COMMUNITY IN THE ELEMENTARY MATHEMATICS CLASSROOM: STUDENT ENGAGEMENT IN FACE-TO-FACE AND ONLINE ENVIRONMENTS

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ABSTRACT

This qualitative study reveals how nineteen students in a grade five classroom engaged in community interactions to solve meaningful mathematics problems. In my experience, students often defer their sense of mathematical authority and autonomy to teachers and textbooks. The rationale for this study stems from an interest to investigate student membership in a mathematical community, both face-to-face and online, as it serves students to assert their own powers. Participants of this study solved two mathematics problems, interacting in a face-to-face and online community of peers. Analyses of teams' audiotaped face-to-face negotiations, digital chat field comments and physical as well as virtual solutions were undertaken, as was a discussion of students' individual survey comments about their experiences in the two forms of community. These present implications around the negotiation of both social and sociomathematical norms, particularly in the digital environment.

DEDICATION

I dedicate this thesis to the family members whose collective efforts have made its undertaking a reality. My first memory of a mathematics teacher belongs to my uncle, Ravinder Ahuja, who could not have known then that the hours of patient work on a balcony, teaching multiplication to a four-year old child, would inspire a lifelong love of mathematics. There is no endeavor, no journey that I have completed without the unconditional love and support of my parents, Nirmala and Bhupen Karia. My sister, Sangeeta McAuley, has been my mentor and the inspiration for entering the teaching profession to fill my days with gratifying work.

Throughout the four years it has taken me to complete course work, conduct research and write this thesis, I have found great sustenance in the love and strength of my own family. My husband, Tushar, is the most positive person I know, fostering our family's ability to see life's challenges as great opportunities. That he and I walk this path together is my blessing. Our three children, Anjali, Vikrum and Rohun, have given me unimaginable joy. I bring them, and all that they have taught me, to the classroom each day.

I would like to express my deep appreciation for the guidance and encouragement of my Supervisor, Dr. Ami Mamolo, my first professor in the Master's program at York University. She has set the foundation upon which I hope to build many more years of asking questions about teaching mathematics. Also, I am grateful to Dr. Jen Jenson, for her review and thoughtful contributions.

My final thanks go to Colin, Heather and Lisa, whose support, empathy and good humour have seen me through moments of doubt. Beyond your individual contributions as exemplary educators, I admire the incredible people that you are.

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Introduction & Rationale

Community offers potential for inspiration, identity and collaboration. Senge et al. (2000) propose, "A community of people is a place, rooted in the biosphere, rife with activity, mutual respect, and the recognition that everyone in that place is responsible for and accountable to one another, because the lives of all are interdependent" (p. 461). A system, if you will. The rationale for this thesis stems from my curiosity about models of community and how they may be reflected in the elementary mathematics classroom, particularly in a digital space, to support student claims of their own mathematical authority. A number of experiences have brought me to this rationale.

When I enrolled in graduate studies at York University, I was a Mathematics Coach, working in the diverse urban setting of one of Ontario's Board of Education. As part of my assignment to two families of schools, I was responsible for facilitating collaborative inquiries amongst the teachers with whom I worked. This role provided ongoing opportunities to observe students, from Kindergarten to Grade eight, as they engaged in the study of mathematics. Often, I noticed that students deferred their own mathematical authority to teachers and textbooks. So, upon completing the first three courses in mathematics education, my focus became the potential of a classroom community to restore authority and empowerment to the hands of students. How could the proverbial 'strength in numbers' be fostered in the mathematics classroom?

Having participated in a Board-wide project for students to engage in mathematics through the use of mobile devices, I completed two courses in digital technologies, studying web tools and the use of gaming in education. Again, my interests lay in how digital tools and spaces could serve community, and ultimately, student assertion of mathematical authority.

Three years ago, I returned to my own classroom of elementary students, motivated to foster a mathematical community for young children, and to continue my study of how digital technologies could serve such a community. Through course readings and assignments, as well as my own professional experiences, the focus and underlying motivation of my inquiry have become clear. Specifically, this study aims to investigate the following:

1) What are some of the salient features of students' community engagement in face-to-face and virtual problem-solving environments?
2) How do students describe their experiences in both face-to-face and virtual communities and how can this inform future research and practice?

To address these research questions, I developed both face-to-face and virtual environments in which students would engage to solve mathematical problems, so that I could study their interactions and elicit feedback about their participation in each environment. In what follows, I provide the contextual basis for this study via a review of literature in Chapter 2. The theoretical framework and research method are discussed in Chapter 3, with focus upon key considerations in the design of community environments. In response to the second research question, Chapter 4 reveals students' survey comments with regards to each community experience, and discusses their implications. Then, Chapter 5 addresses the first research question by examining students' lines of questioning, particularly questions of correctness, and those of mathematical conditions. Chapter 6 indicates the limitations of this study, and suggests methodological considerations for subsequent studies. Finally, Chapter 7 describes potential avenues for future research and discusses the conclusions arising from this study of elementary students' mathematical communities.

Chapter 2

Literature Review

In this chapter, I present the literature that informed the basis of my study's theoretical foundation. Included in this review are classical seminal works, as well as contextualizing research in mathematics and digital education. First, I will consider the definition of community, then discussing its desirability, and finally propose key features of a mathematical community.

Before envisioning *how* community would manifest in the context of mathematics education, or *why* this would even be desirable, it seems necessary to revisit my use of the term 'community', bearing in mind Getzels' (1978) caution that over ninety definitions exist. In analyzing the paradox between sameness and diversity of community, Furman (1998) proposed that, "Postmodern community is community of difference. It is based on the ethics of acceptance of otherness…" (p. 312). McCabe (2001) extended this to consider classroom communities where each participant's differences are respected in the common pursuit of a scholarly purpose. Through shared values, mutual respect and the recognition and use of diversity, the classroom community is a setting where sameness and difference can flourish.

Given this working definition of classroom community, a consideration of its necessity should be undertaken, based on one's perspective of learning. After all, no matter how compelling the arguments of its desirability, if your philosophical stance of learning does not require community, then why seek its establishment? Greeno, Collins and Resnick (1996) reviewed behaviourist, cognitive and situative perspectives of learning, hoping that they would be used conjunctively. The authors referenced the works of Skinner to describe the behaviourist's focus on individualized stimuli and response. Similarly, they pointed to Piaget's theory of constructivism to relay the cognitivist focus on an individual's ability to reason and problem-solve in the process of building conceptual understanding. The situative perspective presented in this work was described as a context-based, joint enterprise between an individual and his environment, which includes objects, technology and community. Each of the perspectives presented designates the role that community plays in learning, defining its necessity. Arguably, Piaget's emphasis on activity and cooperation, as the means by which students build their own understanding, necessitates community. Through his social-constructivist stance and Zone of Proximal Development, Vygotsky (1978) recognized a state of interdependence in his statement, "…learning awakens a variety of internal developmental processes that are able to operate only when a child is interacting with people in his environment and in cooperation with his peers" (p. 35). So, in perspectives other than a behaviourist stance, community is necessary. Now, why is it desirable?

As a start, consider the affective value of belonging and purpose that community can deliver, that "feel good" component. The sense of safety fostered through mutual respect has the potential to promote risk-taking by community members as they collectively solve problems and face challenges.

But, the literature presented additional justification, revealing facets of community that include cultivation, illumination and promotion. First of all, community is the cultivating ground for a collaborative spirit. Beaufait, Lanvin and Tomei (2008) proposed that collaboration reflected greater interdependence amongst group members than cooperative efforts. Similarly, Bruner (1996) recommended, "For the agentive mind is not only active in nature, but it seeks out dialogue and discourse with other active minds" (p. 93). A classroom community of interdependent members building upon each other's ideas will mirror the collaborative functioning of the "real-life" mathematical community. So, the gap between in- and out-of school learning noted by Resnick (1987), and by Moschkovich (2002), can be narrowed. Wouldn't the development of collaborative skills also exemplify Dewey's (1938) "enduring attitude" (p. 48), a collateral learning experience? The fact that Ontario's Provincial Report Card promotes students' collaborative effort as a Learning Skill, while the Partnership for 21st Century Learning includes collaboration as one of Learning and Innovation skills in its framework, supports this proposition.

Alongside its powers of cultivation lies the potential for community to shed light on misunderstanding. Erlwanger's (1973) study of a remedial program, Individually Prescribed Instruction (IPI) Mathematics, revealed the misconceptions that a sixth-grade student, Benny, had as he self-studied number operations. It is in reference to this study that Lappan and Ferrini-Mundy (1993) suggested, "The creation of a community in which a student's private world is made public will potentially challenge the learner's currently held views and lead to the construction of more acceptable and powerful views" (p. 630). In its destructive and reconstructive duality, illumination ultimately serves the cultivation of understanding.

Through the processes of cultivation and illumination, community sustains and promotes mathematical values. Bruner (1960) likened a mathematical attitude of discovery to the speaker in a conversation, differentiating speaker from listener on the basis of the level of activity involved in each role. By that logic, the active nature of a community would underpin mathematical discovery. Schoenfeld (1992) argued that it is a community's enculturation process by which a mathematical disposition, part of "knowing" mathematics, is fostered.

In reference to Article 26 of the Universal Declaration of Human Rights, Piaget (1973) interpreted the directive that an education should develop the human personality, "…is really to create individuals capable of intellectual and moral autonomy and of respecting this autonomy in others…" (p. 91). The image painted by this interpretation is of individuals who are self-reliant, possessing the confidence to ask questions, and assess arguments. At this point, I suggest that as it cultivates understanding and disposition, community serves Piaget's greater overarching purpose: intellectual autonomy.

On first glance, it would appear that autonomy and community contradict one another. How would self-reliance be nurtured in a state of interdependence? But, as Piaget (1932) explained, "Cooperation alone leads to autonomy…thanks to the mutual control which it introduces, it suppresses…egocentrism and the blind faith in adult authority. Thus, discussion gives rise to reflection and objective verification…cooperation becomes the source of constructive values" (p. 410). Self-reliance is not necessarily self-sufficiency: individuals still need the dialogue and diversity of community to construct the understanding that underlies their autonomy.

At this point, it is necessary to draw the connection between autonomy and authority, both matters of power. The Oxford Dictionary defines autonomy as, "the right or condition of self-government"[1] and authority as, "The power to influence others, especially because of one's commanding manner or one's recognized knowledge about something"[2]. Benne (1970) further classified authority by its source. Team sports exemplified rule authority, where players obeyed the rules of the sport as the ultimate authority. Expert authority emerged from specialization, as in the case of a doctor-patient relationship, where there would be no way to bridge the gap between the expert provider and the user of the expertise. Applying Benne's categorization to the mathematics classroom, teachers and textbooks would appear as manifestations of rule and expert authorities to which a student yields her own powers of intuition and influence believing that "…they possess knowledge…that the students themselves lack" (Amit & Fried, 2005, p. 148). By all of these definitions, it would seem that authority is about power applied external to oneself, while autonomy is a matter of power applied within. However, confidence underpins them both. So, as the states of autonomy and authority refer to student confidence and empowerment, they will be used conjunctively: community serves the assertion of either.

Having reflected upon definition and desirability of community, we are faced with the challenge of its design. As educators, how do we structure mathematics environments that empower students to be intellectually authoritative and autonomous? To visualize and articulate the features of a mathematics classroom community, one can review the portraits painted in previous research for commonality and generalizability.

There is Wenger's (2000) review of a situation theory that proposed communities of practice as a combination of three components: common purpose; mutual interaction through established norms; and a shared collection of tools and resources. As a distinction, Barab and Duffy's (2000) definition of practice fields situated learners in realistic contexts as preparation for life beyond school, but still envisioned collaboration and reflection in inquiries of domain-related problems. Completing two years of work in Australia's secondary classroom communities, Goos (2004) began with the recognition that, "In a sense, *all* classrooms are communities of practice" (p. 259). However, her proposal called for a classroom community of inquiry, making sense of mathematics through conjecture, argument, mathematical dialogue and reflection.

This being the case, there were works that specifically afforded me a view to mathematical argumentation and authority in the classroom. Describing Schoenfeld's undergraduate problem-solving course, Arcavi (1998) portrayed a collaborative, creative and reflective community, relocating authority to its own hands through the processes of reasoning and communication. In her three-year study of lessons with fifth-graders, Lampert (1990) wondered if and how features of the mathematics discipline, like risk-taking through conjecture and the revision process between conjecture and proof, could be fostered in a classroom setting. She analyzed the use of authentic tasks to promote student formulation of conjecture and argument, both of which were seen as integral in shifting mathematical authority to the hands of learners. In Clarke's (2001) Classroom Learning Project through the University of Melbourne, secondary mathematics and science lessons served as the locus for his study of uncertainty, its resolution and the opportunity for negotiation. Chazan and Ball (1995) considered authority within elementary and secondary communities through their analysis of the teacher's role in telling.

What are notable features of a mathematics classroom community? Based on a review of the research, I propose that they are as follows:

a) Active engagement in mathematics through solving meaningful mathematical problems;
b) Ongoing and reflective classroom discourse that implicates learning as a social endeavor;
c) Negotiation as a necessary aspect of membership in the community; and
d) The role of the teacher as facilitator.

Having generalized these as necessary features of a mathematical community, Chapter 3 of this thesis will present the developmental process taken to embed such features in both face-to-face and online communities of elementary school students. I am aware that each of these features is significant in its own right, and so, I will define the scope of this thesis by focusing on participants' discourse and the process of negotiation as potential means of fostering student authority in mathematics learning.

The digital environment presents additional terrain that demands negotiation. Initially, I was discouraged by the dearth of relevant research around elementary students' mathematical efforts in the online space. I noted that much of the existing literature about virtual communities was:

A) a general discussion of organizational issues or practices (e.g. Paloff, 2005; Resta & Laferriere, 2007); or B) an analysis of how a theoretical foundation or construct manifested in the digital realm (e.g. Hung, 2001; Radford, 2013). In this way, it exemplified the ongoing tension between the theory and pragmatism of educational research. Furthermore, many of the works were written about: secondary or undergraduate communities (e.g. Beaufait, Lanvin & Tomei, 2008; Tozzo, 2011); about gaming and mathematics (e.g. van den Heuvel-Panhuizen, Kolovou, & Robitzsch, 2013); or about dynamic software for mathematics (e.g. Sinclair, 2006).

More relevant to my own research interests around students' problem solving in digital space, there was Freiman and Lirette-Pitre's (2009) presentation of CASMI, Université de Moncton's virtual community of mathematics students and educators, in which participants solved rich problems and received feedback from fellow community members. Also, Nussbaum et al. (2009) studied the collaborative efforts of student groups solving problems using handheld devices loaded with an application, CollPad, which acted like digital paper. What rang true was Lagrange and Kynigos' (2013) caution, that without attention to the context of the research, "outcomes cannot be really insightful outside each context respectively" (p. 382). Paying heed to this, but also recognizing that this was, by no means, an exhaustive look at the landscape, I turned to Joubert's (2012) summary of three themes faced by the mathematics education community in a digital age, namely:

i) a heightened connectedness that the Internet allows learners;

ii) effective coordination of physical and technological resources to facilitate learning; and,

iii) the establishment of new contexts to be served through technology.

In its comparison of elementary students' engagement to solve mathematics problems in both face-to-face and digital classroom communities, this study has the potential to add unique value to the research that targets the three themes noted above. As part of the methodology, student participants will access an online whiteboard to solve a mathematics problem with a team of peers. Ultimately, this may encourage open student access to a global community of young mathematical minds through the Internet. Furthermore, this research may facilitate educators in establishing learning communities of students, exercising their autonomy in both face-to-face and digital environments, through the negotiation of social and sociomathematical norms.

Chapter 3

Theoretical Framework & Methodology

In this chapter, I outline the theoretical framework around which this study's design and analysis were built. Subsequently, the design considerations for establishing both forms of community engagement will be described. As well, participants of the study and my data collection method will be reviewed. I recognize that the presentation of a study's theoretical framework is usually separate from one of methodology. However, I have chosen to combine these two elements, given that the theoretical framework has been central to all decisions made when designing students' community environments and will ultimately be the basis of the data analysis presented in subsequent chapters.

Theoretical Framework

The theoretical framework for this thesis is based upon a symbolic interactionist perspective, combined with constructs of Toulmin's *argumentation scheme* as presented by Krummheuer (2007) as well as Cobb and Yackel's (1996) *social and sociomathematical norms*. The overarching ideology of this work is informed by Piaget's *intellectual autonomy*.

In a comparison of the theories of Piaget and Vygotsky, Lourenco (2012) highlighted that while both theories incorporated an element of social interaction, Piaget's goal was a rejection of authority and a claim of autonomy through cooperation with an equal peer, whereas Vygotsky's theoretical stance was constructed upon an individual's interaction with a more capable peer. This comparison exposed the source of personal tension that lies in my own definition and negotiation of the continuum between autonomy and heteronomy in the mathematics classroom: I have been trying to envision individual empowerment, within the social realm of community.

So, it needs to be stated that my focus lies in the interaction amongst peers, whether equal or not, to create the personal mathematical meanings which allow a student to gain and assert confidence. This being the case, I see shifts on the continuum between autonomy and heteronomy as ongoing, and necessary. Perhaps when a student shifts her autonomy to the will of a peer(s) in one context, this shift promotes her reclamation of autonomy in another. As new problems present new experiences, the student's position between heteronomy and autonomy will not remain static. However, it is when students shift their confidence to their peer(s) without merit that proves problematic, something to be explored later in Chapter 5.

The focus on student interaction has led me to the work of Blumer (1969) and his definition of *symbolic interactionism* which "…sees group life as a process in which people, as they meet in their different situations, indicate lines of action to each other and interpret the indications made by others" (p. 52). While he credited Mead with laying this theory's foundation, Blumer's work aimed to explicate its three underpinnings: object, interaction, and interpretation. In his analysis, Blumer (1969) argued that a person ascribes meaning to an object, be it physical, social or abstract, based on her social interactions. She uses and changes these meanings by interpreting that which her social interactions indicate. In communicating such an interpretation to herself, she forms and alters her actions accordingly.

Applied in the context of mathematics education, the object for which meaning is sought may be a problem or concept, one peer or the collective team, even the digital space. Which social interactions facilitate a student making meaning? Cobb, Perlwitz and Underwood-Gregg (1998) noted that both constructivist and sociocultural perspectives "agree that the construction and validation of mathematical concepts are collective as well as individual activities and that they occur via a process of argumentation with a community" (p. 72). If, as Cobb and his colleagues advocated, argumentation is the interaction in Blumer's definition, then schemes of such argumentation should inform the basis on which student interpretations of their interactions lie. In my study, I relied on two perspectives, one being Krummheuer's (2007) application of Toulmin's scheme, and the other being Cobb and Yackel's (1996) construct of *social and sociomathematical norms*. Krummheuer's analysis of Toulmin's (1958) argument layout is the first perspective from which to view students' mathematical arguments. Cobb and Yackel's (1996) norms present another view of students' arguments, particularly how they define a problem's condition, interpret its context or determine the difference between solutions. Both perspectives influenced task design decisions, and will be instrumental to data analysis.

It was Krummheuer who applied Toulmin's (1958) argumentation scheme, as shown below in Figure 3.1, for purposes of mathematics education. In Krummheuer's (2007) elaboration of the diagram, the relationship between a conclusion and its data, an irrefutable fact, must be proven through the process of argumentation. Validity of this relationship is supported by a warrant, and the "unquestionable basic convictions" (Krummheuer, 2007. p. 65) that render the warrant applicable are its backing.

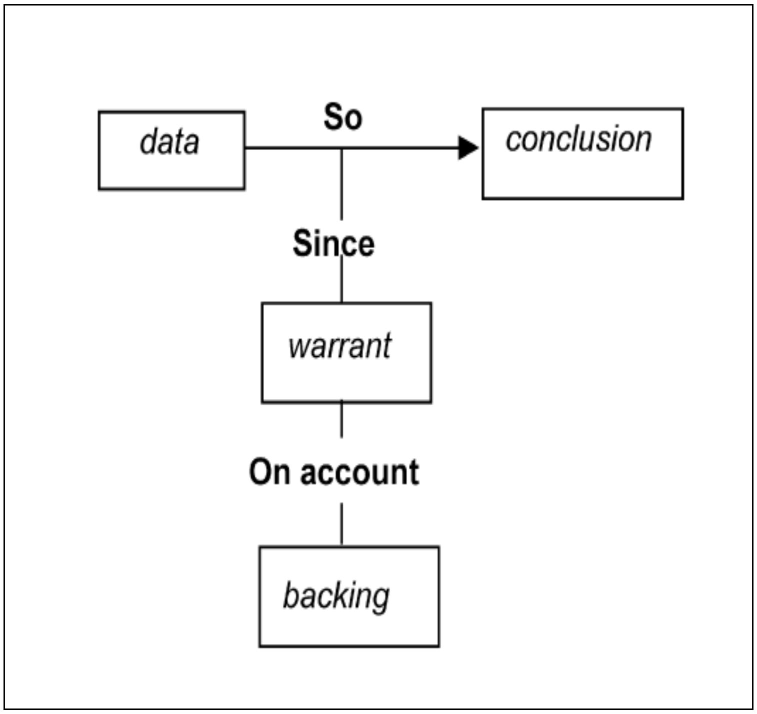


Figure 3.1 - Toulmin's Argumentation Scheme (Krummheuer, 2007, p. 65)

Krummheuer (2009) cautioned that an argument may not necessarily contain each component, and that component parts may not be apparent in an analysis of student dialogue. As such, this scheme of argumentation is not necessarily for explicit, direct teaching, but has value as a perspective with which to examine student interaction. Accordingly, I used the diagram from Krummheuer's analysis to study students' arguments as they negotiated mathematical conditions or their problem's context.

On an ongoing basis, students recognize patterns in their interactions with peers, such that they are able to negotiate the standards of classroom activity. Undertaking projects in second- and third-grade classrooms, Cobb and Yackel (1996) distinguished between the social norms applicable in all classroom interactions, and sociomathematical norms that were particular to students' mathematical activity. While the act of explaining one's thinking represents a social norm as a requirement in many classroom contexts, sociomathematical norms are specific to a student's mathematical context and include, "what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation" (Cobb and Yackel, 1996, p. 178). For the purposes of this study, social and sociomathematical norms influenced the design of each form of community, as well as the choice of mathematical problems. This perspective was also used to analyze participants' survey comments about their community engagement and student arguments about mathematical condition and context.

Having established the theoretical framework of this thesis, I will now detail the design of both face-to-face and online community environments.

Considerations in the Design of Community Environment

Social norms and argumentation provide the lenses with which to study student interactions within communities of their peers. As outlined in Chapter 2's Literature Review, four salient features of a mathematical community can be generalized as:

a) Active engagement in mathematics through solving meaningful mathematical problems;
b) Ongoing and reflective classroom discourse that implicates learning as a social endeavor;
c) Negotiation as a necessary aspect of membership in the community; and
d) The role of the teacher as facilitator.

In designing community environments, I relied on these four salient features. Moreover, I envisioned creating a space in which students could negotiate both social and sociomathematical norms, and in which I could observe their argumentation. I will now discuss each in turn.

Active Engagement Through Solving Meaningful Mathematical Problems. Given her suggestion that the act of conjecture involved an element of risk-taking, for Lampert (1990) "The most important criterion in picking a problem was that it be the sort of problem that would have the capacity to engage all of the students in the class in making and testing mathematical hypotheses" (p. 158). She went further to propose that since these types of problems were not solved simply by algorithm, and given that there may be many potential roads to follow, the problem-solver could navigate the journey.

Construction of mathematical concepts and skills *through* a problem differs from students simply being given a sequence of problem-solving strategies, each attached to its own specific practice problem. Moreover, if students are encouraged to solve problems only at the end of a unit of study, after the "basic skills" have been taught, consider what messages they receive about values of risk, struggle and perseverance. Watson (2006) argued that "watering down" mathematics did not demonstrate a high expectation of low-achieving learners. Requiring students to actively, yet patiently, participate in making conjectures and solve meaningful problems exemplifies teaching mathematics "harder and higher" (Watson, 2006), and meets Goldenberg's (1991) definition of *mathematics worth doing*.

This leads to the ongoing responsibility of seeking worthy problems for study, within the context of Ontario's mathematics curriculum. In his outline of the five features that characterized problems selected by Professor Schoenfeld, Arcavi (1998) described them to be: relevant and relatable in context; open, such that they can be undertaken in various ways; based on a "big" mathematical idea; promoters of generality; and free of trickery. Consider the two problems chosen for the purposes of this study, the first of which was taken from the University of Waterloo's Problem of the Week resource (Figure 3.2), and the second (Figure 3.3) which came from Small's (2009) resource with its bank of open problems.

Problem of the Week
Problem B

Sweet Child of Mine

Fei has earned $8.00 this week helping his mother with housework. He wants to buy her some chocolates for her birthday, and has decided on orange creams at $0.60 each, or truffles at $0.80 each, or a mix of both.

If he spends all of his earnings, what are the possible different combinations of chocolates and/or truffles Fei can purchase?

Figure 3.2 – Problem 1

For the Grade 6 Graduation Ceremony:

You are arranging more than 100 chairs into equal rows. Choose a number where there are many possibilities. Find them all and describe your strategies.

Figure 3.3 – Problem 2

While in alignment with grade five Number Sense and Numeration expectations in Ontario's mathematics curriculum document (2005), the two problems meet Schoenfeld's criteria, each presenting a real-world context from which several possible solutions can emerge. In fact, Small's Problem 2 adds an additional element, that of choice. With the freedom to choose the number of chairs for the graduation ceremony comes the responsibility of justification. Problem 1 includes a condition that Fei use the entire $8.00 of his earnings, and Problem 2 presents an opportunity to negotiate both the use of context and what constitutes a *different* solution. The two problems require students to apply mathematical content as well as process, and while challenging, are not impossible for grade five students. Participants' problem-solving strategies and negotiation during both problems will be further explored in Chapters 5 and 6.

Reflective Classroom Discourse and Learning as a Social Endeavor. Cobb, Yackel and Wood (1992) emphatically declared, "… we do not believe that mathematical learning can ever be natural, if by natural we mean the unconstrained organic growth of mathematical knowledge independent of social and cultural circumstance" (p. 27). Social interaction makes it possible for students to engage in making mathematical meaning through communication. It is with this is mind that the structure of social interaction in both face-to-face and online engagement was designed.

First of all, small teams of three participants (as well as one team of four) were formed, attempting to establish homogeneous ability groupings while recognizing the social dynamics of pairing certain individuals. Student achievement in mathematical assessments across the five strands of Ontario's (2005) mathematics curriculum became the basis of measuring ability for the purpose of establishing groupings. For its face-to-face interactions, each team would be given a separate physical space in an adjoining classroom where members would talk and co-construct a solution to the assigned problem.

I developed a website for our class community's online space, the Wordpress blog, Strength-In-Numbers (http://room210math.wordpress.com). The site contains various rooms for different teams to solve problems using an interactive whiteboard space purchased from Scribblar.com (exhibits below). This particular interactive whiteboard was chosen after searching for an age-appropriate, digital tool that would achieve students' problem-solving, communication and privacy needs. Each team's whiteboard space, shown in Figures 3.4 – 3.6 below, is password-protected, so users need to sign-in to their room, creating a separate digital space within which to engage in problem solving. Online interactions would mirror the face-to-face interactions since students could create physical representations of their solution on the interactive whiteboard, as well as share mathematical talk in the room's chat field, a space that holds up to 400 characters. The room also includes a microphone, if students prefer its use to written text in the chat field. Both the chat field and microphone tool appear in Figure 3.7.

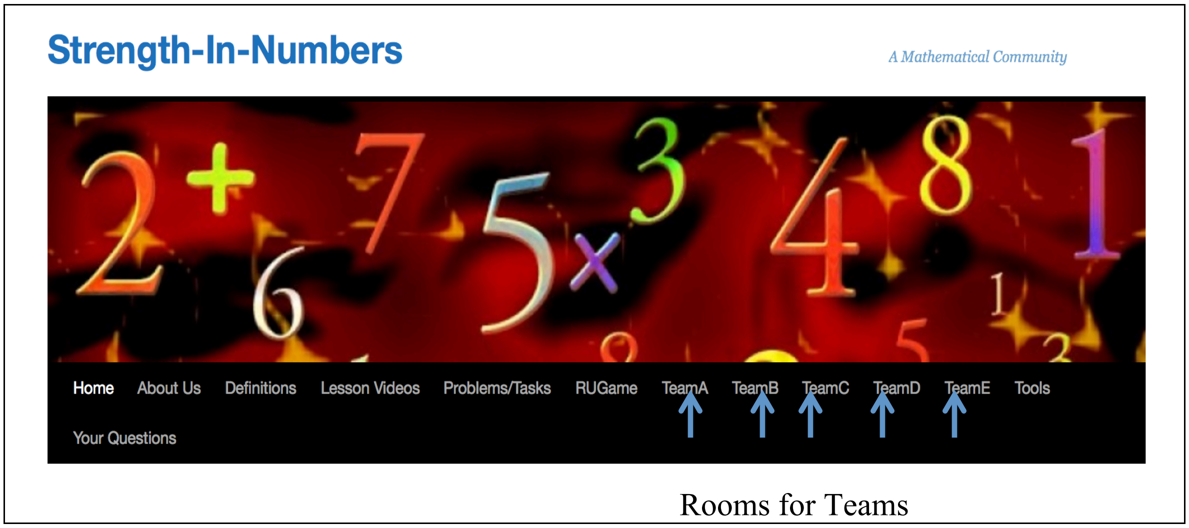


Figure 3.4 – Strength-In-Numbers Website – Problem-Solving Rooms

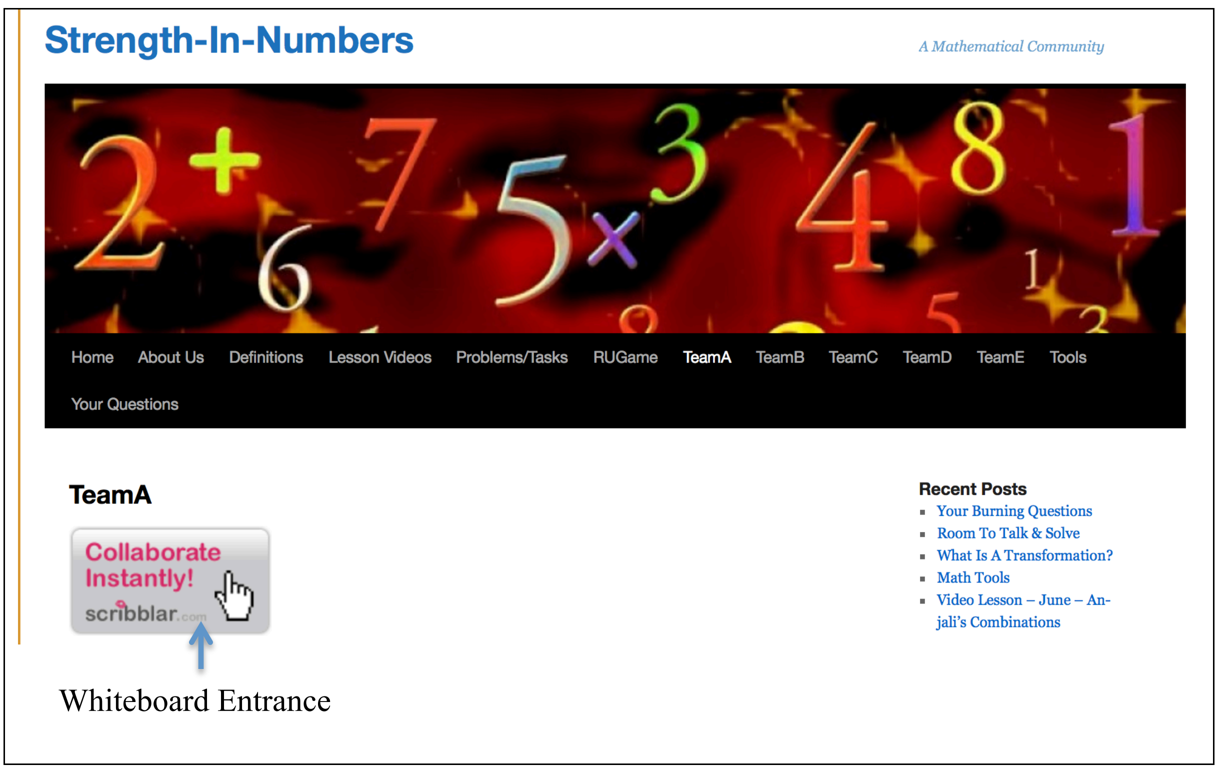
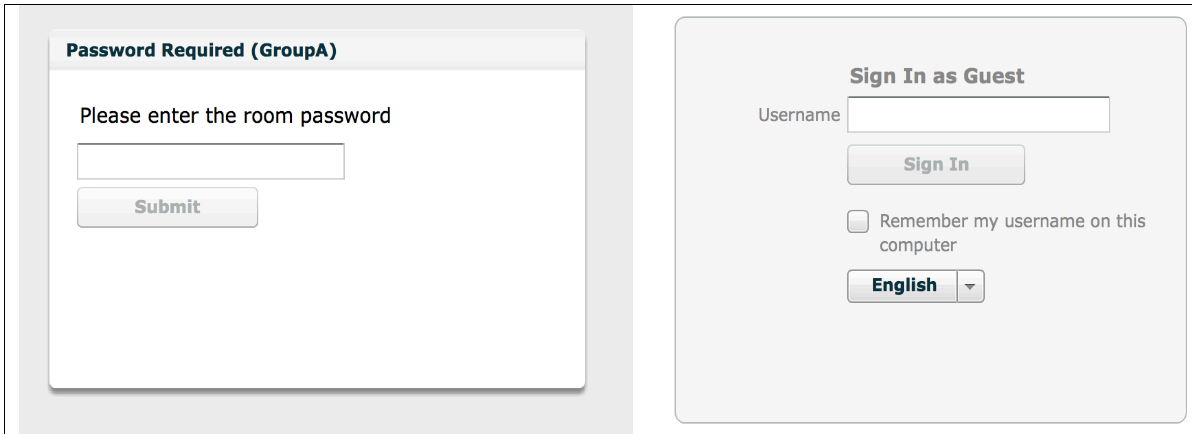


Figure 3.5 – Entrance to Scribblar Whiteboard Space



Entry, Sign In with Password

Figure 3.6 – Entrance into Whiteboard Space – Password-Protected

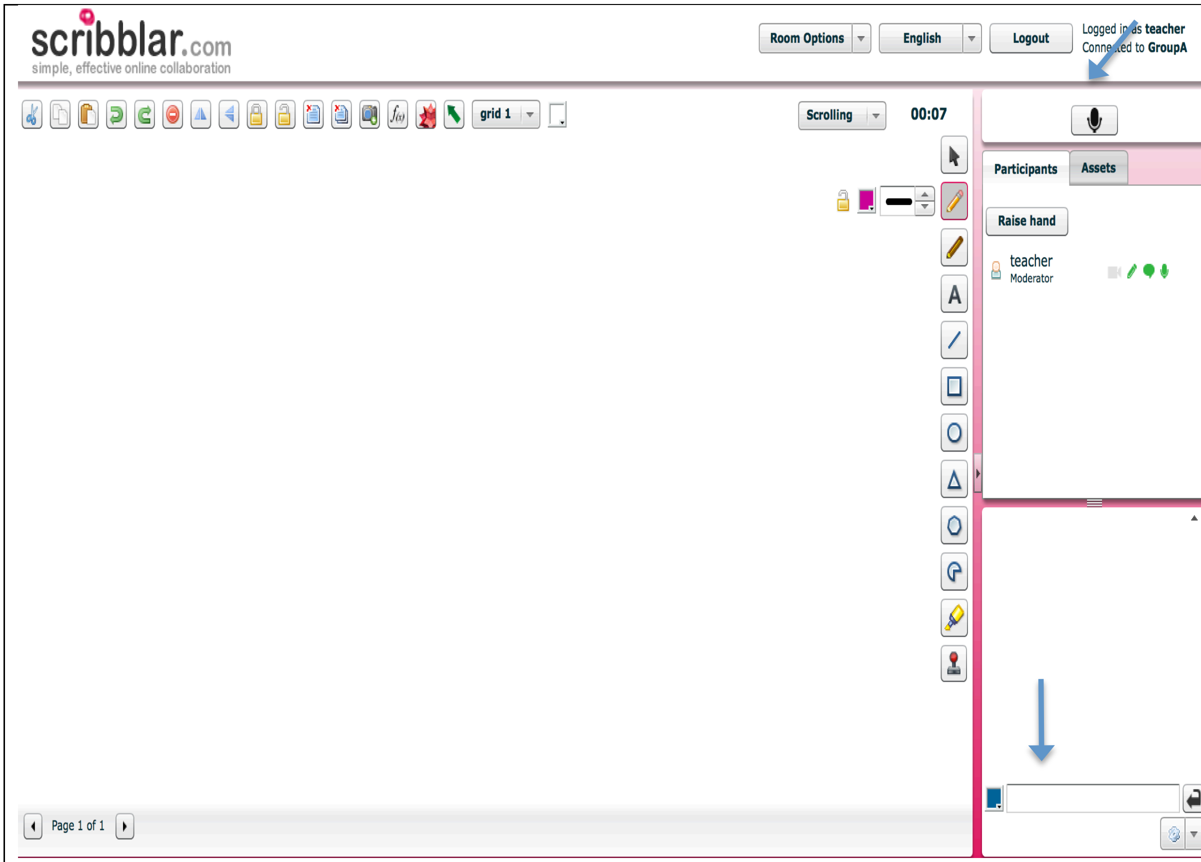


Figure 3.7 – Scribblar Whiteboard Space – Microphone, Chat Field

Negotiation as a Necessary Aspect of Community Membership. Clarke (2001) required that, "For negotiation to occur something must be at issue, unresolved, uncertain. In the classroom, one characterization of learning as experienced by a student is as a progression of emergent uncertainties and the resolution of those uncertainties"(p. 34). Furthermore, he proceeded to categorize the types of uncertainty including those of fact, method, interpretation, and validity of strategy or solution (Clarke, 2001). The ideal of uncertainty, in a discipline often viewed as logical, structured, and rule-based may seem counter-intuitive and uncomfortable. However, if doing mathematics is only about implementing the prescribed, known methods of others, and students are certain that this path leads to a correct answer, is this really *mathematics worth doing* (Goldenberg, 1991)? In this way, I suggest that uncertainty is a requirement, not only for negotiation, but also for deep engagement in mathematics.

Furthermore, the author suggested that such uncertainties could be resolved by appealing to an individual, text, argument, or evidence (Clarke, 2001). It is such an appeal that will demand justification or explanation, depending on whether it is a call for accountability or for clarification (Cobb, Yackel, Wood and McNeal, 1992).

It was anticipated that in both face-to-face and online interaction, students could be required to justify or account for their thinking, or explain their ideas to clarify their intentions to team members. Face-to-face, this could be achieved through pictorial representation or mathematical computation. If classroom routines were any indication, justification or explanation would be undertaken through mathematical talk. To support the drawings, written and oral communication in a face-to-face interaction, students were accustomed to using concrete tools for demonstrating and validating their thinking. As a result, participants had access to concrete manipulative tools for their face-to-face interaction.

Attempts to embed all of these capabilities into the online space were made. As mentioned earlier, the chat field was designed to hold both written appeals and subsequent justification or explanation, up to 400 characters. Secondly, the whiteboard space would enable students to justify/explain their ideas in pictorial form using the drawing tools appearing in Figure 3.8.

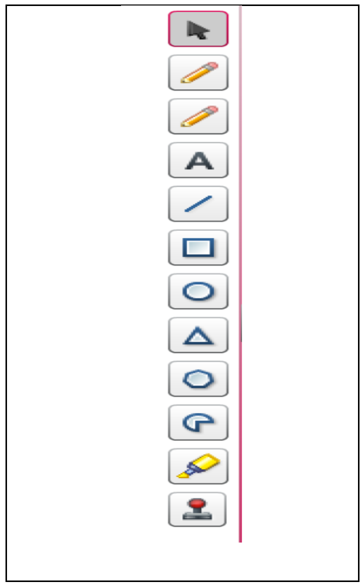


Figure 3.8 - Scribblar Whiteboard – Drawing Tools

Another feature of the whiteboard, as seen in Figure 3.9, is the Equation Editor with which equations could be written and solved as justification/explanation.

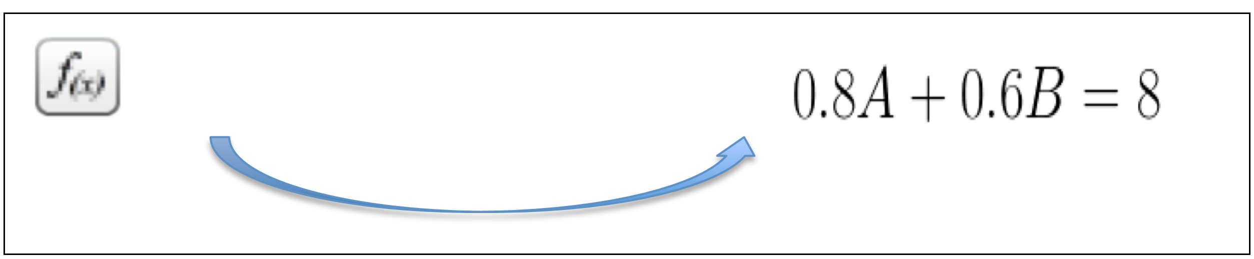


Figure 3.9 - Scribblar Whiteboard – Equation Editor

In addition, Scribblar's space also has a Wolfram Alpha feature (Figure 3.10) that would support students to define terms or verify calculations.

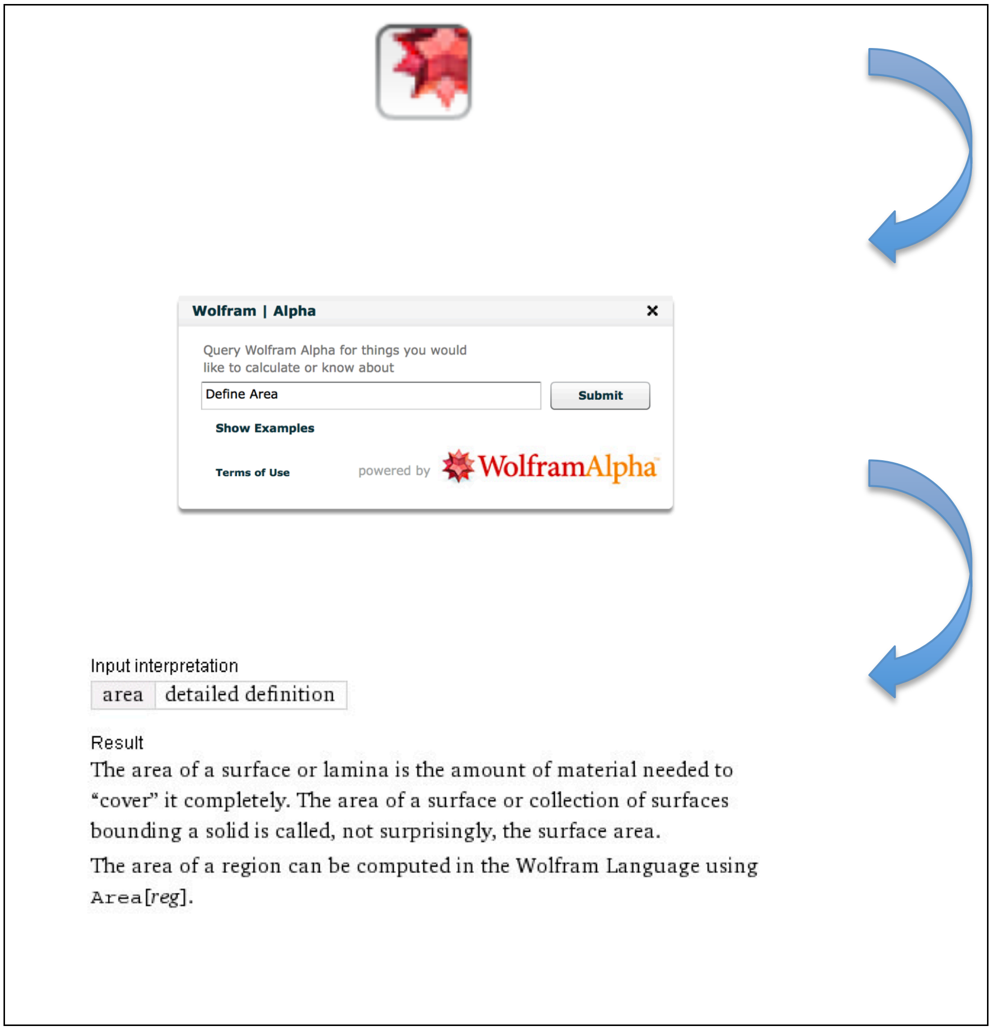


Figure 3.10 – Scribblar Whiteboard - Wolfram Alpha Tool

Finally, in its Tools page, the Strength-In-Numbers website embeds a scientific calculator (Figure 3.11), as well as a link to the National Library of Virtual Manipulatives (www.nlvm.usu.edu), a bank of digital manipulative tools for mathematics. Granted that use of this Tools page would require participants to come out of their team's room, perhaps in the middle of their problem-solving experience. However, the option was made available given students' previous use of the NLVM site, and familiarity with digital calculator tools.

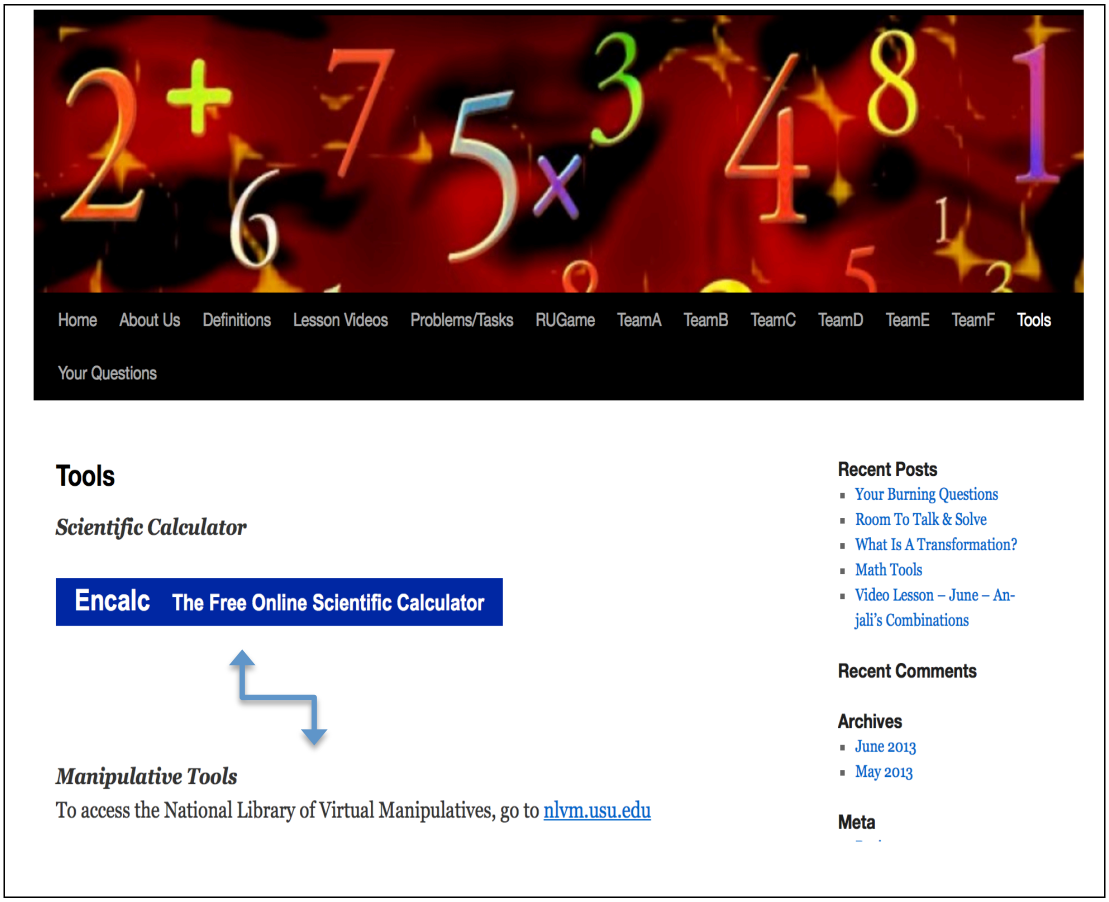


Figure 3.11 – Scribblar Whiteboard – Calculator and Virtual Manipulative Tools

Role of the Teacher as Facilitator. Based on three preceding features that I believe to be salient to a classroom mathematical community, it is clear that the role of the teacher is complex. It is the teacher who responsibly chooses meaningful problems with which to deeply engage students in mathematics. This will require that she also engage in such open-ended tasks, unafraid of her own uncertainty, to anticipate student responses. In this way, she becomes an active inquirer of the community, a role students need to see and appreciate. But, this active participation and unpredictable result may be daunting for some teachers, given the demands placed on their time and energy. In addition, teachers must balance the need to control the curriculum being studied with the need for students to drive their own inquiries. This is a difficult balance to achieve, particularly in the absence of confidence and support to do so.

A second role that teachers must play is that of facilitator. Student discourse and the negotiation of classroom values, norms and mathematical solutions require the teacher to provide the language and conventions that moves the classroom community towards the larger mathematical community. Furthermore, the facilitation role requires that a teacher be an attentive listener and an effective generator of questions in response to student discourse. As Goos (1996) admitted, “A teacher is far from being an irresponsible or passive participant in the classroom: rather, he or she is the representative of the culture into which students seek entry, and is responsible for structuring the cognitive and social opportunities for students to experience mathematics in a meaningful way” (p. 15).

In designing the environments for student interaction, I sought to facilitate teams’ problem-solving experiences in much the same way as I would in daily mathematical interactions within our classroom community. So, I would periodically enter each team’s face-to-face activity, asking questions to clarify or redirect student thinking. Similarly, I had authorized access to each team’s Scribblar whiteboard space, making possible online facilitation of a team’s discourse.

Now, having presented the considerations in designing community space for student interaction in both face-to-face and digital contexts, I will describe the participants of this study and the data collection methodology.

Participants of the Study and Data Collection

Participants. Participants of this study were nineteen grade five students in a culturally diverse public elementary school (Kindergarten - Grade 6) in an urban setting of Ontario, Canada. I was their teacher between September 2013 and June 2014. An Informed Consent document was obtained from each student prior to participation. Participant involvement was twofold:

A) To address the above-noted Question 1) of this investigation, participants worked in small, like-ability groupings to solve two mathematics problems, one in each of face-to-face and online community contexts; and

B) In response to Question 2) of this investigation, all participants were asked to independently complete a written survey after each problem- solving task, providing data about their reflections on their experiences in different modes of interaction.

Data Collection. All six of the student teams would be required to solve Problems 1 and 2, one through face-to-face interaction, the other online. Prior to each episode of their community engagement, students were provided with written copies of the problem.

In the course of daily classroom routines, students were accustomed to face-to-face engagement. Recognizing that the online space was a new environment that would, at the onset, present participants with navigational challenges, I facilitated six practice sessions prior to the data collection phase of this study. As part of these sessions, students were shown features and tools within the interactive whiteboard space, and given an opportunity to explore and test these features as they solved mathematics problems within peer groupings.

Early into these practice sessions, students noted their ability to delete objects from the whiteboard space, including those that had been placed there by peers. Upon contacting Scribblar's Customer Service representatives, I was advised to show students how to lock all objects they placed on the whiteboard space. Despite this, students communicated in subsequent practice sessions that they had been able to unlock and delete the locked objects created by their group members, though this ability was tempered by the presence of an 'Undo' command. This phenomenon will be discussed further in Chapter 4, as part of the social norms of online engagement.

From the perspective of data collection, the online environment would provide data comparable to that gathered in the face-to-face environment: a record of the community's talk; its solution to the assigned problem; and any field notes taken during such engagement.

The specific mathematics problems were chosen closer to the date of data collection, and as explained earlier, struck a balance between grade five expectations from the Ontario's mathematics curriculum document (2005) with pre-defined qualities of a rich task. For each of the two problems, half of the teams worked face-to-face, while the other half worked online, so that ultimately, each team experienced two modes of engagement. An example for the work done by Teams 1 and 3 is shown below:

Mode	**Problem One**	**Problem Two**
Face-to-Face	Team 1	Team 3
Online	Team 3	Team 1

Table 3.1 – Schedule of Community Engagement

Face-to-Face Interaction. Constraints of the school timetable usually allowed student teams to work in a face-to-face context for approximately twenty minutes as part of daily classroom routines. However, for the purpose of this study, teams were given freedom to determine when the assigned problem was complete, with the result that teams worked face-to-face for an average of about forty-five minutes. I facilitated each team's interaction two or three times over its duration, though teams worked independently for the majority of the time. All face-to-face conversations were audiotaped and team solutions were retained. Field notes of any notable occurrences during a team's interaction were collected in the process of the problem-solving task.

Online Interaction. Using independent devices (laptops/computers), participants accessed the classroom community's website, Strength-In-Numbers. For the purposes of this study, each participant was given a student number, to maintain anonymity, and each team was provided the password for its room only. Although participants were in the same physical location in the school's Computer Lab, they were reminded that all team interaction was to occur through the website's affordances. As in the face-to-face engagement, many teams took between forty and fifty minutes to complete their assigned task, at which point students were asked to log out of the online space. Although I remained in their physical space during online efforts, examining some whiteboard representations, questioning student thinking as deemed necessary, I did not enter each team's whiteboard space to moderate the chat field. Upon completion, chat conversations were printed, and student solutions, as they appeared on the whiteboard space, were printed as part of the collected data.

Student Reflections. After each problem-solving interaction, participants were provided in-class time to independently complete a survey in which they reflected upon their experiences. The survey questions were as follows:

1) What, if anything, do you like about solving math problems face-to-face (online) with your team?
2) What, if anything, is difficult about solving math problems face-to-face (online) with your team?
3) How would your describe the way your team worked together to solve your problem? Circle your choice (Very Well, Well, Okay, Not Very Well), and explain it.
4) Tell a student in another class what you think they will need to do to work well with others face-to-face (online).
5) Tell that same student what you think they shouldn't do when they are working face-to-face (online) with a team.
6) How should working face-to-face (online) to solve math problems be changed to make it better?

Data Conventions. All data of this study were anonymised for presentation in subsequent chapters. Each participant was assigned a student number for the purposes of logging into the Scribblar whiteboard, and transcribed notes of audiotapes referred to students by their student number. Also, to be faithful to students' voices and truly reflect their comments, transcribed comments and student survey comments have been presented as written, or as spoken. Chat field comments have also been left in their original written form.

Chapter 4

The Social Norms of Community

Social norms, as defined by Cobb and Bauersfeld (1995), are "Classroom norms that involve conventions describing how to collaborate with others, and obligations describing how to react socially to a mistake" (p. 297). The co-construction of social norms within a classroom community is often undertaken at the onset of a school year, and then presumed to be taken-as-shared understanding. Examples of these taken-as-shared understandings include: defining respectful behaviour; demonstrating active listening; and proposing strategies to resolve conflicts and reach consensus. However, co-constructed understandings do not necessarily remain static. Negotiated conventions and obligations described in the above-noted definition are centered upon fluid social interactions. As such, they are subject to testing and may require renegotiation in the face of new classroom contexts. This chapter focuses upon the social norms of students' face-to-face and online community engagement. An examination of sociomathematical norms and argumentation is reserved for Chapter 5.

From the analysis of student responses on both face-to-face and online survey instruments, four themes can be constructed, each of which I will now discuss in turn. They are as follows:

a) Contribution
b) Consensus
c) Concentration
d) Communication

<u>Contribution</u>

With respect to face-to-face engagement, the theme of contribution is noted in students' survey comments about workload and its distribution as part of the negotiation of responsible contribution. Consider three such comments below:

STUDENT31: "Be fair don't hog all the work."

STUDENT33: "Don't exclude people and do all the work but also don't just relax and not do any work."

STUDENT22: "Don't have one person do all the work and you just sit back."

Such statements implicate an ideal of fairness and equity, of balance on a continuum between "do all the work…and not do any work". This ideal demands that students negotiate two definitions: one of fairness, one of work. Arguably, a student's vision of "doing all the work" can be any or all of: recording the group's solution using symbolic notations; posing conjectures and suggesting strategies; argumentation. Is "work" a product or a process? The response determines the unit of measure by which team members weigh contribution and judge the fairness of each other's effort. A student's perception that their peers "just sit back" will be subsequently explored in the context of *mulling* (Mason, Burton & Stacey, 2010).

ELL participants' ability to contribute represents a second facet of this theme in face-to-face engagement. It may be daunting for English Language Learners to choose and then speak the words of an unfamiliar language in the process of conveying their indications to others. While doing so, they are also required to hear the spoken indications of others, translate and interpret these indications to make personal meaning of objects around them. Fortunately, mathematical discourse encompasses the use of numerical, algebraic and geometric representations, many of which are common to the global mathematical community, not confined within a cultural boundary. While the discipline's common ground can offer some relief, it cannot eliminate the linguistic challenges faced by the ELL student in their efforts to contribute to a community of their peers.

In response to the survey question of what was difficult about solving mathematics problems face-to-face, an ELL participant of this study wrote, “Some times I don’t really understand the problems or don’t understand their way to solve the problems.” Further, to justify a rating of “well” for the group’s collective effort, this student stated, “when I face-to-face with team, I would have a bit nevous. Scare if my answer is wrong or scare to talk.” This comment seems to align with the student in Cobb’s (1995) univocal explanation, who simply accepts a partner’s judgment that they have made a mistake. It seems that the oral language barrier feeds this ELL participant’s assumption that his undertaking of all mathematical processes is incorrect.

Intuitively, the digital space, with its short, abbreviated text and use of emoticons would level the mathematical playing field between English-speaking and English-learning students even further. In the online chat field, the linguistic disparity between participants need not be apparent. The same ELL student rated the team’s online effort as “very well” for the reason, “because I won’t be nevous to talk. (wink emoticon)”. While this individual expressed greater comfort in the online space, the team’s chat field revealed that some of his questions sought permission from teammates. One might view written appeals such as “can i change the 9” and “can i delete it?” as attempts to meet the social norm of treating others’ contributions with respect. But, declarative statements like, “I think we should change the 9” would still achieve the social norm of respectful contribution. The act of seeking permission betrays this ELL student’s lack of confidence, even in online community engagement.

In contrast to an ELL participant’s language barrier, the *Interrupted* student who cannot complete her attempts to contribute is representative of the third dimension of this theme in face-to-face engagement. Two such instances below, one in Team 2 and one in Team 5, demonstrate how students interrupted one another in mid-sentence:

STUDENT12 (1:05): Wait, so that's eight dollars…
STUDENT22 (01:07): (Interrupts) No, this is what I did. I did eight divided…wait, wait, eight point zero zero divided by…

STUDENT15 (02:51): So, let's just say our maximum would be…
STUDENT45 (03:02): (Interrupts) It said they…
STUDENT15 (03:03): (Interrupts) No, our least amount would be a hundred and two chairs and our greatest would probably be (inaudible).

Subsequently, both STUDENT36's feedback about the difficulties of working face-to-face as well as STUDENT21's recommendation intimated respectful contribution.

STUDENT36: "Some people are cuting people of and not letting them share there ideas."

STUDENT21: "You shouldn't cut off one of your classmates when they are speaking because they can add on to the solution."

Michaels, O'Connor and Resnick (2008) presented three facets of accountability that, with ongoing normative negotiation, needed to be present in classroom talk. The first facet, Accountability to the Community, was envisioned as, "When talk is accountable to the community, participants listen to others and build their contributions in response to those of others" (Michaels, O'Connor and Resnick, 2008, p. 286). Student voices silenced by a more capable peer exemplify a lack of accountability to the community. Moreover, if a less capable, but more aggressive peer disregards voices of reason, the team's collective effort may be subject to inappropriate, inefficient reasoning. In fact, Cobb's (1995) four case studies concluded that univocal explanation, in which one partner seemed to dominate and another partner conceded to being mistaken, was not a particularly rich learning experience for either party of the interaction. In this way, such acts of interruption and the unheard student voice implicate norms around accountability that require renegotiation within our classroom, in an effort to move students towards multivocal (Cobb, 1995) learning opportunities.

A final aspect of student contribution in face-to-face interactions relates to personal versus public contribution, and the timing of student contribution. In a study conducted by Cobb, Stephan, McClain, and Gravemeijer (2001), students solved mathematics problems either independently or in small groups, prior to whole-class engagement. Similarly, in the Collpad project undertaken by Nussbaum et al. (2009), collaboration was scaffolded, first requiring students to independently solve problems on their device, before moving into small-groups to reach consensus. Two of the less vocal participants of my classroom community included in their survey comments a request for some independent time before making public contributions in face-to-face community engagement. These *First Me, Then We* students suggested:

STUDENT35: "You should change it (working face-to-face) by maybe give each groupmate a chance to do the question by themselves then putting them together."

STUDENT12: "Have a group work on separate pages while still discussing and after, mix all the solutions into one."

Perhaps, these students were less vocal because they needed independent time and space to *get involved* with the problem, so as to "…come thoroughly to grips with the question, to sort out meanings and relationships, to specialize in various ways so that the question comes off the page and gets inside you. In fact, the question becomes your own" (Mason, Burton & Stacey, 2010, p. 112). I see this stage as the means to foster intuition and a capacity to conjecture.

A *First Me, Then We* student may have appeared quiet or unfocused because they are *mulling*, described by Mason, Burton and Stacey (2010) as pulling away from the problem to see it in a different light. So, providing students with *getting involved* or *mulling* time may facilitate their active engagement with the problem, and serve as the basis for richer, and more focused dialogue when they enter their small-group discussions. Though their voices may have been heard less frequently than their peers in audiotaped conversations, the two participants above appear to be motivated to make meaningful contributions to their group in due course, rather than to avoid responsibility altogether. Their suggestions may be more an issue of the timing of their public contributions. I propose that in their attempts to rely on their own abilities, organize and then share their own ideas with the group, these individuals are moving towards intellectual autonomy. In fact, a possibility for future lessons would be to sacrifice some discourse time in order to provide each student with an opportunity to individually engage with the assigned task before community engagement.

Neither of the two *First Me, Then We* students made the suggestion of independence in the context of their online interaction. The Scibblar whiteboard would not necessarily offer team members an independent space to *get involved*, or *mull*, because each member of the team is required to be on the same page of the space. But, perhaps the online space offers students an opportunity to detach from the group, by not posting chat or adding to the whiteboard solution, so as to reflect on the assigned task.

If phenomenon around ELL participants, the *Interrupted* student and the *First Me, Then We* student were less pronounced, even absent in the online environment, how was the theme of responsible contribution built from that context? In students' digital engagement, this theme was shaped by an act of deletion. Of nineteen student survey instruments for online engagement, fifteen made mention of such an act. As described in Chapter 3, during practice sessions leading up to the actual investigations, participants noted that objects or notations they made on the virtual whiteboard could be deleted by their teammates. In fact, both the 'Clear Page' and 'Clear Chat' commands on Scribblar's whiteboard, shown in Figures 4.1 and 4.2 below, are irrevocable.

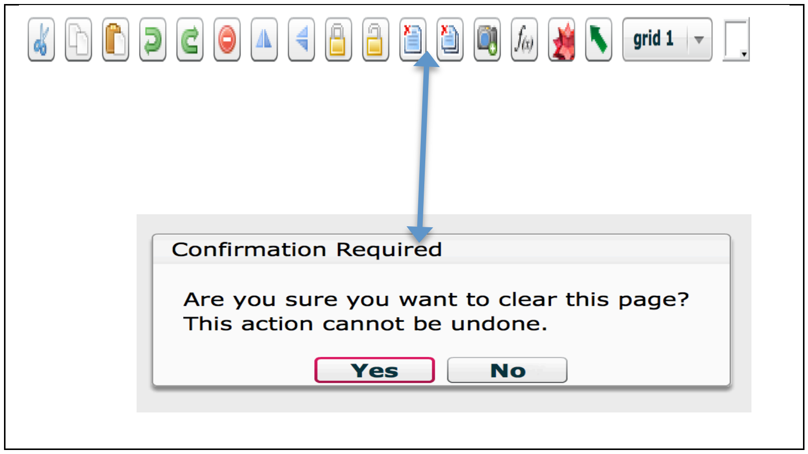


Figure 4.1 – Scribblar Whiteboard - ‘Clear Page’ Command

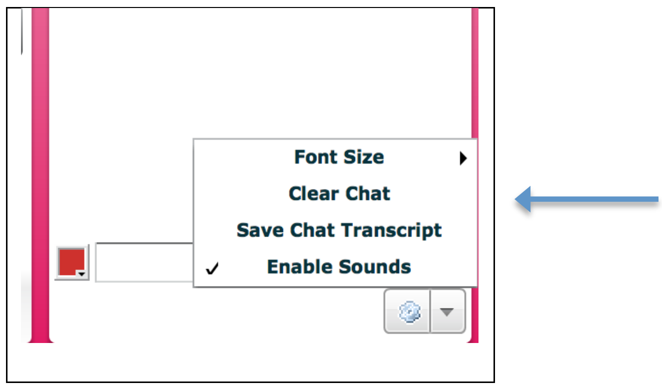


Figure 4.2 – Scribblar Whiteboard – ‘Clear Chat’ Command

Despite whole-class negotiation that only the individual who locked an object would have authority to unlock and delete it from the whiteboard space, incidents of deletion, whether accidental or intentional, continued to be problematic and often remained unresolved. As evidenced by the comments in chat fields of two teams below, this was a source of frustration for students. In the case of Team 1’s chat, there was no claim of responsibility for the deletion:

STUDENT31: WHO GOT RID OF THAT
STUDENT21: y is it gone
STUDENT11: aaaaaaaaah
STUDENT11: what happen?
STUDENT31: OK WHO IS GETTING RID OF THIS!!!

STUDENT21: ok i finally wrote it down without problems
STUDENT11: dk
STUDENT31: OK I SWEAR I AM NOT GETTING RID OF THIS

In contrast, the following chat field for Team 3 showed STUDENT13 admitting responsibility for the deletion:

STUDENT33: thats not me
STUDENT33: im not deleting ur work
STUDENT13: its me ok! sorry
STUDENT13: but it was to big!!

Relevant to this discussion is a description of this particular group of children and their ability to nurture and accept each other, despite individual quirks of habit. Everyone had a place in this community, and generally, students treated each other with consideration. In their prior face-to-face experiences, social norms did not include taking the work of a peer and destroying it. So, they were taken aback by such a reality as it presented itself in the online environment, a violation of the social norm they took as established from their face-to-face routines. In the case of Team 1 noted above, the fact that a student did not claim responsibility implies that the deletion was not considered to be positive act and did not align with the community's established social norm of responsible participation. As for Team 3, STUDENT13's apology indicates an understanding that deletion was inappropriate.

In a third instance, as a group approached the end of its online efforts, one of its members deleted all comments from the chat field of the whiteboard space. When questioned further, this participant admitted to deleting the entire conversation because specific comments seemed off-task. Consequently, only the team's computations were available for data analysis, with the contents of the student chat field irretrievable. In face-to-face engagement, this particular student would whisper ideas, as though not wanting to be heard for fear of being negatively judged. In light of such conduct when working face-to-face, the online equivalent would be to delete the contents of the chat field. However, while this student's actions were true to a personal norm of interaction, teammates may have perceived the deletion as a violation of the social norm of the community. In this act of online deletion, not only had the individual student withheld his own contribution; he had unilaterally decided to withhold the contributions of all teammates.

The episode above also raises concerns about students' perceptions of other social norms, like what constitutes on-task behaviour, and what constitutes an appropriate piece of work. Did this participant perceive that any act of being off-task was unacceptable, an imperfection to be hidden?

As a result of the above-noted interactions, many students wrote about this barrier to their contribution in the survey instrument, either as: a difficulty they encountered in the environment; an aspect they would change about the online experience; advice for how not to behave in the online context. Consider the following comments, addressing social norms related to online contribution, as they appeared on the survey instrument:

> STUDENT36: "It is also difficlet because when your working online people can delete your work but on paper face to face you can't realy delete it."
>
> STUDENT25: "You shouldn't just delete work without asking. Because that will start a fight."
>
> STUDENT14: "When people lock their work you shouldn't be able to unlock it."
>
> STUDENT32: "You get frustrated when people mess up work that you're doing. Make the lock system so only the person that created the shape or thing can unlock it."
>
> STUDENT23: "When I do type something, other people delete it…Not to touch my work."

STUDENT36's comparison of online and face-to-face realities around the deletion of one's contribution supports the earlier statement that this emerged as a social norm requiring negotiation. Face-to-face conversation cannot be unspoken once stated, in the same way that the written text in Scribblar's chat field can be deleted once filled. Still, if students do not want to hear or to be heard, it is possible in both environments. Although every written representation in students' face-to-face interaction is not necessarily incorporated within the group's final solution, students expect that the physical presence of each representation, as a validation of their individual effort, remains intact. In contrast, the presence of an individual's contribution on Scribblar's whiteboard space can be easily wiped from existence through the act of deletion, a shortcoming of this software interface that STUDENT14 targeted via the recommendation that, "When people lock their work you shouldn't be able to unlock it."

By definition of a participatory culture, "Not every member must contribute, but all must believe they are free to contribute when ready and that what they contribute will be appropriately valued" (Jenkins, 2009, p. 6). It is granted that each of us may differently describe the ideal of "appropriate value". Still, STUDENT32's comment, "You get frustrated when people mess up work that you're doing", as well as STUDENT25's caution, "…Because that will start a fight", indicate that the deletion of whiteboard objects is unsettling and undermines students' expectations that their efforts will receive appropriate value. Furthermore, Team 1's unsolved case of deletion necessitates a definition of responsible contribution be established. Our day-to-day online experiences provide demonstrations that anonymity promotes bravery to take paths that one may not take in face-to-face interactions. Even young children recognize this, as indicated by STUDENT23's difficulty with arguing in face-to-face interactions where, "one hear the emotion in talking but on text u can't." Given the invisibility that an online presence offers, students will need ongoing opportunities to define the freedom and constraints of responsible contribution in this form of community engagement. This presents a future opportunity to study the processes students take to negotiate and establish online social norms around contribution, and to observe how the social norms in their face-to-face engagement influence those of their online communities. If community members regard the destruction of others' contribution as disrespectful, or unethical in their face-to-face interactions, how will ethical contribution manifest in online community?

In its negation of the virtual existence and appropriate value of individual contribution, deletion elicits an emotional response. Beyond the emotional implications, it is an act of judgment that arguably, runs counter to Toulmin's (1958) scheme of argumentation as applied by Krummheuer (2007). An object on the whiteboard space, whether drawing, numerical or text symbols, represents a claim. In the due process of argumentation, all claims should remain intact until data, warrants and backing can be reasoned and judged to confirm or refute their validity.

STUDENT23's closing command, "…Not to touch my work", presents another facet of contribution in the digital space worth negotiation: the issue of ownership. This mirrors the global community's ongoing negotiation of ownership, modification, and use of idea through Creative Commons licensing. While Hsi's (2007) Digital Kid fluidly uses remix processes so "The digital objects created are dynamically changing in their representations as they are created and changed by online learners" (p. 1514), educational assessment practices contradict this reality leaving individual students concerned about demarcating their work as separate entities.

To close this discussion of contribution and deletion in the online environment, I am compelled to raise the implications of such an act for the teacher-researcher. First, it is worth considering which aspects of the classroom's culture fuel student ego, such that the greatest emphasis is placed upon "being right". Second, if individuals have the ability to destroy any or all of their online contributions, including the chat fields, the type and quantity of data collected, as well as the researcher's subsequent data analysis, are potentially unrepresentative, even misrepresentative of the phenomena under study. Limitations of particular research tools and methodologies require balanced analysis prior to their use in data collection, and present an avenue for future study.

Consensus

Calls for reaching consensus, were heard in both online and face-to-face surveys. The noteworthy difference in these appeals is that consensus in the online environment seems to be centered around the *deletion* of objects, whereas consensus for face-to-face interaction is related to the *inputting and merging* of student ideas as part of the group's collective solution. For example, consider the following three comments that participants made about their online interactions:

> STUDENT11: "if someone want to Delete your own work, there should be a warning to said to you Then if you agree to delete it, Then they can delete it."
>
> STUDENT31: "Have An asking button, you chick an asking button before you can get rid of stuff.
>
> STUDENT22: "Voting for things to be on the page."

Interpreting the "things to be" component of STUDENT22's statement as "things to remain" is not unreasonable given that this group's chat field indicated disagreement with items already placed on the page.

> STUDENT12: dont delete my work
> STUDENT12: stop thats not nessesary
> STUDENT32: stop dont draw big lines

Based on the three above-noted statements about online activity, consensus is deemed necessary for the *removal* of objects from the solution space. This is distinguished from the following comments that recommended compromise and consensus about which ideas to *bring forward* to the group's solution in face-to-face interactions.

STUDENT13: "let others give their opinion, it can't always be <u>your</u> way."

STUDENT35: "Without group mates agreeing don't do anything."

I suggest that this distinction between *consensus for removal* and *consensus for bringing forward* makes sense given that in the online space, idea generation and idea representation appear as a combined action, not divided into two separate acts of oral brainstorming followed by visual representation in symbolic form. The process of reaching consensus in both environments is displayed in Figure 4.3 below.

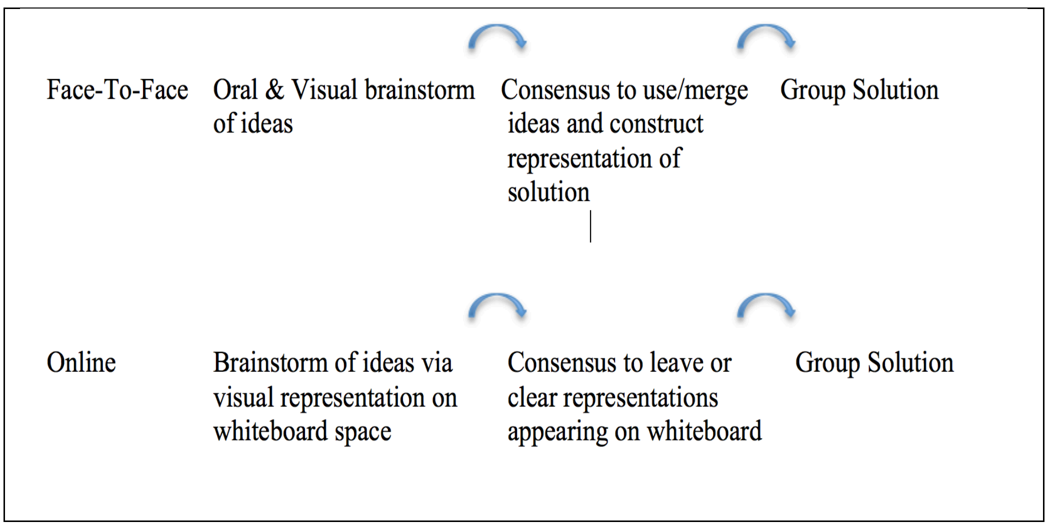


Figure 4.3 – Consensus in Different Forms of Engagement

To demonstrate the constructive potential of reaching consensus in a face-to-face engagement, I have chosen an example from Team 3's transcripts. Similarly, to demonstrate the destructive nature of online consensus, I have also analyzed chat from Team 3's whiteboard.

In the case of the chair arrangement in Problem 2, examine the lengthy process of oral brainstorming undertaken by Team 3. First, each team member made an initial proposal:

STUDENT23 (01:01): We can do it just 100.
STUDENT33 (01:04): No, it has to be…
STUDENT13 (01:04): We could do 400.
STUDENT33 (01:05): …it has to be more than…
STUDENT23 (01:06): 110 then.

Then, consideration was given to the upper limits of the problem, which recognized that the problem's context of a graduation ceremony was important.

STUDENT33 (02:09): It has to be more than a hundred.
STUDENT23 (02:11): Yeah but, if it goes a hundred into a million, where's the stop?
STUDENT13 (02:16): Well, let's be reasonable.
STUDENT33 (02:18): Let's just be reasonable…
STUDENT23 (02:20): There's no million.

There was even a suggestion to test various possibilities:

STUDENT13 (02:55): Yo! You know there's a possibility. What is just do both, one fifty and one twenty and then we can see which one has the most possibilities? And then we'll do…use that one?
STUDENT33 (03:10): I'm pretty sure they have equal., 'cause one twenty can go, it has twenty, but fifty can't go through. But, one fifty, twenty can't do it, but fifty can. But two hundred both can.

An agreement was reached:

STUDENT33 (03:34): Both of them, twenty and fifty can go in. Plus five, ten…
STUDENT23 (03:42): So let's do two hundred.

Compare this to Team 3's online attempt of Problem 1, which included mention of work being deleted, but did not mention a deletion of the chat field. I believe the chat field to be intact, given that at the onset, students greeted each other, and by the end, team members had concluded that there were three solutions, a statement which corresponded to the whiteboard space. Also, STUDENT13's repeated efforts to clarify the condition of this problem suggest little possibility that the group had reached consensus, and then deleted the chat field. If, in fact, consensus had been previously negotiated and deleted, STUDENT13 would have been certain about the team's whiteboard solutions. For these reasons, I accept the chat field to be a complete representation of the team's online interaction.

Towards the start of the transcript, STUDENT13 stated his interpretation of the problem's condition.

> STUDENT33: now we have $7.60 we have $0.40 left
> STUDENT13: what
> STUDENT13: see
> STUDENT13: but the question says to use all the money
> STUDENT13: there can not be change

Nearing the end of the chat field's comments, STUDENT33 replied that the team's last whiteboard page contained three solutions. Appearing in Figure 4.4 below is this page with its three solutions, which I have circled for the reader's consideration.

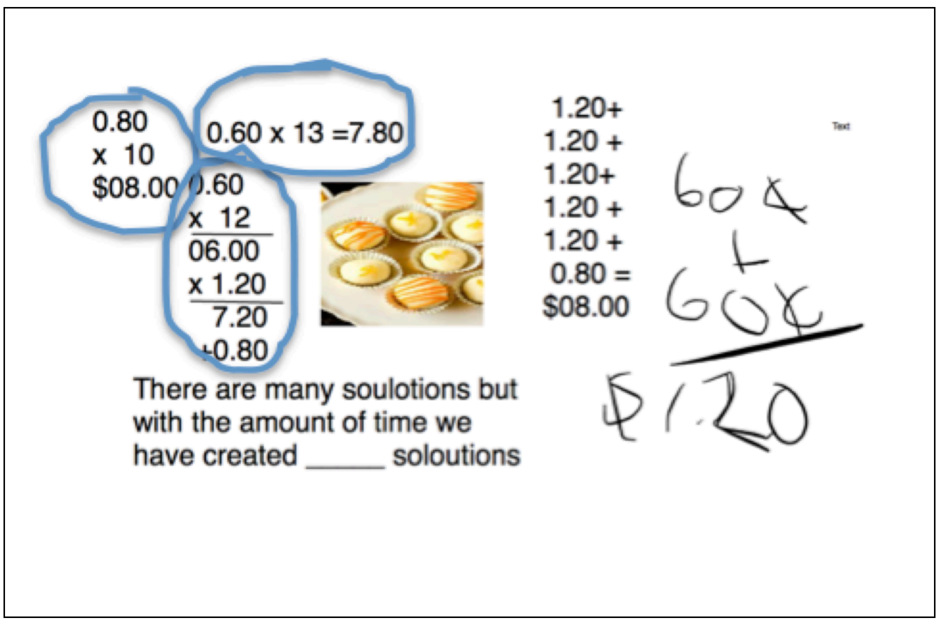


Figure 4.4 – Team 3 Solution for Problem 2

In response to the three solutions circled, STUDENT23 stated, "we are not aloud change", and consensus between STUDENT13 and STUDENT23 seems to have been reached.

In its face-to-face interaction, Team 3 embarks upon filling the solution space after reaching consensus of two hundred chairs, whereas online, Team 3's solution space is filled with a conclusion awaiting consensual refutation or validation. Had STUDENT33 agreed with STUDENT13 and STUDENT23's interpretation of the problem that change was not allowed, the claim on the solution space, a solution of $7.80, would have required clearing. So, while consensus may have been a constructive force in this team's face-to-face interactions, there is potential for it to have been destructive in online problem solving.

As evidenced by STUDENT31's recommendation for an 'Ask' button in the online space and STUDENT22's suggestion for 'Voting', members of the community have proactively initiated the negotiation of this social norm. But, as Kruger (1993) argued, "In no study does simple agreement or simple disagreement relate to outcome. In all studies success is predicted by engaged discussion of the issues, including explanation, clarification or revision of ideas" (p. 166). Students' understanding lies beyond the surface comfort of reaching consensus with their peers; it lies in the shared meaning that emerges from negotiation.

Examine Team 4's negotiation to reach consensus through the lens of Krummheuer's (2007) analysis of Toulmin's argumentation scheme.

STUDENT24 (01:16): No, it can be a hundred and one…
STUDENT14 (01:18): No, a thousand.
STUDENT24 (01:19): …It's more than a hundred.
STUDENT14 (01:20): A hundred and one won't have any equal rows.
STUDENT24 (01:23): A hundred and two?
STUDENT34 (01:24): A hundred and one will have equal…
STUDENT14 (01:25): Yeah, but that's only one. (Laughing) A hundred and one rows of one. That's it!
STUDENT34 (01:33): Yeah, but that's still equal rows. (They all laugh)

When STUDENT24 proposed 101 chairs for this problem, a teammate challenged this. In this case, the consensus was about inappropriateness, rather than appropriateness of the conclusion.

Ultimately, the warrant that a choice of 101 chairs would not result in equal rows, other than a backing of 101 rows of 1, the team appeared to abandon this proposal. Despite the backing, it was unclear whether the consensus was based on a taken-as-shared (Cobb, Wood, Yackel, & McNeal, 1992), or whether the consensus was merely on social grounds. Consider this interaction from the perspective of Krummheuer's (2007) analysis of Toulmin's argumentation scheme as previously displayed in Figure 3.1.

Data	The answer should have many possibilities, as stated in the problem
Warrant	A Choice of 101 chairs won't have equal rows
Backing	Except 101 rows of 1
Consensual Conclusion	We should not choose to use 101 chairs

It is not that the group abandoned the use of 101 chairs, but it is the negotiation of this conclusion that exemplified Kruger's (1993) above-mentioned characterization of success.

How did argumentation appear in the online environment? In a third page of its whiteboard solution, Team 2's members showed a solution of 12-by-12. In the chat field, the team decided to move to page 4 of the whiteboard. Then, building upon a 12-by-12 conclusion constructed by a teammate on the previous page, STUDENT32 suggested, "les do 13 by 13", to which STUDENT12 expressed agreement. Subsequently, page 4 of the team's whiteboard space in Figure 4.5 showed 13 marks started in what appeared as an incomplete solution.

STUDENT22: bring us to 4
STUDENT32: we r
STUDENT12: we r at 4
STUDENT32: les do 13 by 13
STUDENT12: k'

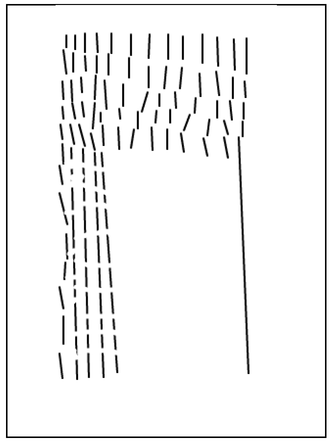


Figure 4.5 – Team 2 Solution – Page 4 – 13-by-13

Using the definitions given in Krummheuer's (2007) analysis of Toulmin's scheme, the absence of a teammate's challenge may have been the warrant to justify a conclusion of 13-by-13, using the previous solution of 12-by-12 as data. As explained, "Arguments can be chained together in such a way that an accepted conclusion can function again as data for a subsequent new argument" (Krummheuer, 2007, p. 65).

Data	Another teammate's accepted conclusion of 12-by-12.
Warrant	STUDENT12 agreed when faced with the proposal, "les to 13 by 13".
Consensual Conclusion	13-by-13 was another appropriate solution.

However, the lack of Kruger's (1993) "engaged discussion of the issues, including explanation, clarification or revision of ideas" (p. 166) does not render STUDENT12 and STUDENT32's consensual conclusion to be successful. In fact, none of the members in this team questioned why STUDENT32's proposed solution of 13-by-13 with its total of 169 chairs was appropriate given the previous 12-by-12 solution of 144 chairs. Page 3 of this team's whiteboard justifies the solution of 12-by-12 as having equal rows and being over 100 chairs, an interpretation of "equal" which then leads to a proposal of 13-by-13. Despite their consensus, the lack of argumentation led to the team's solution containing two different numbers for total chairs, rather than being two different ways to arrange the same number of chairs.

Concentration

The survey comments of many students conveyed a sense of ease around face-to-face engagement. In this context, the absence of a keyboarding component to their interaction permitted the comfortable exchange of ideas and opinions, as indicated by the following comments:

STUDENT31: "You get to talk more about your ideas and they can talk back…And some words you can't spell…It is more easyer talking face to face."

STUDENT13: "I like that you can ask questions easily and almost right away you can get an answer right away."

STUDENT36: "It's a better discution."

STUDENT15: "I like it because its easier to speak each others minds."

Although they were positive about this, six of nineteen participants were equally honest in admitting that their face-to-face conversations had gone astray into off-topic territories, an admission that was verifiable through transcripts of their audiotaped interactions. In addition, another five of nineteen participants made a recommendation on their survey that intimated staying on-task. Consider Team 4's audiotaped conversation:

STUDENT24 (08:42): When I score a goal, I'm going to be like…
STUDENT34 (08:45): (Interrupts) No, I'm going to go like this.
STUDENT14 (08:47): No, do this.
STUDENT34 (08:50): No, the best celebration is this. (S14 laughs)
STUDENT24 (08:57): No, this is.
STUDENT34 (08:59): Whenever you go to six, go like this.
STUDENT14 (09:00): We're supposed to be doing math.

Each member of the above-noted team subsequently rated their collective effort as 'Okay' on the survey with similar justifications:

STUDENT14: "Our group/team got off-topic a lot of times. We weren't able to work till the end."

STUDENT24: "Okay, because we went off topic a lot because we all are friends."

STUDENT34: "Like I said before we got distracted many times…"

As for the online engagement, one team's distraction took the form of an argument between two students, something that was raised in the survey comments of their two teammates.

STUDENT45: STOP
STUDENT15: HI
STUDENT15: YOU STOP
STUDENT45: STOP 15
STUDENT45: U STOP
STUDENT15: IM NOT DOING ANYTHING
STUDENT25: plz stop fighting

Similarly, the online whiteboard space of a different team contained what two of its members described as "random pictures", unrelated to the task at hand. What had participants perceived as an acceptable addition to an online mathematical solution? In our classroom's daily face-to-face interactions, placing such "random pictures" would likely have been challenged in a whole-class examination, where questions of justification would be posed during the team's communication of its solution. Perhaps this result would have been different had I moderated the chat field, even logging in to make my presence felt. However, if students refrain from such behaviour simply in the presence of a "Big Brother" teacher, are they denied an opportunity to negotiate as well as exercise their moral and intellectual autonomy?

The above-noted incidents also call attention to the perception of on-task behaviour in both face-to-face and online engagement. Student definition of its indicators requires questioning, given that the appearance of one's being on-task may contradict the reality. An individual that appears distracted may be in the process of *mulling*, described by Mason, Burton and Stacey (2010) as removing oneself from a problem to think about it differently. For that matter, how does one's appearance manifest in the digital environment? By Hsi's (2007) account of the Digital Kid, "multi-tasking behaviour and attention switching is common" (p. 1514). If this is so, students will need to negotiate an acceptable "margin of occurrence" around their definition of off-task behaviour. Finally, they will require a repertoire of strategies to re-establish focus when it is lost in either face-to-face or online interactions.

<u>Communication</u>

In their face-to-face interactions, participants identified the theme of communication in their comments about explanation, such as:

> STUDENT11: "Say out loud your opions. to told you team what you think and told they what your way to solve the problems.

> STUDENT23: "They need to explain what there doing for they can get helped by other teammates."

For three participants, communication was emphasized as a matter of justification:

> STUDENT32: "Good reasons to deny an answer."

> STUDENT33: "Also, look at why the other person thinks something is right."

> STUDENT12: "Try working out on each others decision."

In contrast, although ten of the nineteen participants mentioned the communication theme for the online environment, many were vague in their comments, often making simple suggestions in their survey, such as:

> STUDENT45: "Talk on the chat box."

However, two participants did convey the concerns below over the use of text convention in the chat field:

> STUDENT21: "I think they shouldn't use too many shortforms because they might not know what it means or they might think of the insult differently."

> STUDENT15: "Some people don't use the text box apropoly like using 20 exsplanation marks. the text tool you have to keep extending it."

Finally, one student did raise the notion of "listening" online, stating:

> STUDENT13: "Listen to what the type…And try to check in on what they might have said/typed every few minutes."

An aspect of this theme that requires negotiation is what constitutes listening, an act defined differently in a digital context. At the onset of the school year, our class co-constructed criteria for attentive listening which included: making eye contact with the speaker; not speaking simultaneously; gestures such as turning towards the speaker and nodding; and paraphrasing the speaker's words. Mapping these criteria onto the digital workspace results in a different sensory experience such that students will need to "Listen to what they type". Then, there is the connection between listening and the act of reflecting. Dewey, as cited by Mason and Johnston-Wilder (2004), suggested that to thoughtfully respond "…we must be willing to sustain and protract that state of doubt which is the stimulus to thorough inquiry…not to accept an idea or make a positive assertion of a belief until justifying reasons have been found" (p. 281), which explicates the mathematical acts of explanation and justification. Though Cobb, Wood, Yackel and McNeal (1992) have distinguished justification, an obligation to provide proof, from explanation, an act to clarify one's intent for a known audience, how this distinction comes to be a taken-as-shared norm by students is worthy of investigation. Particularly, research potential lies in a study of which form, if any, data, warrants and backings will take in the classroom's online community as it co-constructs mathematical meaning.

Summary

In summary, four themes were shaped from the survey comments of the grade five students who participated in this study: contribution; consensus; concentration; and communication. Each of these themes represents a social norm that demands student negotiation and renegotiation. It is reasonable to expect that a negotiation of norms in the online context is necessary, given that interactions in this context are unchartered waters (See Chapter 3). Students need to ascertain if and how to map their face-to-face problem-solving identities and experiences to their online presence. But, survey comments indicated that even in their face-to-face interactions, restatement or renegotiation of certain norms was necessary, perhaps because such interactions occurred with different group members, over a longer time span.

As Blumer (1969) has argued, "Out of a process of mutual indications common objects emerge – objects that have the same meaning for a given set of people and are seen in the same manner by them" (p. 11). Students, in the course of their interactions, will assign shared meaning to various social norms. Through their engagement in face-to-face and online communities, they must be presented with opportunities to co-construct the meaning of: responsible contribution to the community, negotiating the experiences articulated by ELL participants, the *Interrupted* student and the *First Me, Then We* student; achieving consensus through argumentation and negotiation; what constitutes on-task behaviour; and effective, even appropriate mathematical communication. Subsequently, by a symbolic interactionist account, students must be able to interpret these meanings and act in accordance with their own interpretations, which perhaps will initiate the negotiation process once more.

Chapter 5

Questioning within Community Engagement

Chapter 3 of this thesis presented the design and methodological decisions made in the process of preparing for data collection. To address the second component of this study's rationale, Chapter 4 shared students' survey comments of their experiences in both face-to-face and online community engagement, arriving at four themes around which our classroom's social norms require renegotiation.

To begin this chapter, I propose that if argumentation is the social interaction that supports student construction of meaning, then as Clarke (2001) advocated, an uncertainty must be present and subsequently resolved through an appeal. By posing questions, students have potential to trigger argumentation, negotiate meaning and resolve uncertainty. In this chapter, I will examine participants' lines of questioning in both face-to-face and online environments, since questions represent potential for negotiation, with others and within themselves.

At the outset, consider the quantitative difference between face-to-face and online talk. Most teams engaged in face-to-face and online environments for comparable amounts of time. Despite this, the transcribed notes of each team's face-to-face conversations well exceeded the printouts of their corresponding online chat field, a result that was consistent with student survey remarks about the ease of their face-to-face engagement. Accordingly, the number of questions posed face-to-face also surpassed those posed online.

Several factors may have contributed to these results. First, the online chat field presents a limit of 400 characters, unlike oral communication. Secondly, extended online chat requires greater keyboarding proficiency than that possessed by most of the grade-five participants of this study. Finally, students are familiar with text conventions, such as short statements, abbreviations and acronyms, through their use of other digital devices, like cellphones and tablets. Acknowledging quantitative differences in participants' questions brings me to a consideration of qualitative differences. What types of appeals did students make in each form of their community engagement?

First, I will compare participants' use of questions to resolve the uncertainty that their own interpretations are valid. These include questions of proposal that are natural manifestations of student attempts to develop what Cuoco, Goldenberg and Mark (1996) refer to as *Habits of Mind*, such as guessing, and experimenting. Such habits have been encouraged in our classroom community, as part of individual and small-group problem solving, as well as in whole-class discussions of solutions and methods. So, how such questions appear in both forms of community engagement will be analyzed.

The second line of questioning I will examine requires my return to Cobb and Yackel's (1996) often-cited definition of a sociomathematical norm as one that includes "what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation" (Cobb and Yackel, 1996, p. 178). Though this may already be implied in the given definition, I am making an explicit extension to this definition such that a sociomathematical norm also refers to what counts as *a mathematical condition or constraint*. While some may suggest that this is an aspect of defining an assigned task, something that is applicable to many classroom contexts to be regarded as a social norm, I respectfully disagree. Mathematical problems often incorporate constraints that must be interpreted and satisfied in mathematical ways. If Cobb and Yackel's (1996) statement that, "…what counts as an acceptable mathematical explanation and justification is a sociomathematical norm" (p. 461) rings true, then on that basis, what counts as a mathematical condition or constraint, is also a sociomathematical norm. By questioning and meeting the conditions and constraints of specific problems, students will ultimately be in a position to negotiate what constitutes a mathematical condition, and what counts as an appropriate mathematical response to such a condition. So, a study of students' interactions to solve a problem should encompass a focus on their negotiation of just what constitutes that problem in the first place. Specifically, students' questions around their interpretations of the problem's conditions will be examined.

Face-to-Face Questioning of Correctness

A form of uncertainty outlined by Clarke (2001) is the uncertainty that one's own position is correct. As opposed to the requests for justification and explanations as part of questioning the correctness of others, we make appeals to resolve our own doubts that indeed, our own interpretations, results, processes are accurate, even appropriate. In their face-to-face engagement, participants in each team made these types of appeals, some seeking independent verification of their own mathematical guesses. For example:

STUDENT24 (17:50): I wrote one thousand divided by nine hundred and eighty. Did you do that?

STUDENT45 (15:57): (Interrupts) How about forty? How about forty? Guys, how about forty…

Others sought the reassurance of team members' validating their efforts:

STUDENT16 (29:01): We did a lot of work, right?

As well, students articulated questions of correctness they had directed inwards:

STUDENT21 (04:25): Yeah. So then, it's four eighty, right? Four eighty…continue down here. Five…forty? Five forty.

As an educator, I have encouraged students to make mathematical guesses and suggest experiments, honouring my own value for such *Habits of Mind* (Cuoco et al., 1996). But, my hope is that students will first be their own guess-checkers, internally negotiating and validating themselves, before appealing to the judgment of others for validation. In that way, they will exercise autonomy before approaching community negotiations. Listening to one's own arguments, like listening to the arguments of others, is an act of *Intellectual Courage* (Polya, 1957), for it may require that you change your own thinking. I propose that questions of correctness directed at oneself demand an internal negotiation that demonstrates *Intellectual Courage*, rather than an act that implicates a fear of failure.

While STUDENT45 could have internally directed the question, "How about forty? How about forty? Guys, how about forty…" prior to an appeal to team members, STUDENT21's "Five…forty? Five forty." statement suggests the negotiation of an inward appeal, one that did not require further external validation. Furthermore, consider STUDENT23's utterance, "I have one. Two hundred divided by fifty?" which suggests an internal check prior to the subsequent proposal, "Two hundred divided by fifty?" That proposal then invited team negotiation. So, questions of correctness in students' face-to-face engagement appear to be of four broad types, as follows:

1) reassuring statements of validation from peers;
2) peer verification of a guess before self-verification;
3) inward appeals of correctness that are self-negotiated; and
4) proposals presented to the group after inward questioning of correctness.

Online Questioning of Correctness

As in face-to-face interactions, some participants still explicitly sought validation from their online community members, as in the cases of Team 3's STUDENT13 and Team 2's STUDENT12, who sought reassurance as follows:

STUDENT13: 4 truffles=4.00x2=8.00
STUDENT13: get it
STUDENT13: helllllllllllllllllloooooooooooo
STUDENT13: anyone?

STUDENT12: do we need the stage?
STUDENT32: not really
STUDENT12: it doesnt seem nessecsariey

Teams' online chat fields revealed questions of correctness in the form of declarative statements. Below, STUDENT34's ongoing guesswork in the form of declarative utterances implies a request that STUDENT14 verify each guess.

STUDENT34: 14
STUDENT14: 14 truffles dont work
STUDENT34: then thirteen
STUDENT14: nope
STUDENT34: then twelve

The virtual chat field does not hold a record of students' inward appeals of correctness. The nature of the environment implies chat as an action between at least two people in the designated space. Also, rather than explicitly questioning peers about the correctness of one's own ideas, the following digital chat comments show students making statements of proposition and inviting negotiation through those declarations. Instead of explicitly questioning correctness, STUDENT32 boldly proposed, "les do 13 by 13", after STUDENT22's previous 12 by 12 solution. The proposal invited negotiation, but STUDENT12 merely agreed, "k". It is curious to me why agreement came so easily. Was it because of an implied sense of certainty in the declarative statement? Had STUDENT32 instead posed a question of correctness, "Should we do 13 by 13?" would this have led team members into a negotiation of why not to do so? Given that the "k" agreement is almost dismissive, as though it was more convenient for STUDENT12 to agree than to reflect upon the proposal, my focus turns to whether STUDENT32 reflected upon her own proposal. This remains unknown because the digital space's chat field does not include self-chat.

In contrast to the team above-noted, Team 1 decided to work with 120 chairs in its solution to Problem 2's graduation ceremony. Accordingly, STUDENT11 was bold to propose, "13333333 times 9= 120", rather than ask a question of correctness, "What do you think about 13333333 times 9 = 120?" But, STUDENT11's proposal was still taken as an invitation to negotiate, to which STUDENT21 replied as follows:

STUDENT21: uhh are u sure
STUDENT11: i think
STUDENT11: yeah i am sure
STUDENT21: i think its 1.3333333x9

Table 5.1 that follows summarizes the appeals around correctness in both face-to-face and online environments.

Types of Appeals	**Face-To-Face**	**Online**
Reassurance/Validation from Teammates	✔	✔
Request for Team Verification of Guess Without Inward Appeal	✔	✔
Inward Appeals, Self-Negotiated	✔	✖
Proposal for Negotiation	Question	Declaration

Table 5.1 – Questions of Correctness – Face-to-Face and Online

Note that Team 1's negotiation above offers additional insight into questions of correctness in the online space. If many of these types of questions in face-to-face interactions were uncertainties of computational correctness, then use of the whiteboard's Wolfram Alpha tool may eliminate these types of uncertainties, so that students can confidently offer propositional statements to team members after internally verifying their guesses. Whether this represents a reliance on technology or a confidence in technology on Kiran and Verbeek's (2010) spectrum of trust, it is still a matter of trust in technology.

Although participants were provided access to calculators in their face-to-face engagement, it is not evident to what extent they were used. Is there a difference in the level of confidence inspired by Wolfram Alpha versus calculator technologies? Or, perhaps the physical movement between chat field, whiteboard space and Wolfram Alpha tool in the online space is achieved more fluidly than putting aside a written solution to grasp and use calculators or other concrete manipulative tools in face-to-face engagement. This proposition finds support in Hsi's (2007) criteria that "Digital kids demonstrate fluency by simultaneously operating and managing multiple devices and multiple media types…" (p. 1514). The ability to multi-task and switch attention was discussed in Chapter 4, as part of the theme of Concentration. Arguably, it is through multi-tasking in the online space that students internally verify their guesses. What may appear as off-task behaviour may ultimately serve *Habits of Mind* (Cuoco et al., 1996) and Piaget's (1973) goal of intellectual autonomy.

To summarize, the online environment did include guesswork without self-verification, as well as explicit requests for validation, as in face-to-face engagement. But, students' online chat contained proposals in declarative form. Whether this is so because fewer uncertainties of computational correctness existed in the online space, or whether students directed questions of correctness inward prior to making these proposals, is not evident. So, although participants appeared to more boldly propose ideas in the form of declarative statements, perhaps a window to the student's mind afforded by face-to-face engagement is lost in online space.

Now, students' lines of questioning as they pertain to a mathematics problem's conditions will be examined for both face-to-face and online community interactions.

Face-to-Face Questioning of the Problem's Condition

Team 1. From the start, members of this team noted the significance of the $8.00 stated in the problem.

STUDENT21 (00:40): So, total money was eight dollars. (student whispers, "So that's eight dollars") So, we'll do all of…
STUDENT31 (01:02): …eight dollars. (voices echo, "Eight dollars")

Then, STUDENT21 raised the possibility of an $8.00 limit, with the words "add to".

STUDENT21 (01:15): So, you start with all orange creams? (Scribbling noises) Wait, how many orange creams are…add to eight dollars?

Subsequently, STUDENT11 took the $8.00 as given, and proposed a halving and doubling strategy.

STUDENT11 (01:55): For me, I can cut the eight dollar into half.

But, later in the conversation, after undertaking a repeated addition of orange creams, each valued at $0.60, STUDENT21 revisited an interpretation of the $8.00 condition, through a question.

STUDENT21 (05:50): Wait, does it have to…does it have to…have to add up to eight dollars?
STUDENT31 (05:56): Yeah.
STUDENT21 (05:57): Oh, but this can't work. Might not work.

STUDENT31 agreed with STUDENT21's interpretation of the condition, though the rationale for this agreement was not stated. From this affirmation, STUDENT21 realized that a solution of all orange creams "can't work. Might not work." The team's dialogue proceeded with members constructing and deconstructing $8.00 using combinations of orange creams and truffles.

The final solution included five possible solutions, four of which are correct (two that were the same), and a statement that "all creams don't work because it will go over $8.00". Although interpretation of the $8.00 condition seemed to be shared given that each of the combinations in the team's solution came to a total of $8.00 only, what remains unclear is how the team came to share this interpretation. Possibly, members interpreted the $8.00 as a condition because it was the only numeric value stated in the problem. Alternatively, each student deemed $8.00 (and only $8.00) as a condition because of the phrase "If he spends all of his earnings", but did not explicitly state this. Finally, it is possible that individual members felt social discomfort around challenging the interpretation of their peers. In this team, STUDENT21 did question his interpretation of the problem's condition, and the team solution seemed to indicate a shared understanding to fulfill such a condition, but why this was so did not necessarily implicate an appropriate line of reasoning. Using Krummheuer's (2007) analysis of Toulmin's scheme, Team 1's interaction is represented in Figure 5.1 below.

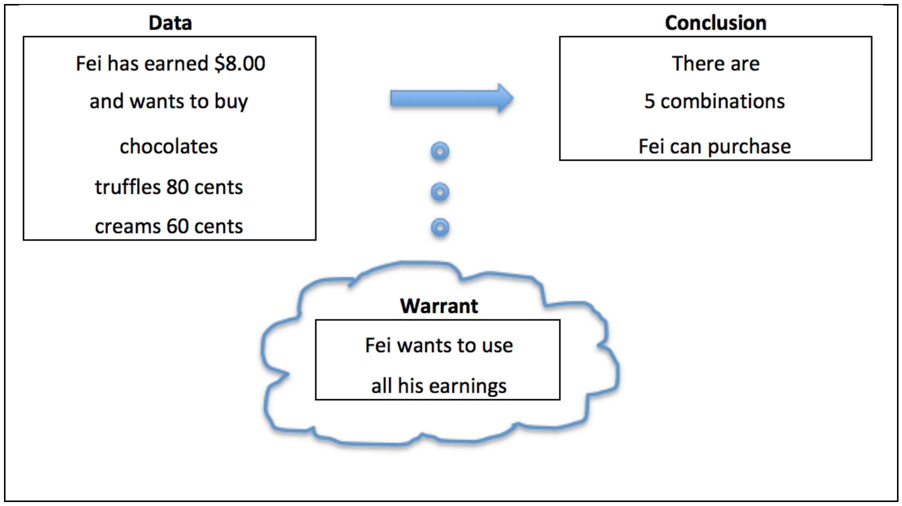


Figure 5.1 – Team 1 Solution – Face-to-Face

While the team's conclusion of "5 combinations" was supported by data from the problem's wording, "Fei has earned $8.00", its recognition or use of the condition that "Fei wants to use all his earnings" was unclear. STUDENT21's question had potential to elicit a statement of the rationale that would explicate the warrant, but this potential remained unrealized. So, the warrant statement in the diagram above was placed in a thought bubble to represent how its presence, though not explicitly stated, may have contributed to the team solution. Note that a diagram of the team's argument may differ from an individual team member's internal argument. Sharing the same conclusion would not necessitate a shared recognition and use of the warrant statement. What the individuals of this team interpreted from the team's interactions and how these interpretations would influence their individual acts in an independent solution would require further study.

Team 6. Team 6 also worked in face-to-face engagement to solve the chocolate purchase presented in Problem 1. Early in their interaction, two members read aloud the problem, but after each reading, students proposed a computational strategy and busied themselves in using classroom calculators. In the second facilitation of their discussion presented below, I assumed the responsibility of questioning the team's interpretation of the problem's conditions.

> KPain (18:17): How much has he earned? (The group says, "Eight dollars")
> KPain (18:20): And how much does he want to use?
> STUDENT26 (18:23): It doesn't say. (Pauses and rereads problem aloud)

Even in the face of my pointed questions like, "What does that last paragraph say?" team members were unsure. Examine STUDENT26's following response when I asked how much of Fei's earnings he wished to use.

> STUDENT26 (19:26): As much as he can? (Pauses while STUDENT36 says, "All of it") Probably not all of it, because it says, "If he spends all of his earnings".

Then, STUDENT16 asked the question below with regards to the problem's statement, "*If he spends all of his earnings…*"

> STUDENT16 (20:05): 'All', hold on, that one's giving us a clue. 'All'. Right?

Despite the appeal "Right?" the group did not seem to reach a shared interpretation of the $8.00 condition. Later, STUDENT36 appeal tried to once more make sense of the problem's condition.

> STUDENT36 (21:52): It doesn't say like an actual price. It says, "If he spends all his earnings". He wants to spend all of it then?
> STUDENT16 (22:00): Yeah!
> STUDENT26 (22:03): No, he doesn't. (Three students speak simultaneously)
> STUDENT36 (22:10): It says, "If he spends all of his money". The "all" gives you a clue.
> STUDENT26 (22:13): But, it says "if". "If" means maybe, too.

Confusion lay in the language of probability: certainty implied by the word "all" clashed with a level of possibility implied by "if". This team's subsequent solution reflected this contradiction. On one hand, STUDENT26 correctly calculated 10 truffles as a solution, meeting the problem's condition. On the other hand, the group's conclusion that, "You can only get 13 chocolate creams which is 780 because 14 X 60 = 840 so you can get 13." did not meet the condition that only the full $8.00 can be spent. Figure 5.2 below represents Team 6's argument with respect to Krummheuer's (2007) scheme of data, conclusion and warrant.

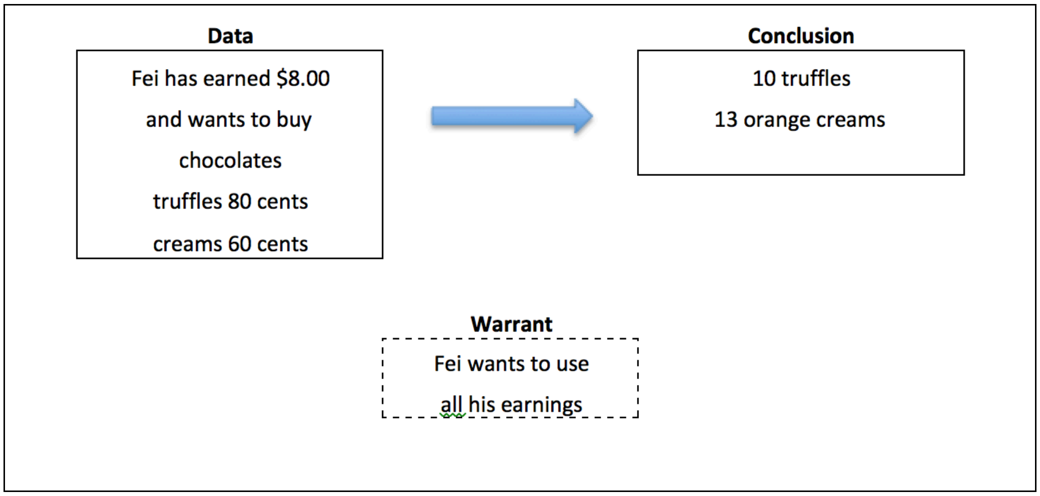


Figure 5.2 – Team 6 Solution – Face-to-Face

In their attempts to make sense of the problem's "clue", team members accepted a growing responsibility to question their peers. However, they were unable to reconcile the language of 'if' and 'all' to reach a shared understanding of the condition as a precursor to determining how to meet this condition. This inability was reflected in the teams' solution of two combinations, one which corresponded to 'all' and the other which corresponded to 'if'.

Even before the problem's condition could be regarded as the warrant that, upon satisfaction, linked data to conclusion, this team needed to construct a shared understanding of the condition. For this reason, the diagram in Figure 5.2 above shows a dotted border around the warrant statement still being constructed, and does not indicate a sense of direction from the warrant to other elements of the team's argument.

Team 2. Team 2 was the third of three teams to solve Problem 1 through face-to-face engagement. Team members attempted to negotiate the problem's condition by asking questions of each other.

STUDENT32 (06:50): Yeah, he has to use all of his money. Did he use all of his money?
STUDENT22 (06:53): He did. Does he need…all of his money?
STUDENT12 (06:54): No, wait, no. (students speaking simultaneously)
STUDENT32 (06:55): Yeah, he needs to use all of his money.
STUDENT22 (06:57): All of his money?

As demonstrated by the following transcript, STUDENT32 repeatedly questioned solutions proposed by STUDENT22 in an effort to revisit STUDENT22's interpretation of the problem's condition.

STUDENT32 (07:51): No, no, no. Um…he spends eight…like one truffle's eighty cents. But, where does it say that he used the exact amount of money?
STUDENT32 (29:01): Oh, so it's not, but that's not full money. 'Cause then how would we be able to do that?
STUDENT32 (30:37): Yeah, but how will this equal up to eight dollars?

Despite STUDENT32's ongoing efforts to negotiate and use the problem's condition, a shared interpretation of this condition did not appear to have emerged, as indicated by STUDENT22's question to clarify this once more.

STUDENT32 (31:30): Yeah, you can. But, you're not…but it says, "He uses all (emphasis) of his earnings."
STUDENT22 (31:55): He uses all of his earnings?

In the team's solution, appearing in Figure 5.3, only one of three solutions presented, that of "10 truffles", is correct.

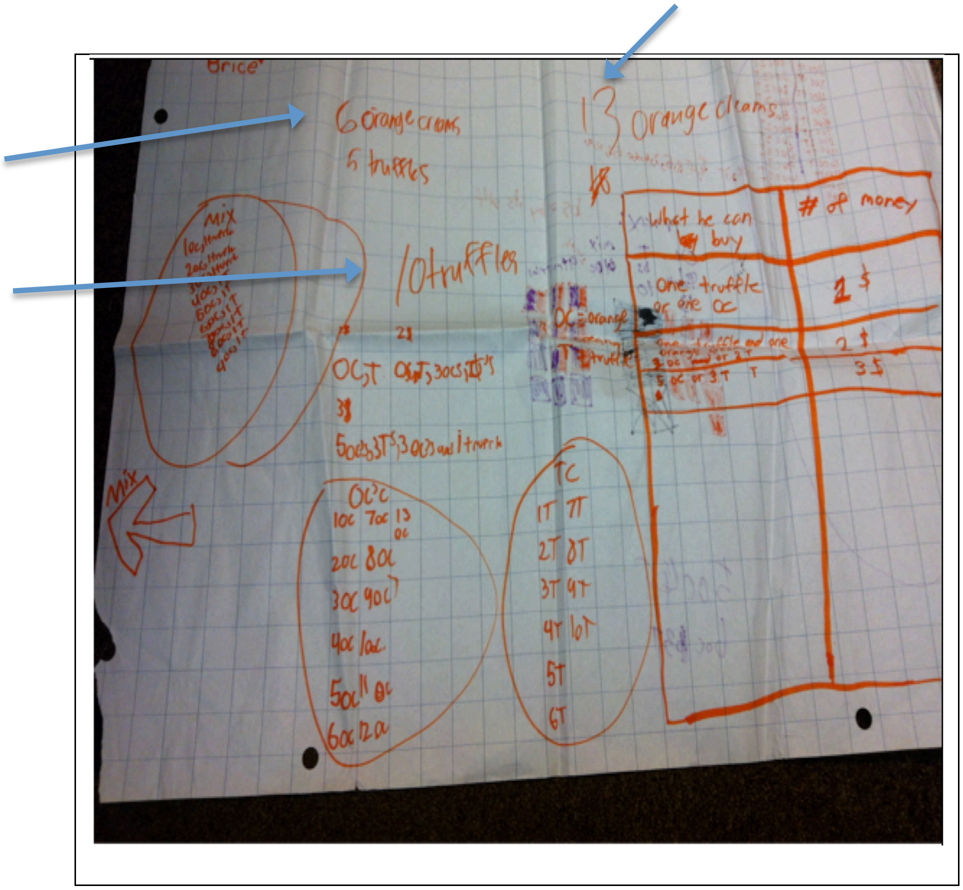


Figure 5.3 – Team 2 Solution

However, it is unclear whether the three combinations written on the team's solution page were put forth as the team's final solution in light of the realization made by STUDENT22 close to the end of the team's audiotape.

> STUDENT32 (50:11): Like, he has to spend all…
> STUDENT22 (50:14): (Interrupts) He has to spend all of his money to do this.
> STUDENT32 (50:15): …instead of…just some.
> STUDENT22 (50:16): So, that was a mistake. I didn't read through the thing properly, when we're doing that. So, that's the reason why we tried doing that. And that didn't work out.

Team 2's interaction as mapped onto Krummheuer's (2007) analysis in Figure 5.4, shows a missing link as indicated by the star symbol. STUDENT32 not only questioned the team's interpretation of the problem's condition, she also asked questions to challenge the validity of solutions proposed by the team. Challenges such as, “Yeah, but how will this equal up to eight dollars?” sought proof that solutions met the problem’s condition of $8.00. STUDENT32 used the data from the problem’s statement and together with the condition that the entire amount earned had to be spent, concluded that $8.00 should be spent in the purchase of chocolates, with no change. Had the team used STUDENT32’s conclusion towards its own data, it may have reached conclusions that correctly satisfied the problem’s condition. However, this did not happen. Instead, the team’s data remained unmodified from the problem’s original statement, and the team reached conclusions supported by a different warrant, one of purchasing only truffles, only orange creams or purchasing a mix. There lies a gap between STUDENT32’s own conclusion and that of the team’s, the result of different interpretations of the problem’s $8.00 condition.

This difference was pervasive throughout most of this team’s problem-solving session. By the end of their interaction, however, STUDENT22 and STUDENT32 seemed to have agreed upon an interpretation whereby only solutions using the entire $8.00 of earnings were acceptable. But, the voice of one remaining team member, STUDENT12, was absent in this negotiation. For this reason, it would be premature to suggest that all the team’s members ended with a shared interpretation of the problem’s condition.

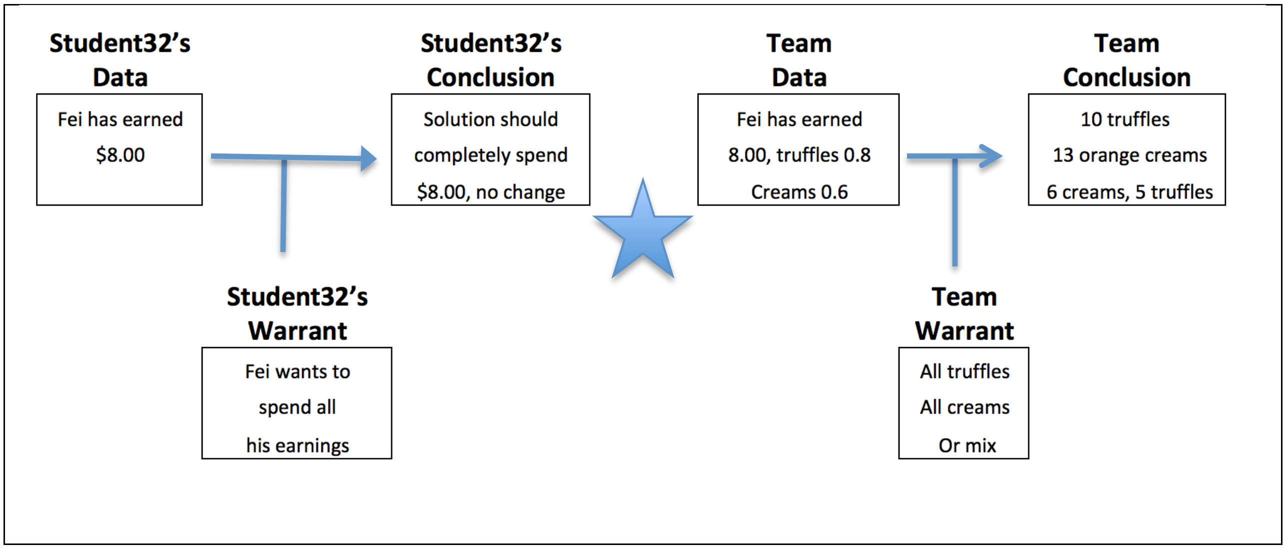


Figure 5.4 – Team 2 Solution – Face-to-Face

In reviewing Figure 5.4, I wonder just why there was a missing link between STUDENT32's conclusion and the team's data, especially in spite of repeated efforts made by STUDENT32 to return the team's attention to the problem's condition. If children hear each other's messages, what keeps them from listening for meaning? Admittedly, this question presumes that vocabulary is shared and that language (or other) barriers do not block messages spoken from being heard. But, given a common language, perhaps students ignore their peers' indications in favour of ego that prevents them from admitting what they fail to understand. Maybe there are elements of bias that lessen the credibility of certain speakers, and dull the strength with which we listen to their messages. Alternatively, students may abandon efforts to reflect deeply on the messages they receive, overwhelmed by the demands that such an act of reflection might entail. It is no small feat to perform processes of inference, application and connection simultaneously, sometimes instantly. In identifying potential reasons for the missing link between hearing and listening for meaning, I humbly admit that the school day is fraught with such missed opportunities. How we, as educators, facilitate students to recognize and benefit from the messages of a classroom community presents an avenue of future study.

The distinction between univocal and multivocal explanation provided by Cobb (1995), presents another view of the interactions between Team 2's members. In transcript notes shown below, dialogue between STUDENT12 and STUDENT22 appear as univocal explanations in which, "one child judges that the partner either does not understand or has made a mistake, and the partner accepts this judgment" (Cobb, 1995, p. 53). Consider the following transcript:

> STUDENT12 (26:06): How much does it take?
> STUDENT22 (26:09): Okay, there you go. One truffle, there you go. So, you got… the one truffle's there. Now, write in a separate spot, one orange cream.
> STUDENT12 (26:21): Oh, you have to do orange cream now?

Although STUDENT22 was equally determined in interactions with STUDENT32, the difference constituted a multivocal explanation where, "…both children attempt to advance their perspectives by explicating their own thinking and challenging that of the partner" (Cobb, 1995, p. 53). In the author's case studies, it was multivocal explanations, like those between STUDENT22 and STUDENT32 of this study, which presented greater learning opportunities than univocal ones. On one hand, Team 2's members had not reached a shared understanding of the problem's $8.00 condition. On the other hand, through STUDENT32's repeated challenges of STUDENT22's position, both children may have engaged in a stronger learning experience. To shift community interactions from univocal positioning to those that are multivocal is also a topic of future study.

At this point, having analyzed students' face-to-face questioning of the conditions of Problem 1, I will use this focus to examine the online interactions for three teams as they solved the same problem.

Online Questioning of the Problem's Condition

Team 3. Online chat from Team 3's whiteboard space indicates STUDENT13's three attempts to negotiate the problem's condition. First, STUDENT13 questioned STUDENT33's proposed solution totaling $7.60 (shown in Figure 5.5 below) with a simple "what", justifying the challenge by stating, "but the question says to use all the money…there cannot be change…too low". As indicated in the following section of the chat field, when team members ignored a second attempt to question the problem's condition, STUDENT13 shared a potential solution, but asked for clarification once more.

STUDENT13: canthere be change?
STUDENT13: sorry guys!
STUDENT13: i am showing 13 creams = $7.80
STUDENT13: canthere bechange
STUDENT23: nmoooo
STUDENT13: sooo no change?

STUDENT23's reply "nmooo", provided no rationale, still leaving STUDENT13 uncertain. But, through his later suggestion to "write that there would be more (solutions) if there could be change", it could be inferred STUDENT13 had resolved this uncertainty. Morever, STUDENT23 appeared to share this interpretation when addressing STUDENT33's statement below that there were three solutions on the final page of the team's whiteboard solution.

STUDENT33: 3 on the last page
STUDENT23: we are not aloud change

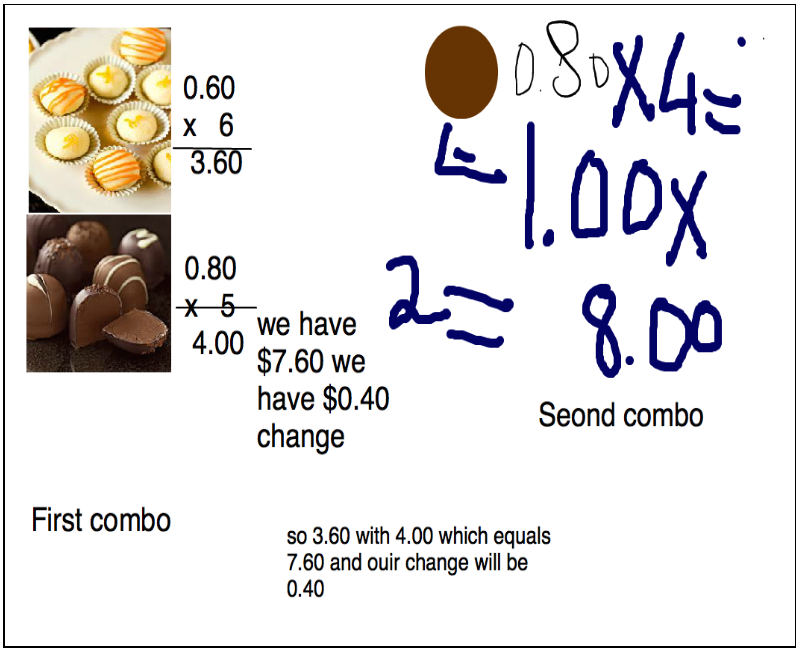


Figure 5.5 – Team 3 Online Solution Screen 3

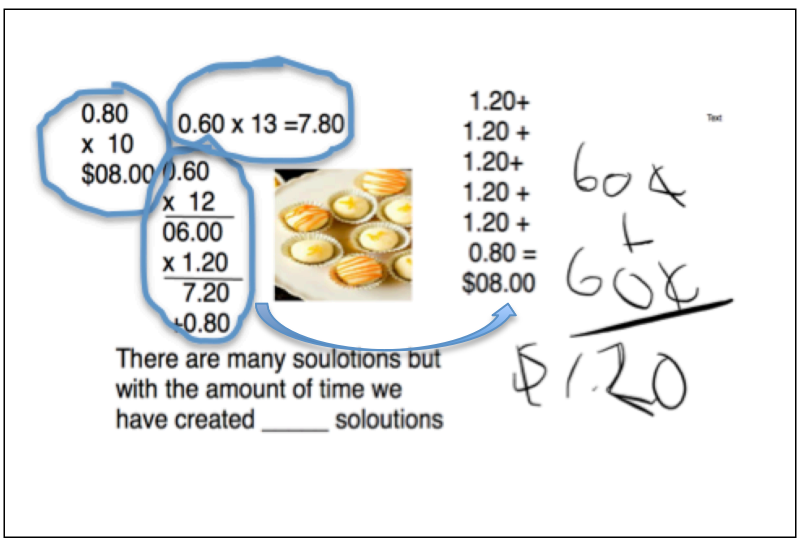


Figure 5.6 – Team 3's Three Solutions - Online, Screen 4

The solution in Figure 5.6 above, with 12 orange creams and 1 truffle, fulfills the $8.00 condition, as does the solution of 10 truffles, each costing $0.80. However, the third solution of 13 orange creams each for $0.60, results in $0.20 in change, and so, does not correctly fulfill the condition of this problem. Though not explicitly referred to by STUDENT23, it is reasonable to infer that his comment, "we are not aloud change" applies to the third solution of 13 orange creams. What remains uncertain is whether STUDENT33 agreed with this interpretation of the problem's condition that had come to be shared by teammates STUDENT13 and STUDENT23.

Members of this group did question the problem's condition in their online chat, and in stating rationale from the problem's statement, "but the question says to use all the money", they negotiated what appeared to approach a shared interpretation of the problem's condition. The explication of members' rationale is a distinguishing feature between this team's interaction, and that of Team 1 in the face-to-face environment. Team 1 did accomplish a complete solution with all its possible combinations meeting the problem's condition; however, the team's rationale for what appeared to be a shared understanding of the condition, was unstated. Although Team 3's solution did not encompass all possible chocolate combinations given the problem's condition of $8.00, this may have been an issue of time, as indicated by the conclusion "but with the amount of time" appearing in Figure 5.6 above.

Team 4. Similar to Team 3, one member in Team 4's online chat posed a question to negotiate the problem's $8.00 condition with other team members, and provided a rationale for her own interpretation, using the text from the problem's statement.

STUDENT34: 14 question says if fay or fei spend all his earning (names student in another group) tell me no change i tink she rit
STUDENT34: wadyatink
STUDENT34: (what do you think)

STUDENT34 above claimed additional support for this interpretation through corroboration with a member of a different team. Figure 5.7 below uses Krummheuer's (2007) analysis to present a diagram of STUDENT34's argument with its data, warrant and conclusion. Corroboration with another team was the data, which when joined by the warrant of the problem statement, led to STUDENT34's conclusion of a purchase with no change.

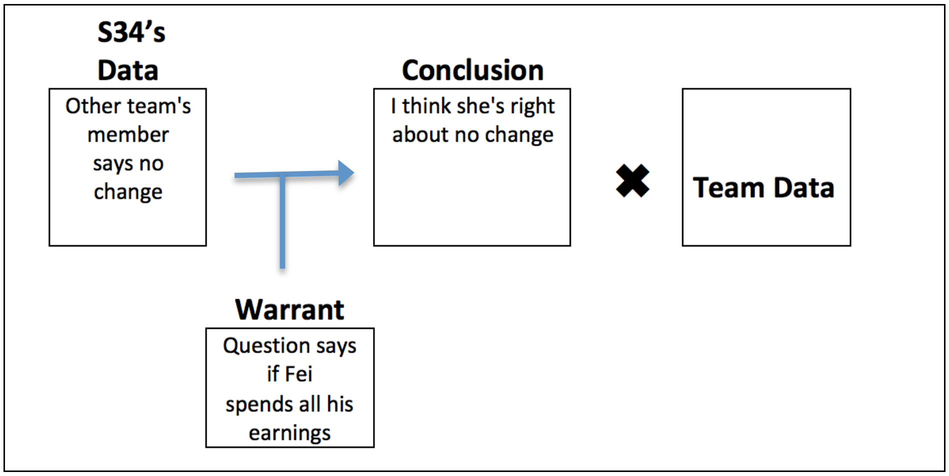


Figure 5.7 – STUDENT 34's Online Argument

By Krummheuer's (2007) proposal, STUDENT34's personal conclusion may have become the data that served the formulation of the whole team's conclusion. But, there were two indicators to suggest that this likely did not transpire. First, the team's chat field showed no response to her question and its accompanying rationale. This fact, in and of itself, would not have been enough to indicate that STUDENT34's individual conclusion was not considered, particularly in light of the deletion issue around online contribution as noted in Chapter 4. But, the second indicator comes by way of the team's whiteboard space, below in Figure 5.8. Solutions on this space contradict a shared interpretation of the problem's condition of "no change", as concluded by STUDENT34.


1st combination:
13 orange creams: $7.80
Total: $7.80
Change: $0.20

2nd combination:
10 truffles: $0.80
Total: $8.00
Change: $0.00

3rd combination:
6 orange creams: $3.60
5 truffles: $4.00
Total: $7.60
Change: $2.40

4th combination:
1 orange cream:
$0.60
9 truffles: $7.20
Total: $7.80
Change: $2.20

5th combination:

Figure 5.8 – Team 4 Online Solution Page Screen 10

Note that of the team's four solutions on the left-hand side of Figure 5.8 above, only the second combination of "10 truffles" meets the requirement of fully using the $8.00 of earnings. The remaining solutions are calculations of a box of orange creams only, and two boxes of mixed chocolates, each costing over $7.00. However, they do not reflect use of STUDENT34's earlier conclusion. I argue that the data for the formulation of the team's conclusion remain incomplete, leading to an incorrect solution. While the problem's condition had been questioned, members did not appear to have engaged in its negotiation. Consequently, as evidenced by the team conclusions in its solution space, a shared interpretation of the condition is not evident. This team's online interaction seems comparable to that of Team 2 in the face-to-face environment given the missing link between one member's conclusion, and the team's data.

Team 5. A question was also posed at the start of Team 5's chat field, but it was one that questioned a *requirement of representation*, rather than a condition. STUDENT25 sought to negotiate what the team was required to demonstrate in its representation of the solution, rather than the terms that need to be fulfilled in arriving at that solution. Moreover, it is not clear in STUDENT25's use of "explain", whether the student was proposing a clarification of some sorts, or a statement to account for the team's thinking, as differentiated by Cobb, Yackel, Wood and Macneal (1992).

STUDENT 25: dont we have to explain
STUDENT15: NO
STUDENT45: YES

Unlike the other two teams working in an online environment, no member of this team appeared to have questioned the problem's condition. The whiteboard space, as shown in Figure 5.9, does include a notation of "8.00 is the total" and states, "so if he used all of his money…" in correctly calculating a solution of only truffles. Still, this does not implicate a shared interpretation that <u>all</u> solutions had to use the full $8.00 of earnings. In fact, the whiteboard space from Figure 5.9 contains the following two calculations that do not meet the problem's $8.00 condition:

"0.60 times 13 = 7.80 cents" and,

"he can get three chocolates and three truffles left over 60 cents left over"

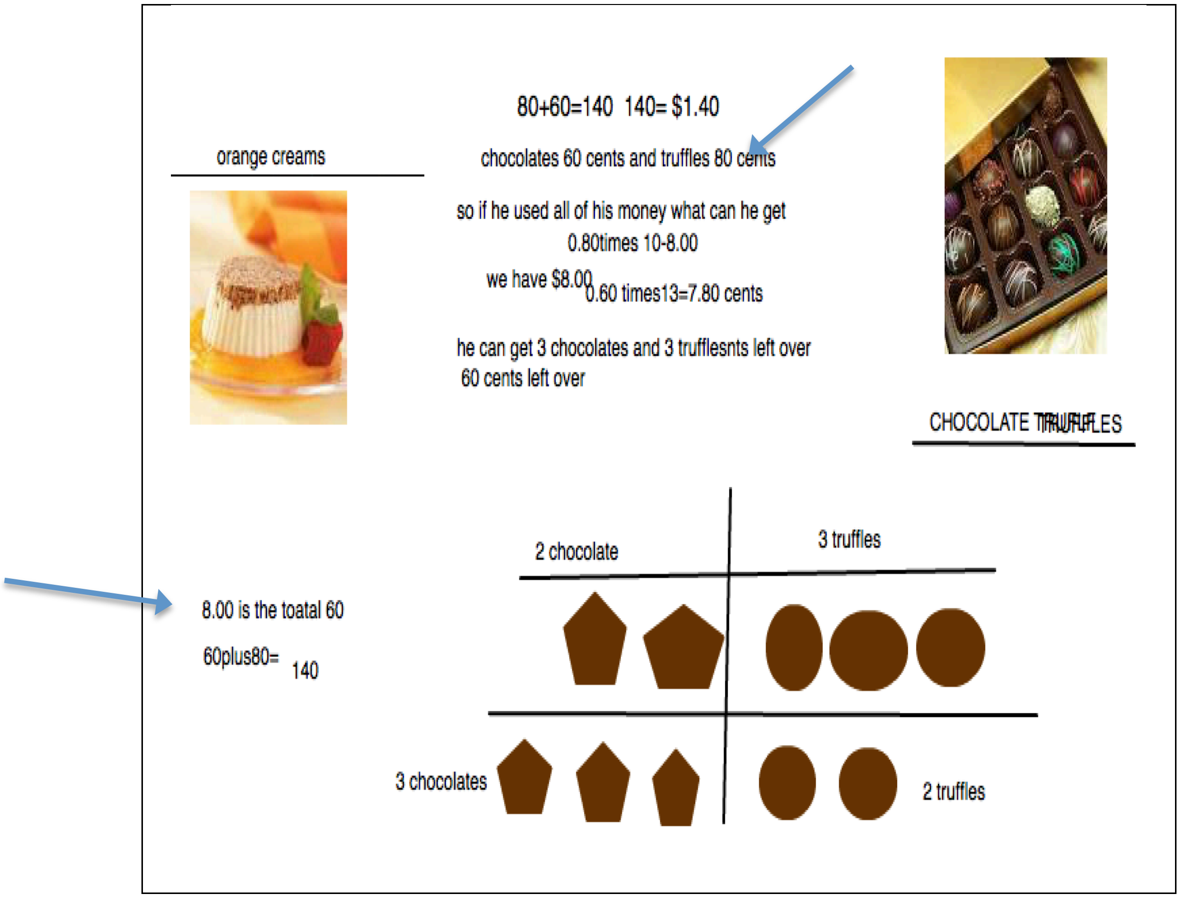


Figure 5.9 – Team 5 – Online Solution

It was not evident at any point in this team's interaction its members questioned the problem's condition amongst themselves. I suggest this is because the $8.00 condition was interpreted as a piece of data, rather than a warrant that would legitimize the team's conclusions. It should be noted that two members of this team used the chat field to engage in an argument unrelated to the mathematical task. Quite possibly, these two individuals paid little notice to the whiteboard space's solutions which, had they engaged, may have led to some fruitful dialogue around interpretation of the problem's condition. As it stands, it is unclear from the online space whether all members shared the same incorrect interpretation of the problem's condition.

Summary

In the act of posing questions and communicating their uncertainty, students invite mathematical negotiation, one of the four salient features of a mathematical community, as outlined in Chapter 2. In particular, students' questioning of a problem's conditions facilitates their ability to negotiate an extension to Cobb and Yackel's (1996) definition of a sociomathematical norm, namely what counts as a mathematical condition, and what is an acceptable response to such a condition. In the preceding analysis, both face-to-face and online environments were examined from the perspective of student questioning of a problem's condition. The results of this analysis have been summarized in Table 5.2 below.

Team	**Environment**	**Questioning the Problem's Condition**	**Negotiation of the Problem's Condition**	**Application of the Problem's Condition (>1 correct solution)**
Team 1	FTF	X		X
Team 2	FTF	X	X	
Team 3	O	X	X	X
Team 4	O	X		
Team 5	O			
Team 6	FTF	X	X	

Table 5.2 – Summary of Student Questioning of Conditions

Encouragingly, the environment of a team's interactions did not prevent questions from being posed, as indicated by the observation that members in five of six teams asked questions about

Problem 1's conditions. An example of such a question in a face-to-face environment was demonstrated by STUDENT16's appeal, "'All', hold on, that one's giving us a clue. 'All'. Right?", while in the online environment, STUDENT13 simply asked, "what" and then proceeded to justify this challenge. Of importance is Team 5's lack of questioning around the problem's condition, suggesting that the $8.00 condition was considered as a piece of data, rather than as a warrant statement to legitimize the team's conclusion.

The presence of a question around a problem's condition does not automatically translate to a negotiation and appropriate application of this condition. As a start, consider three teams' face-to-face engagement. In the case of Team 1, the transcripts and the team solution implicate a shared understanding and fulfillment of the problem's condition, but it is unclear on what basis this shared understanding emerged because there was no explicit negotiation of the condition. Though members of Team 6 did engage in negotiating the implications of 'if' and 'all', they were unable to share the meaning of the condition's language, resulting in an unsuccessful application. Similarly, members in Team 2 did argue about their interpretation of the condition, but there remained a gap between one individual's conclusion and the team's conclusion such that application of the problem's condition was incorrect. Such a gap was also identified in Team 4's online chat field, although the difference between this interaction and that of Team 2 lay in the absence of negotiation by Team 4's members. On a positive note, of the two teams that questioned, negotiated and correctly applied the problem's condition, one did so via face-to-face engagement, and one did so through online interaction. Although Team 3's online solution was quantitatively less complete than Team 1's solution generated in face-to-face interaction, this appears to be a function of time.

If two of three teams asked questions about the problem's condition in their online engagement, why was it that only one of the two entered negotiation of those questions? From an analysis of the online environment emerges a sense that the interactive whiteboard may have represented a *common* space rather than a *shared* one. Before explaining the last statement, it is necessary to return to Blumer (1969) and symbolic interactionism.

Let me say, for example, that STUDENTA asks STUDENTB and STUDENTC a question to interpret the problem's conditions. STUDENTB and STUDENTC will respond, indicating their intent, at which point STUDENTA will need to interpret their indications to make meaning of the problem's condition. Based on the meaning that comes out of interpretation, STUDENTA will act on the condition, making indications that STUDENTB and STUDENTC must then interpret.

For students to convey indications to their peers in the online environment, consider Brown's (2006) description of multimedia literacy, which states, "For example, we all know how to make arguments in text, but how should we make an argument visually? More generally, how can we communicate effectively using image, text, sound, movement, sequence, and interactivity?" (p. 21). Furthermore, an act of interpretation requires reflection: each of the three students in the above example will need to reflect on the indications of the others to interpret these indications and make meaning of objects, in this case, the problem's condition. One person's reflection, interpretation and the resulting action must then be reflected upon and interpreted by another person in determining their own action. This interactivity is what makes a *shared* space, a space of between-ness. Without reflection, interpretation and action between us, the space we occupy together is simply a *common* one.

In the digital environment, students occupied the same space on the whiteboard, but do not appear to have reflected upon each other's indications to make meaning of the problem's condition and determine their own action accordingly. I have some hypotheses around this appearance, and why perhaps, they are not prepared to receive the indications sent by their peers. Like their spoken words in face-to-face engagement, written text can be put into a digital space, but lingers there until someone uses them. If students are in the process of tuning the different combination of senses required to function in their community's new locale, they may be unprepared to receive the indications of their community members. Also, the novelty of the virtual space may encourage students' independent explorations to define what they can accomplish in that space before being prepared to join the community. However, there is potential for this to change given time and opportunity to negotiate the space, as well as the social and sociomathematical norms of their online engagement.

Another line of questioning examined in this chapter has been around students' uncertainty of correctness. In both environments, team members demonstrated guesswork without self-verification before posing such appeals of correctness of their peers, and made explicit requests for validation. But, the online space reveals propositions written in declarative form, though whether this was the result of uncertainties removed by digital tools, or whether students internally verified their proposals prior to placing them in the chat field, remains unclear and is worth future study.

Chapter 6

Limitations of The Study

This study was designed with the rationale discussed in Chapter 1 and undertaken in the spirit of informing mathematics and/or digital education. Chapters 4 and 5 analyzed the data that were collected as students solved mathematics problems through face-to-face and online community engagement. In this chapter, I address the limitations of my research.

Students' ability to delete objects and chat field comments in the online space presents a methodological limitation of this study, a question of the completeness of data collected. Participants' survey comments and the normative implications of deletion have been duly noted in Chapter 4. With regards to Chapter 5's analysis of students' online questioning of a problem's condition, the completeness of data collected is relevant only to results where an absence of online negotiation or application is noted, which is only the case for Team 5. Given that this team's whiteboard and chat fields are consistent, and that they include the non-mathematical arguments of two individuals, something that could have easily been eliminated had team members chosen to do so, I believe the chat field data to be intact.

Heersmink (2011) defines extension theory as "…a set of related frameworks that argue that technology is somehow an extension of the human organism" (p. 121). In the design of the online space for individuals to engage in mathematical problem-solving tasks, I sought to extend the student community and its interactions. Although Scribblar's whiteboard software allows me to extend the salient features of a mathematical community to the virtual realm, it does not yet have the capability to extend our classroom community's negotiated social norms around deletion and respectful contribution to the digital space. This is a significant normative and methodological shortcoming, rendering student discourse and whiteboard contributions insecure.

In selecting a different online interface for the classroom community, I would extend students' face-to-face engagement to a virtual environment that includes: a personal space within the community for the *First Me, Then We* student to organize and manipulate her own ideas; access to a translation tool for ELL participants; a voice-to-text tool to scribe student discourse; and three-dimensional sketching capabilities to represent the team's geometric reasoning.

Time represents a second limitation of this study. Most participants had engaged in face-to-face problem-solving tasks most days of the week, over the course of an entire school year. In contrast, students' online engagement had taken place a handful of times over two months. Consequently, participants were on different continuums in their negotiation of social norms and sociomathematical norms for each environment.

A third limitation is the length of this study, which comprised a single problem-solving task in each environment. Social and sociomathematical norms are negotiated through a variety of mathematical problems. So, future studies that collect data over an extended period will provide students time and mathematical opportunities to negotiate norms in both environments and may offer greater generalizability about their community engagement.

The small sample size of this study presents its final limitation. Nineteen participants were placed into six teams, only half of which worked in each form of community engagement at a time. Larger-scale studies in the future, with a greater sample size of participants, have the potential to more meaningfully represent student populations.

In summary, although the methodological limitations of Scribblar's digital whiteboard are largely mitigated, future research should investigate the use of a different software interface to extend community interactions into virtual space. Longer and large-scale studies will promote in-depth data analysis and greater generalizability for mathematics and digital education.

Chapter 7

Future Avenues of Study and Conclusions

The previous chapter analyzes how students question the conditions of their mathematical problems in face-to-face and online interactions, working towards a taken-as-shared sociomathematical norm (Cobb, Wood, Yackel & McNeal, 1992), the definition of mathematical condition. In this chapter, I will begin to examine students' engagement to negotiate the sociomathematical norm of how to use a problem's context, recognizing that this negotiation is incomplete and presents a path for future research.

Face-to-Face Questioning of The Problem's Context

While Problem 1 challenged participants to argue about a mathematical condition, Problem 2 presented its own opportunity for negotiation. To solve Problem 2, teams were required to choose a number of chairs, exceeding 100, and arrange this number into equal rows. Students in all three teams posed questions and argued about using the problem's context to arrive at their solutions.

Team 3. Team 3 decided upon 200 chairs as its choice for the graduation ceremony. In fact, the group acknowledged the context of this problem early on in their comments around choosing a reasonable number.

STUDENT33 (02:20): ...you don't want to get two thousand…
STUDENT23 (02:22): …or a million.
STUDENT13 (02:22): That must be a really big graduation ceremony.

Then, STUDENT33 questioned the team's solution of 10X20.

STUDENT33 (05:30): Wait, does this one mean there are ten rows, or ten things in each row?

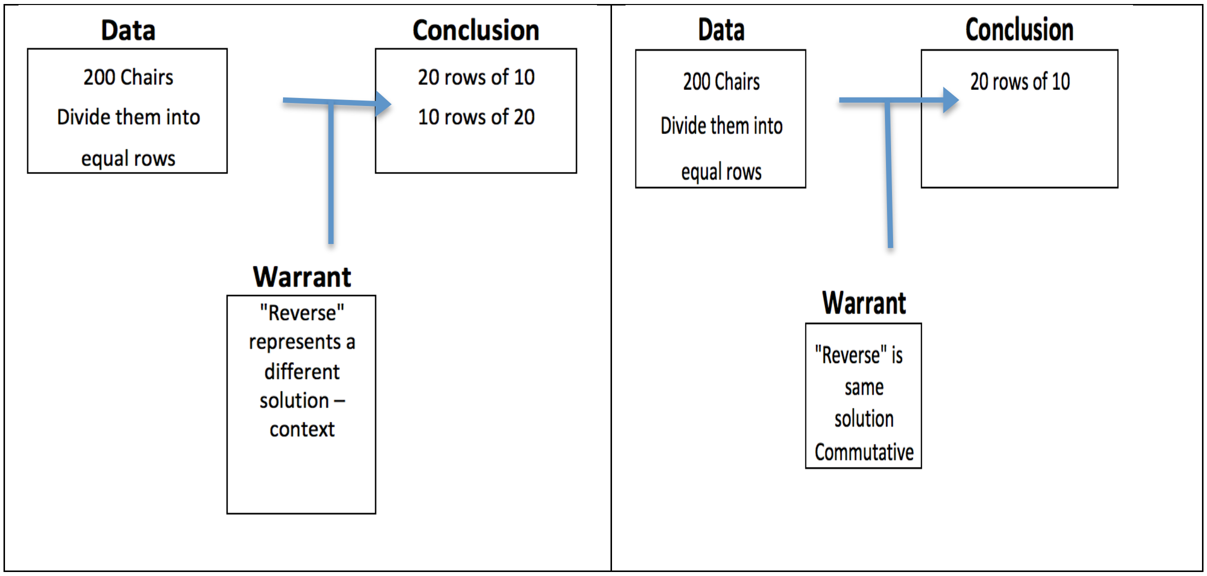
In their response, STUDENT23 and STUDENT13 argued that by the commutative property of number, 10X20 and 20X10 arrived at the same product. For this reason, they were the same solution.

> STUDENT23 (06:20): 'Cause either way…either way it equals the same thing.
> STUDENT33 (06:22): This is…
> STUDENT23 (06:23): Twenty times ten equals two hundred.
> STUDENT33 (06:24): This is…
> STUDENT23 (06:25): …and then ten times twenty equals two hundred.

But, STUDENT33 brought a contextual view to the negotiation, arguing that a configuration of 20 rows, each with 10 chairs was different in appearance from a configuration of 10 rows, each with 20 chairs. STUDENT23's calculator example below that alluded to the object's area. Even if the object were to undergo a transformation, such as "flip", the area would remain constant.

> STUDENT23 (09:19): It's like saying, it's like saying, "Oh look, when the calculator's faced forward, (clicking sound) that's one possibility. Now when I flip it over, it's a second possibility". No, it's the same thing.

Placing the two arguments side-by-side in Krummheuer's (2007) scheme, Figure 6.2 reveals the validity of each. How students define *different* becomes the basis of their arguments' warrant statements, which in turn, accounts for their respective conclusions. STUDENT33 argued that *different configurations* in the context of chairs represented *different solutions*. With this definition as a warrant statement, solutions of 10X20 and 20X10 were seen as different. Alternatively, STUDENT13 and STUDENT23 agreed that if, by the commutative property, rows of chairs had exactly the same area, then this was not a case of different solutions. For these two students, 10X20 and 20X10 appeared to represent the same solution. This team uses the problem's context to negotiate Cobb and Yackel's (1996) sociomathematical norm of what constitutes a *different* solution.



STUDENT33's Argument STUDENT13 and STUDENT23's Argument

Figure 7.1 - Team 3's Solution of Problem 2 – Face-to-Face

Team 4. Members of this team decided to arrange 1000 chairs for the graduation ceremony, justifying their choice by the many possible solutions it would invite. Upon making this choice, they began to divide 1000 by various other numbers, intent on a complete solution. At times, there appeared to be some recognition of the problem's context, like STUDENT24 recognition, "That's a lot of people coming". The team also discussed the inappropriateness of solutions with decimals, given that chairs could not be cut. However, in proposing solutions like, "Twenty rows of fifty. Fifty rows of twenty", team members did not question whether such solutions were different. So, as part of my facilitation of their interaction, I questioned their definition by asking, "Well so, are they different or are they the same?"

Only STUDENT34 maintained her argument that a solution like two rows of five hundred chairs (2X500) was different from a solution of five hundred rows of two chairs (500X2).

STUDENT24 (35:42): Wait, wait. This is the same, but when it's shown, it's in a different way.
STUDENT34 (35:47): So, that's different!

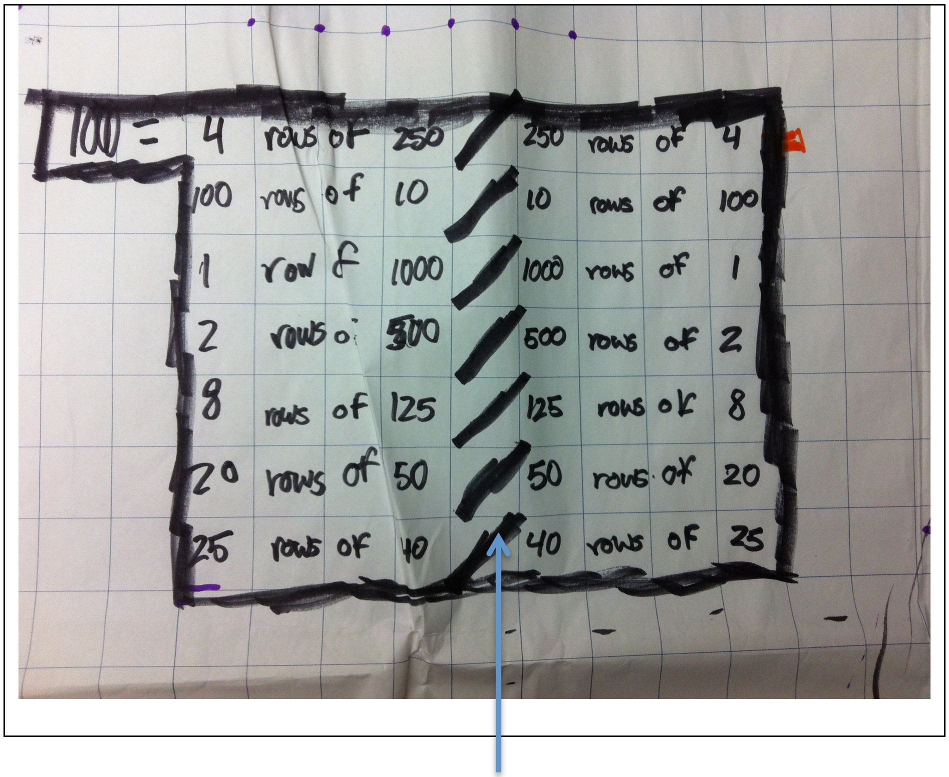


Figure 7.2 – Team 4's Solution of Problem 2 – Face-to-Face

The team's final solution appears in Figure 6.3 above, with its use of backslash symbols to indicate 'or' between two configurations of chairs, and its conclusion that, "They are the same because they all have 1000 chairs in total. They are different because it is showed differently…set up differently". Most striking about this team's interaction is its focus on the mathematical structure of this problem, with less emphasis on the reasonableness of their solutions based on the problem's context. As with Team 3 above, Team 4 has used the problem's context to negotiate a sociomathematical norm of what constitutes a *different* solution

(Cobb & Yackel, 1996); however, this team explicitly uses the problem's context in its determination that only whole-number solutions are possible, since chairs cannot be cut.

Team 5. When the team members of Team 5 first considered a configuration of 60X2 for their choice of 120 chairs, they appear to have distinguished between two rows of sixty chairs and sixty rows of two chairs. STUDENT45's question articulates the team's dismissal of the latter configuration because of its unreasonableness in this context.

STUDENT35 (06:50): You can't do sixty rows. There's sixty chairs (laughs).
STUDENT15 (06:58): You gotta do two rows of sixty. It's two rows of just sixty.
STUDENT45 (07:05): You can't have sixty rows of chairs….Why would you have sixty rows of chairs? For pieces of chairs.

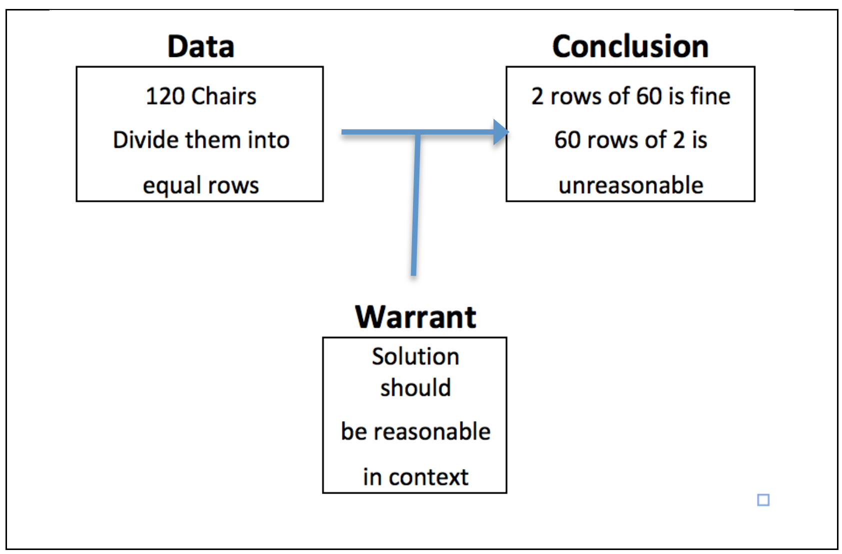


Figure 7.3 – Team 5 Warrant of Reasonableness – Face-to-Face

Representing Team 5's argument in Krummheuer's (2007) analysis (Figure 6.4) shows a warrant statement that appears to have been negotiated about what is reasonable in the problem's given context. Students referred to two solutions as being different, two rows of sixty chairs and sixty rows of two chairs. Despite regarding these two configurations as different, Team 5's members did not take both to their final solution, deeming that sixty rows of two chairs was unreasonable in the context of a graduation ceremony. Team 5 appears to position the problem's context as central to defining a *reasonable* solution, a use that would need to be presented when the community negotiates a sociomathematical norm (Cobb & Yackel, 1996) of how to apply a problem's context.

<u>Online Questioning of The Problem's Context</u>

Of the three teams that attempted Problem 2 in an online environment, only the chat fields of two teams can be examined for student negotiation of how their team will use the problem's context (the third chat field being deleted as previously explained). Upon such an examination, it is noted that none of the students in Team 1 or Team 2 appear to have asked questions that invited a negotiation of this sociomathematical norm (Cobb & Yackel, 1996).

To begin, Team 2's members generated only one complete solution on their whiteboard, that of a 12-by-12 (Figure 4.4) configuration, with chairs drawn as individual lines. Although a 13-by-13 configuration (Figure 4.5) had been started, it remained incomplete. With a partial second solution, the issue of what constitutes a *different* solution, or a *reasonable* solution in this context had not become relevant for the team, given that members had not yet discussed the *different total number of chairs* represented by both 12-by-12 and 13-by-13 solutions.

Figure 6.5 below shows three of the nine configurations in which members of Team 1 arranged their choice of 120 chairs.

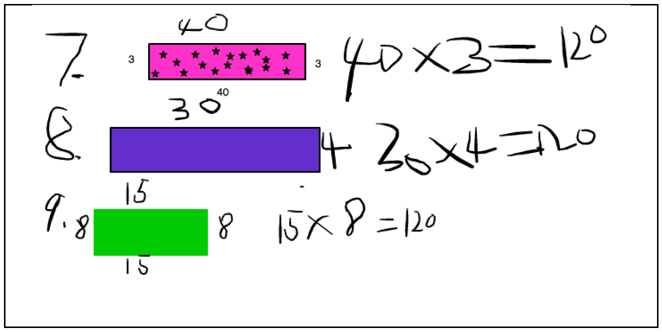


Figure 7.4 – Team 1's Solution to Problem 2 – Online

Remaining screens of the team solution contained rectangles with lengths and widths with a product of 120. The last screen on the whiteboard, as shown in Figure 6.6, contained a message of the team's focus on area, a focus intimating the possibility that members had treated a 40X3 rectangle to be the same solution as a 3X40 rectangle.

we found 9 possibilities to this problem when we picked 120 to be the area.

Figure 7.5 – Team 1 Solution Based on Area – Online

In fact, none of the team's configurations were mirrors of one another, and members made no reference to rows or chairs in the solution. The problem's context appears to have made no difference, as long as each rectangle shown calculated an area of 120 square units. Possibly, this team had implicitly negotiated the sociomathematical norm (Cobb & Yackel, 1996) of what constitutes a *different* solution in the online environment through its focus on area. Also possible was that team members had explicitly negotiated as much in face-to-face contact prior to logging in. Notably, these students had opened their online chat, already using the 120 as given, and had not justified this choice in the field. While the absence of explicit questioning and negotiation in the chat field does not necessarily mean that it did not occur, this team's sociomathematical negotiation of how the problem's context is to be used remains unclear.

<u>Summary of Students' Negotiation of the Problem's Context</u>

Members of each of the three teams who solved Problem 2 in face-to-face interaction engaged in a negotiation of how to apply the problem's context, a sociomathematical norm by Cobb and Yackel's (1996) definition. Team 3 used the problem's context to construct opposing but justifiable definitions of a *different* solution, which in turn, are the basis for the warrant and backing statements that support the team's conclusions. In face-to-face negotiation, Team 4 used the problem's context to limit its solutions to whole numbers, while Team 5 used context as the basis of arriving at a *reasonable* solution. This analysis serves as a reminder that the mathematical problems assigned to students need a context of substance, if context is to be used to evaluate the difference between solutions, or the reasonableness of a solution.

A similar negotiation of the problem's context was not evident in any of the online interactions. However, drawing implications from an analysis of Team 2's incomplete solution, or from Team 1's implicit or covert negotiation is premature. So, it is presented in this chapter as the basis of future study.

Student negotiation of social and sociomathematical norms in both the face-to-face and online environments are at different points on a continuum. Through their ongoing face-to-face interactions, participants of this classroom's community have had time to define social norms around contribution, consensus, concentration and communication; although these social norms require periodic renegotiation, they are more firmly established than their online counterparts.

This being the case, when students engage face-to-face, they are in a position to focus their attention on the negotiation of sociomathematical norms, such as those around a problem's condition, a problem's context, what constitutes a *different* solution, and what constitutes a *reasonable* solution.

In contrast, the online environment is a newer space in which students have engaged as problem-solving communities. Time is required to define social norms and map what students have negotiated in face-to-face interactions to the digital realm. Participant effort seems to have been directed at using available digital tools to simply represent a solution on the interactive whiteboard. While team members may have recognized a problem's condition in the process of undertaking a solution, representing this solution on the virtual space appears to have taken priority over a negotiation of what represents a *different* or *reasonable* solution. This is the point at which students find themselves on a continuum of negotiating online norms. Beyond a future consideration of a different software interface, there is opportunity to investigate how students, once given time to negotiate social norms and master their online representations of a solution, will negotiate the sociomathematical norm of using a problem's context.

Conclusions

This study was designed to investigate the features of elementary students' engagement as they solved mathematics problems in both face-to-face and online communities. In addition, participants were asked to describe their experiences in the two forms of community, both of which were designed as environments in which students would solve meaningful mathematics problems through ongoing discourse and negotiation, with my role as one of facilitation.

Through survey comments of their experiences in both forms of community interaction, students conveyed their need to negotiate, even renegotiate, social norms around the four themes of contribution, consensus, concentration and communication. In face-to-face interaction, barriers to respectful contribution were expressed by ELL participants, the *First Me, Then We* student, and the *Interrupted* student, whereas in the online space, acts of deletion contradicted students' perception of the social norm around contribution. In face-to-face engagement, the second theme of consensus was about the input and merge processes of building a collective solution, while online consensus was about removal of objects from the whiteboard space. In both environments, participants expressed concern about demonstrating on-task behaviour and remaining concentrated on an assigned task, despite the social and multimodal facets of community interactions. Lastly, while students demanded that face-to-face communication include explanations and justification of one's ideas and strategies, comments around online communication were more vague, suggesting that they need time to visualize and establish social norms of what it means to "listen" to each other in online space. From this analysis, what remains to be seen is whether students will align social norms between forms of their community engagement, or whether they establish distinct social norms for each of their face-to-face and online environments.

From an examination of students' lines of questioning, it was observed that they posed questions of correctness in both forms of community interaction, asking peers to validate their efforts and verify their guesses. However, in the online environment, participants presented proposals for negotiation as declarative statements, perhaps suggesting a confidence in the digital tools available to them. Also, examples of students' inward appeals of correctness, as indications of autonomy, were absent in the digital space, possibly because the virtual chat field implicates external, rather than internal conversation.

A sociomathematical negotiation undertaken by five of the six student teams was one around the problem's condition. The environment of a team's interactions did not prevent questions about the problem's condition from being posed, though not all teams negotiated and applied a problem's condition after such questioning. Mapping team arguments onto Krummheuer's (2007) analysis of Toulmin's argumentation scheme showed a case in each environment where an individual student's conclusion did not get used towards the team's data. This implicates a need for students' literacy skills to include how to convey indications to others, as part of Blumer's (1969) symbolic interactionism, and how to reflect upon others' indications. Still, of the two teams that questioned, negotiated and correctly applied their problem's condition, one did so via face-to-face engagement, and one did so through online interaction, demonstrating that students are capable of sociomathematical negotiation around condition in either form of community engagement. While this may be true, their sociomathematical negotiation of a problem's context in online engagement requires further study. Community members require time to negotiate and establish social norms, as well as master representation skills before they can focus on this aspect of their sociomathematical negotiation.

Future research should address the limitations of this study through design and use of a different software interface, given Scribblar's methodological and normative shortcomings. Also, future studies should provide participants with ample time and opportunity to engage in community environments, so that they may negotiate social norms and develop proficiency with representing solutions and using the tools of an environment. To promote generalizability for mathematics and digital educators, subsequent studies should span a longer term and investigate a larger sample size. While it is true that future study is required to address these limitations, consider the potential for community engagement in the online space, as recognized by the following comment:

> STUDENT25: "I like how you get to use chat field to talk and you don't have to be in the same room."

This comment, together with the above-noted implications of this small-scale and short study on students' negotiation of social and sociomathematical norms, offers possibility. Perhaps linking students across a city, a nation, or across the globe, into a digital mathematical community will serve my original rationale to facilitate their assertion of intellectual autonomy, so that they may realize their own mathematical power.

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NOTES

1 Definition from:
http://www.oxforddictionaries.com/us/definition/american_english/autonomy

2 Definition from:
http://www.oxforddictionaries.com/us/definition/american_english/authority