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Highlights

- Behavior in the centipede game when players are not expected utility maximizers;
- Players choose under uncertainty in a probabilistic manner;
- A core deterministic decision theory is embedded in a model of probabilistic choice;
- We consider, inter alia, a constant error/trembles and quantal response equilibrium;
- Players adopt non-linear decision weights/overweight the likelihood of rare events.


# Behavior in the Centipede Game: a Decision-Theoretical Perspective 

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#### Abstract

The centipede game is a two-player finite game of perfect information where a unique subgame perfect Nash equilibrium appears to be intuitively unappealing and descriptively inadequate. This paper analyzes behavior in the centipede game when a traditional game-theoretical assumption that players maximize expected utility is relaxed. We demonstrate the existence of a descriptively adequate subgame perfect equilibrium under two standard decision-theoretical assumptions. First, players choose under uncertainty in a probabilistic manner as captured by embedding a core deterministic decision theory in a model of probabilistic choice. Second, players adopt non-linear decision weights and overweight the likelihood of rare events as captured, for example, by rank-dependent utility or prospect theory.


## JEL Classification Codes: C72, D81

Keywords: Game of perfect information; Subgame perfect Nash equilibrium; Centipede game; Probabilistic choice; Rank-dependent utility

## Behavior in the Centipede Game: a Decision-Theoretical Perspective

## 1. The Centipede Game

The centipede game is a famous finite game of perfect information. In this game two players move sequentially one after another. At the beginning of the game, player 1 can either terminate the game immediately (in which case both players receive a small payoff of 1) or pass the move to player 2. Player 2 can then either terminate the game (in which case player 1 receives nothing and player 2 receives a payoff of 2 ) or pass the move back to player 1 . Player 1 can then either terminate the game (in which case both players receive a payoff of 2 ) or pass the move back to player 2 . The game continues in this fashion for many rounds with payoffs gradually increasing. If the last round is reached, player 2 must decide between option $R$ where both players receive a payoff of 100 and option D where player 1 receives a payoff of 98 and player 2 receives a payoff of 101.

The centipede game is presented in the extensive form on Figure 1. The idea of this game can be traced back to Rosenthal (1981, Figure 3, p.96). In all Nash equilibria of the centipede game player 1 chooses $D$ in the first decision node. In a unique subgame perfect Nash equilibrium both players choose D in all decision nodes, which can be established by backward induction. This outcome appears counterintuitive-both players can get much higher utility in the later nodes if they do not terminate the game in the first node. Experimental evidence (e.g., McKelvey and Palfrey, 1992) suggests that most people do not stop at the first node but terminate the game at some intermediate node (the game is rarely played till the last node).

## 2. Introducing Models of Probabilistic Choice

Traditional game theory assumes that players maximize expected utility (e.g., von Neumann and Morgenstern 1944). In expected utility theory a decision maker chooses in a deterministic manner (except for a special case of indifference). Numerous laboratory experiments, however, establish that revealed choices under uncertainty are often probabilistic, e.g. Hey and Orme (1994), Ballinger and Wilcox (1997). Thus, a more descriptively adequate modeling approach is to embed a deterministic decision theory (expected utility or a generalized non-expected utility theory) in a model of probabilistic choice (cf. Loomes and Sugden, 1998; Blavatskyy and Pogrebna, 2010).

In traditional game theory players always pick the strategy that yields the highest expected utility. In this section we only assume that players are more likely to pick the strategy that yields the highest expected utility (but they do not necessarily always choose this strategy). With a model of probabilistic choice as a primitive we have a natural foundation for a mixed strategy equilibrium. In
this equilibrium, players randomize not to keep the opponent indifferent but because it is in their nature to select better choice options with a higher probability but not all the time.

Several models of probabilistic choice were proposed in the literature but not all of them are promising candidates for analysing the centipede game. Random utility or random preferences e.g. Falmagne $(1985)^{1}$, the models of Fishburn (1978) and Blavatskyy $(2007,2011)$ assume that choice under certainty is deterministic. The implications of these models are the same as in the subgame perfect Nash equilibrium described in Section 1. In the last node player 2 decides between option $D$ that yields utility of 101 with certainty and option $R$ that yields utility of 100 with certainty. If choice under certainty is deterministic, player 2 always chooses D in the last node. Knowing this, player 1 always chooses option $D$ in the before-last node and so forth until we arrive at the conclusion that player 1 choses option $D$ in the first node of the game.

One of the simplest models not assuming deterministic choice under certainty is a constant error or tremble model (e.g., Harless and Camerer, 1994). In this model, a decision maker chooses the option with a higher expected utility with probability 1- $\tau$ and the option with a lower expected utility - with probability $\tau \in(0,0.5)$. Thus, in the last node of the centipede game, player 2 chooses option D with probability 1- $\tau$ and option R -with probability $\tau$. Knowing this, in the before-last node of the game, player 1 chooses option $D$ with probability 1- $\tau$ and option $R$-with probability $\tau$. Going by backward induction we establish that in all nodes both players chose option $D$ with probability 1- $\tau$ and option $\mathrm{R}-$ with probability $\tau$. Thus, a constant error/tremble model predicts that the centipede game most likely ends in the first node but the play might also terminate in one of the intermediate nodes or even in the last node (though the chances are small).

A more sophisticated model of probabilistic choice is Fechner (1860) model of random errors. ${ }^{2}$ In this model, a decision maker chooses option $D$ over option $R$ with probability
(1) $\quad P(D, R)=\Phi_{0, \sigma}(U(D)-U(R))$
where $\Phi_{0, ब}: \mathbb{R} \rightarrow[0,1]$ is the cumulative distribution function of the normal distribution with zero mean and constant variance $\sigma>0$ and $U:\{R, D\} \rightarrow \mathbb{R}$ is the expected utility of the corresponding choice option. Thus, in the last node of the centipede game, player 2 chooses option D with probability $\Phi_{0, \sigma}(1)>0.5$ and option $R$ with probability $\Phi_{0, \sigma}(-1)<0.5$. Knowing this, in the before-last node, player 1 chooses option $D$ with probability $\Phi_{0, \sigma}\left(2 \cdot \Phi_{0, \sigma}(1)-1\right)>0.5$ and option R with

[^0]probability $\Phi_{0, \sigma}\left(1-2 \cdot \Phi_{0, \sigma}(1)\right)<0.5 \cdot{ }^{3}$ Proceeding by backward induction we establish that the probability of choosing $R$ increases for both players (moving from the last to the first node) but never exceeds 0.5 . Dashed lines on figures 2 and 3 plot the probability of choosing $R$ in a subgame perfect equilibrium derived from the Fechner model with $\sigma=1$ for player 1 and 2 respectively.

Luce (1959) choice model assumes that people first detect and delete dominated alternatives. Second, they chose in a probabilistic manner among the remaining non-dominated alternatives. The prediction of this model coincides with the subgame perfect Nash equilibrium described in Section 1. Yet, in most microeconomic applications, the first stage of Luce's choice model is typically ignored and the mathematical formula of the second stage is applied to all choice alternatives. In application to game theory Luce's choice model is known as logit quantal response equilibrium (McKelvey and Palfrey, 1995). A decision maker chooses option D over option $R$ with probability
(2) $\quad P(D, R)=\frac{e^{\lambda U(D)}}{e^{\lambda U(D)}+e^{\lambda U(R)}}$
where $\lambda>0$ is a noise parameter. Models (1) and (2) generate nearly identical choice patterns. In a logit quantal response equilibrium the probability of choosing $R$ increases (but never exceeds 0.5 ) for both players as we move from the last to the first node. Grey lines on figures 2 and 3 plot the probability of choosing $R$ in a logit quantal response equilibrium with $\lambda=1.6$ for player 1 and 2 respectively.

Wilcox $(2008,2011)$ recently proposed "contextual utility" model of probabilistic choice. When one choice option first-order stochastically dominates the other option, Wilcox (2011, p.94) assumes that a decision maker chooses as if making a constant error/tremble: the dominant option is chosen with probability $1-\omega / 2$ and the dominated option is chosen with probability $\omega / 2$, for small probability $\omega \in(0,0.5)$. In the centipede game, this happens only when player 2 decides in the last decision node. In all other nodes of the centipede game neither option stochastically dominates the other option. In this case, Wilcox (2011, p.96) assumes that a decision maker chooses option D over R with probability (3) $\quad P(D, R)-(1-\omega) \Phi_{0, C}\left(\frac{\tilde{U}(D)-\Gamma(R)}{\bar{u}-\underline{u}}\right)+\frac{\omega}{2}$
where $\bar{u}$ denotes the highest utility payoff that a player can receive in choice options D and R and $\underline{u}$ denotes the lowest utility payoff that a player can receive in choice options D and R.

In the last node of the centipede game, model (3) implies that player 2 choses option D with probability $1-\omega / 2>0.5$. Knowing this, in the before-last node, player 1 chooses option $D$ with probability $(1-\omega) \Phi_{0, \sigma}(0.5-\omega / 2)+\omega / 2>0.5$ and option R with probability $(1-\omega) \Phi_{0, \sigma}(\omega / 2-0.5)+\omega / 2<0.5$. Proceeding by backward induction we establish that the subgame perfect equilibrium derived from model (3) is qualitatively similar to logit quantal response equilibrium and the subgame perfect equilibrium derived from model (1). Dotted lines on figures 2 and 3 plot the probability of choosing $R$

[^1]in a subgame perfect equilibrium derived from Wilcox $(2008,2011)$ model with $\sigma=0.5$ and $\omega=0.01$ for player 1 and 2 respectively.

Blavatskyy (2012, p. 49) recently proposed to model the probability of choosing D over R as
(4) $\quad P(D, R)=\eta\left(\frac{U(D)-U(R)}{U(D \vee R)-U(D / R)}\right)$
where $D \vee R$ denotes the least upper bound on D and R in terms of the state-wise dominance; $D \wedge R$ denotes the greatest lower bound on D and R in terms of the state-wise dominance; and function $\eta:[-1,1] \rightarrow[0,1]$, which satisfies $\eta(v)+\eta(-v)=1$ for all $v \in[-1,1]$, can be interpreted as the cumulative distribution function of relative random errors. An act $D \vee R$ is simply a choice option R modified so that all utility payoffs lower than $U(D)$ are replaced with $U(D)$. An act $D \wedge R$ is simply a choice option R modified so that all utility payoffs greater than $U(D)$ are replaced with $U(D)$.

If $\eta(-1)=0$ and $\eta(1)=1$ then model (4) always respects the state-wise dominance ( $c f$. Blavatskyy 2012, p. 48, Axiom 4). In this case, the prediction of model (4) for the centipede game is the same as the subgame perfect Nash equilibrium described in Section 1 (player 1 terminates the game in the first node and both players end up with a payoff of one). Yet, if $\eta(-1)>0$ and, consequently, $\eta(1)<1$ then model (4) can generate rare violations of state-wise dominance (cf. Blavatskyy, 2014, p. 269, footnote 4) and its prediction differs from the subgame perfect Nash equilibrium. We consider model (4) when function $\eta($.$) is the cumulative distribution function of the normal distribution with zero$ mean and constant variance $\sigma>0$, i.e. $\eta(v)=\Phi_{0, \sigma}(v)$ for all $v \in[-1,1]$.

In the last node of the centipede game, the implications of model (4) coincide with those of model (1) and (2). In the before-last node, the implications of model (4) coincide with those of model (1). Proceeding by backward induction we establish that the subgame perfect equilibrium derived from model (4) is qualitatively similar to subgame perfect equilibria derived from models (1), (2) and (3). Long dashed lines on figures 2 and 3 plot the probability of choosing $R$ in a subgame perfect equilibrium derived from Blavatskyy (2012) model with $\sigma=1$ for player 1 and 2 respectively.

Subgame perfect equilibria derived from models (1)-(4) improve upon the subgame perfect Nash equilibrium. Probability of choosing $R$ is close to zero in the last decision nodes for both players. This probability increases in the intermediate nodes but, unfortunately, it never exceeds 0.5 . Such an observation resembles a conclusion from the decision theoretical literature. Expected utility theory embedded in models of probabilistic choice can rationalize certain types of the common ratio effect (Loomes, 2005, pp.303-305) and violations of the betweenness axiom (Blavatskyy, 2006) but not when a modal choice switches across pairs of decision problems. Similarly, expected utility theory embedded in standard models of probabilistic choice cannot generate a switch in the modal choice from $D$ to $R$ in the centipede game.

## 3. Introducing Non-Linear Decision Weights (Overweighting of Rare Events)

Subgame perfect equilibria derived from models (1)-(3) and QRE improve upon the subgame perfect Nash equilibrium. Probability of choosing R is close to zero in the last nodes for both players. This probability increases in the intermediate nodes but, unfortunately, it never exceeds 0.5 . When models of probabilistic choice (1)-(3) are combined with expected utility (as a core decision theory) they all have the following property: in a binary choice an option with a lower expected utility can never be chosen with a probability greater than $0.5 .{ }^{4}$ Thus, if option $R$ is chosen with probability close to zero (i.e., less than 0.5 ) in the last node, it is also chosen with probability less than 0.5 in the before-last node and so forth. Formally, consider any node such that in all subsequent nodes $R$ is chosen with probability less than 0.5 . In this node, players face a binary choice between $D$ that yields utility $u \in\{1,2, \ldots\}$ for certain and an uncertain option $R$ that yields expected utility not greater than

$$
\begin{equation*}
U(R)<\frac{u-1}{2}+\frac{u+1}{4}+\frac{u}{8}+\frac{u+2}{16}+\frac{u+1}{32}+\frac{u+3}{64}+\ldots=u-\frac{1}{2}+\sum_{n=1}^{\infty} \frac{n}{4^{n}}+\frac{1}{8} \sum_{n=1}^{\infty} \frac{n}{4^{n}}=u \tag{5}
\end{equation*}
$$

Thus, the expected utility of $R$ never exceeds the utility of $D$ and the probability of choosing $R$ can never be greater than the probability of choosing $D$ (as long as in all subsequent decision nodes of the centipede game the probability of choosing $R$ does not exceed 0.5).

An uncertain act R resembles the St. Petersburg lottery-it yields the prospect of receiving an ever greater utility payoff with an ever smaller probability. Thus, if a decision maker overweights the likelihood of rare events, which can be modeled as non-linear decision weights, the utility of $R$ increases (cf. Blavatskyy, 2005) and can become greater than $U(D)$. One popular and descriptively successful non-expected utility theory with non-linear decision weights is rank-dependent utility (Quiggin, 1981). In the context of the centipede game, in which all utility payoffs are non-negative, rank-dependent utility coincides with cumulative prospect theory (Tversky and Kahneman, 1992).

We shall consider the implications of embedding rank-dependent utility theory into models of probabilistic choice described in Section 2 for the subgame perfect equilibrium in the centipede game. Thus, we use formulas (1)-(4) where function $U($.$) is now rank-dependent utility rather than$ expected utility. We use rank-dependent utility function with Quiggin (1981) non-linear probability weighting function $w(p)=p^{\gamma} /\left(p^{\gamma}+(1-p)^{\gamma}\right)^{1 / \gamma}, p \in[0,1], \gamma=$ const .

For example, consider the subgame perfect equilibrium of the centipede game when rankdependent utility is embedded into model (1). In the last node, player 2 chooses D with probability $\Phi_{0, \sigma}(1)$ and R with probability $\Phi_{0, \sigma}(-1)$. Knowing this, in the before-last node, player 1 faces a binary

[^2]choice between $D$ that yields 99 for sure and $R$ that yields 100 with probability $\Phi_{0, \sigma}(-1)$ and 98 -with probability $\Phi_{0, \sigma}(1)$. Rank-dependent utility of $D$ is simply $U(D)=99$. Rank-dependent utility of $R$ is given by $U(R)=98+2 \cdot w\left(\Phi_{0, \sigma}(-1)\right)$, where $w:[0,1] \rightarrow[0,1]$ is the probability weighting function. Thus, in the before-last node, player 1 chooses D with probability $q=\Phi_{0, \sigma}\left(1-2 \cdot w\left(\Phi_{0, \sigma}(-1)\right)\right)$. Note that $q>0.5$ if $w(p)<0.5$ for all $p<0.5$.

In the before-before-last node, player 2 then chooses between $D$ that yields 100 for sure and R that yields 99 with probability $q, 101$-with probability $(1-q) \Phi_{0, o}(1)$ and 100 with probability $(1-q) \Phi_{0, \sigma}(-1)$. Rank-dependent utility of D is $U(D)=100$. Rank-dependent utility of R is given by $U(R)=99+w(1-q)+w\left((1-q) \Phi_{0, \sigma}(1)\right)$. Thus, in the before-before-last node, player 2 chooses D with probability $r=\Phi_{0, \sigma}\left(1-w(1-q)-w\left((1-q) \Phi_{0, \sigma}(1)\right)\right)$. Clearly, $r<\Phi_{0, \sigma}(1)$ but we still have $r>0.5$ if $w(p)<0.5$ for all $p<0.5$.

Proceeding by backward induction we establish that in the before-before-before-last node player 1 chooses $D$ with probability $\Phi_{0, \sigma}\left(1-w(1-r)-w\left((1-r)\left[\Phi_{0, \sigma}(-1)+q \Phi_{0, \sigma}(1)\right]\right)-w\left((1-r)(1-q) \Phi_{0, \sigma}(-1)\right)\right)$. This probability can be less than 0.5 (even if $q>0.5$ and $r>0.5$ ) when players have an inverse $S$-shaped probability weighting function (that overweights the likelihood of rare events). Thus, rank-dependent utility with an inverse $S$-shaped probability weighting function imbedded in a model of probabilistic choice can generate an intuitively appealing and descriptively adequate subgame perfect equilibrium.

Figures 4 and 5 plot the probability of choosing $R$ for player 1 and 2 respectively in a subgame perfect equilibrium derived from rank-dependent utility with $\gamma=0.61$ (a median value found in the experiment of Tversky and Kahneman, 1992) embedded in various models of probabilistic choice described in Section 2.

Figures 4 and 5 shows that rank-dependent utility with a typical inverse S-shaped probability weighting function (that overweights the likelihood of rare events), when imbedded in all standard models of probabilistic choice expect for the simplest tremble model, can generate an intuitively appealing and descriptively adequate subgame perfect equilibrium in the centipede game. For both players the probability of choosing $R$ is almost one in the initial and intermediate decision nodes of the game. Only in the last 10-20 decision nodes the probability of choosing $R$ gradually decreases to almost zero (in the last node). Thus, we can construct a reasonable subgame perfect equilibrium using two standard decision theoretical assumptions: overweighting of rare events and probabilistic choice under uncertainty. Note that we need both assumptions simultaneously. Assuming that players maximize rank-dependent utility without any model of probabilistic choice leads to the subgame perfect Nash equilibrium described in Section 1. Assuming that players choose under uncertainty in a probabilistic manner but do not overweight rare events leads to the subgame perfect equilibrium where option $R$ is chosen in every decision node with a probability not greater than 0.5 .

## 4. Conclusion

Traditionally, game theory was built on the premise that people always choose a lottery with the highest utility and only in a special case when several lotteries yield the highest utility people could choose any of them with any probability. Recent insights from descriptive decision theory (e.g., Loomes and Sugden, 1998) show that revealed preference in choice under risk are often inconsistent across repeated trials. This empirical finding motivated the development of models of probabilistic choice. This paper applies these recent advances in decision theory to model behavior in the centipede game. Such an application is quite straightforward since strategic games traditionally use the notion of a mixed strategy (corresponding to probabilistic choice in decision theory).

Several models of probabilistic choice coupled with a standard decision-theoretical assumption (overweighting of rare events) can generate a reasonable subgame perfect equilibrium where both players choose to cooperate in the initial and intermediate decision nodes of the game (cooperation inevitably breaks down in the last nodes). Such a result suggests that the paradoxical subgame perfect Nash equilibrium of the centipede game could be the product of an unrealistic assumption of the traditional game theory that people maximize expected utility and not the failure of backward induction. In fact, backward induction appears to work quite successfully when a deterministic expected utility theory is replaced with a generalized non-expected utility theory embedded in a model of probabilistic choice. The present paper also illustrates that it is possible to construct a cooperative equilibrium in the centipede game from "selfish" decision theoretical assumptions without invoking altruistic preferences.

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Figure 2 Probability of choosing $\mathbf{R}$ for player 1 (last $\mathbf{3 0}$ decision nodes) in a subgame perfect equilibrium derived from expected utility embedded in various models of probabilistic choice


Figure 3 Probability of choosing $\mathbf{R}$ for player 2 (last 30 decision nodes) in a subgame perfect equilibrium derived from expected utility embedded in various models of probabilistic choice


Figure 4 Probability of choosing $\mathbf{R}$ for player 1 (last 40 decision nodes) in a subgame perfect equilibrium derived from rank-dependent utility embedded in various models of probabilistic choice


Figure 5 Probability of choosing R for player 2 (last 40 decision nodes) in a subgame perfect equilibrium derived from rank-dependent utility embedded in various models of probabilistic choice


[^0]:    ${ }^{1}$ See also Loomes and Sugden (1995) for an application to decision theory and Gul and Pesendorfer (2006) for a behavioral characterization.
    ${ }^{2}$ See Hey and Orme (1994) for an application to decision theory and Blavatskyy (2008) for a behavioral characterization.

[^1]:    ${ }^{3}$ We make a simplifying assumption that both players have the same variance $\sigma>0$ of random errors.

[^2]:    ${ }^{4}$ This problem is also discussed in Loomes (2005, pp.303-305) in the context of the common ratio effect and in Blavatskyy (2006) in the context of violations of the betweenness axiom. For example, expected utility embedded in models (1)-(3) can generate a statistically significant asymmetry between fanning-in and fanning-out patterns in two common ratio problems but not a switching modal choice (when a safer lottery is predominantly chosen in one problem but not in the other).

