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# Intertemporal Choice with Different Short-Term and Long-Term Discount Factors

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**Abstract**: This paper proposes a new axiomatic model of intertemporal choice that allows for dynamic inconsistency. We weaken the classical assumption of stationarity into two related axioms: stationarity in the short-term and stationarity in the long-term. We obtain a model with two independent discount factors, which is flexible enough to capture different time preferences, including a greater impatience for more immediate outcomes (when a long-term discount factor exceeds a compounded short-term discount factor). Our proposed model can accommodate some experimental results that cannot be rationalized by other existing models of dynamic inconsistency (such as quasi-hyperbolic discounting and generalized hyperbolic discounting).

**Keywords**: Discounted Utility; Time Preference; Dynamic Consistency; Stationarity; Discount Factor; Intertemporal Consumption

**JEL Classification Codes**: D00; D01; D90

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**Intertemporal Choice with Different Short-Term and Long-Term Discount Factors**

Discounted utility (Samuelson, 1937) is the most popular model of intertemporal choice. The main behavioral assumption of constant (exponential) discounting is stationarity (Koopmans, 1960, postulate 4, p. 294). Yet, empirical evidence suggests that decision makers may violate stationarity (*e.g.*, Loewenstein and Prelec, 1992, section II, pp. 574-578). Several generalizations of discounted utility were proposed in the literature including quasi-hyperbolic discounting (Phelps and Pollak, 1968), generalized hyperbolic discounting (Loewenstein and Prelec, 1992), the similarity theory (Rubinstein, 2003), subadditive discounting (Scholten and Read, 2010) and liminal discounting (Pan et al., 2013).

This paper proposes a new model of intertemporal choice that allows for dynamic inconsistency. Our approach is to weaken the classical assumption of stationarity into two related behavioral assumptions: stationarity in the short-term and stationarity in the long-term. We obtain a model with two different discount factors. We can think of one of them as a long-term discount factor and the other—as a short-term discount factor.

Intuitively, our proposed model works as follows. Suppose that time periods are measured in days and seven days (*i.e.*, one week) constitute one short term period. Within each week, a decision maker discounts daily utilities using one (short-term) discount factor. This generates weekly utilities. A decision maker then discounts these weekly utilities using another (long-term) discount factor.

A model with two different discount factors is useful in several applied problems. For example, consider a customer with a line of credit for a certain time period (*e.g.*, one month). The customer must pay a high interest rate if a payment is deferred for a longer period. In this case, it is rather natural to assume that the customer distinguishes between the short term (within one credit period) and the long term. The customer may discount little (or not at all) the time periods falling within one credit period. At the same time, the customer may use a lower discount factor in the long term (for time periods extending beyond the length of the credit period).

As another example, consider a taxpayer whose income/revenue is accounted for within a certain fiscal period (*e.g.*, one year). The taxpayer may differentiate between time periods falling within one fiscal period (the short term) and falling into different fiscal periods (the long term). Under progressive taxation, the taxpayer may use a lower discount factor in the short term since an increased income/revenue within the current fiscal period increases the tax burden. The taxpayer may use a higher discount factor in the long term since income/revenue received in different time periods is taxed with a lower tax rate.

In a related example, consider an organization operating on an annual budget. Again, such a decision maker may use one discount factor in the short term (within one year) when its approved budget funds are known with certainty. The same organization, however, may use a different discount factor in the long term (several years) since the availability of budget funds is uncertain/unknown across different budget periods.

Finally, a model with two discount factors is a useful framework in situations where intertemporal impatience is compounded with a certain survival probability. For example, a decision maker may use one discount factor in the short term that reflects his or her intertemporal impatience. The same decision maker may use another discount factor in the long term that reflects his or her perceived survival probability. For individuals, this can be the perception of own mortality rate. For firms, this can be the estimated probability of remaining on the market.

In our proposed model, two discount factors are independent of each other. This creates a flexible framework for capturing different types of time preferences. Specifically, our proposed model can accommodate dynamically consistent preferences (when a long-term discount factor coincides with a compounded short-term discount factor), a greater impatience for immediate outcomes (when a long-term discount factor is greater than a compounded short-term discount factor) and a greater patience, possibly even no discounting at all, within a short term period (when a long-term discount factor is smaller than a compounded short-term discount factor).

The remainder of the paper is organized as follows. Section 1 introduces the mathematical notation and our proposed model. Bleichrodt et al. (2008) recently provided a behavioral characterization (axiomatization) of the discounted utility model of Samuelson (1937). We adopt the framework of Bleichrodt et al. (2008) for characterizing the behavioral properties of our proposed model in section 2. Section 3 compares our proposed model with other generalizations of discounted utility such as quasi-hyperbolic discounting (Phelps and Pollak, 1968), generalized hyperbolic discounting (Loewenstein and Prelec, 1992), the similarity theory (Rubinstein, 2003) and liminal discounting (Pan et al., 2013). Section 4 applies our proposed model to several behavioral regularities in intertemporal choice (experiment I reported in Rubinstein (2003) and the common difference effect of Loewenstein and Prelec (1992)). Section 5 concludes.

## 1. Notation and the model

There is a connected and separable set $X$. The elements of $X$ are called *outcomes*. An outcome can be a monetary payoff, a consumption bundle, a financial portfolio, a health state etc. A *program* is an infinite sequence of outcomes $\{x_t\}_{t=1}^{\infty}$, where $x_t \in X$ is an outcome received in time period $t \in \mathbb{N}$. The set of all programs is denoted by $\mathbb{P}$.

For a compact notation let $y_T p \in \mathbb{P}$ denote a program that yields outcomes $\{y_1, y_2, \ldots, y_T\} \in X^T$ in the first $T$ periods, for some $T \in \mathbb{N}$ and the same outcome as program $p \in \mathbb{P}$ in all subsequent periods $t > T$. An *ultimately constant* program $y_T c \in \mathbb{P}$ yields outcomes $\{y_1, y_2, \ldots, y_T\} \in X^T$ in the first $T$ periods and the same outcome $x_t = c$ for all $t > T$, $c \in X$. A *constant* program that yields the same outcome $c \in X$ in all periods is denoted by $\boldsymbol{c} \in \mathbb{P}$.

A decision maker has a *preference relation* $\succcurlyeq$ on $\mathbb{P}$. As usual, the symmetric part of $\succcurlyeq$ is denoted by $\sim$ and the asymmetric part of $\succcurlyeq$ is denoted by $\succ$. The preference relation $\succcurlyeq$ is represented by a function $U$:$\mathbb{P} \to \mathbb{R}$ if $p \succcurlyeq q$ implies $U(p) \geq U(q)$ and vice versa for all $p, q \in \mathbb{P}$. We consider utility function (1).

(1) $$U\left(\{x_t\}_{t=1}^{\infty}\right) = \sum_{t=1}^{\infty} \delta^{t-1}\left[\sum_{\tau=1}^{T} \beta^{\tau-1} u\left(x_{(t-1)\cdot T+\tau}\right)\right]$$

In formula (1), a standard utility function $u: X \to \mathbb{R}$ is continuous, bounded, non-constant on $X$ and determined up to an increasing linear transformation. Discount factors $\delta \in (0,1)$ and $\beta > 0$ are unique and $T \in \mathbb{N}$ denotes the number of time periods in the short term.

If $T=1$ then utility function (1) becomes a conventional discounted utility with one constant discount factor: $U\left(\{x_t\}_{t=1}^{\infty}\right) = \sum_{t=1}^{\infty} \delta^{t-1} u(x_t)$. Thus, we can interpret discount factor $\delta \in (0,1)$ as a standard (long-term) discount factor. According to formula (1), utility of outcomes in the first $T \in \mathbb{N}$ periods is $u(x_1) + \beta \cdot u(x_2) + \ldots + \beta^{T-1} \cdot u(x_T)$. Thus, we can interpret discount factor $\beta > 0$ as a short-term discount factor.

If $\delta = \beta^T$ then the short-term discount factor is consistent with the long-term discount factor and a decision maker behaves as if maximizing a standard discounted utility (with one constant discount factor). If $\delta > \beta^T$ then a decision maker exhibits greater impatience for the immediate outcomes in the short term (*i.e.*, he or she is more patient in the long term). Finally, if $\delta < \beta^T$ then a decision maker is more patient in the short term. In fact, model (1) allows for the possibility that a decision maker does not discount outcomes within the first $T$ periods ($\beta = 1$).

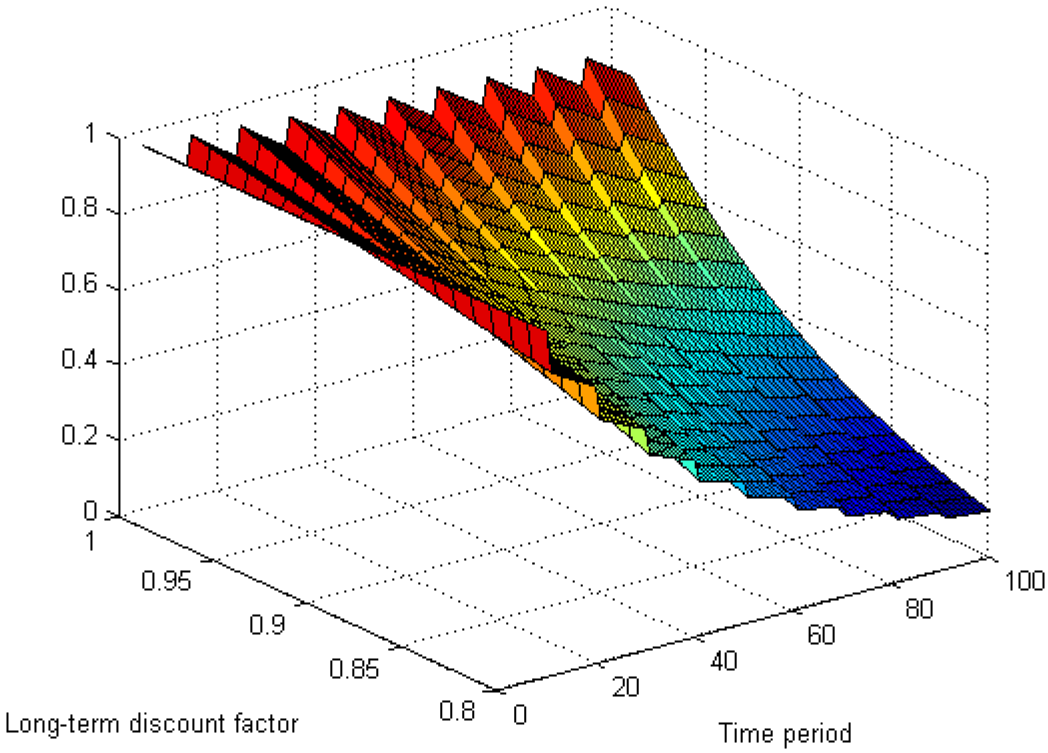


**Figure 1 One unit of utility received in period $t\in\{1, 2, ...,100\}$ evaluated according to (1) for different values of the long-term discount factor $\delta\in[0.8,0.99]$ and a fixed short-term discount factor $\beta$=0.99 as well as a fixed length of the short term period $T$=10.**

Figure 1 illustrates model (1). Figure 1 plots the present value of one unit of utility received in period $t\in\{1, 2, ...,100\}$ when it is evaluated by formula (1). We fixed the short-term discount factor $\beta$=0.99 and the length of the short term period $T$=10 but allowed for a varying long-term discount factor in the range [0.8,0.99]. When the long-term discount factor is $\delta^*=0.99^{10}\cong0.904$ then the future units of utility are evaluated by standard discounted utility with a constant discount factor $\beta$=0.99. When the long-term discount factor is greater than $\delta^*$ a decision maker exhibits greater impatience in the short term (at any point in time the slope of the surface on figure 1 is steeper than the slope of its asymptotic trend). When the long-term discount factor is smaller than $\delta^*$ a decision maker exhibits greater impatience in the long term (the slope of the asymptotic trend of the surface on figure 1 is steeper than the slope of the surface at any point in time).

## 2. Behavioral Characterization

Our behavioral characterization of model (1) is similar to the axiomatization of discounted utility by Bleichrodt et al. (2008).

**Axiom 1** (*Completeness*) For all $p, q \in \mathbb{P}$ either $p \succcurlyeq q$ or $q \succcurlyeq p$ (or both).

**Axiom 2** (*Transitivity*) For all $p, q, r \in \mathbb{P}$ if $p \succcurlyeq q$ and $q \succcurlyeq r$ then $p \succcurlyeq r$.

**Axiom 3** (*First Period Sensitivity*) There exist $p \in \mathbb{P}$ and $x, y \in X$ such that $x_1 p \succ y_1 p$.

**Axiom 4** (*Independence*) For all $p, q \in \mathbb{P}$, $t \in \mathbb{N}$ and $x, y \in X^t$ we have $x_t p \succcurlyeq y_t p$ if and only if $x_t q \succcurlyeq y_t q$.

**Axiom 5** (*Long-Term Stationarity*) There exists $T \in \mathbb{N}$ such that $y_T p \succcurlyeq y_T q$ if and only if $p \succcurlyeq q$ for all $p, q \in \mathbb{P}$ and $y \in X^T$.

**Axiom 6** (*Ultimate Continuity*) For any ultimately constant program $y_T c \in \mathbb{P}$ the sets $\{(z_1, \ldots, z_T) \in X^T : y_T c \succcurlyeq z_T c\}$ and $\{(z_1, \ldots, z_T) \in X^T : z_T c \succcurlyeq y_T c\}$ are closed with respect to the product topology on $X^T$.

**Axiom 7** (*Constant-Equivalence*) For all $p \in \mathbb{P}$ there exists $c \in X$ such that $p \sim \boldsymbol{c}$.

**Axiom 8** (*Tail-Robustness*) For all $p \in \mathbb{P}$ and $c \in X$ such that $p \succ \boldsymbol{c}$ ($\boldsymbol{c} \succ p$) there exist $t \in \mathbb{N}$ such that $p_T c \succ \boldsymbol{c}$ ($\boldsymbol{c} \succ p_T c$) for all $T \geq t$.

Axioms 1-4 are standard. Axioms 1 and 2 are necessary for any real-valued representation. Axiom 3 rules out a degenerate case when a decision maker does not care about the present outcome. Axiom 4 is required for intertemporal separability of utility. Axiom 4 may be replaced with an axiom known as the Thomsen-Blaschke condition (Debreu, 1960, p.18, Assumption 1.3) or double cancelation (Kranz et al. 1971, section 6.2.1., p. 250, Definition 3).

Next, we weaken the classical axiom of stationarity to axiom 5, which we call long-term stationarity. Axiom 5 states that whenever two programs yield the same outcomes in the first $T$ periods, $T \in \mathbb{N}$, then a decision maker reveals the same preference between these programs in periods one and $T+1$. In other words, a common consumption vector can be dropped altogether without affecting the preferences of a decision maker. By iteration, this implies that whenever two programs yield the same outcomes in the first $nT$ periods, $n \in \mathbb{N}$, then these outcomes do not affect the preference relation (and they can be disregarded). Axiom 5 becomes the standard stationarity condition when $T=1$.

Axioms 6-8 are standard. Axiom 6 is required to derive utility representation for ultimately constant programs. Alternatively, axiom 6 may be replaced with solvability and Archimedean axioms. [1] Axioms 7 and 8 are required for extending any utility representation from ultimately constant programs to all programs.

**Proposition 1** The preference relation $\succcurlyeq$ satisfies axioms 1-8 if and only if it admits representation (2), where utility function $v:X^T\rightarrow\mathbb{R}$ is continuous, bounded and determined up to an increasing linear transformation, discount factor $\delta\in(0,1)$ is unique and $T\in\mathbb{N}$.

(2) $$U\left(\{x_t\}_{t=1}^{\infty}\right)=\sum_{t=1}^{\infty}\delta^{t-1}v\left(x_{(t-1)\cdot T+1},x_{(t-1)\cdot T+2},\dots,x_{t\cdot T}\right)$$

The proof is analogous to the proof of theorem 2 in Bleichrodt et al. (2008, p.343).

Representation (2) becomes standard discounted utility (with a constant discount factor) when $T=1$. When $T>1$ we need an additional behavioral assumption to characterize function $v:X^T\rightarrow\mathbb{R}$.

Let $\{xy\}_Tp$ denote a program that yields an outcome $x\in X$ in the first period, and outcomes $y\equiv\{y_1, y_2, \dots, y_{T-1}\}\in X^{T-1}$ in periods 2, 3,… $T$ for some $T\in\mathbb{N}$, and the same outcome as program $p\in\mathbb{P}$ in all subsequent periods $t>T$. Similarly, let $\{yx\}_Tp$ denote a program that yields outcomes $\{y_1,y_2, \dots, y_{T-1}\}\in X^{T-1}$ in periods 1,2,… $T$-1 and an outcome $x\in X$ in period $T$, and the same outcome as program $p\in\mathbb{P}$ in all subsequent periods $t>T$. With this notation we can state our last behavioral assumption.

**Axiom 9** (*Short-Term Stationarity*) For all $p\in\mathbb{P}$, $y,z\in X^{T-1}$ and $x\in X$ we have $\{xy\}_Tp\succcurlyeq\{xz\}_Tp$ if and only if $\{yx\}_Tp\succcurlyeq\{zx\}_Tp$.

Axiom 9 can be interpreted as a classical stationarity condition for a limited time period $T\in\mathbb{N}$.[2] Therefore, we refer to axiom 9 as a short-term stationarity. According to axiom 9, if two programs yield the same outcome in the first period then the decision maker's preference between them is not affected by dropping this common outcome and advancing the subsequent $T$-1 outcomes by one period. Axiom 9 holds trivially if $T=1$. Note that period $T$ in axiom 9 is the same as period $T$ in axiom 5.

**Proposition 2** The preference relation $\succcurlyeq$ satisfies axioms 1-9 if and only if it admits representation (1).

The proof is presented in the appendix.

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[1] See Blavatskyy (2013, section 3) for details in the context of choice under uncertainty.

[2] Conversely, a classical stationarity is the limiting case of axiom 9 when $T\rightarrow\infty$.

## 3. Comparison with other models

Quasi-hyperbolic discounting (Phelps and Pollak, 1968) models dynamic inconsistency by introducing the present bias. Specifically, the classical stationarity condition is assumed to hold starting from the second (rather than the first) period. Thus, quasi-hyperbolic discounting coincides with standard discounted utility from the second period onwards. In terms of our proposed model, we can think of quasi-hyperbolic discounting as a model with an initial short-term period of length $T=2$ and a degenerate short-term period of length $T=1$ afterwards.

Loewenstein and Prelec (1992, p. 580, equation 15) proposed to discount the future units of utility using the functional form of a generalized hyperbola. This functional form departs from constant (exponential) discounting so that the periods in the immediate future are relatively more heavily discounted (cf. figure I in Loewenstein and Prelec, 1992, p. 581). Thus, the qualitative difference between model (1) and the model of Loewenstein and Prelec (1992) is the following. Model (1) allows for a long-term discount factor to differ from the short-term discount factor (not only in the near future but also in the distant future periods). The model of Loewenstein and Prelec (1992) can be interpreted as a model with an increasing long-term discount factor.

The similarity theory (Rubinstein, 2003, p.1210) postulates that a decision maker first applies the monotonicity axiom—a decision maker chooses the program that yields better outcomes in all periods. Otherwise, when there is an intertemporal tradeoff, a decision maker attempts to simplify programs—if one program is similar to a program that monotonically dominates the other then it is chosen. Finally, if none of the programs even seemingly dominates the other, "the choice is made using a different criterion". Somewhat unexpectedly, model (1) can generate alike behavior as the similarity theory. Model (1) satisfies the monotonicity axiom. When the short-term discount factor is in the small neighborhood of one, model (1) effectively aggregates outcomes in the short term without much discounting. In this case, a decision maker behaves as if an outcome received in time $t_1$ is "similar" to the same outcome received in time $t_2$ when $t_1$ and $t_2$ belong to the same short term period; but the decision maker would consider these outcomes "dissimilar" when $t_1$ and $t_2$ belong to different short term periods.

Similar to the model presented in this paper, the model of liminal discounting (Pan et al., 2013) also employs two discount factors. A liminal discounter uses one discount factor for outcomes received up to a certain time period in the future and the other discount factor—for outcomes received in all subsequent time periods. In contrast, model (1) employs one discount factor for outcomes received within one short-term period and the other discount factor—for outcomes received in different short-term periods.

## 4. Examples

**Example 1** (Experiment I reported in Rubinstein, 2003, section 3.1, p.1211)

In the first question, the majority of subjects prefer receiving \$607.07 on 17.06.2005 instead of receiving \$467 on 17.06.2004. In the second question, the majority of subjects prefer receiving \$467 on 16.06.2005 instead of receiving \$467.39 on 17.06.2005. This majority choice is inconsistent with classical discounted utility (with a constant discount factor), quasi-hyperbolic discounting and generalized hyperbolic discounting (Rubinstein, 2003, p. 1212).

Let us now analyze this example with model (1). Let one time period to be one day. If 16.06.2005 and 17.06.2005 happen to be in the same short term period, then the majority choice in the second question implies inequality (3).

$$u(\$467) > \beta u(\$467.39) \tag{3}$$

If 16.06.2005 and 17.06.2005 happen to be in different short term periods, then the majority choice in the second question implies inequality (4).

$$\beta^{T-1} u(\$467) > \delta u(\$467.39) \tag{4}$$

Let $N \in \mathbb{N}$ be the highest integer number not exceeding $365/T$. The majority choice in the first question then implies inequality (5).

$$\delta^N \beta^{365-NT} u(\$607.07) > u(\$467) \tag{5}$$

If $T>1$ then model (1) can rationalize the majority choice in example 1. For instance, let $T=5$ (so that $N=73$) and let utility function be linear. Inequality (3) then implies an upper bound on the short-term discount factor $\beta<0.999166$. On the other hand, inequality (5) implies a lower bound on the long-term discount factor $\delta>0.996413$. Thus, if the short-term discount factor is sufficiently low and the long-term discount factor is sufficiently high, inequalities (3) and (5) can hold simultaneously.

Inequalities (4) and (5) can also hold simultaneously. We already established that (5) implies $\delta>0.996413$. Given this result, inequality (4) implies a lower bound on the short-term discount factor $\beta>0.999310$. Thus, (4) and (5) can hold at the same time.

**Example 2** The common difference effect (Loewenstein and Prelec, 1992, p. 574)

A decision maker chooses outcome $x$ today over outcome $y$ tomorrow but prefers to receive outcome $y$ in time period $t+1$ rather than outcome $x$ in time period $t$ for some $t \in \mathbb{N}$. If $T=1$ then model (1) becomes classical discounted utility with a constant discount factor and it cannot rationalize such a choice pattern. If $T>1$ then choosing outcome $x$ today over outcome $y$ tomorrow implies inequality (6).

(6) $$u(x) > \beta u(y)$$

If $t$ and $t+1$ happen to be in the same short term period then choosing outcome $y$ in time $t+1$ rather than outcome $x$ in time $t$ implies inequality (6) with a reversed sign. In this case, model (1) cannot rationalize the common difference effect. Yet, if $t$ and $t+1$ are in different short term periods, choosing outcome $y$ in time $t+1$ rather than outcome $x$ in time $t$ implies inequality (7).

(7) $$\delta u(y) > \beta^{T-1} u(x)$$

Inequalities (6) and (7) can hold simultaneously if $\delta > \beta^T$. Thus, model (1) can rationalize the common difference effect if $t$ and $t+1$ are in different short term periods and a decision maker exhibits greater impatience in the short term ($\delta > \beta^T$).

## 5. Conclusion

This paper presents a new model of intertemporal choice. The model allows a short-term discount factor to be different from the long-term discount factor. The model has a natural application in situations when intertemporal choice in the short term is qualitatively different from intertemporal choice in the long term (for example, due to an accounting practice, progressive taxation, a credit period etc.)

The main advantage of the new model is parsimony. Compared to the classical discounted utility, our proposed model has only two additional parameters—a short-term interest rate and the length of the short term period. Compared to the popular quasi-hyperbolic discounting, our proposed model has only one additional parameter—the length of the short term period.

Our proposed model is flexible in capturing different time preferences (*cf.* figure 1). In particular, it allows for greater impatience in the short-term as well as greater impatience in the long-term.

The model has two different discount factors. This may have several possible interpretations. For example, we can view a short-term discount factor $\beta$ as a standard coefficient of intertemporal impatience (converting the desirability of outcomes across time periods). At the same time, we can think of a long-term discount factor $\delta$ as a "survival probability" that a decision maker remains in a position to take decisions after $T$ time periods.

We provide a behavioral characterization of the new model. The model is derived by breaking the standard stationarity assumption into two related assumptions: stationarity in the long term and in the short term (axioms 5 and 9). Thus, arguably, the model has an intuitive appeal.

The model can accommodate some experimental results that cannot be rationalized by other existing models such as quasi-hyperbolic discounting and generalized hyperbolic discounting (example 1). Yet, there are some experimental results that contradict our model. For example, our proposed model, like any model with a time-invariant utility function, cannot accommodate the absolute magnitude effect (Loewenstein and Prelec, 1992, p. 575). Also, our model, like any model built on the consequentialist premise, cannot accommodate any framing effects such as the delay-speedup asymmetry (Loewenstein, 1988).

The model presented in this paper allows for two different discount factors. Such a model can be further extended to allow for three (or more) different discount factors (*e.g.*, in short term, medium term and long term). In fact, a model with three discount factors fits naturally into our system of time measurement (in days, months and years).

Our proposed model introduces a distinction between discounting in the short and long term. In contrast, the popular quasi-hyperbolic discounting model differentiates between discounting in the current and subsequent periods. We can approximate quasi-hyperbolic discounting within our proposed model by using a short term period of length $T=2$ and restricting a long-term discount factor to be greater than a short-term discount factor.

In case when the long-term discount factor is greater then a compounded short-term discount factor ($\delta > \beta^{T-1}$) our proposed model implies that a decision maker may prefer to receive the same outcome later (*e.g.* at time period $T+1$) rather than sooner (*e.g.* at time period $T$). Discounted utility and all its generalizations, except for the model of Blavatskyy (2015), can imply a similar preference for a delayed outcome. For illustration, let us consider the following example from Blavatskyy (2015). A decision maker who receives two million now obtains utility $u$(\$2m), where $u$(.) is a concave utility function. The same decision maker who receives one million now as well as one million dollars at a later moment of time $t$ obtains utility $u$(\$1m)+$D$($t$)$u$(\$1m), where $D$($t$) is a discount function such that $D$($t$) converges to 1 when time period $t$ is sufficiently close to the present (*cf.* Figure 1 in Loewenstein and Prelec, 1992, p. 581). Thus, when time period $t$ is sufficiently close to the present, a decision maker with a concave utility function prefers to receive one million at a later time period $t$ (utility $u$(\$1m)+$D$($t$)$u$(\$1m) converges to 2*$u$(\$1m), which is greater than $u$(\$2m) due to Jensen's inequality).

## Appendix

### Proof of proposition 2

It is relatively straightforward to show the necessity of axioms 1-9. We prove only their sufficiency. If axioms 1-8 hold then preferences admit representation (2) due to proposition 1. Moreover, if axioms 1, 2, 4 and 6 hold then utility function $v: X^T \to \mathbb{R}$ in (2) is separable so that we can write

(8) $$v(x_1, x_2, \ldots, x_T) = \sum_{\tau=1}^{T} u_\tau(x_\tau)$$

where utility functions $u_\tau: X \to \mathbb{R}$ are continuous and unique up to a positive affine transformation for all $\tau \in \{1, \ldots, T\}$.

If axiom 9 holds then any standard sequence of outcomes is invariant across time periods (see Kranz et al. (1971), section 6.11.2, p.305) so that utility functions $u_\tau: X \to \mathbb{R}$ are identical expect for a multiplication by a positive constant. If axiom 3 holds then the constant associated with utility function in the first period cannot be zero and we can divide all utility functions by this constant to obtain

(9) $$v(x_1, x_2, \ldots, x_T) = u(x_1) + \sum_{\tau=2}^{T} a_\tau \cdot u(x_\tau)$$

where utility function $u: X \to \mathbb{R}$ is continuous and unique up to a positive affine transformation and $a_\tau > 0$ for all $\tau \in \{1, \ldots, T\}$. Let us set $\beta = a_2$. If $T=2$ then we immediately obtain (1).

If $T \geq 3$ then for any $x, y \in X$, we can construct a program that yields $y$ in one period $\tau \in \{2, \ldots, T\}$ and $x$—in all other periods. We can also find an outcome $z \in X$ such that a decision maker is indifferent between this program and a program that yields $z$ in period $\tau+1$ and $x$—in all other periods. Thus, we have

(10) $$a_\tau \cdot [u(y) - u(x)] = a_{\tau+1} \cdot [u(z) - u(x)]$$

If axiom 9 holds, then a decision maker is also indifferent between a program that yields $y$ in period $\tau$-1 (and $x$—in all other periods) and a program that yields $z$ in period $\tau$ (and $x$—in all other periods). Thus, we also have

(11) $$a_{\tau-1} \cdot [u(y) - u(x)] = a_\tau \cdot [u(z) - u(x)]$$

Multiplying both sides of (21) with $a_\tau$ and using (20) we obtain $a_{\tau+1} = a_\tau^2 / a_{\tau-1}$. Solving by iteration we obtain $a_\tau = \beta^{\tau-1}$ for all $\tau \in \{2, \ldots, T\}$. Plugging this result into formula (19) and formula (19)—into (2) yields representation (1). *Q.E.D.*

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