17:50:26 OCA PAD AMENDMENT - PROJECT HEADER INFORMATION 11/06/90

Active

Project #: E-24-655 Cost share #: E-24-368 Rev #: 3
Center # : R6606-0A0 Center shr #: F6606-0A0 OCA file #:
Work type : RES
Contract#: 14-08-0001-G1629 Mod #: 01 Document : GRANT
Prime #: Contract entity: GTRC

Subprojects ? : N
Main project #:

Project unit: ISYE Unit code: 02.010.124
Project director(s):
ESOGBUE A O ISYE (404)894-2323

Sponsor/division names: US DEPT OF INTERIOR / GEOLOGICAL SURVEY
Sponsor/division codes: 111 / 002

Award period: 880930 to 910929 (performance) 911130 (reports)

Sponsor amount	New this change	Total to date
Contract value	0.00	103,352.00
Funded	0.00	103,352.00
Cost sharing amount		103,352.00

Does subcontracting plan apply ?: N

Title: FLOOD PLAIN MANAGEMENT, WATER QUALITY MANAGMENT, MATHEMATICAL MODELS

PROJECT ADMINISTRATION DATA

OCA contact: E. Faith Gleason 894-4820

Sponsor technical contact	Sponsor issuing office
ALLEN FORD, PROJECT OFFICER	FRANCINE H. GRIER, CONT. SPECIALIST
(703)648-6806	(703)648-7386
U.S. GEOLOGICAL SURVEY	U.S. GEOLOGICAL SURVEY
WATER RESOURCES DIVISION -- MS 426	OFF. OF PROCURE & CONTRACTS--MS205C
12201 SUNRISE VALLEY DRIVE	12201 SUNRISE VALLEY DRIVE
RESTON, VA 22092	RESTON, VA 22092

Security class (U,C,S,TS) : U ONR resident rep. is ACO (Y/N): N
Defense priority rating : N/A N/A supplemental sheet
Equipment title vests with: Sponsor GIT X
NONE PROPOSED
Administrative comments -
REVISION 01 EXTENDS THROUGH 9/29/91 & CHANGES THE CONTRACT & TECHNICAL REPS.
BUDGET REVISION DATED 10/26/90 SHIFTS FUNDING BETWEEN BUDGET CATEGORIES.

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 01/27/92

Project No. E-24-655 Center No. R6606-0A0

Project Director ESOGBUE A O School/Lab ISYE

Sponsor US DEPT OF INTERIOR/GEOLOGICAL SURVEY

Contract/Grant No. 14-08-0001-G1629 Contract Entity GTRC

Prime Contract No. __________

Title FLOOD PLAIN MANAGEMENT, WATER QUALITY MANAGMENT, MATHEMATICAL MODELS

Effective Completion Date 910929 (Performance) 911130 (Reports)

Closeout Actions Required:	Y/N	Date Submitted
Final Invoice or Copy of Final Invoice	Y	
Final Report of Inventions and/or Subcontracts	Y	
Government Property Inventory & Related Certificate	Y	
Classified Material Certificate	N	
Release and Assignment	N	
Other	N	

Comments __________

Subproject Under Main Project No. __________

Continues Project No. __________

Distribution Required:	
Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
Procurement/Supply Services	Y
Research Property Managment	Y
Research Security Services	N
Reports Coordinator (OCA)	Y
GTRC	Y
Project File	Y
Other	N
	N

NOTE: Final Patent Questionnaire sent to PDPI.

E-24-655

Proposal for a No Cost Extension

on Project Grant No. 14-08-0001-G 1629

by

Augustine O. Esogbue
School of Industrial and Systems Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0205 U.S.A.

Project Title: WATER QUALITY ENHANCEMENT VIA INTEGRATIVE MANAGEMENT OF NON-POINT SOURCE WATER POLLUTION AND FLOOD DAMAGE REDUCTION CONTROL STRATEGIES

I. Remaining Work: Revised Work Schedule for the Period 10/1/90 - 9/30/91

To complete the project, we need to

1. Produce a final version of a Mathematical Model for the flood control problem
2. Develop a computational algorithm for solving the problem posed in 1
3. Discuss the data needs of the algorithm
4. Design a preliminary data collection scheme in keeping with data needs
5. Develop and debug a high level computer program for processing the algorithm
6. Pretest the data collection instrument
7. Produce a final version of the data collection instrument
8. Test the algorithm with sample - perhaps synthetic - data
9. Modify the optimal flood control algorithm for use in the Non Point Source Water Pollution (BMPs) Control Problem
10. Design data collection instrument for the BMPs
11. Pretest the instrument

12. Develop a high level computer program for processing the algorithm
13. Test the algorithm with synthetic data
14. Analyze the results and finally
15. Write a final report.

II. Reason for Time Extension

In a separate communication we had requested a time extension from the completion time of September 29, 1990 to a new completion time of September 30, 1991. We reiterate that the principal reasons for this request are as follows:

1. The funding of the project was not effected until late in the Fall of 1988.
2. The PI was away on leave of absence to the University of California, Berkeley as the Chancellors Distinguished Visiting Professor of Engineering and Management Science.
3. The PI was sick for a greater part of 1989 and as such could not commit significant effort to the research project. A considerable effort was made to catch up during the Summer and Fall quarters of 1989.

We note that in all of our previous Technical Progress Reports we had alluded to the possibility of the project life extending beyond the date stated in the original proposal. We had also given the above reasons in support of this need.

III. Revised Budget for a Time Extension

		FEDERAL	GIT MATCHING
1.	Personal Services P.I., A.O. Esogbue	1,535.30	2,312.60
2.	Fringe Benefits @ 26.3% of P.I.	403.78	608.22
3.	Graduate Research Assistant (1) Beg. Ph.D. @ 2348/Qtr.	-	2,248.00
4.	Materials and Supplies (includes computer, software supplies, publication, etc.)	-	19,075.11
5.	Total Direct Costs	1,938.08	24,243.93
6.	Overhead @ 62.5% of direct costs	1,211.93	15,152.46
7.	Total Cost ($42,547.40)	3,151.01	39,396.39

E-24-655

Technical Progress Report #1

on Project Grant No. 14-08-0001-G 1629

by

Augustine O. Esogbue
School of Industrial and Systems Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0205 U.S.A.

Project Title: WATER QUALITY ENHANCEMENT VIA INTEGRATIVE MANAGEMENT OF NON-POINT SOURCE WATER POLLUTION AND FLOOD DAMAGE REDUCTION CONTROL STRATAGIES

(Key words: Flood plain management, water quality management, mathematical models)

I. Research Objectives

1. Statement

The objectives of the research are to i) develop new and useable planning methodologies which would enable water resources planners to select a combination of structural and non structural measures both for the twin problems of non-point source water pollution and flood control measures over time and space so as to maximize the expected discounted value of reduction in damages to any regions' water resources due to the almost inseparable problems of non-point source pollution and flood in urban and urbanizing areas over some future planning horizon, (ii) implement the methodologies on a digital computer, and (iii) to test and assess the feasibility and utility of the methodologies in a real-world setting such as the Chattahoochee River Corridor in Fulton County and the Bear Creek watershed located immediately south of the City of Douglasville in Douglas County in Georgia. The latter is much less developed than a typical urban area although it has many of the sedimentation problems of such an area. In short, the difficulties inherent in planning and management of complex socio-technical systems involving imprecise and usually vague data will be minimized via the tools we propose to develop. We hope to develop tools which utilize data in their natural occurring setting exploiting the tendencies of the data to be vaguely stated.

2. Analysis

The foregoing objectives still remain valid. We have, however, enlarged the objective and scope to include developing a model applicable to planning both on a national, regional and local levels. On the other hand, we feel that implementation of the model on a digital computer may be somewhat ambitious and beyond the scope of this phase. We will ultimately do this, but perhaps in a future effort. We will, however, illustrate the operation of the model with various examples and additionally sketch a computational algorithm.

II. Research Approach (Task and Methodology)

1. Statement

The research will begin with an update on the BMP studies in the areas involved in the 1983 study by the principal investigator followed by an inventory of flood control management strategies in use in these areas. Much of this is hard data. Data on damages due to these two types of problems will be collected. In general, such data is essentially vague, imprecise or qualitative. Most people are unable to precisely state these effects. Thus fuzzy set theoretic methods will be invoked to design a data collection and analysis program.

The methodology will be tested first on the flood control project and then adapted for the BMP component. This will be complemented by tools from multi-attribute decision theory and the theory of approximate reasoning. We have applied these to previous studies involving non-point source water

pollution control planning in urban areas [21].

The optimization methodology will be via mathematical programming and heuristics. Decomposition techniques will also be used to solve the problem. Specifically, Benders' decomposition will be employed since the project interdependencies and their competitive nature lead to a classic form for which Benders' decomposition approach has proven to be especially powerful. That is, the problem contains decision variables which are "complicating" in the sense that if they are fixed at some level, then the problem becomes much easier to solve.

In the optimal mix of adjustments to flood problems, the complicating variables correspond to the nonstructural adjustments. For a fixed level of nonstructural flood control, the problem reduces to a classic project sequencing problem in the structural measures.

2. Analysis

Our initial efforts focussed on the development of a general and new philosophical approach to the problem. We next embarked on a mathematical model, based on fuzzy sets theory and the theory of approximate reasoning as envisaged in our proposal, for dealing with the flood control planning problem. We are interested in a robust model applicable to both the national, regional and local levels. This model, although focussed on the flood control problem, is capable of being applied to the nonpoint source water pollution control problem. It is a hybrid fuzzy dynamic programming and branch and bound type algorithm.

III. Summary of Our Efforts and Results

1. Major Output

We have rigorously analyzed the problem and previous related models for the flood control problem. We discarded any temptation to resort to simple quick fixes involving direct modifications. Rather, we have developed a philosophically and mathematically different model. The results of our effort are reflected in the attached technical paper entitled "A Fuzzy Methodology and Algorithm for the Flood Control Problem". This paper is under revision and will be submitted to one of the following journals:

i) Journal of Fuzzy Sets and Systems (Journal of the International Fuzzy Systems Association) Special Issue on Operations Research Applications

ii) Water Resources Bulletin, Journal of the American Water Resources Association

2. We have also updated the Best Management Strategies in use in our study areas.

3. We have attended three conferences where methodologies and applications germane to the research mission were presented. These are

i) Fall National Meeting, Operations Research Society, Denver, Colorado, October 1988

ii) Fall National Meeting, Operations Research Society, New York New York, October 1989

iii) Third World Congress, International Fuzzy Systems Association, Seattle, Washington, July 1989

IV. Future Work

We outline in the sequel the remaining activities necessary for the successful completion of the project.

1. Revise the Mathematical Model
2. Analyze model and refine as necessary
3. Develop a computational algorithm
4. Analyze data needs of the algorithm
5. Design a data collection scheme in keeping with data needs identified in 4.
6. Collect data on flood control strategies and best management strategies from planners at such agencies as the Atlanta Regional Commission, EPA and the State Environmental Planning Division
7. Test the algorithm with sample data
8. Write report.

We expect to attend three or four future meetings to present and discuss some of our findings. The proposed ones are:

i) Conference and Workshop on Stormwater and NonPoint Source Water Management at the University of Louisville, Kentucky, March 1990

ii) Joint Canadian-American Water Resources Association Conference on Water Problems, Toronto, Canada, April 1990

iii) International Federation of Operations Research Societies Conference, Athens, Greece, June 1990

V. Analysis of Results & Problems

We feel we have made some useful beginnings and progress on this project. The project is however behind the original schedule for the following principal reasons:

i) The funding of the project was not effected until late in the Fall of 1988

ii) the PI was away on leave of absence to the University of California, Berkeley as the Chancellors Distinguished Visiting Professor of Engineering and Management Science

iii) The PI was sick for the first half of 1989 and as such could not commit significant effort to the research project. An attempt was made to catch up during the Summer and Fall 1989 quarters.

In view of the above, we expect that the project life may be extended beyond the date in the proposal. We hope however, to continue to make significant efforts and progress towards successfully completing the project as close to schedule as possible.

E-24-655

Technical Progress Report #2

on Project Grant No. 14-08-0001-G 1629

by

Augustine O. Esogbue
School of Industrial and Systems Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0205 U.S.A.

Project Title: WATER QUALITY ENHANCEMENT VIA INTEGRATIVE MANAGEMENT OF NON-POINT SOURCE WATER POLLUTION AND FLOOD DAMAGE REDUCTION CONTROL STRATEGIES

(Key words: Flood plain management, water quality management, mathematical models)

I. Research Objectives

1. Statement

The objectives of the research are to i) develop new and useable planning methodologies which would enable water resources planners to select a combination of structural and non structural measures both for the twin problems of non-point source water pollution and flood control measures over time and space so as to maximize the expected discounted value of reduction in damages to any regions' water resources due to the almost inseparable problems of non-point source pollution and flood in urban and urbanizing areas over some future planning horizon, (ii) implement the methodologies on a digital computer, and (iii) to test and assess the feasibility and utility of the methodologies in a real-world setting such as the Chattahoochee River Corridor in Fulton County and the Bear Creek watershed located immediately south of the City of Douglasville in Douglas County in Georgia. The latter is much less developed than a typical urban area although it has many of the sedimentation problems of such an area. In short, the difficulties inherent in planning and management of complex socio-technical systems involving imprecise and usually vague data will be minimized via the tools we propose to develop. We hope to develop tools which utilize data in their natural occurring setting exploiting the tendencies of the data to be vaguely stated.

2. Analysis

The foregoing objectives still remain valid. We have, however, enlarged the objective and scope to include developing a model applicable to planning both on a national, regional and local levels. On the other hand, we feel that implementation of the model on a digital computer may be somewhat ambitious and beyond the scope of this phase. We will ultimately do this, but perhaps in a future effort. We will, however, illustrate the operation of the model with various examples and additionally sketch a computational algorithm.

II. Research Approach (Task and Methodology)

1. Statement

The research will begin with an update on the BMP studies in the areas involved in the 1983 study by the principal investigator followed by an inventory of flood control management strategies in use in these areas. Much of this is hard data. Data on damages due to these two types of problems will be collected. In general, such data is essentially vague, imprecise or qualitative. Most people are unable to precisely state these effects. Thus fuzzy set theoretic methods will be invoked to design a data collection and analysis program.

The methodology will be tested first on the flood control project and then adapted for the BMP component. This will be complemented by tools from multi-attribute decision theory and the theory of approximate reasoning. We have applied these to previous studies involving non-point source water

pollution control planning in urban areas.

III. Summary of Our Efforts and Results Since Technical Progress Report No. 1

1. Major Output

We revised and updated the technical paper entitled "A Fuzzy Methodology and Algorithm for the Flood Control Problem". The revised version entitled, "On the Application of Fuzzy Sets Theory to Water Resources: The Optimal Flood Control Problem" was submitted to the Journal of Fuzzy Sets and Systems - the original and official journal of the International Fuzzy Sets Association. This is being considered for the Special Issue on Operations Research.

We are proceeding with the analysis of the mathematical algorithm and are revising it as necessary to respond more realistically to operating characteristics of regional flood control management agencies. The data needs of the algorithm are being siphoned out and critically examined.

Efforts have been made to collect data on flood strategies and best management strategies from planners at the Atlanta Regional Commission, EPA, State Environmental Planning Division of the Department of Natural Resources, FEMA, etc. These efforts have yielded very little fruit. We have been frustrated by the complete lack of data (especially at the local and regional levels) for a comprehensive analysis of our model.

The import of the foregoing is that a carefully designed data collection scheme to quantify the fuzzy, imprecisely stated and subjective data has to be embarked upon. To do this correctly requires project team members reasonably experienced in the theory and use of fuzzy sets. The graduate student assistants are therefore currently studying the subject under the PI's direction and guidance. We hope to accomplish this by the summer when the PI is budgeted to spend more time on the project.

IV. Future Work

The major aspects of this phase listed in our first Technical Report still remain to be done. These are:

1. Revise the Mathematical Model
2. Analyze model and refine as necessary
3. Develop a computational algorithm
4. Analyze data needs of the algorithm
5. Design a data collection scheme in keeping with data needs identified in 4
6. Collect data on flood control strategies and best management strategies from planners at such agencies as the Atlanta Regional Commission, EPA and the State Environmental Planning Division
7. Test the algorithm with sample data
8. Write report.

Conferences

We expect to attend three or four future meetings to present and discuss some of our findings:

1. The Association of State Flood Plain Managers 14th Annual Conference in Asheville, North Carolina is scheduled for June 11-14, 1990 and not March as stated in the previous report.
2. International Federation of Operations Research Societies Conference, Athens, Greece, June 25-29, 1990
3. We plan to attend the IPMU Conference of Fuzzy Logic, Algorithms and Knowledge Engineering in France. This conference comes immediately after the IFORS Conference. We have been invited to present our work entitled "Aspects of a Fuzzy Sets Methodology and Algorithm for the Flood Control Problem." We expect to receive valuable inputs from the participants there.

4. We did not attend the Joint AWRA and Canadian Conference on Water Problems in Toronto because we felt that it would be of minimal utility to the project.

V. Analysis of Results and Problems

We repeat our comments on the project contained in the last report. We will definitely need a time extension. Although some progress has been made, the project is quite behind schedule. The major reasons stated earlier are:

1. The funding of the project was not effected until late in the Fall of 1988.
2. The PI was away on leave of absence to the University of California, Berkeley as the Chancellors Distinguished Visiting Professor of Engineering and Management Science.
3. The PI was sick for the first half of 1989 and as such could not commit significant effort to the research project. An attempt was made to catch up during the Summer and Fall 1989 quarters.

In view of the above, we expect that the project life may be extended beyond the date in the proposal. We hope however, to continue to make significant efforts and progress towards successfully completing the project as close to schedule as possible.

E-24655

Technical Progress Report #3
on Project Grant No. 14-08-0001-G 1629

Since the last technical progress report we have done the following:

1) We wrote a proposal for a cost and time extension which inter alia contained a report on the technical accomplishment on the project up to that time.

2) We subsequently wrote a proposal for time extension. (See Appendix A of this report). This was accepted. As can be seen, it contains an outline of the remaining work mission. We will therefore use it as the fundamental reference for this report.

We have produced two research papers which are based on the project mission. One is a substantial revision and extension of a previously submitted paper. It contains a new and implementable model for the flood control problem as well as a detailed computational algorithm. Additionally, the performance of the algorithm has been tested with synthetic data. The title of the paper is: "On the Application of Fuzzy Sets Theory to the Optimal Flood Control Problem Arising in Water Resources Systems." It is being considered for publication in the Special Issue of the Journal of Fuzzy Sets and Systems on Operations Research.

The second document is a paper entitled "Computational Aspects and Applications of a Branch and Bound Algorithm for Multistage Decision Processes" submitted to the Journal of Computers and Mathematics and Applications.

This paper deals with an old version of an algorithm we were using for the flood control model. It discusses the computational aspects of this algorithm. The paper has now been accepted and is scheduled to appear next year.

3) To summarize, with respect to the work schedule contained in the accepted proposal for a no cost-time extension, the following have now been accomplished:

   i) We have produced a final version of the mathematical model for the flood control problem.

   ii) We have developed a computational algorithm for 33 solving the problem posed in 1)

   iii) We have developed and debugged a high level computer program for processing the algorithm

   iv) We have done some preliminary testing with synthetic data; however for more realistic results we will need to do more of this.

Essentially then, we have accomplished the work items 1,2,3,4, 5,6, and 8. of the proposal. What remains therefore, are items 4, 7, and 9 through 15. We hope to complete the remaining aspects on schedule next year.

4) Additionally, we wrote a proposal entitled "A Practical Tool for the Optimal and Conjunctive Planning of Non Point Source Water Pollution and Flood Damage Control Systems" which will extend and concretize our current work. This was submitted to USGS and assigned Proposal #1228.

E-24-655

GEORGIA TECH 1885-1985

DESIGNING TOMORROW TODAY

Georgia Institute of Technology
School of Industrial and Systems Engineering
Atlanta, Georgia 30332-0205
(404) 894-2300

January 12, 1990

Mr. Allen Ford
Office of External Research
United States Department of the
Interior
U.S. Geological Survey
WGS-Mail Stop 424
Reston, VA 22092

RE: Technical Progress Report on
Grant No. 14-08-0001-G1629

Dear Mr. Ford:

Enclosed please find a copy of the Technical Report on the above reference Grant.

I regret the delay in submitting it. I was however hampered by several difficulties discussed in Section V of the report.

Thank you for your understanding.

Sincerely yours,

Augustine O. Esogbue, Ph.D.
Professor and
Principal Investigator

AOE/jl
Enclosures

A FUZZY PROCEDURE AND ALGORITHM FOR THE OPTIMAL FLOOD CONTROL PROBLEM

1. INTRODUCTION

It is becoming quite apparent that floods are the most widespread geophysical hazard in the United States today. Data on their impacts show that they account for very significant annual property losses. Of great concern is the fact that the total amount of annual national flood damages keeps increasing at an accelerated pace despite the substantial expenditures that have been made for their control.

It is generally accepted that structural measures such as storage reservoirs, floodwalls, levees, and channel improvements do not, by themselves, provide the necessary security from flood damages. Hence, the role of non-structural measures, such as floodplain zoning, land-use allocation, flood proofing, flood insurance, and emergency procedures has begun to receive attention as an integral part of any flood damage mitigation planning. A mix of these approaches is of interest. However, determining an optimal mix of structural and nonstructural measures is very difficult as a consequence of both the interdependencies between them and the considerable variety of their feasible combinations.

It is necessary that a methodology be developed for an optimal solution to this mix of adjustment measures problem. In the past, due to a variety of needs and different considerations a number of flood control models were developed. Most of these, however, were derived with a particular application in mind and thus are not adaptable to more general cases.

Exploiting the underlying scheduling nature of the problem Morin et al. (1981) proposed a dynamic programming formulation that is suitable to any specific application. Their objective was to minimize the total annual flood damages over a long planning horizon as well as the present worth of the optimal sequence of the structural and nonstructural measures undertaken. The recursive equations of the dynamic programming formulation led to the selection of an optimal sequence of the structural measures.

According to their point of view the nonstructural measures complimented a given set of structural measures in terms of damage reduction. Thus, for any year of the planning horizon and any set of structural measures, the optimal levels of the nonstructural measures are determined by some simulation/optimization procedure. It is also claimed that the levels of the nonstructural measures could determine the optimal timings for the structural measures, since they are variable.

Compared to previous algorithms, the computational efficiency of this one was improved somewhat by the use of a so called sieve strategy in modifiying the hybrid DP - B&B algorithm. This approach efficiently generated feasible solutions with near optimal objective values while at the same time providing strong bounds on the optimal value.

One of the shortcomings of the above approach is its local nature and a difficulty, computational and otherwise, to apply it on a regional or national level. A more serious concern is its inability to incorporate directly persistent and pervasive systemic variables which are intrinsically fuzzy and imprecise. In other words, Morin et al.'s approach is a crisp model.

In the present effort we propose a novel approach to the Flood Control Problem (FCP), by recourse to the tools of Fuzzy Sets and Possibility Theory. The driving force for this approach is the strong belief that in the environmental systems analysis field a substantive departure from the conventional crisp quantitative way of modeling is needed. Such an approach would provide the researcher with a more close-to-reality representation of complex or ill-defined phenomena as employed by planners. This should lead to more effective common sense control policies for a wide variety of practical problems.

The FCP integrates engineering, economic, environmental, social and management aspects and therefore deals with entities and relations which are often not precisely known or difficult to quantify. A fuzzy approach appears to be more natural and appropriate than classical methods. In particular, the difficulty of disassociating crisply the impacts (benefits) of interacting control strategies usually the case with non-structural measures will be minimized by allowing the use of fuzzy variables or descriptors.

2. THE TWO-PHASE OPTIMIZATION PROCEDURE

Our approach is as follows: As soon as the flood hazard areas are determined on the basis of some hydrologic and hydraulic analyses, a group of National Flood Insurance Program (NFIP) specialists from each Federal Emergency Management Agency (FEMA) Regional Office is appointed. This group then meets with community officials and a study contractor to discuss the places within the region that have to be studied. We call this the time and cost meeting. A set of structural and nonstructural measures is proposed according to the particular geological and hydrological characteristics of the area. Thus at this stage, the types of measures, characteristics (scale, etc.) and locations have been determined.

The procedure we propose consists of two phases. The first phase of the optimization procedure consists of determining the optimal sequencing and the optimal timings of combinations of structural and nonstructural measures in each region in order to reduce the regional flood damages to a minimal or at least to an acceptable level within some budget limitations. A fuzzy dynamic programming optimization procedure is proposed for this phase as detailed in Section 3. In this phase, the stage of the DP formulation will be determined each time a new measure is included and tested (in order to be either accepted and realized or rejected) in any current combination of measures. Thus, for each region we obtain a set of the K best policies for reducing flood damages. This set of controls which now constitutes the control space for each region then becomes an input to the second phase of the optimization process.

The second optimization phase determines the optimal scheduling and sequencing of flood protection measures on a national scale. Here, each region comprises the stage of the DP formulation. The goal is to maximize a weighted average of flood damage reductions in each and every of the 10 regions that correspond to a Federal Emergency Management Agency. The weights will be determined by National Flood Insurance Program (NFIP) specialists on the basis of emergency priorities and other political considerations.

3. FUZZY FORMULATION OF THE FLOOD CONTROL PROBLEM

We may view the system under control as a geographical region of the U.S. in which structural and nonstructural measures are to be constructed so as to minimize the total amount of flood damages encountered.

The region is presumed to be represented as a fuzzy system. Its state is equated with an index describing the level of the total flood damages that is expected to be attained after a combination of structural and/or nonstructural measures has been selected and has been put into use.

When defining the system, imprecision is experienced in two ways:

(i) We are not able to assess exactly damages in monetary terms especially when loss of human lives and of other non-materialistic factors is involved.

(ii) It is not possible to measure as well as predict precisely the utility (effects) of the structural and nonstructural measures constructed.

This is particularly the case with nonstructural measures.

Both of these two sources of fuzziness are important in determining what is to be called the state of the system; thus, the system must appropriately be considered to be fuzzy.

One could argue that a combined approach of stochastic dynamic program and Fuzzy Set Theory [5] would be more close-to-reality and ultimately more efficient due to the probabilistic nature of hydrological and hydraulic phenomena. However, the actual hydrological and hydraulic data would be different from the average ones and thus the results from the optimization procedure should be revised in order to lead to valid conclusions. Moreover, since the evaluation of safety and economic efficiency is subjective and qualitative the regular fuzzy dynamic approach is, for practical purposes, preferable and sufficient.

The input (control) to the system is the decision about what mix of structural and/or nonstructural measures will be used at different times in the planning horizon and at different areas of the USA to mitigate flood damage effects.

The state variable, 'level of overall flood damages' will be defined over the fuzzy sets: 'significant flood damage level', 'moderate flood damage level' or 'insignificant flood damage level'.

The evolution of the system is governed by a set of functional equations developed in a subsequent section.

The output (immediate return) of the system is the flood damage reductions achieved. The returns are also defined over the fuzzy sets: 'significant flood damage reductions', 'moderate flood damage reductions', 'insignificant flood damage reductions'. The reason for the returns treated as fuzzy variables is that the utility of any measure can only be approximately estimated in the real world as it is greatly dependent on future hydrological occurrences, the strategies already in place, as well as the combination of strategies under consideration. Clearly, these confounding interdependencies obviate the ability to provide crisp reliable qualitative estimates, even by a so called expert.

The constraints imposed on the controls concern the following:

(i) Limitations in financing.

The budgeting constraints are deterministic. The amount of money available to each state or to each of the 10 FEMA (each FEMA is responsible for a number of states) is known exactly or at least the total amount made available by the National Flood Insurance Program is known. However, the constraints applied on the controls in the DP formulation will be expressed via fuzzy set terminology.

There are two reasons justifying such a preference. The construction of a structural measure involves a fixed cost given its particular characteristics and assuming precise knowledge of future economic conditions. However, the latter is rarely the case and hence if we want to be as close to real conditions as possible we should incorporate this source of imprecision into our model. On the other hand, the actual cost and benefits involved with the nonstructural measures, such as adoption of tax incentives to encourage wise use of the flood plain land, placement of warning signs in the flood plain to discourage development, installation of flood forecast and warning systems with an appropriate evacuation plan, can never be estimated accurately nor precisely, thus contributing as an additional source of imprecision (fuzziness) of information. For this reason, we define the cost of any structural and/or nonstructural combination over the fuzzy sets 'high', 'medium', 'low' cost that may correspond to discretized financing levels. Then, the membership function values can be interpreted as the degree of willingness of the planners to invest the corresponding amount of money for the construction of a given mix of measures.

In the case that the financial constraints are not rigid, i.e. they are of the form: in region A, we do not want to spend more than x dollars or we are willing to spend at least y dollars for region B or the expenditure for region C should be roughly between pre-selected bounds, then the membership function values would indicate the degree that each alternative (control action) satisfies these predetermined restrictions.

(ii) Timing preferences

It is assumed that the timing of any measure to be undertaken is independent of any other's and it is furthermore not known beforehand. It is related to the existing environmental, social, political and other considerations. A membership function with values dependent on these constraints indicates the most preferable for a measure to be put into use.

The fuzzy goal at each stage is concerned with the desired flood damage reductions to be attained as a result of an optimal mix of structural and nonstructural flood control programs.

A fuzzy decision is the intersection of the fuzzy constraints and the fuzzy goals. An optimal policy is a sequence of controls maximizing the membership value of the system in the fuzzy set of 'significant flood damage reductions'.

4. MULTISTAGE CONTROL OF A FUZZY SYSTEM IN A FUZZY ENVIRONMENT

The behavior of the fuzzy system is governed by the following state and output equations:

$$x_{i+1} = f(x_i, u_i),$$

where x_i, $x_{i+1} \in X$ are fuzzy states and times t_i and t_{i+1} respectively denoting the level of flood damages before and after the control u_{i+1} in region (i+1) has been put into use. The function $f: X * U \longrightarrow X$ is a function from the product space of U and X to the space of the fuzzy sets in X and

$$y_{i+1} = g_{i+1} (x_{i+1}),$$

where y_{i+1} is the return from region (i+1) at time t_{i+1}. It denotes the flood damage reduction achieved due to the control action u_{i+1}.

As was mentioned in the optimization phase I, the process moves from one stage to the next every time we add a new measure, structural or nonstructural, to the existing combination of measures in any given region. Note that in phase II of the optimization procedure, the stage is the region in which we are going to construct the most appropriate measure from among those already determined in phase one.

It is easy to see that the same scheduling algorithm is applicable to both optimization phases.

5. FUZZY CONSTRAINTS, FUZZY GOALS AND FUZZY DECISIONS

The fuzzy environment is represented by the fuzzy constraints and the fuzzy goals.

The control, essentially the expendture on a selected combination of structural and/or nonstructural measure, u is subjected to a fuzzy economic constraint $c^i(u_i/x_i)$, which may be derived as shown in figure 1.

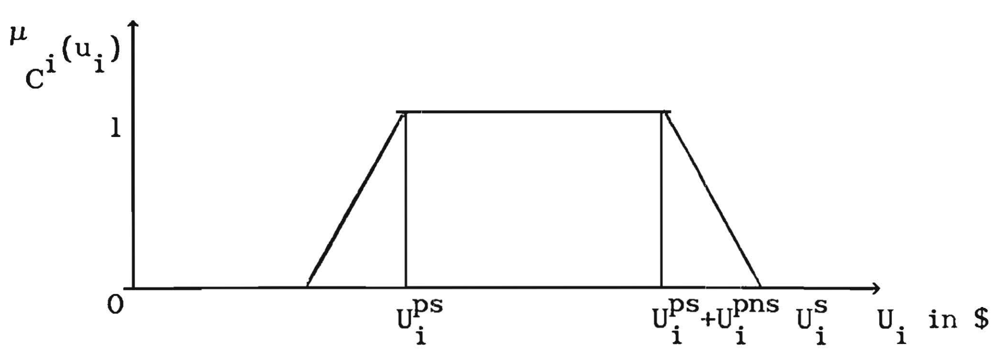

fig. 1.

Let U_i^{ps} = the planned investment for the construction of a structural measure in region 1,

U_i^{pns} = the additional planned investment for the construction of a nonstructural measure in region 1

U_i^{s} = the maximal emergency expenditure

$\mu_{c_i}(u_i)$ = the degree of willingness of the planners to invest in a particular measure or combination of measures

The state dependence of these constraints does not change further considerations in the formulation.

Also, there is a fuzzy knowledge about the most appropriate time that a combination of measures is put into use. Thus, the end of the construction for a structural/nonstructural measure is not known beforehand. Time is considered to be a continuous variable. The intervals between the completion of two measures may not be of equal duration. Figure 2 suggests how the membership function for the timing constraints might be. Itexpresses an evaluation of what is considered to be the most preferable time for the completion of the construction of a measure of a combination of measures.

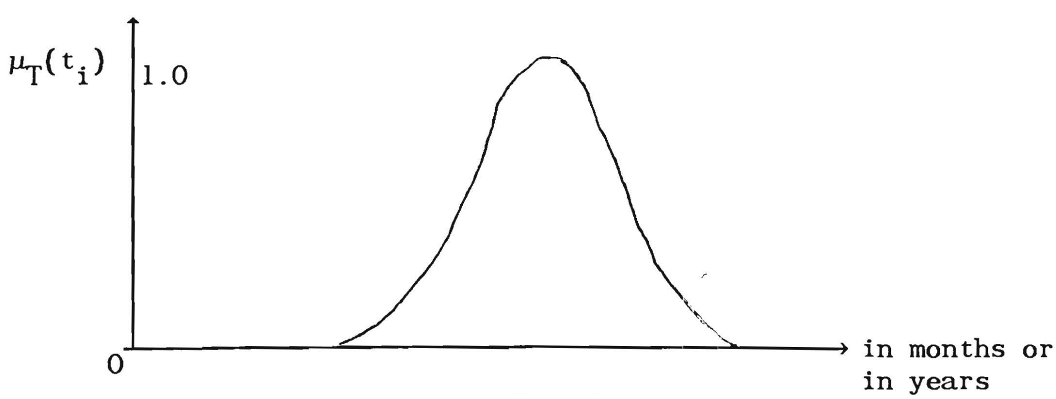

fig. 2.

The fuzzy goals are imposed at all intermediate stages of the planning horizon and concern the flood damage reductions achieved at any stage. The membership function $\mu_{G^i}(y_i)$ of a fuzzy goal G at stage i may have the form shown in figure 3.

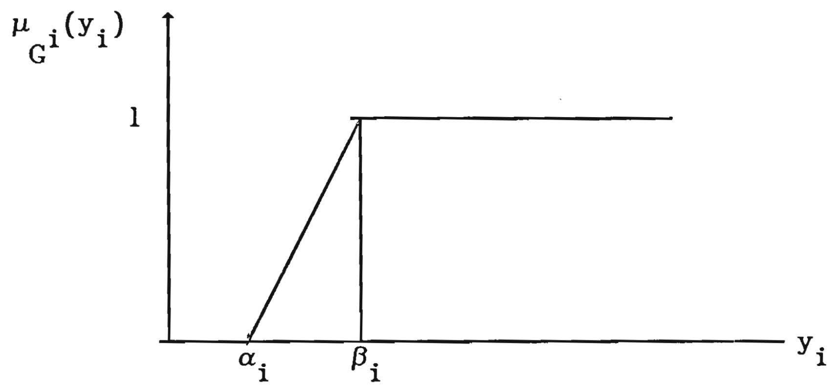

fig. 3

where y_i is the reduction achieved at region i,

α_i is the lowest acceptable level of flood damage reduction, and

β_i is the highest possible damage reduction in region i according to our estimations.

Assuming that x_o is the initial state (initial level of flood damages), the fuzzy decision $\mu_D\,(u_1,u_2,\ldots,u_N/x_o)$ is the intersection of the fuzzy constraints and the fuzzy goals, ie.

$$\mu_D\,(u_1,u_2,\ldots,u_N/x_o) = \min_{u_i \epsilon U_i} \{\mu_{C^1}(u_1)\ \mu_{G^1}(y_1)\ \mu_T(t_1),\ldots,$$

$$\mu_{C^N}(u_N)\ \mu_{G^N}(y_n)\ \mu_T(t_N)\ )$$

for i = 1,2,...N.

where N is the number of regions in the country and U_i is the control space for the i-th region.

We note that the control space U_i is determined in the optimization phase 1 and consists of the K best policies selected for the i-th region.

At this point more attention should be given to the form and the derivation of the membership function of the fuzzy goal G^i. Recall that the fuzzy goal expresses the desired level of flood damage reductions. Its membership function evaluation takes into consideration the following:

(i) the hydrological characteristics of the flood that determine the damage level, such as (1) the depth of the flood, (2) the intensity of the flood, (3) the duration of the flood. These variables may be defined over fuzzy sets as well.

(ii) the different damage reduction effects induced by different combination of measures.

The total damge reduction achieved at a stage is the result of a combination of interdependent effects that can be expressed in the form of 'reduction factors'. These factors when incorporated into the membership function of the fuzzy goal $G^i(y_i)$, the latter may take the form

$\mu_{G^i}(y_i)$ = (reduction factor due to a structural measure) *

(reduction factor due to a nonstructural measure) *

(effects of depth, intensity, duration of flooding)

6. THE DECISION-MAKING PROBLEM AND ITS SOLUTION

The problem is to find the maximizing decision, ie. a sequence of inputs $u_1^*, u_2^*, \ldots, u_N^*$ at times $t_1, t_2, \ldots, t_N$ that will yield the maximal flood damage reductions:

$$\mu_D(u_1^*, u_2^*, \ldots, u_N^*) = \max_{u_i, t_i} \{\min \{\mu_{C^1}(u_1)\ \mu_{G^1}(y_1)\ \mu_T(t_1), \ldots, \mu_{C^N}(u_N)\ \mu_{G^N}(y_N)\ \mu_T(t_N))\ \} \quad (1)$$

for $i = 1, 2, \ldots, N$

At each stage i (i.e. inclusion of a new measure to the current combination in optimization phase 1 or a new region in optimization phase 2) at time t_i a fuzzy goal G^i is set and the aim of the control U_i to obtain the return of the system y_i as close as possible to predetermined one given by G^i. As a measure of the closeness between y_i and G^i at time t_i we use the relative distance $d(y_i, G^i)$ between the two fuzzy sets:

$$d(y_i, G^i) = (1/L) * \left| \sum_{i=1}^{L} (y_i) - \mu_{G^i}(y_i) \right| , \qquad (2)$$

where L is the number of all possible states that the system can be in.

For solving the optimization problem as it appears in (1) a solo use of dynamic programming was initially proposed but this approach was obviated by the non-uniqueness of backward transition from y to x. Thus, instead of using the usual recurrence equations

$$\mu_{G^i}(y_i) = \max_{y_i} (\mu_{G^i}(y_i)\ \mu_T(t_i) \wedge \mu_{G^{i+1}}(y_{i+1})\ ) \quad \text{and}$$

$$y_{i+1} = g(x_{i+1}) = g(x_i, u_i),$$

We use a modified hybrid dynamic programming and branch-and-bound procedure. An approach akin to this was also employed by Morin et al. [11] in their crisp model.

The idea of the method is based on the following property:

$$\min \{\mu_{C^1}(u_1)\ \mu_{G^1}(y_1)\ \mu_T(t_1), \ldots, \mu_{C^k}(u_k)\ \mu_{G^k}(y_k)\ \mu_T(t_k)\ \}$$

$$\leq \min \{\mu_{C^1}(u_1)\ \mu_{G^1}(y_1)\ \mu_T(t_1), \ldots, \mu_{C^m}(u_m)\ \mu_{G^m}(y_m)\ \mu_T(t_m)\ ) \quad \text{for } k \leq m.$$

We branch via the controls applied at particular control stages and we bound as follows:

At the k-th control stage, we add that control that will maximize the fuzzy decision function at that stage.

The set of controls is finite $U = \{U_1, U_2, \ldots U_N)$. The decision process can be represented by a decision tree. The root of the tree is the initial state x_o, the edges correspond to specific control values and the nodes to the resulting states of the system.

Before we expand on the branch-and-bound technique we would like to establish mathematically the temporal evolution of the system in terms of membership functions. It is noticeable in our formulation that we use the term 'minimization of flood damage level' interchangeably with the term 'maximization of the flood damage reductions'.

In terms of membership function the temporal evolution of the system is governed by

$$\mu_{X_{i+1}}(x_{i+1}) = \max_{x_i} \{\mu_{X_i}(x_i) \wedge \mu_{X_{i+1}}(x_{i+1}/x_i,u_i) \} \tag{3}$$

$$\mu_{X_{i+2}}(x_{i+2}) = \max_{x_{i+1}} \{\max_{x_i} \{\mu_{X_i}(x_i) \wedge \mu_{X_{i+1}}(x_{i+1}/x_i,u_i) \} \wedge \mu_{X_{i+2}}(x_{i+2}/x_{i+1},u_{i+1}) \} \tag{4}$$

and generally,

$$\mu_{X_{i+n}}(x_{i+1}) = \max_{x_{i+n-1}} (\max_{x_{i+n-2}} (\ldots \{ \max_{x_i} \mu_{X_i}(x_i) \wedge \mu_{X_{i+1}}(x_{i+1}/x_i,u_i) \wedge \mu_{X_{i+2}}(x_{i+2}/x_{i+1},u_{i+1}) \} \wedge \ldots) \wedge \mu_{X_{i+n}}(x_{i+n}/x_{i+n-1},u_{i+n-1})) \tag{5}$$

For finite state and control spaces equations (3)-(5) can be written more compactly. For each input $u \in U$, let $M(u)$ denote a matrix whose (i,j) element is given by

$$M_{k\ell}(u_i) = \mu_{X_k}(x/x_\ell,u_\ell) \tag{6}$$

and $\tilde{x}_{i+1}$ and $\tilde{x}_i$ denote the column vectors whose i-th elements are $\mu_{X_{i+1}}(x_{i+1})$ and $\mu_{X_i}(x_i)$ respectively, evaluated at x_{i+1} and x_i equal to x_i for $i = 1,2,\ldots,$ max number of states.

Equation (3) can be written as

$$\tilde{x}_{i+1} = M(u_i)\tilde{x}_i \tag{7}$$

where $M(u_i)\tilde{x}_i$ is the max-min matrix product of M(u) and x. Similarly,

$$\tilde{x}_{i+2} = M(u_{i+1})M(u_1)\tilde{x}_i \tag{8}$$

$$\tilde{x}_{i+n} = M(U_{i+n-1})M(u_{i+n-2})\ldots M(u_i)\tilde{x}_i \tag{9}$$

We will make use of these operations when illustrating the hybrid-DP branch-and-bound technique with an example.

Let the set of controls be U - $\{a_1,a_2,\ldots,a_m)$. The decision process can conveniently be represented by a decision tree. The root of the tree is the initial state of the system x, the edges are associated with the particular values of the controls applied and the nodes are associated with subsequent states attained. Let $X_{k\ell m\ldots w}$ denote the state of the system attained at stage k from state x_o through the sequence of controls $a_\ell, a_m, \ldots a_w$.

We will consider the general case where we have N goals, N timing constraints and N financing constraints.

Any sequence $u_1,u_2,\ldots,u_N$ will be called a decision and any subsequence $u_1,u_2,\ldots u_i$, $i \leq N$, will be called a partial decision at stage i and it will be denoted by d_i.

The value of equation (1) will be called the value of decision $u_1,u_2,\ldots u_N$ and it is its grade of membership in the fuzzy decision D.

Similarly, the membership function value of the partial decision will be the following equation

$$v_i = v_i(d_i) = \mu_{C^1}(u_1)\ \mu_{G^1}(y_1)\ \mu_T(t_1)\ \Lambda\ldots\Lambda\ \mu_{C^i}(u_i)\ \mu_{G^i}(y_i)\ \mu_T(t_i) \tag{10}$$

We also denote

$$v_i' = v_i'(d_i) = \mu_{C^1}(u_1)\ \mu_{G^1}(y_1)\mu_T(t_1)\ \Lambda\ldots\Lambda\ \mu_{C^i}(u_i)\ \mu_T(t_i) \tag{11}$$

which represents the value of the partial decision at stage i but without considering the fuzzy goal G^i at this stage.

The problem is to determine a maximizing decision, ie. the partial decision d_N with the best value.

If we consider consecutively partial decisions at successive stages t=1,2,...,N we should take into account only those found so far that have the highest value. Thus, we apply only to the best partial decision a further control and proceed to a future state. The process is terminated when we obtain a complete decision d with value greater than all those considered so far. Evidently, it need not be unique.

6a. EXAMPLE. We illustrate the foregoing with an example [7].

In this case there are N fuzzy constraints and N fuzzy goals. Let X = $\{\sigma_1, \sigma_2, \ldots \sigma_5\}$ and U = $\{a_1, a_2, a_3\}$ and the system under control is equated with a conditioned fuzzy set: $\mu_{X_{i+1}}(x_{i+1}|x_i, u_i)$

u = a_1

	$x_i \backslash x_{i+1}$	σ_1	σ_2	σ_3	σ_4	σ_5
	σ_1	1	0.1	0.9	0.1	0.2
	σ_2	0.8	0.5	0.7	0.3	0.5
$M(\sigma_1)$	σ_3	0.7	0.9	0.5	0.5	0.7
	σ_4	0.5	0.7	0.7	0.3	0.4
	σ_5	0.2	0.3	0.9	0.7	0.3

u = a_2

	$x_i \backslash x_{i+1}$	σ_1	σ_2	σ_3	σ_4	σ_5
	σ_1	0.3	0.9	1	0.4	0.6
	σ_2	0.5	0.7	0.5	0.2	0.3
$M(\sigma_2)$	σ_3	0.8	0.5	0.3	0.5	0.2
	σ_4	0.9	0.7	0.7	0.9	0.5
	σ_5	0.7	0.9	0.7	1	0.7

u = a_3

	$x_i \backslash x_{i+1}$	σ_1	σ_2	σ_3	σ_4	σ_5
	σ_1	0.5	0.7	0.7	1	0.7
	σ_2	0.7	0.8	0.1	0.5	0.9
$M(\sigma_3)$	σ_3	0.8	0.1	0.2	0.3	1
	σ_4	0.9	0.2	0.3	0.5	0.8
	σ_5	1	0.5	0.4	0.7	0.4

$$X_o = 0.1/\sigma_1 + 0.2/\sigma_2 + 0.3/\sigma_3 + 0.7/\sigma_4 + 1/\sigma_5$$

$$C^1 = 0.3/a_1 + 0.7/a_2 + 1/a_3$$

$$C^2 = 0.5/a_1 + 1/a_2 + 0.7/a_3$$

$$C^3 = 1/a_1 + 0.8/a_2 + 0.6/a_3$$

$$G^1 = 0.7/\sigma_1 + 1/\sigma_2 + 0.7/\sigma_3 + 0.4/\sigma_4 + 0.1/\sigma_5$$

$$G^2 = 0.2/\sigma_1 + 0.5/\sigma_2 + 0.7/\sigma_3 + 0.8/\sigma_4 + 1/\sigma_5$$

$$G^3 = 0.4/\sigma_1 + 0.7/\sigma_2 + 1/\sigma_3 + 0.7/\sigma_4 + 0.4/\sigma_5$$

Starting from X_o and applying a_1, a_2, a_3 we obtain

$v_1'(a_1) = 0.3$

$v_1'(a_2) = 0.7$ $\quad\quad X_o \circ C^1$

$v_1'(a_3) = 1$

Thus, we consider a_3 and proceed to X_{13} which is equal to

$$X_{13} = 1/\sigma_1 + 0.5/\sigma_2 + 0.4/\sigma_3 + 0.7/\sigma_4 + 0.7/\sigma_5$$

$$\mu_G{}^1 = 1-\ell\ (X_{13}, G^1) = 1 - \frac{1}{5}(0.3 + 0.5 + 0.3 + 0.6) = 0.6$$

and $v_1(a_3) = 1 \wedge 0.6 = 0.6$

Thus, we consider a_2 and proceed to X_{12} given by

$$X_{12} = 0.7/\sigma_1 + 0.9/\sigma_2 + 0.7/\sigma_3 + 1/\sigma_4 + 0.7/\sigma_5$$

Now

$$\mu_G{}^1 = 1-\ell(X_{12}, G^1) = 1 - \frac{1}{5}(0.1 + 0.6 + 0.6) = 0.74$$

and $v_1(a_2) = 0.7 \wedge 0.74 = 0.7$

Thus we start from X_{12} and applying a_1, a_2, a_3 we obtain

$v_2'(a_2, a_1) = 0.7 \wedge 0.5 - 0.5$

$v_2'(a_2, a_2) = 0.7 \wedge 7 = 0.7$ $\quad\quad X_{12} \circ C^2$

$v_2'(a_2, a_3) = 0.7 \wedge 0.7 = 0.7$

We proceed to X_{222} and X_{223}, given by

$X_{222} = 0.9/\sigma_1 + 0.7/\sigma_2 + 1/\sigma_3 + 0.9/\sigma_4 + 0.7/\sigma_5$

$X_{223} = 0.9/\sigma_1 + 0.8/\sigma_2 + 0.7/\sigma_3 + 0.7/\sigma_4 + 0.9/\sigma_5$

Now for X_{222}, $\mu_{G^2} = 1-\ell(X_{222}, G^2) = 0.68$

and for X_{223}, $\mu_{G^2} = 1-\ell\,(X_{223}, G^2) = 0.76$

and $v_2(a_2,a_2) = 0.7 \wedge 0.68 = 0.68$

and $v_2(a_2,a_3) = 0.7 \wedge 0.76 = 0.7$

Thus we start from X_{223} and applying a_1,a_2,a_3 we obtain

$v'_3(a_2,a_3,a_1) = 0.7$, $v'_3(a_2,a_3,a_2) = 0.7$, $v'_3(a_2,a_3,a_3) = 0.6$

We therefore proceed to X_{3231}, and X_{3232} given by

$X_{3231} = 0.9/\sigma_1 + 0.7/\sigma_2 + 0.9/\sigma_3 + 0.7/\sigma_4 + 0.7/\sigma_5$

$X_{3232} = 0.7/\sigma_1 + 0.9/\sigma_2 + 0.9/\sigma_3 + 0.9/\sigma_4 + 0.7/\sigma_5$

for X_{3231}, $\mu_{G^3} = 1-\ell(X_{3231}, G^3) = 0.82$ and $V_3(a_2,a_3,a_1) = 0.7$

for X_{3232}, $\mu_{G^3} = 1-\ell(X_{3232}, G^3) = 0.78$ $\quad V_3(a_2,a_3,a_2) = 0.7$

Since there is no other partial decision with higher value, these two (a_2,a_3,a_1) and (a_2,a_3,a_2) are the maximizing ones.

7. CONCLUDING REMARKS

The model presented determines the optimal flood damage reduction policies using the fuzzy dynamic programming methodology and bounding the solution space by a branch-and-bound procedure. The primary goal is to apply that sequence of flood controls (structural and/or nonstructural) that will yield the highest flood damage reductions. The finite set of controls $U = \{U_1, U_2, \ldots, U_N)$ includes a selected number of combinations of measures for each region. The nonstructural measures are not treated as a simple augmentation of the structural ones. At each stage after determining which control or combination of controls is to be applied, an optimization procedure of less extent is performed to reveal the optimal timing for the completion of the construction.

Prior to deciding to use this optimization methodology other formulations were also considered. For example, a fuzzy linear programmming formulation with two-component objective function (minimizing the total flood damage level as well as the financial expenditures induced by the properly selected flood mitigation measures undertaken) was considered. Also, a trial was attempted to rank the multi-aspect alternatives using fuzzy sets. However, these approaches were obviated by the economic and physical nonlinearities involved in such a complex problem.

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Technical Progress Report

on Project Grant No. 14-08-0001-G 1629

by

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Project Title: WATER QUALITY ENHANCEMENT VIA INTEGRATIVE MANAGEMENT OF NON-POINT SOURCE WATER POLLUTION AND FLOOD DAMAGE REDUCTION CONTROL STRATAGIES

(Key words: Flood plain management, water quality management, mathematical models)

I. Research Objectives

1. Statement

The objectives of the research are to i) develop new and useable planning methodologies which would enable water resources planners to select a combination of structural and non structural measures both for the twin problems of non-point source water pollution and flood control measures over time and space so as to maximize the expected discounted value of reduction in damages to any regions' water resources due to the almost inseparable problems of non-point source pollution and flood in urban and urbanizing areas over some future planning horizon, (ii) implement the methodologies on a digital computer, and (iii) to test and assess the feasibility and utility of the methodologies in a real-world setting such as the Chattahoochee River Corridor in Fulton County and the Bear Creek watershed located immediately south of the City of Douglasville in Douglas County in Georgia. The latter is much less developed than a typical urban area although it has many of the sedimentation problems of such an area. In short, the difficulties inherent in planning and management of complex socio-technical systems involving imprecise and usually vague data will be minimized via the tools we propose to develop. We hope to develop tools which utilize data in their natural occurring setting exploiting the tendencies of the data to be vaguely stated.

2. Analysis

The foregoing objectives still remain valid. We have, however, enlarged the objective and scope to include developing a model applicable to planning both on a national, regional and local levels. On the other hand, we feel that implementation of the model on a digital computer may be somewhat ambitious and beyond the scope of this phase. We will ultimately do this, but perhaps in a future effort. We will, however, illustrate the operation of the model with various examples and additionally sketch a computational algorithm.

II. Research Approach (Task and Methodology)

1. Statement

The research will begin with an update on the BMP studies in the areas involved in the 1983 study by the principal investigator followed by an inventory of flood control management strategies in use in these areas. Much of this is hard data. Data on damages due to these two types of problems will be collected. In general, such data is essentially vague, imprecise or qualitative. Most people are unable to precisely state these effects. Thus fuzzy set theoretic methods will be invoked to design a data collection and analysis program.

The methodology will be tested first on the flood control project and then adapted for the BMP component. This will be complemented by tools from multi-attribute decision theory and the theory of approximate reasoning. We have applied these to previous studies involving non-point source water

pollution control planning in urban areas [21].

The optimization methodology will be via mathematical programming and heuristics. Decomposition techniques will also be used to solve the problem. Specifically, Benders' decomposition will be employed since the project interdependencies and their competitive nature lead to a classic form for which Benders' decomposition approach has proven to be especially powerful. That is, the problem contains decision variables which are "complicating" in the sense that if they are fixed at some level, then the problem becomes much easier to solve.

In the optimal mix of adjustments to flood problems, the complicating variables correspond to the nonstructural adjustments. For a fixed level of nonstructural flood control, the problem reduces to a classic project sequencing problem in the structural measures.

2. Analysis

Our initial efforts focussed on the development of a general and new philosophical approach to the problem. We next embarked on a mathematical model, based on fuzzy sets theory and the theory of approximate reasoning as envisaged in our proposal, for dealing with the flood control planning problem. We are interested in a robust model applicable to both the national, regional and local levels. This model, although focussed on the flood control problem, is capable of being applied to the nonpoint source water pollution control problem. It is a hybrid fuzzy dynamic programming and branch and bound type algorithm.

III. Summary of Our Efforts and Results

1. Major Output

We have rigorously analyzed the problem and previous related models for the flood control problem. We discarded any temptation to resort to simple quick fixes involving direct modifications. Rather, we have developed a philosophically and mathematically different model. The results of our effort are reflected in the attached technical paper entitled "A Fuzzy Methodology and Algorithm for the Flood Control Problem". This paper is under revision and will be submitted to one of the following journals:

i) Journal of Fuzzy Sets and Systems (Journal of the International Fuzzy Systems Association) Special Issue on Operations Research Applications

ii) Water Resources Bulletin, Journal of the American Water Resources Association

2. We have also updated the Best Management Strategies in use in our study areas.

3. We have attended three conferences where methodologies and applications germane to the research mission were presented. These are

i) Fall National Meeting, Operations Research Society, Denver, Colorado, October 1988

ii) Fall National Meeting, Operations Research Society, New York New York, October 1989

iii) Third World Congress, International Fuzzy Systems Association, Seattle, Washington, July 1989

IV. Future Work

We outline in the sequel the remaining activities necessary for the successful completion of the project.

1. Revise the Mathematical Model
2. Analyze model and refine as necessary
3. Develop a computational algorithm
4. Analyze data needs of the algorithm
5. Design a data collection scheme in keeping with data needs identified in 4.
6. Collect data on flood control strategies and best management strategies from planners at such agencies as the Atlanta Regional Commission, EPA and the State Environmental Planning Division
7. Test the algorithm with sample data
8. Write report.

We expect to attend three or four future meetings to present and discuss some of our findings. The proposed ones are:

i) Conference and Workshop on Stormwater and NonPoint Source Water Management at the University of Louisville, Kentucky, March 1990

ii) Joint Canadian-American Water Resources Association Conference on Water Problems, Toronto, Canada, April 1990

iii) International Federation of Operations Research Societies Conference, Athens, Greece, June 1990

V. Analysis of Results & Problems

We feel we have made some useful beginnings and progress on this project. The project is however behind the original schedule for the following principal reasons:

i) The funding of the project was not effected until late in the Fall of 1988

ii) the PI was away on leave of absence to the University of California, Berkeley as the Chancellors Distinguished Visiting Professor of Engineering and Management Science

iii) The PI was sick for the first half of 1989 and as such could not commit significant effort to the research project. An attempt was made to catch up during the Summer and Fall 1989 quarters.

In view of the above, we expect that the project life may be extended beyond the date in the proposal. We hope however, to continue to make significant efforts and progress towards successfully completing the project as close to schedule as possible.

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May 15, 1991

Mr. Allen Ford
Office of External Research
United States Department of
the Interior
U.S. Geological Survey
WGS-Mail Stop 424
Reston, VA 22092

RE: Technical Progress Report on
Grant No. 14-08-0001-G1629

Dear Mr. Ford:

Enclosed please find a copy of the Technical Report on the above referenced Grant as well as copies of two papers which are based on the research mission.

With best regards.

Yours sincerely,

Augustine O. Esogbue, Ph.D.
~~Professor and~~
Principal Investigator

AOE/jl
Enclosures

TECHNICAL PROGRESS REPORT ON USGS PROJECT NO. 14-08-0001-G1629

1.0 Accomplishments

Since our last progress report and our request for a No Cost Time Extension on the project, although no man power time was budgeted this quarter for the project, we have accomplished quite a lot towards a final completion of the project.

i) We wrote a new proposal for an extension to allow us concretize and practilize our algorithms.

ii) We have received acceptance of our paper submitted to the Journal of Fuzzy Sets and Systems. See the attached.

iii) We wrote, submitted and received acceptance of another paper entitled, "Computational Aspects and Applications of a Branch and Bound Algorithm for Fuzzy Multistage Decision Processes" from the Journal of Computers and Mathematics with Applications. See the attached.

iv) We have either prepared or are preparing the following invited papers for presentations at these conferences:

a) "Optimization of Nonpoint Source Water Pollution Control Planning Using Fuzzy Mathematical Programming", International Fuzzy Systems Association, 1991 World Congress, Brussels, July 7-12, 1991

b) "Fuzzy Sets Theory and Applications: A Guided Tour", An invited hour lecture , XIV Systems Engineering Meeting, Santiago, Chile, July 12, 1991

c) "Disaster Control Planning Via Fuzzy Mathematical Programming," An Invited Tutorial, Joint International Meeting, TIMS-SOBRAPO, Rio, Brazil, July 15,-17 1991

d) "Fuzzy Sets Modelling and Optimization as an Aid to Disaster Control Systems Planning", an invited paper, International Fuzzy Engineering Symposium, Yokohama, Japan, November 13-15, 1991

2. FUTURE WORK

During the summer, we plan to follow our proposed task outline and i) finish the adaptation of the models to the water pollution control problem, ii) complete the questionnaire design for data collection for both the flood control problem and the water pollution control problem, and finally write up the report.

November 1991

WATER QUALITY ENHANCEMENT VIA INTEGRATIVE PROCEDURES FOR URBAN NONPOINT SOURCE WATER POLLUTION AND FLOOD CONTROL

by

Augustine O. Esogbue

School of Industrial and Systems Engineering
GEORGIA INSTITUTE OF TECHNOLOGY
Atlanta, Georgia 30332-0205

ISyE Tech # 1-91-11 November 1991

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WATER QUALITY ENHANCEMENT VIA INTEGRATIVE PROCEDURES FOR URBAN NONPOINT SOURCE WATER POLLUTION AND FLOOD CONTROL

by

Augustine O. Esogbue

Technical Completion Report

USGS 14-08-001-G1629
(Also Technical Report No. J-91-11)

Initiated: October 1, 1988

Completed: September 31, 1991

Acknowledgement and Disclaimer

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School of Industrial and Systems Engineering

GEORGIA INSTITUTE OF TECHNOLOGY

Atlanta, Georgia 30332-0205

BIOGRAPHICAL SKETCH OF THE AUTHOR, AUGUSTINE O. ESOGBUE

Dr. Augustine O. Esogbue is a full professor in the School of Industrial and Systems Engineering at Georgia Tech and has held appointments as Adjunct Professor of Community Medicine, Morehouse School of Medicine and in the Department of Mathematical and Computer Sciences at Atlanta University.

Professor Esogbue, whose areas are Operations Research and Systems Engineering, is a recognized authority in dynamic programming and fuzzy logic applications, and is pursuing research in the application of neural networks in fuzzy decision and control processes.

Professor Esogbue has extensive research and professional consulting experience applying operations research and systems engineering methods in health care, water resources and pollution as well as urban systems, both in the U.S. and internationally. He is currently consultant for several agencies and serves on panels of the National Research Council, National Academy of Sciences and the National Science Foundation. Professor Esogbue is an associate editor of the Journal of Mathematical Analysis and Applications, advisory editor of the International Journal on Fuzzy Sets and Systems, co-author of the book Mathematical Aspects of Scheduling and Applications (Pergammon Press, 1982), and author of Dynamic Programming for Optimal Water Resources Analysis (Prentice Hall, 1989). Professor Esogbue is a Fellow of the American Association for the Advancement of Science and is listed in various Who's Whos including Who's Who in the World, and Who's Who in Engineering and American Men and Women of Science.

He received his PhD in industrial and systems engineering (and operations research) under the supervision of Dr. Richard E. Bellman from the University of Southern California, Los Angeles in 1968, with a minor in control theory. He completed a M.S. degree in industrial engineering and operations research at Columbia University, New York in 1965, and his B.S. in electrical engineering from the University of California at Los Angeles in 1964.

Professor Esogbue recently served as the Chancellor's Distinguished Professor of Industrial Engineering and Operations Research and Management Sciences at the University of California. He is the current Chairman, Health Applications Technical Section of the Operations Research Society of America; formerly, Chair, George Nicholson Student Paper Competition, and Chairman, Distinguished Visiting Lecturer Program of the Operations Research Society and The Institute of Management Sciences.

ACKNOWLEDGEMENTS

This completion report of USGS Project No. 14-08-001-G1629 is the result of several years of research effort. During the evolutionary periods, a number of professionals and organizations made invaluable inputs which contributed in varying degrees to the success of the project. Space and other considerations however, obviate the enumeration of all that impacted favorably on this project. In spite of this constraint, I wish to specifically acknowledge Dr. Thomas E. Stelson, former Vice President for Research and subsequently Executive Vice President at Georgia Tech whose office provided the requisite matching funds for the project. The staff of the U.S. Geological Survey and in particular, Mr. Allen Ford, Project Monitor, must be credited with immense understanding of the circumstances necessitating a no cost time extension to complete the project.

My professional staff at the Georgia Institute of Technology made invaluable contributions. Of all the research assistants who worked on the project at various stages, special credits must go to Mr.Kejiao Guo for programming and modeling assistance and Maria Theologidu for inputs into the preliminary modeling development. Secretarial support was provided by the School of Industrial and Systems Engineering at Georgia Tech.

Augustine O. Esogbue, Ph.D.
Professor and Project Principal
Investigator

ABSTRACT

It has now become well known that when urbanization occurs within a watershed, the rate and volume of runoff generally increase. The higher flow rates also result in increased flooding of areas downstream of the developed area. Additionally, the increased rates of runoff, together with the destruction of the natural vegetation, lead to increased erosion. The resultant erosion, besides causing problems such as stream bank caving and gullying, can also result in the deposition of large quantities of sediment in downstream areas and other water quality problems. These twin problems of non-point source pollution and flooding create problems which have serious impacts on both quantity and quality problems in water resources management. Despite various attempts to deal with them, serious difficulties continue to be encountered by water resources managers. This has led to the call for novel approaches in a recent NSF study. This report is the result of a project geared towards providing a response to this call.

Planning for the effective control of non-point source water pollution in urban areas is considerably more complicated than the situation for agricultural, forestal and mining areas. An additional source of difficulty arises from the fact that it is not easy to isolate non-point source water pollution from that caused by other urban guidance systems.

For non-point source water pollution, an acceptable approach proposed in the Atlanta Region Areawide Wastewater Management Plan in 1978 and updated twice since then is the so called Best Management Practices. These approaches are also nationally utilized to combat the

deleterious effects of non-point source water pollution. These strategies include structural and nonstructural measures.

Hitherto, however, no attempts had been made to coordinate the use of both structural and non-structural measures in an integrated plan in the management of non-point water pollution problem. Such an approach would not only make sense (especially from a cost-effectiveness perspective) but appears unavoidable.

With regards to the twin problem of flood control strategies, some efforts had been made in other regions of the country. However, the determination of an optimal mix of adjustments to floods had been hitherto impossible both because of the sheer size and complexity of the problem and the inherent interdependencies between the structural and nonstructural adjustments.

The research effort was aimed at providing decision techniques which would assist the water resources planner in quantitatively evaluating and choosing an "optimum" from the myriad of feasible combinations of structural and nonstructural measures over time and space in terms of mitigation of future water-caused damages, and in particular, the degradation of the quality of both surface and underground water due to flooding and erosion.

One of the shortcomings of the most notable previous effort is its local nature and an inherent difficulty, computational and otherwise, to apply it on a regional or national level. A more serious concern is its inability to incorporate satisfactorily and directly persistent as well as pervasive systemic variables which are intrinsically fuzzy and imprecise. In other words, Morin et al.'s approach suffers from all the well known objections to the use of crisp models to represent sociotechnical systems.

In the present effort, we have proposed a novel approach to the Flood Control Problem (FCP), as well as the Non Point Source Water Pollution Control Problem by recourse to the tools of Fuzzy Sets and Possibility Theory, Mathematical Programming and Utility Theory. The driving force for this approach is the strong belief that in the environmental systems analysis field a substantive departure from the conventual crisp quantitative way of modeling is needed. Such an approach would provide the researcher with a more close-to-reality representation of complex or ill-defined phenomena as employed by planners. This should lead to more effective common sense control policies for a wide variety of practical problems.

The FCP Integrates engineering, economic, environmental, social and management aspects and therefore deals with entities and relations which are often not precisely known or difficult to quantify. A fuzzy approach appears to be more natural and appropriate than classical methods. In particular, the difficulty of dis-associating crisply the impacts (benefits) of interacting control strategies usually the case with non-structural measures is minimized by allowing the use of fuzzy variables or descriptors.

The report is organized as follows: In Chapter One, we motivate the problem, review previous studies, and state both the project objectives and our project design. In Chapter Two, we present our fuzzy mathematical model of the problem for both the flood control and nonpoint source pollution control problems. We focus however, on the flood control problem using it as the leitmotif for our studies. The problem is modelled as a fuzzy hierarchical multi stage resource allocation problem. Version One treated in this chapter employs a modification of a branch and bound solution algorithm first proposed by Kacpryzk. In Chapter Three, we develop a second version of this fuzzy model solved in a multi-level hierarchical mode and requiring data inputs in their simplest and most natural occurring setting. A three phase procedure is proposed with the first two dealing with regional and national allocation models and a third playing the role of coordination. In Chapter Four, we exercise our two versions of the algorithm on a flood control problem while its equivalent water pollution model is discussed in Chapter Five. Data issues critical to the successful implementation of the models in a real world setting are discussed and treated in Chapter Six. The report is concluded with an Appendix containing the flow charts for the Algorithms, the attendant computer algorithms and other related project issues.

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CHAPTER ONE

INTRODUCTION TO THE PROBLEM AND RESEARCH MISSION

1.1 INTRODUCTION

It has now become well known that when urbanization occurs within a watershed, the rate and volume of runoff generally increase. The higher flow rates also result in increased flooding of areas downstream of the developed area. Additionally, the increased rates of runoff, together with the destruction of the natural vegetation, lead to increased erosion. The resultant erosion, besides causing problems such as stream bank caving and gullying, can also result in the deposition of large quantities of sediment in downstream areas and other water quality problems. These twin problems of non-point source pollution and flooding create problems which have serious impacts on both quantity and quality problems in water resources management. Despite various attempts to deal with them, serious difficulties continue to be encountered by water resources managers. This has led to the call for novel approaches in a recent NSF study report [35].

Planning for the effective control of non-point source water pollution in urban areas is considerably more complicated than the situation for agricultural, forestal and mining areas. An additional source of difficulty arises from the fact that it is not easy to isolate non-point source water pollution from that caused by other urban guidance systems.

For non-point source water pollution, an acceptable approach proposed in the Atlanta Region Areawide Wastewater Management Plan in 1978 and updated twice since then is the so called Best Management Practices [3]. These approaches are also nationally utilized to combat the deleterious effects of non-point source water pollution. The USGS had, for example, sponsored several demonstration projects in various parts of the country to test the effectiveness of storm water control strategies. These strategies include structural and nonstructural measures. The <u>structural</u> areas identified both in Georgia and nationally in our 1983 study include: Landsmoothing, Filter Berm, Sediment Barrier, Level Spreader, Top Soiling, Riprap, Gabion, Vertical Drain, Toe Berm, Haulageway, Construction Exit, Subsurface Drains, Sediment Trap, Storm Drain Outlet Protection, Dikes, Temporary Seeding, Mulching, Sediment Basin, Buffer Zone, Downdrain Structures while the nonstructural ones include: Retention of Natural Vegetation, Proper Storage of Deicing Materials, Disposal of Unused Pesticides, Reduction of Vehicle Miles Traveles, Establishment of New Vegetation, Proper Maintenance of Deicing Equipment, Leaf Disposal, Proper Timing of Fertilizer Application, Preventive Care for Vehicles, Storage Containers, Alternatives to Pesticides, Soil Testing, Legal Requirements for Pesticide Application, Public Education, Street Sweeping, Litter Control, Street Flushing. We had evaluated their effectiveness as used both nationally and in Georgia. This is well documented in Esogbue [21].

Hitherto, however, no attempts had been made to coordinate the use of both structural and non-structural measures in an integrated plan in the management of non-point water pollution problem. Such an approach would not only make sense (especially from a cost-effectiveness perspective) but appears unavoidable.

With regards to the twin problem of flood control strategies, some efforts had been made in other regions of the country. However, the determination of an optimal mix of adjustments to floods had been hitherto impossible both because of the sheer size and complexity of the problem and the inherent interdependencies between the structural and nonstructural adjustments. That is, the number, size and timing of structural measures for flood control such as reservoirs, flood walls and channel improvements to add to an existing system is both dependent upon and competitive in the economic sense with existing and planned nonstructural measures such as flood proofing, flood zoning, flood insurance and outright purchase of portions of the flood plain, and *vice versa*. Furthermore, both the structural and nonstructural measures directly affect and are affected by current and future land-use patterns, anticipated flood loadings, and flood plain management strategies.

The research effort was aimed at providing decision techniques which would assist the water resources planner in quantitatively evaluating and choosing an "optimum" from the myriad of feasible combinations of structural and nonstructural measures over time and space in terms of mitigation of future water-caused damages, and in particular, the degradation of the quality of both surface and underground water due to flooding and erosion.

Exploiting the underlying scheduling nature of the problem, Morin et al. [1981] proposed a dynamic programming formulation that is suitable to any specific application. Their objective was to minimize the annual flood damages over a long planning horizon as well as the present worth of the optimal sequence of the structural and nonstructural measures undertaken. The recursive equations of the dynamic programming formulation led to the selection of the optimal sequencing of the structural measures.

According to their point of view, the nonstructural measures complimented a given set of structural measures in terms of damage reduction. Thus, for any year of the planning horizon and any set of structural measures the optimal levels of the nonstructural measures are determined by some simulation/optimization procedure. They also mentioned that the levels of the nonstructural measures may determine the optimal timings for the structural measures, since they are variable.

Compared to previous algorithms the computational efficiency of this one has been improved by the use of a so called "sieve strategy" which modified the hybrid dynamic programming and branch and bound algorithm. This approach efficiently generated feasible solutions with near optimal objective values while at the same time provided strong bounds on the optimal value.

1.2 GENERAL MATHEMATICAL MODEL OF THE PROBLEM

A very general version of the flood control problem treated by Morin et al. [1981] may be stated as follows: Find a combination $(\vec{x},\vec{y})$ of structural ($\vec{x}$) and nonstructural ($\vec{y}$) measures so as to

$$\max f(\vec{x},\vec{y})$$

$$\text{subject to} \quad (\vec{x},\vec{y}) \in G(\vec{x},\vec{y}) \qquad (1)$$
$$\vec{x} \in X$$
$$\vec{y} \in Y,$$

in which ($\vec{x}$) = (x_1, x_2,...,x_N) is the vector of structural measures, where x_j = 1 if structural measure j is selected and 0 if not, $\vec{y}$ = (y_1,y_2,...,y_K) is the vector of nonstructural measures, where y_k is the level of the k^{th} nonstructural measure selected, $f(\vec{x},\vec{y})$ is the objective function, e.g., the discounted net reduction in flood damages resulting from plan ($\vec{x},\vec{y}$), $G(\vec{x},\vec{y})$ is the set of feasible plans ($\vec{x},\vec{y}$) i.e., those satisfying the planning, financial, engineering, and social constraints, and X and Y, respectively, are the sets of feasible structural and nonstructural measures.

If, as in the literature, it is assumed that non-structural measures essentially complement a given set of structural measures as far as damage reduction is concerned and further, that they may vary over time, then (1) reduces to the determination of $\vec{y}^*$ so as to minimize the expected damages in year t. This can be expressed as

$$P(I,t) = \min_{\vec{y} \in Y \cap \Phi(I)} \{D(I,\vec{y},t) + C(\vec{y},t)\} \qquad (2)$$

where $D(I,\vec{y},t)$ is the annual flood damage in the t^{th} year for a given combination I of the structural measures with the level of non-structural measures at $\vec{y}$; $C(\vec{y},t)$ is the annual cost incurred in the t^{th} year with the levels of the non-structural measures at $\vec{y}$; and Φ and Y are similar to those defined in the (FCP) problem. P(I,t) denotes the minimal sum of the net annual flood damages and the non-structural measure costs in the t^{th} year for the combination I of the structural measures.

Following Erlenkotter and Rogers [6], Morin et al. [28] considered this problem as a discrete time sequencing problem. They made the following assumptions:

(a) A finite number, m, of structural measures may be undertaken, with each project indexed by an $l \epsilon I^* = \{1,2,\ldots,m\}$. The investment cost for project l is given by $c_i > 0$. This also includes an allowance for the present value of maintenance, replacement and other fixed operating costs.

(b) I denotes an arbitrary subset of project indices while ℓ denotes the power (or ground) set consisting of all the possible 2^m subsets I. The variable operating cost rate (annual net flood damage as a function of the non-structural measures) in year t for the project set I is expressed by $P(I,t) \geq 0$. Furthermore, for each I, some project $l \epsilon I$ must be established and added to I no later than the time $T(I) \geq 0$, where $T(I-i) \leq T(I)$ for all $i \epsilon I$ and $T(I^*) = +\infty$.

(c) Costs are continuously discounted at a constant rate, $r > o$, leading to a discount factor of e^{-rt} from time t to the initial time 0.

(d) Sequencing and timing decisions for the projects are to be selected so as to minimize the total net discounted damages over an infinite horizon.

In the foregoing, I[k] is the project index assigned to the k-th position in a sequence; {I[k]} is the complete assignment of project indices for a particular sequence, where $k = 1,2,\ldots,m$; S_I is the set of all permutations of project indices in I; I_k = the set of first k project indices for a particular sequence, where $I_o = \emptyset$, $I_{k+1} = I_k \cup_I (k+1)$ for $k = 0,1,\ldots,m-1$, and $I_m = I^*$; τ_k = the establishment time for the k^{th} project in a sequence, where $\tau_o = 0$, $\tau_k \leq \tau_{k+1}$, and $\tau_{m+1} = +\infty$; $C(I^*,\infty)$ equals the total net flood damages over the time interval $[0,\infty]$ discounted to time 0 for a minimum-damage sequencing.

The following model of the Optimal Mix of Adjustments (OMA) Problem then results

$$C(I^*,\infty) = \min_{\{i[k]\}\epsilon S_I*} \min_{\{\tau_k\}} \sum_{k=o}^{m} \sum_{t=\tau_k}^{\tau_{k+1}-1} P(I_k,t)\, e^{-rt} + \sum_{k=1}^{m} c_{i[k]}\, e^{-r\tau_k}, \quad (3)$$

where

$$0 = \tau_o \leq \tau_1 \leq \ldots \tau_m < \tau_{m+1} \text{ and } \tau_{k+1} \leq T(I_k),\ k = 0,1,\ldots,m-1.$$

It must be noted that in this model it is possible not to establish some projects at all since an establishment time equal to a very large value implies indefinite postponement, which is tantamount to eliminating that project from consideration.

If the non-structural measures are allowed to change their level only with the construction of a new structural measure, then the foregoing reduces to:

$$C(I^*,\infty) = \min_{\{i[k]\}\in S_I*} \min_{\{\tau_k\}} \sum_{k=o}^{m} \sum_{t=\tau_k}^{\tau_{k+1}-1} [D(I,y_k,t) + C(y_k,t)e^{-rt} + \sum_{k=1}^{m} c_{i[k]}\, e^{-rt_k}] \qquad (4)$$

where

$$\tau_o \leq \tau_1 \leq \ldots \leq \tau_m < \tau_{m+1} \text{ and}$$

$$\tau_k \leq T(I_{k-1}),\ y_k \in G(I_k) \cap Y,\ k = 1,\ldots,m.$$

and y_k, the new level of non-structural measures accompanying the construction of structural measure 1[k], with y_o as the initial level of the measures.

Morin et al. then proposed a dynamic programming algorithm for the minimization of the total annual flood damages over some long planning horizon as well as the present worth of the optimal sequence of the structural and non-structural measures undertaken in one specific region.

Compared to previous algorithms, the computational efficiency of their approach was somewhat improved by the use of a so-called sieve strategy in modifying the hybrid dynamic programming and branch and bound algorithm.

Although this approach 'efficiently' generated feasible solutions with near optimal objective values while at the same time providing strong bounds on the optimal value, its modeling and computational complexity is still foreboding.

1.3 ANALYSIS

One of the shortcomings of the above approach is its local nature and an inherent difficulty, computational and otherwise, to apply it on a regional or national level. A more serious concern is its inability to incorporate satisfactorily and directly persistent as well as pervasive systemic variables which are intrinsically fuzzy and imprecise. In other words, Morin et al.'s approach suffers from all the well known objections to the use of crisp models to represent sociotechnical systems.

In the present effort, we propose a novel approach to the Flood Control Problem (FCP), by recourse to the tools of Fuzzy Sets and Possibility Theory. The driving force for this approach is the strong belief that in the environmental systems analysis field a substantive departure from the conventional crisp quantitative way of modeling is needed. Such an approach would provide the researcher with a more close-to-reality representation of complex or ill-defined phenomena as employed by planners. This should lead to more effective common sense control policies for a wide variety of practical problems.

The FCP integrates engineering, economic, environmental, social and management aspects and therefore deals with entities and relations which are often not precisely known or difficult to quantify. A fuzzy approach appears to be more natural and appropriate than classical methods. In particular, the difficulty of dis-associating crisply the impacts (benefits) of interacting control strategies usually the case with non-structural measures is minimized by allowing the use of fuzzy variables or descriptors.

1.4 RESEARCH PROJECT OBJECTIVES

The original objectives of the research effort were to i) develop new and useable planning methodologies which would enable water resources planners to select a combination of structural and non structural measures both for the twin problems of non-point source water pollution and flood control measures over time and space so as to maximize the expected discounted value of reduction in damages to any regions' water resources due to the almost inseparable problems of non-point source pollution and flood in urban and urbanizing areas over some future planning horizon, (ii) implement the methodologies on a digital computer, and (iii) test and assess the feasibility and utility of the methodologies in a real-world setting such as the Chattahoochee River Corridor in Fulton County and the Bear Creek watershed located immediately south of the City of Douglasville in Douglas County in Georgia. The latter is much less developed than a typical urban area although it has many of the sedimentation problems of such an area. In short, the difficulties inherent in planning and management of complex socio-technical systems involving imprecise and usually vague data would be minimized via the tools we proposed to develop. It was hoped that the tools to be developed would utilize data in their natural occurring setting exploiting the tendencies of the data to be vaguely stated.

The foregoing objectives still remained valid. However, the project mission and scope were broadened and modified as necessary.

1.5 RESEARCH APPROACH (TASK AND METHODOLOGY)

The research began with a revisit to the BMP studies in the areas involved in the 1983 study [21] followed by an inventory of flood control management strategies normally utilized in these areas. Much of this is hard data. Data on damages due to these two types of problems might be needed to implement any resultant models. In general, such data is essentially vague, imprecise or qualitative. Most people are unable to precisely state these effects. Collection of such data via conventional methodologies is considered to be inadvisable. Thus novel approaches such as those based on fuzzy set theoretic methods might be invoked to design a data collection and analysis program.

The methodology was tested first on the central problem, namely flood control project and then adapted for the BMP component. This was complemented by tools from multi-attribute decision theory and the theory of approximate reasoning. It must be noted that such tools had been applied to previous studies involving non-point source water pollution control planning in urban areas [21].

The basic optimization methodology consisted of mathematical programming specifically dynamic programming and branch and bound as well as heuristics. Decomposition techniques were used to break the problem into hierarchical levels for analysis and solution. Specifically, Benders' decomposition and Saaty's concepts of heuristics were employed since the project interdependencies and their competitive nature lead to a classic form for which Benders' decomposition approach has proven to be especially powerful. That is, the original problem contained decision variables which are "complicating" in the sense that once they were fixed at some level, then the problem became comparatively easier to solve.

In the optimal mix of adjustments to flood problems, for example, the complicating variables correspond to the nonstructural adjustments. For a fixed level of nonstructural flood control, the problem reduced to a classic project sequencing problem in the structural measures.

Our initial model development efforts were expanded to include a general and new philosophical approach to the problem. We were interested in a robust model application to both the national, regional and local levels. This model, although focused on the flood control problem, is capable of being applied to the nonpoint source water pollution control problem and similar planning and control problems. It is a hybrid fuzzy dynamic programming and branch and bound type algorithm.

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CHAPTER TWO

FUZZY MATHEMATICAL MODELS AND ALGORITHMS FOR THE FLOOD CONTROL PROBLEM: VERSION 1

2.1 THE DECOMPOSITION OPTIMIZATION PROCEDURE

Following our model applied to the cancer research allocation process [3] and Saaty's analytic hierarchy process [11] we decompose the problem into levels or phases for analysis. Our approach is as follows: As soon as the flood hazard areas are determined on the basis of some hydrologic and hydraulic analyses, a group of specialists such as those at the National Flood Insurance Program (NFIP) from each Federal Emergency Management Agency (FEMA) Regional Office is appointed. This group then meets with community officials and a study contractor to discuss the places within the region that have to be studied. We call this the time and cost meeting. A set of structural and non-structural measures is proposed according to the particular geological and hydrological characteristics of the area. Thus at this stage, the types of measures, characteristics (scale, etc.) and locations will be determined.

The procedure we propose essentially decomposes the problem into two phases complemented by a third. The first phase of the optimization procedure consists of determining the optimal sequencing and the optimal timings of combinations of structural and non-structural measures in each region in order to reduce the regional flood damages to a minimal or at least to an acceptable level within some budget limitations. A fuzzy dynamic programming-type optimization procedure is proposed for this phase as detailed in Section 2.5. In this phase, the stage of the dynamic programming formulation will be determined each time a new measure is included and tested (in order to be either accepted and realized or rejected) in any current combination of measures. Thus, for each region we obtain a set of the K best policies for reducing flood damages. This set of controls which now constitutes the control space for each region then becomes an input to the second phase of the optimization process.

The second optimization phase determines the optimal scheduling and sequencing of flood protection measures on a national scale. Here, each region comprises the stage of the dynamic programming formulation. The goal is to maximize a weighted average of flood damage reductions in each and every of the 10 regions that correspond to a Federal Emergency Management Agency (FEMA). The weights will be determined by National Flood Insurance Program (NFIP) specialists on the basis of emergency priorities, budget and other political considerations.

The third is basically a linkage program. It consists of a model for coordination between the input-output phases of the preceding two to produce the desired system's outputs.

We consider a generic model useful in treating the problem at either the regional or national level.

2.2 FUZZY FORMULATION OF THE FLOOD CONTROL PROBLEM

Suppose the system under control is a geographical region of a country (the U.S.) in which structural and non-structural measures are to be constructed so as to minimize the total amount of flood damages encountered. The region is presumed to be represented as a fuzzy system. Its state may then be equated with an index describing the level of the total flood damages that is observed or expected to be attained before and after a combination of structural and/or non-structural measures has been selected and put into use respectively.

When defining the system, imprecision is experienced in at least two ways:

(i) We are not able to assess exactly or probabilistically damages in monetary terms especially when loss of human lives and of other non-materialistic factors is involved.

(ii) It is not possible to measure as well as predict precisely the utility (effects) of the structural and non-structural measures constructed. This is particularly the case with non-structural measures.

Both of these two sources of fuzziness are important in determining what is to be called the state of the system; thus, the system must appropriately be considered to be fuzzy.

One could argue that a combined approach of stochastic dynamic program [7] and Fuzzy Set Theory [13] would be closer to reality and ultimately more efficient due to the probabilistic nature of hydrological and hydraulic phenomena. However, the actual hydrological and hydraulic data would be different from the average ones and thus the results from the optimization procedure should be revised in order to lead to valid conclusions. Moreover, since the evaluation of safety and economic efficiency is subjective and qualitative the regular fuzzy dynamic approach is, for practical purposes, preferable and sufficient. We have shown this to be the case first in connection with our work with medical diagnosis [4] where the fuzzy model out performed the existing computerized Bayesian based models, and in our major effort in non point source water pollution control planning.

The <u>input</u> (control) to the system is the decision about what mix of structural and/or non-structural measures will be used at different times in the planning horizon and at different areas of the country (USA) to mitigate flood damage effects.

The <u>state</u> variable, 'level of overall flood damages' will be defined over the fuzzy sets: 'significant flood damage level', 'moderate flood damage level' or 'insignificant flood damage level'.

The evolution of the system is governed by a set of functional equations developed in a subsequent section.

The <u>output</u> (immediate return) of the system is the flood damage reductions achieved. The returns are also defined over the fuzzy sets: 'significant flood damage reductions', 'moderate flood damage reductions', 'insignificant flood damage reductions'. Alternatively, the output can be measured in terms of the difference between output and input states or flood damage levels before and after the application of controls. The reason for treating the returns as fuzzy variables is that the utility of any measure can only be approximately estimated in the real world as it is greatly dependent on future hydrological occurrences, the strategies already in place, as well as the combination of strategies under consideration.

Clearly, these confounding interdependencies obviate the ability to provide crisp reliable qualitative estimates, even by a so called expert.

The constraints imposed on the controls concern the following:

(i) Limitations in financing.

The budgeting constraints are deterministic. The amount of money available to each state or to each of the 10 FEMA (each FEMA is responsible for a number of states) is known exactly or at least the total amount made available by the National Flood Insurance Program is known. However, the constraints applied on the controls in the DP formulation will be expressed via fuzzy set terminology.

There are two reasons justifying such a preference. The construction of a structural measure involves a fixed cost given its particular characteristics and assuming precise knowledge of future economic conditions. However, the latter is rarely the case and hence if we want to be as close to real conditions as possible we should incorporate this source of imprecision into our model. On the other hand, the actual cost and benefits involved with the non-structural measures, such as adoption of tax incentives to encourage wise use of the flood plain land, placement of warning signs in the flood plain to discourage development, installation of flood forecast and warning systems with an appropriate evacuation plan, can never be estimated accurately nor precisely, thus contributing as an additional source of imprecision (fuzziness) of information. For this reason, we define the cost of any structural and/or non-structural combination over the fuzzy sets 'high', 'medium', 'low' cost that may correspond to discretized financing levels. Then, the membership function values can be interpreted as the degree of willingness of the planners to invest the corresponding amount of money for the construction of a given mix of measures.

If, however, the financial constraints are not rigid, i.e. they are of the form: in region A, we do not want to spend more than x dollars or we are willing to spend at least y dollars for region B or the expenditure for region C should be roughly between pre-selected bounds, then the membership function values would indicate the degree that each alternative (control action) satisfies these predetermined restrictions.

(ii) Timing preferences

It is assumed that the timing of any measure to be undertaken is independent of any other's and it is furthermore not known beforehand. It is related to the existing environmental, social, political and other considerations. A membership function with values dependent on these constraints indicates the most preferable for a measure to be put into use.

The fuzzy goal at each stage is concerned with the desired flood damage reductions to be attained as a result of an optimal mix of structural and non-structural flood control programs. Alternatively, it is the desirable flood damage levels as a consequence of applied controls.

A fuzzy decision is the intersection of the fuzzy constraints and the fuzzy goals while an optimal policy is a sequence of controls maximizing the membership value of the system in the fuzzy set of 'significant flood damage reductions' or 'minimal flood damage levels'. The foregoing concepts and operations were first proposed in Bellman and Zadeh [2] and amplified by Esogbue and Bellman [6] as well as various writing of others but specifically Kacprzyk [8], [9]. They are sharpened further in a forthcoming review paper on theory and applications by Kacprzyk and Esogbue [10].

2.3 FUZZY DECISION PROCESSES

What is now known as a fuzzy decision process with the system under control, the goals, decisions, and constraints defined over fuzzy sets may be formally stated as follows:

Given a set of $X = \{x\}$ of alternatives; a fuzzy goal G and a fuzzy constraint C, all defined over X, i.e. $G \subset X$ and $C \subset X$, then the fuzzy decision D defined also over the space X is simply the intersection of goals and constraints, i.e.

$$D = G \cap C \qquad (1)$$

Another way to represent (1) in terms of its membership function, $\mu_D(x)$ is

$$\mu_D(x) = \mu_G(x) \wedge \mu_C(x) = \min\{\mu_G(x), \mu_C(x)\} \qquad (2)$$

An optimal policy is a sequence of controls which optimizes the value of the membership function.

In a completely fuzzy system operating in a fuzzy environment, we may assume that the usual system descriptors of state, decision, transformation and return functions as well as the termination time are fuzzified. For such a system then, we may expect the usual issues and questions normally discussed in their non fuzzy analog to be of concern. Indeed, they have been raised by various authors such as Esogbue and Ramesh [3], Kacprzyk [7] [8], Stein [11], Esogbue and Bellman [5], Baldwin, et al. [1], etc. The seminal work by Bellman and Zadeh [2] provides the foundation for all work in this area.

2.4. FUZZY MULTISTAGE DECISION PROCESSES

A review of processes of this genre is provided by Esogbue and Bellman [5] with an update emphasizing applications by Kacprzyk and Esogbue [9]. Briefly and for simplicity let us for the moment focus attention on the following time-invariant, finite-state deterministic automaton A = [U, X, f}, where $U = [\alpha_1, \alpha_2, \ldots, \alpha_m\}$, $X = \{\sigma_1, \sigma_2, \ldots, \sigma_n\}$ are finite sets known as the input (control), and state spaces respectively, and $f: X \times U \rightarrow X$.

The temporal evolution of A is described by the state equation

$$x_{t+1} = f((x_t, u_t)), \quad t = 0, 1, \ldots, N-1$$

where $x_0 \in X$ is the initial state and N is the final or termination time which we assume to be fixed.

Let us assume that $\forall$ t, $\exists$ i) a fuzzy constraint $C^t \subseteq X$. Given an initial state X_o, we are interested in finding a maximizing decision via dynamic programming.

We can at once express the decision, a decomposable fuzzy set in U x U x ... x U as

$$R = C^o \cap C^1 \cap \ldots \cap C^{N-1} \cap G^{-N}$$

where G^{-N} is the fuzzy set in U x U x ... x U which induces G^N in X.

In terms of membership functions, we have

$$\mu_D(u_o, u_1, \ldots, u_{N-1}) = \min(\mu_{C^o}(u_o), \mu_{C^1}(u_1), \ldots, \mu_{C^{N-1}}(u_{n-1}), \mu_{G^N}(x_N) \quad (5)$$

where X_N is expressible as a function of x_o and $u_o, \ldots, u_{N-1}$.

We may rephrase the problem as: find the sequence of inputs $u_o, \ldots, u_{N-1}$ which maximizes μ_D of (5). The solution may be conveniently expressed in terms of Π_t the policy function with

$$U_t = \Pi_t(x_t), \quad t = 0, 1, 2, \ldots, N-1$$

Dynamic programming may then be employed to obtain both the Π_t and the maximizing decisions $U_0^M, \ldots, U_{N-1}^M$.

More specifically, this reduces to

$$\mu_D(U_0^M, \ldots, U_{n-1}^M) = \mathrm{Max}_{u_0}, \ldots, u_{N-2}\ \mathrm{Max}_{u_{N-1}} (\mu_0(u_0) \wedge \ldots \mu_{N-2}(u_{n-2})$$
$$\wedge\ \mu_{N-1}(u_{N-1}) \wedge \mu_{G^N}(f(x_{N-1}, u_{N-1})) \qquad (6)$$

Now, if γ is a constant and g is any function of u_{N-1}, we have the identity

$$\mathrm{Max}\ \mu_{N-1}\ (\quad g(u_{N-1})) = \mathrm{Max}\ u_{N-1}\ g\ (u_{N-1}).$$

Consequently, (6) may be rewritten as

$$\mu_D(u_0^M, \ldots, u_{N-1}^M) = \mathrm{Max}_{u_0}, \ldots, u_{N-1}\ (\mu_0(u_0) \wedge \cdots \wedge \mu_{N-2}(u_{N-2})$$
$$\wedge\ \mu_{G^{N-1}}(x_{N-1}) \qquad (7)$$

where

$$\mu_{G^{N-1}}(x_{N-1}) = \mathrm{Max}_{\mu_{N-1}} (\mu_{N-1}(u_{N-1}) \wedge \mu_{G^N}(f(x_{N-1}, u_{N-1}))) \qquad (8)$$

may be regarded as the membership function of a fuzzy goal at time $t = N - 1$ which is induced by the given goal G^N at time $t = N$.

On repeating this backward iteration, which is a simple instance of dynamic programming, we obtain the set of recurrence equations

$$\mu_{G^{N-v}}(x_{N-v}) = \mathrm{Max}_{\mu_{N-v}} (\mu(U_{N-v}) \wedge \mu_{G^{N-v+1}}(x_{N-v+1}))$$
$$x_{N-v+1} = f(x_{N-v}, U_{N-v}), \qquad v = 1, \ldots, N, \qquad (9)$$

which yield the solution to the problem. Thus, a maximizing decision $u^M_0, \ldots, u^M_{N-1}$ is given by the successive maximizing values of u_{N-v} in (9), with u_{N-v}^M defined as a function of X_{N-v}, $v = 1, \ldots, N$.

2.5. A BRANCH AND BOUND ALGORITHM FOR THE FUZZY DECISION PROBLEM

The fuzzy dynamic program presented in the foregoing, as well as its various variants, has applications in many real life situations. For example, its use in resource allocation and scheduling are well documented in Esogbue and Bellman [4] and recently Kacpryzk and Esogbue [8]. The solution approaches proposed for such models include variations of dynamic programming algorithms, branch and bound procedures, and hybrid dynamic programming-branch and bound algorithms. In the sequel, we sketch aspects of one such branch and bound algorithm proposed by Kacprzyk [2] for the multistage fuzzy decision problem.

Consider a fuzzy multistage decision problem such as was described in Section 2.3. The system under control may be represented as a conditioned fuzzy set whose membership function is given by

$$\mu_{X_{t+1}}(x_{t+1} | x_t, u_t)$$

The system's dynamics is then governed by

$$\mu_{X_{t+1}}(x_{t+1}) = \max_{x_t} \{\mu_{X_t}(x_t) \wedge \mu_{X_{t+1}}(x_{t+1} | x_t, u_t)\} \tag{10}$$

$$\mu_{X_{t+2}}(x_{t+2}) = \max_{x_{t+1}} \{\max_{x_t} \{\mu_{X_t}(x_t) \wedge \mu_{X_{t+1}}(x_{t+1} | x_t, u_t)\} \wedge \mu_{X_{t+2}}(x_{t+2} | x_{t+1}, u_{t+1})\} \tag{11}$$

and, in general

$$\mu_{X_{t+n}}(x_{t+n}) = \max_{x_{t+n-1}} (\max_{x_{t+n-2}} (\ldots\{ \max_{x_t} \mu_{X_t}(x_t) \wedge \mu_{X_{t+1}}(x_{t+1} | x_t, u_t) \wedge \mu_{X_{t+2}}(x_{t+2} | x_{t+1}, u_{t+1}) \wedge \ldots\}) \wedge \mu_{X_{t+n}}(x_{t+n} | x_{t+n-1}, u_{t+n-1})) \tag{12}$$

If both the state and control spaces are finite then (10)-(12) can be written more compactly. Let $M(u_t)$ denote a matrix whose (i,j) element is given by

$$M_{ij}(u_t) = \mu_{X_i}(x_i | x_j, u_t), \qquad u_t \in U \tag{13}$$

and $\bar{x}_{t+1}$ and $\bar{x}_t$ denote the column vectors whose i-th elements are $\mu_{X_{t+1}}(x_{t+1})$ and $\mu_{X_t}(x_t)$ respectively, evaluated at x_{t+1} and x_t equal to x_i, for i = 1,2..., max number of states, say n.

Rewriting equation (13) in matrix terms results in

$$\tilde{x}_{t+1} = M(u_t)\tilde{x}_t \tag{14}$$

with $M(u_t)\bar{x}_t$, the max-min matrix product of $M(u_t)$ and $\bar{x}_t$. In general then,

$$\tilde{x}_{t+n} = M(u_{t+n-1})M(u_{t+n-2})\ldots M(u_t)\tilde{x}_t \tag{15}$$

We will make use of these operations when illustrating the hybrid dynamic programming branch-and-bound technique with an example.

Recall that the objective of the decision making problem is to seek the sequence of inputs $u_1^*, u_2^*, \ldots, u_N^*$ that will yield the maximal membership functions. Thus, we need to find

$$\mu_D(u_1^*, u_2^*, \ldots, u_N^*) = \max_{u_i, t_i} \{\min \{\mu_{C^1}(u_1)\ \mu_{G^1}(x_1), \ldots, \mu_{C^N}(u_N)\ \mu_{G^N}(x_N)\}\} \quad (16)$$

for $i = 1, 2, \ldots, N$

It is assumed that at each stage i a fuzzy goal G^i with membership function $\mu_{G^i}(x_i)$, is set and the aim of the control u_i is to return the state of the system x_i as close as possible to a predetermined one given by G^i. As a measure of the closeness between X_N and G^N we may use the relative distance $d(X_N, G^N)$ between the two fuzzy sets:

$$d(X_N, G^N) = (1/n) \left(\sum_{i=1}^{n} | \mu_{X^i}(x_i) - \mu_{G^i}(x_i) | \right), \quad (17)$$

where n is the number of all possible states that the system can be in. Note further that the $\mu_G(x)$ in equation [16] is given by $\mu_{G^N}(x) = 1 - d(X_N, G^N)$.

Let the set of controls be $U = \{a_1, a_2, \ldots, a_m\}$. The decision process can conveniently be represented by a decision tree whose root is the initial state of the system X_o. The edges are associated with the particular values of the controls applied while the nodes are associated with subsequent states attained. Let $X_{klm\ldots w}$ denote the state of the system attained at stage k from state X_o through the sequence of controls $a_1, a_m, \ldots, a_w$.

Now consider a general case where we have N goals and N constraints.

Let the sequence $u_1, u_2, \ldots, u_N$ be called a decision while the subsequence $u_1, u_2, \ldots, u_i$, $i \le N$, the partial decision at stage i, be denoted by d_i. Correspondingly, let the value of equation (16), which is also its grade of membership in the fuzzy decision D, be called the value of the decision $u_1, u_2, \ldots, u_N$.

Similarly, let the membership function value of the partial decision be the following equation

$$v_i = v_i(d_i) = \mu_{C^1}(u_1)\ \mu_{G^1}(x_1) \wedge \ldots \wedge \mu_{C^i}(u_i)\ \mu_{G^i}(x_i) \qquad (18)$$

For the value of the partial decision at stage i but without considering the fuzzy goal G^i at this stage, the value v_i is given by

$$v_i' = v_i'(d_i) = \mu_{C^1}(u_1)\ \mu_{G^1}(x_1) \wedge \ldots \wedge \mu_{C^i}(u_i) \qquad (19)$$

The problem is to determine a maximizing decision, i.e. the partial decision d_N with the best membership function value in equation (16).

The principal idea of the method is based on the following property:

For $k \leq m$,

$$\min \{\mu_{C^1}(u_1)\ \mu_{G^1}(x_1), \ldots, \mu_{C^k}(u_k)\ \mu_{G^k}(x_k)\}$$
$$\geq \{\mu_{C^1}(u_1)\ \mu_{G^1}(x_1), \ldots, \mu_{C^m}(u_m)\ \mu_{G^m}(x_m)\}. \qquad (20)$$

We branch via the controls applied at particular control stages and we bound as follows:

At the k-th control stage, we add that control that will maximize the fuzzy decision function at that stage.

If we consider consecutively partial decisions at successive stages 1=1,2,...,N, we should take into account only those found so far that have the highest value. We note that both v_i and v_i are monotone nonincreasing functions of increasing i. Thus, we apply only to the best partial decision a further control and proceed to a future state, obtain a new partial decision, compute its value and compare it with the existing one, choosing only for further considerations, the one with the highest value. The process is terminated when we obtain a complete decision d with value greater than all those considered so far. Evidently, it need not be unique.

2.6. COMPUTATIONAL ASPECTS

Kacprzyk considered two versions of this problem. The first version considered N fuzzy constraints with the fuzzy goal applied only at the Nth stage. The second one considers N fuzzy goals. In the first example, the maximizing decision was unique. In the second example with three goals, two decisions, i.e. (a_2, a_3, a_1) and (a_2, a_3, a_2) were obtained. Note that in each example, the same fuzzy matrix was applied to all stage transitions.

Although this illustrates the nonuniqueness of this solution, the wrong solution was obtained. We will show that computational errors in Kacprzyk's example can be avoided by a correct application of the algorithm.

Suppose we have a multistage decision process with N fuzzy constraints as well as N fuzzy goals. Following the foregoing model, let the state of the system be given by $X = \{\sigma_1, \sigma_2, \ldots, \sigma_5\}$ while the controls are $U = \{a_1, a_2, a_3\}$. Let the system under control be equated with a conditioned fuzzy set: $\mu_{x_{i+1}}(x_{i+1}, x | x_i, u_i)$. Thus, we have at each stage five possible states and three possible controls that can be applied. Consider the following three matrices $M(a_1)$, $M(a_2)$ and $M(a_3)$ as required by equation (13) which show for each of the three controls $U(a_1,a_2,a_3)$ the membership functions for possible limitations from x_i to x_{i+1} for each of the various stages.

$u = a_1$

$M^T(a_1)$: x_i \ x_{i+1}	σ_1	σ_2	σ_3	σ_4	σ_5
σ_1	1	0.1	0.9	0.1	0.2
σ_2	0.8	0.5	0.7	0.3	0.5
σ_3	0.7	0.9	0.5	0.5	0.7
σ_4	0.5	0.7	0.7	0.3	0.4
σ_5	0.2	0.3	0.9	0.7	0.3

$u = a_2$

$M^T(a_2)$: x_i \ x_{i+1}	σ_1	σ_2	σ_3	σ_4	σ_5
σ_1	0.3	0.9	1	0.4	0.6
σ_2	0.5	0.7	0.5	0.2	0.3
σ_3	0.8	0.5	0.3	0.5	0.2
σ_4	0.9	0.7	0.7	0.9	0.5
σ_5	0.7	0.9	0.7	1	0.7

$u = a_3$

$M^T(a_3)$: x_i \ x_{i+1}	σ_1	σ_2	σ_3	σ_4	σ_5
σ_1	0.5	0.7	0.7	1	0.7
σ_2	0.7	0.8	0.1	0.5	0.9
σ_3	0.8	0.1	0.2	0.3	1
σ_4	0.9	0.2	0.3	0.5	0.8
σ_5	1	0.5	0.4	0.7	0.4

In addition to the foregoing, we are provided the following data on the system

i) fuzzy initial state

$X_o = 0.1/\sigma_1 + 0.2/\sigma_2 + 0.3/\sigma_3 + 0.7/\sigma_4 + 1/\sigma_5$

ii) the fuzzy constraints

$C^1 = 0.3/a_1 + 0.7/a_2 + 1/a_3$

$C^2 = 0.5/a_1 + 1/a_2 + 0.7/a_3$

$C^3 = 1/a_1 + 0.8/a_2 + 0.6/a_3$

iii) the fuzzy goals

$G^1 = 0.7/\sigma_1 + 1/\sigma_2 + 0.7/\sigma_3 + 0.4/\sigma_4 + 0.1/\sigma_5$

$G^2 = 0.2/\sigma_1 + 0.5/\sigma_2 + 0.7/\sigma_3 + 0.8/\sigma_4 + 1/\sigma_5$

$G^3 = 0.4/\sigma_1 + 0.7/\sigma_2 + 1/\sigma_3 + 0.7/\sigma_4 + 0.4/\sigma_5$

We can now perform our computations to determine the optimal control policy.

Starting from X_o and applying controls a_1, a_2, a_3 we obtain using equations (18) and (19)

$v_1'(a_1) = 0.3$

$v_1'(a_2) = 0.7$

$v_1'(a_3) = 1$

Thus, working backwards we consider a_3 and proceed to calculate X_{13}, $\mu^1{}_G$ and $v_1(a_2)$. The result is

$X_{13} = 1/\sigma_1 + 0.5/\sigma_2 + 0.4/\sigma_3 + 0.7/\sigma_4 + 0.7/\sigma_5$

$\mu_{\tilde{G}}{}^1 = 1\text{-}d(X_{13},G^1) = 1 - \frac{1}{5}(0.3 + 0.5 + 0.3 + 0.6) = 0.6$

and $v_1(a_3) = 1 \wedge 0.6 = 0.6$ (from equation 18)

Next, we consider a_2 and proceed to X_{12} given by

$X_{12} = 0.7/\sigma_1 + 0.9/\sigma_2 + 0.7/\sigma_3 + 1/\sigma_4 + 0.7/\sigma_5$

As before $\mu_{\tilde{G}}1$ and $v_1(a_3)$ are computed as

$\mu_{\tilde{G}}{}^1 = 1\text{-}d(X_{12},G^1) = \frac{1}{5} - \frac{1}{7}(0.1 + 0.6 + 0.6) = 0.74$

and $v_1(a_2) = .7 \wedge 0.74 = 0.7$

Thus, we start from X_{12} and applying a_1, a_2, a_3 we obtain the values of the partial decisions.

$v_2'(a_2,a_1) = 0.7 \wedge 0.5 - 0.5$

$v_2'(a_2,a_2) = 0.7 \wedge 7 = 0.7$

$v_2'(a_2,a_2) = 0.7 \wedge \; 0.7 = 0.7$

We next proceed to compute X_{222} and X_{223}. These are given respectively by

$X_{222} = 0.9/\sigma_1 + 0.7/\sigma_2 + 0.7/\sigma_3 + 0.9/\sigma_4 + 0.7/\sigma_5$

$X_{223} = 0.9/\sigma_1 + 0.8/\sigma_2 + 0.7/\sigma_3 + 0.7/\sigma_4 + 0.9/\sigma_5$

Now for X_{222}, $\mu_{\bar{G}^2} = 1\text{-}d(X_{222},G^2) = 1 \_ (0.7 + 0.2 + 0 + 0.1 + 0.3) =$
and for X_{223}, $\mu_{\bar{G}^2} = 1\text{-}d(X_{223},G^2) = 1 - r\ (0.7 + 0.3 + 0 + 0.1 +$
while $v_2(a_2,a_2) = 0.7 \wedge 0.74 = 0.7$
and $v_2(a_2,a_3) = 0.7 \wedge 0.76 = 0.7$
We may now compute the values of the partial decisions as done previously. Thus we start from X_{223} and applying a_2,a_2,a_3 we obtain

$v_3'(a_2,a_2,a_1) = 0.7 \wedge 1 = 0.7$

$v_3'(a_2,a_2,a_2) = 0.7 \wedge 0.8 = 0.7$

$v_3'(a_2,a_2,a_2) = 0.7 \wedge 0.6 = 0.6$

$v_3'(a_2,a_3,a_2) = 0.7 \wedge 1 = 0.7$

$v_3'(a_2,a_2,a_2) = 0.7 \wedge 0.8 = 0.7$

$v_3'(a_2,a_3,a_3) = 0.7 \wedge 0.6 = 0.6$

Finally, we proceed to compute X_{3231}, X_{3222}, X_{3231} and X_{3232} respectively as

$X_{3221} = 0.9/\sigma_1 + 0.7/\sigma_2 + 0.9/\sigma_3 + 0.7/\sigma_4 + 0.7/\sigma_5$

$X_{3232} = 0.9/\sigma_1 + 0.9/\sigma_2 + 0.9/\sigma_3 + 0.9/\sigma_4 + 0.7/\sigma_5$

$X_{3231} = 0.9/\sigma_1 + 0.7/\sigma_2 + 0.9/\sigma_3 + 0.7/\sigma_4 + 0.7/\sigma_5$

$X_{3232} = 0.7/\sigma_1 + 0.9/\sigma_2 + 0.9/\sigma_3 + 0.9/\sigma_4 + 0.7/\sigma_5$

At this stage, we need to find μ_{G_3} and v_3 for X_{3221}, X_{3222}, X_{3231} and X_{3232}

for X_{3221}, $\mu_{\bar{G}^3} = 1\text{-}d(X_{3221},G^3) = 0.82$ and $v_3(a_2,a_2,a_1) = 0.7 \wedge 0.82 = 0.7$
for X_{3222}, $\mu_{\bar{G}^3} = 1\text{-}d(X_{3231},G^3) = 0.74$ and $v_3(a_2,a_2,a_2) = 0.7 \wedge 0.74 = 0.7$
for X_{3231}, $\mu_{\bar{G}^3} = 1\text{-}d(X_{3231},G^3) = 0.82$ and $v_3(a_2,a_3,a_1) = 0.7 \wedge 0.82 = 0.7$
for X_{3232}, $\mu_{\bar{G}^3} = 1\text{-}d(X_{3232},G^3) = 0.78$ and $v_3(a_2,a_3,a_2) = 0.7 \wedge 0.78 = 0.7$

Since there is no other partial decision with higher value, these four $(a_2,a_2,a_2)(a_2,a_2,a_2)$, (a_2,a_3,a_1) and (a_2,a_3,a_2) are the maximizing ones. We note that the four values are equal in this example in contrast to the two obtained by Kacprzyk. As correctly pointed out by Kacprzyk, however, the solutions need not be unique.

To aid in the ease of computational realization of this algorithm, especially when dealing with real life data that may involve large matrices, we have developed a high level Fortran computer program. The program has been debugged and tested with synthetic data. The flow chart is given in Fig 2.1 while a complete computer listing is provided in the Appendix.

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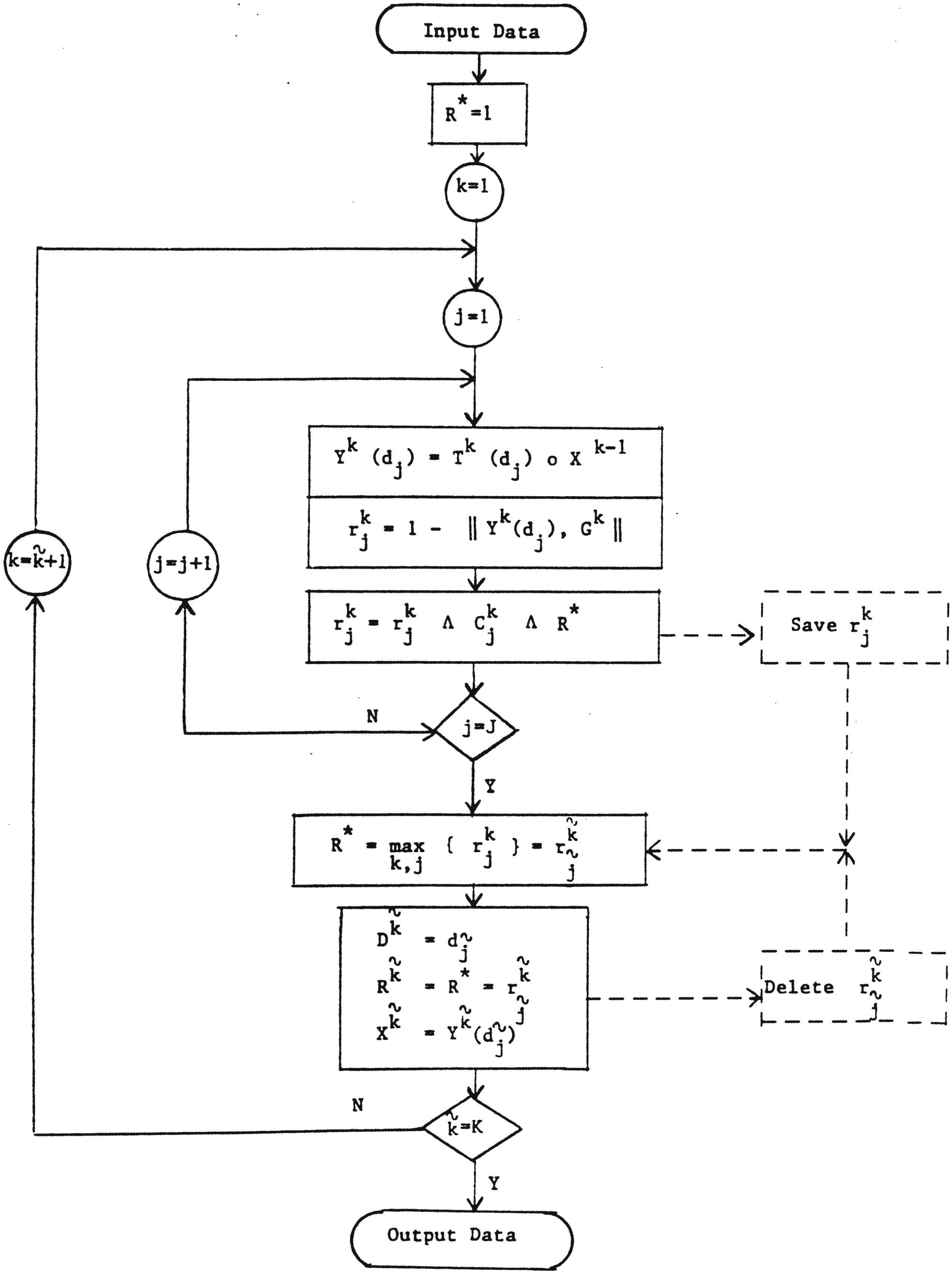
Fig 2.1. Flow Chart for Version 1 Model [Decision Tree Algorithm]



CHAPTER THREE

FUZZY MATHEMATICAL MODELS AND ALGORITHMS FOR THE FLOOD CONTROL PROBLEM: VERSION II

3.1 INTRODUCTION

This version is philosophically different from that of version one discussed in Chapter 2. It always provides a unique optimal solution. It may however, be computationally more tedious than the foregoing unless careful steps are taken in programming the model.

In the sequel, we provide models of the flood control problem viewed as fuzzy multistage decision processes. The organization of the developments is as follows: We begin with the definition of symbols and notation employed in the models as well as in the flow charts that accompany them. We next present the models for the regional, national and coordination phases. For the first and second phases, we first show the core model and then provide an expanded version along with a practical algorithm for its implementation.

3.2 FUZZY CONTROL MODEL

We define the following symbols employed in the models

- n: the index of region
- k: the index of flood control measure
- j: the index of flood control investment level
- i: the index of flood damage level

At the national level, Phase 2, the following are used.

- $C(j)$: the membership function of constraint for the nation
- $G(j)$: the membership function of goal for the nation
- $C_n(j)$: the membership function of constraint for region n
- $G(j)$: the membership function of goal for region n
- $\bar{J}$: the upper bound of total investment for the nation while
- W_n: the weight or critically of region n

In the foregoing, $C(j)$ and $G(j)$ are defined on the set of all of the possible investment levels for the nation, $C_n(j)$ and $G_n(j)$ are defined on the set of all of the possible investment levels for region n. When used in regions, the symbols have the following additional meanings:

$I_n(i)$: the membership function of initial states in region n

$F_n(i)$: the membership function of final states in region n

$G_n(i)$: the membership function of goal of states in region n

$\bar{J}_n$: the upper bound of total investment for region n

$C_{nk}(j)$: the membership function of constraint for measure k in region n

Here $I_n(i)$, $F_n(i)$ and $G_n(i)$ are defined on the state space (all of the possible flood damage levels for region n). While $C_{nk}(j)$ is defined on the decision space (all of the possible investment levels for measure k in region n).

Additionally, let:

$T_{nkj}(i,i)$: the fuzzy matrix of state transform for measure k in region n with investment level j

Here $T_{nkj}(i,i)$ is an I*I matrix, where I is the dimension of the state space (all of the possible flood damage levels for region n), and represents the fuzzy relation between the membership function of states before and after measure k has been put into use at the investment level j.

The essential aspects of a very general model of a fuzzy decision system solved by branch and bound method was first proposed by Kacprzyk [18]. Because of the simple structure of the model, the solution algorithm involved only a single directional search down the branch of a decision tree.

3.3 CORE FUZZY MODEL OF FLOOD CONTROL FOR REGIONS - PHASE I

A general description of the ensuing model is that of a multi-stage decision-making process for a fuzzy system in a fuzzy environment. The usual concepts of stage, decision, and state are defined respectively as follows:

Let

Stage - the (structural or non-structural) measure for flood control

Decision - the level of investment for measure (in \$), and

State - the level of flood damage (in \$)

The necessary data for the model are the following:

$I_n(i)$, the membership function of initial states;

$G_n(i)$, the membership function of goal of states;

$C_{nk}(j)$, the membership function of constraint for measure k (k=1,2,...,K)

and $T_{nkj}(i,i)$, the fuzzy matrix of state transform for measure k with investment level j, (j=0,...,j; k=1,...,K)

We may then postulate the following fuzzy mathematical model of the problem as

$$\Phi_n = \bigvee_{j_{n1},\dots,j_{nK}} \left([C_{n1}(j_{n1}) \wedge \dots \wedge C_{nk}(j_{nk}) \dots \wedge C_{nk}(j_{nK})] \wedge \tilde{G}(F_n) \right) \quad (6)$$

s.t.

$$F_n = T_{nKj_{nK}} * \dots * T_{nkj_{nk}} * \dots * T_{n1j_{n1}} * I_n \quad (7)$$

$$\tilde{G}(F_n) = 1. - \| G_n, F_n \| \quad (8)$$

Where in the foregoing * is the max-min product operator,

F_n is the membership function of final states and

$\| G_n, F_n \|$ is a relative distance between G_n and F_n

Solution of the above model will provide the following output data for use in the next optimization phase.

j_{nk}^*, the optimal investment level for measure k (k=1,...,K) in region n and

Φ_n, the optimal effect of flood control program for region n,

We call this the core model. Note that for each measure, the decision set includes a 'null' decision, i.e. investment level $j_{nk} = 0.$, which means measure k will not be used at all. Correspondingly, the grade of membership function of constraint $C_{nk}(0) = 1.$, and the matrix of state transform

$T_{nko} = I$ (unit matrix) which keeps the membership functions of states identical before and after stage k.

3.4 THE EXPANDED FUZZY MODEL OF FLOOD CONTROL FOR REGIONS - PHASE 1

We may now expatiate on the core model and provide a practical algorithm which gives one an insight into the general solution procedure.

The basic idea behind the expansion is the following. Due to budgeting constraints, it may be necessary to impose a (crisp) limit to the7 total investment available for region n, namely j_n . Thus, the model should be modified to reflect this constraint. The resultant model is therefore equations (6),(7)(8) and (9).

$$j_{n1} + \dots + j_{nk} + \dots + j_{nK} \le j_n \qquad (9)$$

Additionally, we need the data on the maximum possible or the upper bound of total investment for region n. Let this be denoted $\bar{J}_n$. This responds to the budgetary constraint of the fuzzy resource allocation problem.

Let us sketch the essential steps of a global and fractional algorithm for implementing the foregoing model in Phase 1. It may be broken into five basic steps:

Step 0: Repeat Steps 1,..., 4 for n=1,...,10.

Step 1: Determine the scale of possible level of total investment for region n, namely $[0.,\bar{J}_n]$, by using $\bar{J}_n$ defined above.

Step 2: For each j_n within $[.0.,\bar{J}_n]$, run the Expanded Model above to obtain $\Phi_n(j_n)$ and j_{nk}^* (k=1,...K).

Step 3: Construct $G_n(j_n)$, the membership function of goal for region n. as follows:

$$G_n(j_n) = \begin{cases} 0 & j_n < 0. \\ \Phi_n(j_n) & 0. \le j_n \le J_n \\ 0 & J_n < j_n \end{cases} \qquad (10)$$

Step 4: Send $G_n(j_n)$ to Phase 2 and store all j_{nk}^* for each j_n.

3.5 A CORE FUZZY MODEL OF FLOOD CONTROL FOR THE NATIONAL LEVEL - PHASE 2

The core model for the problem at the national level or phase 2 may be viewed as that of a multi-stage decision-making process for a non-fuzzy system in fuzzy environment. In this phase, the usual concepts of stage, decision and state are defined as follows:

Stage - the region for flood control

Decision - the level of total investment for region (in $) and

State - the effect of flood control for region

As before, we define the following necessary input data.

$C_n(j)$ - the membership function of constraint for region n $(n=1,2,\ldots,10)$

$G_n(j)$ - the membership function of goal for region $n(n=1,\ldots,10)$ and

W_n - the weight or relative importance of region $n(n=1,\ldots,10)$

The fuzzy mathematical program to be solved here may then be stated as:

$$\Phi = \bigvee_{j_1 \ldots j_{10}} (R_1(j_1) + \ldots + R_n(j_n) + \ldots + R_{10}(j_{10})) \quad (11)$$

$$\text{s.t. } R_n(j_n) = [\, G_n(j_n) \wedge C_n(j_n) \,] * W_n \quad , n = 1,\ldots, 10 \quad (12)$$

$$W_1 + \ldots + W_n + \ldots + W_{10} = 1. \quad (13)$$

where * is the algebraic product operator and $R_n(j_n)$ is the return function for stage n, i.e., region n. Solution of the foregoing generates the output data j_n^* and Φ where j_n^* is the optimal investment level for region n $(n=1,\ldots,10)$ and Φ is the optimal weighted-sum of effect of flood control for the nation.

3.6 AN EXPANDED FUZZY MODEL OF FLOOD CONTROL FOR THE NATIONAL LEVEL - PHASE 2

As before, we proceed to expand on the model. Basically, the presence of a budget availability constraint, on a national level, for flood control management necessitates a model modification.

If we assume a (crisp) limit to the total investment for the country, namely j, then the model should be equations (11)(12)(13) and (14).

$$j_1 + \ldots + j_n + \ldots + j_{10} \leq j \quad (14)$$

Let $\Phi(j)$ be the optimal weighted-sum of effect of flood control for the country which depends on the j. With $\bar{J}$, the upper bound on total investment on flood control program for the country. The following four step practical algorithm shows how the model developed for phase 2 may be realized.

Step 1: Determine the scale of possible level of total investment of the nation, namely $[0,\bar{J}]$, by using $\bar{J}$ defined above.

Step 2: For each $j \in [0.,\bar{J}]$, run the expanded model above to obtain $\Phi(j)$ and j_n^* (n=1,...,10).

Step 3: Construct G(j), the membership function of goal at the national level, as follows:

$$G(j) = \begin{cases} 0 & j < 0 \\ \Phi(j) & 0. \leq j \leq \bar{J} \\ 0. & \bar{J} < j \end{cases} \quad (15)$$

Step 4: Send G(j) to Phase 3 and store all j_n^* for each j.

3.7 FUZZY MODEL FOR COORDINATION - PHASE 3

Finally, we present a linkage program for coordinating the preceding two phases. This phase is basically a single-stage decision-making process for a non-fuzzy system in a fuzzy environment by standard fuzzy decision-making. Before presenting the model, let us define the following which are essentially input data to the model.

C(j) = the membership function of constraint at the national level

and G(j) = the membership function of national flood control goal.

The mathemetical program is then

$$\Phi = \bigvee_{j\in[0,J]} [G(j) \wedge C(j)] \quad (16)$$

Solution of this optimization problem leads to the output data.

j^*, the optimal investment level for flood control management for the country, and

Φ, the degree to which the optimal flood control plan satisfies the national objective.

A three step practical algorithm for Phase 3 model follows.

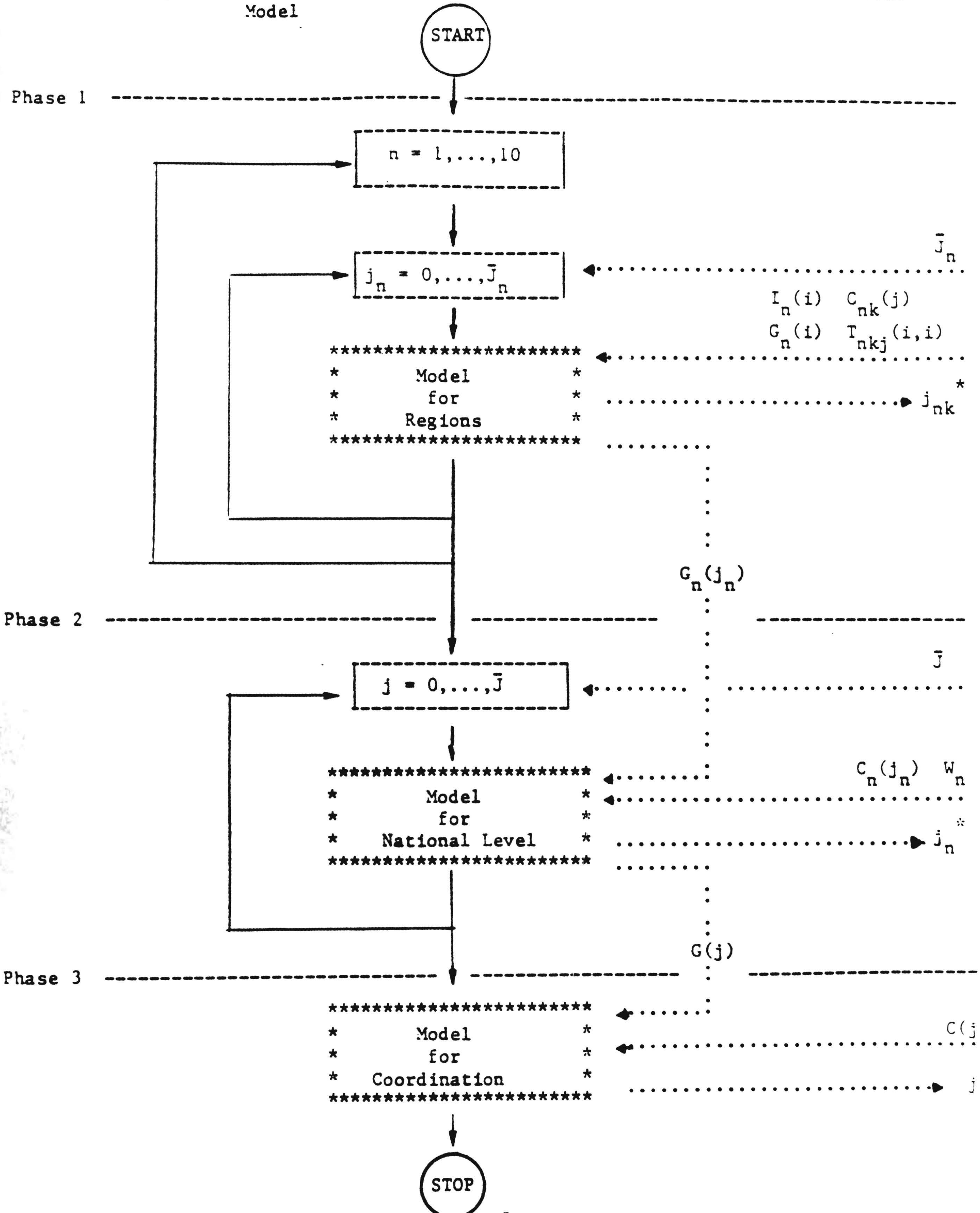
Step 1: Run the model for coordination of (16) to get j^*.

Step 2: Using j^* and the solution stored in Phase 2, find $_n^*$ for region n (n=1,...,10).

Step 3: Using j_n^* and the solution stored in Phase 1, find j_{nk}^* for measure k in region n (k=1,...,K; n=1,...,10).

A schematic view of this three phase solution procedure showing the interactions and data flows is given in Fig. 3.1.

Fig. 3.1 Flow Chart and Information Transmission for the Hierarchical Model



3.8 COMPUTATIONAL ASPECTS

Let us now develop computational algorithms for the implementation of the models presented in Section 2.6. We first consider the regional level analysis and then the national. Since the third phase, namely the coordination model is a simple one stage fuzzy decision model, its computational algorithm is routine and will not be presented here. We remark that the method of solution in both cases in the branch and bound procedure for a fuzzy multistage decision problem. A version of this was first presented by Kacprzyk [18]. The reader is referred to that reference for its exposition. Since our model is more complex than that used to illustrate Kacprzyk's algorithm, a different form of the branch and bound procedure is utilized.

3.8.1 The Algorithm for the Regional Model

To motivate our presentation, we first recapitulate the model of equations (6),(7) and (8) where equation (8) is replaced by a specific norm in equation (19). Justification of this measure of closeness between the goal G_n and the final state F_n is given in Kacprzyk and is acceptable here. Examples of other measures are given in Kaufman and Gupta [22] and Klir and Folger [23].

$$\Phi_n(j_n) = \bigvee_{j_{n1}\cdots j_{nK}} \{ [C_{n1}(j_{n1}) \wedge\cdots\wedge C_{nk}(j_{nK}) \wedge C_n(j_{nK})] \wedge \tilde{G}(F_n) \} \tag{17}$$

$$\text{s.t. } F_n = T_{nKj_{nK}} * \ldots * T_{nkj_{nk}} * \ldots * T_{n1j_{n1}} * I_n \tag{18}$$

$$\tilde{G}(F_n) = 1 - \frac{1}{I} \sum_{i=1}^{I} \| G_n(i) - F_n(i) \| \tag{19}$$

$$j_{n1} + \ldots + j_{nk} + \ldots + j_{nK} \leq j_n \tag{20}$$

Note that in the foregoing * is the max-min product operator and $j_{n1} \ldots j_{nK}$ are the decision variables.

The basis of our algorithm is the following analysis of the foregoing model.

We may view the objective function as being made up of two components. The first part is:

$$C = C_{n1}(j_{n1}) \wedge\cdots\wedge C_{nk}(j_{nk}) \wedge \cdots C_{nK}(j_{nK}) \tag{21}$$

Denote $C^{(k)} = C_{n1}(j_{n1}) \wedge\cdots\wedge C_{nk}(j_{nk})$ (22)

then $C^{(k)}$ is a non-increasing function of k. i.e.

$$C^{(k)} \geq C^{(k+1)} \quad , \quad k = 1,\ldots,K-1 \tag{23}$$

However, because the second part, namely $\bar{G}$, is non monotonic in k, we may employ the Bound-Branch method to search in the decision tree by using $C_{nk}(j_{nk})$ as the upper bound of the branch. If we are in stage k and $C_{nk}(j_{nk})$ is not greater than the present best solution, the branch will be fathomed. Meanwhile, that branch which does not satisfy constraint (20) will also be fathomed. When the end of the branch in the tree is reached, i.e. at stage K, both F_n and $\bar{G}(.)$ are evaluated successively by (18) and (19) respectively and C is evaluated by (21). The result is used to combine C with $\bar{G}$ to get a new decision solution. The present best solution is updated if the new solution is better than the old one.

We note that computations for region n (n=1,...,10) can be performed independently; thus, the subscript n in this section can be omitted without loss of generality. A detailed flow chart for this computational algorithm is given in Fig. 3.2.

3.8.2 <u>A Branch and Bound Algorithm for the National Model</u>

We restate the optimization problem to be solved for this phase.

$$\Phi(j) = \bigvee \left(R_1(j_1) + \dots + R_n(j_n) \dots + R_{10}(j_{10})\right) \tag{24}$$

$$\text{s.t. } R_n(j_n) = [\, G_n(j_n) \wedge C_n(j_n) \,] * W_n \;, \qquad n = 1,\dots,10 \tag{25}$$

$$W_1 + \dots + W_n + \dots + W_{10} = 1. \tag{26}$$

$$j_1 + \dots + j_n + \dots + j_{10} \le j \tag{27}$$

As before where * is the algebraic product operator and $j_1,\dots j_{10}$ are the decision variables.

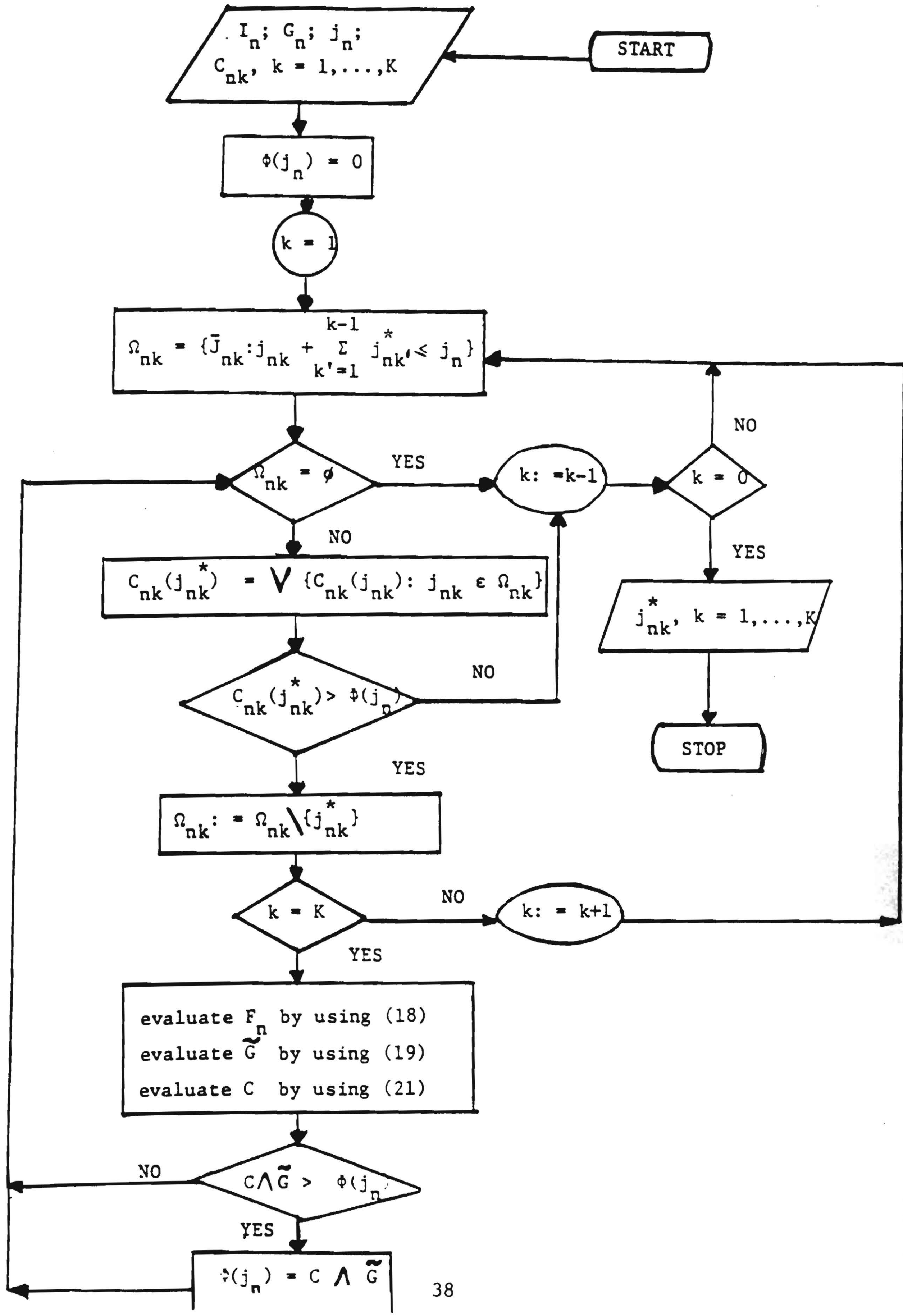
The principal algorithm employed here is the branch and bound procedure. As in phase 1, the basic idea behind the algorithm is to decompose the objective function into two parts. The first part is the sum of the terms from R_1 to R_n while the second is the sum of the remaining terms. The upper bound of the branch, when in stage n, should be the sum of the first part and the upper bound of the second part which can be defined as follows:

$$H^{(10)} = 0$$

$$H^{(n)} = \sum_{n'=n+1}^{10} \max_{j} \, [R_{n}'(j)\,] \qquad n=1,\dots,9 \tag{28}$$

When the sum of the first part and $H^{(n)}$ is not greater than present best solution the branch is fathomed. Meanwhile, the branch which does not satisfy constraint (27) is also fathomed. When the end of branch in the tree is reached, i.e. stage 10, a new solution is obtained and the present

Fig. 3.2 Flow Chart for Phase 1 Optimization



best solution is updated. Fig. 3.3 shows the detailed flow chart for performing the computations in this phase.

3.9 DISCUSSION

The performance of these algorithms has been investigated through computer implementation and experiments. They have been tested using synthetic data for both the regional and national level problems. These are discussed fully in Chapter 4.

Clearly, the solutions for both levels are dependent on the membership functions prescribed for the state, goals, and constraints while on the regional and national levels, the state transform matrix and the weights are respectively additional sources of influence. The algorithms overcome the concern for high storage while at the same time are quite fast. The computation times on the IBM PC are quite negligible for the regional phase and take only a few minutes for the national phase.

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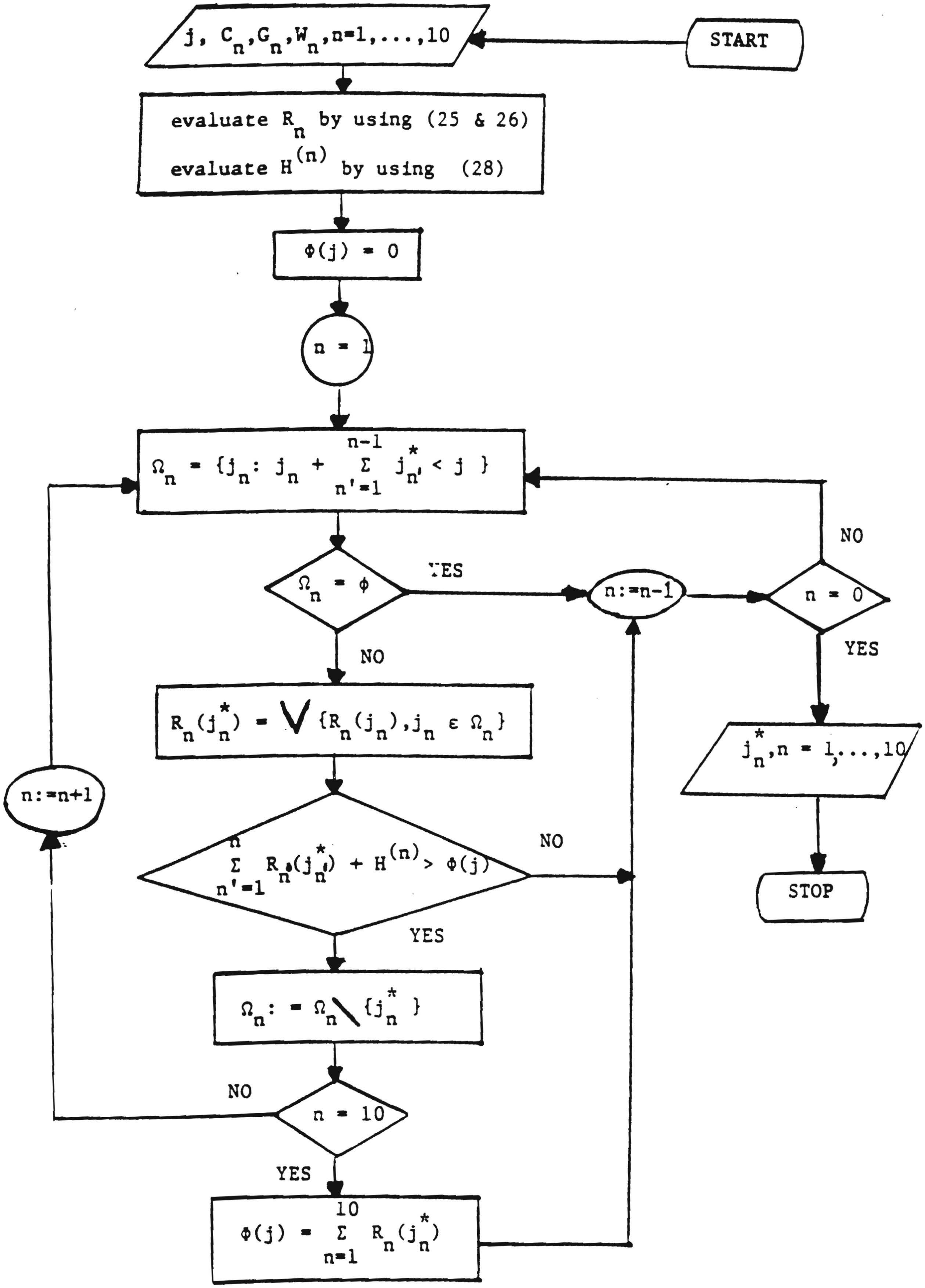
Fig. 3.3 Flow Chart for Phase 2 Optimization



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CHAPTER FOUR

APPLICATIONS TO FLOOD CONTROL PLANNING

4.1 INTRODUCTION

In this chapter, we wish to exercise the two versions of the algorithms on a flood control problem described extensively in Chapters 2 and 3. We reiterate the generality of our models and their application to other types of disaster control planning problems arising in various sectors including non point source water pollution.

4.2 FLOOD CONTROL ALGORITHM: VERSION 1

We consider two examples. The first illustrates a scenario where a unique optimal policy may be obtained, while the second shows non uniqueness. In both examples, we have a fuzzy state of flood damage representing five levels: no damage, slight damage, moderate damage, severe damage and disastrous damage. The decision space concerns three investment levels for each of the three flood control measures (structural and/or non-structural). These measures represent the three stages of the model. There are three fuzzy goals, different for each control measure, and expressed in terms of membership functions. Similarly, we have three fuzzy constraints, expressed in terms of membership functions, for each measure. Additionally, we are given the membership function for the fuzzy initial state. The problem is to determine the optimal combination of controls or measures together with the associated funding levels to put in place so as to minimize the damage levels due to incipient floods. We state parenthetically, that fuzzy set theory is used to model these systems because usually the damage levels and goals can not be stated precisely in such flood control systems.

Note that we have the same fuzzy initial state and the same goal for the first measure, in the two examples but different goals and constraints for the other measures in the two examples. The first example led to a single unique optimal decision solution while the second generated two optimal solutions. The examples and computations are given below in Tables 4.1 and 4.2.

Table 4.1.1: Fuzzy Flood Control Model
Version 1, Example 1

Example 1 : a problem with only one optimal solution

****

STATE SPACE : the flood damage level
{ no, slight, moderate, severe, disatrous }

DECISION SPACE : the investment level for the measures
{ low, medium, high }

****

(1) The Membership Function of Initial State :

X0 = 0.1/no + 0.4/slight + 0.7/moderate + 1.0/severe + 0.8/disatrous

(2) The Membership Function of Goal State :

G1 = 0.4/no + 0.6/slight + 0.6/moderate + 0.7/severe + 0.5/disatrous
G2 = 0.7/no + 0.8/slight + 0.5/moderate + 0.4/severe + 0.2/disatrous
G3 = 1.0/no + 0.7/slight + 0.4/moderate + 0.1/severe + 0.0/disatrous

(3) The Membership Function of Constraint For Measures :

C1 = 0.35/low + 0.85/medium + 0.60/high
C2 = 0.25/low + 0.50/medium + 0.75/high
C3 = 1.00/low + 0.70/medium + 0.40/high

(4) The Fuzzy Transform Matrix :

T1(low) =

0.3	0.8	0.5	0.3	0.1
0.2	0.3	0.8	0.5	0.3
0.1	0.2	0.3	0.8	0.5
0.0	0.1	0.2	0.3	0.8
0.0	0.0	0.1	0.2	0.3

T1(medium) =

0.6	0.9	0.4	0.1	0.0
0.1	0.6	0.9	0.4	0.1
0.0	0.1	0.6	0.9	0.4
0.0	0.0	0.1	0.6	0.9
0.0	0.0	0.0	0.1	0.6

T1(high) =

0.4	0.6	0.8	0.2	0.1
0.3	0.4	0.6	0.8	0.2
0.2	0.3	0.4	0.6	0.8
0.1	0.2	0.3	0.4	0.6
0.0	0.1	0.2	0.3	0.4

T2(low) =

0.4	0.7	0.5	0.3	0.1
0.3	0.4	0.7	0.5	0.3
0.2	0.3	0.4	0.7	0.5
0.1	0.2	0.3	0.4	0.7
0.0	0.1	0.2	0.3	0.4

T2(medium) =

0.5	0.8	0.4	0.2	0.1
0.3	0.5	0.8	0.4	0.2
0.1	0.3	0.5	0.8	0.4
0.0	0.1	0.3	0.5	0.8
0.0	0.0	0.1	0.3	0.5

T2(high) =

0.5	0.6	0.8	0.4	0.1
0.3	0.5	0.6	0.8	0.4
0.2	0.3	0.5	0.6	0.8
0.1	0.2	0.3	0.5	0.6
0.0	0.1	0.2	0.3	0.5

T3(low) =

0.3	0.7	0.6	0.4	0.2
0.2	0.3	0.7	0.6	0.4
0.1	0.2	0.3	0.7	0.6
0.0	0.1	0.2	0.3	0.7
0.0	0.0	0.1	0.2	0.3

T3(medium) =

0.7	0.9	0.3	0.2	0.0
0.4	0.7	0.9	0.3	0.2
0.1	0.4	0.7	0.9	0.3
0.0	0.1	0.4	0.7	0.9
0.0	0.0	0.1	0.4	0.7

T3(high) =

0.3	0.6	0.9	0.2	0.1
0.2	0.3	0.6	0.9	0.2
0.2	0.2	0.3	0.6	0.9
0.1	0.2	0.2	0.3	0.6
0.0	0.1	0.2	0.2	0.3

****

SOLUTION :

```
stage 1 -------------------------------------------------------------------

v1'(low)    = 0.35
v1'(medium) = 0.85 ***
v1'(high)   = 0.60

X1m = 0.4/no + 0.7/slight + 0.9/moderate + 0.8/severe + 0.6/disatrous

1.0 - D[X1m,G1] = 1.0 - [ 0.0 + 0.1 + 0.3 + 0.1 + 0.1 ] / 5 = 0.88

v1(medium) = 0.85 /\ 0.88 = 0.85


stage 2 -------------------------------------------------------------------

v2'(low)    = 0.25 /\ 0.85 = 0.25
v2'(medium) = 0.50 /\ 0.85 = 0.50
v2'(high)   = 0.75 /\ 0.85 = 0.75 ***

X2mh = 0.8/no + 0.8/slight + 0.6/moderate + 0.6/severe + 0.5/disatrous

1.0 - D[X2mh,G2] = 1.0 - [ 0.1 + 0.0 + 0.1 + 0.2 + 0.3 ] / 5 = 0.86

v2(high) = 0.75 /\ 0.86 = 0.75


stage 3 -------------------------------------------------------------------

v3'(low)    = 1.00 /\ 0.75 = 0.75 ***
v3'(medium) = 0.70 /\ 0.75 = 0.70
v3'(high)   = 0.40 /\ 0.75 = 0.40

X3mhl = 0.7/no + 0.6/slight + 0.6/moderate + 0.5/severe + 0.3/disatrous

1.0 - D[X3mhl,G3] = 1.0 - [ 0.3 + 0.1 + 0.2 + 0.4 + 0.3 ] / 5 = 0.74

v3(low) = 0.75 /\ 0.74 = 0.74

---------------------------------------------------------------------------

            The optimal solution is thus [ medium,high,low ].

---------------------------------------------------------------------------
```

Example 2 : a problem with two optimal solutions

****

STATE SPACE : the flood damage level
{ no, slight, moderate, severe, disatrous }

DECISION SPACE : the investment level for the measures
{ low, medium, high }

****

(1) The Membership Function of Initial State :

X0 = 0.1/no + 0.4/slight + 0.7/moderate + 1.0/severe + 0.8/disatrous

(2) The Membership Function of Goal State :

G1 = 0.4/no + 0.6/slight + 0.6/moderate + 0.7/severe + 0.5/disatrous
G2 = 0.9/no + 0.7/slight + 0.5/moderate + 0.3/severe + 0.1/disatrous
G3 = 1.0/no + 0.8/slight + 0.4/moderate + 0.1/severe + 0.0/disatrous

(3) The Membership Function of Constraint For Measures :

C1 = 0.45/low + 0.85/medium + 0.65/high
C2 = 1.00/low + 0.80/medium + 0.60/high
C3 = 0.50/low + 0.70/medium + 0.90/high

(4) The Fuzzy Transform Matrix :

T1(low) =

0.3	0.8	0.5	0.3	0.1
0.2	0.3	0.8	0.5	0.3
0.1	0.2	0.3	0.8	0.5
0.0	0.1	0.2	0.3	0.8
0.0	0.0	0.1	0.2	0.3

T1(medium) =

0.6	0.9	0.4	0.1	0.0
0.1	0.6	0.9	0.4	0.1
0.0	0.1	0.6	0.9	0.4
0.0	0.0	0.1	0.6	0.9
0.0	0.0	0.0	0.1	0.6

T1(high) =

0.4	0.6	0.8	0.2	0.1
0.3	0.4	0.6	0.8	0.2
0.2	0.3	0.4	0.6	0.8
0.1	0.2	0.3	0.4	0.6
0.0	0.1	0.2	0.3	0.4

T2(low) =

0.4	0.7	0.5	0.3	0.1
0.3	0.4	0.7	0.5	0.3
0.2	0.3	0.4	0.7	0.5
0.1	0.2	0.3	0.4	0.7
0.0	0.1	0.2	0.3	0.4

T2(medium) =

0.5	0.8	0.4	0.2	0.1
0.3	0.5	0.8	0.4	0.2
0.1	0.3	0.5	0.8	0.4
0.0	0.1	0.3	0.5	0.8
0.0	0.0	0.1	0.3	0.5

T2(high) =

0.5	0.6	0.8	0.4	0.1
0.3	0.5	0.6	0.8	0.4
0.2	0.3	0.5	0.6	0.8
0.1	0.2	0.3	0.5	0.6
0.0	0.1	0.2	0.3	0.5

T3(low) =

0.3	0.7	0.6	0.4	0.2
0.2	0.3	0.7	0.6	0.4
0.1	0.2	0.3	0.7	0.6
0.0	0.1	0.2	0.3	0.7
0.0	0.0	0.1	0.2	0.3

T3(medium) =

0.7	0.9	0.3	0.2	0.0
0.4	0.7	0.9	0.3	0.2
0.1	0.4	0.7	0.9	0.3
0.0	0.1	0.4	0.7	0.9
0.0	0.0	0.1	0.4	0.7

T3(high) =

0.3	0.6	0.9	0.2	0.1
0.2	0.3	0.6	0.9	0.2
0.2	0.2	0.3	0.6	0.9
0.1	0.2	0.2	0.3	0.6
0.0	0.1	0.2	0.2	0.1

****

SOLUTION :

```
stage 1 ----------------------------------------------------------------

v1'(low)     = 0.45
v1'(medium)  = 0.85 ***
v1'(high)    = 0.65

X1m = 0.4/no + 0.7/slight + 0.9/moderate + 0.8/severe + 0.6/disatrous
1.0 - D[X1m,G1] = 1.0 - [ 0.0 + 0.1 + 0.3 + 0.1 + 0.1 ] / 5 = 0.88

v1(medium) = 0.85 /\ 0.88 = 0.85

stage 2 ----------------------------------------------------------------

v2'(low)     = 1.00 /\ 0.85 = 0.85 ***
v2'(medium)  = 0.80 /\ 0.85 = 0.80 **
v2'(high)    = 0.60 /\ 0.85 = 0.60

X2ml = 0.7/no + 0.7/slight + 0.7/moderate + 0.6/severe + 0.4/disatrous
1.0 - D[X2ml,G2] = 1.0 - [ 0.2 + 0.0 + 0.2 + 0.3 + 0.3 ] / 5 = 0.80

X2mm = 0.7/no + 0.8/slight + 0.8/moderate + 0.6/severe + 0.5/disatrous
1.0 - D[X2mm,G2] = 1.0 - [ 0.2 + 0.1 + 0.3 + 0.3 + 0.4 ] / 5 = 0.74

v2(low)     = 0.85 /\ 0.80 = 0.80 ***
v2(medium)  = 0.80 /\ 0.74 = 0.74 **

stage 3 ----------------------------------------------------------------

v3'(low)     = 0.50 /\ 0.80 = 0.50
v3'(medium)  = 0.70 /\ 0.80 = 0.70
v3'(high)    = 0.90 /\ 0.80 = 0.80 ***

X3mlh = 0.7/no + 0.6/slight + 0.6/moderate + 0.4/severe + 0.3/disatrous
1.0 - D[X3mlh,G3] = 1.0 - [ 0.3 + 0.2 + 0.2 + 0.3 + 0.3 ] / 5 = 0.74

v3(high) = 0.80 /\ 0.74 = 0.74


v3'(low)     = 0.50 /\ 0.74 = 0.50
v3'(medium)  = 0.70 /\ 0.74 = 0.70
v3'(high)    = 0.90 /\ 0.74 = 0.74 ***

X3mmh = 0.8/no + 0.6/slight + 0.6/moderate + 0.5/severe + 0.3/disatrous
1.0 - D[X3mmh,G3] = 1.0 - [ 0.2 + 0.2 + 0.2 + 0.4 + 0.3 ] / 5 = 0.74

v3(high) = 0.74 /\ 0.74 = 0.74


------------------------------------------------------------------------
The optimal policies are thus
                    both [ medium,low,high ] and [ medium,medium,high ].

------------------------------------------------------------------------
```

Table 4.2.1 Fuzzy Flood Control Model, Version 2

4.1 Example : **Phase I (Regional Level Allocation Problem)**

4.2.1a Data

STATE SPACE : the flood damage level quantuzed as or via fuzzy descriptors, i.e. { 1,2,3,4,5 } or { no, slight, moderate, severe, disastrous }

DECISION SPACE : the measure investment level with fuzzy descriptors, i.e. { 0,1,2,3 } or { no, low, medium, high }

THE LIMIT TO TOTAL INVESTMENT = { Dimension of State Space } - 1 = 4

The Membership Function of Initial State : X0 =
0.13/no + 0.45/slight + 0.79/moderate + 1.00/severe + 0.88/disastrous

The Membership Function of Goal State : G =
1.00/no + 0.75/slight + 0.50/moderate + 0.25/severe + 0.00/disastrous

The Membership Function of Constraint For Measures i (i = 1,...4) :

C1 = 1.00/no + 0.92/low + 0.64/medium + 0.37/high
C2 = 1.00/no + 0.62/low + 0.83/medium + 0.44/high
C3 = 1.00/no + 0.35/low + 0.71/medium + 0.89/high
C4 = 1.00/no + 0.75/low + 0.85/medium + 0.48/high

The Fuzzy Transform Matrix :

T1(no) = T2(no) = T3(no) = T4(no) = I (unit matrix)

T1(low) =

$$\begin{pmatrix} .6 & .9 & .7 & .5 & .1 \\ .1 & .6 & .9 & .7 & .5 \\ .0 & .1 & .6 & .9 & .7 \\ .0 & .0 & .1 & .6 & .9 \\ .0 & .0 & .0 & .1 & .6 \end{pmatrix}$$

T1(medium) =

$$\begin{pmatrix} .5 & .8 & .6 & .4 & .1 \\ .1 & .5 & .8 & .6 & .4 \\ .0 & .1 & .5 & .8 & .6 \\ .0 & .0 & .1 & .5 & .8 \\ .0 & .0 & .0 & .1 & .5 \end{pmatrix}$$

T1(high) =

$$\begin{pmatrix} .4 & .7 & .5 & .3 & .2 \\ .2 & .4 & .7 & .5 & .3 \\ .1 & .2 & .4 & .7 & .5 \\ .0 & .1 & .2 & .4 & .7 \\ .0 & .0 & .1 & .2 & .4 \end{pmatrix}$$

T2(low) =

$$\begin{pmatrix} .6 & .7 & .3 & .1 & .0 \\ .1 & .6 & .7 & .3 & .1 \\ .0 & .1 & .6 & .7 & .3 \\ .0 & .0 & .1 & .6 & .7 \\ .0 & .0 & .0 & .1 & .6 \end{pmatrix}$$

T2(medium) =

$$\begin{pmatrix} .5 & .9 & .5 & .3 & .1 \\ .2 & .5 & .9 & .5 & .3 \\ .0 & .2 & .5 & .9 & .5 \\ .0 & .0 & .2 & .5 & .9 \\ .0 & .0 & .0 & .2 & .5 \end{pmatrix}$$

T2(high) =

$$\begin{pmatrix} .4 & .7 & .3 & .1 & .0 \\ .2 & .4 & .7 & .3 & .1 \\ .1 & .2 & .4 & .7 & .3 \\ .0 & .1 & .2 & .4 & .7 \\ .0 & .0 & .1 & .2 & .4 \end{pmatrix}$$

T3(low) =

$$\begin{pmatrix} .6 & .5 & .4 & .1 & .0 \\ .1 & .6 & .5 & .4 & .1 \\ .0 & .1 & .6 & .5 & .4 \\ .0 & .0 & .1 & .6 & .5 \\ .0 & .0 & .0 & .1 & .6 \end{pmatrix}$$

T3(medium) =

$$\begin{pmatrix} .6 & .9 & .3 & .2 & .1 \\ .5 & .6 & .8 & .3 & .2 \\ .1 & .5 & .6 & .8 & .3 \\ .0 & .1 & .5 & .6 & .8 \\ .0 & .0 & .1 & .5 & .6 \end{pmatrix}$$

T3(high) =

$$\begin{pmatrix} .7 & .9 & .5 & .3 & .1 \\ .5 & .7 & .9 & .5 & .3 \\ .3 & .5 & .7 & .9 & .5 \\ .1 & .3 & .5 & .7 & .9 \\ .0 & .1 & .3 & .5 & .7 \end{pmatrix}$$

T4(low) =

$$\begin{pmatrix} .3 & .9 & .6 & .3 & .1 \\ .2 & .3 & .9 & .6 & .3 \\ .0 & .2 & .3 & .9 & .6 \\ .0 & .0 & .2 & .3 & .9 \\ .0 & .0 & .0 & .2 & .3 \end{pmatrix}$$

T4(medium) =

$$\begin{pmatrix} .5 & .6 & .5 & .3 & .2 \\ .3 & .5 & .6 & .5 & .3 \\ .1 & .3 & .5 & .6 & .5 \\ .0 & .1 & .3 & .5 & .6 \\ .0 & .0 & .1 & .3 & .5 \end{pmatrix}$$

T4(high) =

$$\begin{pmatrix} .4 & .6 & .7 & .4 & .1 \\ .3 & .4 & .6 & .7 & .4 \\ .2 & .3 & .4 & .6 & .7 \\ .1 & .2 & .3 & .4 & .6 \\ .0 & .1 & .2 & .3 & .4 \end{pmatrix}$$

4.2.1b Solution

STAGE	CONSTRAINT	GOAL	DECISION	STATE : X(1),X(2),--------------,X(I)				
0		.3820		.130	.450	.790	1.000	.880
1	.9200	.6060	1	.700	.790	.900	.880	.600
2	.8300	.6820	2	.790	.900	.880	.600	.500
4	.7500	.8240	1	.900	.880	.600	.500	.300

[OPTIMAL SOLUTION] .7500 = [CONSTRAINT] .7500 /\ [GOAL] .8240

where , [GOAL] = 1.0 - [DISTANCE BETWEEN G AND X]
[OPTIMAL SOLUTION] = MIN [CONSTRAINT] /\ FINAL [GOAL]

The optimal solution for this region is thus [1,2,0,1] or [low, medium, no, low], i.e. the 1st and 4th measures will be invested in at level 1 (or low money), the 2nd measure at level 2 (or medium money) and the 3rd measure will not be invested in or funded.

Table 4.2.2 Fuzzy Flood Control Model. Version 2

4.2.2 Example : Phase 2 (National Level Allocation Problem)

4.2.2a Data

DECISION SPACE : the regional investment level quantized as or via fuzzy descriptors, i.e. (0,1,2,3,4,5) or (no, little, low, medium, much, high)

THE LIMIT TO TOTAL INVESTMENT = 16

THE WEIGHTS FOR 10 REGIONS : 0.080, 0.105, 0.120, 0.095, 0.130, 0.070, 0.117, 0.083, 0.112 and 0.088

The Membership Function of Goal for Region n, G(n), and the Membership Function of Constraint for Region n, C(n) :

G(1)= 0.6/no + 0.8/little + 1.0/low + 0.9/medium + 0.8/much + 0.7/high
C(1)= 1.0/no + 1.0/little + 0.8/low + 0.6/medium + 0.4/much + 0.2/high

G(2)= 0.4/no + 0.5/little + 0.6/low + 0.7/medium + 0.8/much + 0.9/high
C(2)= 0.8/no + 0.9/little + 0.8/low + 0.7/medium + 0.6/much + 0.5/high

G(3)= 0.1/no + 0.4/little + 0.7/low + 0.9/medium + 1.0/much + 1.0/high
C(3)= 0.6/no + 1.0/little + 0.9/low + 0.8/medium + 0.7/much + 0.6/high

G(4)= 0.2/no + 0.4/little + 0.6/low + 0.8/medium + 1.0/much + 0.9/high
C(4)= 1.0/no + 1.0/little + 0.9/low + 0.7/medium + 0.5/much + 0.3/high

G(5)= 0.5/no + 0.8/little + 0.9/low + 1.0/medium + 0.9/much + 0.8/high
C(5)= 1.0/no + 0.8/little + 0.5/low + 0.2/medium + 0.1/much + 0.0/high

G(6)= 0.2/no + 0.6/little + 0.7/low + 0.8/medium + 0.8/much + 0.8/high
C(6)= 0.5/no + 0.8/little + 1.0/low + 0.7/medium + 0.4/much + 0.1/high

G(7)= 0.3/no + 0.4/little + 0.7/low + 0.9/medium + 0.8/much + 0.7/high
C(7)= 0.6/no + 0.7/little + 0.8/low + 0.8/medium + 0.7/much + 0.6/high

G(8)= 0.1/no + 0.4/little + 0.7/low + 0.8/medium + 0.9/much + 0.8/high
C(8)= 0.7/no + 0.9/little + 0.6/low + 0.4/medium + 0.2/much + 0.0/high

G(9)= 0.3/no + 0.4/little + 0.5/low + 0.6/medium + 0.7/much + 0.8/high
C(9)= 0.5/no + 1.0/little + 0.9/low + 0.8/medium + 0.5/much + 0.2/high

G(10)=0.3/no + 0.5/little + 0.7/low + 0.9/medium + 1.0/much + 1.0/high
C(10)=0.5/no + 0.9/little + 1.0/low + 0.8/medium + 0.6/much + 0.4/high

4.2.2b Solution

STAGE	WEIGHT (	CONSTRAINT /\	GOAL) =	RETURN	OBJECTIVE	DECISION
1	.080	1.0000	.8000	.06400	.06400	1
2	.105	.8000	.4000	.04200	.10600	0
3	.120	.8000	.9000	.09600	.20200	3
4	.095	.9000	.6000	.05700	.25900	2
5	.130	.8000	.9000	.10400	.36300	1
6	.070	.8000	.6000	.04200	.40500	1
7	.117	.8000	.9000	.09360	.49860	3
8	.083	.6000	.7000	.04980	.54840	2
9	.112	.5000	.3000	.03360	.58200	0
10	.088	1.0000	.7000	.06160	.64360	2

The optimal solution is thus [1,0,3,2,1,1,3,2,0,2] or [little, no, medium, low, little, little, medium, low, no, low], i.e. the 1st, 5th and 6th regions will be invested in at level 1 (or little money), the 4th, 8th and 10th regions at level 2 (or low money), the 3rd and 7th regions at level 3 (or medium money), and the 2nd and 9th regions will not be invested in during the current plan.

4.3 FUZZY CONTROL ALGORITHM: VERSION 2

We now turn our attention to the exemplification of the second version of the algorithm proposed in Chapter 3 using the flood control problem as the leitmotif of our discussion. The examples show how the algorithm would perform using data for both the regional and national levels. The computations are presented in Tables 4.2.1 and 4.2.2.

The solutions for both levels are clearly dependent on the membership functions for state, goals, and constraints. Additional effects on these solutions are engendered by the nature of the state transform matrix at the regional level, and the weights provided by experts at the national level. This algorithm overcomes the concern for high storage while at the same time runs quite fast. The computation times on the IBM PC are also quite negligible for the regional level problem and take only a few minutes for the national phase.

4.4 DISCUSSION

The field of water resources systems analysis and management is replete with complex problems usually multi faceted, multi dimensional, and multi criteria that exist in a complex web of socio-technical variables. While numerous optimization and multi-criteria based models exist in the literature, we believe that fuzzy sets theory offers considerable promise in elevating the state of the art in mathematical modeling and optimization of water resources systems. Planners often have to cope with a system replete with non linearities and qualitative variables of economic, social and political origin. While numerous areas can benefit from the fuzzy modeling viewpoint, flood damage control and water pollution issues are of particular interest and form the major theme of this project.

The model presented takes off from a model of the flood control problem treated as basically a variation of the project sequencing problem in capacity expansion planning by Morin et al. It determines the optimal flood damage reduction policies using the fuzzy dynamic programming-type methodology but solved by bounding the solution space via a branch-and-bound procedure. The primary goal is to determine an action plan through funding decisions on flood control strategies (structural and/or non-structural) that will yield the minimal flood damage. The finite set of controls $U = (u_1, u_2, \ldots, u_N)$ includes a selected number of combinations of measures and their associated funding levels for each region in a national effort to control the deleterious effects of flood. The non-structural measures are not treated as a simple augmentation of the structural ones. At each stage after determining which control or combination of controls is to be applied, an optimization procedure of the branch and bound variety is performed to reveal the optimal funding levels.

The special procedure we have used for this problem was inspired by that utilized by Kacprzyk [18]. It is quite different, however, because while the objective function in Kacprzyk's model is separable, monotonic, and non-increasing in k, only one part of our two component objective function is similarly behaved. The effect of this difference is that in the node search procedure utilized by Kacprzyk, one always proceeds forward from the currently 'best' node and never backtracks. The record keeping demands are however horrendous since all the nodes which have been reached previously as well as those not yet 'developed' need to be recorded. In our algorithm, on the other hand, the fathoming criterion and the bounding strategies are more complex because of the non monotonicity of the objective function. Our search procedure involves backtracking. For aspects of this and the details of the extended branch and bound procedures, see Bellman, et al. [4].

Prior to deciding to use this optimization methodology other formulations and approaches were also considered. For example, a fuzzy linear programming formulation [15] with two-component objective function (minimizing the total flood damage level as well as the financial expenditures induced by the properly selected flood mitigation measures undertaken) were considered. Also, an attempt was made to rank the multi-aspect alternatives using fuzzy sets as proposed by Bass and Kwakernaak [2]. However, these approaches were obviated by the economic as well as physical nonlinearities involved in such a complex problem.

4.5 REFERENCES

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CHAPTER FIVE

EXTENSIONS TO WATER POLLUTION CONTROL PLANNING

5.1 INTRODUCTION

One of the primary motives behind the development of our mathematical and generic models discussed in detail in Chapters 2 and 3 was flexibility and adaptability to various hazard control planning problems arising in other similar environments. In particular, an objective of this research effort is the treatment of flood control problems as well as non point source water pollution control planning problems. In the sequel, we sketch how these generic models can be adapted to treat the water pollution control problem.

5.2 BEST MANAGEMENT PRACTICES (BMPs) AS NONPOINT SOURCE CONTROL

BMPs are accepted procedures for the practical controls of most nonpoint source water pollution problems. We had identified these, both nationally and locally-Georgia and the Metropolitan Atlanta Area- in our 1983 study.We first investigated the extent of their usage in these areas and then analyzed them from both an effectiveness and cost-effectiveness standpoint. The major tool for these studies were fuzzy set theoretic mathematical model which in addition to being hierarchical allowed us to quantify intrinsically qualitative evaluations of experts.

In Tables 5.1 through 5.6, we present synopses of the principal results from the studies cited previously. The mathematical model has been modified as appropriate for the non point source water pollution problem. This is given in the Appendix. In Chapter 6, we discuss a data collection instrument which we have designed for collecting relevant data as well as processing them.

5.3 REFERENCES

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Table 5.1 Extent of Usage of Structural BMPs in a National Survey

Rank	Types of BMPs	Percentage of Usage
1	Sediment Barrier	100
	Sediment Basin	
	Haulageways	
	Dikes	
	Downdrain Structures	
	Storm Drain Outlet Protection	
	Temporary Seeding	
	Mulching	
	Topsoiling	
2	Sediment Trap	83.33
3	Riprap	80.00
	Subsurface Drains	
	Buffer Zones	
4	Toe Berms	75.00
	Construction Exits	
5	Gabion	60.00
	Vertical Drains	
	Landsmoothing	
6	Level Spreader	40.00
7	Filter Berms	25.00

Table 5.2. Extent of Usage of Nonstructural BMPs in a National Survey (Source [3])

Rank	Types of BMPs	Percent Of Usage
1	Street Sweeping Storage Containers Leaf Disposal Retention of Natural Vegetation Establishment of New Vegetation	100
2	Proper Storage of Deicing Materials	83.00
3	Public Education	80.00
4	Litter Control Alternatives to Pesticides	75.00
5	Disposal of Unused Pesticides	66.67
6	Reduction of Vehicle Miles Traveled (VMT) - e.g. promotion of public transportation use	57.14
7	Street Flushing Soil Testing Proper Maintenance of Deicing Equipment	50.00
8	Preventive Care for Vehicles	37.50
9	Proper Timing of Fertilizer Application	25.00
10	Legal Requirements for Pesticide Application	20.00

Table 5.3. Mean System Effectiveness of Structural BMPs (Source [3])

We now summarize the data on BMP system effectiveness obtained in the national survey. We first consider structural BMPs and then non-structural BMPs. The table below gives a listing of the structural BMPs with their corresponding mean system effectiveness values.

BMP	Mean System Effectiveness, (%)
Land Smoothing	24.0
Filter Berm	38.0
Sediment Barrier	39.0
Level Spreader	44.0
Top Soiling	44.6
Riprap	45.1
Gabion	45.5
Vertical Drains	46.4
Toe Berm	47.0
Haulageway	52.0
Construction Exit	56.0
Subsurface Drains	62.0
Sediment Trap	63.0
Storm Drain Outlet Protection	66.0
Dikes	72.02
Temporary Seeding	72.07
Mulching	73.0
Sediment Basin	78.0
Buffer Zone	83.0
Downdrain Structures	86.0

Table 5.4. Mean System Effectiveness of Nonstructural BMPs (Source [3])

Let us now turn our attention to nonstructural BMPs. We follow the same approach as the preceding section in summarizing our results. The table below lists the nonstructural BMPs with their corresponding mean system effectiveness values. They are listed in a descending order of effectiveness.

BMP	Mean System Effectiveness, (%)
Retention of Natural Vegetation	93.0
Proper Storage of Deicing Materials	88.0
Disposal of Unused Pesticides	82.0
Reduction of Vehicle Miles Traveled	81.0
Establishment of New Vegetation	73.0
Proper Maintenance of Deicing Equipment	72.0
Leaf Disposal	68.8
Proper Timing of Fertilizer Application	68.6
Preventive Care for Vehicles	64.3
Storage Containers	63.5
Alternative to Pesticides	59.0
Soil Testing	54.0
Legal Requirements for Pesticide Application	52.0
Public Education	51.7
Street Sweeping	49.0
Litter Control	33.0
Street Flushing	27.0

Table 5.5. Mean Cost-Effectiveness of Structural BMPs (Source [3])

While system effectiveness is important, it is even more so important to consider cost-effectiveness. This important concept, however, is more difficult to evaluate accurately. We summarize the results of our survey a la the approach of section 5.4.1 of [3]. We first give the data on the mean cost-effectiveness values of structural BMPs.

BMP	Mean Cost Effectiveness, (%)
Landsmoothing	28.0
Gabion	29.0
Vertical Drains	44.2
Haulageways	44.6
Topsoiling	45.0
Riprap	49.0
Construction Exit	51.0
Subsurface Drains	52.0
Sediment Trap	52.8
Downdrain Structures	53.4
Sediment Basin	55.6
Filter Berms	56.1
Level Spreader	57.0
Toe Berm	59.0
Storm Drain Outlet Protection	71.0
Sediment Barrier	72.0
Mulching	79.0
Dikes	81.0
Temporary Seeding	82.0
Buffer Zone	91.0

Table 5.6. Mean Cost-Effectiveness of Nonstructural BMPs (Source [3])

We now turn our attention to nonstructural BMPs. As usual, we follow the aproach we employed previously in section 5.5.1 of [3] while dealing with structural BMPs. The following table gives a summary of the nonstructural BMPs with their corresponding mean cost effectiveness values arranged in an ascending order of effectiveness.

BMP	Mean Cost Effectiveness, (%)
Street Flushing	23.0
Street Sweeping	31.0
Storage Containers	43.0
Litter Control	45.0
Legal Requirements for Pesticide Application	49.0
Leaf Disposal	54.0
Disposal of Unused Pesticides	59.0
Establishment of New Vegetation	60.0
Public Education	62.7
Alternatives to Pesticides	63.5
Soil Testing	63.8
Proper Maintenance of Deicing Equipment	64.2
Reduction of Vehicle Miles Traveled	75.0
Preventive Care for Vehicles	83.0
Proper Timing of Fertilizer Application	84.0
Retention of Natural Vegetation	85.0
Proper Storage of Deicing Materials	86.0
Dikes	81.0
Temporary Seeding	82.0
Buffer Zone	91.0

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CHAPTER SIX

DATA ACQUISITION AND PROCESSING

6.1 INTRODUCTION

The mathematical and computer models that we have developed and implemented in the foregoing chapters require various resources for their implementation in the real world. There are at least two primary issues. The first deals with data requirements of the models and some suggested procedures for their generation. The second is process oriented and addresses computational realization of the models considering computational complexity and computer resource requirements. The purpose of this chapter is to provide some tour of these problems and concerns.

6.2. GENERATION OF ESSENTIAL DATA

As a vehicle for these inquiries, we provide a global flow chart of the two phases program, omitting without loss of generality, the linkage program. This chart is given in Fig. 6.1 and followed by some brief notes isolating the kernels of the program. Central to the first phase is the Core Program.

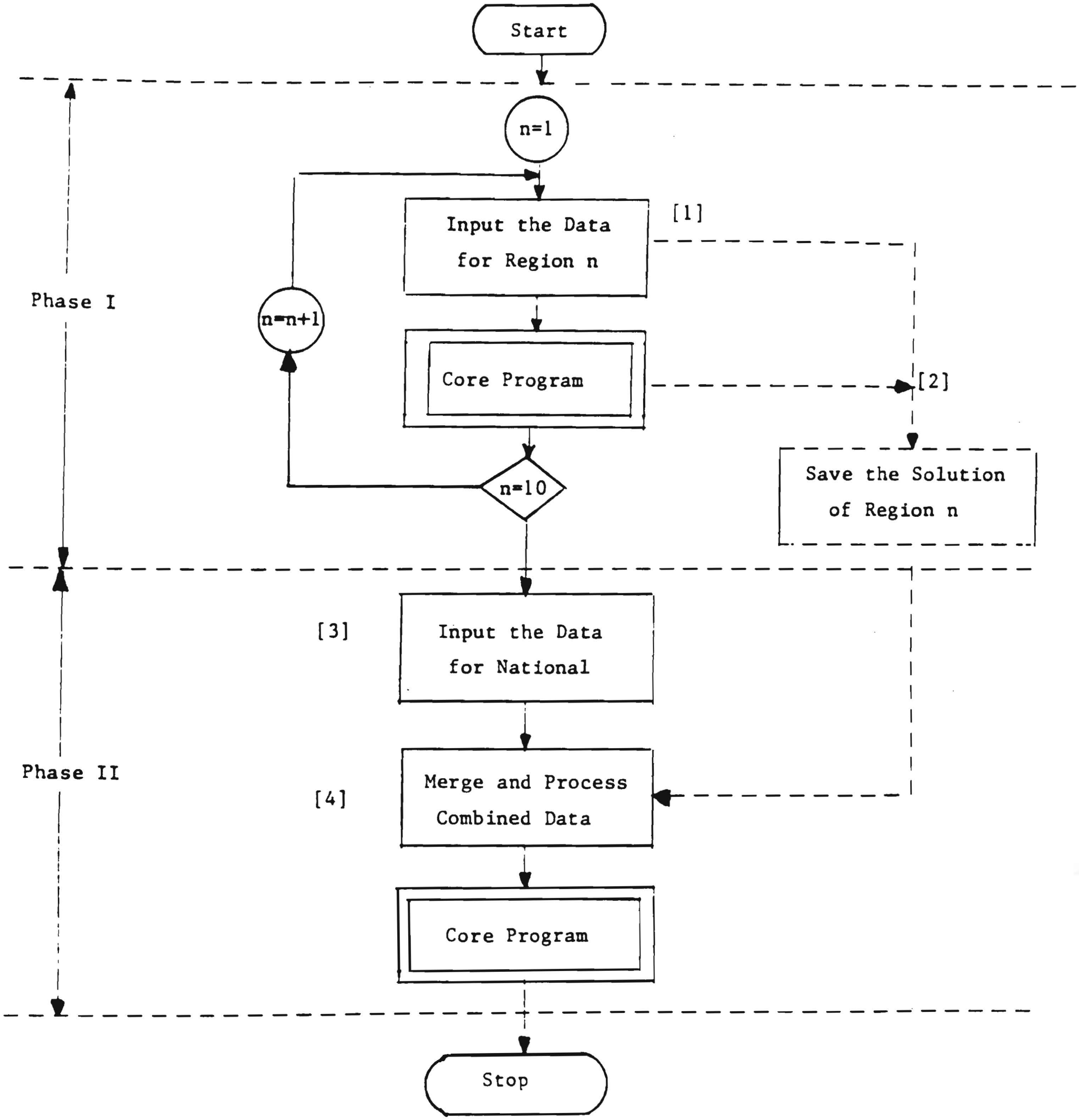
The essentials of this program are summarized in Fig 6.2.

The most significant data packets required to implement the models in each of the phases have been isolated and summarized for ease of reference in Fig. 6.3. Equally important, is the concern about the various methods for the acquisition of these data as well as some possible source for them. We provide this information in Fig. 6.4

In Chapter 5, we discussed the application of the models to the non point source water pollution control problem. The role of data was also discussed. In Fig.6.5, we present a design of a data collection questionnaire for use in obtaining the type of information which the model calls for.

It must be recalled that the models represented the various fuzzy variables such as state, goals, constraints, decisions and transitions in terms of their respective membership functions. The determination and measurement of these functions in the real world has always been somewhat of a thorny problem in the use of fuzzy sets in systems modelling. We have suggested an instructive algorithm for their generation in this project.

Fig. 6.1. The Two Phases Program



4. Notes for Flowchart*

[1] C^k, $k=1,...,K$;

G^k, $k=1,...,K$;

$T^k(d_j)$, $j=1,...,J$, $k=1,...,K$;

X^o.

[2] D^k, $k=1,...,K$; where D^k is the optimum control level among d_j for k-th measure.

[3] W_n, $n=1,...,10$;

G^n, $n=1,...,10$;

$\hat{K}$, where $\hat{k}$ is the number of best measures determined in Phase I.

[4] (1) $X^o = \sum_{n=1}^{10} W_n X^o(n)$

where $X^o(n)$ is the X^o of region n

(2) $T^n(d_n) = T^k[D]$, $k=1,...,\hat{k}$, $n=1,...,10$
where D^k is the optimum control level for the k-th measure in region n. $T^k[D^k]$ is the transform matrix for the k-th measure with control level D^k in region n.

(3) $G^n = W_n G^n$, $n=1,...,10$

---

* Also see ((Core Program)) for more details.

Fig. 6.2. Summary of Essentials of the Core Program

§1 Notations

⟨1⟩ State: S_i, i = 1,...,I; Decision: d_j, j = 1,...,J; Stage: k = 1,...,K.

⟨2⟩ Membership Function (k = 1,...,K)

state : $X^k = (x_1^k \dots x_i^k \dots x_I^k)^T$ i.e. $\mu_{X^k}(s) = x_1^k/s_1 + \dots + x_i^k/s_i + \dots + x_I^k/s_I$

constraint: $C^k = (c_1^k \dots c_j^k \dots c_J^k)^T$ i.e. $\mu_{C^k}(d) = c_1^k/d_1 + \dots + c_j^k/d_j + \dots + c_J^k/d_J$

goal : $G^k = (g_1^k \dots g_i^k \dots g_I^k)^T$ i.e. $\mu_{G^k}(s) = g_1^k/s_1 + \dots + g_i^k/s_i + \dots + g_I^k/_i$

⟨3⟩ Transform Matrix

$T^k(d_j)$, j = 1,...,J, k = 1,...,K

$Y^k(d_j) = T^k(d_j) \circ X^{k-1}$, where $\circ$ = max–min product operator

⟨4⟩ Return Function

$$r_j^k = 1 - \|Y^k(d_j), G^k\| = 1 - \frac{1}{I}\sum_{i=1}^{I} |[Y^k(d_j)]_i - g_i^k| \quad j = 1,\dots,J;\ k = 1,\dots,K$$

$$R^k = \max_j \{r_j^k\}, \quad k = 1,\dots,K$$

$R^* = \min_k \{R^K\}$, $\bar{R}, \underline{R}$ = Upper and Lower bounds of R respectively

§2. Data

⟨1⟩ Input

C^k, k = 1,...,K

G^k, k = 1,...,K

$T^k(d_j)$, j = 1,...,J, k = 1,...,K

X^0

⟨2⟩ Output

D^k, k = 1,...,K

R^k, k = 1,...,K

X^k, k = 1,...,K

R^*, $\bar{R} = \underline{R}$

Fig. 6.3. Data Requirements Summary

Phase	Notation	Meaning	
0	S_i, $i=1,\ldots,I$ d_j, $j=1,\ldots,J$	• Discretize {state, control} level	for each region
I	C^k, G^k, $k=1,\ldots,K$ $T^k(d_j)$, $j=1,\ldots,J$, $k=1,\ldots,K$ X^o	• Constraint, • Goal } for each measure k • Transform Matrix for each measure k at control level d_j • Initial State	for each region
II	G^n, $n=1,\ldots,10$ W_n, $n=1,\ldots,10$	• Goal for each region, n • Weight for each region, n	

Fig. 6.4. Possible Methods and Sources of Data Stipulated in the Model

Data	Method	Source
X^o	Predictions or forecast (e.g., time series analysis)	Hydrologic and hydraulic experts
W	AHP (Analytic Hierarchy Process)	NFIP specialists
C,G,T.	Delphi	FEMA regional officials

Fig. 6.5. Sample Data Collection Questionnaire for Evaluators

For National Level

⟨1⟩ Investment for Nationwide (TYPE B or TYPE A Measures)

- L : What is the least amount of budget fot the N.S.W.P. control plan in the nation, that we can have?
- U : What is the maximum budget that we can expect to get?
- M : Has the budget been determined right now? If so, how much is it?

For each region:

⟨2⟩ Initial States (TYPE C+D)

- M : If no N.S.W.P. control action is implemented in this region, what water quality level do you think, will be most likely?
- X : In estimating M above, what was your confidence level?

⟨3⟩ Goal of States (TYPE C+D)

- M : After an optimal combination of possible actions for N.S.W.P. control (structural and nonstructural) has been implemented, what is the highest water quality level that we can possibly expect to occur?
- X : In estimating M above, what was your confidence level?

For example, the label for ⟨2⟩ and ⟨3⟩ may be displayed as

```
            Water Quality                         Confidence Level

        0.                   1.              0%                     100%
        |---------M---------|               |-------------X-----|
   polluted                very          not sure              extremely
   thoroughly              good           at all                 sure
```

⟨4⟩ Investment for Region (TYPE B or TYPE A Measures)

- L : What budget for N.S.W.P. control plan in this region can you be sure to
- U : What is the maximum budget that you may expect to get?
- M : Has the budget been determined right now? If so, how much is it?

For each measure:

⟨5⟩ Investment for Structural Measure (TYPE A)

- M : For putting this measure into use most efficiently, how much investment do you prefer?
- L : What is the minimum investment level needed to implement this measure?
- U : What is the upper bound for the investment if the budget were not a constraint?

⟨6⟩ Investment for Non structural Measure (TYPE B)

- L : In your opinion, what is the minimum investment level that is necessary to use this measure most efficiently?
- U : What is the upper bound of this investment or, what is its economic scale?

⟨7⟩ Overall Effectiveness of Measure (TYPE C+D)

E: In using this measure, what level of effectiveness for the improvewment of water quality can we expect? In other words, what percent of the water pollution can be removed with this control measure?

X : In estimating E above, what was your confidence level?

For example, the label for ⟨7⟩ may be displayed as

```
         Effectiveness Level                         Confidence Level

     0.                         1.            0%                          100%
     |----------E---------|                   |---------------X------|
not effective           very              not sure                  extremely
  at all              effective            at all                     sure
```

For each measure with each investment level:

⟨8⟩ Cost Effectiveness of Measure (TYPE C)

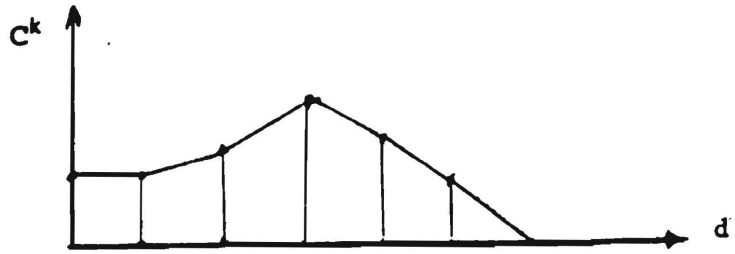
M : For this measure, with each possible investment level, try to determine the most possible effectiveness degrees, where label 1. means that with this investment level using the measure will be most effective and the label 0. means that with this investment level using the measure will be least effective.

For example, the form and labels for cost effectiveness may be displayed as

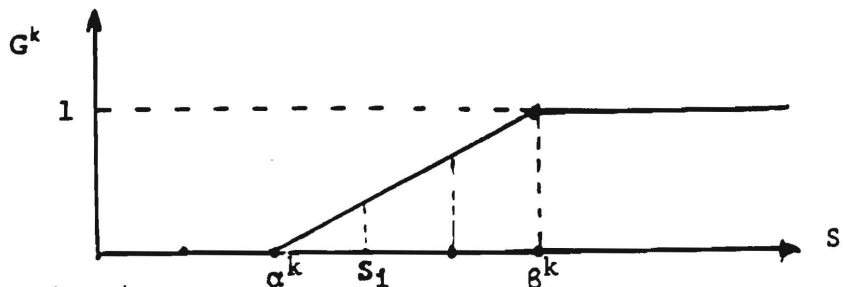
```
Effectiveness  .
               |
            1.+--+--+--+--+--+--+--+--+--+--M--M
              |  :  :  :  :  :  :  :  M  M  :  :
              |  :  :  :  :  :  M  M  :  :  :  :
              |  :  :  :  :  M  :  :  :  :  :  :
              |  :  :  :  M  :  :  :  :  :  :  :
              |  :  :  M  :  :  :  :  :  :  :  :
              |  :  M  :  :  :  :  :  :  :  :  :
              |  :  :  :  :  :  :  :  :  :  :  :
              |  M  :  :  :  :  :  :  :  :  :  :
              |  :  :  :  :  :  :  :  :  :  :  :
            0.M--+--+--+--+--+--+--+--+--+--+--+---> $ Investment
              0. 1Δ 2Δ ...                   ... JΔ

    ***   Cost-Effectiveness of Measure k in Region n   ***
```

(1) C^k: The degree of willingness to invest d_j on the k-th measure



(2) G^k: The degree of belief that the state level will be attained when using the k-th measure



where α^k, β^k are the lowest and highest state levels respectively that we can expect when using the k-th measure.

(3) $T^k(d_j)$: The relation between the state of the system before and after the k-th measure has been employed at the d_j investment level. i.e.

$$X^{k+1} = T^k(d_j) \circ X_k$$

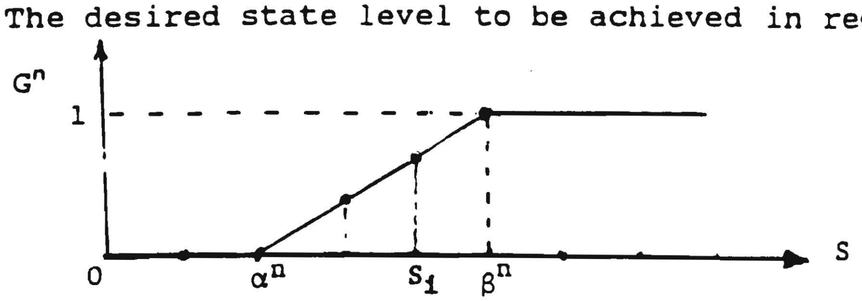
where X^{k+1}, X^k are the states after and before; o is the max-min product operator.

(4) X^o: The degree of belief that the state level will be at a certain value using any measure i.e. initial state level.

(5) W_n: The relative importance of the n-th region in the scheme of things.

$$\sum_{n=1}^{10} W_n = 1$$

(6) G^n: The desired state level to be achieved in region n



where α^n is the lowest acceptable state level and β^n is the

Essentially, the data acquired from the experts and specialists are converted to membership functions through the algorithm displayed in Fig. 6.7.

6.3. COMPUTER PROCESSING OF MODEL

We developed two versions of the fuzzy mathematical model for both the flood control problem and the non point source water pollution control problem. The models are hybrid dynamic programming and branch and bound procedures. The first version uses a decision tree search procedure while the second uses the classical algorithm. The flow charts for these procedures are displayed in Figs. 6.8 and 6.9 respectively.

6.4. MODEL VALIDATION

The models have been computerized, debugged and tested using synthetic data. Both the data and results are given in Table 6.1. The detailed computer programs are displayed in the Appendix.

Fig. 6.7. An Algorithm for the Construction of Membership Functions

Step 1: Label for each evaluator as follows and consider two types of evaluation (A & B)

TYPE A : L--+--+--L--+--M--+--+--U--+--+--> j
0. 1Δ 2Δ ... ... JΔ

TYPE B : --+--+--+--L--+--+--+--U--+--+--> j
0. 1Δ 2Δ ... ... JΔ

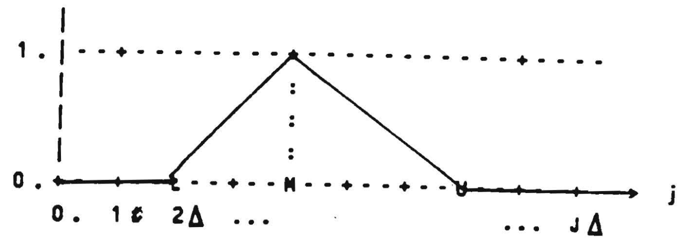
TYPE C : |----------M----------| Quality Scale
0. 1.

TYPE D : |--------------X------| Confidence Level
0% 100%

where
Δ: the interval between two value points
L: the label for the lower bound of the value
U: the label for the upper bound of the value
M: the label for the most possible/preferable value
X: the degree of confidence for value labeled

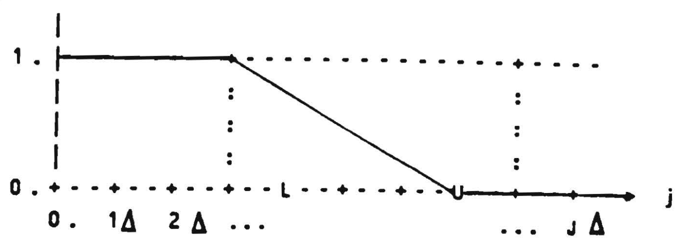
Step 2: Construct the membership function for each evaluator as follows:

Type A: u(j)



or
$$u(j) = \begin{cases} 0. & j < L \\ [j - (L - \Delta)]/[M - (L - \Delta)] & L \le j < M \\ 1. & j = M \\ [(U + \Delta) - j]/[(U + \Delta) - M] & M < j \le U \\ 0. & U < j \end{cases}$$

Type B: u(j)

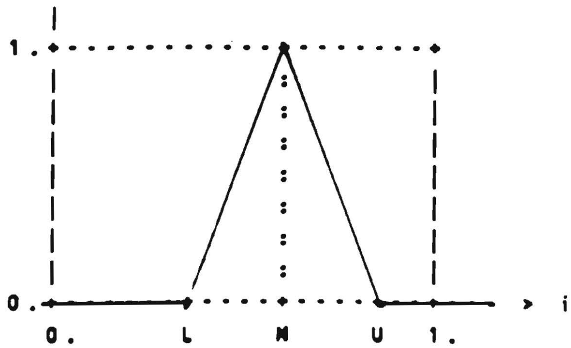


or
$$u(j) = \begin{cases} 1. & j < L \\ [(U + \Delta) - j]/[(U + \Delta) - (L - \Delta)] & L \le j \le U \\ 0. & U < j \end{cases}$$

Step 3: Data Processing: Amalgamation of Quality Scale and Confidence Level

Step 3.1: Combine the factors, quality scale and confidence level, i.e., C+D as follows:

Type C+D: u(i)



$$\text{or} \quad u(i) = \begin{cases} 0. & i < L \\ [i - L]/[M - L] & L \leq i < M \\ 1. & i = M \\ [U - i]/[U - M] & M < i \leq U \\ 0. & U < i \end{cases}$$

where $U = M + (1. - X)$ and $L = M - (1. - X)$

Step 3.2: **Combine the membership functions from all evaluators as follows:**

$$u(j) = [u_1(j) + \ldots + u_p(j) + \ldots + u_{\underline{P}}(j)] / \underline{P}$$

where $u_p(j)$: the membership function for p^{th} evaluator

$\underline{P}$: total number of evaluators

Fig. 6.8. Flow Chart for Version 1 Model [Decision Tree Algorithm]

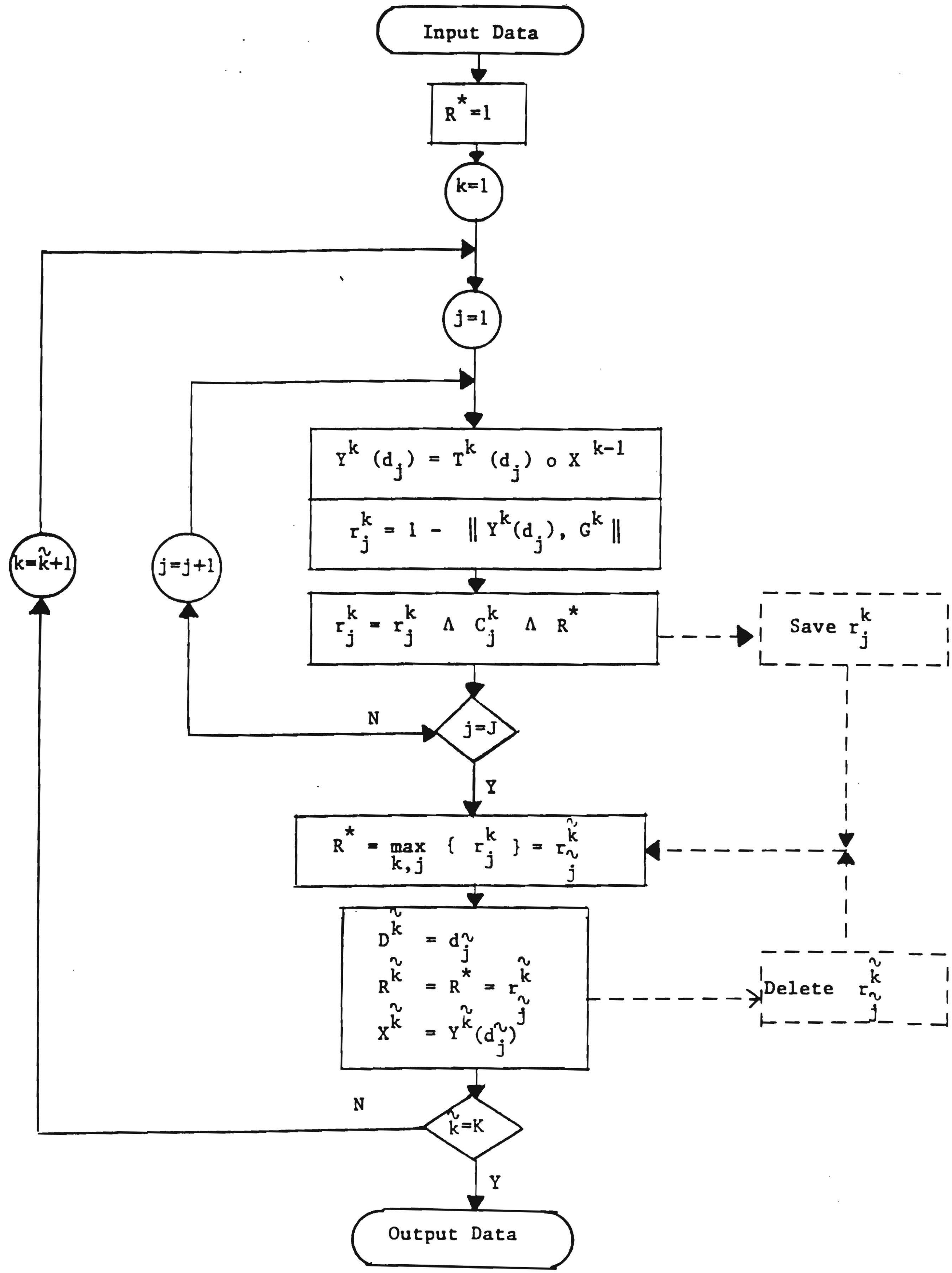


Fig. 6.9. Flow Chart for Version 2 Model [Classical Branch & Bound]

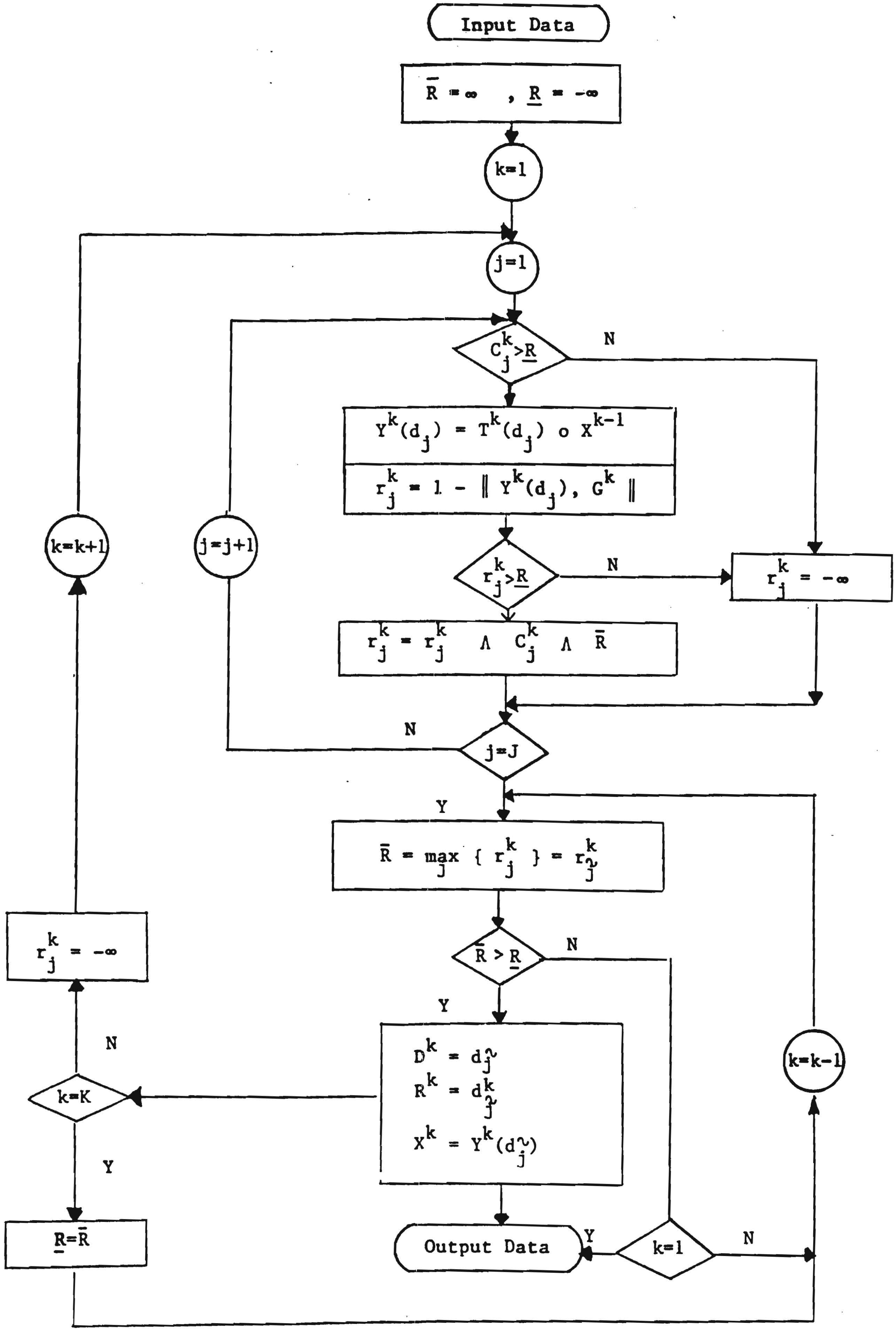


Table 6.1 Example for Model Validation

EXAMPLE

(1) Input Data

5,3,3,	I,J,K
0.1,0.2,0.3,0.7,1.0,	X^o
0.7,1.0,0.7,0.4,0.1,	G^1
0.3,0.7,1.0	C^1
1.0,0.8,0.7,0.5.0.2,	
0.1,0.5,0.9,0.7,0.3,	
0.9,0.7,0.5,0.7,0.9,	$T^1(d_i)$
0.1,0.3,0.5,0.3,0.7,	
0.2,0.5,0.7,0.4,0,3,	
0.3,0.5,0.8,0.9,0.7,	
0.9,0.7,0.5,0.7,0.9,	
1.0,0.5,0.3,0.7,0.7,	$T^1(d_2)$
0.4,0.2,0.5,0.9,1.0,	
0.6,0.3,0.2,0.5,0.7,	
0.5,0.7,0.8,0.9,1.0,	
0.7,0.8,0.1,0.2,0.5,	$T^1(d_3)$
0.7,0.1,0.2,0.3,0.4,	
1.0,0.5,0.3,0.5,0.7,	
0,7,0.9.1.0,0.8,0.4,	
0.2,0.5,0.7,0.8,1.0,	G^2
0.5,1.0,0.7,	C_2
1.0,0.8,0.7,0.5,0.2,	
0.1,0.5,0.9,0.7,0.3,	
0.9,0.7,0.5,0.7,0.9,	$T^2(d_1)$
0.1,0.3,0.5,0.3,0.7,	
0.2,0.5,0.7,0.4,0.3,	
0.3,0.5,0.8,0.9,0.7,	
0.9,0.7,0.5,0.7,0.9,	$T^2(d_2)$
1.0,0.5,0.3,0.7,0.7,	
0.4,0.2,0.5,0.9,1.0,	
0.6,0.3,0.2,0.5,0.7,	
0.5,0.7,0.8,0.9.1.0,	
0.7,0.8,0.1.0.2,0.5,	
0.7,0.1,0.2,0.3,0.4,	$T^2(d_3)$
1.0,0.5,0.3,0.5,0,7,	
0.7,0.9,1.0,0.8,0.4,	

Table 6.1 Cont'd.

EXAMPLE (Cont'd.)

```
0.4,0.7,1.0,0.7,0.4,                    G3
1.0,0.8,0.6,                            C3

1.0,0.8,0.7,0.5,0.2,
0.1,0.5,0.9,0.7,0.3,
0.9,0.7,0.5,0.7,0.9,                    T3(d1)
0.1.0.3,0.5,0.3.0.7,
0.2,0.5,0.7,0.4,0.3,

0.3,0.5,0.8,0.9,0.7,
0.9,0.7,0.5,0.7,0.9,                    T3(d2)
1.0,0.5,0.3,0.7,0.7,
0.4,0.2,0.5,0.9,1.0,
0.6,0.3,0.2,0.5,0.7,

0.5,0.7,0.8,0.9,1.0,
0.7,0.8,0.1,0.2,0.5,                    T3(d3)
0.7,0.1,0.2,0.3,0.4,
1.0,0.5,0.3,0.5,0.7,
0.7,0.9,1.0,0.8,0.4,
```

(2) Output Data

```
*******************************************************************

STAGE-RETURN-CONTROL---STATE : X(1), X(2),--------------,X(N)--

   0    1.000    0     .10  .20  .30  .70  1.00
   1     .700    2     .70  .90  .70 1.00   .70
   2     .700    2     .90  .70  .70  .90   .70
   3     .700    1     .90  .70  .90  .70   .70
*******************************************************************
```

APPENDICES

ppendix

4. PROGRAM {DECISION TREE METHOD]

```
************************************************************************
*
      PROGRAM FUZZY DP BY DECISION TREE
      PARAMETER(MAXI=5,MAXJ=5,MAXK=5,MAXN=5,MAXN=MAXJ**MAXK)
      COMMON//NI,NJ,NK,NODE
      COMMON/III/C(MAXK,MAXJ),G(MAXK,MAXI),T(MAXK,MAXJ,MAXI,MAXI)
      COMMON/000/C(MAXD/(0:MAXK)RETURN(0:MAXK),STATE(0:MAXK,MAXI)
      PARAMETER(NUIN=10,NUMID1=21,NUMID2=22,NUMID3=23=NUOUT=30)
C
             OPEN(NUIN,FILE='DATA',STATUS='OLD')
             OPEN(NUMID1,ACCESS='DIRECT',RECL=MAXN)
             OPEN(NUMID2,ACCESS='DIRECT',RECL=MAXN)
             OPEN(NUMID3,ACCESS='DIRECT',RECL=MAXN)
             OPEN(NUOUT,FILE='SOLUTION')
C
      CALL INPUT(NUIN)
      CALL TREE(NBEST,NUMID1,NUMID2,NUMID3)
      CALL PATH(NBEST)
      CALL OUTPUT(NUOUT)
      STOP
      END
*
*****************************************************************
```

```
*
*****************************************************************
*
      SUBROUTINE INPUT(NUIN)
      PARAMETER(MAXI=5,MAXJ=5,MAXK=5,MAXN=MAXJ**MAXK)
      COMMON//NI,NJ,NK,NODE
      COMMON/III/C(MAXK,MAXJ),G(MAXK,MAXI),T(MAXK,MAXJ,MAXI,MAXI)
      COMMON/000/MAXD(0:MAXK),RETURN(0:MAXK),STATE(0:MAXK,MAXI)
C
      READ(NUIN,*) NI,NJ,NK
      NODE=NK**NK
C
      MAXD(0)=0
      RETURN(0)=1.
      READ(NUIN,*) (STATE(0,I), I=1,NI)
C
      DO 1000 K=1,NK
           READ(NUIN,*)  (G(K,I),I=1,NI)
           READ(NUIN,*)  (C(K,J),J=1,NJ)
      DO 1000 J=1,NJ
        READ(NUIN,*)  ((T(K,J,I1,I2),I2=1,NI),I1=1,NI)
1000 CONTINUE
C
     END
```

```
*
**********************************************************************
*
      SUBROUTINE TREE(NBEST,NUMID1,NUMID2,NUMID3)
      PARAMETER(MAXI=5,MAXJ=5,MAXK=5,MAXN=MAXJ**MAXK)
      COMMON//NI,NJ,NK,NODE
      COMMON/III/C(MAXK,MAXJ),G(MAXK,MAXI),T(MAXK,MAXJ,MAXI,MAXI)
      COMMON/000/MAXD(0:MAXK),RETURN(0:MAXK),STATE(0:MAXK,MAXI)
      REAL Y(MAXI),X(MAXI)
      PARAMETER(SMALL=1,E-25,GREAT=1,E25)
C
      K=0
      DO 100 N=1,NODE
      WRITE(NUMID1,REC=N) K
 100  CONTINUE
C
      RBEST=GREAT
      NBEST=1
      K=1
      DO 200 I=1,NI
      X(I)=STATE(0,I)
 200  CONTINUE
C
2000  CONTINUE
C
      INTER=NODE/(NJ**K)
C
           DO 4000 J=1,NJ
C
                  DO 300 I=1,NI
                  Y(I)=GREAT
                  DO 300 L=1,NI
                  YYY=MIN(T(K,J,I,L),X(L))
                  Y(I)=MAX(Y(I),YYY)
 300              CONTINUE
C
                  R=0.
                  DO 400 I=1,NI
                  R=R+ABS(Y(I)-G(K,I))
 400              CONTINUE
                  R=1.-(R/NI)
                       R=MIN(R,C(K,J))
                       R=MIN(R,RBEST)
C
      N=NBEST+INTER*(J-1)
           WRITE(NUMID1,REC=N) K
           WRITE(NUMID2,REC=N) R
           WRITE(NUMID3,REC=N) (Y(I),I=1,NI)
C
4000       CONTINUE
C
```

```
      RBEST=GREAT
C
      DO 6000 N=1,NODE
           READ(NUMID1,REC=N) K
      IF(K.EQ.0) GOTO 6000
           READ(NUMID2,REC=N) R
      IF(R-RBEST).LE.SMALL)  GOTO 6000
           NBEST=N
           KBEST=K
           RBEST=R
6000  CONTINUE
C
      READ(NUMID3,REC=NBEST)  (X(I),I=1,NI)
      K=KBEST+1
      IF(K.LE.NK) GOTO 2000
C
      END

*
*****************************************************************
*
      SUBROUTINE PATH(NBEST)
      PARAMETER(MAXI=5,MAXJ=5,MAXK=5,MAXN=MAXJ**MAXK)
      COMMON//NI,NJ,NK,NODE
      COMMON/III/C(MAXK,MAXJ),G(MAXK,MAXI),T(MAXK,MAXJ,MAXI,MAXI)
      COMMON/000/MAXD(0:MAXK),RETURN(0:MAXK),STATE(0:MAXK,MAXI)
      PARAMETER(SMALL=1,E-25,GREAT=1,E25)
C
      N=NBEST
C
      DO 1000 K=1,NK
      INTER=NODE/(NK**K)
      J=(N-1)/INTER+1
      N=MOD(N-1,INTER)+1
C
           MAXD(D)=J
C
           DO 100 I=1,NI
                STATE(K,I)=GREAT
           DO 100 L=1,NI
                SSS=MIN(T(K,J,I,L),STATE(K-1,L))
                STATE(K,I)=MAX(STATE(K,I),SSS)
 100       CONTINUE
C
           R=0.
           DO 200 I=1,NI
           R=R+ABS(STATE(K,I)-G(K,I)
 200       CONTINUE
           R=1.-(R/NI)
           R=MIN(R,C(K,J))
           RETURN(K)=MIN(RETURN(K-1),R)
1000  CONTINUE
C
      END
```

```
        RBEST=GREAT
C
        DO 6000 N=1,NODE
                READ(NUMID1,REC=N) K
        IF(K.EQ.0) GOTO 6000
                READ(NUMID2,REC=N) R
        IF(R-RBEST).LE.SMALL)  GOTO 6000
                NBEST=N
                KBEST=K
                RBEST=R
6000    CONTINUE
C
        READ(NUMID3,REC=NBEST)  (X(I),I=1,NI)
        K=KBEST+1
        IF(K.LE.NK) GOTO 2000
C
        END

*
*****************************************************************
*
        SUBROUTINE PATH(NBEST)
        PARAMETER(MAXI=5,MAXJ=5,MAXK=5,MAXN=MAXJ**MAXK)
        COMMON//NI,NJ,NK,NODE
        COMMON/III/C(MAXK,MAXJ),G(MAXK,MAXI),T(MAXK,MAXJ,MAXI,MAXI)
        COMMON/000/MAXD(0:MAXK),RETURN(0:MAXK),STATE(0:MAXK,MAXI)
        PARAMETER(SMALL=1,E-25,GREAT=1,E25)
C
        N=NBEST
C
        DO 1000 K=1,NK
        INTER=NODE/(NK**K)
        J=(N-1)/INTER+1
        N=MOD(N-1,INTER)+1
C
                MAXD(D)=J
C
                DO 100 I=1,NI
                         STATE(K,I)=GREAT
                DO 100 L=1,NI
                         SSS=MIN(T(K,J,I,L),STATE(K-1,L))
                         STATE(K,I)=MAX(STATE(K,I),SSS)
 100            CONTINUE
C
                R=0.
                DO 200 I=1,NI
                R=R+ABS(STATE(K,I)-G(K,I)
 200            CONTINUE
                R=1.-(R/NI)
                R=MIN(R,C(K,J))
                RETURN(K)=MIN(RETURN(K-1),R)
1000    CONTINUE
C
        END
```

```
*
*****************************************************************
*
      SUBROUTINE OUTPUT(NUOUT)
      PARAMETER(MAXI=5,MAXJ=5,MAXK=5,MAXN-MAXJ**MAXK)
      COMMON//NI,NJ,NK,NODE
      COMMON/000/MAXD(0:MAXK),RETURN(0:MAXK),STATE(0:MAXK,MAXI)
C
                                WRITE(NUOUT,'(/1X,70(1H*)/)')
      WRITE(NUOUT,200)
      DO 1000 K=0,NK
      WRITE(NUOUT,201) K,RETURN(K),MAXD(K),(STATE(K,I),I=1,NI)
1000  CONTINUE
                                WRITE(NUOUT,'(/1X,70(1H*)/)')
C
 200  FORMAT(1X,'STAGE-RETURN-CONTROL---STATE :',
   +        ' X(1),X(2),---------------------,X(NJ)--'/)
 201  FORMAT(1X,I3,F8.3,I6,3X,(10F5.2))
      END
*
*******************************************************************
```

4. PROGRAM [CLASSICAL "BRANCH AND BOUND" METHOD]

```
*****************************************************************
*
      PROGRAM FUZZY DP BY BRANCH AND BOUND
      PARAMETER(MAXI-10,MAXJ=10,MAXK=20)
      COMMON//NI,NJ,NK
      COMMON/III/C(MAXK,MAXJ),G(MAXK,MAXI),T(MAXK,MAXJ,MAXI,MAXI)
      COMMON/000/MAXD(0:MAXK),RETURN(0:MAXK),STATE(0:MAXK,MAXI)
      COMMON/KEEP/M(MAXK,R(MAXK,MAXJ),S(MAXK,MAXJ,MAXI)
      COMMON/WORK,X(MAXI),Y(MAXI),RUPPER,RLOWER,LABEL(MAXK,MAXJ)
      PARAMETER(SMALL=1,E-25,GREAT=1.E25)
      PARAMETER(NUIN-10,NUOUT=30)
C
           OPEN(NUIN,FILE='DATA',STATUS='OLD')
           OPEN(NUOUT,FILE='SOLUTION')
C
      CALL INPUT(NUIN)
C
      RUPPER=GREAT
      RLOWER=GREAT
           DO 100 I=1,NI
           X(I)=STATE(0,I)
 100       CONTINUE
      KK=0
C
1000  CONTINUE
      CALL BOUND(KK,*2000,*3000)
2000  CONTINUE
      RLOWER=RUPPER
           DO 200 K=1,NK
           MAXD(K)
           RETURN(K)=R(K,M(K))
           DO 200 I=1,NI
           STATE(K,I)=S(K,M(K),I)
 200       CONTINUE
3000  CONTINUE
      CALL BRANCH(KK,*1000,*4000)
4000  CONTINUE
C
      CALL OUTPUT(NUOUT)
C
      STOP
      END
*
*********************************************************************
```

```
*
**********************************************************************
*
        SUBROUTINE INPUT(NUIN)
        PARAMETER(MAXI=10,MAXJ=10,MAXK=20)
        COMMON/NI,NJ,NK
        COMMON/III/C(MAXK,MAXJ),G(MAXK,MAXI),T(MAXK,MAXJ,MAXI,MAXI)
        COMMON/000/MAXD(0:MAXK),RETURN(0:MAXK),STATE(0:MAXK,MAXI)
C
        READ(NUIN,*) NK,NJ,NK
C
        MAXD(0)=0
        RETURN(0)=1.
        READ(NUIN,*) (STATE(0,I), I=1,NI)
C
        DO 1000 K=1,NK
                READ(NUIN,*) (G(K,I),I=1,NI)
                READ(NUIN,*) (C(K,J),J=1,NJ)
        DO 1000 J=1,NJ
                READ(NUIN,*) ((T(K,J,I1,I2),I2=1,NI),I1=1,NI)
1000    CONTINUE
C
        END
```

```
*
**********************************************************************
*
        SUBROUTINE BOUND(K,*,*)
        PARAMETER(MAXI=10,MAXJ=10,MAXK=20)
        COMMON//NI,NJ,NK
        COMMON/III/C(MAXK,MAXJ),G(MAXK,MAXI),T(MAXK,MAXJ,MAXI,MAXI)
        COMMON/KEEP/M(MAXK),R(MAXK,MAXJ),S(MAXK,MAXJ,MAXI)
        COMMON/WORK/X(MAXI),Y(MAXI),RUPPER, RLOWER,LABEL(MAXK,MAXJ)
        PARAMETER(SMALL=1,E-25,GREAT=1.E25)
C
1000    CONTINUE
C
        K=K+1
        M(K)=0
        RNEW=RLOWER
C
                DO 4000 J=1,NJ
                LABEL(K,J)=0
C
                IF((C(K,J)-RLOWER).LE.SMALL) GOTO 4000
C
                        DO 200 I=1,NI
                        Y(I)=-GREAT
                        DO 200 L=1,NI
                        YYY=MIN(T(K,J,I,L)X(L))
                        Y(I)=MAX(Y(I),YYY)
 200                    CONTINUE
C
                        RRR=0.
                        DO 300 I=1,NI
                        RRR=RRR+ABS(Y(I)-G(K,I))
 300                    CONTINUE
                        R(K,J)=2,-(RRR/NI)
C
                IF((R(K,J)-RLOWER).LE.SMALL) GOTO 4000
C
                LABEL(K,J)=1
                        R(K,J)=MIN(R(K,J),C(K,J))
                        R(K,J)=MIN(R(K,J),RUPPER)
                        DO 400 I=1,NI
                        S(K,J,I)=Y(I)
 400                    CONTINUE
C
                IF((R(K,J)-RNEW).LE.SMALL) GOTO 4000
                        RNEW=R(K,J)
                        M(K)=J
4000    CONTINUE
C
        RUPPER=RNEW
C
        IF(M(K).EQ.0) RETURN 2
        IF(K.EQ.NK) RETURN 1
C
```

```
      DO 500 I-1,NI
      X(I)=S(K,M(K),I)
 500  CONTINUE
      GOTO 1000
      END
*
******************************************************************
*
      SUBROUTINE BRANCH(K,*,*)
      PARAMETER(MAXI=10,MAXJ=10,MAXK=20)
      COMMON//NI,NJ,NK
      COMMON/KEEP/M(MAXK),R(MAXK,MAXJ),S(MAXK,MAXJ,MAXI)
      COMMON/WORK/X(MAXI),Y(MAXI),RUPPER,RLOWER,LABEL(MAXK,MAXJ)
      PARAMETER(SMALL=1,E-25,GREAT=2,E25)
C
1000  CONTINUE
C
      K=K-1
      IF(K,EQ.O) RETURN 2
      LABEL(K,M(K))=0
      M(K)=0
C
            DO 700 J=1,NJ
            IF(LABEL(K,J),EQ.O) GOTO 700
            IF((R(K,J)-RUPPER).LE.SMALL) GOTO 700
            RUPPER=R(K,J)
            M(K)=J
 700        CONTINUE
C
      IF(M(K).EQ.0) GOTO 1000
C
      DO 800 I=1,NI
      X(I)=S(K,M),I)
 800  CONTINUE
      RETURN 1
C
      END
```

```
*
***********************************************************************
*
      SUBROUTINE OUTPUT(NUOUT)
      PARAMETER(MAXI=10,MAXJ=10,MAXK=20)
      COMMON//NI,NJ,NK
      COMMON/000/MAXD(0:MAXK),RETURN(0:MAXK),STATE(0:MAXK,MAXI)
C
                                 WRITE(NUOUT,'(/1X,70(1H*)/)')
      WRITE(NUOUT,200)
      DO 1000 K=0,NK
      WRITE(NUOUT,201) K,RETURN(K),MAXD(K),(STATE(K,I),I=1,NI)
1000  CONTINUE
                                 WRITE(NUOUT,'(/1X,70(1H*)/)')
C
 200  FORMAT(1X,'STAGE-RETURN-CONTROL---STATE :',
   +         ' X(1),X(2),--------------------------,X(N)--'/)
 201  FORMAT(1X,I3,F8.3,I6,3X,(10F5.2))
*     END
***********************************************************************
```

```
**********************************************************************
*                                                                    *
*                      PROGRAM IN FORTRAN 77                         *
*                                                                    *
*                      FLOOD CONTROL PHASE 1                         *
*                                                                    *
*              MULTI-STAGE DECISION-MAKING PROCESSES                 *
*                               FOR                                  *
*                FUZZYSYSTEM IN FUZZY ENVIRONMENT                    *
*                              WITH                                  *
*        CONSTRAINT FOR EACH STAGE ( OVER DECISION SPACE )           *
*            GOAL FOR FINAL STAGE ( OVER STATE SPACE )               *
*            ADDITION OPERATOR FOR STATE TRANSFORMATION              *
*                               AND                                  *
*              MAX-MIN TYPE GLOBAL OBJECTIVE FUNCTION                *
*                    LIMIT TO SUM OF DECISIONS                       *
*                                BY                                  *
*                 CLASSICAL BRANCH & BOUND METHOD                    *
*                       MAY STOP AT ANY TIME                         *
*                                                                    *
*                          JULY 27, 1991                             *
*                                                                    *
**********************************************************************
```

```
**********************************************************************
*
*                     THE NOTATION FOR ARRAY
*
* NI : NO. OF STATE
* NJ : NO. OF DECISION
* NK : NO. OF STAGE
* LIMIT : LIMIT TO SUM OF DECISION
*
* G : MEMBERSHIP FUNCTION OF GOAL ( FINAL STATE )
* C : MEMBERSHIP FUNCTION OF CONSTRAINT ( STAGE, DECISION )
* T : MEMBERSHIP FUNCTION OF TRANSFORM ( STAGE, DECISION, STATE )
*
* L : LABEL FOR OPTIMAL DECISION ( STAGE )
* S : MEMBERSHIP FUNCTION OF STATE BY OPTIMAL DECISION ( STAGE, STATE )
* D : DISTANCE BETWEEN GOAL AND STATE ( STAGE )
*
* U : UPPER BOUND OF BRANCH ( STAGE, DECISION )
* MODE : MODE OF BRANCH ( STAGE, DECISION )
*          1 : THE BRANCH IS ACTIVE
*          0 : THE BRANCH IS IDLE
*         -1 : THE BRANCH IS DEAD
*
* REAL : PRESENT BEST SOLUTION
* COME : COMING SOLUTION
* HOPE : UPPER BOUND FOR SOLUTION
*
**********************************************************************
```

```
************************************************************************
*
*                     THE FUNCTION OF SUBROUTINES
*
* INPUT   : READ DATA FROM FILE
* OUTPUT  : WRITE DATA TO FILE
* BEGIN   : INITIALIZE VARIABLES
* BOUND   : SEARCH FORWARD TO FIND NEW LOWER BOUND ( FEASIBLE SOLUTION )
* FINAL   : DETERMINE NEW SOLUTION AND COMPARE WITH PRESENT BEST ONE
*             IF BETTER SOLUTION FOUND, GOTO 'OUTPUT'
*             IF NOT, GOTO 'BRANCH'
* RENEW   : DELETE SOME BRANCHES BY NEW SOLUTION
* BRANCH  : SEARCH BACKWARD TO FIND NEW BRANCH
*             IF NEW BRANCH FOUND, GOTO 'BOUND'
*             IF NOT, STOP
*
************************************************************************
*                                                                      *
        PROGRAM FLOOD CONTROL PHASE 1
        PARAMETER(MAXI=40,MAXJ=25,MAXK=20)
        COMMON//NI,NJ,NK,LIMIT
        COMMON/PARA/G(MAXI),C(MAXK,0:MAXJ),T(MAXK,0:MAXJ,1-MAXI:MAXI-1)
        COMMON/VERI/L(MAXK),S(0:MAXK,MAXI),D(0:MAXK)
        COMMON/WORK/U(MAXK,0:MAXJ),MODE(MAXK,0:MAXJ),REAL,COME,HOPE
        PARAMETER(NUIN=10,NUOUT=30)
C
        CALL INPUT(NUIN)
        CALL BEGIN(K)
C
1000    CONTINUE
        CALL BOUND(K)
        CALL FINAL(*2000)
        CALL OUTPUT(NUOUT)
        CALL RENEW(*3000)
C
2000    CONTINUE
        CALL BRANCH(K,*1000)
C
3000    CONTINUE
        STOP
        END
*                                                                      *
************************************************************************
```

```
*************************************************************************
*
*  INPUT NO. OF STATE, DECISION AND STAGE
*  INPUT MEMBERSHIP FUNCTION OF GOAL AND INITIAL STATE
*  INPUT MEMBERSHIP FUNCTION OF CONSTAINT FOR EACH STAGE
*  INPUT MEMBERSHIP FUNCTION OF TRANSFORM FOR EACH STAGE AND DECISION
*
*************************************************************************
*                                                                       *
      SUBROUTINE INPUT(NUIN)
      PARAMETER(MAXI=40,MAXJ=25,MAXK=20)
      COMMON//NI,NJ,NK,LIMIT
      COMMON/PARA/G(MAXI),C(MAXK,0:MAXJ),T(MAXK,0:MAXJ,1-MAXI:MAXI-1)
      COMMON/VERI/L(MAXK),S(0:MAXK,MAXI),D(0:MAXK)
C
      OPEN(NUIN,FILE='PHASE1.DAT',STATUS='OLD')
C
      READ(NUIN,*) NI,NJ,NK
C
      READ(NUIN,*) (S(0,I), I=1,NI)
      READ(NUIN,*) (G(I),I=1,NI)
C
      DO 1000 K=1,NK
              READ(NUIN,*) (C(K,J),J=0,NJ)
      DO 1000 J=0,NJ
              READ(NUIN,*) (T(K,J,I),I=1-NI,NI-1)
1000  CONTINUE
C
      CLOSE(NUIN)
C
      END
*                                                                       *
*************************************************************************
```

```
**************************************************************************
*
* (1) SET 0 FOR PRESENT BEST SOLUTION
*     SET 1 FOR COMING SOLUTION
* (2) CALCULATE UPPER BOUND FOR SOLUTION
*     CALCULATE DISTANCE BETWEEN GOAL AND INITIAL STATE
* (3) SET ALL BRANCHES IDLE
*     SET LIMIT TO SUM OF DECISION
*
**************************************************************************
*                                                                        *
        SUBROUTINE BEGIN(K)
        PARAMETER(MAXI=40,MAXJ=25,MAXK=20)
        COMMON//NI,NJ,NK,LIMIT
        COMMON/PARA/G(MAXI),C(MAXK,0:MAXJ),T(MAXK,0:MAXJ,1-MAXI:MAXI-1)
        COMMON/VERI/L(MAXK),S(0:MAXK,MAXI),D(0:MAXK)
        COMMON/WORK/U(MAXK,0:MAXJ),MODE(MAXK,0:MAXJ),REAL,COME,HOPE
        PARAMETER(EPSILON=1.E-5,INFINITE=10**10)
C
        REAL=0.
        COME=1.
C
        HOPE=INFINITE
        DO 200 K=1,NK
        CMAX=-INFINITE
        DO 100 J=0,NJ
        CMAX=MAX(CMAX,C(K,J))
 100    CONTINUE
        HOPE=MIN(HOPE,CMAX)
 200    CONTINUE
C
        DISTAN=0.
        DO 400 I=1,NI
        DISTAN=DISTAN+ABS(S(0,I)-G(I))
 400    CONTINUE
        D(0)=1.-(DISTAN/NI)
C
        DO 500 K=1,NK
        DO 500 J=0,NJ
        MODE(K,J)=0
 500    CONTINUE
C
        K=0
        LIMIT=NI-1
C
        END
*                                                                        *
**************************************************************************
```

```
*************************************************************************
*
* (1) BYPASS DEAD BRANCH
*     SET OTHER BRANCH ACTIVE AND CALCULATE UPPER BOUND
* (2) FIND BRANCH HAVING MAX UPPER BOUND AS OPTIMAL DECISION
* (3) CALCULATE STATE UNDER OPTIMAL DECISION
*     CALCULATE DISTANCE BETWEEN GOAL AND STATE
* (4) REPEAT STEP (1)--(3) FOR NEXT STAGE
*     UNTIL LAST STAGE
*
*************************************************************************
*                                                                       *
      SUBROUTINE BOUND(K)
      PARAMETER(MAXI=40,MAXJ=25,MAXK=20)
      COMMON//NI,NJ,NK,LIMIT
      COMMON/PARA/G(MAXI),C(MAXK,0:MAXJ),T(MAXK,0:MAXJ,1-MAXI:MAXI-1)
      COMMON/VERI/L(MAXK),S(0:MAXK,MAXI),D(0:MAXK)
      COMMON/WORK/U(MAXK,0:MAXJ),MODE(MAXK,0:MAXJ),REAL,COME,HOPE
      PARAMETER(EPSILON=1.E-5,INFINITE=10**10)
C
1000  CONTINUE
C
      K=K+1
      IF(K.EQ.NK) RETURN
C
      UMAX=-INFINITE
C
              DO 4000 J=0,MIN(NJ,LIMIT)
C
              IF(MODE(K,J).EQ.-1) GOTO 4000
C
                      U(K,J)=MIN(C(K,J),COME)
                      MODE(K,J)=1
C
              IF((U(K,J)-UMAX).LE.EPSILON) GOTO 4000
C
                      UMAX=U(K,J)
                      L(K)=J
C
4000          CONTINUE
C
      COME=UMAX
      LIMIT=LIMIT-L(K)
C
      DO 300 I=1,NI
      S(K,I)=0.
      DO 300 II=1,NI
      S(K,I)=MAX(S(K,I),MIN(T(K,L(K),II-I),S(K-1,II)))
 300  CONTINUE
C
      DISTAN=0.
      DO 400 I=1,NI
      DISTAN=DISTAN+ABS(S(K,I)-G(I))
 400  CONTINUE
      D(K)=1.-(DISTAN/NI)
```

```
C
      GOTO 1000
C
      END
*                                                                        *
**************************************************************************
```

```
**************************************************************************
*
* (1) FOR EACH DECISION
*         CALCULATE FINAL STATE
*         CALCULATE DISTANCE BETWEEN GOAL AND FINAL STATE
*         CALCULATE OBJECTIVE FUNCTION
* (2) FIND OPTIMAL DECISION
* (3) COMPARE SOLUTION UNDER OPTIMAL DECISION WITH PRESENT BEST ONE
*         IF BETTER SOLUTION FOUND, RETURN
*         IF NOT, EXIT
*
```

```
*****************************************************************************
*                                                                           *
      SUBROUTINE FINAL(*)
      PARAMETER(MAXI=40,MAXJ=25,MAXK=20)
      COMMON//NI,NJ,NK,LIMIT
      COMMON/PARA/G(MAXI),C(MAXK,0:MAXJ),T(MAXK,0:MAXJ,1-MAXI:MAXI-1)
      COMMON/VERI/L(MAXK),S(0:MAXK,MAXI),D(0:MAXK)
      COMMON/WORK/U(MAXK,0:MAXJ),MODE(MAXK,0:MAXJ),REAL,COME,HOPE
      DIMENSION X(MAXI)
      PARAMETER(EPSILON=1.E-5,INFINITE=10**10)
C
      OMAX=-INFINITE
C
              DO 1000 J=0,MIN(NJ,LIMIT)
C
              IF(MODE(NK,J).EQ.-1) GOTO 1000
C
                      DO 100 I=1,NI
                      X(I)=0.
                      DO 100 II=1,NI
                      X(I)=MAX(X(I),MIN(T(NK,J,II-I),S(NK-1,II)))
 100                  CONTINUE
C
                      DISTAN=0.
                      DO 200 I=1,NI
                      DISTAN=DISTAN+ABS(X(I)-G(I))
 200                  CONTINUE
                      DISTAN=1.-(DISTAN/NI)
C
              IF((DISTAN-REAL).LE.EPSILON) GOTO 1000
C
                      OBJECT=MIN(DISTAN,C(NK,J))
C
              IF((OBJECT-OMAX).LE.EPSILON) GOTO 1000
C
                      OMAX=OBJECT
                      L(NK)=J
                      DO 300 I=1,NI
                      S(NK,I)=X(I)
 300                  CONTINUE
                      D(NK)=DISTAN
C
1000          CONTINUE
C
      OMAX=MIN(OMAX,COME)
C
      IF((OMAX-REAL).LE.EPSILON) RETURN 1
C
      REAL=OMAX
      COME=MIN(COME,C(NK,L(NK)))
C
      END
*                                                                           *
*****************************************************************************
```

```
*************************************************************************
*
* FOR EACH STAGE, PRINT
*    GRADE OF MEMBERSHIP FUNCTION OF CONSTRAINT UNDER OPTIMAL DECISION
*    DISTANCE BETWEEN GOAL AND STATE UNDER OPTIMAL DECISION
*    LABEL FOR OPTIMAL DECISION
*    GRADE OF MEMBERSHIP FUNCTION OF STATE UNDER OPTIMAL DECISION
*
*************************************************************************
*                                                                       *
      SUBROUTINE OUTPUT(NUOUT)
      PARAMETER(MAXI=40,MAXJ=25,MAXK=20)
      COMMON//NI,NJ,NK,LIMIT
      COMMON/PARA/G(MAXI),C(MAXK,0:MAXJ),T(MAXK,0:MAXJ,1-MAXI:MAXI-1)
      COMMON/VERI/L(MAXK),S(0:MAXK,MAXI),D(0:MAXK)
      COMMON/WORK/U(MAXK,0:MAXJ),MODE(MAXK,0:MAXJ),REAL,COME,HOPE
C
      OPEN(NUOUT,FILE='PHASE1.SOL')
                                WRITE(NUOUT,'(/1X,70(1H*)/)')
      K=0
      WRITE(NUOUT,201) K,D(K),(S(K,I),I=1,NI)
C
      DO 1000 K=1,NK
      IF(L(K).EQ.0) GOTO 1000
      WRITE(NUOUT,202) K,C(K,L(K)),D(K),L(K),(S(K,I),I=1,NI)
1000  CONTINUE
C
      WRITE(NUOUT,203) REAL,COME,D(NK)
                                WRITE(NUOUT,'(/1X,70(1H*)/)')
      CLOSE(NUOUT)
C
 201  FORMAT(1X,'STAGE CONSTRAINT GOAL  DECISION  STATE :',
    +        ' X(1),X(2),--------------,X(I)'/
    +        /1X,I3,10X,F8.4,9X,5F8.3:/1X,30X,5F8.3)
 202  FORMAT(1X,I3,2X,2F8.4,I7,2X,5F8.3:/1X,30X,5F8.3)
 203  FORMAT(/1X,'¡OPTIMAL SOLUTION¿',F8.4,' =  ¡CONSTRAINT¿',F8.4,
    + '  /Ñ ¡GOAL¿',F8.4)
      END
*                                                                       *
*************************************************************************
```

```
************************************************************************
*
* (1) COMPARE NEW SOLUTION WITH UPPER BOUND
*     IF NEW SOLUTION MEETS UPPER BOUND, EXIT
*     IF NOT, CONTINUE
* (2) RESET MODE FOR EACH BRANCH
*     IF UPPER BOUND LOWER THAN NEW SOLUTION, SET BRANCH IDLE
*     IF CONSTRAINT LOWER THAN NEW SOLUTION, SET BRANCH DEAD
*
************************************************************************
*                                                                      *
      SUBROUTINE RENEW(*)
      PARAMETER(MAXI=40,MAXJ=25,MAXK=20)
      COMMON//NI,NJ,NK,LIMIT
      COMMON/PARA/G(MAXI),C(MAXK,0:MAXJ),T(MAXK,0:MAXJ,1-MAXI:MAXI-1)
      COMMON/WORK/U(MAXK,0:MAXJ),MODE(MAXK,0:MAXJ),REAL,COME,HOPE
      PARAMETER(EPSILON=1.E-5,INFINITE=10**10)
C
      IF((HOPE-REAL).LE.EPSILON) RETURN 1
C
      DO 100 K=1,NK-1
      DO 100 J=0,NJ
      IF(MODE(K,J).EQ.-1) GOTO 100
      IF((U(K,J)-REAL).LE.EPSILON) MODE(K,J)=0
      IF((C(K,J)-REAL).LE.EPSILON) MODE(K,J)=-1
 100  CONTINUE
C                                       WRITE(*,'(/1X,70(1H*)/)')
      WRITE(*,201) HOPE,REAL,100.*(HOPE-REAL)/HOPE
C     WRITE(*,202)
C     DO 1000 J=0,NJ
C     WRITE(*,203) (MODE(K,J),K=1,NK-1)
1000  CONTINUE
C
 201  FORMAT(1X,'UPPER BOUND = ',F7.5,5X,'BEST RESULT = ',F7.5,5X,
     +          'REMINDER : ',F5.2,' %')
 202  FORMAT(1X,'MODE OF BRANCH :')
 203  FORMAT(1X,20I2)
      END
*                                                                      *
************************************************************************


************************************************************************
*
* (1) SET PRESENT BRANCH IDLE
* (2) FOR PRESENT STAGE
*     IF ACTIVE BRANCH FOUND,
*            CHOOSE ONE HAVING MAX UPPER BOUND AS OPTIMAL DECISION
*            CALCULATE NEW STATE
*            CALCULATE DISTANCE BETWEEN GOAL AND NEW STATE
*            EXIT
*     IF NO ACTIVE BRANCH, GO BACK ONE STAGE
* (3) REPEAT (1)--(2) UNTIL FIRST STAGE
*
```

```
****************************************************************************
*                                                                          *
        SUBROUTINE BRANCH(K,*)
        PARAMETER(MAXI=40,MAXJ=25,MAXK=20)
        COMMON//NI,NJ,NK,LIMIT
        COMMON/PARA/G(MAXI),C(MAXK,0:MAXJ),T(MAXK,0:MAXJ,1-MAXI:MAXI-1)
        COMMON/VERI/L(MAXK),S(0:MAXK,MAXI),D(0:MAXK)
        COMMON/WORK/U(MAXK,0:MAXJ),MODE(MAXK,0:MAXJ),REAL,COME,HOPE
        PARAMETER(EPSILON=1.E-5,INFINITE=10**10)
C
        COME=-INFINITE
C
1000    CONTINUE
C
        K=K-1
        LIMIT=LIMIT+L(K)
        MODE(K,L(K))=0
        L(K)=-INFINITE
C
                DO 100 J=0,NJ
                IF(MODE(K,J).NE.1) GOTO 100
                IF((U(K,J)-COME).LE.EPSILON) GOTO 100
                        COME=U(K,J)
                        L(K)=J
 100            CONTINUE
C
        IF(L(K).GE.0) THEN
C
                LIMIT=LIMIT-L(K)
C
                DO 300 I=1,NI
                S(K,I)=0.
                DO 300 II=1,NI
                S(K,I)=MAX(S(K,I),MIN(T(K,L(K),II-I),S(K-1,II)))
 300            CONTINUE
C
                DISTAN=0.
                DO 400 I=1,NI
                DISTAN=DISTAN+ABS(S(K,I)-G(I))
 400            CONTINUE
                D(K)=1.-(DISTAN/NI)
C
                RETURN 1
C
        END IF
C
        IF(K.GT.1) GOTO 1000
C
        END
*                                                                          *
****************************************************************************
```

FILE PHASE1.DAT

```
5,3,4,

0.13,0.45,0.79,1.00,0.88,
1.00,0.75,0.50,0.25,0.00,


1.00,0.92,0.64,0.37,

0.0,0.0,0.0,0.0,1.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.1,0.6,0.9,0.7,0.5,0.1,
0.0,0.0,0.0,0.1,0.5,0.8,0.6,0.4,0.1,
0.0,0.0,0.1,0.2,0.4,0.7,0.5,0.3,0.2,

1.00,0.62,0.83,0.44,

0.0,0.0,0.0,0.0,1.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.1,0.6,0.7,0.3,0.1,0.0,
0.0,0.0,0.0,0.2,0.5,0.9,0.5,0.3,0.1,
0.0,0.0,0.1,0.2,0.4,0.7,0.3,0.1,0.0,

1.00,0.35,0.71,0.89,

0.0,0.0,0.0,0.0,1.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.1,0.6,0.5,0.4,0.1,0.0,
0.0,0.0,0.1,0.5,0.6,0.8,0.3,0.2,0.1,
0.0,0.1,0.3,0.5,0.7,0.9,0.5,0.3,0.1,

1.00,0.75,0.85,0.48,

0.0,0.0,0.0,0.0,1.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.2,0.3,0.9,0.6,0.3,0.1,
0.0,0.0,0.1,0.3,0.5,0.6,0.5,0.3,0.2,
0.0,0.1,0.2,0.3,0.4,0.6,0.7,0.4,0.1,
```

FILE PHASE1.SOL

```
*********************************************************************

STAGE CONSTRAINT GOAL  DECISION  STATE : X(1),X(2),-------------,X(I)

  0              .3820             .130    .450    .790   1.000   .880
  1      .9200   .6060      1      .700    .790    .900    .880   .600
  2      .8300   .6820      2      .790    .900    .880    .600   .500
  4      .7500   .8240      1      .900    .880    .600    .500   .300

¡OPTIMAL SOLUTION¿   .7500  =  ¡CONSTRAINT¿   .7500  /Ñ ¡GOAL¿   .8240

*********************************************************************
```

```
************************************************************************
*                                                                      *
*                      PROGRAM IN FORTRAN 77                           *
*                                                                      *
*                      FLOOD CONTROL PHASE 2                           *
*                                                                      *
*             MULTI-STAGE DECISION-MAKING PROCESSES                    *
*                              FOR                                     *
*             NON-FUZZY SYSTEMIN FUZZY ENVIRONMENT                     *
*                              WITH                                    *
*       CONSTRAINT FOR EACH STAGE ( OVER DECISION SPACE )              *
*          GOAL FOR EACH STAGE ( OVER DECISION SPACE )                 *
*                              AND                                     *
*        MAX-WEIGHTED-SUM TYPE GLOBAL OBJECTIVE FUNCTION               *
*                   LIMIT TO SUM OF DECISIONS                          *
*                              BY                                      *
*                CLASSICAL BRANCH & BOUND METHOD                       *
*                     MAY STOP AT ANY TIME                             *
*                                                                      *
*                         JULY 27, 1991                                *
*                                                                      *
************************************************************************


************************************************************************
*
*                     THE NOTATION FOR ARRAY
*
* NI : NO. OF STATE
* NJ : NO. OF DECISION
* NK : NO. OF STAGE
* LIMIT : LIMIT TO SUM OF DECISION
*
* G : MEMBERSHIP FUNCTION OF GOAL ( STAGE, DECISION )
* C : MEMBERSHIP FUNCTION OF CONSTRAINT ( STAGE, DECISION )
* W : WEIGHT ( STAGE )
*
* L : LABLE FOR OPTIMAL DECISION ( STAGE )
* R : RETURN FUNCTION ( STAGE, DECISION )
* O : OBJECTIVE FUNCTION UNDER OPTIMAL DECISION ( STAGE, DECISION )
*
* MODE : MODE OF BRANCH ( STAGE, DECISION )
*          1 : THE BRANCH IS ACTIVE
*          0 : THE BRANCH IS IDLE
*         -1 : THE BRANCH IS DEAD
*
* REAL : PRESENT BEST SOLUTION
* COME : COMING SOLUTION
*
* BEFORE : UPPER BOUND FOR SOLUTION OF PART BEFORE ( STAGE )
* AFTER  : UPPER BOUND FOR SOLUTION OF PART AFTER ( STAGE )
*
************************************************************************
```

```
************************************************************************
*
*                     THE FUNCTION OF SUBROUTINES
*
* INPUT   : READ DATA FROM FILE
* OUTPUT  : WRITE DATA TO FILE
* BEGIN   : INITIALIZE VARIABLES
* BOUND   : SEARCH FORWARD TO FIND NEW FEASIBLE SOLUTION
*           IF BETTER SOLUTION FOUND, GOTO 'OUTPUT'
*           IF NOT, GOTO 'BRANCH'
* RENEW   : DELETE SOME BRANCHES BY NEW SOLUTION
* BRANCH  : SEARCH BACKWARD TO FIND NEW BRANCH
*           IF NEW BRANCH FOUND, GOTO 'BOUND'
*           IF NOT, STOP
*
************************************************************************
*                                                                      *
        PROGRAM FLOOD CONTROL PHASE 2
        PARAMETER(MAXI=40,MAXJ=40,MAXK=10)
        COMMON//NI,NJ,NK,LIMIT
        COMMON/PARA/G(MAXK,0:MAXJ),C(MAXK,0:MAXJ),W(MAXK)
        COMMON/KEEP/L(MAXK),R(MAXK,0:MAXJ),O(MAXK,0:MAXJ)
        COMMON/WORK/MODE(MAXK,0:MAXJ),COME,REAL,BEFORE(MAXK),AFTER(MAXK)
        PARAMETER(NUIN=10,NUOUT=30)
C
        CALL INPUT(NUIN)
        CALL BEGIN(K)
C
1000    CONTINUE
        CALL BOUND(K,*2000)
        CALL OUTPUT(NUOUT)
        CALL RENEW
C
2000    CONTINUE
        CALL BRANCH(K,*1000)
        STOP
        END
*                                                                      *
************************************************************************
```

```
***********************************************************************
*
*  INPUT NO. OF STATE, DECISION AND STAGE
*  INPUT WEIGHT FOR EACH STAGE
*  INPUT MEMBERSHIP FUNCTION OF GOAL FOR EACH STAGE AND DECISION
*  INPUT MEMBERSHIP FUNCTION OF CONSTAINT FOR EACH STAGE AND DECISION
*
***********************************************************************
*                                                                     *
        SUBROUTINE INPUT(NUIN)
        PARAMETER(MAXI=40,MAXJ=40,MAXK=10)
        COMMON//NI,NJ,NK,LIMIT
        COMMON/PARA/G(MAXK,0:MAXJ),C(MAXK,0:MAXJ),W(MAXK)
C
        OPEN(NUIN,FILE='PHASE2.DAT',STATUS='OLD')
C
        READ(NUIN,*) NI,NJ,NK
C
        READ(NUIN,*) (W(K),K=1,NK)
C
        DO 1000 K=1,NK
        READ(NUIN,*) (G(K,J),J=0,NJ)
        READ(NUIN,*) (C(K,J),J=0,NJ)
1000    CONTINUE
C
        END
*                                                                     *
***********************************************************************
```

```
************************************************************************
*
* (1) CALCULATE RETURN FUNCTION FOR EACH STAGE AND DECISION
* (2) CALCULATE UPPER BOUND OF BEFORE AND AFTER STAGE FOR EACH STAGE
* (3) SET 0 FOR PRESENT BEST SOLUTION
*     SET 0 FOR COMING SOLUTION
* (4) SET ALL BRANCHES IDLE
*     SET LIMIT TO SUM OF DECISION
*
************************************************************************
*                                                                      *
      SUBROUTINE BEGIN(K)
      PARAMETER(MAXI=40,MAXJ=40,MAXK=10)
      COMMON//NI,NJ,NK,LIMIT
      COMMON/PARA/G(MAXK,0:MAXJ),C(MAXK,0:MAXJ),W(MAXK)
      COMMON/KEEP/L(MAXK),R(MAXK,0:MAXJ),O(MAXK,0:MAXJ)
      COMMON/WORK/MODE(MAXK,0:MAXJ),COME,REAL,BEFORE(MAXK),AFTER(MAXK)
C
      DO 100 K=1,NK
      DO 100 J=0,NJ
      R(K,J)=MIN(G(K,J),C(K,J))*W(K)
 100  CONTINUE
C
              DO 200 K=1,NK
                      BEFORE(K)=0.
                      AFTER(K)=0.
              DO 200 J=0,NJ
                      BEFORE(K)=MAX(BEFORE(K),R(K,J))
                      AFTER(K)=MAX(AFTER(K),R(K,J))
 200          CONTINUE
              DO 300 K=1,NK-1
              BEFORE(K+1)=BEFORE(K+1)+BEFORE(K)
              AFTER(NK-K)=AFTER(NK-K)+AFTER(NK-K+1)
 300          CONTINUE
              DO 400 K=1,NK-1
              BEFORE(NK-K+1)=BEFORE(NK-K)
              AFTER(K)=AFTER(K+1)
 400          CONTINUE
                      BEFORE(1)=0.
                      AFTER(NK)=0.
C
      COME=0.
      REAL=0.
C
      DO 500 K=1,NK
      DO 500 J=0,NJ
      MODE(K,J)=0
 500  CONTINUE
C
      K=0
      LIMIT=NI-1
C
      END
*                                                                      *
************************************************************************
```

```
*************************************************************************
*
* (1) FOR PRESENT STAGE
*     IF ALL BRANCH DEAD, EXIT
*     OTHERWISE SET OTHER BRANCH ACTIVE
* (2) CALCULATE COMING SOLUTION FOR ACTIVE BRANCH
* (3) FIND BRANCH HAVING MAX COMING SOLUTION AS OPTIMAL DECISION
* (4) REPEAT STEP (1)--(3) FOR NEXT STAGE
*     UNTIL LAST STAGE
*
*************************************************************************
*                                                                       *
        SUBROUTINE BOUND(K,*)
        PARAMETER(MAXI=40,MAXJ=40,MAXK=10)
        COMMON//NI,NJ,NK,LIMIT
        COMMON/PARA/G(MAXK,0:MAXJ),C(MAXK,0:MAXJ),W(MAXK)
        COMMON/KEEP/L(MAXK),R(MAXK,0:MAXJ),O(MAXK,0:MAXJ)
        COMMON/WORK/MODE(MAXK,0:MAXJ),COME,REAL,BEFORE(MAXK),AFTER(MAXK)
        PARAMETER(EPSILON=1.E-5,INFINITE=10**10)
C
1000    CONTINUE
C
        K=K+1
        OMAX=-INFINITE
        L(K)=-INFINITE
C
                DO 4000 J=0,MIN(NJ,LIMIT)
C
                IF(MODE(K,J).EQ.-1) GOTO 4000
C
                        O(K,J)=COME+R(K,J)
C
                IF((O(K,J)+AFTER(K)-REAL).LE.EPSILON) GOTO 4000
C
                        MODE(K,J)=1
C
                IF((O(K,J)-OMAX).LE.EPSILON) GOTO 4000
C
                        OMAX=O(K,J)
                        L(K)=J
C
4000            CONTINUE
C
        IF(L(K).LT.0) RETURN 1
C
        IF(K.LT.NK) THEN
                COME=OMAX
                LIMIT=LIMIT-L(K)
                GOTO 1000
        END IF
C
        REAL=OMAX
        END
*                                                                       *
*************************************************************************
```

```
************************************************************************
*
* FOR EACH STAGE, PRINT
*     WEIGHT FOR STAGE
*     GRADE OF MEMBERSHIP FUNCTION OF CONSTRAINT UNDER OPTIMAL DECISION
*     GRADE OF MEMBERSHIP FUNCTION OF GOAL UNDER OPTIMAL DECISION
*     RETURN FUNCTION UNDER OPTIMAL DECISION
*     OBJECTIVE FUNCTION UNDER OPTIMAL DECISION
*     LABLE FOR OPTIMAL DECISION
*
************************************************************************
*                                                                      *
        SUBROUTINE OUTPUT(NUOUT)
        PARAMETER(MAXI=40,MAXJ=40,MAXK=10)
        COMMON//NI,NJ,NK,LIMIT
        COMMON/PARA/G(MAXK,0:MAXJ),C(MAXK,0:MAXJ),W(MAXK)
        COMMON/KEEP/L(MAXK),R(MAXK,0:MAXJ),O(MAXK,0:MAXJ)
C
        OPEN(NUOUT,FILE='PHASE2.SOL')
                                    WRITE(NUOUT,'(/1X,70(1H*)/)')
        WRITE(NUOUT,200)
        DO 1000 K=1,NK
        WRITE(NUOUT,201) K,W(K),C(K,L(K)),G(K,L(K))
     +                  ,R(K,L(K)),O(K,L(K)),L(K)
1000    CONTINUE
                                    WRITE(NUOUT,'(/1X,70(1H*)/)')
        CLOSE(NUOUT)
C
 200    FORMAT(1X,'STAGE : WEIGHT ( CONSTRAINT /Ñ GOAL ) =  ',
     +          'RETURN --> OBJECTIVE DECISION'/)
 201    FORMAT(1X,I4,F9.3,F12.4,F10.4,2F12.5,I8)
        END
*                                                                      *
************************************************************************
```

```
************************************************************************
*
* RESET MODE FOR EACH BRANCH
*     IF ( OBJECTIVE FUNCTION + UPPER BOUND AFTER )
*        LOWER THAN NEW SOLUTION, SET BRANCH IDLE
*     IF ( RETURN FUNCTION + UPPER BOUND BEFORE + UPPER BOUND AFTER )
*        LOWER THAN NEW SOLUTION, SET BRANCH DEAD
*
************************************************************************
*                                                                      *
        SUBROUTINE RENEW
        PARAMETER(MAXI=40,MAXJ=40,MAXK=10)
        COMMON//NI,NJ,NK,LIMIT
        COMMON/KEEP/L(MAXK),R(MAXK,0:MAXJ),O(MAXK,0:MAXJ)
        COMMON/WORK/MODE(MAXK,0:MAXJ),COME,REAL,BEFORE(MAXK),AFTER(MAXK)
        DIMENSION NSTILL(MAXK)
C
        DO 100 K=1,NK-1
        DO 100 J=0,NJ
        IF(MODE(K,J).LT.1) GOTO 100
        IF((O(K,J)+AFTER(K)-REAL).LE.EPSILON) MODE(K,J)=0
        IF((R(K,J)+BEFORE(K)+AFTER(K)-REAL).LE.EPSILON) MODE(K,J)=-1
 100    CONTINUE
C
        DO 200 K=1,NK
        NSTILL(K)=0
        DO 200 J=0,NJ
        IF(MODE(K,J).EQ.1) NSTILL(K)=NSTILL(K)+1
 200    CONTINUE
C                                       WRITE(*,'(/1X,70(1H*)/)')
        WRITE(*,201) REAL,(NSTILL(K),K=1,MIN(NK,10))
C       WRITE(*,202)
C       DO 1000 J=0,NJ
C       WRITE(*,203) (MODE(K,J),K=1,NK)
1000    CONTINUE
C
 201    FORMAT(1X,'BEST RESULT = ',F7.5,8X,'REMINDER :',10I3)
 202    FORMAT(1X,24X,'MODE OF BRANCH :')
 203    FORMAT(1X,40X,10I3)
        END
*                                                                      *
************************************************************************
```

```
************************************************************************
*
* (1) SET PRESENT BRANCH IDLE
* (2) FOR PRESENT STAGE
*     IF ACTIVE BRANCH FOUND,
*          CHOCIE ONE HAVING MAX OBJECTIVE FUNCTION AS OPTIMAL DECISION
*          EXIT
*     IF NO ACTIVE BRANCH, GO BACK ONE STAGE
* (3) REPEAT (1)--(2) UNTIL FIRST STAGE
*
************************************************************************
*                                                                      *
        SUBROUTINE BRANCH(K,*)
        PARAMETER(MAXI=40,MAXJ=40,MAXK=10)
        COMMON//NI,NJ,NK,LIMIT
        COMMON/KEEP/L(MAXK),R(MAXK,0:MAXJ),O(MAXK,0:MAXJ)
        COMMON/WORK/MODE(MAXK,0:MAXJ),COME,REAL,BEFORE(MAXK),AFTER(MAXK)
        PARAMETER(EPSILON=1.E-5,INFINITE=10**10)
C
1000    CONTINUE
C
        K=K-1
        LIMIT=LIMIT+L(K)
        MODE(K,L(K))=0
C
        COME=-INFINITE
        L(K)=-INFINITEC
                DO 100 J=0,NJ
                IF(MODE(K,J).LT.1) GOTO 100
                IF((O(K,J)-COME).LE.EPSILON) GOTO 100
                        COME=O(K,J)
                        L(K)=J
 100            CONTINUE
C
        IF(L(K).GE.0) THEN
                LIMIT=LIMIT-L(K)
                RETURN 1
        END IF
C
        IF(K.GT.1) GOTO 1000
C
        END
*                                                                      *
************************************************************************
```

FILE PHASE2.DAT

```
16,5,10,

0.080,0.105,0.120,0.095,0.130,0.070,0.117,0.083,0.112,0.088,

0.6,0.8,1.0,0.9,0.8,0.7,
1.0,1.0,0.8,0.6,0.4,0.2,

0.4,0.5,0.6,0.7,0.8,0.9,
0.8,0.9,0.8,0.7,0.6,0.5,

0.1,0.4,0.7,0.9,1.0,1.0,
0.6,1.0,0.9,0.8,0.7,0.6,

0.2,0.4,0.6,0.8,1.0,0.9,
1.0,1.0,0.9,0.7,0.5,0.3,

0.5,0.8,0.9,1.0,0.9,0.8,
1.0,0.8,0.5,0.2,0.1,0.0,

0.2,0.6,0.7,0.8,0.8,0.8,
0.5,0.8,1.0,0.7,0.4,0.1,

0.3,0.4,0.7,0.9,0.8,0.7,
0.6,0.7,0.8,0.8,0.7,0.6,

0.1,0.4,0.7,0.8,0.9,0.8,
0.7,0.9,0.6,0.4,0.2,0.0,

0.3,0.4,0.5,0.6,0.7,0.8,
0.5,1.0,0.9,0.8,0.5,0.2,

0.3,0.5,0.7,0.9,1.0,1.0,
0.5,0.9,1.0,0.8,0.6,0.4,
```

FILE PHASE2.SOL

```
*************************************************************************

STAGE : WEIGHT ( CONSTRAINT /Ñ GOAL ) =  RETURN --> OBJECTIVE DECISION

   1      .080      1.0000      .8000      .06400      .06400        1
   2      .105       .8000      .4000      .04200      .10600        0
   3      .120       .8000      .9000      .09600      .20200        3
   4      .095       .9000      .6000      .05700      .25900        2
   5      .130       .8000      .8000      .10400      .36300        1
   6      .070       .8000      .6000      .04200      .40500        1
   7      .117       .8000      .9000      .09360      .49860        3
   8      .083       .6000      .7000      .04980      .54840        2
   9      .112       .5000      .3000      .03360      .58200        0
  10      .088      1.0000      .7000      .06160      .64360        2

*************************************************************************
```

****

ON MODEL FOR

NONPOINT SOURCE WATER POLLUTION CONTROL

(N.S.W.P.)

****

============== CONTENTS ===============

==

#0. Data Classification and Notation

Re : << ON DATA FOR NONPOINT SOURCE WATER POLLUTION CONTROL >>

<0> n : the index of N.S.W.P. control region
k : the index of N.S.W.P. control measure
j : the index of investment level
i : the index of water quality level

Used in National Level

<1> $C(j)$: the membership function of constraint for the nation
$G(j)$: the membership function of goal for the nation

Here $C(j)$ and $G(j)$ are defined on the set of all of the possible total investment levels for the nation.

<2> J : the upper bound of total investment for the nation

<3> $C_n(j)$: the membership function of constraint for region n
$G_n(j)$: the membership function of goal for region n

Here $C_n(j)$ and $G_n(j)$ are defined on the set of all of the possible total investment levels for region n.

<4> W_n : the weight of region n which represents its relative importance

Used in Regional Level

<5> $I_n(i)$: the membership function of initial states in region n
$F_n(i)$: the membership function of final states in region n
$G_n(i)$: the membership function of goal of states in region n

Here $I_n(i)$, $F_n(i)$ and $G_n(i)$ are defined on the state space (all of the possible water quality levels for region n).

<6> J_n : the upper bound of total investment for region n

<7> $C_{nk}(j)$: the membership function of constraint for measure k in region n

Here $C_{nk}(j)$ is defined on the decision space (all of the possible investment levels for measure k in region n).

<8> $T_{nkj}(i)$: the fuzzy state transform function for measure k in region n with investment level j

Here $T_{nkj}(i)$ is defined in the state space (all of the possible water quality levels in region n), and represents the fuzzy relationship between the membership functions of states before and after measure k being put into use with investment level j.

#1. Core Model for Regions

<0> Brief Description

MULTI-STAGE DECISION-MAKING PROCESSES for FUZZY SYSTEM in FUZZY ENVIRONMENT by BRANCH & BOUND METHOD

<1> Concepts

Stage : the (structural or non-structural) measure for N.S.W.P. control
Decision : the level of investment for the measure (in $)
State : the level of water quality

<2> Input Data

$I_n(i)$: the membership function of initial states
$G_n(i)$: the membership function of goal of states
$C_{nk}(j)$: the membership function of constraint for measure k (k= 1,...,K)
$T_{nkj}(i)$: the fuzzy state transform function for measure k with investment level j (j= 0,...,J ; k= 1,...,K)

<3> Formulas

$$Z_n = \bigvee_{j_{n1}\cdots j_{nK}} \{ [C_{n1}(j_{n1}) \wedge \cdots \wedge C_{nk}(j_{nk}) \wedge \cdots \wedge C_{nK}(j_{nK})] \wedge D(F_n) \}$$

$$\text{s.t. } F_n = T_{nKj_{nK}} (+) \cdots (+) T_{nkj_{nk}} (+) \cdots (+) T_{n1j_{n1}} (+) I_n$$

$$D(F_n) = 1. - || G_n, F_n ||$$

---

where, (+) : the fuzzy addition operator
F_n : the membership function of final states
$|| G_n, F_n ||$: a relative distance between G_n and F_n

<4> Output Data

j_{nk}^* : the optimal investment level for measure k (k= 1,...,K) in region n
Z_n : the highest satisfactory degree for N.S.W.P. control in region n

---

Note : For each measure, the decision set includes a 'null' decision, i.e. investment level j_{nk} = 0., which means measure k will not be used at all. Correspondingly, the grade of membership function of constraint $C_{nk}(0)$ = 1., and the state transform function $T_{nk0}(i)$ = 1. for all i which keeps the membership functions of states identical before and after stage k.

#2. Expanded Model for Regions

<1> Basic Idea

When there is a (crisp) limit to total investment for region n, namely j_n, the model should be :

$$Z_n(j_n) = \bigvee_{j_{n1}\cdots j_{nK}} \{ [C_{n1}(j_{n1}) /\backslash \cdots /\backslash C_{nk}(j_{nk}) /\backslash \cdots /\backslash C_{nK}(j_{nK})] /\backslash D(F_n) \}$$

$$\text{s.t. } F_n = T_{nKj_{nK}} (+) \cdots (+) T_{nkj_{nk}} (+) \cdots (+) T_{n1j_{n1}} (+) I_n$$

$$D(F_n) = 1. - || G_n, F_n ||$$

$$j_{n1} + \cdots + j_{nk} + \cdots + j_{nK} =< j_n$$

---

where, $Z_n(j_n)$: the highest satisfactory degree for N.S.W.P. control in region n which depends on j_n.

<2> Added Data

J_n : the upper bound of total investment for region n

<3> Practical Algorithm (Phase I)

+++

Step 0 : Repeat Step 1,...,4 for n= 1,...,N.

Step 1 : Determine the scale of possible level of total investment for region n, namely [0.,J_n], by using J_n in <2>.

Step 2 : For each j_n within [0.,J_n], run the Expanded Model in <1> to get $Z_n(j_n)$ and j_{nk}^* (k= 1,...,K).

Step 3 : Construct $G_n(j_n)$, the membership function of goal for region n, as follows :

$$G_n(j_n) = \begin{cases} 0. & j_n < 0. \\ Z_n(j_n) & 0. =< j_n =< J_n \\ 0. & J_n < j_n \end{cases}$$

Step 4 : Send $G_n(j_n)$ to Phase II. Store all j_{nk}^* for each j_n.

+++

#3. Core Model for Nationwide

<0> Brief Description

MULTI-STAGE DECISION-MAKING PROCESSES for NON-FUZZY SYSTEM in FUZZY ENVIRONMENT by BRANCH & BOUND METHOD

<1> Concepts

Stage : the region for N.S.W.P. control
Decision : the level of total investment for the region (in $)
State : the degree of N.S.W.P. control effect to the region

<2> Input Data

$C_n(j)$: the membership function of constraint for region n (n= 1,...,N)
$G_n(j)$: the membership function of goal for region n (n= 1,...,N)
W_n : the weight of region n (n= 1,...,N)

<3> Formulas

$$Z = \bigvee_{j_1 \cdots j_N} \{ R_1(j_1) + \cdots + R_n(j_n) + \cdots + R_N(j_N) \}$$

$$\text{s.t. } R_n(j_n) = [G_n(j_n) \wedge C_n(j_n)] * W_n \qquad n = 1,\ldots,N$$

$$W_1 + \cdots + W_n + \cdots + W_N = 1.$$

---

where, $R_n(j_n)$: the return function of stage n

<4> Output Data

j_n^* : the optimal investment level for region n (n= 1,...,N)
Z : the highest weighed-sum degree of N.S.W.P. control effect to N regions

#4. Expanded Model for Nationwide

<1> Basic Idea

When there is a (crisp) limit to total investment for the nation, namely j, the model should be :

$$Z(j) = \bigvee_{j_1 \cdots j_N} \{ R_1(j_1) + \ldots + R_n(j_n) + \ldots + R_N(j_N) \}$$

$$\text{s.t. } R_n(j_n) = [\, G_n(j_n) \wedge C_n(j_n) \,] * W_n \qquad n = 1,\ldots,N$$

$$W_1 + \ldots + W_n + \ldots + W_N = 1.$$

$$j_1 + \ldots + j_n + \ldots + j_N \leq j$$

---

where, Z(j) : the highest weighted-sum degree of N.S.W.P. control effect to N regions which depends on j.

<2> Added Data

J : the upper bound of total investment for the nation

<3> Practical Algorithm (Phase II)

+++

Step 1 : Determine the scale of possible level of total investment for the nation, namely [0.,J], by using J in <2>.

Step 2 : For each j within [0.,J], run the Expanded Model in <1> to get Z(j) and j_n^* (n= 1,...,N).

Step 3 : Construct G(j), the membership function of goal for the nation, as follows :

$$G(j) = \begin{cases} 0. & j < 0. \\ Z(j) & 0. \leq j \leq J \\ 0. & J < j \end{cases}$$

Step 4 : Send G(j) to Phase III. Store all j_n^* for each j.

+++

#5. Model for Coordination

<0> Brief Description

SINGLE-STAGE DECISION-MAKING for NON-FUZZY SYSTEM in FUZZY ENVIRONMENT by STANDARD FUZZY DECISION-MAKING

<1> Input Data

C(j) : the membership function of constraint for the nation
G(j) : the membership function of goal for the nation

<2> Formulas

$$Z = \bigvee_{j} [G(j) \wedge C(j)]$$

<3> Output Data

j^* : the optimal investment level for the nation
Z : the highest satisfactory degree for N.S.W.P. control in the nation

<4> Practical Algorithm (Phase III)

Step 1 : Run the Model for Coordination in <2> to get j^*.

Step 2 : Using j^* and the solution stored in Phase II, find j_n^* for region n (n= 1,...,N).

Step 3 : Using j_n^* and the solution stored in Phase I, find j_{nk}^* for measure k in region n (k= 1,...,K ; n= 1,...,N).

#6. Flow Chart & Information Transmission