SAFT: Split-Algorithm for Fast T2 Mapping

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INTRODUCTION:

Quantitative Magnetic Resonance Imaging (qMRI) provides a measure of physical tissue properties that are ideally independent from scanner hardware and the employed sequence. This allows a better inter- and intra-patient comparison, thus bearing the potential to be a good biomarker for pathology. However, long acquisition times are usually required for qMRI in comparison to conventional MRI, a disadvantage which is an obstacle for its use in clinical research and routine. Several iterative reconstruction methods have been developed to accelerate qMRI sequences; these algorithms usually go along with long computation times and a reduced robustness compared to a direct Fourier transform. Here, we propose to split the optimization problem of a model-based reconstruction of T2 maps into smaller sub-problems, with the purpose of increasing its robustness as well as decreasing its computational cost. This can be generalized to other, similarly posed problems.

THEORY:

It is common practice for model-based methods to define a cost-function that incorporates the model behavior directly within a data fidelity term[1-3]. This is also done in the "Model-Based Accelerated Relaxometry by Iterative Nonlinear Inversion" (MARTINI)[2], a T2 mapping algorithm using a multi-echo spin-echo (MESE) sequence. MARTINI's cost-function is defined as follows:

$$\phi(M_0, T2) = \frac{1}{2}\sum_{c=1}^{N}\sum_{t\in TE}\left\| P\ F\left\{S_c\ M_0 \exp\left(-\frac{t}{T2}\right)\right\} - Y_{t,c}\right\|_2^2 \quad ,$$

with TE being the echo times, N the number of coil elements, P a binary mask representing the sampling pattern, F the Fourier transform operator, S the coil sensitivities, M_0 the equilibrium magnetization, T2 the transverse relaxation and Y the acquired k-space data. Minimizing this cost-function will result in an estimation of T2 and M_0. However, the minimization of this nonlinear problem is numerically challenging and may lead to image artifacts and long reconstruction times. We suggest splitting up the problem similarly to what was proposed for compressed sensing[4], resulting in a 2-step algorithm which we term "Split-Algorithm for Fast T2 mapping" (SAFT).

Step 1: The MESE magnetization is calculated based on an initial guess of T2 and M_0 using the forward signal model:

$$\widehat{M}_t = M_0 \exp\left(-\frac{t}{T2}\right)$$

Using this first guess of the magnetization, the following problem is solved with a linear least-squares algorithm:

$$\phi_1(M) = \frac{1}{2}\sum_{c=1}^{N}\sum_{t\in TE}\left\| P\ F\{S_c M_t\} - Y_{t,c}\right\|_2^2 + \sum_{t\in TE}\alpha\left\|M_t - \widehat{M}_t\right\|_2^2 \quad ,$$

estimating the magnetization M that best fits the acquired data. The second l2-norm of the cost function forces the magnetization to be similar to the previously calculated $\widehat{M}$ with the similarity weighted by α.

Step 2: The MESE signal model is fitted onto the previously estimated M by solving the following problem with a nonlinear least-squares algorithm,

$$\Phi_2(T2,\ M_0) = \frac{1}{2}\sum_{t\in TE} \left\| M_t - \ M_0 \exp\left(-\frac{t}{T2}\right)\right\|_2^2 \ ,$$

yielding a new estimate of T2 and M_0. Subsequently, steps 1 and 2 are iteratively repeated until the algorithm converges to a minimum, providing an approximation of T2 and M_0. Optionally, a spatial regularization can be added to Φ_2. Here, we performed an additional reconstruction using a wavelet sparsity constraint for both T2 and M_0.

MATERIALS & METHODS:

After obtaining written consent, three whole-brain MESE datasets (TA 3:28min, acq. matrix 260x512, resolution 0.75x0.45x3mm³, slice gap 0.3mm, TR/ΔTE 4000/10.9ms, Number of echoes/slices/concatenations 16/43/2) of healthy volunteers were acquired at 3T (MAGNETOM Skyra, Siemens Healthcare, Germany) using commercially available 20- and 32-channel head/neck coils. The used prototype MESE sequence was 10x undersampled according to a GRAPPATINI sampling pattern[5]. The datasets were reconstructed using MARTINI[2] and the proposed algorithm, with and without a spatial regularization (SAFT vs. regularized SAFT).

RESULTS & DISCUSSION:

To compare the convergence of MARTINI and SAFT, the cost value of each iteration according to $\Phi(T2,M_0)$ is plotted in Figure 1. For a fair comparison, all algorithms were initialized with the same guess for T2 and M_0. The plot demonstrates that both SAFT reconstructions converge smoothly and reach a minimum after ~25 iterations. MARTINI's cost values jump initially until the algorithm starts converging at ~15 iterations and reaches a minimum similar to SAFT after ~60 iterations. Example T2 maps, reconstructed using MARTINI, non-regularized and regularized SAFT, are illustrated in Figure 2. It can be seen that the non-regularized SAFT reconstruction resembles the results of MARTINI. The wavelet-regularized SAFT reconstruction yields similar results but with less noise in the parameter maps. It should be noted that an even faster computation performance can be achieved solving Φ_2 with a log-linear regression, because then the algorithm comprises only two linear problems. However, we used a nonlinear approach to avoid noise-induced T2 overestimation[6].

CONCLUSION:

We suggest a method that splits the nonlinear problem of model-based reconstruction into smaller sub-problems which are solved alternately. The splitting results in a convex convergence, improving the robustness of the inverse reconstruction problem. Furthermore, we demonstrate that the improved robustness can be used to add non-convex regularization (e.g. sparsity constraints) to the optimization in order to further improve the estimated quantitative maps.

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Synopsis:
Numerous iterative reconstruction techniques have been published in the past, facilitating the calculation of quantitative parameter maps based on undersampled k-space data. Model-based approaches, for example, iteratively minimize a cost function that comprises a formulation of the signal behavior. Minimizing this non-linear problem yields the quantitative parameter maps, but is numerically challenging and thus accompanied with reduced robustness and long reconstruction times compared to a direct Fourier transform. Here we suggest a method to split the optimization problem of a model-based T2 mapping into sub-problems which are solved alternately. The splitting results in a more robust reconstruction with less computational cost.

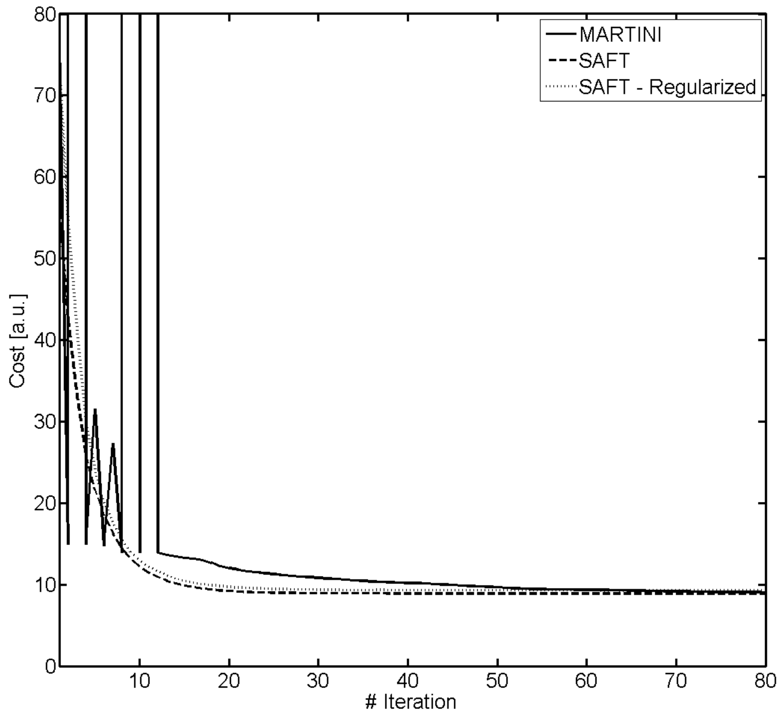

Figure 1: *The cost value within each iteration for SAFT (with and without regularization) and MARTINI.*

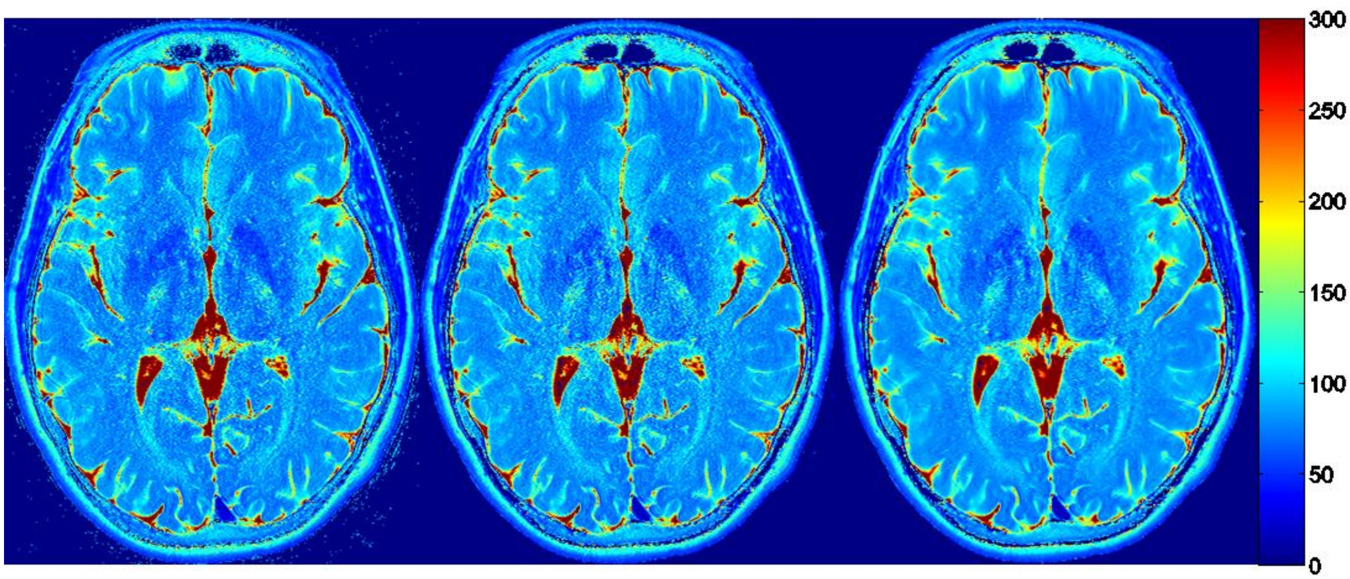

Figure 2: *The reconstructed T2 maps [ms] using MARTINI (left) and non-regularized (middle) and regularized (right) SAFT.*