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ANALYZING HUMAN-SWARM INTERACTIONS USING CONTROL LYAPUNOV FUNCTIONS AND OPTIMAL CONTROL

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Abstract. A number of different interaction modalities have been proposed for human engagement with networked systems. In this paper, we establish formal guarantees for whether or not a given such human-swarm interaction (HSI) strategy is appropriate for achieving particular multi-robot tasks, such as guiding a swarm of robots into a particular geometric configuration. In doing so, we define what it means to impose an HSI control structure on a multi-robot system. Control Lyapunov functions are used to establish feasibility for a user to achieve a particular geometric configuration with a multi-robot system under some selected HSI control structure. Several examples of multi-robot systems with unique HSI control structures are provided to illustrated the use of CLFs to establish feasibility. Additionally, we also uses these examples to illustrate how to use optimal control tools to compute three metrics for evaluating an HSI control structure: attention, effort, and scalability.

1. Introduction.

Many applications require human intervention to guide autonomous robots through complicated tasks. For example, we often rely on and benefit from a human operator's ability to decide where robots should focus their efforts [1]. And even if autonomous robots do not require human guidance, humans and robots will continue to coexist in most environments (e.g., manufacturing floors [14], disaster areas [6]) and to interact with each other. Existing interfaces have focused on supporting human interactions with one or a few robots (for example, [19]); however, as the number of robots involved in the task grows large, such interfaces can become less effective or even unusable due to a lack of scalability in the corresponding interaction [12]. Therefore, there has been a growing effort to understand human-swarm interactions (HSI) and devise interactions that are amenable to having humans interact with swarms of robots easily and effectively (for example, see [20], [17], and other types of interactions cited below). Figure 1 illustrates two such interaction strategies, namely leader-follower and Eulerian interactions.

We are interested in being able to select appropriate interaction strategies by establishing guarantees that the proposed strategies are indeed expressive enough to solve the given task. In particular, we investigate if the provided HSI allows a user to guide a swarm of robots into some geometric configuration. Human-swarm interactions come in a variety of flavors, such as gesture-based methods [22], [25], mode selection [18], music instrument interfaces [7], broadcast control [2], [8],

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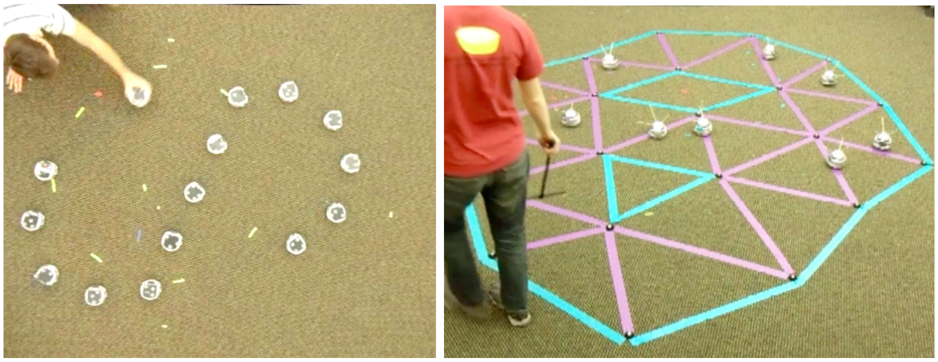


(A) Leader-follower interaction. (B) Eulerian interaction.

FIGURE 1. Two interaction strategies: leader-follower networks and Eulerian swarms.

deformable media [10], and biologically inspired interactions [13], just to cite and name a few. The commonalities amongst these interactions are two-fold: the user's interaction with the swarm happens alongside the interactions amongst the robots in the swarm, and each HSI imposes a specialized structure on the possible inputs to limit the complexity of the interaction. Some of these HSI control structures are tailored to making it possible to achieve particular types of geometric configurations with swarms of robots, such as rendezvous, flocking, coverage, and formations, which beckons the question, *if we are given a particular HSI control structure for a multi-robot system, is it feasible to use the corresponding interaction to achieve a desired geometric configuration with a swarm of robots?*

In this paper, as we have done in [9], we will show that control Lyapunov functions can be used to answer this question. While providing proofs of convergence of an HSI-enabled multi-agent systems to a geometric configuration is standard (see, for example, [26]), the novelty in our work is that we start with a formal definition of what it means to impose an HSI control structure on the dynamics of the multi-robot system that represents a swarm of robots. And then, we use a CLF approach to show convergence of the HSI-structured multi-robot system to some geometric configuration to demonstrate that it is feasible for the user to achieve the desired geometric configuration with such a swarm of robots. In addition to our previous work in [9], we will propose attention, effort, and scalability as metrics for evaluating a user's interactions with an HSI-enabled swarm of robots during a specific task. These metrics can be measured after the task, or approximated before the task to evaluate and improve an HSI. We will demonstrate that in the latter approach, we can use optimal control tools to generate an approximation of user control input under the assumption that users with training will act almost optimally. Consequently, we are able to quantify the answer to the questions, *if we are given a particular HSI control structure for a swarm of robots, does it provide an interaction that requires low attention and effort, and does it scale well as the swarm increases in size?*

2. **Definitions.** Our objective is to determine whether it is possible for a human operator to use a particular human-swarm interaction (HSI) to achieve some geometric configuration with a swarm of mobile robots. To establish feasibility, we

first need to know what a HSI represents in terms of the structure it imposes on a multi-robot system, and what it means for a human operator to achieve a particular geometric configuration with the robotic swarm.

2.1. **Human-swarm interaction structure.** In general, we consider continuous-time, time-invariant systems with inputs, which represent robotic swarms that can be externally controlled (or interacted with) by a user. The dynamics of such multi-robot systems can be defined as $\dot{x}(t) = f(x(t), u(t))$, where $x(t) \in \mathcal{X}$ is the state of the system at time t and $u(t) \in \mathcal{U}$ is the input to the system at time t. In fact, $x(t)$ will represent the stacked vector of the states belonging to all robots at time t, while $x_i(t)$ will refer to the state of robot i at time t. For example, $x(t)$ will typically represent the position or pose of all robots together at time t.

More importantly, the differentiable function $f : \mathcal{X} \times \mathcal{U} \to T\mathcal{X}$, where $T\mathcal{X}$ is a tangent space, is structured according to the network topology of the multi-robot system. The network topology is given by a graph $\mathcal{G} = (V, E)$, where V is the set of vertices representing the agents, and E is the set of edges representing information exchange between agents via communication links or due to sensor footprints (see, for example, [21]). Specifically, $f \in \text{sparse}_{\mathcal{X}}(\mathcal{G})$ conveys that state information in the multi-robot system can only flow between agents that are linked in the network topology. Consequently,

$$f \in \text{sparse}_{\mathcal{X}}(\mathcal{G}) \Leftrightarrow \left(j \notin \overline{N}(i) \Rightarrow \frac{\partial f_i(x, u)}{\partial x_j} = 0, \ \forall x, u \right), \tag{1}$$

where $N(i)$ is the so-called neighborhood of robot i, i.e., $j \in N(i)$ if $(i, j) \in E, i, j \in V$, and $\overline{N}(i) = N(i) \cup \{i\}$.

By picking a particular HSI control structure, we are being specific about the structure of $\mathcal{U}$, i.e., how the user can interact with the robotic swarm. Our definition is as follows:

Definition 2.1. an HSI control structure is a map

$$H : \mathcal{X} \times \mathcal{V} \to \mathcal{U}, \tag{2}$$

where $\mathcal{V}$ is some set of admissible inputs to make the corresponding robotic swarm more amenable to human control. Additionally,

$$f(x, H(x, v)) = f_H(x, v) \in \text{sparse}_{\mathcal{X}}(\mathcal{G}), \tag{3}$$

which means that the dynamics f under this map H needs to observe the sparsity structure imposed by the network topology.

This definition of an HSI control structure implies that the control input to the system is really a combination of state feedback and a restricted set of inputs from the user, which respects the constraints imposed by the network topology. Consequently, the dynamics of a multi-robot system under such an HSI control structure are

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t)) \\ &= f(x(t), H(x(t), v(t)) \\ &= f_H(x(t), v(t)). \end{aligned} \tag{4}$$

Therefore, an HSI control structure is a very specific way in which the user controls the multi-robot system, i.e., interacts with the robotic swarm.

For example, suppose that a robotic swarm consists of n mobile robots positioned on a rail ($x_i(t) \in \mathbf{R}$) with single-integrator dynamics,

$$\dot{x}_i(t) = u_i(t),\ i = \{1, \ldots, n\}, \tag{5}$$

where the control input for the first $n-1$ robots is

$$u_i(t) = \sum_{j \in N(i)} (x_j(t) - x_i(t)). \tag{6}$$

$N(i)$ denotes the neighborhood of robot i, which is the set of all its immediate neighbors in the network topology derived from communication links or sensor footprints.

The control input for the n-th robot is

$$u_n(t) = \sum_{j \in N(i)} (x_j(t) - x_i(t)) + v(t),\ v(t) \in \mathcal{V}, \tag{7}$$

which corresponds to the user directly controlling the position of the n-th robot. This HSI control structure is commonly referred to as a *single-leader network* (see, for example, [24]), because the user interacts with the swarm of robots by guiding a "leader" robot, while the other robots follow the leader and each other according to the consensus dynamics in (6) (see [23] for more on consensus).

If we stack all $x_i(t)$'s into a state vector $x(t) \in \mathbf{R}^n$ and all $u_i(t)$'s into an input vector $u(t) \in \mathbf{R}^n$, then the ensemble dynamics of our example system are

$$\dot{x}(t) = u(t) = -Lx(t) + lv(t),\ l = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \mathbf{R}^n \tag{8}$$

where is the graph Laplacian as defined in [21] (and commonly used in multi-robot control). Consequently, the single-leader network HSI control structure is a particular structuring of the control input $u(t)$ in (8) given by the function H, such that

$$u(t) = H(x(t), v(t)) = -Lx(t) + lv(t), \tag{9}$$

where $v(t) \in \mathbf{R}$ is the user input.

2.2. Achieving geometric configurations.

Definition 2.2. When a multi-robot system under some HSI control structure can asymptotically converge to a state, a subset of states, or all states in a specification set $\mathcal{S}$ and stay in this set, then

$$\limsup_{t \to \infty} d(x(t), \mathcal{S}) = 0, \tag{10}$$

where,

$$d(x(t), \mathcal{S}) = \inf_{s \in \mathcal{S}} \|x(t) - s\|. \tag{11}$$

If this is true, then we say that the user can *achieve* some or all of the geometric configurations described by $\mathcal{S}$ with the robotic swarm.

The *specification set* is the set of geometric configurations that we want the user to achieve with the robotic swarm, in the sense that the user should be able to form a geometric configuration with the swarm and keep it in this configuration. For example, a specification set could be defined as

$$\mathcal{S} = \left\{ x \in \mathbf{R}^n \mid x_i = x_j,\ i, j = 1, \ldots, n \right\}, \tag{12}$$

which merely states that all components of the state should be equal, or $\mathcal{S} = \text{span}\{\mathbf{1}\}$. For example, the specification set for consensus problems with multi-robot teams is typically defined in this way. Or, we may want the user to guide a single-leader network, such that all robots in the swarm rendezvous at a specific location, i.e., $\alpha \in \mathbf{R}, \mathcal{S} = \alpha\mathbf{1}$.

3. **Feasibility.**

We have shown that the function $H : \mathcal{X} \times \mathcal{V} \to \mathcal{U}$ encodes a particular HSI control structure into the dynamics of a multi-robot system, and that if this combination of multi-robot system and HSI control structure, $(\mathcal{X}, \mathcal{V}, f_H, x_0)$, can asymptotically converge to a specification set $\mathcal{S}$ (or a subset thereof) from the initial conditions, x_0, then we say that it is feasible for a user to use this HSI control structures to achieve some desired geometric configuration with a robotic swarm. More formally,

Definition 3.1. It is feasible to achieve a specification set $\mathcal{S}$ under an HSI control structure defined by H if there exists $v(t)$ such that, $v(t) \in \mathcal{V},\ \forall t \geq t_0$, and

$$\limsup_{t\to\infty} d(x(t), \mathcal{S}) = 0,$$

when $\dot{x}(t) = f_H(x(t), v(t)), x(t_0) = x_0$.

We will use control Lyapunov functions (CLFs) [27] to determine this feasibility.

3.1. **Control Lyapunov functions.**

Let us denote $D \subset \mathcal{X}$ as a domain of the state space containing the quasi-static equilibrium point z for some $w \in \mathcal{V}$, such that $\dot{x}(t) = f_H(z, w) = 0$.

Definition 3.2. A continuously differentiable $V : D \to \mathbf{R}$ with

$$V(z) = 0 \text{ and } V(x) > 0 \text{ in } D - \{z\}$$

is a control Lyapunov function (CLF), if there exists a $v \in \mathcal{V}$ for each $x \in D$, such that

$$\dot{V}(x, v) = \nabla V(x) \cdot f_H(x, v) < 0 \text{ in } D - \{z\} \tag{13}$$

and $\dot{V}(z, w) = 0$.

If such a control Lyapunov function exists, then any trajectory starting in some compact subset $\Omega_c = \{x \in \mathcal{X} \mid V(x) \leq c,\ c > 0\} \subset D$ will approach z as $t \to \infty$.

Theorem 3.3. *If there exists a CLF as defined in Definition 3.2 for the system $(\mathcal{X}, \mathcal{V}, f_H, x_0)$ and the specification set $\mathcal{S}$ is some quasi-static equilibrium point $z \in D$, then it is feasible to converge to z as $t \to \infty$.*

Proof. By Definition 3.2, the existence of a CLF guarantees that if $x_0 \in \Omega_c$, then there exists $v(t) \in \mathcal{V}$, such that the multi-robot system converges to z asymptotically, i.e. $\lim_{t\to\infty} x(t) = z$. Since $z = \mathcal{S}$, it is true that

$$\left(\limsup_{t\to\infty} d(x(t), z) = 0\right) \Rightarrow \left(\limsup_{t\to\infty} d(x(t), \mathcal{S}) = 0\right),$$

which by Definition 3.1 confirms that for this particular multi-robot system and HSI control structure, the user can achieve the geometric configuration in the specification set $\mathcal{S}$ with the corresponding robotic swarm. □

Using this formulation of CLFs allows us to test the feasibility of achieving, for example, rendezvous at a specific location or a formation at a specific location with a specific rotation and assignment to positions. However, we would also like to capture formations that can translate and rotate, like cyclic pursuit, or rendezvous at any arbitrary location. Therefore, our definition of CLFs needs to include sets of quasi-static equilibrium points and limit cycles.

Suppose that $D \subset \mathcal{X}$ is a domain of the state space that contains all or part of the specification set $\mathcal{S}$.

Definition 3.4. A continuously differentiable $V : D \to \mathbf{R}$ (and locally positive definite as before) is a control Lyapunov function, if there exists $v \in \mathcal{V}$ such that

$$\dot{V}(x, v) = \nabla V(x) \cdot f_H(x, v) \leq 0 \tag{14}$$

for each x in some compact set $\Omega \subset D$, for example, Ω_c. By LaSalle's invariance principle [16], if M is the largest invariant set in $\left\{x \in \Omega \,\middle|\, \dot{V}(x, v) = 0,\ v \in \mathcal{V}\right\}$, then any trajectory starting in Ω will approach M as $t \to \infty$.

Consequently, we must ensure that our choice of CLF satisfies $M \subseteq \mathcal{S}$, otherwise we cannot show that it is feasible to achieve any of the geometric configurations in the specification set $\mathcal{S}$.

Theorem 3.5. *If there exists a CLF as defined in Definition 3.4 for the system defined by $(\mathcal{X}, \mathcal{V}, f_H, x_0)$ and $M \subseteq \mathcal{S}$, then it is feasible to asymptotically converge to M from any $x(t_0) \in \Omega$.*

Proof. The proof is similar to what was shown in the first theorem. By Definition 3.4, the existence of a CLF guarantees that if $x_0 \in \Omega$, then there exists $v(t) \in \mathcal{V}$, such that the multi-robot system converges to the invariant set M asymptotically. Therefore,

$$\limsup_{t\to\infty} d(x(t), M) = 0$$
$$\limsup_{t\to\infty} \inf_{m \in M} \|x(t) - m\| = 0.$$

If $M \subseteq \mathcal{S}$, then any $m \in M$ is also in $\mathcal{S}$, which means that

$$\limsup_{t\to\infty} \inf_{m \in \mathcal{S}} \|x(t) - m\| = 0$$
$$\limsup_{t\to\infty} d(x(t), \mathcal{S}) = 0,$$

which satisfies our definition of feasibility. □

In the context of HSIs, feasibility establishes that it is possible for the user to achieve the specification set with the swarm of mobile robots; however, it does not guarantee that the user will actually use a control input that guarantees stability. Therefore, one will want to either restrict the user to a set of stable trajectories, or demonstrate robustness.

4. **Attention, effort and scalability.** Just because a human-swarm interaction strategy is feasible, it does not follow that it is useful or even possible for a human operator to effectively employ due to the complexity associated with the strategy. This is sometimes referred to as the user experience, and the user's experience is typically explored through user-studies, where measures such as attention or effort are evaluated. Unfortunately, such user-studies tend to be time-consuming and focus on individual experiments and in this paper, we circumvent this problem by

formally investigating what attention and effort are required to accomplish a given task with a given HSI control structure.

4.1. **Attention and effort.** Attention and effort are common metrics by which one can evaluate the user's experience in some task [15]. These metrics can be gathered through experiments in a user study, but the concept of attention and effort can also be formulated in a control-theoretic context. Roger Brockett introduced the notion of a minimum attention controller [4] by solving an optimal control problem that minimized the total variation of the control signal over time as well as over the state of the system, e.g., the attention functional would take the form

$$\int_{\mathbf{T}} \int_{\mathbf{X}} \phi \left(x, t, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial t} \right) dx dt, \tag{15}$$

where $\mathbf{X}$ and $\mathbf{T}$ are the state space and time domains over which the controller is defined, and u is the control signal.

However, attention is only part of the story. Optimization problems often minimize $u(t)$ itself (see, for example, [5]), meaning that it is desirable to complete some task with minimal effort. Consequently, we propose that a cost on the user control input should be both in terms of attention and effort. One way to define the attention-effort cost for an HSI control structure, $f_H(x(t), v(t))$, is

$$J_{AE}(v) = \int_0^\infty \left(\|v(t)\|^2 + \|\dot{v}(t)\|^2 \right) dt, \tag{16}$$

which encodes a cost on the magnitude of the user input v (effort) and the variation in v over time (attention).

It should be noted that such a functional computes the joint attention-effort cost for a particular $v(t)$ and not for a particular HSI-structure. To overcome this problem, we focus instead on a particular choice of control signal – the optimal one, v^* – as a proxy for the signal the user might indeed employ. Rather than measuring the user control input in a user study, we will assume that v^* is a good benchmark for a trained user.

While a user is likely to try to complete a task as fast and accurately as possible, a user also likely choses to minimize attention and effort. Too much attention or effort required to complete a task is likely undesirable. Consequently, we propose to compute the optimal control v^* using a cost function that encodes accuracy, effort, and attention simultaneously. For example, in Section 5.1.1 we minimize the following cost with respect to $\dot{v}$:

$$\begin{aligned} \min_{w} \quad & J(w) = \frac{1}{2} \int_0^\infty \left((x - \alpha \mathbf{1})^T (x - \alpha \mathbf{1}) + v^T v + w^T w \right) dt \\ \text{s.t.} \quad & \dot{x} = -Lx + lv \\ & \dot{v} = w \\ & x(0) = x_0, \; v(0) = 0 \end{aligned} \tag{17}$$

The first term penalizes any swarm configuration that is not in the specification set, while the second and third terms penalize effort and attention. Computing $\dot{v}^*$ allows us to construct v^*, which we will use in evaluating the attention-effort cost. We will demonstrate this example in full in Section 5 for two different HSI control structures.

Optimal control solutions are implicitly a function of the initial conditions; therefore, different initial conditions are likely to result in different attention-effort costs.

Consequently, we recommend to either average the cost over a sampling of the initial conditions, or use attention-effort cost to compare two different HSI control structures in the same task with the same initial conditions.

4.2. **Scalability.** If $J_{AE}^{N}(v^*)$ is the attention-effort cost for N robots under some HSI control structure, then $J_{AE}^{N+1}(\nu^*)$ is the attention-effort cost for $N+1$ robots under the same control structure. Scalability captures the increase in attention and effort required to interact with a swarm of more robots. Consequently, scalability could be defined as the raw increase (or decrease) in cost from N to $N+1$ robots. We chose to define scalability as $\Sigma(N)$, an approximation of the rate of growth of the attention-effort cost as the number of robots involved in the task are increased. For example, if we were to compute the attention-effort cost for some task with N robots from 10 to 100, and we could fit this data with a line, then $\Sigma(N) = cN, c \in \mathbf{R}$ would encode the change in the attention-effort cost with respect to the swarm size. It is important to note that when approximating scalability, v^* is not necessarily equal to ν^*, because adding more robots to the swarm changes the dynamics and the initial conditions. Consequently, it may be useful to compute the average cost, $\bar{J}_{AE}(v^*)$, over a sampling of the space of initial conditions for each N.

5. **Examples.** In this section, we will provide several examples of HSI control structures imposed on multi-robot systems for which we can find CLFs and show that a user can achieve a particular geometric configuration with a swarm of robots. First, we will revisit our previous example of a single-leader network, where the user guides the swarm of robots to a common rendezvous location. Then, we will revisit a broadcast control HSI control structure proposed in a previous paper [8] in the context of the approach established in this paper.

5.1. **Rendezvous with single-leader networks.** Rendezvous is similar to consensus in that all robots meet up at the same location; however, let us suppose rendezvous captures the additional constraint that all robots should meet up at a particular location. The specification set that encodes this objective is $\mathcal{S} = \{x \in \mathbf{R}^n \mid x_i = \alpha,\ \alpha \in \mathbf{R},\ i = \{1, 2, \ldots, n\}\}$, or more concisely, $\mathcal{S} = \alpha\mathbf{1}$, where α is the rendezvous location.

We chose a candidate CLF [24] given by

$$V(x) = \frac{1}{2}\|x - \alpha\mathbf{1}\|^2, \tag{18}$$

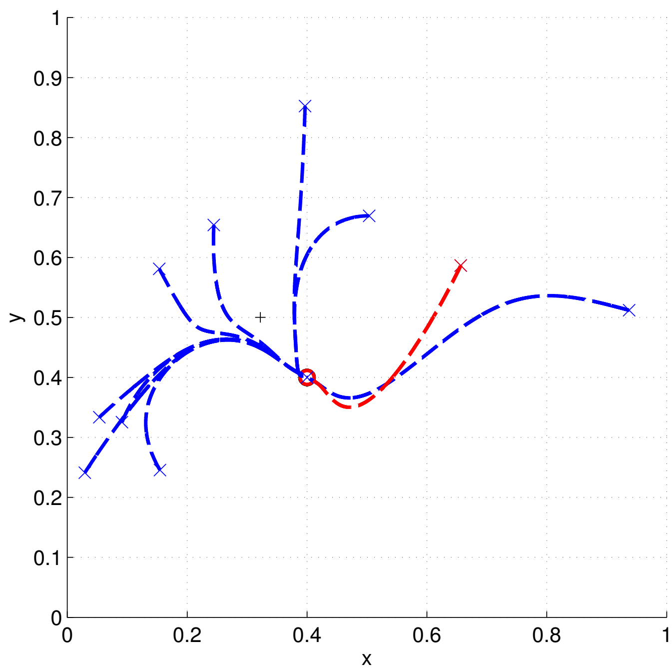
which captures the disagreement between the current state of the robotic swarm and the rendezvous location. $V(x)$ is positive definite everywhere except at the desired equilibrium point $x = \alpha\mathbf{1}$ and is radially unbounded ($\|x\| \to \infty \Rightarrow V(x) \to \infty$).

Next, we need to compute $\dot{V}(x, v)$, which is defined by

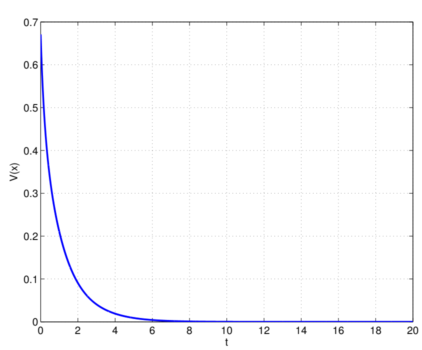
$$\begin{aligned} \dot{V}(x, v) &= \nabla V(x) \cdot f_H(x, v) \\ &= (x - \alpha\mathbf{1})^T(-Lx + lv) \\ &= -(x - \alpha\mathbf{1})^T Lx - (\alpha - x_n)v. \end{aligned} \tag{19}$$

If $V(x)$ is a CLF, then it must be true that for each $x \in \mathbf{R}^n$, there exists $v \in \mathcal{V}, \mathcal{V} = \mathbf{R}$ such that $\dot{V}(x, v) < 0$ when $x \neq \alpha\mathbf{1}$ and $\dot{V}(x, v) = 0$ when $x = \alpha\mathbf{1}$. In Equation (19), we can see that even if $-(x - \alpha\mathbf{1})^T Lx$ is positive, we can always chose $v \in \mathbf{R}$, such that $\dot{V}(x, v) < 0$. Therefore, $V(x)$ is a CLF that guarantees that there exists

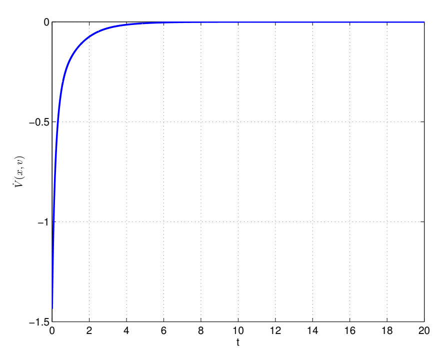
$v(t) \in \mathcal{V}$, such that the user can guide the swarm of robots from $x(t_0) \in \mathbf{R}^n$ to $x = \alpha\mathbf{1}$ as $t \to \infty$.



(A) Trajectories in $\mathbf{R}^2$



(B) $V(x, y)$



(C) $\dot{V}(x, y, v)$

FIGURE 2. A user is guiding a swarm of 10 robots to rendezvous at $(0.4, 0.4)$ by interacting with the leader robot.

Figure 2 is a demonstration of rendezvous with a single-leader network. To aid in the visualization, the above candidate CLF and the single-leader network system have been extended to $\mathbf{R}^2$. Since the robots are single integrators, the dynamics along each dimension, x and y, are decoupled.

Figure 2a shows the trajectories of ten robots that are organized over an arbitrary connected, static network topology. The solid, red trajectory belongs to the leader robot that is controlled by the user, while the dashed, blue trajectories belong to all other robots in the swarm. $\times$ denotes their starting location, while $\circ$ denotes the rendezvous location if $v(t) = 0, \forall t$, and $\circ$ illustrates the robots' actual final position.

We see in Figure 2a that by guiding the leader robot to the origin, the user can change the rendezvous location of the swarm of robots to the origin. Figure 2b shows that the CLF $V(x, y)$ is positive, but "energy" dissipates as robots converge on the rendezvous location, while Figure 2c shows that $\dot{V}(x, y, v)$ remains negative during the interaction. Consequently, it is feasible for the user to use this HSI control structure to chose the rendezvous location of a swarm of robots. Similarly, this combination of multi-robot system and HSI control structure would be effective in setting the flocking direction if the state x were the orientation θ of each robot, rather than its position.

5.1.1. *Attention, effort, and scalability.* We discussed in a previous section that in place of measuring $v(t)$, we compute v^* using optimal control tools. We solve the following optimization problem:

$$\begin{aligned} \min_{w} \quad & J(w) = \frac{1}{2}\int_0^\infty \left((x-\alpha\mathbf{1})^T(x-\alpha\mathbf{1}) + v^T v + w^T w\right) dt \\ \text{s.t.} \quad & \dot{x} = -Lx + lv \\ & \dot{v} = w \\ & x(0) = x_0,\ v(0) = 0 \end{aligned} \tag{20}$$

This is a continuous-time, infinite horizon linear quadratic regulator-like (LQR-like) problem, which can be solved in the following manner. First, the first order necessary conditions (FONC) for optimality [3] are:

$$\begin{aligned} H &= \frac{1}{2}\left((x-\alpha\mathbf{1})^T(x-\alpha\mathbf{1}) + v^T v + w^T w\right) + \lambda^T \dot{x} + \mu^T \dot{v} \\ \frac{\partial H}{\partial w} &= w^T + \mu^T = 0 \Rightarrow w = -\mu \\ \dot{\lambda} &= -\frac{\partial H}{\partial x} = L^T\lambda - x + \alpha\mathbf{1} \\ \dot{\mu} &= -\frac{\partial H}{\partial v} = -l^T\lambda - v \end{aligned} \tag{21}$$

It is important to note here that the co-state dynamics, $\dot{\lambda}$, include an extra affine term that is typically not present in a standard LQR problem. For convenience, let us stack states and co-states into single variables in the following way:

$$\begin{aligned} z &= \begin{bmatrix} x \\ v \end{bmatrix},\ \dot{z} = \begin{bmatrix} -L & l \\ 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w = A_z z + B_z w \\ \eta &= \begin{bmatrix} \lambda \\ \mu \end{bmatrix},\ \dot{\eta} = \begin{bmatrix} -L^T & 0 \\ l^T & 0 \end{bmatrix} \eta - z + \begin{bmatrix} \alpha\mathbf{1} \\ 0 \end{bmatrix} = -A_z^T \eta - z + \Psi \end{aligned} \tag{22}$$

We propose that $\eta(t) = S(t)z(t) + P(t)$ is the solution to the stacked co-state equations. The affine component, $P(t)$, is to account for the affine component that is tracked in the cost. If we start from the proposed solution, then

$$\begin{aligned} \eta &= Sz + P \\ \dot{\eta} &= \dot{S}z + S\dot{z} + \dot{P} \\ -A_z^T\eta - z + \Psi &= \dot{S}z + SA_z z + SB_z w + \dot{P} \\ -A_z^T Sz - A_z^T P - z + \Psi &= \dot{S}z + SA_z z - SB_z B_z^T Sz - SB_z B_z^T P + \dot{P} \\ -\dot{P} - (A_z^T - SB_z B_z^T)P + \Psi &= \Big(\dot{S} + SA_z + A_z^T S - SB_z B_z^T S + I\Big) z \end{aligned} \tag{23}$$

Since this LQR-like problem is computed over an infinite horizon, we can compute the steady state $\hat{S}$ and $\hat{P}$, when $\dot{S} = 0$ and $\dot{P} = 0$. Consequently, to satisfy Equation 23, we must solve

$$\begin{aligned} \hat{P} &= (A_z^T - SB_z B_z^T)^{-1}\Psi \\ 0 &= \hat{S}A_z + A_z^T\hat{S} - \hat{S}B_z B_z^T\hat{S} + I \end{aligned} \tag{24}$$

The second equation is the continuous time algebraic Ricatti equation, while P can be solved for directly. Finally, we are able to compute $\dot{v}^* = w$,

$$\begin{aligned} w &= -\mu \\ &= -B_z^T(\hat{S}z + \hat{P}). \end{aligned} \tag{25}$$

Consequently, the optimal user control input signal is

$$v^*(t) = \int_0^t w(\tau)d\tau,\ v^*(0) = 0. \tag{26}$$

Figure 2a was generated by controlling the single-leader network with the optimal user control input $v^*(t)$.

Scalability can be calculated by augmenting the task by adding more robots. In this example, the new robot is added to the swarm in a random location. Figure 3 illustrates the increased attention, effort, and attention-effort cost of completing the "same" task with more robots. The increase in cost is mainly attributed to an increase in effort as shown by the red dashed line in Figure 3, while attention has only marginally increased as shown by the black dash-dotted line in the same figure. The scalability metric for this particular task is approximated by a linear fit to the attention-effort cost. The slope of this linear fit is $\Sigma(n) = 1.05n$, which is an increase in the attention-effort cost for every robot. However, the exact coefficient of $\Sigma(n)$ is only meaningful once it is compared to different HSI control structures under this same geometric task.

5.1.2. *Comparison to other HSI control structures.* Suppose that we have two other HSI control structures for achieving rendezvous. The first HSI control structure relies on broadcasting an input signal. The dynamics of the swarm are

$$\dot{x}(t) = -Lx(t) + \mathbf{1}v(t) \tag{27}$$

where L is once again the graph Laplacian, and $\mathbf{1} \in \mathbf{R}^n$ is a vector of all ones. The second HSI control structure supposes that the user could somehow control all robots individually. The control structure of this *concurrent control* approach is

$$\dot{x}(t) = -Lx(t) + v(t), \tag{28}$$

where $v(t)$ is a N-dimensional input vector, because then input to each robot is computed separately. The procedure described earlier in this section can again be used to demonstrate feasibility and compute an optimal control signal for achieving

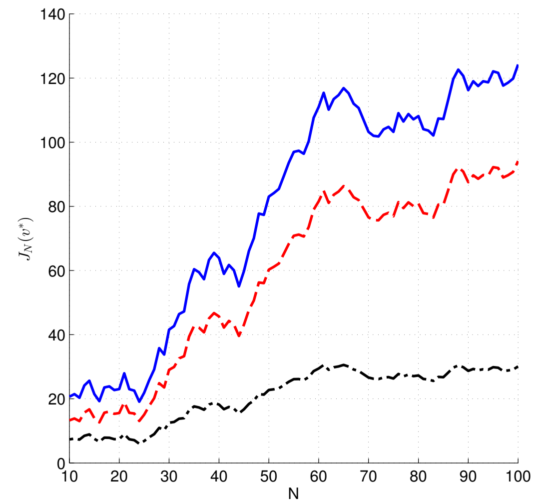


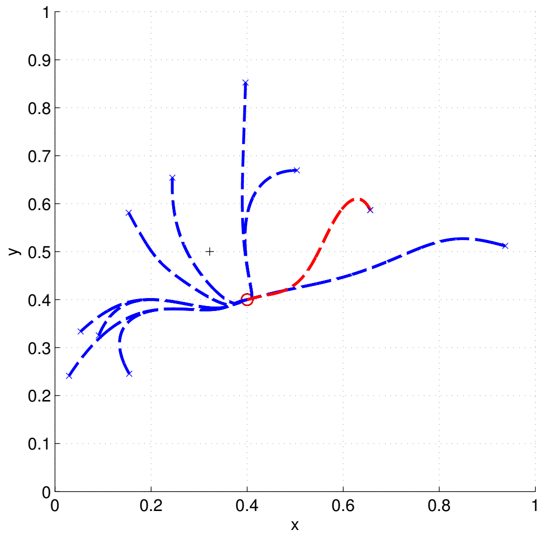
FIGURE 3. Growth of attention (black dash-dotted), effort (red dashed), and attention-effort (blue solid) cost for guiding a single-leader network of N robots to the rendezvous location.

rendezvous with the swarm of mobile robot. Figures 4a and 4b illustrate rendezvous using broadcast and concurrent control.

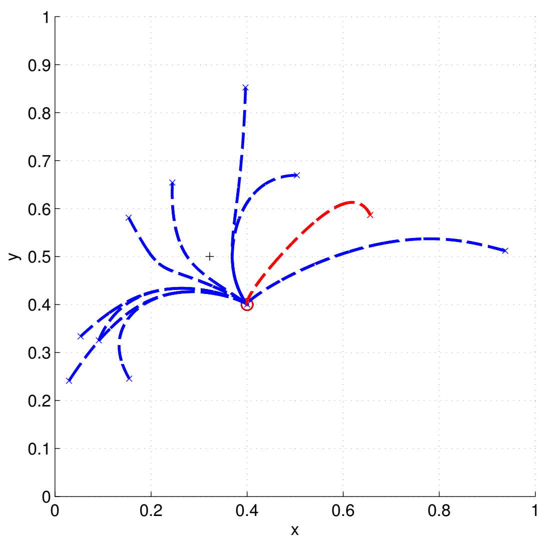
Demonstrating feasibility for these two HSI control structures and the single-leader network precludes us from choosing one control structure over the other. However, we can use attention, effort, and scalability as metrics for making this decision. Figure 5 contains plots of the attention-effort cost, attention, and effort on the interval $t \in [0, 20]$ of all three tasks, where single-leader network is solid blue, concurrent control is dashed red, and broadcast control is dash-dotted black.

If we focus on Figure 5a, then it is evident that using a single leader network for rendezvous incurred the greatest attention-effort cost, while broadcast control incurred the least attention-effort cost. The effort required for rendezvous under concurrent control and a single leader network is almost the same at its greatest in Figure 5b, but the effort is sustained longer for the single leader network. On the other hand, attention is less for the single leader network than concurrent control as shown in Figure 5c. Broadcast control required less attention and effort compared to the other two control structures.

If the broadcast control-based task requires less attention and effort compared to concurrent the control-based and single-leader network-based task, is this also true for a larger number of robots? Scalability describes the growth rate of the attention-effort cost when modifying the task by adding more robots to the swarm. Figure 6 illustrates the effect of increasing the swarm size from ten to 100 robots on attention, effort, and the combined attention-effort cost. The procedure for increasing the swarm size was to add each new robot to the workspace by choosing its location from a uniform distribution that covers the entire workspace, $\mathcal{X} = \{x, y \mid x \in [0, 1],\ y \in [0, 1]\}$. Figure 6a includes a linear fit to the attention-effort cost data, i.e., $\Sigma(n) = cn$, where $c = 0.31$ for concurrent control, $c = 1.05$ for single leader networks, and $c = 0.13$ for broadcast control. Comparing Figure 6b and 6c



(A) Broadcast control.

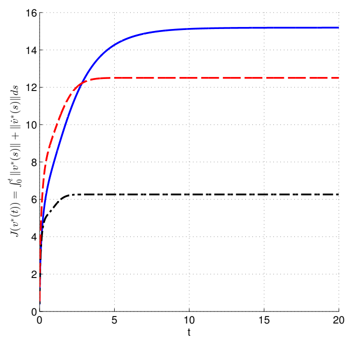


(B) Concurrent control.

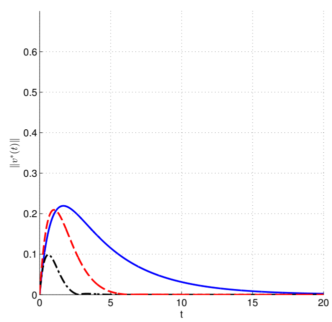
FIGURE 4. These trajectories in $\mathbf{R}^2$ illustrate rendezvous at $(0.4, 0.4)$ using broadcast and concurrent control.

reveals that these control structures are more differentiated by effort than attention, and that attention levels off after $N \approx 60$.

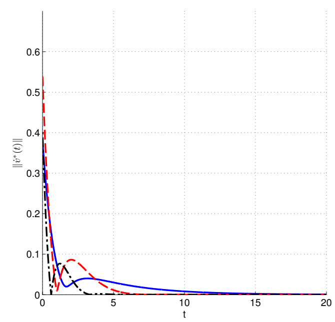
The result of this comparison is that broadcast control outperforms concurrent control, while concurrent control outperforms single leader networks with respect to attention, effort, and scalability. However, this comparison omits one important



(A) Cost $J(v^*)$ up to time t (Attention-Effort)



(B) $\|v^*(t)\|$ (Effort)

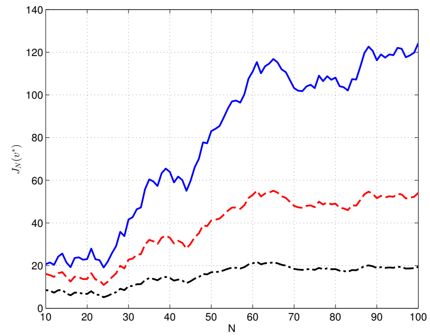


(C) $\|\dot{v}^*(t)\|$ (Attention)

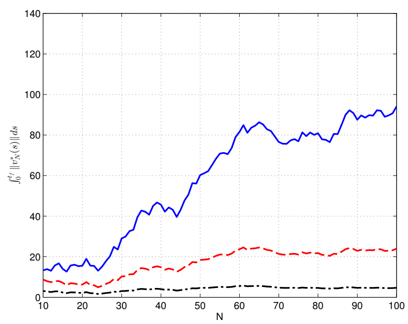
FIGURE 5. A user's estimated attention and effort while guiding a swarm of 10 robots to rendezvous at a specific location with different control structures: single-leader network (blue solid), broadcast control (black dash-dotted), and concurrent control (red dashed).

factor that differentiates concurrent control from single-leader networks (and broadcast control), which is the fact that the dimension of the former control structure grows linearly in the size of the swarm, while the latter control structures require the user to decide only a two dimensional input (akin to a joystick). Consequently, if a single user could yield 100 joystick, or gather 100 co-operators, or rely on a computer (perhaps, the user simply specifies a goal location with a point-and-click interface), then concurrent control is better than a single-leader network. Regardless, broadcast control is the better control structure for this particular rendezvous task.

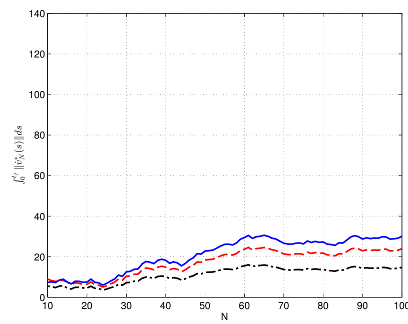
5.2. **Separation with a broadcast signal.** In a previous paper [8], we showed that it is possible to use a broadcast signal to separate a swarm of heterogeneous robots, but let us revisit this problem in the context of this paper. Suppose that broadcasting an input signal is a HSI for a swarm of two types of robots, and we would like to know if it is feasible to separate the two types of robots by a distance of Δ by broadcasting an input signal. Each robot $i \in \mathcal{N}$, where $\mathcal{N} = \{1, \ldots, n\}$ is the set of all robots, belongs to one of two classes in $\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2\}$. A class membership



(A) Cost $J_N(v^*)$ for 10-100 robots (Attention-Effort)



(B) Cumulative effort for 10-100 robots



(C) Cumulative attention for 10-100 robots

FIGURE 6. Growth of a user's estimated attention and effort while guiding a swarm of 10-100 robots to rendezvous at a specific location with different control structures: single-leader network (blue solid), broadcast control (black dash-dotted), and concurrent control (red dashed).

function $\pi : \mathcal{N} \to \mathcal{C}$ maps each robot i into one of the two classes. The dynamics of each robot are

$$\begin{aligned} \dot{x}_i(t) &= u_i(t) \\ &= \gamma_{\pi(i)} \left(\sum\nolimits_{j \in N(i)} (x_j(t) - x_i(t)) + v(t) \right), \end{aligned} \tag{29}$$

where $j \in N(i)$ if robot i and robot j are separated by a distance less than Δ and $v(t) \in \mathbf{R}_+ \cup \{0\}$ is the broadcast input signal. If we use the initial conditions $x_i(t_0) = x_j(t_0),\ \forall i, j \in \mathcal{C}_1$ and $x_i(t_0) = x_j(t_0),\ \forall i, j \in \mathcal{C}_2$, which corresponds all robots of same type starting together, then we can simplify the dynamics (as shown in the paper) to

$$\begin{aligned} \dot{\chi}_1 &= -\gamma_1(N_2(\chi_1 - \chi_2) - v) = f_{H,1}(\chi, v) \\ \dot{\chi}_2 &= \gamma_2(N_1(\chi_1 - \chi_2) + v) = f_{H,2}(\chi, v), \end{aligned} \tag{30}$$

where $\chi_i \in \mathbf{R}$ represents the shared position of all robots of type $\mathcal{C}_i$, and $\chi = [\chi_1, \chi_2]^T$.

A specification set that encodes a separation distance of Δ between the two types of robots is $\mathcal{S} = \{x \in \mathbf{R} \mid \|x_i - x_j\| = \Delta,\ \forall i,j,\ i \in \mathcal{C}_1, j \in \mathcal{C}_2\}$. Consequently, we pick a candidate CLF [11]

$$V(\chi) = \frac{1}{4}\left(\|\chi_1 - \chi_2\|^2 - \Delta^2\right)^2, \tag{31}$$

which is positive definite everywhere except at the quasi-static equilibrium points, where $\|\chi_1 - \chi_2\| = \Delta$. Next, we need to show that

$$\dot{V}(\chi, v) = \begin{bmatrix} \frac{\partial V}{\partial \chi_1} \\ \frac{\partial V}{\partial \chi_2} \end{bmatrix}^T \begin{bmatrix} f_{H,1}(\chi, v) \\ f_{H,2}(\chi, v) \end{bmatrix} < 0 \tag{32}$$

Suppose that in this example the domain is $D = \left\{\chi \in \mathbf{R}^2 \;\middle|\; 0 \leq \|\chi_1 - \chi_2\| \leq \Delta, \chi_1 \leq \chi_2\right\}$, that all robots of the same type start at the same location $[\chi_1(t_0), \chi_2(t_0)]^T \in D$, that the "weights" of the the two types of robots are ordered $0 < \gamma_1 < \gamma_2$, and that

$$\begin{aligned} \dot{V}(\chi, v) = &-(\gamma_1 N_2 + \gamma_2 N_1)(\chi_1 - \chi_2)^2(\|\chi_1 - \chi_2\|^2 - \Delta^2) \\ &- (\gamma_2 - \gamma_1)(\chi_1 - \chi_2)(\|\chi_1 - \chi_2\|^2 - \Delta^2)v, \end{aligned} \tag{33}$$

then for every $\chi \in D$,

$$v \geq \frac{\gamma_1 N_2 + \gamma_2 N_1}{\gamma_2 - \gamma_1}(\chi_2 - \chi_1) \tag{34}$$

will ensure that $\dot{V}(\chi, v) \leq 0$, where $\dot{V}(\chi, v) = 0$ only whenever $\|\chi_1 - \chi_2\| = \Delta$ or $\chi_1 = \chi_2$. By LaSalle's invariance principle, this system will converge to the largest invariant set M in $\left\{\chi \in \Omega \;\middle|\; \dot{V}(\chi, v) = 0\right\}$ as $t \to \infty$, where Ω is the compact subset

$$\left\{\chi \in \mathbf{R}^2 \;\middle|\; V(\chi) \leq \frac{1}{4}\Delta^4 - \epsilon, \epsilon > 0\right\} \subset D. \tag{35}$$

The largest invariant set M is

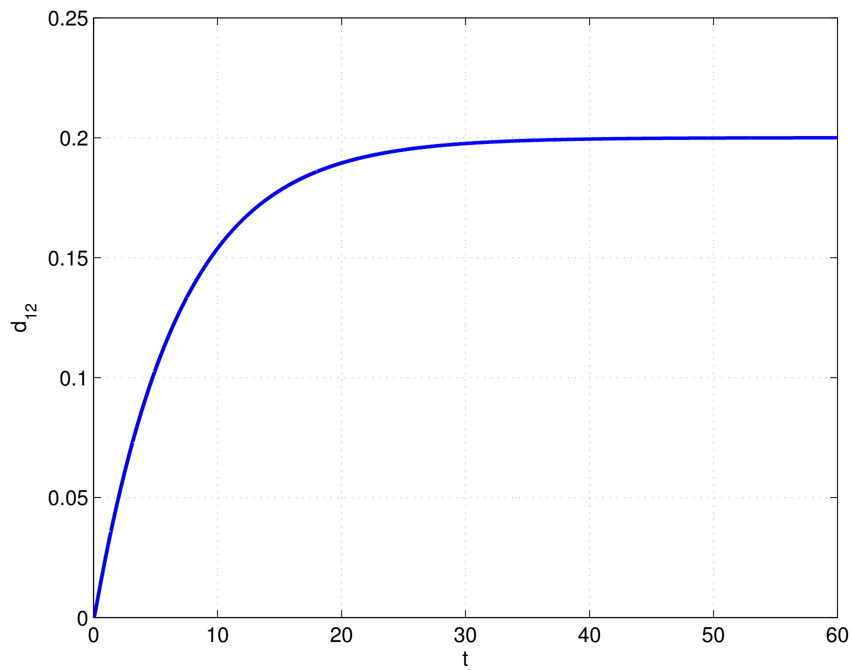
$$\left\{\chi \in \Omega \;\middle|\; \dot{V}(\chi, v) = 0,\ \|\chi_1 - \chi_2\| = \Delta, \right. \\ \left. v = \frac{\gamma_1 N_2 + \gamma_2 N_1}{\gamma_2 - \gamma_1}\Delta\right\}, \tag{36}$$

because for this particular $v \in \mathcal{V}$, $\dot{\chi}_2 - \dot{\chi}_1 = 0$, such that $\|\chi_1 - \chi_2\| = \Delta$ will always hold and thus $\dot{V}(\chi, v) = 0$ and $V(\chi) = 0$. $M \subseteq S$; therefore, it is feasible for the user to use this broadcast control HSI control structure to separate the two types of robots by a distance Δ if the system starts at $\chi(t_0)$ in Ω.

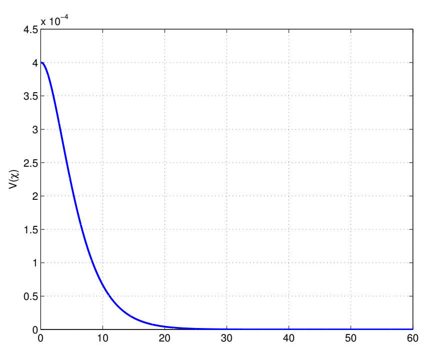
Figure 7 illustrates separation of a swarm of ten robots of $\mathcal{C}_1$ and five robots of $\mathcal{C}_2$ by a distance $\Delta = 0.4$. The user is applying a constant, positive broadcast signal

$$v = \frac{(\gamma_1 N_2 + \gamma_1 N_1)}{\gamma_2 - \gamma_1}\Delta,$$

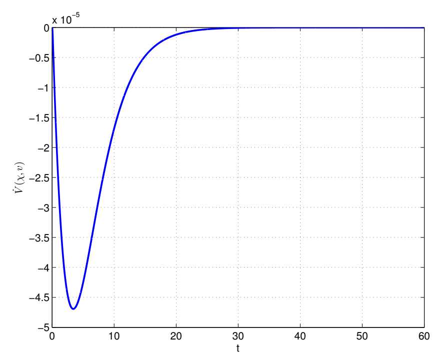
analogous to using a wind tunnel to move robots (on a rail) with mass inversely proportional to γ_i. Figure 7a indicates the starting location of the $\mathcal{C}_1$ (blue) robots and $\mathcal{C}_2$ (red) robots by $\times$ and their final positions by $\bullet$. Initially, the separation between the two types of robots is less than Δ, but eventually, their separation equals Δ. This plot is confirmed by Figure 7b which shows that the CLF $V(\chi)$ is positive, but "energy" dissipates as the desired separation distance is achieved, while Figure 7c shows that $\dot{V}(\chi, v)$ remains negative during the interaction. Consequently,



(A) Separation distance, d_{12}, in $\mathbf{R}$



(B) $V(\chi)$



(C) $\dot{V}(\chi, v)$

FIGURE 7. A user is separating a swarm of ten robots of $\mathcal{C}_1$ ($\gamma_1 = 0.2$) and 30 robots of $\mathcal{C}_2$ ($\gamma_2 = 0.9$) by a distance $\Delta = 0.2$ with a broadcast signal v.

it is feasible for the user to use a strong enough broadcast signal to separate the two types of robots in the example by a distance of Δ.

5.2.1. *Attention, effort, and scalability.* Following the developments in the previous single-leader network example, we compute v^* for this separation problem using optimal control tools. We solve the following optimization problem:

$$\begin{aligned}
\min_{w} \quad & J(w) = \frac{1}{2}\int_0^\infty \left(\kappa(d_{12}-\Delta)^T(d_{12}-\Delta) + w^T w\right) dt \\
\text{s.t.} \quad & \dot{x} = -(\gamma_1 N_2 + \gamma_2 N_1)d_{12} + (\gamma_2 - \gamma_1)v = Ad_{12} + Bv \\
& \dot{v} = w \\
& d_{12}(0) = d_{12,0},\ v(0) = 0,
\end{aligned} \tag{37}$$

where d_{12} denotes the separation between the two classes of robots, and $\kappa > 1$ weighs the tracking error stronger than the cost on attention. The effort cost, $v^T v$, is missing, because in this particular task, it is crucial to exert enough effort to separate the two classes of robots.

This is again a continuous-time, infinite horizon linear quadratic regulator-like (LQR-like) problem, which can be solved in the following manner. First, the first order necessary conditions (FONC) for optimality are:

$$\begin{aligned}
H &= \frac{1}{2}\left((d_{12}-\Delta)^T(d_{12}-\Delta) + w^T w\right) + \lambda^T \dot{x} + \mu^T \dot{v} \\
\frac{\partial H}{\partial w} &= w^T + \mu^T = 0 \Rightarrow w = -\mu \\
\dot{\lambda} &= -\frac{\partial H}{\partial x} = A^T\lambda - \kappa d_{12} + \kappa\Delta \\
\dot{\mu} &= -\frac{\partial H}{\partial v} = -B^T\lambda
\end{aligned} \tag{38}$$

It is important to note here that the co-state dynamics, $\dot{\lambda}$, include an extra affine term that is typically not present in a standard LQR problem. For convenience, let us stack states and co-states into single variables in the following way:

$$\begin{aligned}
z &= \begin{bmatrix} d_{12} \\ v \end{bmatrix},\ \dot{z} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w = A_z z + B_z w \\
\eta &= \begin{bmatrix} \lambda \\ \mu \end{bmatrix},\ \dot{\eta} = \begin{bmatrix} A^T & 0 \\ B^T & 0 \end{bmatrix} \eta - \begin{bmatrix} \kappa & 0 \\ 0 & 0 \end{bmatrix} z + \begin{bmatrix} \kappa\Delta \\ 0 \end{bmatrix} = -A_z^T \eta - C_z z + \Psi
\end{aligned} \tag{39}$$

We propose that $\eta(t) = S(t)z(t) + P(t)$ is the solution to the stacked co-state equations. The affine component, $P(t)$, is to account for the affine component that is tracked in the cost. If we start from the proposed solution, then

$$\begin{aligned}
\eta &= Sz + P \\
\dot{\eta} &= \dot{S}z + S\dot{z} + \dot{P} \\
-A_z^T\eta - C_z z + \Psi &= \dot{S}z + SA_z z + SB_z w + \dot{P} \\
-A_z^T S z - A_z^T P - C_z z + \Psi &= \dot{S}z + SA_z z - SB_z B_z^T S z - SB_z B_z^T P + \dot{P} \\
-\dot{P} - (A_z^T - SB_z B_z^T)P + \Psi &= \left(\dot{S} + SA_z + A_z^T S - SB_z B_z^T S + C_z\right) z
\end{aligned} \tag{40}$$

Since this LQR-like problem is computed over an infinite horizon, we can compute the steady state $\hat{S}$ and $\hat{P}$, when $\dot{S} = 0$ and $\dot{P} = 0$. Consequently, to satisfy Equation 40, we must solve

$$\begin{aligned}
\hat{P} &= (A_z^T - SB_z B_z^T)^{-1}\Psi \\
0 &= \hat{S}A_z + A_z^T\hat{S} - \hat{S}B_z B_z^T\hat{S} + C_z
\end{aligned} \tag{41}$$

The second equation is the continuous time algebraic Ricatti equation, while P can be solved for directly. Finally, we are able to compute $\dot{v}^* = w$,

$$\begin{aligned} w &= -\mu \\ &= -B_z^T(\hat{S}z + \hat{P}). \end{aligned} \tag{42}$$

Consequently, the optimal user control input signal is

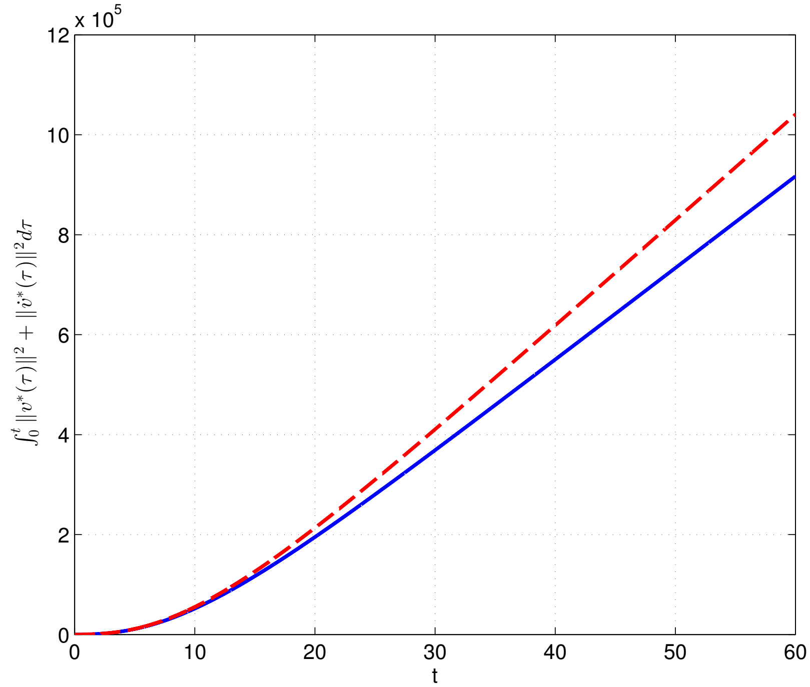
$$v^*(t) = \int_0^t w(\tau)d\tau, \ v^*(0) = 0. \tag{43}$$

Figure 8 was generated by separating a swarm of two classes with the optimal broadcast user input $v^*(t)$. The attention-effort cost is illustrated in Figure 8a, while Figure 8b and 8c illustrate the instantaneous effort and attention. The attention-effort cost increases steadily, because a constant input signal is required to keep the two classes of robots separated. The instantaneous effort ramps up to separate the two classes of robots by a distance of $\Delta = 0.2$, which also requires some attention. Once the two classes are separated, the (instantaneous) attention is zero, while effort is constant, but non-zero.

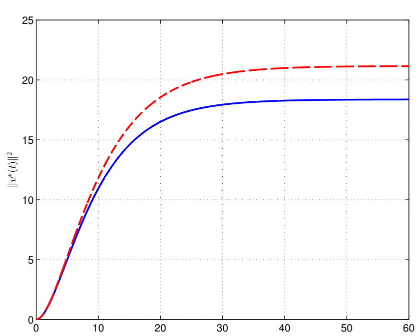
Scalability can be calculated by adding one more robot to each class. In this example, the new robots are initially located at the centroid of the other robots in their class. Figure 8a illustrates the increased attention-effort cost of completing the “same” task with an extra robot via the dashed line. The increase in cost is mainly attributed to an increase in effort as shown in Figure 8b, while attention has only marginally increased as shown in Figure 8c. The scalability metric for this particular task has to be approximated, because the constant non-zero effort continuously increases the attention-effort cost. Therefore, we will approximate scalability by examining the attention-effort costs at a point in time when separation has been achieved, attention is zero, and effort is constant. Once again, we approximate $\Sigma(n)$ by the slope of a linear fit on the attention-effort cost. Consequently, $\Sigma(n) = 5010.4n$, which translates to a 2.2% cost increase over the original cost for every new robot. In comparison, broadcast control in the rendezvous task incurred a 1.65% cost increase over the original cost for every new robot.

5.3. **Remarks.** The two examples in the section show that a CLF approach is useful to show convergence of the HSI-structure multi-robot system to a specification set. In fact, our definition of an HSI control structure allows us to use CLFs directly, and the CLFs themselves can typically be constructed by inspecting the specification set. The specification set is also useful when adding a tracking cost to the optimal control problem. The optimal control problems may be different for each HSI control structure and task; however, we have shown that a general guideline is to include a tracking, effort, and attention cost when computing v^*. Consequently, v^* will likely serve as a good proxy (or benchmark) for the user control input, v, when evaluating attention, effort, and scalability; however, there is an opportunity to improve the cost in the optimal control problem to better model the user, and therefore, generate a better approximation of a user’s control input.

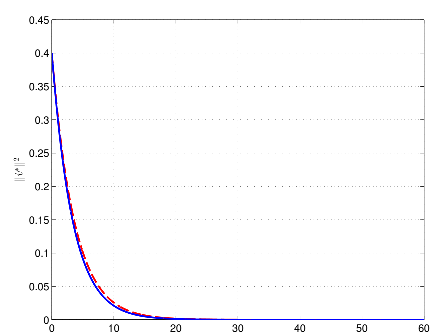
6. **Conclusion.** In this paper, we have provided a precise definition for what it means to impose a human-swarm interaction (HSI) control structure on a multi-robot system and to achieve a geometric configuration with a swarm of robots. With these two definitions in hand, we defined that feasibility in this context implies that



(A) Cost $J_{AE}(v^*)$ up to time t (Attention-Effort)



(B) $\|v^*(t)\|^2$ (Effort)



(C) $\|\dot{v}^*(t)\|^2$ (Attention)

FIGURE 8. A user's estimated attention and effort while separating swarm of $|\mathcal{C}_1| = 10, |\mathcal{C}_2| = 30$ robots (solid) and $|\mathcal{C}| = 11, |\mathcal{C}_2| = 31$ robots (dashed).

a user can successfully guide a swarm of robots into some desired geometric configuration. We have also shown that finding a control Lyapunov function (CLF) implies feasibility, such that CLFs can be used to show that a particular combination of multi-robot system and HSI control structure is appropriate for achieving a particular geometric configuration or set of configurations as demonstrated by the included examples. Additionally, we proposed attention, effort, and scalability as metrics for evaluating a user's interactions with an HSI-enabled swarm of robots during a specific task. We demonstrated in two examples how to use optimal control tools to generate an approximation of the user control input, which allowed us to evaluate (and possibly improve) an HSI control structure before users have to interact with the swarm of robots. Moreover, we were able to demonstrate that our definition for HSI control structures and this set of tools allows us to make a quantitative comparison between different HSI control structures, and decide which control structure to pick for a particular task.

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