]	3.1593	0.64	-2217.55	3.1593	0.63	-1080.49
[3.2017,3.2937]	3.2017	0.64	-2216.98	3.2017	0.63	-1080.34
[3.2938,3.3352]	3.2938	0.65	-2221.81	3.2938	0.64	-1082.66
[3.3353,3.4012]	3.3353	0.65	-2222.33	3.3353	0.64	-1082.98
[3.4013,3.4745]	3.4013	0.65	-2217.77	3.4013	0.64	-1084.23
[3.4746,3.7894]	3.4746	0.65	-2221.52	3.4746	0.64	-1082.93
[3.7895,3.9237]	3.7895	0.65	-2235.22	3.7895	0.64	-1089.56
[3.9238,4.0000]	3.9238	0.66	-2237.21	3.9238	0.65	-1090.70

D Estimated Market Outcomes

Here we present disaggregated results for the maximum likelihood estimation for the entire data set and the initial fifteen rounds. These are broken down into types of movement, i.e., first and second period simultaneous movement and sequential movement. The notation is the same employed in the paper. For example, in tables referring to first period movement, 8,8 refers to each subject playing quantity 8 in that period (Cournot equilibrium); in tables referring to sequential movement, 12,6 refers to the leader playing quantity 12 in the first period and the follower playing quantity 6 in the second period (Stackelberg equilibrium).

Table 16: Estimation results for first period movement (entire set)

Pairs	Data	Bench	Alt	Pairs	Data	Bench	Alt
3,3	0.00%	0.00%	0.00%	**7,7**	0.67%	0.03%	0.02%
3,4	0.00%	0.00%	0.00%	**7,8**	0.90%	0.13%	0.32%
3,5	0.00%	0.00%	0.00%	**7,9**	0.67%	0.25%	0.13%
3,6	0.00%	0.01%	0.01%	**7,10**	2.70%	0.52%	0.35%
3,7	0.00%	0.01%	0.01%	**7,11**	0.45%	0.25%	0.13%
3,8	0.00%	0.02%	0.09%	**7,12**	0.67%	0.13%	0.07%
3,9	0.00%	0.04%	0.04%	**7,13**	0.45%	0.07%	0.04%
3,10	0.00%	0.09%	0.10%	**7,14**	0.00%	0.04%	0.03%
3,11	0.00%	0.04%	0.04%	**7,15**	0.22%	0.02%	0.02%
3,12	0.00%	0.02%	0.02%	**8,8**	2.47%	0.12%	1.27%
3,13	0.00%	0.01%	0.01%	**8,9**	1.57%	0.47%	1.06%
3,14	0.00%	0.01%	0.01%	**8,10**	5.39%	0.98%	2.75%
3,15	0.00%	0.00%	0.00%	**8,11**	3.37%	0.47%	1.06%
4,4	0.00%	0.00%	0.00%	**8,12**	0.67%	0.24%	0.57%
4,5	0.00%	0.01%	0.01%	**8,13**	0.00%	0.13%	0.32%
4,6	0.00%	0.01%	0.01%	**8,14**	0.22%	0.07%	0.20%
4,7	0.00%	0.02%	0.01%	**8,15**	0.22%	0.04%	0.14%
4,8	0.00%	0.03%	0.11%	**9,9**	0.45%	0.46%	0.22%
4,9	0.00%	0.06%	0.04%	**9,10**	2.02%	1.89%	1.14%
4,10	0.22%	0.12%	0.12%	**9,11**	0.67%	0.91%	0.44%
4,11	0.00%	0.06%	0.04%	**9,12**	0.45%	0.47%	0.24%
4,12	0.00%	0.03%	0.02%	**9,13**	0.00%	0.25%	0.13%
4,13	0.00%	0.02%	0.01%	**9,14**	0.00%	0.14%	0.08%
4,14	0.00%	0.01%	0.01%	**9,15**	0.00%	0.08%	0.06%
4,15	0.00%	0.01%	0.01%	**10,10**	2.92%	1.96%	1.49%
5,5	0.00%	0.00%	0.00%	**10,11**	0.67%	1.89%	1.14%
5,6	0.00%	0.01%	0.01%	**10,12**	1.12%	0.98%	0.61%
5,7	0.00%	0.02%	0.02%	**10,13**	0.45%	0.52%	0.35%
5,8	0.00%	0.04%	0.14%	**10,14**	0.22%	0.29%	0.21%
5,9	0.00%	0.08%	0.06%	**10,15**	0.22%	0.18%	0.15%
5,10	0.00%	0.18%	0.15%	**11,11**	0.22%	0.46%	0.22%
5,11	0.00%	0.08%	0.06%	**11,12**	0.67%	0.47%	0.24%
5,12	0.00%	0.04%	0.03%	**11,13**	0.00%	0.25%	0.13%
5,13	0.00%	0.02%	0.02%	**11,14**	0.00%	0.14%	0.08%
5,14	0.00%	0.01%	0.01%	**11,15**	0.22%	0.08%	0.06%
5,15	0.00%	0.01%	0.01%	**12,12**	0.67%	0.12%	0.06%
6,6	0.22%	0.01%	0.01%	**12,13**	0.00%	0.13%	0.07%
6,7	0.00%	0.04%	0.03%	**12,14**	0.00%	0.07%	0.04%
6,8	2.25%	0.07%	0.20%	**12,15**	0.00%	0.04%	0.03%
6,9	0.90%	0.14%	0.08%	**13,13**	0.00%	0.03%	0.02%
6,10	0.90%	0.29%	0.21%	**13,14**	0.22%	0.04%	0.03%
6,11	0.45%	0.14%	0.08%	**13,15**	0.00%	0.02%	0.02%
6,12	0.67%	0.07%	0.04%	**14,14**	0.00%	0.01%	0.01%
6,13	0.00%	0.04%	0.03%	**14,15**	0.00%	0.01%	0.01%
6,14	0.22%	0.02%	0.02%	**15,15**	0.00%	0.00%	0.00%
6,15	0.00%	0.01%	0.01%	**Total**	37.75%	17.86%	17.69%

Table 17: Estimation results for second period movement (entire set)

Pairs	Data	Bench	Alt	Pairs	Data	Bench	Alt
3,3	0.00%	0.01%	0.01%	**7,7**	0.45%	0.87%	0.70%
3,4	0.00%	0.02%	0.02%	**7,8**	1.35%	3.62%	3.74%
3,5	0.00%	0.04%	0.04%	**7,9**	0.67%	1.73%	1.41%
3,6	0.00%	0.08%	0.07%	**7,10**	0.67%	0.89%	0.73%
3,7	0.00%	0.15%	0.14%	**7,11**	0.00%	0.46%	0.39%
3,8	0.00%	0.31%	0.37%	**7,12**	0.00%	0.25%	0.23%
3,9	0.00%	0.15%	0.14%	**7,13**	0.00%	0.15%	0.14%
3,10	0.00%	0.08%	0.07%	**7,14**	0.00%	0.09%	0.10%
3,11	0.00%	0.04%	0.04%	**7,15**	0.00%	0.07%	0.08%
3,12	0.00%	0.02%	0.02%	**8,8**	2.02%	3.74%	4.97%
3,13	0.00%	0.01%	0.01%	**8,9**	2.02%	3.62%	3.74%
3,14	0.00%	0.01%	0.01%	**8,10**	1.80%	1.85%	1.95%
3,15	0.00%	0.01%	0.01%	**8,11**	0.67%	0.97%	1.05%
4,4	0.00%	0.02%	0.02%	**8,12**	0.22%	0.53%	0.60%
4,5	0.00%	0.07%	0.06%	**8,13**	0.00%	0.31%	0.37%
4,6	0.00%	0.13%	0.12%	**8,14**	0.00%	0.20%	0.26%
4,7	0.00%	0.25%	0.23%	**8,15**	0.00%	0.14%	0.21%
4,8	0.00%	0.53%	0.60%	**9,9**	0.45%	0.87%	0.70%
4,9	0.22%	0.25%	0.23%	**9,10**	0.90%	0.89%	0.73%
4,10	0.00%	0.13%	0.12%	**9,11**	0.45%	0.46%	0.39%
4,11	0.00%	0.07%	0.06%	**9,12**	0.00%	0.25%	0.23%
4,12	0.00%	0.04%	0.04%	**9,13**	0.00%	0.15%	0.14%
4,13	0.00%	0.02%	0.02%	**9,14**	0.00%	0.09%	0.10%
4,14	0.00%	0.01%	0.02%	**9,15**	0.22%	0.07%	0.08%
4,15	0.00%	0.01%	0.01%	**10,10**	0.00%	0.23%	0.19%
5,5	0.00%	0.06%	0.06%	**10,11**	0.22%	0.24%	0.21%
5,6	0.00%	0.24%	0.21%	**10,12**	0.00%	0.13%	0.12%
5,7	0.00%	0.46%	0.39%	**10,13**	0.00%	0.08%	0.07%
5,8	0.22%	0.97%	1.05%	**10,14**	0.00%	0.05%	0.05%
5,9	0.00%	0.46%	0.39%	**10,15**	0.00%	0.03%	0.04%
5,10	0.00%	0.24%	0.21%	**11,11**	0.00%	0.06%	0.06%
5,11	0.00%	0.12%	0.11%	**11,12**	0.00%	0.07%	0.06%
5,12	0.00%	0.07%	0.06%	**11,13**	0.00%	0.04%	0.04%
5,13	0.00%	0.04%	0.04%	**11,14**	0.22%	0.03%	0.03%
5,14	0.00%	0.03%	0.03%	**11,15**	0.00%	0.02%	0.02%
5,15	0.00%	0.02%	0.02%	**12,12**	0.00%	0.02%	0.02%
6,6	0.22%	0.23%	0.19%	**12,13**	0.00%	0.02%	0.02%
6,7	0.90%	0.89%	0.73%	**12,14**	0.00%	0.01%	0.02%
6,8	0.45%	1.85%	1.95%	**12,15**	0.00%	0.01%	0.01%
6,9	0.22%	0.89%	0.73%	**13,13**	0.00%	0.01%	0.01%
6,10	0.22%	0.45%	0.38%	**13,14**	0.00%	0.01%	0.01%
6,11	0.45%	0.24%	0.21%	**13,15**	0.00%	0.01%	0.01%
6,12	0.45%	0.13%	0.12%	**14,14**	0.00%	0.00%	0.00%
6,13	0.00%	0.08%	0.07%	**14,15**	0.00%	0.00%	0.01%
6,14	0.00%	0.05%	0.05%	**15,15**	0.00%	0.00%	0.00%
6,15	0.00%	0.03%	0.04%	**Total**	15.73%	33.28%	33.57%

Table 18: Estimation results for sequential movement (entire set)

Pairs	**Data**	**Bench**	**Alt**	**Pairs**	**Data**	**Bench**	**Alt**
3,3	0.00%	0.00%	0.00%	**5,6**	0.00%	0.02%	0.02%
3,4	0.00%	0.00%	0.00%	**5,7**	0.00%	0.03%	0.03%
3,5	0.00%	0.00%	0.00%	**5,8**	0.00%	0.06%	0.05%
3,6	0.00%	0.01%	0.01%	**5,9**	0.00%	0.12%	0.10%
3,7	0.00%	0.01%	0.01%	**5,10**	0.45%	0.24%	0.27%
3,8	0.00%	0.02%	0.02%	**5,11**	0.00%	0.12%	0.10%
3,9	0.00%	0.03%	0.04%	**5,12**	0.00%	0.06%	0.05%
3,10	0.00%	0.06%	0.07%	**5,13**	0.00%	0.03%	0.03%
3,11	0.22%	0.13%	0.18%	**5,14**	0.00%	0.02%	0.02%
3,12	0.00%	0.06%	0.07%	**5,15**	0.00%	0.01%	0.01%
3,13	0.00%	0.03%	0.04%	**6,3**	0.00%	0.01%	0.01%
3,14	0.00%	0.02%	0.02%	**6,4**	0.00%	0.02%	0.01%
3,15	0.00%	0.01%	0.01%	**6,5**	0.00%	0.03%	0.02%
4,3	0.00%	0.00%	0.00%	**6,6**	0.90%	0.05%	0.04%
4,4	0.00%	0.00%	0.01%	**6,7**	0.00%	0.10%	0.08%
4,5	0.00%	0.01%	0.01%	**6,8**	0.00%	0.19%	0.15%
4,6	0.00%	0.01%	0.01%	**6,9**	1.12%	0.40%	0.39%
4,7	0.00%	0.02%	0.02%	**6,10**	0.00%	0.19%	0.15%
4,8	0.00%	0.04%	0.04%	**6,11**	0.00%	0.10%	0.08%
4,9	0.00%	0.08%	0.08%	**6,12**	0.00%	0.05%	0.04%
4,10	0.00%	0.17%	0.21%	**6,13**	0.00%	0.03%	0.02%
4,11	0.00%	0.08%	0.08%	**6,14**	0.00%	0.02%	0.01%
4,12	0.00%	0.04%	0.04%	**6,15**	0.00%	0.01%	0.01%
4,13	0.00%	0.02%	0.02%	**7,3**	0.00%	0.03%	0.02%
4,14	0.00%	0.01%	0.01%	**7,4**	0.00%	0.05%	0.04%
4,15	0.00%	0.01%	0.01%	**7,5**	0.00%	0.09%	0.07%
5,3	0.00%	0.00%	0.01%	**7,6**	0.00%	0.18%	0.12%
5,4	0.22%	0.01%	0.01%	**7,7**	2.70%	0.35%	0.24%
5,5	0.00%	0.01%	0.01%				

Table 19: Estimation results for sequential movement (entire set) - cont.

Pairs	**Data**	**Bench**	**Alt**	**Pairs**	**Data**	**Bench**	**Alt**
7,8	2.47%	0.72%	0.64%	**9,11**	0.00%	0.34%	0.22%
7,9	0.67%	0.35%	0.24%	**9,12**	0.00%	0.18%	0.13%
7,10	0.00%	0.18%	0.12%	**9,13**	0.00%	0.11%	0.08%
7,11	0.00%	0.09%	0.07%	**9,14**	0.00%	0.07%	0.06%
7,12	0.00%	0.05%	0.04%	**9,15**	0.00%	0.05%	0.04%
7,13	0.00%	0.03%	0.02%	**10,3**	0.22%	0.38%	0.33%
7,14	0.00%	0.02%	0.02%	**10,4**	0.00%	0.71%	0.58%
7,15	0.00%	0.01%	0.01%	**10,5**	0.00%	1.35%	1.08%
8,3	0.00%	0.06%	0.19%	**10,6**	0.00%	2.64%	2.08%
8,4	0.00%	0.09%	0.30%	**10,7**	4.49%	5.41%	5.44%
8,5	0.00%	0.17%	0.53%	**10,8**	1.35%	2.64%	2.08%
8,6	0.00%	0.33%	0.98%	**10,9**	1.12%	1.35%	1.08%
8,7	0.00%	0.65%	1.89%	**10,10**	3.15%	0.71%	0.58%
8,8	9.89%	1.35%	5.03%	**10,11**	0.90%	0.38%	0.33%
8,9	2.25%	0.65%	1.89%	**10,12**	0.00%	0.22%	0.21%
8,10	0.22%	0.33%	0.98%	**10,13**	0.00%	0.14%	0.14%
8,11	0.00%	0.17%	0.53%	**10,14**	0.00%	0.10%	0.11%
8,12	0.00%	0.09%	0.30%	**10,15**	0.22%	0.08%	0.10%
8,13	0.00%	0.06%	0.19%	**11,3**	0.00%	0.19%	0.13%
8,14	0.00%	0.04%	0.13%	**11,4**	0.00%	0.34%	0.22%
8,15	0.00%	0.03%	0.10%	**11,5**	0.00%	0.65%	0.41%
9,3	0.00%	0.11%	0.08%	**11,6**	0.22%	1.27%	0.80%
9,4	0.00%	0.18%	0.13%	**11,7**	2.02%	2.62%	2.09%
9,5	0.00%	0.34%	0.22%	**11,8**	1.12%	1.27%	0.80%
9,6	0.00%	0.64%	0.41%	**11,9**	0.45%	0.65%	0.41%
9,7	0.00%	1.26%	0.79%	**11,10**	0.00%	0.34%	0.22%
9,8	1.80%	2.62%	2.09%	**11,11**	0.22%	0.19%	0.13%
9,9	0.90%	1.26%	0.79%	**11,12**	0.45%	0.11%	0.08%
9,10	0.67%	0.64%	0.41%				

Table 20: Estimation results for sequential movement (entire set) - cont.

Pairs	**Data**	**Bench**	**Alt**	**Pairs**	**Data**	**Bench**	**Alt**
11,13	0.22%	0.07%	0.06%	**13,15**	0.00%	0.01%	0.01%
11,14	0.00%	0.05%	0.04%	**14,3**	0.00%	0.11%	0.09%
11,15	0.22%	0.04%	0.04%	**14,4**	0.00%	0.22%	0.17%
12,3	0.45%	0.18%	0.12%	**14,5**	0.00%	0.40%	0.39%
12,4	0.00%	0.35%	0.23%	**14,6**	0.00%	0.22%	0.17%
12,5	0.00%	0.68%	0.44%	**14,7**	0.00%	0.11%	0.09%
12,6	0.90%	1.35%	1.12%	**14,8**	0.00%	0.06%	0.05%
12,7	0.00%	0.68%	0.44%	**14,9**	0.00%	0.03%	0.03%
12,8	0.45%	0.35%	0.23%	**14,10**	0.00%	0.02%	0.02%
12,9	0.22%	0.18%	0.12%	**14,11**	0.00%	0.01%	0.01%
12,10	0.45%	0.10%	0.07%	**14,12**	0.00%	0.01%	0.01%
12,11	0.45%	0.06%	0.04%	**14,13**	0.00%	0.01%	0.01%
12,12	1.57%	0.04%	0.03%	**14,14**	0.00%	0.01%	0.01%
12,13	0.22%	0.03%	0.02%	**14,15**	0.00%	0.00%	0.01%
12,14	0.00%	0.02%	0.02%	**15,3**	0.00%	0.07%	0.06%
12,15	0.00%	0.02%	0.02%	**15,4**	0.00%	0.13%	0.11%
13,3	0.00%	0.10%	0.07%	**15,5**	0.22%	0.24%	0.27%
13,4	0.00%	0.18%	0.13%	**15,6**	0.00%	0.13%	0.11%
13,5	0.00%	0.36%	0.25%	**15,7**	0.00%	0.07%	0.06%
13,6	0.00%	0.72%	0.64%	**15,8**	0.00%	0.03%	0.03%
13,7	0.00%	0.36%	0.25%	**15,9**	0.22%	0.02%	0.02%
13,8	0.22%	0.18%	0.13%	**15,10**	0.00%	0.01%	0.01%
13,9	0.00%	0.10%	0.07%	**15,11**	0.00%	0.01%	0.01%
13,10	0.22%	0.05%	0.04%	**15,12**	0.00%	0.00%	0.01%
13,11	0.00%	0.03%	0.02%	**15,13**	0.00%	0.00%	0.00%
13,12	0.00%	0.02%	0.02%	**15,14**	0.00%	0.00%	0.00%
13,13	0.00%	0.01%	0.01%	**15,15**	0.00%	0.00%	0.00%
13,14	0.00%	0.01%	0.01%	**Total**	46.52%	48.76%	48.74%

Table 21: Estimation results for first period movement (first half)

Pairs	Data	Bench	Alt	Pairs	Data	Bench	Alt
3,3	0.00%	0.00%	0.00%	**7,7**	0.00%	0.03%	0.02%
3,4	0.00%	0.00%	0.00%	**7,8**	0.89%	0.13%	0.32%
3,5	0.00%	0.00%	0.00%	**7,9**	0.44%	0.25%	0.13%
3,6	0.00%	0.00%	0.01%	**7,10**	2.22%	0.53%	0.35%
3,7	0.00%	0.01%	0.01%	**7,11**	0.44%	0.25%	0.13%
3,8	0.00%	0.02%	0.09%	**7,12**	0.00%	0.13%	0.07%
3,9	0.00%	0.03%	0.04%	**7,13**	0.00%	0.07%	0.04%
3,10	0.00%	0.06%	0.10%	**7,14**	0.00%	0.03%	0.02%
3,11	0.00%	0.03%	0.04%	**7,15**	0.44%	0.02%	0.02%
3,12	0.00%	0.02%	0.02%	**8,8**	3.11%	0.12%	1.26%
3,13	0.00%	0.01%	0.01%	**8,9**	1.78%	0.49%	1.04%
3,14	0.00%	0.00%	0.01%	**8,10**	5.78%	1.02%	2.80%
3,15	0.00%	0.00%	0.00%	**8,11**	3.11%	0.49%	1.04%
4,4	0.00%	0.00%	0.00%	**8,12**	0.89%	0.25%	0.56%
4,5	0.00%	0.00%	0.01%	**8,13**	0.00%	0.13%	0.32%
4,6	0.00%	0.01%	0.01%	**8,14**	0.00%	0.07%	0.20%
4,7	0.00%	0.01%	0.01%	**8,15**	0.44%	0.04%	0.14%
4,8	0.00%	0.02%	0.11%	**9,9**	0.44%	0.48%	0.21%
4,9	0.00%	0.04%	0.04%	**9,10**	2.67%	2.01%	1.15%
4,10	0.00%	0.09%	0.12%	**9,11**	1.33%	0.96%	0.43%
4,11	0.00%	0.04%	0.04%	**9,12**	0.89%	0.49%	0.23%
4,12	0.00%	0.02%	0.02%	**9,13**	0.00%	0.25%	0.13%
4,13	0.00%	0.01%	0.01%	**9,14**	0.00%	0.13%	0.08%
4,14	0.00%	0.01%	0.01%	**9,15**	0.00%	0.07%	0.06%
4,15	0.00%	0.00%	0.01%	**10,10**	2.22%	2.11%	1.56%
5,5	0.00%	0.00%	0.00%	**10,11**	1.33%	2.01%	1.15%
5,6	0.00%	0.01%	0.01%	**10,12**	1.33%	1.02%	0.62%
5,7	0.00%	0.02%	0.02%	**10,13**	0.00%	0.53%	0.35%
5,8	0.00%	0.04%	0.14%	**10,14**	0.44%	0.28%	0.22%
5,9	0.00%	0.07%	0.06%	**10,15**	0.00%	0.16%	0.15%
5,10	0.00%	0.16%	0.15%	**11,11**	0.44%	0.48%	0.21%
5,11	0.00%	0.07%	0.06%	**11,12**	0.89%	0.49%	0.23%
5,12	0.00%	0.04%	0.03%	**11,13**	0.00%	0.25%	0.13%
5,13	0.00%	0.02%	0.02%	**11,14**	0.00%	0.13%	0.08%
5,14	0.00%	0.01%	0.01%	**11,15**	0.44%	0.07%	0.06%
5,15	0.00%	0.01%	0.01%	**12,12**	0.89%	0.12%	0.06%
6,6	0.00%	0.01%	0.01%	**12,13**	0.00%	0.13%	0.07%
6,7	0.00%	0.03%	0.02%	**12,14**	0.00%	0.07%	0.04%
6,8	1.78%	0.07%	0.20%	**12,15**	0.00%	0.04%	0.03%
6,9	0.44%	0.13%	0.08%	**13,13**	0.00%	0.03%	0.02%
6,10	0.44%	0.28%	0.22%	**13,14**	0.44%	0.03%	0.02%
6,11	0.00%	0.13%	0.08%	**13,15**	0.00%	0.02%	0.02%
6,12	0.89%	0.07%	0.04%	**14,14**	0.00%	0.01%	0.01%
6,13	0.00%	0.03%	0.02%	**14,15**	0.00%	0.01%	0.01%
6,14	0.00%	0.02%	0.02%	**15,15**	0.00%	0.00%	0.00%
6,15	0.00%	0.01%	0.01%	**Total**	36.89%	18.12%	17.69%

Table 22: Estimation results for second period movement (first half)

Pairs	Data	Bench	Alt	Pairs	Data	Bench	Alt
3,3	0.00%	0.00%	0.01%	**7,7**	0.89%	0.88%	0.68%
3,4	0.00%	0.02%	0.02%	**7,8**	0.00%	3.69%	3.77%
3,5	0.00%	0.03%	0.04%	**7,9**	0.89%	1.76%	1.36%
3,6	0.00%	0.07%	0.07%	**7,10**	0.89%	0.89%	0.71%
3,7	0.00%	0.13%	0.14%	**7,11**	0.00%	0.45%	0.38%
3,8	0.00%	0.27%	0.38%	**7,12**	0.00%	0.24%	0.22%
3,9	0.00%	0.13%	0.14%	**7,13**	0.00%	0.13%	0.14%
3,10	0.00%	0.07%	0.07%	**7,14**	0.00%	0.07%	0.10%
3,11	0.00%	0.03%	0.04%	**7,15**	0.00%	0.05%	0.08%
3,12	0.00%	0.02%	0.02%	**8,8**	2.22%	3.74%	5.21%
3,13	0.00%	0.01%	0.01%	**8,9**	3.11%	3.69%	3.77%
3,14	0.00%	0.01%	0.01%	**8,10**	1.33%	1.87%	1.96%
3,15	0.00%	0.00%	0.01%	**8,11**	0.44%	0.95%	1.06%
4,4	0.00%	0.02%	0.02%	**8,12**	0.44%	0.50%	0.61%
4,5	0.00%	0.06%	0.06%	**8,13**	0.00%	0.27%	0.38%
4,6	0.00%	0.12%	0.11%	**814**	0.00%	0.16%	0.27%
4,7	0.00%	0.24%	0.22%	**8,15**	0.00%	0.10%	0.21%
4,8	0.00%	0.50%	0.61%	**9,9**	0.89%	0.88%	0.68%
4,9	0.44%	0.24%	0.22%	**9,10**	0.44%	0.89%	0.71%
4,10	0.00%	0.12%	0.11%	**9,11**	0.89%	0.45%	0.38%
4,11	0.00%	0.06%	0.06%	**9,12**	0.00%	0.24%	0.22%
4,12	0.00%	0.03%	0.04%	**9,13**	0.00%	0.13%	0.14%
4,13	0.00%	0.02%	0.02%	**9,14**	0.00%	0.07%	0.10%
4,14	0.00%	0.01%	0.02%	**9,15**	0.44%	0.05%	0.08%
4,15	0.00%	0.01%	0.01%	**10,10**	0.00%	0.22%	0.18%
5,5	0.00%	0.06%	0.05%	**10,11**	0.44%	0.23%	0.20%
5,6	0.00%	0.23%	0.20%	**10,12**	0.00%	0.12%	0.11%
5,7	0.00%	0.45%	0.38%	**10,13**	0.00%	0.07%	0.07%
5,8	0.44%	0.95%	1.06%	**10,14**	0.00%	0.04%	0.05%
5,9	0.00%	0.45%	0.38%	**10,15**	0.00%	0.02%	0.04%
5,10	0.00%	0.23%	0.20%	**11,11**	0.00%	0.06%	0.05%
5,11	0.00%	0.12%	0.11%	**11,12**	0.00%	0.06%	0.06%
5,12	0.00%	0.06%	0.06%	**11,13**	0.00%	0.03%	0.04%
5,13	0.00%	0.03%	0.04%	**11,14**	0.00%	0.02%	0.03%
5,14	0.00%	0.02%	0.03%	**11,15**	0.00%	0.01%	0.02%
5,15	0.00%	0.01%	0.02%	**12,12**	0.00%	0.02%	0.02%
6,6	0.44%	0.22%	0.18%	**12,13**	0.00%	0.02%	0.02%
6,7	1.33%	0.89%	0.71%	**12,14**	0.00%	0.01%	0.02%
6,8	0.89%	1.87%	1.96%	**12,15**	0.00%	0.01%	0.01%
6,9	0.44%	0.89%	0.71%	**13,13**	0.00%	0.00%	0.01%
6,10	0.44%	0.45%	0.37%	**13,14**	0.00%	0.01%	0.01%
6,11	0.89%	0.23%	0.20%	**13,15**	0.00%	0.00%	0.01%
6,12	0.44%	0.12%	0.11%	**14,14**	0.00%	0.00%	0.00%
6,13	0.00%	0.07%	0.07%	**14,15**	0.00%	0.00%	0.01%
6,14	0.00%	0.04%	0.05%	**15,15**	0.00%	0.00%	0.00%
6,15	0.00%	0.02%	0.04%	**Total**	19.11%	32.73%	33.57%

Table 23: Estimation results for sequential movement (first half)

Pairs	**Data**	**Bench**	**Alt**	**Pairs**	**Data**	**Bench**	**Alt**
3,3	0.00%	0.00%	0.00%	**5,6**	0.00%	0.01%	0.02%
3,4	0.00%	0.00%	0.00%	**5,7**	0.00%	0.03%	0.03%
3,5	0.00%	0.00%	0.00%	**5,8**	0.00%	0.05%	0.05%
3,6	0.00%	0.00%	0.01%	**5,9**	0.00%	0.10%	0.10%
3,7	0.00%	0.01%	0.01%	**5,10**	0.89%	0.21%	0.28%
3,8	0.00%	0.01%	0.02%	**5,11**	0.00%	0.10%	0.10%
3,9	0.00%	0.02%	0.04%	**5,12**	0.00%	0.05%	0.05%
3,10	0.00%	0.04%	0.07%	**5,13**	0.00%	0.03%	0.03%
3,11	0.44%	0.08%	0.18%	**5,14**	0.00%	0.01%	0.02%
3,12	0.00%	0.04%	0.07%	**5,15**	0.00%	0.01%	0.01%
3,13	0.00%	0.02%	0.04%	**6,3**	0.00%	0.01%	0.01%
3,14	0.00%	0.01%	0.02%	**6,4**	0.00%	0.01%	0.01%
3,15	0.00%	0.01%	0.01%	**6,5**	0.00%	0.02%	0.02%
4,3	0.00%	0.00%	0.00%	**6,6**	0.44%	0.05%	0.04%
4,4	0.00%	0.00%	0.01%	**6,7**	0.00%	0.09%	0.07%
4,5	0.00%	0.00%	0.01%	**6,8**	0.00%	0.18%	0.14%
4,6	0.00%	0.01%	0.01%	**6,9**	0.89%	0.38%	0.40%
4,7	0.00%	0.02%	0.02%	**6,10**	0.00%	0.18%	0.14%
4,8	0.00%	0.03%	0.04%	**6,11**	0.00%	0.09%	0.07%
4,9	0.00%	0.06%	0.08%	**6,12**	0.00%	0.05%	0.04%
4,10	0.00%	0.13%	0.21%	**6,13**	0.00%	0.02%	0.02%
4,11	0.00%	0.06%	0.08%	**6,14**	0.00%	0.01%	0.01%
4,12	0.00%	0.03%	0.04%	**6,15**	0.00%	0.01%	0.01%
4,13	0.00%	0.02%	0.02%	**7,3**	0.00%	0.02%	0.02%
4,14	0.00%	0.01%	0.01%	**7,4**	0.00%	0.05%	0.04%
4,15	0.00%	0.00%	0.01%	**7,5**	0.00%	0.09%	0.07%
5,3	0.00%	0.00%	0.01%	**7,6**	0.00%	0.17%	0.12%
5,4	0.44%	0.00%	0.01%	**7,7**	0.89%	0.34%	0.23%
5,5	0.00%	0.01%	0.01%				

Table 24: Estimation results for sequential movement (first half) - cont.

Pairs	Data	Bench	Alt	Pairs	Data	Bench	Alt
7,8	1.33%	0.71%	0.64%	**9,11**	0.00%	0.34%	0.21%
7,9	0.00%	0.34%	0.23%	**9,12**	0.00%	0.18%	0.12%
7,10	0.00%	0.17%	0.12%	**9,13**	0.00%	0.10%	0.08%
7,11	0.00%	0.09%	0.07%	**9,14**	0.00%	0.06%	0.05%
7,12	0.00%	0.05%	0.04%	**9,15**	0.00%	0.04%	0.04%
7,13	0.00%	0.02%	0.02%	**10,3**	0.00%	0.37%	0.33%
7,14	0.00%	0.01%	0.02%	**10,4**	0.00%	0.71%	0.59%
7,15	0.00%	0.01%	0.01%	**10,5**	0.00%	1.39%	1.09%
8,3	0.00%	0.05%	0.19%	**10,6**	0.00%	2.76%	2.09%
8,4	0.00%	0.09%	0.30%	**10,7**	5.78%	5.61%	5.70%
8,5	0.00%	0.17%	0.52%	**10,8**	0.89%	2.76%	2.09%
8,6	0.00%	0.33%	0.97%	**10,9**	0.89%	1.39%	1.09%
8,7	0.00%	0.66%	1.86%	**10,10**	3.11%	0.71%	0.59%
8,8	8.00%	1.38%	5.13%	**10,11**	1.78%	0.37%	0.33%
8,9	2.67%	0.66%	1.86%	**10,12**	0.00%	0.20%	0.21%
8,10	0.44%	0.33%	0.97%	**10,13**	0.00%	0.12%	0.15%
8,11	0.00%	0.17%	0.52%	**10,14**	0.00%	0.07%	0.12%
8,12	0.00%	0.09%	0.30%	**10,15**	0.00%	0.05%	0.10%
8,13	0.00%	0.05%	0.19%	**11,3**	0.00%	0.18%	0.12%
8,14	0.00%	0.03%	0.13%	**11,4**	0.00%	0.34%	0.22%
8,15	0.00%	0.02%	0.10%	**11,5**	0.00%	0.67%	0.40%
9,3	0.00%	0.10%	0.08%	**11,6**	0.00%	1.32%	0.77%
9,4	0.00%	0.18%	0.12%	**11,7**	3.56%	2.73%	2.11%
9,5	0.00%	0.34%	0.21%	**11,8**	1.78%	1.32%	0.77%
9,6	0.00%	0.66%	0.40%	**11,9**	0.44%	0.67%	0.40%
9,7	0.00%	1.30%	0.76%	**11,10**	0.00%	0.34%	0.22%
9,8	2.22%	2.73%	2.11%	**11,11**	0.00%	0.18%	0.12%
9,9	1.33%	1.30%	0.76%	**11,12**	0.44%	0.10%	0.08%
9,10	0.44%	0.66%	0.40%				

Table 25: Estimation results for sequential movement (first half) - cont.

Pairs	Data	Bench	Alt	Pairs	Data	Bench	Alt
11,13	0.00%	0.06%	0.05%	**13,15**	0.00%	0.01%	0.01%
11,14	0.00%	0.04%	0.04%	**14,3**	0.00%	0.10%	0.08%
11,15	0.44%	0.03%	0.04%	**14,4**	0.00%	0.20%	0.16%
12,3	0.89%	0.18%	0.12%	**14,5**	0.00%	0.38%	0.40%
12,4	0.00%	0.35%	0.22%	**14,6**	0.00%	0.20%	0.16%
12,5	0.00%	0.69%	0.43%	**14,7**	0.00%	0.10%	0.08%
12,6	1.78%	1.38%	1.13%	**14,8**	0.00%	0.05%	0.04%
12,7	0.00%	0.69%	0.43%	**14,9**	0.00%	0.03%	0.03%
12,8	0.44%	0.35%	0.22%	**14,10**	0.00%	0.01%	0.02%
12,9	0.00%	0.18%	0.12%	**14,11**	0.00%	0.01%	0.01%
12,10	0.00%	0.09%	0.07%	**14,12**	0.00%	0.01%	0.01%
12,11	0.00%	0.05%	0.04%	**14,13**	0.00%	0.00%	0.01%
12,12	0.89%	0.03%	0.03%	**14,14**	0.00%	0.00%	0.01%
12,13	0.00%	0.02%	0.02%	**14,15**	0.00%	0.00%	0.01%
12,14	0.00%	0.01%	0.02%	**15,3**	0.00%	0.06%	0.06%
12,15	0.00%	0.01%	0.02%	**15,4**	0.00%	0.11%	0.11%
13,3	0.00%	0.09%	0.07%	**15,5**	0.44%	0.21%	0.28%
13,4	0.00%	0.18%	0.13%	**15,6**	0.00%	0.11%	0.11%
13,5	0.00%	0.36%	0.24%	**15,7**	0.00%	0.06%	0.06%
13,6	0.00%	0.71%	0.64%	**15,8**	0.00%	0.03%	0.03%
13,7	0.00%	0.36%	0.24%	**15,9**	0.00%	0.02%	0.02%
13,8	0.00%	0.18%	0.13%	**15,10**	0.00%	0.01%	0.01%
13,9	0.00%	0.09%	0.07%	**15,11**	0.00%	0.00%	0.01%
13,10	0.00%	0.05%	0.04%	**15,12**	0.00%	0.00%	0.01%
13,11	0.00%	0.03%	0.02%	**15,13**	0.00%	0.00%	0.00%
13,12	0.00%	0.01%	0.02%	**15,14**	0.00%	0.00%	0.00%
13,13	0.00%	0.01%	0.01%	**15,15**	0.00%	0.00%	0.00%
13,14	0.00%	0.01%	0.01%	**Total**	44.00%	48.70%	48.74%

E Different Specifications for $L0$ Behavior

This appendix studies the impact of different specifications for $L0$ behavior. In our benchmark model a $L0$ player chooses a random quantity and plays in the first period with 50% probability. We start by analyzing the implications of different probabilities of first period play for the $L0$ player. After that we analyze the implications of different probabilities of playing quantity 8 in the first period.

Tables 26, 27, and 28, respectively, display the maximum likelihood estimates, the aggregate results, and the predicted market outcomes for 50%, 75% and 99% probability that a $L0$ player plays in the first period. As we can see from the tables, different levels of first period play by the $L0$ players bear a small impact on the estimates for tau and epsilon, the maximum likelihood values, aggregate results and predicted market outcomes. This happens because (i) the percentage of $L0$ players is quite small for the estimated taus and (ii) the maximum likelihood taus are obtained in intervals for which the players' behavior is exactly the same–all players move in the second period (and play quantity 8 if no move is observed) except for the $L2$s and the $L4$s who move in the first with quantity 10. Therefore, the only impact on results is the slightly higher levels of τ which are insufficient to generate appreciable differences.

Table 26: ML estimates for different probabilites of first period play of a L0 player

x	τ	ε	ML
50%	2.86	0.65	−2257.49
75%	2.97	0.65	−2254.03
99%	3.06	0.65	−2254.19

Tables 29, 30, and 31, respectively, display the estimates, the aggregate results, and the predicted market outcomes for 50%, 75%, and 99% probability that a $L0$ player who moves in the first period chooses quantity 8.

Table 27: Aggregate results for different probabilites of first period play of a L0 player

	Both players in period 1	Explicit followers	Both players in period 2
Entire set			
$x = 50\%$ (benchmark model)			
Average quantity	9.89	7.25	8.10
% of observations	42.2%	24.4%	33.3%
$x = 75\%$			
Average quantity	9.87	7.29	8.11
% of observations	43.1%	24.5%	32.3%
$x = 99\%$			
Average quantity	9.85	7.32	8.12
% of observations	43.7%	24.6%	31.6%

Table 28: Market outcomes for different probabilites of first period play of a L0 player

Market outcomes	HMN	$x = 50\%$	$x = 75\%$	$x = 99\%$
Cournot:				
1st period	4.5	1.0	1.1	1.1
Sequential	14.8	5.9	6.0	6.1
2nd period	4.5	8.2	8.5	8.8
Stackelberg:				
Leader 12, follower 6	0.9	1.4	1.4	1.4
Leader 11, follower 7	2.0	2.6	2.7	2.7
Leader 10, follower 7	4.5	5.4	5.5	5.6
First mover punished or rewarded:				
Stackelberg leader punished	11.9	9.3	9.1	9.0
Stackelberg leader rewarded	0.2	3.9	3.9	3.9
Cournot punished	0.9	1.8	1.7	1.7
Cournot rewarded	0.0	2.9	2.9	2.9
Stackelberg and Cournot in 1st period	12.6	5.0	5.1	5.1
Double Stackelberg leadership	6.3	5.9	6.0	6.0
Collusion:				
Collusion successful	6.1	2.7	2.9	3.0
Collusion failed	10.6	2.0	2.1	2.2
Collusion exploited	4.3	2.3	2.4	2.5
Other	16.0	39.7	38.7	37.9
$\sqrt{\sum(o_i - \hat{o}_i)^2}$		29.1	28.2	27.5

Table 29 shows that an increase in y raises the estimates for τ (particularly for $y =$ 75%) and the estimates for ε. The maximum likelihood value increases with y. Thus, a specification of $L0$ that places the highest probability on playing quantity 8 in the first period offers the best explanation for observed behavior.

Table 29: ML estimates for different probabilites of a L0 playing 8 in the first period

y	τ	ε	ML
50%	1.74	0.59	-2176.01
75%	2.34	0.62	-2157.14
99%	2.13	0.62	-2152.69

Table 30 shows that when both players move in the 1st period, an increase in y raises the predicted average quantity, moving away from the data (9.15), and raises the percentage of simultaneous play in the 1st period, moving closer to the data (61%). When both players move in the 2nd period, an increase in y reduces predicted average quantity, moving away from the data (8.40), and reduces the percentage of simultaneous play in the 2nd period, moving closer to the data (16%).

Table 31 shows that an increase in y improves market outcomes predictions since it increases predicted play of market outcomes "Cournot 1st period," "Cournot sequential," and "Stackelberg and Cournot in 1st period," and reduces predicted play of market outcomes "Cournot 2nd period," and "others."

Table 30: Aggregate results for different probabilites of a L0 playing 8 in the first period

	Both players in period 1	Explicit followers	Both players in period 2
Entire set			
$y = 50\%$ (alternative model)			
Average quantity	9.45	7.49	8.12
% of observations	42.1%	24.4%	33.6%
$y = 75\%$			
Average quantity	9.63	7.31	8.07
% of observations	46.8%	25.0%	28.3%
$y = 99\%$			
Average quantity	9.48	7.34	8.05
% of observations	49.0%	25.0%	26.0%

Table 31: Market outcomes for different probabilities of a L0 playing 8 in the first period

Market outcomes	HMN	$y = 50\%$	$y = 75\%$	$y = 99\%$
Cournot:				
1st period	4.5	2.5	2.7	4.2
Sequential	14.8	9.8	9.3	11.3
2nd period	4.5	9.4	7.9	7.5
Stackelberg:				
Leader 12, follower 6	0.9	1.1	1.2	1.1
Leader 11, follower 7	2.0	2.1	2.4	2.3
Leader 10, follower 7	4.5	5.4	5.8	5.5
First mover punished or rewarded:				
Stackelberg leader punished	11.9	6.9	8.0	7.3
Stackelberg leader rewarded	0.2	2.9	3.5	3.3
Cournot punished	0.9	2.6	2.5	2.9
Cournot rewarded	0.0	4.1	4.1	5.0
Stackelberg and Cournot in 1st period	12.6	6.2	7.4	9.4
Double Stackelberg leadership	6.3	3.8	5.8	5.3
Collusion:				
Collusion successful	6.1	2.2	2.1	2.0
Collusion failed	10.6	1.6	2.0	1.9
Collusion exploited	4.3	1.9	1.9	1.6
Other	16.0	37.6	32.8	29.4
$\sqrt{\sum(o_i - \hat{o}_i)^2}$		26.9	22.3	19.3