09:08:33 OCA PAD AMENDMENT - PROJECT HEADER INFORMATION 09/15/95

Active

Project #: E-24-666 Cost share #: E-24-331 Rev #: 9
Center # : 10/24-6-R7463-0A0 Center shr #: 10/22-1-F7463-0A0 OCA file #:
Work type : RES
Contract#: SES-9122674 Mod #: OPAS Document : GRANT
Prime #: Contract entity: GTRC

Subprojects ? : Y CFDA: 47.051
Main project #: PE #:

Project unit: ISYE Unit code: 02.010.124
Project director(s):
NEMHAUSER G L ISYE (404)894-2306

BARNHART C ISYE (404)894-2324

Sponsor/division names: NATL SCIENCE FOUNDATION / GENERAL
Sponsor/division codes: 107 / 000

Award period: 920415 to 960930 (performance) 961231 (reports)

Sponsor amount	New this change	Total to date
Contract value	0.00	150,000.00
Funded	0.00	150,000.00
Cost sharing amount		7,500.00

Does subcontracting plan apply ?: N

Title: COLUMN GENERATION FOR AIRLINE PROBLEMS

PROJECT ADMINISTRATION DATA

OCA contact: Jacquelyn L. Bendall 894-4820

Sponsor technical contact

N. JOHN CASTELLAN
(202)357-7417

NATIONAL SCIENCE FOUNDATION
1800 G STREET, NW
WASHINGTON, DC 20550

Sponsor issuing office

STEPHEN G. BURNISKY
(202)357-9653

NATIONAL SCIENCE FOUNDATION
1800 G STREET, NW
WASHINGTON, DC 20550

Security class (U,C,S,TS) : U ONR resident rep. is ACO (Y/N): N
Defense priority rating : supplemental sheet
Equipment title vests with: Sponsor GIT X

Administrative comments -
ISSUED TO EXTEND PROJECT TERMINATION DATED TO SEPTEMBER 30, 1996 WITH THE FINAL REPORT DUE DECEMBER 31, 1996.

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

4

Closeout Notice Date 01/02/97

Project No. E-24-666 Center No. 10/24-6-R7463-0A0

Project Director NEMHAUSER G L School/Lab ISYE

Sponsor NATL SCIENCE FOUNDATION/GENERAL

Contract/Grant No. SES-9122674 Contract Entity GTRC

Prime Contract No.

Title COLUMN GENERATION FOR AIRLINE PROBLEMS

Effective Completion Date 960930 (Performance) 961231 (Reports)

Closeout Actions Required:	Y/N	Date Submitted
Final Invoice or Copy of Final Invoice	N	
Final Report of Inventions and/or Subcontracts	N	
Government Property Inventory & Related Certificate	N	
Classified Material Certificate	N	
Release and Assignment	N	
Other	N	

Comments
LETTER OF CREDIT APPLIES. 98A SATISFIES PATENT REPORT.

Subproject Under Main Project No.

Continues Project No.

Distribution Required:

Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
Procurement/Supply Services	Y
Research Property Managment	Y
Research Security Services	N
Reports Coordinator (OCA)	Y
GTRC	Y
Project File	Y
Other	N
	N

E-24-666

Georgia Tech

School of Industrial and Systems Engineering

Georgia Institute of Technology
Atlanta, Georgia 30332-0205
USA
404•894•2300
404•894•2301 FAX

April 14, 1994

Dr. Robin A. Cantor
Program Director
Decision Risk & Management Science
National Science Foundation
4201 Wilson Boulevard
Arlington, VA 22230

RE: SES 9122674
Cynthia Barnhart/George L. Nemhauser

Dear Robin:

Enclosed is a first-year progress report on grant "Column Generation for Airline Problems". I believe that our work is having a major impact in the airline industry.

I have also enclosed a request for second-year funding of $75,000. The budget contains a subcontract with MIT because the co-principal investigator Dr. Cynthia Barnhart has moved there. The delay in requesting second-year funding was due to a change in companies that are providing the matching funds. With the financial crises going on in the airline industry it became necessary to find some new companies. We now have agreements with three new companies: United Airlines, Praxair (letters attached and contract signed), and AT&T. The letter from AT&T will be sent under separate cover.

I have also enclosed copies of the reports prepared under the grant.

Sincerely,

George L. Nemhauser
Institute Professor and
Chandler Chair

GLN/yk
Enclosures

APPENDIX D

ANNUAL NSF GRANT PROGRESS REPORT

NSF Program: Decision Risk & Management Science
NSF Award Number: SES-9122674
PI Name: Cynthia Barnhart
George L. Nemhauser
Period Covered By This Report:
PI Institution: Georgia Institute of Technology
Date:
PI Address: ISyE - Groseclose Bldg.
765 Ferst Street
Atlanta, GE 30332-0205

☑ **Check if Continued Funding is Requested**

Please include the following information:

1. Brief summary of progress to date and work to be performed during the succeeding period;
2. Statement of funds estimated to remain unobligated —if more than 20%— at the end of the period for which NSF currently is providing support (not required for participants in the Federal Demonstration Project);
3. Proposed budget for the ensuing year in the NSF format, **only** if the original award letter did not indicate specific incremental amounts or if adjustments to a planned increment exceeding the greater of 10% or $10,000 are being requested;
4. Current information about other research support of senior personnel, if changed from the previous submission;
5. Any other significant information pertinent to the type of project supported by NSF or as specified by the terms and conditions of the grant;
6. A statement describing any contribution of the project to the area of education and human-resource development, if changed from any previous submission; and
7. Updated information on animal care and use, Institutional Biohazard Committee and Human Subject Certification, if changed substantially from those originally proposed and approved.

I certify that to the best of my knowledge (1) the statements herein (excluding scientific hypotheses and scientific opinions) are true and complete, and (2) the text and graphics in this report as well as any accompanying publications or other documents, unless otherwise indicated, are the original work of the signatories or individuals working under their supervision. I understand that the willful provision of false information or concealing a material fact in this report or any other communication submitted to NSF is a criminal offense (U.S. Code, Title 18, Section 1001.)

P.I. Signature: _____

NSF Form 1328 (1/94)

FIRST YEAR PROGRESS REPORT
April 1994

National Science Foundation
Grant No. : SES-9122674
Project Title: Column Generation for Airline Problems
Principal Investigators: Cynthia Barnhart and George L. Nemhauser

I. Research Accomplished

The objective of this grant is to develop column generation methods for solving large-scale integer programs and to apply them to optimization problems associated with air transportation and other industrial problems. The importance of large-scale mixed-integer programming is discussed in [7]. In the first year, matching funds were received from American, Delta and Northwest Airlines. For each of these companies, we studied a different problem. We also developed a general methodology, called branch-and-price, for solving integer programs with a very large number of variables by column generation.

A. Branch-and-Price

This methodology applies to integer programs with a huge number of variables. Sometimes it is desirable to formulate an integer program with a huge number of variables even though there are alternative formulations with a much smaller number of variables because the bigger formulation gives a much tighter linear programming relaxation and eliminates certain symmetries which make the smaller formulation impossible to solve. The cutting-stock problem is a well-known example. In other situations, such as crew scheduling, modeling is not possible unless a huge number of variables is considered.

In solving LP relaxations at each node of the branch-and-bound tree, it is necessary to use special procedures to price the non-basic variables since there are so many of them. For example, pricing may be done implicitly by solving an optimization subproblem. However, the subproblem may be complicated greatly by the choice of branching rules. One of our main contributions here are some approaches for how to branch while keeping the subproblem tractable. This work is described in [3]. Its application to the cutting stock problem is given in [9].

B. American Airlines

We developed a new model and solution procedure for solving the airline crew scheduling problem. It is based on breaking the decision problem into two stages. In the first stage, we select feasible one-day sets of flight segments for crews that cover all of the flight segments. These are called duty sets. In the second stage we combine the duty sets into pairings, which constitute from one to four or five days of flying that begin and end at the crew's base. We present a decomposition algorithm for solving this model and present results for test problems provided by American Airlines. The results are promising because our approach gives a much tighter linear programming relaxation than the standard set partitioning problem. However our linear programs are very difficult to solve and more work is needed in this area. This work is described in [8].

We have developed models and algorithms for the crew scheduling problem in international operation [1,2]. This is called the long-haul crew pairing problem. It differs from the domestic crew pairing problem in that its network structure is not hub-and-spoke and its flight segments are typically long distance and not operated on a daily schedule. These characteristics allow us to determine an optimal solution to the linear programming relaxation of the crew pairing problem using column generation, with a (constrained) shortest path subproblem solved to generate pairings as needed. Improved solutions to the linear program are determined using a deadhead selection procedure that identifies flights on which crews fly as passengers in order to reposition them for better utilization. An integer solution to the crew pairing problem is determined using the pairing in the solution to the linear programming relaxation, augmented with additional (nonbasic) pairings.

C. Delta Airlines

We have worked with Delta on the fleet assignment problem, which is to decide which type of airplane to assign to each flight segment. Our work on the formulation and solution of the basic fleet assignment problem is described in [6]. Results on the complexity of fleet assignment and related issues are given in [5]. In [4], we study how to include some complex issues concerning maintenance and crew planning into the basic model. After fleeting is done, routes must be determined for the airplanes in each fleet. We are also studying this rotation problem and will report on it in the near future. Finally, work is in progress on a new decomposition/column generation approach to the fleeting problem which allows many special constraints to be included at outlier stations.

D. Northwest Airlines

When delays occur because of malfunctioning equipment or bad weather, it may be necessary to reschedule some crews. This is called the crew recovery problem. Unlike fleet assignment or crew planning, which have reasonable lead times for their solution, crew recovery is an operational problem that must be solved with short lead times. Our approach to crew recovery is to consider all of the available crews whose assignments could be changed to help manage the delay and the reserve crews. The current assignments of these crews are temporarily relaxed and a new crew assignment problem is solved. When the number of delays is small, which is typically the case when delays are caused by malfunctioning equipment but not bad weather, the crew assignment problem is small and can be solved quickly to get a good reassignment. A report on this work will be available in the near future. Crew recovery for multiple delays is likely to involve cancellations and requires a different approach which we are also beginning to study.

II. Publications

1. C. Barnhart, L.Hatay and E.L. Johnson, "Deadhead Selection for the Long-Haul Crew Pairing Problem," to appear in *Operations Research.*, 1994.

2. C. Barnhart, E.L. Johnson, R. Anbil and L. Hatay, "A Column Generation Technique for the Long-Haul Crew Assignment Problem", *Optimization in Industrial Environments*, Vol. 2, T. Ciriani and R. Leachman, eds., John Wiley and Sons, Ltd., 1994.

3. C. Barnhart, E.L. Johnson, G. L. Nemhauser, M. Savelsbergh and P. Vance, "Branch-and-Price: Column Generation for Solving Huge Integer Programs", Working Paper COC-94-03.

4. L.W. Clarke, C.A. Hane, E.L. Johnson and G.L. Nemhauser, "Modeling Issues in the Fleet Assignment Problem", Working Paper COC-94-05.

5. Z. Gu, E.L. Johnson, G.L. Nemhauser and Y. Wang, "Some Properties of the Fleet Assignment Problem", Working Paper COC-92-06, to appear in *Operations Research Letters*, 1994.

6. C.A. Hane, C. Barnhart, E.L. Johnson, R.E. Marsten, G.L. Nemhauser and G. Sigismondi, "The Fleet Assignment Problem: Solving a Large-Scale Integer Program", Working Paper COC-92-04, to appear in *Mathematical Programming.*

7. G.L. Nemhauser, "The Age of Optimization: Solving Large-Scale Real World Problems", Working Paper COC-93-04, to appear in *Operations Research*, 1994.

8. P. Vance, "Crew Scheduling Cutting Stock and Column Generation: Solving Huge Integer Programs", Ph.D. Dissertation, 1993.

9. P. Vance, C. Barnhart, E.L. Johnson, and G.L. Nemhauser, "Solving Binary Cutting Stock Problems by Column Generation and Branch-and-Bound," Working Paper COC-92-09, to appear in *Computational Optimization and Applications.*

III. Scientific Personnel

A. Faculty Supported by NSF or Matching Funds

George L. Nemhauser, Institute Professor and Chandler Chair
Cynthia Barnhart, Assistant Professor, (currently at MIT)
L.W. Clarke, Assistant Professor
Ram Pandit, Visiting Assistant Professor
(currently with Consolidated Freight)

B. Other Faculty

Ellis L. Johnson, Coca-Cola Professor
Martin W.P. Savelsbergh, Associate Professor

C. Postdoctoral Associates Supported by NSF or Matching Funds

Natashia Boland
Lloyd W. Clarke
Christopher A. Hane (currently visiting Assistant Professor, Univ. of British Columbia)

D. Students Supported by NSF or Matching Funds

Pamela Vance, Ph.D. Sept. 1993 (currently Assistant Professor, Auburn University)
John Zhu, Ph.D expected June 1994
Stephen Querido, M.S., June 1993
Zonghao Gu, Ph.D. expected 1995
Jinhua Wang, Ph.D. expected 1995

IV. Second Year

A. Matching Support

United Airlines is providing $25,000 for research on the weekly exceptions problem of crew planning. This problem deals with crew pairing for flights that are not scheduled on a daily basis.

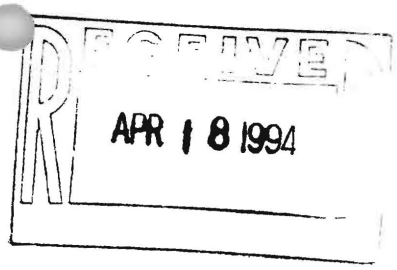
Praxair is providing $28,215 for research on the optimization of a production/distribution system for gas products.

AT&T is providing $36,000 for research on production of scheduling fiber-optic cable.

B. NSF Budget

A budget for $75,000 for the second year of NSF funding is attached.

UNITED AIRLINES



Executive Offices

Joint NSF/Private Sector Research Opportunities Initiative

LETTER OF SUPPORT

This letter is to confirm that United Airlines agrees to be a Cooperating Organization in the Joint Private Sector Research Opportunities Initiative. United Airlines is actively participating in joint research with the Georgia Institute of Technology in solving the Crew-Pairing Problem with Exceptions, and is providing Georgia Tech with $25,000 in support of this project designated as matching funds to an NSF contribution.

Established in 1926 and incorporated in the State of Delaware, United Airlines Incorporated is a commercial airline with executive offices in Elk Grove Village, Illinois. The Corporate Research and Development Department of United Airlines currently employs a technical staff of over 25, all with advanced degrees (M.S., PH.D) in Operations Research, Industrial Engineering and related fields.

Successful completion of the joint venture will result in substantial savings to United Airlines, both financially and otherwise. It is estimated that a savings in cost of several million dollars could be had by using improved techniques to generate crew pairings with exceptions. Additionally, the experience gained from this project can be used to solve other airline problems.

The key contact person for the project is Dr. Deepa Mahidhara, a manager in the Operations Research Group. Dr. Mahidhara has worked for almost five years developing tools for improved airline crew scheduling and, as chairperson of the Crew Scheduling Study group of the Airline Group of the International Federation of Operations Research Societies, is recognized as one of the main practitioners in the field of crew optimization.

If you need more information, please feel free to contact me at (708) 952-4874.

Sincerely,

Richard L. Wysong
Director, Corporate Research and Development

SA

PRAXAIR

Praxair, Inc.
Post Office Box 44
Tonawanda, NY 14151-0044
Tel (716) 879-2000

April 14, 1994

George L. Nemhauser
Institute Professor and Chandler Chair
School of Industrial an Systems Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0205

Dear George,

I appreciate you considering Praxair for matching funds from the NSF. We are happy to be a Cooperating Organization in the Joint Private Sector Research Opportunities Initiative. We look forward to executing the "Praxair Production Distribution Model" project as outlined in the contract that provides Georgia Tech with $28,215 in research monies to be matched by the NSF. I am confident that the matching funds will lead to further improvements in Praxair's competitive position, as well as to productive new research topics for the Logistics Optimization Center.

I will remain the key contact person throughout this effort and look forward to a growing relationship with the Georgia Institute of Technology.

Sincerely,

Michael D. Jordan
Manager of Operations Research

SECOND YEAR FUNDS REQUEST

SUMMARY PROPOSAL BUDGET

FOR NSF USE ONLY		
PROPOSAL NO.	DURATION (MONTHS) Proposed	Granted
AWARD NO.		

ORGANIZATION: GEORGIA TECH RESEARCH CORPORATION

PRINCIPAL INVESTIGATOR/PROJECT DIRECTOR: Dr. George L. Nemhauser

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-mos. CAL	ACAD	SUMR	Funds Requested By Proposer	Funds Granted By NSF (If Different)
1. George L. Nemhauser, PI		1		$ 13,818	$
2.					
3.					
4.					
5.					
6. ( ) OTHERS (LIST INDIVIDUALLY ON BUDGET EXPLANATION PAGE)					
7. (1) TOTAL SENIOR PERSONNEL (1-6)		1		13,818	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. ( ) POST DOCTORAL ASSOCIATES					
2. ( ) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)					
3. (1) GRADUATE STUDENTS @ 1/3 time (2900) for 4 qtrs				11,600	
4. ( ) UNDERGRADUATE STUDENTS					
5. (1) SECRETARIAL - CLERICAL				2,500	
6. ( ) OTHER					
TOTAL SALARIES AND WAGES (A+B)				27,918	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS) @25.1% of A1 and B5				4,096	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A+B+C)				32,014	
D. PERMANENT EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $1,000.)					
TOTAL PERMANENT EQUIPMENT					
E. TRAVEL 1. DOMESTIC (INCL. CANADA AND U.S. POSSESSIONS)				4,000	
2. FOREIGN					
F. PARTICIPANT SUPPORT COSTS					
1. STIPENDS $					
2. TRAVEL					
3. SUBSISTENCE					
4. OTHER					
( ) TOTAL PARTICIPANT COSTS					
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				2,608	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION					
3. CONSULTANT SERVICES					
4. COMPUTER (ADPE) SERVICES					
5. SUBCONTRACTS 2nd yr funding for CoPI Cynthia Barnhart at MIT				14,260	
6. OTHER Graduate Assistant Tuition @ 22.0% of salary				2,552	
TOTAL OTHER DIRECT COSTS				19,420	
H. TOTAL DIRECT COSTS (A THROUGH G)				55,434	
I. INDIRECT COSTS (SPECIFY RATE AND BASE)					
TOTAL INDIRECT COSTS @37.0% of direct cost, less tuition and equipment				19,566	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				75,000	
K. RESIDUAL FUNDS (IF FOR FURTHER SUPPORT OF CURRENT PROJECTS SEE GPM 252 AND 253)					
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ 75,000	$

M. COST SHARING: PROPOSED LEVEL $ AGREED LEVEL IF DIFFERENT $

PI/PD TYPED NAME & SIGNATURE*	DATE	FOR NSF USE ONLY INDIRECT COST RATE VERIFICATION		
INST. REP. TYPED NAME & SIGNATURE*	DATE 4/26/94	Date Checked	Date of Rate Sheet	Initials-ORG

DAVID B. BRIDGES
ASST. TO VP/GEN. MGR.

NSF Form 1030 (1/94) *Supersedes All Previous Editions*

*SIGNATURES REQUIRED ONLY FOR REVISED BUDGET (GPM 233)

E-24-666
#2

OMB Number 345-0058

NATIONAL SCIENCE FOUNDATION
4201 Wilson Blvd.,
Arlington, VA 22230

BULK RATE
POSTAGE & FEES PAID
National Science Foundation
Permit No. G-69

PI/PD Name and Address

George L. Nemhauser OCT 02 REC'D
School of Industrial and Systems Eng.
GA Tech Res Corp - GIT
765 Ferst Drive
Atlanta GA 30332-0205

NATIONAL SCIENCE FOUNDATION
FINAL PROJECT REPORT

PART I - PROJECT IDENTIFICATION INFORMATION

1. Program Official/Org. Robin A. Cantor - SBR

2. Program Name ECONOMIC, DECISION, & MANAGEMENT SCIENCE

3. Award Dates (MM/YY) From: 04/92 To: 09/96

4. Institution and Address
GA Tech Res Corp - GIT
Administration Building
Atlanta GA 30332

5. Award Number 9122674

6. Project Title
Column Generation for Airline Problems

** You are encouraged to submit your Final Project Report electronically **
** through the NSF FastLane home page (www.fastlane.nsf.gov). **

This Packet Contains
NSF Form 98A
And 1 Return Envelope

NSF Grant Conditions (Article 17, GC-1, and Article 9, FDP-11) require submission of a Final Project Report (NSF Form 98A) to the NSF program officer no later than 90 days alter the expiration of the award. Final Project Reports for expired awards must be received before new awards can be made (NSF Grants Policy Manual Section 677).

Below, or on a separate page attached to this form, provide a summary of the completed projects and technical information. Be sure to include your name and award number on each separate page. See below for more instructions.

PART II - SUMMARY OF COMPLETED PROJECT (for public use)

The summary (about 200 words) must be self-contained and intelligible to a scientifically literate reader. Without restating the project title, it should begin with a topic sentence stating the project's major thesis. The summary should include, if pertinent to the project being described, the following items:

- The primary objectives and scope of the project
- The techniques or approaches used only to the degree necessary for comprehension
- The findings and implications stated as concisely and informatively as possible

PART III - TECHNICAL INFORMATION (for program management use)

List references to publications resulting from this award and briefly describe primary data, samples, physical collections, inventions, software, etc. created or gathered in the course of the research and, if appropriate, how they are being made available to the research community. Provide the NSF Invention Disclosure number for any invention.

I certify to the best of my knowledge (1) the statements herein (excluding scientific hypotheses and scientific opinion) are true and complete, and (2) the text and graphics in this report as well as any accompanying publications or other documents, unless otherwise indicated, are the original work of the signatories or of individuals working under their supervision. I understand that willfully making a false statement or concealing a material fact in this report or any other communication submitted to NSF is a criminal offense (U.S. Code, Title 18, Section 1001).

	12/18/96
Principal Investigator/Project Director Signature	**Date**

IMPORTANT:
MAILING INSTRUCTIONS
Return this *entire* packet plus all attachments in the envelope attached to the back of this form. Please copy the information from Part I, Block I to the *Attention block* on the envelope.

PART IV -- FINAL PROJECT REPORT -- SUMMARY DATA ON PROJECT PERSONNEL

(To be submitted to cognizant Program Officer upon completion of project)

The data requested below are important for the development of a statistical profile on the personnel supported by Federal grants. The information on this part is solicited in resonse to Public Law 99-383 and 42 USC 1885C. All information provided will be treated as confidential and will be safeguarded in accordance with the provisions of the Privacy Act of 1974. You should submit a single copy of this part with each final project report. However, submission of the requested information is not mandatory and is not a precondition of future award(s). Check the "Decline to Provide Information" box below if you do not wish to provide the nformation.

Please enter the numbers of individuals supported under this grant.
Do not enter information for individuals working less than 40 hours in any calendar year.

	Senior Staff		Post-Doctorals		Graduate Students		Under-Graduates		Other Participants[1]	
	Male	Fem.	Male	Fem.	Male	Fem.	Male	Fem.	Male	Fem.
A. Total, U.S. Citizens	2	1	2		1	1				
B. Total, Permanent Residents										
U.S. Citizens or Permanent Residents[2]:										
American Indian or Alaskan Native										
Asian.....										
Black, Not of Hispanic Origin.....			1							
Hispanic										
Pacific Islander										
White, Not of Hispanic Origin	2	1	1		1	1				
C. Total, Other Non-U.S. Citizens			1	1	3					
Specify Country										
1. India			1							
2. Australia				1						
3. China Chech Republic					2 1					
D. Total, All participants (A + B + C)	2	1	3	1	4	1				
Disabled[3]										

☐ Decline to Provide Information: Check box if you do not wish to provide this information (you are still required to return this page along with Parts I-III).

[1] Category includes, for example, college and precollege teachers, conference and workshop participants.

[2] Use the category that best describes the ethnic/racial status fo all U.S. Citizens and Non-citizens with Permanent Residency. *(If more than one category applies, use the one category that most closely reflects the person's recognition in the community.)*

[3] A person having a physical or mental impairment that substantially limits one or more major life activities; who has a record of such impairment; or who is regarded as having such impairment. *(Disabled individuals also should be counted under the appropriate ethnic/racial group unless they are classified as "Other Non-U.S. Citizens.")*

AMERICAN INDIAN OR ALASKAN NATIVE: A person having origins in any of the original peoples of North America and who maintains cultural identification through tribal affiliation or community recognition.

ASIAN: A person having origins in any of the original peoples of East Asia, Southeast Asia or the Indian subcontinent. This area includes, for example, China, India, Indonesia, Japan, Korea and Vietnam.

BLACK, NOT OF HISPANIC ORIGIN: A person having origins in any of the black racial groups of Africa.

HISPANIC: A person of Mexican, Puerto Rican, Cuban, Central or South American or other Spanish culture or origin, regardless of race.

PACIFIC ISLANDER: A person having origins in any of the original peoples of Hawaii; the U.S. Pacific territories of Guam, American Samoa, and the Northern Marinas; the U.S. Trust Territory of Palau; the islands of Micronesia and Melanesia; or the Philippines.

WHITE, NOT OF HISPANIC ORIGIN: A person having origins in any of the original peoples of Europe, North Africa, or the Middle East.

Part II - Summary of Completed Project

The objective of this grant is to develop column generation methods for solving large-scale optimization problems with discrete variables and to apply them to airline scheduling problems. Matching funds were received from American, Delta and Northwest and United Airlines. The main result is a general methodology, called branch-and-price, for solving integer programs with a very large number of discrete variables by column generation. We have applied this methodology to models of fleet assignment (which type of plane to assign to each flight segment), routing, crew pairing (assignment of crews to flights) and crew recovery (reassignment of crews after a disruption has occurred). The models and solution algorithms have been validated using real data supplied by the participating airlines.

Part III - Technical Information

The branch-and-price methodology has been implemented in our MINTO software system which is distributed by the Georgia Institute of Technology. Models and algorthms for the fleet and crew assignment problems are described in the publications below.

Publications

C. Barnhart, L. Hatay and E.L. Johnson, "Deadhead Selection for the Long-Haul Crew Pairing Problem," *Operations Research*, 43, 491-499, 1995.

C. Barnhart, E.L. Johnson, R. Anbil and L. Hatay, "A Column Generation Technique for the Long-Haul Crew Assignment Problem," *Optimization in Industrial Environments*, Vol. 2, T. Ciriani and R. Leachman, eds., John Wiley and Sons, Ltd., 1994.

C. Barnhart, E.L. Johnson, G. L. Nemhauser, M. Savelsbergh and P. Vance, "Branch-and-Price: Column Generation for Solving Huge Integer Programs," to appear in *Operations Research.*

H. Chu, E. Gelman and E.L. Johnson, "Solving Large Scale Crew Scheduling Problems," LEC 96-02, Georgia Institute of Technology, 1996.

L.W. Clarke, C.A. Hane, E.L. Johnson and G.L. Nemhauser, "Mainteneance and Crew Considerations in Fleet Assignment," *Transportation Science*, 30, 249-260, 1996.

L.W. Clarke, E.L. Johnson, G.L. Nemhauser and Z. Zhu, "The Aircraft Rotation Problem," LEC 95-03, Georgia Institute of Technology, to appear in *Annals of Operations Research: Mathematics of Industrial Systems 2*, 1996.

Z. Gu, E.L. Johnson, G.L. Nemhauser and Y. Wang, "Some Properties of the Fleet Assignment Problem," *Operations Research Letters*, 15, 59-71, 1994.

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