PAPER REF # 8019
Proceedings: Eighth International Space Syntax Symposium
Edited by M. Greene, J. Reyes and A. Castro. Santiago de Chile: PUC, 2012.

THE IMPACT OF SPATIAL PARAMETERS ON SPATIAL STRUCTURING

AUTHOR: **Tao YANG**
The Bartlett of Graduate Studies, University College London, United Kingdom
e-mail: t.yang@ucl.ac.uk

Bill HILLIER
The Bartlett of Graduate Studies, University College London, United Kingdom
e-mail: b.hillier@ucl.ac.uk

KEYWORDS: *Spatial Network, Weibull Function, Embeddedness Trajectory, Part-whole*

THEME: Architectural Theory and Spatial Analysis

Abstract

How is the spatial structure of a city organised at different scales, varying from connecting one street with its neighbouring streets to aggregating all the streets into a well-structured city as a whole? In order to approach this question, this paper seeks to investigate the sequence of the streets encountered at a series of consecutive radii, from the point of view of any an individual street as a root space, termed as the embeddedness trajectory in this paper. If we clarify such embeddedness trajectory on which each individual street is progressively interconnected with all other streets with regards to its distance to them, this will enable us to better understand the spatial structuring of the whole city, because a collection of the embeddedness trajectories of all the streets of the city can illustrate the entire configuration of the city. Based on the axial and segment representations of the empirical cases, it examines the mathematical relation between node count at the radius of k (NC_Rk), measuring the accumulated number of the new streets encountered up to the radius of k, and radius (Rk). The two-parameter Weibull relation seems to approximate the variation of node count with an increase of radius, which is expressed by the formula of NC_Rk ~ f(Rk; a, b), where 'a' is the scale parameter and 'b' is the shape parameter. Then, a strong linear correlation is found between the parameter of 'a' and mean topological depth (or mean metric depth) at the infinite radius, which suggests that as for each street, the number of the encountered streets up to a constricted radius is influenced by the mean topological/metric depth from that street to all other streets in the system. And meanwhile, the parameter of 'b' is correlated with the average embeddedness pace, meaning the average change rate of node count across all the radii. Thus, as for each street, its embeddedness trajectory is in general impacted on by the parameters of mean depth Rn and the embeddedness pace. From the above analyses, it suggests two things: first, the spatial structuring of a city is influenced by two spatial parameters: the average distance from all streets to all other streets and the average change rate of node count from the local to the global; second, the spatial structuring of all the parts of a city at the local and medium scales are constricted by the emergence of the whole structure – arising from the local structuring - of the city at the global scale (measured by the infinite radius), which supports Hillier's theory of the emergence of urban structure (Hillier, 1996, 2001).

INTRODUCTION

How is the spatial network of a city organised at different scales, varying from locally connecting one street with its neighbouring streets to aggregating all the streets into a well-structured city as a whole? And how is each individual street, as the root space, at first directly connected to its neighbouring spaces, and then reach the further spaces via the neighbouring spaces, and so on until all other spaces within the whole urban network are counted with regards to the distance to the root space? These two questions in fact are related to each other, because when each street is progressively connected to all the other streets or sequentially embedded into the whole network, in terms of its distance to all other spaces (which is termed as the *embeddedness trajectory* in this paper), the whole spatial network and its various sub-networks are created at once. In order to understand such embeddedness trajectory, this paper, based on the axial and segment representations of several empirical cities or regions, investigates the mathematical relation between the accumulated number of the new streets encountered up to the radius of k, namely node count Rk, and the radius of k. The previous empirical studies suggested that node count Rk, given by the processed axial or segment models, has the power-law relation with radius within several constricted radius ranges (Yang & Hillier, 2007). And meanwhile, Park (2007) discovered that within the main radius range[1], 62% axial lines of London have the power-law relation with radius, 26% lines have the exponential law and 12% lines having the super power-law. However, is it possible to describe the embeddedness trajectory of each street within the whole range of radius[2] in a more general way? If so, can we indentify the spatial factors that influence the whole embeddedness trajectory of each street? The answers to these questions would enable us to better understand the spatial structuring of a city as a whole, and its relations to the local configurational patterns.

THE TWO-PARAMETER WEIBULL LAW FOUND IN THE EMBEDDEDNESS TRAJECTORY

This paper began by conducting the empirical study on the axial maps of London[3], the London Docklands, the London Region within the M25 motorway, Beijing, Amsterdam and Chicago, in order to explore the topological embeddedness trajectory. There are three reasons for selecting those cases. First, the axial map of London, with 17,321 lines, mainly represents the historic central districts of London, the London Docklands, with 28,226 lines, primarily demonstrates the newly developed districts in the East London, and the M25, with 100,218 lines, comprises the central districts, the East London and other places, so that those three axial maps show the different parts and scales of London. Second, London, Beijing (with 14,249 lines), Amsterdam (with 8,768 lines) and Chicago (with 30,535 lines) were selected from the different regions of the world and they have different urban development histories. Third, their system size (in terms of the number of axial line) and the maximum radius – meaning the maximum topological distance between any pair of axial lines – vary much (see ***Table 1***). This enables us to establish some general results for the topological embeddedness trajectory. As for each axial line as a root, it investigated the mathematical relation between node count and topological radius, within the range of 1 to the radius at which all the other axial lines are encountered in its outward growth from the root line. The radius discretely rises up at one topological step interval. The node count at the radius of k, denoted as NC_k, is divided by the system

[1] The maximum radius range of the axial map of London is from 1 to 45, and the main range roughly starts from 1 to 20. See Fig.5 in Park (2007).

[2] The whole range of radius of a street means the range from the most localised radius, such as the topological radius 1, to the maximum radius at which the street reaches all other streets within the system as a whole.

[3] This axial map of London is bounded with the north and south circular roads. See Hillier, 1996, p162.

size, that is, the node count at the infinite radius, denoted as NC_n, as a way of scaling the NC_k values into the range of 0 to 1 for all the lines of any a case.

Table 1 The number of axial line and the maximum axial radius of the study cases

Area	Num of Axial Line	Max Ax Radius
London	17,321	45
London Docklands	28,226	85
M25	100,218	126
Beijing	14,249	42
Chicago	30,535	40
Amsterdam	8,768	30

Nonlinear Fit

Response: Line A 17050, Predictor: Pred

Control Panel

Converged in Gradient

Criterion	Current	Stop Limit
Iteration	8	60
Obj Change	2.189745e-10	1e-15
Relative Gradient	1.5965614e-6	0.000001
Gradient	4.7535278e-8	0.000001

Parameter	Current Value	Lock
a	25.618130027	☐
b	4.8044897496	☐

SSE 0.001945355
N 41

Edit Alpha 0.050
Convergence Criterion 0.00001
Goal SSE for CL .

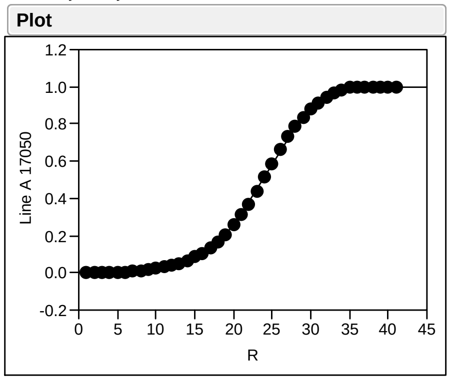
Solution

SSE	DFE	MSE	RMSE
0.001945355	39	4.9881e-5	0.0070626

Parameter	Estimate	ApproxStdErr
a	25.618130027	0.03392606
b	4.8044897496	0.03975937

Solved By: Analytic NR

Plot



Parameter	Estimate	Low	High
a	25.618130027	25.4146	25.8217
b	4.8044897496	4.56593	5.04305

Fig. 1 The non-linear correlation between NC_Rk/NC_Rn and radius k of Line A (the axial reference number is 17050). The two-parameter Weibull law was found.

Then, the non-linear regression analysis was processed to approximate the relation between NC_k/NC_n and radius k, that is, the embeddedness trajectory. ***Fig. 1*** illustrates an example of Line A randomly selected from the London axial map. An explicit form of the accumulated two-parameter Weibull function was deduced from the data of Line A. The follow illustrates the formula of the Weibull function:

$$\frac{NC_{\_Rk}}{NC_{\_Rn}} = 1 - e^{-(\frac{Rk}{a})^{b}} \tag{1}$$

where, Rk denotes the radius of k, NC_Rk denotes the node count value at the radius of k, NC_Rn denotes the node count at the radius n, namely the infinite radius, and 'a' indicating the scale parameter and 'b' meaning the shape parameter.

The formula can be written in another way (see formula 2). This demonstrates that the node count of Line A at a certain radius k is the production of the size of the system and the radius k.

$$NC_{\_Rk} = NC_{\_Rn} \times (1 - e^{-(\frac{Rk}{a})^{b}}) \tag{2}$$

The cumulated two-parameter Weibull fit was then tested for each axial line of each case in the software of MATLAB, in order to see whether the Weibull law can be found for all those individual axial lines, and if so, estimate the two parameters of 'a' and 'b'. ***Table 2*** sums up the goodness of fit, expressed by the R-square of the non-linear regression model, of each case, and the corresponding two parameters, respectively; ***Fig. 2*** demonstrates the distribution patterns of the R-square values, as well as the two parameters of 'a' and 'b' of each case, respectively. The axial lines of all the cases, except for the district of the London Docklands, have the R-square above 0.99. In particular, 99.2% axial lines of London, 82% lines of M25, 69% lines of Beijing and 67% Amsterdam have the R-square above 0.999. The outlier of the London Docklands, the newly developed district of East London, also has 82% lines with the R-square above 0.99, and 100% lines with the R-square above 0.978. The correlations are still very strong, although not perfect. It can be suggested that all the lines in those six cases have the two-parameter Weibull law relation between node count Rk and radius, within the whole range of radius. In other words, the two-parameter Weibull law can be used to approximately describe the topological embeddedness trajectory – from the local to the global – of each line in those six cases.

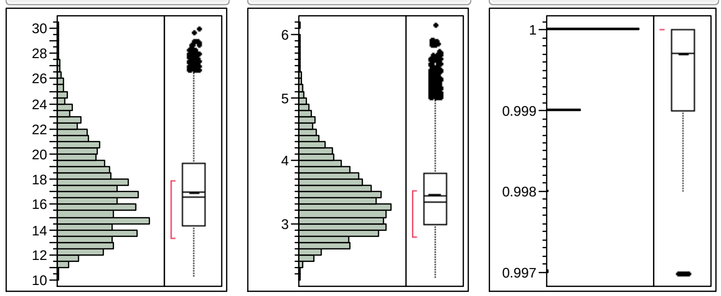
Table 2 The R-square of the non-linear correlation between node count and radius, as well as the parameters of 'a' and 'b' of each study case, based on axial maps.

Area	Max_R2	Mean_R2	Min_R2	Max_a	Median a	Min_a	Max_b	Median b	Min_b
London	1	0.999	0.997	30	16.6	10.3	6.17	3.35	2.15
London Docklands	1	0.994	0.978	66	29.3	21.6	6.94	2.74	1.47
M25	1	0.999	0.996	82.3	47.2	30.4	5.38	2.92	2.11
Beijing	1	0.999	0.995	29.9	15.8	10.2	6.36	3.33	2.07
Chicago	1	0.997	0.993	21.7	9.35	5.8	9.61	3.51	1.95
Amsterdam	1	0.999	0.993	18.9	11.3	6.9	8.92	4.12	2.56

Then, we further examined the metric embeddedness trajectory – meaning how each segment is progressively embedded into the whole network in terms of it metric distance to all other segments – by analysing the segment models of London and Beijing, the two geometrically contrasting cities, one of which visually shows an irregular layout, and the other seems to be an orthogonal structure. The two cities also have the different numbers of segments (London with 61,059 segments and Beijing with 43,523 segments) and the different maximum metric radii – meaning the largest metric distance between any pair of segments – of the system (London with the maximum radius of 32,500m and Beijing with the maximum radius of 53,500m). Since London has much more segments, but has much lower maximum radius, it demonstrates that the segments in London, on average, are metrically closer to each other, and so that London in general is more metrically integrated than Beijing. Can we indentify the similar law influencing the metric embeddedness trajectory in these two geometrically different cities?

Distributions

a	b	R2



Quantiles

		a	b	R2
100.0%	maximum	30.000	6.1700	1.0000
99.5%		26.900	5.3900	1.0000
97.5%		24.700	4.8900	1.0000
90.0%		21.800	4.2800	1.0000
75.0%	quartile	19.300	3.8000	1.0000
50.0%	median	16.600	3.3500	1.0000
25.0%	quartile	14.400	2.9900	0.9990
10.0%		12.900	2.7520	0.9990
2.5%		11.900	2.5500	0.9990
0.5%		11.300	2.4000	0.9980
0.0%	minimum	10.300	2.1500	0.9970

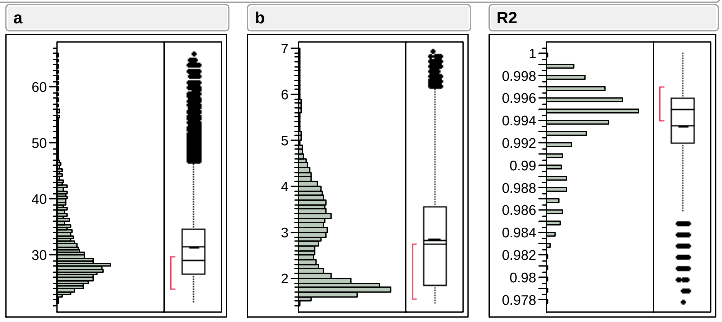
Moments

	a	b	R2
Mean	17.023226	3.4478142	0.9997128
Std Dev	3.4160655	0.6073477	0.0004759
Std Err Mean	0.0259561	0.0046148	3.6157e-6
upper 95% Mean	17.074103	3.4568596	0.9997199
lower 95% Mean	16.97235	3.4387688	0.9997057
N	17321	17321	17321

a. London

Distributions

a	b	R2



Quantiles

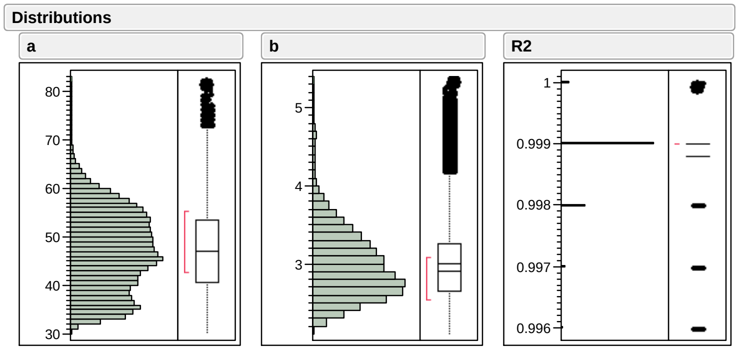
		a	b	R2
100.0%	maximum	66.000	6.9400	1.0000
99.5%		58.000	6.0800	0.9990
97.5%		49.500	5.2800	0.9990
90.0%		41.500	4.1800	0.9980
75.0%	quartile	34.700	3.5700	0.9960
50.0%	median	29.300	2.7400	0.9950
25.0%	quartile	26.700	1.8600	0.9920
10.0%		24.900	1.7200	0.9870
2.5%		23.500	1.6200	0.9840
0.5%		22.700	1.5500	0.9830
0.0%	minimum	21.600	1.4700	0.9780

Moments

	a	b	R2
Mean	31.566694	2.8380305	0.9935878
Std Dev	6.9519185	1.0363533	0.0039279
Std Err Mean	0.041379	0.0061686	2.338e-5
upper 95% Mean	31.647799	2.8501212	0.9936336
lower 95% Mean	31.485589	2.8259399	0.9935419
N	28226	28226	28226

b. the London Docklands

Fig. 2 The distribution patterns of the R-square values, as well as the two parameters of 'a' and 'b' of the Weibull fit of these six cases (London, the London Docklands, M25, Beijing, Amsterdam and Chicago, based on the axial maps) (continued)



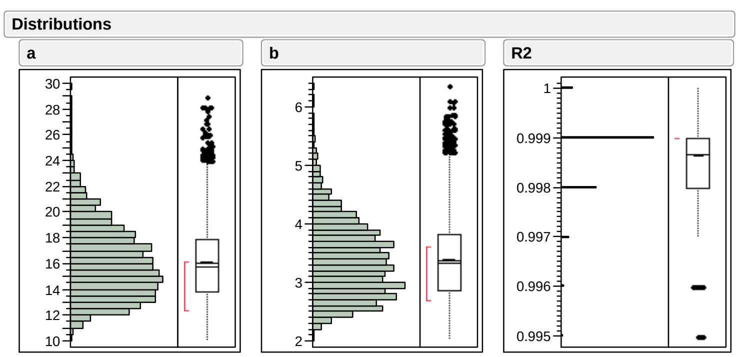
Quantiles

		a	b	R2
100.0%	maximum	82.300	5.3800	1.0000
99.5%		68.800	4.8200	1.0000
97.5%		62.700	4.2100	1.0000
90.0%		57.900	3.6000	0.9990
75.0%	quartile	53.500	3.2700	0.9990
50.0%	median	47.200	2.9200	0.9990
25.0%	quartile	40.700	2.6600	0.9990
10.0%		35.600	2.4900	0.9980
2.5%		33.200	2.3300	0.9970
0.5%		32.000	2.2400	0.9970
0.0%	minimum	30.400	2.1100	0.9960

Moments

	a	b	R2
Mean	47.226782	3.0054516	0.9987976
Std Dev	8.297404	0.4744445	0.0005992
Std Err Mean	0.0262101	0.0014987	1.8928e-6
upper 95% Mean	47.278153	3.008389	0.9988013
lower 95% Mean	47.17541	3.0025142	0.9987939
N	100218	100218	100218

c. London_M25



Quantiles

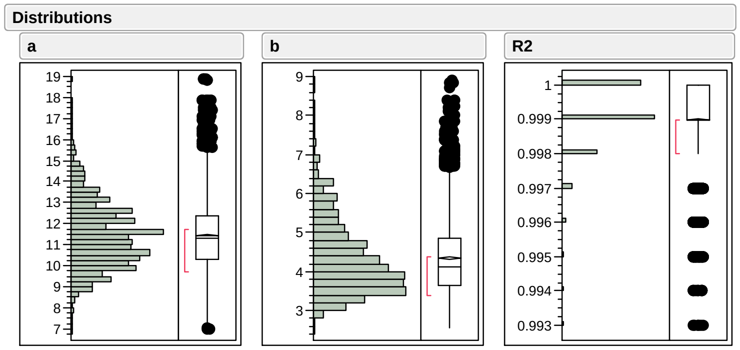
		a	b	R2
100.0%	maximum	29.900	6.3600	1.0000
99.5%		24.100	5.4200	1.0000
97.5%		22.200	4.8500	1.0000
90.0%		19.900	4.2200	0.9990
75.0%	quartile	17.900	3.8200	0.9990
50.0%	median	15.800	3.3300	0.9990
25.0%	quartile	13.900	2.8800	0.9980
10.0%		12.600	2.6000	0.9980
2.5%		11.900	2.4200	0.9970
0.5%		11.300	2.2700	0.9960
0.0%	minimum	10.200	2.0700	0.9950

Moments

	a	b	R2
Mean	16.100814	3.3911432	0.9986605
Std Dev	2.7832671	0.6427515	0.0007923
Std Err Mean	0.0233165	0.0053846	6.6378e-6
upper 95% Mean	16.146517	3.4016977	0.9986735
lower 95% Mean	16.055111	3.3805888	0.9986475
N	14249	14249	14249

d. Beijing

Fig. 2 The distribution patterns of the R-square values, as well as the two parameters of 'a' and 'b' of the Weibull fit of these six cases (London, the London Docklands, M25, Beijing, Amsterdam and Chicago, based on the axial maps) (continued)



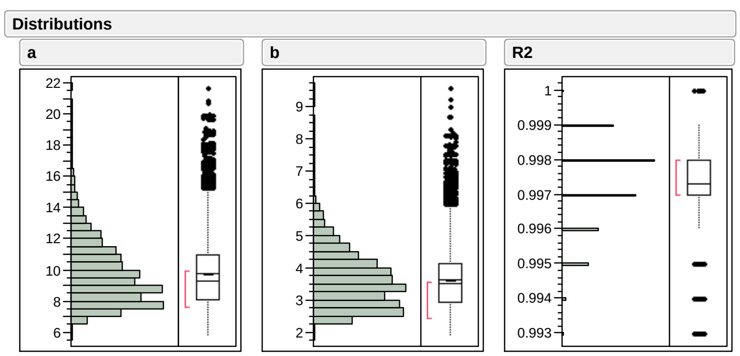
Quantiles

		a	b	R2
100.0%	maximum	18.900	8.9200	1.0000
99.5%		16.500	7.4900	1.0000
97.5%		14.900	6.4800	1.0000
90.0%		13.500	5.7800	1.0000
75.0%	quartile	12.400	4.8500	1.0000
50.0%	median	11.300	4.1200	0.9990
25.0%	quartile	10.300	3.6400	0.9990
10.0%		9.590	3.3600	0.9980
2.5%		8.870	3.1200	0.9960
0.5%		8.190	2.9100	0.9950
0.0%	minimum	6.960	2.5600	0.9930

Moments

	a	b	R2
Mean	11.446848	4.3481068	0.9989992
Std Dev	1.5907162	0.9414057	0.0010382
Std Err Mean	0.016988	0.0100537	1.1087e-5
upper 95% Mean	11.480148	4.3678144	0.9990209
lower 95% Mean	11.413547	4.3283991	0.9989775
N	8768	8768	8768

e. Amsterdam



Quantiles

		a	b	R2
100.0%	maximum	21.700	9.6100	1.0000
99.5%		17.000	6.6000	0.9990
97.5%		14.900	5.6600	0.9990
90.0%		12.600	4.8300	0.9990
75.0%	quartile	11.000	4.1500	0.9980
50.0%	median	9.350	3.5100	0.9980
25.0%	quartile	8.170	2.9300	0.9970
10.0%		7.560	2.5700	0.9950
2.5%		7.010	2.4400	0.9950
0.5%		6.720	2.2900	0.9940
0.0%	minimum	5.810	1.9500	0.9930

Moments

	a	b	R2
Mean	9.7776709	3.6318202	0.9973048
Std Dev	2.0686732	0.8817099	0.0012738
Std Err Mean	0.0118384	0.0050458	7.2895e-6
upper 95% Mean	9.8008746	3.6417101	0.9973191
lower 95% Mean	9.7544671	3.6219303	0.9972905
N	30535	30535	30535

f. Chicago

Fig. 2 The distribution patterns of the R-square values, as well as the two parameters of 'a' and 'b' of the Weibull fit of these six cases (London, the London Docklands, M25, Beijing, Amsterdam and Chicago, based on the axial maps)

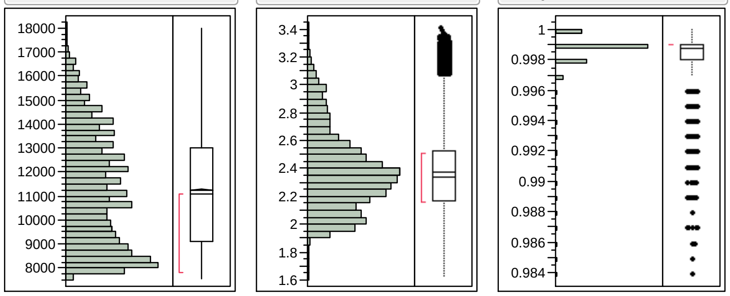
As for each segment as a root space, we explored the mathematical relation between node count and metric radius, within the range of 500m to the maximum metric radius of that segment, equal to the maximum metric distance between the root segment to all other segments. The metric radius discretely increases at an interval of 500m. We use the discrete radii, because the segments are encountered in a discrete way, when we process the segment model in the DepthMap. Like the axial line analysis conducted for those six cities/districts, the cumulated two-parameter Weibull fit between node count Rk and radius k was tested for each segment of London and Beijing in the software of MATLAB, in order to explore whether the Weibull law can be found for all the segments, and if so, estimate the two parameters of 'a' and 'b'.

Table 3 sums up the goodness of fit values, expressed by the R-square of the non-linear regression model, of London and Beijing, and their two parameters, respectively; **Fig. 3** illustrate the distribution patterns of the R-square values, as well as the two parameters of 'a' and ''b. London has 73% segments with the R-square above 0.999, and 99.97% with the R-square above 0.99; and the minimum R-square is 0.984. Beijing has 30% lines with the R-square above 0.99, and 100% lines with R-square above 0.9; and the minimum R-square is 0.924. Although the correlations in the case of Beijing are relative worse than that of London, they are still strong, because the R-square is larger than 0.924. It can be suggested that all the segments in London and Beijing have the two-parameter Weibull relation between node count Rk and metric radius k, within the whole range of metric radius. This demonstrates that the two-parameter Weibull law can be applied to statistically capture the metric embeddedness trajectory of each segment in the two contrasting cities.

Table 3 The R-square of the non-linear correlation between node count and radius, as well as the parameters of 'a' and 'b' of London and Beijing, based on segment models.

City	Max_R2	Mean_R2	Min_R2	Max_a	Median a	Min_a	Max_b	Median b	Min_b
London	1	0.999	0.984	18000	11100	7540	3.41	2.34	1.64
Beijing	0.999	0.973	0.924	26600	15900	11900	3.63	2.55	1.87

Distributions



Quantiles

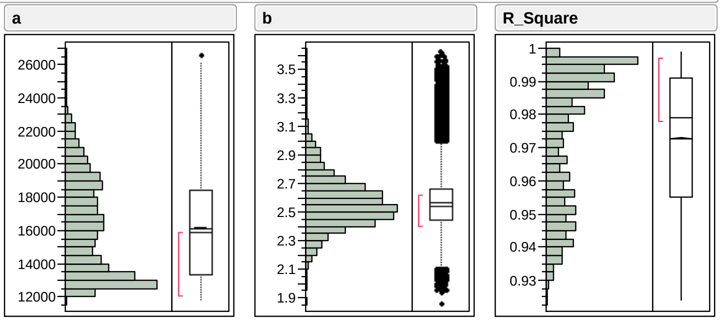
		a	b	R_square
100.0%	maximum	18000	3.4100	1.0000
99.5%		16800	3.2100	1.0000
97.5%		16000	3.0500	1.0000
90.0%		14600	2.8000	1.0000
75.0%	quartile	13000	2.5300	0.9990
50.0%	median	11100	2.3400	0.9990
25.0%	quartile	9090	2.1700	0.9980
10.0%		8240	2.0200	0.9980
2.5%		7900	1.9400	0.9970
0.5%		7740	1.9000	0.9940
0.0%	minimum	7540	1.6400	0.9840

Moments

	a	b	R_square
Mean	11247.208	2.3761416	0.9987831
Std Dev	2374.4196	0.2862809	0.0009567
Std Err Mean	9.6090982	0.0011586	3.8718e-6
upper 95% Mean	11266.042	2.3784124	0.9987907
lower 95% Mean	11228.375	2.3738708	0.9987755
N	61059	61059	61059

a. London

Distributions



Quantiles

		a	b	R_Square
100.0%	maximum	26600	3.6300	0.99900
99.5%		23300	3.2900	0.99800
97.5%		22100	3.0200	0.99700
90.0%		20300	2.8200	0.99500
75.0%	quartile	18500	2.6700	0.99100
50.0%	median	15900	2.5500	0.97900
25.0%	quartile	13400	2.4500	0.95500
10.0%		12700	2.3600	0.94300
2.5%		12300	2.2300	0.93500
0.5%		12100	2.1400	0.92900
0.0%	minimum	11900	1.8700	0.92400

Moments

	a	b	R_Square
Mean	16166.949	2.5712619	0.9728166
Std Dev	2936.0923	0.1946498	0.02018
Std Err Mean	14.073765	0.000933	9.673e-5
upper 95% Mean	16194.533	2.5730906	0.9730062
lower 95% Mean	16139.364	2.5694331	0.972627
N	43523	43523	43523

b. Beijing

Fig. 3 The distribution patterns of the R-square values, as well as the two parameters of 'a' and 'b' of the Weibull fit of London and Beijing, based on the segment models.

THE SPATIAL PARAMETERS CONSTRICTING THE SPATIAL STRUCTURING OF URBAN NETWORK

Then, what is the meaning of the two parameters of 'a' and 'b' in the above analyses? Mathematically, the scale parameter of 'a' determines the range of the possible values of NC_Rk we expect to see. The larger the scale parameter of 'a', the more spread out of the distribution. The parameter of 'b' is the shape parameter that influences the shape of the distribution of NC_Rk. ***Table 2 & 3***, based on either the topological analysis and the metric analysis respectively, show that the median of the parameter of 'a' varies more widely case to case, but the median of the parameter of 'b' varies within a relative narrower band. As for each case, the scale parameter of 'a' varies more widely, but the shape parameter of 'b' varies more narrowly around the median. This suggests that the shape of the curve (***Fig. 1***) denoting the embeddedness trajectory in fact is constricted within a relative narrower range, although the spread of NC_Rk varies much. However, do the two parameters have any relations with the other basic syntactic measures?

As for the axial maps of those six cases, we choose the basic geometric and syntactic variables of line length, connectivity, total depth R3, total depth Radius-radius4, total depth Rn, mean depth R3, mean depth Radius-radius, mean depth Rn, integration R3, integration Radius-radius and integration Rn. Then, the linear regression analysis of each basic geometric or syntactic variable and the parameter of 'a' or 'b' was conducted for each case respectively, as a way of picking out which syntactic factor heavily impacts on the two parameters. The result shows the parameter of 'a' has a nearly perfect linear correlation with mean depth Rn for each case. **Fig. 4** shows the correlation scattergram of each case, of which the x-axis denotes mean depth Rn and the y-axis indicating the parameter of 'a'. The R-square values of London, the London Docklands, M25, Beijing, Chicago and Amsterdam, respectively, are 0.998, 0.995, 0.998, 0.997, 0.996 and 0.994. When we further compared the value range on the x-axis with that on the y-axis, it demonstrated that the mean depth Rn and the parameter of 'a' almost vary within the similar range in each case. And meanwhile, since the 'a' is the scale parameter of the Weibull law, it can be multiplied by a scale factor to be adjusted to approximate the mean depth Rn. To a large extent, it can be concluded that the mean depth Rn of each axial line is in fact the parameter of 'a'.

However, we didn't indentify the strong correlation between those basic geometric/syntactic variables and the parameter of 'b'. But this parameter controls the curve shape (***Fig.1***) that is mathematically related to the change rate of node count. We then propose a conjecture that the shape parameter of 'b' is associated with the pace at which each line is topologically embedded into the surroundings, denoted as *topo-embeddedness pace*. The topo-embeddedness pace (or change rate of node count) is calculated by the following formula (Yang & Hillier, 2007).

$$Emd(Rk) = \frac{\log(NC_{\_Rk}) - \log(NC_{\_Rk-1})}{\log(k) - \log(k-1)} \tag{3}$$

where, Emd(Rk) denotes the topo-embeddedness pace at the radius of k, NC_Rk indicates the node count at the radius of k, and k means the radius.

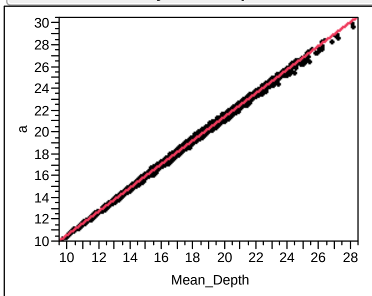
[4] The radius-radius is equal to the mean depth of the most integrated line within a system, and the edge effect that the syntactic analysis of the spaces located at the edge system would be biased by their location can be alleviated at the radius-radius (Hillier, 1996). London, the London Docklands, M25, Beijing, Amsterdam and Chicago, respectively, have the radius-radius of 10, 19, 27, 10, 7 and 6.

The average topo-embeddedness pace of any an axial line is then defined as the mean of a whole range of the topo-embeddedness Rk values, where k starts from 2 to the maximum radius of that axial line. The linear regression analysis of the parameter of 'b' and the average topo-embeddedness pace was then carried out. **Table 4** shows the R-square values of those six cases, and **Fig. 5** illustrates their correlation scattergrams. London, the London Docklands, M25, Beijing, Chicago and Amsterdam, respectively, have the R-square of 0.717, 0.872, 0.751, 0.748, 0.801 and 0.539. This demonstrates that the parameter of 'b' has a strong positive correlation with the average topo-embeddendess pace in each case. It suggests that the parameter of 'b' impacts on the average pace at which the axial lines are topologically embedded into the contextual structures. In other words, the average embeddedness pace is another major spatial parameter influencing the topological embeddedness trajectory.

Table 4. The R-square of the linear correlation between 'a' and mean depth Rn, between 'b' and average embeddedness pace, as well as between 'a' and 'b', based on the axial maps.

Area	R2 of a and MD	R2 of b and Avg Emd	R2 of a and b
London	0.998	0.717	0.772
London Docklands	0.995	0.872	0.612
M25	0.998	0.751	0.756
Beijing	0.997	0.748	0.663
Chicago	0.996	0.801	0.662
Amsterdam	0.994	0.539	0.463

Bivariate Fit of a By Mean_Depth



Linear Fit

Linear Fit

a = -0.128409 + 1.0820237*Mean_Depth

Summary of Fit

RSquare	0.998822
RSquare Adj	0.998822
Root Mean Square Error	0.117267
Mean of Response	17.02323
Observations (or Sum Wgts)	17321

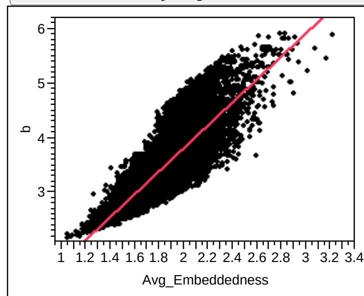
Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	201877.64	201878	14680338
Error	17319	238.16	0.013752	**Prob > F**
C. Total	17320	202115.81		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.128409	0.004564	-28.13	<.0001*
Mean_Depth	1.0820237	0.000282	3831.5	0.0000*

Bivariate Fit of b By Avg_Embeddedness



Linear Fit

Linear Fit

b = -0.390866 + 2.1011075*Avg_Embeddedness

Summary of Fit

RSquare	0.716797
RSquare Adj	0.71678
Root Mean Square Error	0.323221
Mean of Response	3.447814
Observations (or Sum Wgts)	17321

Analysis of Variance

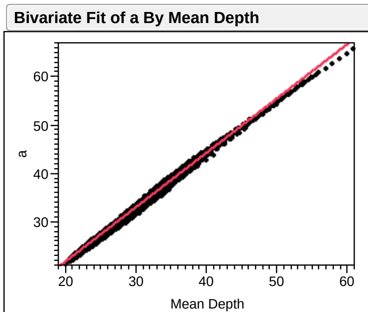
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	4579.5067	4579.51	43834.95
Error	17319	1809.3434	0.10	**Prob > F**
C. Total	17320	6388.8500		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.390866	0.018498	-21.13	<.0001*
Avg_Embeddedness	2.1011075	0.010035	209.37	0.0000*

a. London

Bivariate Fit of a By Mean Depth



Linear Fit

Linear Fit

a = -0.644921 + 1.1262897*Mean Depth

Summary of Fit

RSquare	0.995473
RSquare Adj	0.995473
Root Mean Square Error	0.467736
Mean of Response	31.56669
Observations (or Sum Wgts)	28226

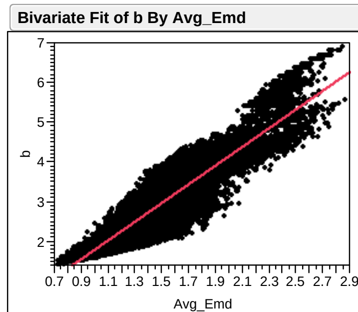
Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	1357916.1	1357916	6206849
Error	28224	6174.8	0.218777	**Prob > F**
C. Total	28225	1364090.8		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.644921	0.013226	-48.76	0.0000*
Mean Depth	1.1262897	0.000452	2491.4	0.0000*

Bivariate Fit of b By Avg_Emd



Linear Fit

Linear Fit

b = -0.550235 + 2.3614979*Avg_Emd

Summary of Fit

RSquare	0.872287
RSquare Adj	0.872282
Root Mean Square Error	0.370368
Mean of Response	2.838031
Observations (or Sum Wgts)	28226

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	26442.887	26442.9	192771.1
Error	28224	3871.555	0.137172	**Prob > F**
C. Total	28225	30314.443		0.0000*

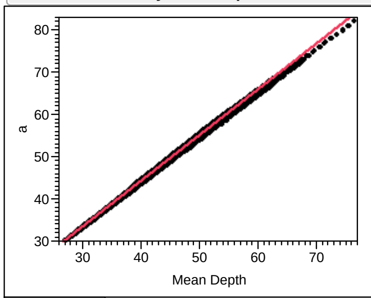
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.550235	0.008026	-68.56	0.0000*
Avg_Emd	2.3614979	0.005379	439.06	0.0000*

b. The London Docklands

Fig. 4 The linear correlation between 'a' and mean depth Rn (LEFT) and between 'b' and the average embeddedness pace (RIGHT), based on the axial maps of London, the London Docklands, M25, Beijing, Chicago and Amserdam, respectively (continued).

Bivariate Fit of a By Mean Depth



Linear Fit

a = 1.2030582 + 1.0833399*Mean Depth

Summary of Fit

RSquare	0.998777
RSquare Adj	0.998777
Root Mean Square Error	0.290228
Mean of Response	47.22678
Observations (or Sum Wgts)	100218

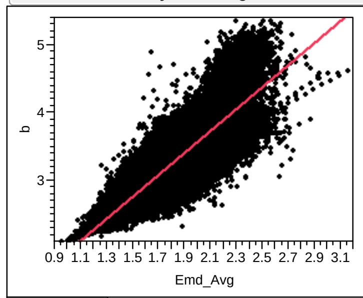
Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	6891189.6	6891190	81811765
Error	100216	8441.4	0.084232	**Prob > F**
C. Total	100217	6899631.0		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	1.2030582	0.00517	232.69	0.0000*
Mean Depth	1.0833399	0.00012	9045.0	0.0000*

Bivariate Fit of b By Emd_Avg



Linear Fit

b = 0.3225934 + 1.6222784*Emd_Avg

Summary of Fit

RSquare	0.750657
RSquare Adj	0.750655
Root Mean Square Error	0.236911
Mean of Response	3.005452
Observations (or Sum Wgts)	100218

Analysis of Variance

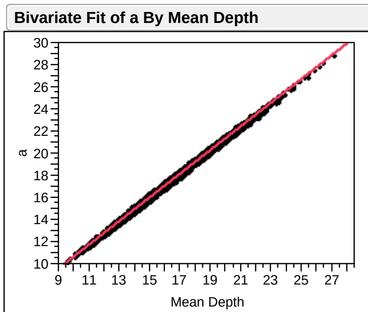
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	16933.779	16933.8	301704.5
Error	100216	5624.826	0.056127	**Prob > F**
C. Total	100217	22558.605		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.3225934	0.004941	65.28	0.0000*
Emd_Avg	1.6222784	0.002953	549.28	0.0000*

c. M25

Bivariate Fit of a By Mean Depth



Linear Fit

a = 0.0562603 + 1.0688146*Mean Depth

Summary of Fit

RSquare	0.997135
RSquare Adj	0.997135
Root Mean Square Error	0.148978
Mean of Response	16.10081
Observations (or Sum Wgts)	14249

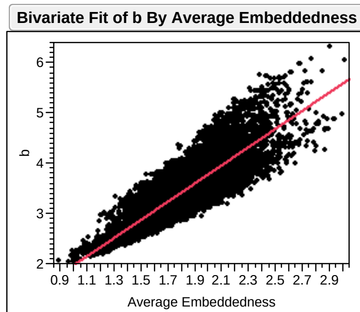
Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	110057.01	110057	4958771
Error	14247	316.20	0.022194	**Prob > F**
C. Total	14248	110373.21		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0562603	0.007312	7.69	<.0001*
Mean Depth	1.0688146	0.00048	2226.8	0.0000*

Bivariate Fit of b By Average Embeddedness



Linear Fit

b = 0.1888692 + 1.801047*Average Embeddedness

Summary of Fit

RSquare	0.747771
RSquare Adj	0.747753
Root Mean Square Error	0.322817
Mean of Response	3.391143
Observations (or Sum Wgts)	14249

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	4401.5807	4401.58	42237.35
Error	14247	1484.6888	0.10	**Prob > F**
C. Total	14248	5886.2695		0.0000*

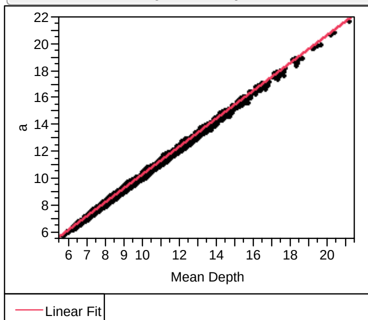
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.1888692	0.015814	11.94	<.0001*
Average Embeddedness	1.801047	0.008763	205.52	0.0000*

d. Beijing

Fig. 4 The linear correlation between 'a' and mean depth Rn (LEFT) and between 'b' and the average embeddedness pace (RIGHT), based on the axial maps of London, the London Docklands, M25, Beijing, Chicago and Amserdam, respectively (continued).

Bivariate Fit of a By Mean Depth



Linear Fit

a = -0.012308 + 1.033283*Mean Depth

Summary of Fit

RSquare	0.995779
RSquare Adj	0.995779
Root Mean Square Error	0.134403
Mean of Response	9.777671
Observations (or Sum Wgts)	30535

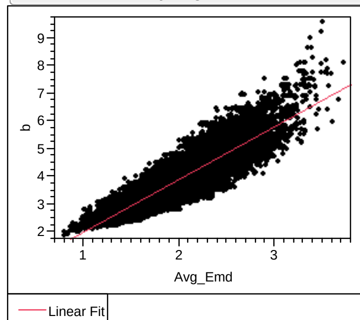
Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	130115.91	130116	7203017
Error	30533	551.55	0.018064	Prob > F
C. Total	30534	130667.46		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.012308	0.003728	-3.30	0.0010*
Mean Depth	1.033283	0.000385	2683.8	0.0000*

Bivariate Fit of b By Avg_Emd



Linear Fit

b = 0.1078694 + 1.9026629*Avg_Emd

Summary of Fit

RSquare	0.800559
RSquare Adj	0.800553
Root Mean Square Error	0.393767
Mean of Response	3.63182
Observations (or Sum Wgts)	30535

Analysis of Variance

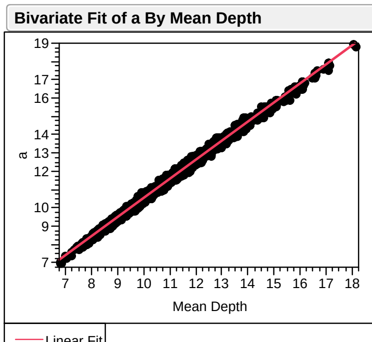
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	19003.285	19003.3	122560.2
Error	30533	4734.224	0.155053	Prob > F
C. Total	30534	23737.508		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.1078694	0.010315	10.46	<.0001*
Avg_Emd	1.9026629	0.005435	350.09	0.0000*

e. Chicago

Bivariate Fit of a By Mean Depth



Linear Fit

a = 0.0783415 + 1.0448442*Mean Depth

Summary of Fit

RSquare	0.993758
RSquare Adj	0.993758
Root Mean Square Error	0.125681
Mean of Response	11.44685
Observations (or Sum Wgts)	8768

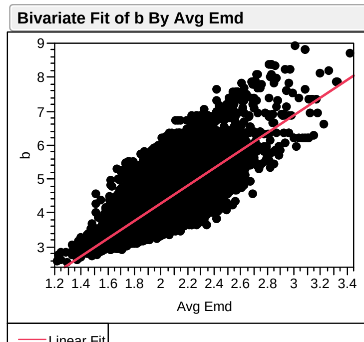
Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	22045.359	22045.4	1395656
Error	8766	138.465	0.015796	Prob > F
C. Total	8767	22183.824		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0783415	0.009716	8.06	<.0001*
Mean Depth	1.0448442	0.000884	1181.4	0.0000*

Bivariate Fit of b By Avg Emd



Linear Fit

b = -0.913997 + 2.5927457*Avg Emd

Summary of Fit

RSquare	0.538797
RSquare Adj	0.538744
Root Mean Square Error	0.639363
Mean of Response	4.348107
Observations (or Sum Wgts)	8768

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	4186.2933	4186.29	10240.81
Error	8766	3583.4138	0.41	Prob > F
C. Total	8767	7769.7072		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.913997	0.052445	-17.43	<.0001*
Avg Emd	2.5927457	0.025621	101.20	0.0000*

f. Amsterdam

Fig. 4 The linear correlation between 'a' and mean depth Rn (LEFT) and between 'b' and the average embeddedness pace (RIGHT), based on the axial maps of London, the London Docklands, M25, Beijing, Chicago and Amserdam, respectively.

In the light of the results of the above topological analyses, we further give another conjecture that the parameter of ‘a’ and ‘b’ of the Weibull relation between node count Rk and metric radius k, based on the segment models, respectively are related to the metric mean depth (meaning the average metric distance from a root segment to all other segments) and metric embeddedness pace (meaning the pace at which a root segment is metrically embedded into the contexts). The metric embeddedness pace is computed by the following formula (Yang & Hillier, 2007).

$$Emd(k,\sigma) = \frac{\log(NC_{-k}) - \log(NC_{-k-\sigma})}{\log(k) - \log(k-\sigma)}$$

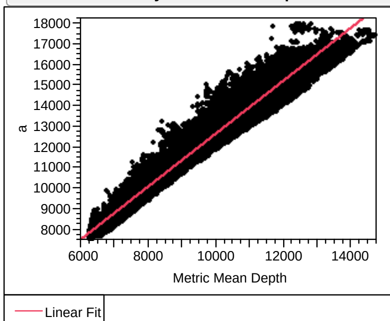
where, Emd(k,σ) denotes the metric-embeddedness pace at the radius of k, NC_k indicates the node count at the radius of k, and σ means the interval of increasing radius.

The average metric embeddedness pace of any a segment is then defined as the mean of an entire range of the metric embeddedness Rk, where k starts from 500m to the maximum metric radius of that segment and the interval σ is 500m. As for the cases of London and Beijing, the average metric embeddedness pace and the metric mean depth Rn values of each segment were given by their segment models respectively. The linear regression analyses of the parameter of ‘a’ and metric mean depth Rn, and of the parameter of ‘b’ and the average metric embeddedness pace were respectively carried out for each case city. ***Table 5*** shows the R-square values of the above correlations of London and Beijing, and ***Fig. 5*** respectively illustrates their correlation scattergrams.

Table 5. The R-square of the linear correlation between ‘a’ and mean depth Rn, between ‘b’ and average embeddedness pace, as well as between ‘a’ and ‘b’, based on the segment maps of London and Beijing.

City	R2 of a and MD	R2 of b and Avg Emd	R2 of a and b
London	0.964	0.665	0.578
Beijing	0.961	0.401	0.234

Bivariate Fit of a By Metric Mean Depth



Linear Fit

a = -90.76675 + 1.2771423*Metric Mean Depth

Summary of Fit

RSquare	0.964428
RSquare Adj	0.964428
Root Mean Square Error	447.8301
Mean of Response	11247.21
Observations (or Sum Wgts)	61059

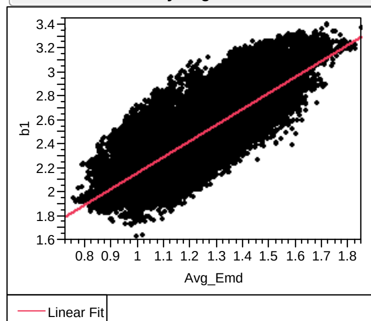
Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	3.3199e+11	3.32e+11	1655392
Error	61057	1.2245e+10	200551.8	**Prob > F**
C. Total	61058	3.4424e+11		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-90.76675	8.996648	-10.09	<.0001*
Metric Mean Depth	1.2771423	0.000993	1286.6	0.0000*

Bivariate Fit of b1 By Avg_Emd



Linear Fit

b1 = 0.8233105 + 1.3389344*Avg_Emd

Summary of Fit

RSquare	0.665149
RSquare Adj	0.665143
Root Mean Square Error	0.165662
Mean of Response	2.376142
Observations (or Sum Wgts)	61059

Analysis of Variance

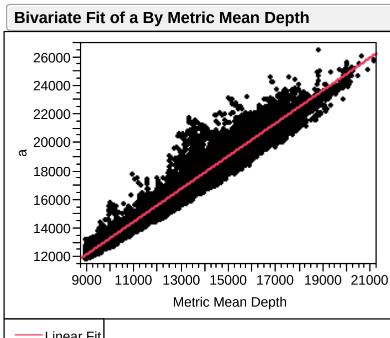
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	3328.4827	3328.48	121283.7
Error	61057	1675.6342	0.027444	**Prob > F**
C. Total	61058	5004.1169		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.8233105	0.004509	182.59	0.0000*
Avg_Emd	1.3389344	0.003845	348.26	0.0000*

a. London

Bivariate Fit of a By Metric Mean Depth



Linear Fit

a = 1828.3667 + 1.1540482*Metric Mean Depth

Summary of Fit

RSquare	0.960679
RSquare Adj	0.960679
Root Mean Square Error	582.2165
Mean of Response	16166.95
Observations (or Sum Wgts)	43523

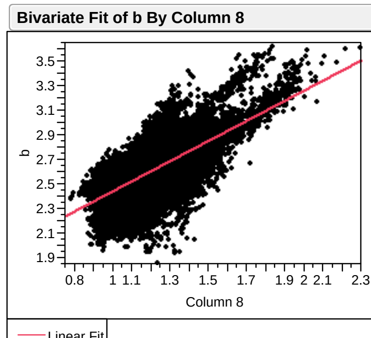
Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	3.6043e+11	3.604e+11	1063305
Error	43521	1.4753e+10	338976.09	**Prob > F**
C. Total	43522	3.7519e+11		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	1828.3667	14.18249	128.92	0.0000*
Metric Mean Depth	1.1540482	0.001119	1031.2	0.0000*

Bivariate Fit of b By Column 8



Linear Fit

b = 1.6310674 + 0.8151094*Column 8

Summary of Fit

RSquare	0.40085
RSquare Adj	0.400837
Root Mean Square Error	0.15067
Mean of Response	2.571262
Observations (or Sum Wgts)	43523

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	660.9962	660.996	29116.95
Error	43521	987.9887	0.023	**Prob > F**
C. Total	43522	1648.9849		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	1.6310674	0.005557	293.51	0.0000*
Column 8	0.8151094	0.004777	170.64	0.0000*

b. Beijing

Fig. 5 The linear correlation between 'a' and mean depth Rn (LEFT) and between 'b' and the average embeddedness pace (RIGHT), based on the segment models of London and Beijing, respectively.

On one hand, the parameter of 'a' has a strong positive correlation with metric mean depth Rn in the cases of London (with an R-square of 0.964) and Beijing (with an R-square of 0.961). This suggests that the scale parameter of 'a' of each segment is heavily influenced by the metric mean depth Rn, that is, the average metric distance from that segment to all other segments. Since the parameter of 'a' is the scale parameter, it can be adjusted to approximate the metric mean depth Rn. In this sense, it can be suggested that the global metric mean depth functions as the scale parameter of the Weibull law governing the metrically structuring of urban networks of London and Beijing.

On the other hand, the parameter of 'b' has a moderate positive correlation with the average metric embeddedness pace in the cases of London (with an R-square of 0.665) and Beijing (with an R-square of 0.401). This demonstrates that to some extent, the shape parameter of 'b' is affected by the average metric embeddedness pace, that is, the average pace at which each segment is metrically embedded into the contextual structure. Mathematically, the parameter of 'b' in fact controls the curve shape of the scattergrams plotting node count Rk against the metric radius of k. To some extent, it can be interpreted that the metric embeddedness pace serves as the shape parameter of the Weibull law governing the metrically structuring of London and Beijing.

In general, the above topological and metric analyses demonstrate that the scale parameter of 'a' can be treated as the average topological/metric distance from each root street, represented by axial line or segment, to all other streets within a system; and the shape parameter of 'b' can be interpreted as the average pace at which each street is topologically/metrically embedded into the whole structure of urban network.

THE LOCAL AND THE GLOBAL RELATION

Then, does the embeddedness trajectory, governed by the two-parameter Weibull law, from the local to the global, affect the spatial configurations at the local or medium scales? We further theoretically investigate the embeddedness trajectory pattern by examining the scattergram of plotting node count Rk, denoted as NC_k, on the vertical axis against radius, indicated as Rk, on the horizontal axis, as ***Fig. 6*** shows. If the radius increases from Rk to Rk+s (s denotes the minor increase of radius), the node count value simultaneously goes up from NC_k to NC_k+s. As a result, the increase of total depth approximates to the value of (NC_k+s – NC_k) * Rk, equal to the area of the shape B1B2O1O2, if s is small enough5. Then, the total depth Rn is the sum of (NC_k+s – NC_k) * Rk, as k rises up from 1 to n. Thus, the total depth Rn is equal to the area of the shape of A1A2A3, coloured in yellow. As for each axial line, the yellow area represents the topological total depth Rn; as for each segment, the yellow area indicates the metric total depth Rn. This shows that the topological/metric total depth Rn is in fact heavily influenced by the embeddedness trajectory from the most localised to the most globalised radius. And the total depth at any a radius of k is equal to the area of the shape A1B1O1 that is also constrained by the curve A1O1 representing the embeddedness trajectory from the radius of 1 to k. This by and large demonstrates that the total depth Rk – the basic syntactic feature – of each line or segment is determined by the embeddedness trajectory pattern at the radius of k.

[5] In the axial analysis, the minimum value of 's' is one step; in the segment analysis, the minimum value of 's' is approximately equal to the average segment length of a system.

As we discussed in the previous section (see ***formula 2***), node count Rk has the two-parameter Weibull relation with radius, so that the embeddedness trajectory pattern at k – meaning the trajectory on which a root space is connected to the contextual spaces from the radius of 1 to k – is heavily influenced by the two spatial parameters: the global mean depth and the average embeddedness pace. Since total depth Rk of each space is shaped by the embeddedness trajectory pattern at k (as we discussed in the previous paragraph), it also suggests that total depth Rk of each space is impacted on by the global mean depth and the average embeddedness pace. In this sense, it demonstrates that to a large extent, the spatial configuration of each space at the local and medium radii (smaller than the largest radius of the whole system) is constrained by the spatial configuration of the whole urban network.

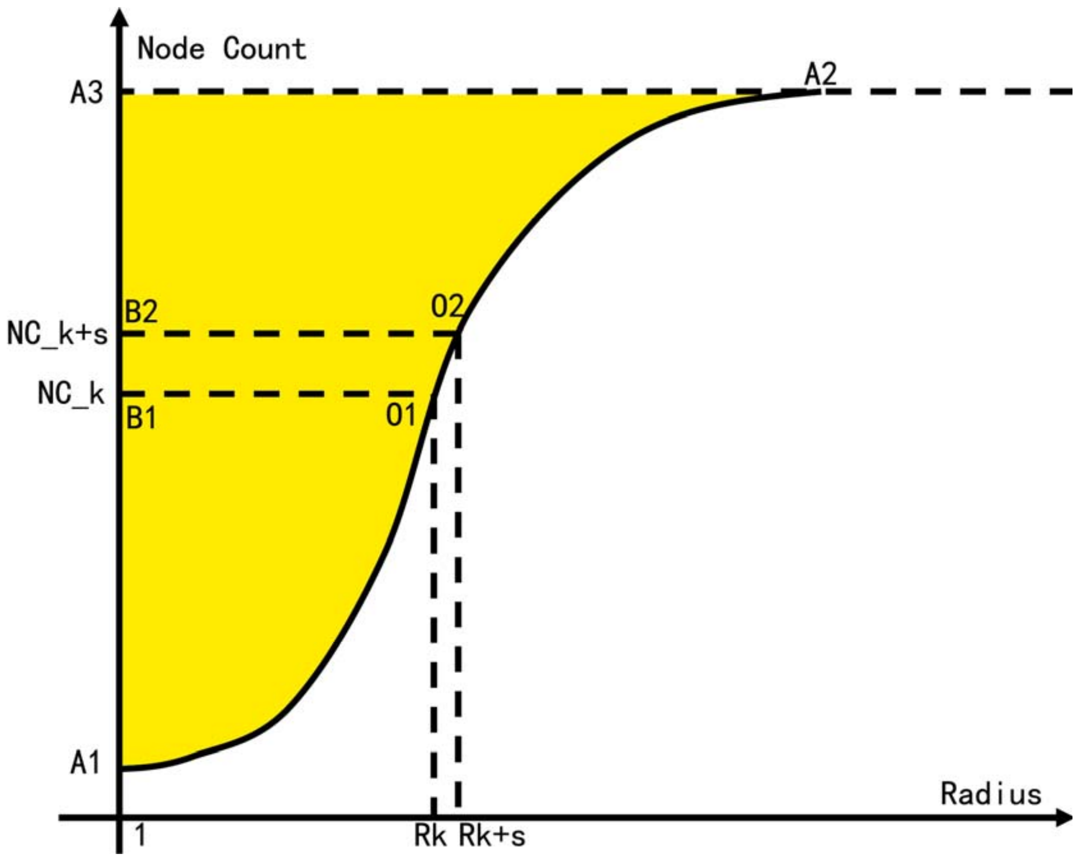


Fig. 6 The calculation of total depth Rn can be illustrated and interpreted by plotting node count against radius

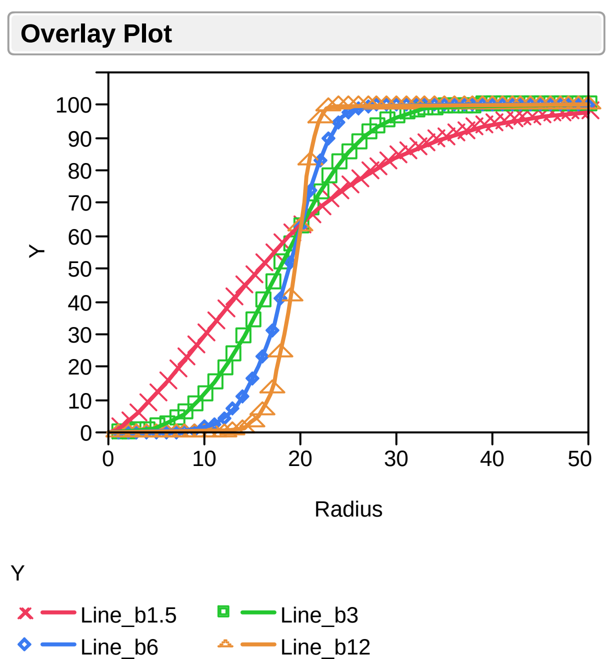


Fig. 7 The scattergram of node count against radius, governed by the two-parameter accumulated Weibull law, in which the parameter 'a' is fixed, but the parameter 'b' varies from 1.5 through 3 and 6 to 12. (Line_b1.5 means the curve with the 'b' of 1.5; Line_b3 denotes the curve with 3; Line_b6 indicates the curve with 6, and Line_b12 means the curve with 12.

In addition, when the topological data of the six cases were further examined, the two global syntactic factors, as the parameters of 'a' and 'b' of the Weibull law, seemed to have a relative strong positive correlation (see ***Table 4***), London, the London Docklands, M25, Beijing, Chicago and Amsterdam, respectively, have the R-square values of 0.772, 0.612, 0.756, 0.663, 0.662 and 0.463. Statistically speaking, larger 'a' is, higher 'b' is. This implies that larger topological mean depth Rn is, faster average embeddedness pace (or average change rate of node count) is. However, we expected that the faster embeddedness pace was associated with smaller topological mean depth Rn, because we intuitively assumed the faster embeddedness pace also indicated the more spaces would be encountered at the lower radii. Why the faster average embeddedness pace is related to the larger mean depth Rn?

The parameter of 'b' is empirically higher than 1.47, as ***Table 4*** shows. Theoretically speaking, when the parameter of 'b' of the Weibull law is higher than the value around 1.5, the curve of node count against radius would have the lower slopes at the lower radii, and this indicates the relative slower change rate of node count (or embeddedness pace) at the lower radii (***Fig. 7***). When the parameter of 'b' increases, say from 1.5 through 3 and 6 to 12, the embeddedness pace at more number of lower radii would slower, and the embeddedness pace at the medium radii would become much faster (***Fig. 7***). This means that less spaces would be encountered at the lower radii, but more spaces would be encountered at the higher radii, and so that the mean depth Rn would become larger. But the average embeddedness pace from the local to the global could become higher, because the embeddedness pace at the medium radii would become much faster. This is the reason why the faster average embeddedness is statistically associated with the larger topological mean depth Rn. And meanwhile, it also suggests that the shape parameter of 'b' captures more complicated change rate of node count from the most localised to the most globalised radius, which needs further study in the future.

And as we discussed the previous section, ***Table 4*** also shows that the shape parameter of 'b' is constrained within a narrower range, although the scale parameter of 'a' varies widely case to case or in one case. This suggests that the shape of the topological embeddedness trajectory does not vary much, and so that it implies that the spatial structuring of urban network is not random process, but is limited within a narrow range of possibilities, as Hillier (1996) argued in Chapter 8 of *Space is the Machine*.

As a result, when we normalise the syntactic values to compare them across the different sized systems, one of the strategies is to divide the syntactic values produced in the different systems by those generated in a/the reference system set within a whole range of possibilities of urban network. In the book of *The Social Logic of Space*, the syntactic value (eg. relative asymmetry6) of the real urban system is divided by the syntactic value (eg. relative asymmetry) of a 'diamond-shaped' pattern7 to normalise that value of the real urban system. When we investigated the topological embeddedness trajectory of the 'diamond-shaped' patterns of those six cases, the two-parameter Weibull law was also found respectively.

[6] See Hiller & Hanson, 1984, p108.

[7] The 'diamond-shaped' pattern indicates 'a justified graph in which there are k spaces at mean depth level, k/2 at one level above and below, k/4 at two levels above and below, and so on until there is one space at the shallowest (the root) and deepest points'. See Hillier & Hanson, 1984, p111-12.

Table 6 the parameters of 'a' and 'b', denoted as D_a and D_b, of the six diamond-shape patterns, as well as the median of the mean depth Rn of those six cities.

Area	D_a	D_b	Mean of mean depth Rn
London	15	10.6	15.9
Beijing	15	10.6	15
Chicago	16	11.3	9.5
Amsterdam	14	9.8	10.9
London Docklands	16	11.3	28.6
M25	18	12.8	42.5

Table 6 shows the parameters of 'a' and 'b', denoted as D_a and D_b, of the six diamond-shape patterns, as well as the median of the mean depth Rn of those six cities, respectively. First, the structuring of the 'diamond-shape' is also governed the two-parameter Weibull law, which is the same as the spatial structuring of the real urban space. Second, the parameter of 'a' – approximating the mean depth Rn – of the 'diamond-shape' patterns of London, Beijing, Amsterdam and Chicago are, respectively, close to the median of the mean depth Rn values of the corresponding cities, although the parameter 'a' values of the London Docklands and M25 are, respectively, even smaller the minimum of the mean depth Rn values of the two regions. This demonstrates that the 'diamond-shape' pattern, to a large extent, lies within a range of possibilities of the spatial structuring of the real cities. To some extent, this explains why the D-value (the relative asymmetry of the 'diamond-shape' pattern) works well in normalising the relative asymmetry of the real cities. However, the further study on the newly developed regions (such as the London Docklands) and the large-scale metropolitan region (such as the M25 comprising urban and rural areas) are needed for better understanding the comparison of the different systems in the light of the two-parameter Weibull law.

DISCUSSION

This paper suggests that the whole embeddedness trajectory of any an individual street is governed by the Weibull law with two parameters, one of which is the global mean depth and the other is the average embeddedness pace. On one hand, when any an individual street of a city is progressively connected to all other streets from the most localised to the most globalised level, the whole network of the city would be generated at once. This suggests that the emergence of urban network as a collective entity results from the embeddedness trajectory of all the individual spaces that constitute the urban network. In fact, this is a basic configurational view used to understand urban structure as a whole. As Hillier (1996: xii) clarified, 'configuration means the relations taking account of other relations'. The justified graph8 of each individual space visualises the configutional relation of that space, as the root space, and all the other spaces, which in fact reflects how the root space is progressively connected to all other spaces with an increase of radius, termed as the embeddedness trajectory in this paper. Once any a justified graph is constructed, the whole network, in spite of that it is observed from the point of the view of the root space, is then created

[8] The difference in the configurational relation between spaces can be easily found by justifying the graph in the following way: a selected node, as the root, is put on the baseline, the nodes one depth away from the root are horizontally aligned immediately above the root, the nodes two depth away from the root above those one depth away, and so on until all other nodes are taken into account in terms of their depth from the root (Hillier & Hanson, 1984).

simultaneously. This reveals a relationship between the emergence of the whole urban network and each individual space.

In this sense, the spatial structuring of the whole urban network is in fact determined by the whole embeddedness trajectory of any an individual space. Since each individual embeddedness trajectory is governed by the two-parameter Weibull law, the spatial structuring of the whole network is also shaped by the two-parameter Weibull law, and meanwhile impacted on by the two parameters as the two spatial facts: average depth from all spaces to all other spaces, as well as the average pace at which the individual spaces are topologically or metrically embedded into the whole network.

On the other hand, it demonstrates that the global configurational features of each individual space, such as those measured by global mean depth and average embeddedness pace, also are constraining parameters that determine the configurational patterns at the local and medium scale, such as that generated by total depth at a certain radius of k, according to the formula (1) and Fig. 5. The embeddedness trajectory from the most localised radius to a specific radius of k is constricted by the global configuration of urban network resulting from the embeddedness trajectory of each individual space. This supports Hillier's part-whole theory that the whole urban grid emerging from the aggregation of the local building forms makes the local places (Hillier, 1996). This enables us to better understand the localised configurational features in the light of the whole urban network.

In addition, the concept of the embeddedness trajectory, governed by the two-parameter Weibull law, also offers a methodological tool for thoroughly investigating the relations between the spatial sub-structures at different scales, ranging from the most localised to the most globalised scale. The features of the parameters of 'a' and 'b', as well as their relations need to be further studied on more empirical cases, in order to more accurately understand the spatial structuring of urban network.

REFERENCE:

Hillier, B. & Hanson, J. (1984) The Social Logic of Space, Cambridge University Press.

Hillier, B. (1996) Space is the Machine, Cambridge University Press.

Hillier, B. (1999) Centrality as A Process: Accounting for Attraction Inequalities in Deformed Grids, Urban Design International 4(3&4), 107-127.

Hillier, B., Turner, A., Yang, T., Park, H. (2007) Metric and Topo-geometric Properties of Urban Street Neworks: Some Covergences, Divergence, and new Results. In: Kubat, A.S. and Ertekin, O. and Guney, Y.I. and Eyuboglu, E., (eds.) 6th International Space Syntax Symposium, 12-15 Jun 2007.

Park,H. (2007) The Structural Similarity of Neighbourhoods in Urban Street Networks:A Case of London. In: Kubat, A.S. and Ertekin, O. and Guney, Y.I. and Eyuboglu, E., (eds.) 6th International Space Syntax Symposium, 12-15 Jun 2007.

Yang,T., Hillier, B. (2007) The fuzzy boundary: the spatial definition of urban areas. In: Kubat, A.S. and Ertekin, O. and Guney, Y.I. and Eyuboglu, E., (eds.) 6th International Space Syntax Symposium, 12-15 Jun 2007.