

Properties of Electricity Prices and the Drivers of  
Interconnector Revenue

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## Abstract

This paper examines the drivers behind revenues of merchant electricity interconnectors and the effect of arbitrage trading over interconnectors on the level and volatility of electricity prices in the connected markets. It sets out a simulation methodology that allows the stochastic and deterministic properties of prices, as well as most model parameters, to be varied freely. The effect of electricity flows over interconnectors on prices and thus on interconnector revenues is modelled explicitly by a mathematical algorithm. It is found that arbitrage can reduce the volatility and to some extent the mean of electricity prices in both markets when two markets with a similar distribution of prices are connected. It is also found that it is possible for interconnectors to generate considerable revenues without any consistent price differences between the connected markets. This shows that interconnectors between seemingly very similar electricity markets can be an attractive proposition for a profit-seeking investor.

**Keywords** merchant interconnectors, electricity prices, price volatility, simulation, bootstrapping

**JEL Classification** C15, C63, G10, L94

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# Properties of Electricity Prices and the Drivers of Interconnector Revenue

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## 1 Introduction

Deregulated electricity markets are characterised by very considerable price volatility. This is mainly due to the fact that electricity is not an easily storable commodity, and is likely to increase as the penetration of wind power in the overall generation mix increases. Such volatility, unless perfectly correlated in any two markets, creates arbitrage opportunities for owners of transmission capacity between those markets and becomes a significant component of the economic rationale for interconnection investment. However, the bulk of existing academic literature on the economics of electricity transmission is theoretical and largely ignores market price volatility to concentrate on consistent price differences between markets<sup>1</sup>. This paper attempts to fill this gap by taking an empirical approach and explicitly modelling volatility in electricity markets in a way that closely mimics its actual observed characteristics.

The model presented here takes its inspiration from the interconnector between the Netherlands and Norway (NorNed), which has been in operation since May 2008, and the interconnector between the Netherlands and the UK (BritNed), which is under construction at the time of writing. However, the methodology and the results are widely applicable to interconnectors built elsewhere. The outputs of the model are distributions of electricity prices in the connected markets after the construction of the interconnectors and the revenues derived from the interconnectors<sup>2</sup>. The predicted revenues determine the prospects for private sector investment in merchant interconnectors. Hence the model presented here helps to determine whether a proposed interconnector is likely to be attractive to profit-seeking investors. The distribution of

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<sup>1</sup>See Section 2 for further details

<sup>2</sup>The size of the interconnectors is allowed to vary in the simulations

electricity prices is the main factor affecting the pattern of investment in generation capacity and is also a determinant of consumer welfare. Since wholesale suppliers generally absorb any short-term variation in prices, only the mean of the distribution of prices directly affects consumer welfare. However, other properties of the price distribution, such as the variance, matter for the pattern of generation investment since some generation plants only produce when the price is very high. If an interconnector reduces price variance in a given market, investment in high variable cost generation becomes less profitable.

Another aim of this paper is to determine some of the main drivers of interconnector revenues. In terms of the motivating example, Norway is rich in reservoir generation while the UK and the Netherlands have a large share of thermal generation, hence the average daily and weekly pattern of electricity prices is generally more flat in Norway than in the Netherlands or the UK as reservoir generators can be expected to arbitrage consistent hourly and daily price differences. This can be seen in Figure 1. One would expect electricity prices in the UK and the Netherlands to be driven by the cost of producing electricity from fossil fuels and electricity prices in Norway to be determined by the deviation of reservoir levels from the seasonal norm. NorNed revenue could therefore be expected to be driven by different patterns of hourly and daily prices in the Netherlands and Norway, prolonged differences in mean prices related to reservoir levels in Norway and changes in fuel prices. The consistent hourly, daily and seasonal price differences between the UK and the Netherlands are much lower. However, the stochastic price shocks in the two countries are not perfectly correlated, meaning that arbitrage opportunities are likely to arise on a regular basis even without a consistent difference in average prices.

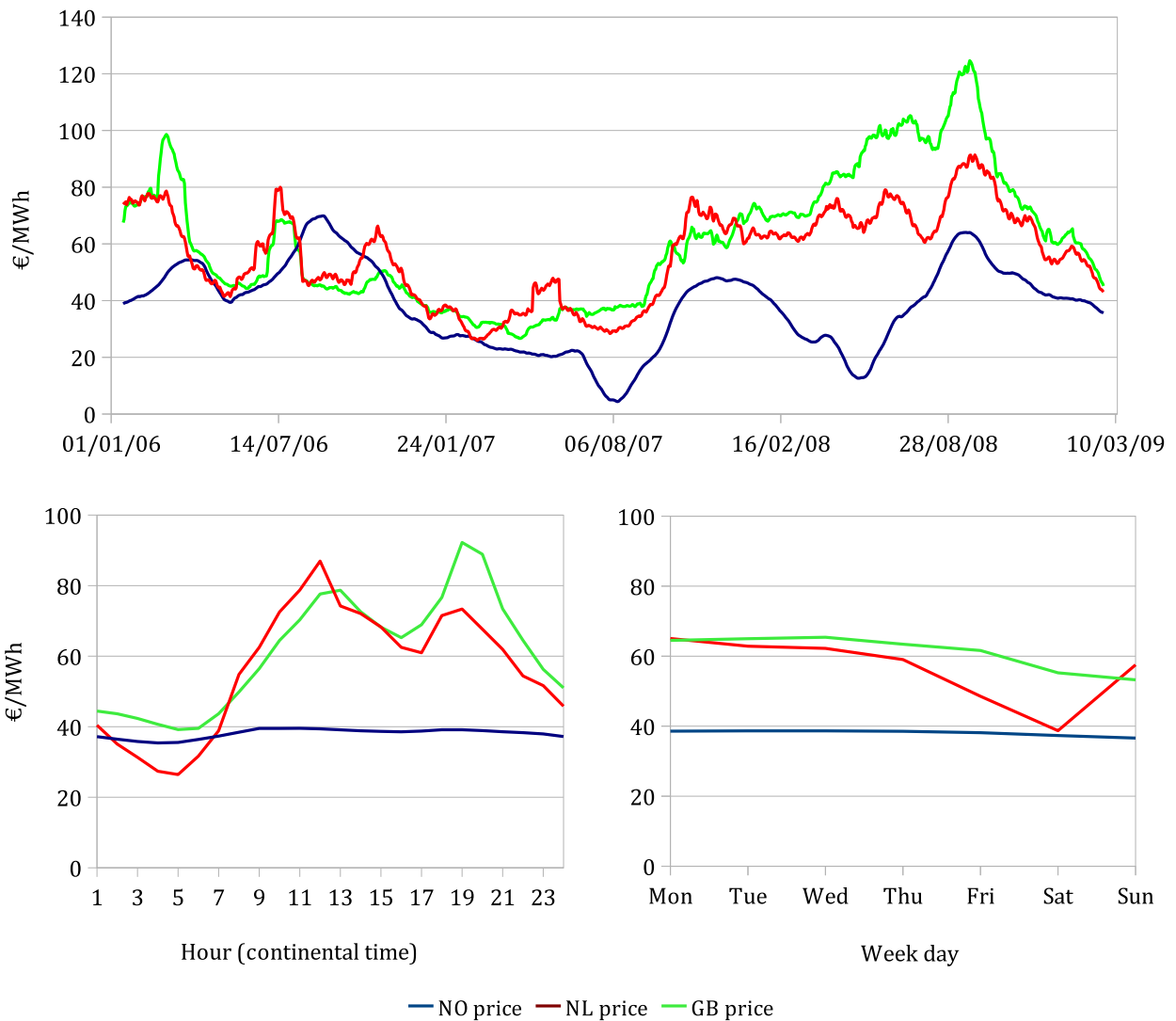


Figure 1: Average historic hourly prices between 1 Jan 2006 and 12 Mar 2009

The methodology adopted here improves on existing empirical literature on interconnectors in a number of ways. For example, Littlechild (2004) concludes that revenue from the Murraylink interconnector in Australia is unlikely to cover its costs on the basis of the absolute difference in mean prices between the two connected regions. He also argues that revenue can be generated by an interconnector without any difference in average prices no formal model of arbitrage on the basis of price volatility is presented. Instead, the author cites the historic financial performance and utilisation ratio of the Murraylink interconnector for a given year. This paper

presents a formal model that does not require onerous simplifying assumptions when simulating electricity prices. It separates prices into their deterministic and stochastic components, allowing complete control over both and making it general enough to accurately simulate the effects of interconnectors built between markets with very different properties. Thus among other questions, this paper also helps to answer the question of how important consistent price differences are relative to stochastic price differences in determining interconnector revenue and examines some of the key drivers of those differences.

The deterministic components of electricity prices, which include predictable hourly, daily and seasonal variations, are estimated through time series regression. The distributions of stochastic components of prices, which include factors such as unexpected changes in fuel prices, are obtained from the residuals of those regressions by bootstrapping, i.e. sampling with replacement, from the empirical distribution of the regression residuals. The stochastic properties of prices are incorporated into the model through the properties of the distribution of bootstrapped residuals. Correlations in stochastic price shocks in different countries are obtained from the empirical distribution of the regression residuals and also incorporated into the model. Both the deterministic and stochastic components of prices are simulated by closely controlling their properties. These properties are put into context firstly by estimating the effect of changes in fuel prices on electricity price levels and secondly by estimating the effect of increasing penetration of wind generation on electricity price volatility. The latter effect is calibrated from the estimates provided in Green and Vasilakos (2009).

The effect of electricity flows over interconnectors on prices and thus on interconnector revenues is modelled explicitly by a mathematical algorithm. Interconnector flows are determined endogenously in the model from the simulated prices in the connected markets. The price effect of flows over an interconnector is calibrated for market size and interconnector capacity from results obtained in Parail (2009). These are estimates of the effect of flows over the 700MW NorNed interconnector on the hourly day ahead electricity prices in the Netherlands and South Norway, obtained using an autoregressive time series econometric model. Assuming that NorNed is used up to its full capacity, the estimated average effect of flows from Norway to the Netherlands is to reduce the price in the Netherlands by 2.6% and to increase the South Norway nodal price by 4.2%. The effect of flows over an interconnector on UK prices is assumed to be the same as for prices in the Netherlands, adjusted for market size, as both markets are characterised by a large share of thermal generation in the overall generation mix.

The rest of the paper is organised as follows. Section 2 puts the paper in the context of related literature. Section 3 firstly sets out the methodology for estimating the deterministic and stochastic components of electricity prices, including details of the bootstrap procedure used to derive the underlying distribution of stochastic shocks. It then explains how electricity prices and the effect of interconnectors on those prices are simulated. Section 4 derives some results on interconnector revenues and the effect of interconnectors on electricity prices, abstracting from the

examples of NorNed and BritNed. Section 5 concludes.

Any reference to 'price' or 'electricity price' in this paper refers to the hourly day-ahead electricity price as determined on the relevant exchange unless specifically stated otherwise. Since electricity is traded on a half-hourly basis in the UK, an average of the two half-hourly prices is taken in order to bring it into line with hourly pricing in the Netherlands and South Norway. For ease of notation, and especially in tables, the Netherlands, South Norway and UK are referred to as 'NL,' 'NO' and 'GB' respectively.

## **2 Literature review**

The literature on trading in electricity between spatially separated markets is related to the theory of international trade. However, when making a comparison between the two different strands of literature, one must be mindful of the properties of electricity markets that differentiate them from other markets. One crucial property is the fact that electricity can only be traded up to the capacity of the relevant transmission cable in any given period. In international trade literature, this is equivalent to a trade quota. One paper that is relevant in this context is Krishna (1989). It models price competition between two firms, one domestic and one foreign. Both make differentiated products that are imperfect substitutes for one another. In this setting, the paper examines the relative welfare implications of trade quotas and tariffs in terms of consumer and domestic producer surplus. The model in the presence of quotas bears close resemblance to Borenstein et al. (2000), except that the latter model quantity rather than price competition. Since electricity is fundamentally a uniform commodity, it cannot be subject to product differentiation between firms.

Krishna's paper is a special case of a whole class of literature on international trade under imperfect competition. The issues relating to the competitiveness of electricity markets are not considered here explicitly. Any exercise of market power by generators is implicitly part of the estimate of the price effects of interconnectors as derived in Parail (2009). This means that if incumbent generators respond to electricity imports over an interconnector by increasing their output, this effect would be incorporated into the econometric estimate of the price effect of those imports.

The electricity industry bears many resemblances to other industries that are divided into spatially separated markets. However, it also displays certain characteristics that are not typical of many other goods. Firstly, it is an infinitely divisible and uniform commodity that is not amenable to product differentiation. Secondly, given the existence of appropriate transmission infrastructure, it can be transported instantaneously and almost costlessly between different

markets. This means that, given the existence of liquid markets, any price differences between spatially separated and adequately connected markets can be arbitrated with ease that resembles arbitrage in financial markets. However, neither financial arbitrage nor trade in goods or commodities are normally subject to the same physical limits on shipping capacity as is the case with electricity transmission. Finally, electricity cannot be stored in high volumes unless significant reservoir capacity is available and hence, unlike for other goods and commodities, arbitrage in electricity in most markets can only take place between different regions and not across time<sup>3</sup>. This means that electricity supplied in one time period does not represent the same product as electricity supplied in another time period.

Most literature dealing with the effects of DC transmission cables between spatially separated markets models the formation of electricity prices as a deterministic process. This applies to Joskow and Tirole (2000), who show that allowing generators to hold physical rights to transmission capacity may give them the incentive to create network congestion. They assume that the difference in market prices between two nodes is constant and results from differing generation costs, hence electricity flows across an interconnector would only ever be one way. It is also true of one of the most frequently quoted papers in this field, Borenstein et al. (2000). They model the effects of connecting two identical monopolistic electricity markets with constant demand and generation costs on the behaviour of incumbent monopolists. Like much of the rest of the literature, and indeed this paper, they adopt the concept of nodal pricing. Under this assumption, if there is no congestion, electricity prices are the same in both markets. When the network is congested, transmission capacity is rationed using the price mechanism and the owner of that capacity collects the associated scarcity rent.

Borenstein et al. (2000) find that when the capacity of the transmission line is above a certain threshold, the two firms act as a duopoly and prices in the two markets are lowered from the monopoly level. No actual power needs to flow down the interconnector to guarantee this outcome, which stems from this being a deterministic model with two identical producers on either side of the interconnector. Below this threshold, there are no pure strategy equilibria, but the expected amount of electricity sold into each market in mixed strategy equilibria is higher than the monopoly level of output and increasing in transmission line capacity. Based on these results, the authors conclude that the social value of transmission capacity may not be closely related to the actual flows of electricity across the interconnector.

Whilst much of the literature in this field takes the existing transmission network as given, some papers have examined the decision to invest in transmission capacity. One such paper is Joskow and Tirole (2005). This paper also assumes that electricity prices, and hence the factors that drive them, are deterministic. The authors show that numerous imperfections in energy markets lead to the social and private benefits of transmission investment diverging. Existence of

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<sup>3</sup>See Newbery (1984) for an examination of price stabilization of storable commodities



market power at one node leads the local generators to withhold capacity relative to the competitive level, driving up the price at that node and increasing the amount of power transmitted to that node. This leads to an incentive to over-invest in transmission capacity. Likewise, they show that lumpiness of transmission investment, or in other words the availability of economies of scale, leads to sub-optimal incentives to invest. This happens for the same reason as the unwillingness of owners of existing transmission capacity to increase that capacity. Increasing capacity tends to reduce their existing congestion rent. Economies of scale make it optimal to invest in transmission capacity non-incrementally. However, the decision on the profit-maximising capacity investment now has to take into account the effect that investment on the margin will have on the revenue from non-marginal capacity investment.

One paper that considers the problem of connecting separate electricity markets characterised by stochastic prices is Anderson et al. (2007). In particular, changes in electricity prices are driven by stochastic demand shocks. It is a natural compliment to this paper because it tests the extent to which such a problem can be solved analytically in contrast to the simulation approach adopted here. In order to make an analytical solution feasible, Anderson et al. make a number of simplifying assumptions. The first assumption is that market players do not behave strategically, and hence their behaviour does not change after the construction of an interconnector. The electricity price at each separate node is determined by a System Operator (SO). Generators submit their cost functions to the SO and buyers submit their demand schedule with quantities corresponding to different prices. The SO then finds the optimum solution which, given the assumption of non-strategic behaviour by market participants, would constitute a Walrasian equilibrium. This optimisation methodology is also employed after the interconnector is built. The assumption that differentiates their paper from this one and also the majority of the literature is that the interconnector has unlimited capacity. Instead of modelling capacity constraints, they model a quadratic loss function that applies to any electricity flows over the interconnector. Finally, for the bulk of their analysis, it is assumed that the distributions of demands at the two nodes on each end of the interconnector are independent.

Since market-based transmission investment is a relatively recent phenomenon, lagging behind electricity market liberalisation, empirical literature on this topic has been constrained by data availability and is still in the early stages of development. One paper that combines theoretical arguments with the use of data to support them is Brunekreeft (2003), where the author sets out to define the natural domains of regulatory and market-based transmission investment. Littlechild (2003) also straddles the line between empirical and theoretical work on this topic by discussing the Australian experience of developing market-based investment alongside more traditional regulatory investment in the case of Murraylink and SNI. The empirical part of the paper makes an estimate of the revenue from the Murraylink interconnector on the basis of the absolute difference in mean prices between the two connected regions, as well as historic financial performance and utilisation ratio. Without making any formal estimates, the paper also

argues that revenue can be generated by an interconnector without any difference in average prices.

Finally, this paper builds on Parail (2009), which estimates the effects of the NorNed interconnector on day-ahead electricity prices in NL and NO using a time series econometric model. The estimates obtained in that paper are used to calibrate the price effects of interconnectors simulated here. The paper uses those estimates to argue that the profits gained by private owners of interconnection capacity do not fall far short of the social benefits, implying that merchant interconnector investment is capable of filling the gap left by regulated cross-border investment.

## **3 Methodology**

### **3.1 Overview**

Rather than relying on restrictive assumptions in order to produce analytical solutions to a simplified set of questions, this paper adopts a simulation methodology that allows a broader set of questions to be answered in a more realistic setting. For the purposes of experiments conducted here, it separates out the deterministic and stochastic components of electricity prices and simulates them separately whilst mimicking their properties as closely as possible. This methodology allows a lot of flexibility in varying model parameters and price properties in order to simulate an almost unlimited number of scenarios. Among other things, this could include simulating the likely effect of increasing wind penetration on electricity price volatility and ultimately on interconnector revenues.

The deterministic components of electricity prices, which include predictable daily and seasonal variations, are obtained through time series regression using historical data between 1 January 2006 and 3 March 2009. The stochastic components of electricity prices are all the determinants of electricity prices that are not explained by the independent variables in the regression, as well as the variation in the effect of those independent variables. These are incorporated into the model through the properties of the distribution of bootstrapped residuals, which are obtained by bootstrapping, i.e. sampling with replacement, from the empirical distribution of the regression residuals. Correlations in stochastic price shocks in different countries are obtained from the empirical distribution of the regression residuals.

Some factors cannot be accurately predicted far in advance, such as the effect of changing fuel prices on the electricity price level or the effect of an increasing share of wind power in a country's overall generation mix on the volatility of electricity prices. This is because fuel prices or the actual level of wind penetration are, to a large extent, unpredictable over a long time horizon.

They are therefore represented as stochastic components in the model. However, forecasters may wish to examine scenarios that make assumptions on these factors. To make this possible within the framework presented here, relationships between gas and electricity prices in the GB and NL are estimated using a Vector Error Correction Model. The effect of wind penetration on electricity prices is calibrated using estimates taken from Green and Vasilakos (2009).

Having obtained a set of prices with the desired stochastic and deterministic properties, this paper simulates the response of those prices to electricity flows across interconnectors using an algorithm described in Section 3.5, with the full derivation being given in Appendix D. The algorithm mimics an efficient market coupling mechanism. Given exogenously determined prices in the connected markets before any connection is simulated, for each period, it calculates the market equilibrium solution and optimum flows over interconnectors given the transmission constraints. The outputs of the model include the distribution of prices on either end of a simulated interconnector and the revenue of that interconnector. Thus the methodology set out here is able to determine how the properties of electricity prices in the connected markets affect the revenue of an interconnector and, given those properties, how the interconnector is likely to affect the entire distribution of prices in those markets. Simulation results are put into context of fuel price changes and the effect of wind power on electricity prices to determine how these factors influence interconnector revenues.

### **3.2 Estimating deterministic and stochastic components of prices**

Since this paper uses NorNed and BritNed interconnectors as an inspiration, the first step in the analysis is to examine the properties and determinants of hourly electricity prices in the three regions that would be connected by those links, namely NL, GB and NO. The aim is to separate out the deterministic and stochastic components of electricity prices and to simulate them separately. This allows us to specify the deterministic and stochastic properties of prices freely for the purposes of simulation, depending on the scenario being simulated.

Deterministic components are deemed to be those that can be predicted far in advance and with a high level of precision. For each set of electricity prices, these are modelled as separate dummy variables that represent each hour of the day, each day of the week, each month of the year and any public holidays that do not fall on a weekend. In the case of NO prices, these are also variables that represent the expected reservoir levels in three different regions of Norway on the basis of historical data<sup>4</sup>.

The stochastic components of electricity prices are deemed to be the residuals that cannot be explained by regressing electricity prices against their deterministic components. As an initial step towards separating out the deterministic and stochastic components of prices, an OLS

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<sup>4</sup>Average reservoir levels calculated for the period between 1 Jan 1990 and 31 Dec 2003

regression of log electricity prices against the full set of observable deterministic variables is carried out. The histograms of residuals from these regressions are given below. In each diagram, the red line represents a plot of a normal distribution with the same mean and variance parameters as the corresponding sample.

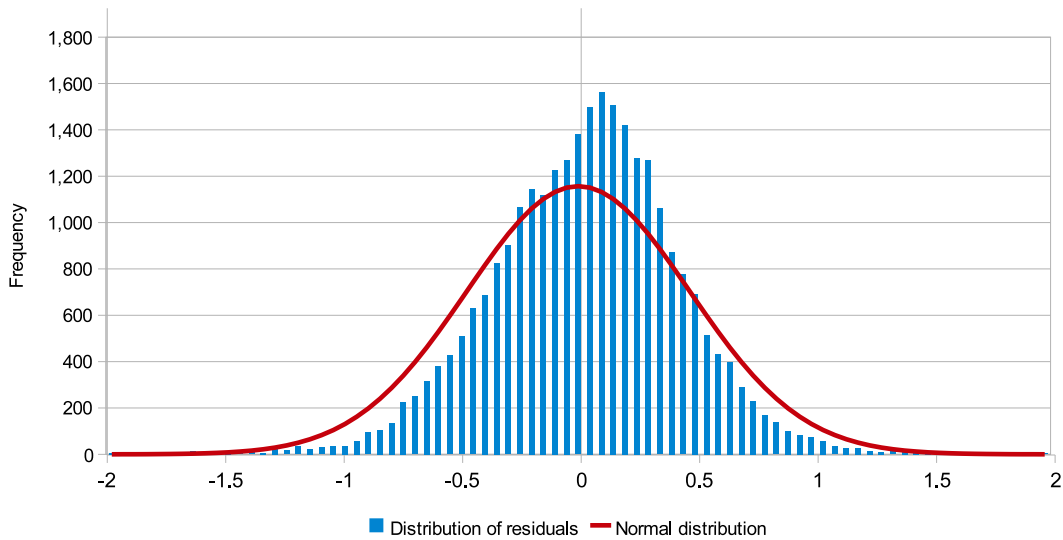


Figure 2: OLS regression residuals for log NL price

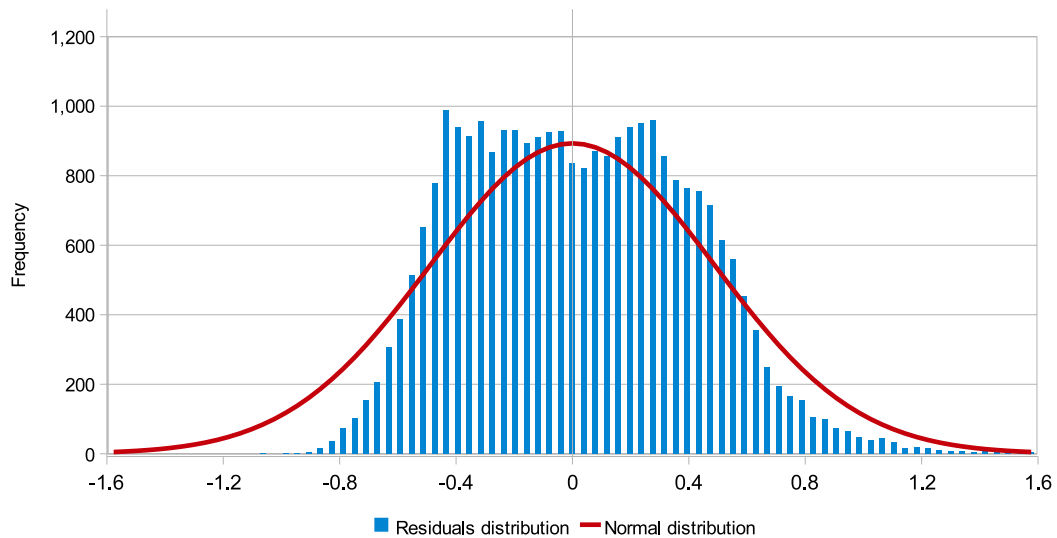


Figure 3: OLS regression residuals for log GB price

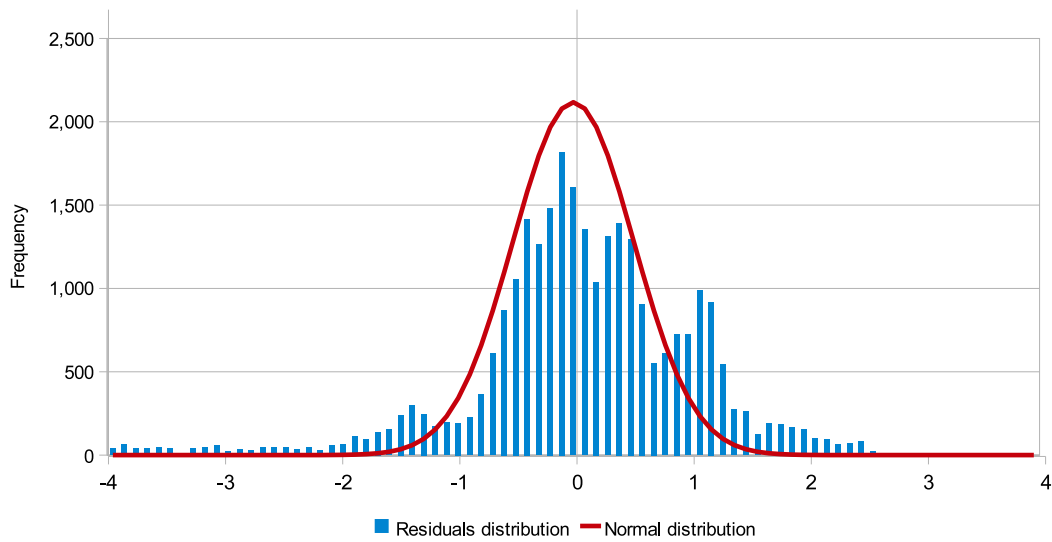


Figure 4: OLS regression residuals for log NO price

It is immediately apparent that, with the possible exception of residuals from the log NL price regression, these distributions are irregular and do not resemble any known parametric distribution.

Figure 5 demonstrates a potential reason why the distributions of OLS residuals for GB and NO prices have such irregular shapes. The autocorrelation functions for those residuals shows a very high degree of persistence even at lags of up to 200 periods. This means that any significant price shocks that are not explained by the exogenous variables in those regressions are likely to persist for a large number of periods, creating local peaks in the distribution of residuals.

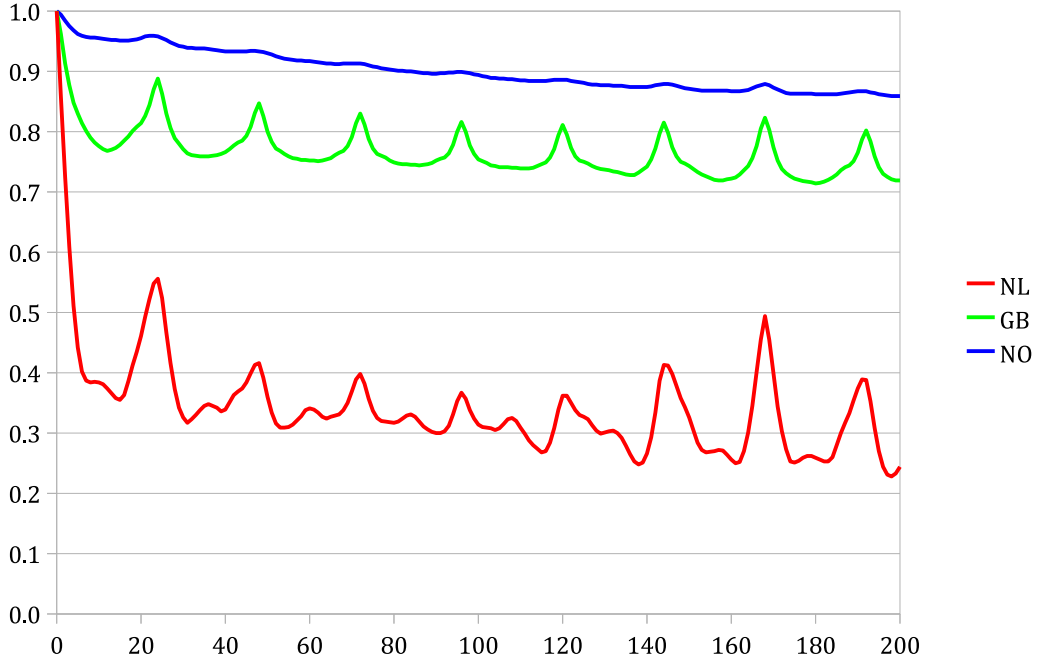


Figure 5: Autocorrelation functions of OLS residuals

Estimating the regressions using an ARMA in the disturbances model gives consistent and unbiased estimates of the deterministic components of electricity prices in the presence of auto-correlated residuals. The model is formulated as follows

$$y_t = \sum_{i=1}^K x_{ti} \beta_i + \mu_t$$

$$\mu_t = \sum_{p=1}^P \phi_p \mu_{t-p} + \sum_{q=1}^Q \theta_q \epsilon_{t-q} + \epsilon_t.$$

The first equation is a structural equation and the second equation specifies the ARMA structure of the disturbances. The explanatory variables in the structural equation are as in the original OLS regression. This model is estimated using conditional maximum likelihood, which, given the large number of observations, should yield the same results as unconditional maximum likelihood. Autoregressive terms in the residuals are included or excluded on the basis of the Akaike Information Criterion (AIC)<sup>5</sup>. Regression outputs for the three ARMA models, including

<sup>5</sup>The AIC is a method of model selection by which the model with the lowest AIC value is chosen as the best. The value of AIC is given by  $2k - 2\ln(L)$ , where  $k$  is the number of model parameters, i.e. explanatory variables and autoregressive terms, and  $\ln(L)$  is the maximum value of the log likelihood function of the estimated model

the estimated coefficients of the deterministic factors and their respective standard errors and confidence intervals are given below. Variable names are defined as follows. *NL*, *NO* and *GB* represent log hourly electricity prices. *H1 – H23* are hourly dummy variables. *mon – sat* are week day dummy variables. *jan – nov* are monthly dummy variables. *nlhol* and *gbhol* are dummy variables for public holidays in NL and GB. *norned* represents flows from NO to NL over the NorNed interconnector in units of 100 MW. *break* is a dummy variable assigned to any period after NorNed comes online. *res* represents the historic average reservoir level in South Norway corresponding to a given weekly period, expressed in percentage terms. *AR* and *MA* prefixes represent autoregressive and moving average terms in the residuals ARMA equation.

No. of observations 27,792      Dependent variable: NL(t)  
 Log likelihood 2161.8

	Coefficient	Standard error	P value	95% confidence interval	
<b>Structural Equation</b>					
H1	-0.140	0.022	0.000	-0.184	-0.096
H2	-0.302	0.038	0.000	-0.376	-0.227
H3	-0.451	0.049	0.000	-0.547	-0.356
H4	-0.620	0.059	0.000	-0.735	-0.505
H5	-0.661	0.065	0.000	-0.789	-0.533
H6	-0.462	0.069	0.000	-0.597	-0.328
H7	-0.295	0.069	0.000	-0.431	-0.159
H9	0.234	0.074	0.002	0.088	0.379
H10	0.398	0.077	0.000	0.247	0.549
H11	0.482	0.082	0.000	0.322	0.642
H12	0.565	0.084	0.000	0.400	0.730
H13	0.451	0.088	0.000	0.279	0.624
H14	0.407	0.090	0.000	0.231	0.583
H15	0.338	0.090	0.000	0.161	0.515
H16	0.253	0.091	0.006	0.074	0.431
H17	0.235	0.086	0.006	0.067	0.404
H18	0.360	0.077	0.000	0.209	0.512
H19	0.414	0.070	0.000	0.277	0.550
H20	0.363	0.062	0.000	0.241	0.485
H21	0.299	0.054	0.000	0.192	0.405
H22	0.171	0.044	0.000	0.086	0.257
H23	0.127	0.028	0.000	0.072	0.182
mon	0.041	0.023	0.080	-0.005	0.086
sat	-0.127	0.017	0.000	-0.160	-0.094
nlhol	-0.085	0.028	0.002	-0.140	-0.031
jan	0.253	0.065	0.000	0.125	0.381
feb	0.205	0.123	0.096	-0.036	0.447
mar	0.228	0.138	0.099	-0.043	0.498
apr	0.311	0.145	0.032	0.027	0.596
jun	0.350	0.149	0.019	0.058	0.642
jul	0.345	0.143	0.016	0.065	0.624
normed	-0.004	0.002	0.053	-0.008	0.000
constant	3.643	0.146	0.000	3.357	3.928
<b>ARMA Equation</b>					
AR1	0.801	0.001	0.000	0.799	0.803
AR4	-0.043	0.002	0.000	-0.046	-0.040
AR7	0.041	0.003	0.000	0.035	0.048
AR16	-0.020	0.005	0.000	-0.031	-0.009
AR17	0.041	0.005	0.000	0.031	0.051
AR21	0.024	0.003	0.000	0.017	0.030
AR23	0.043	0.003	0.000	0.038	0.048
AR24	0.064	0.002	0.000	0.059	0.069
AR26	-0.056	0.002	0.000	-0.061	-0.052
AR27	0.014	0.004	0.001	0.006	0.023
AR28	-0.017	0.004	0.000	-0.024	-0.010
AR48	0.042	0.004	0.000	0.034	0.050
AR49	-0.051	0.004	0.000	-0.059	-0.043
AR72	0.044	0.005	0.000	0.034	0.053
AR73	-0.031	0.005	0.000	-0.042	-0.021
AR96	0.032	0.005	0.000	0.022	0.042
AR97	-0.035	0.005	0.000	-0.045	-0.026
AR120	0.024	0.005	0.000	0.014	0.034
AR121	-0.019	0.005	0.000	-0.028	-0.009
AR143	0.060	0.001	0.000	0.058	0.063
AR144	-0.015	0.002	0.000	-0.019	-0.012
AR167	0.029	0.002	0.000	0.025	0.033
AR168	0.178	0.001	0.000	0.175	0.181
AR169	-0.154	0.002	0.000	-0.157	-0.150
constant	0.050	0.000	0.000	0.050	0.050

Figure 6: NL ARMA regression results



No. of observations 27,792      Dependent variable: GB(t)  
 Log likelihood 24,043

	Coefficient	Standard error	P value	95% confidence interval		
<b>Structural Equation</b>	H3	-0.101	0.061	0.097	-0.220	0.018
	H4	-0.147	0.071	0.037	-0.285	-0.009
	H5	-0.149	0.077	0.053	-0.301	0.002
	H9	0.241	0.077	0.002	0.091	0.391
	H10	0.323	0.074	0.000	0.178	0.467
	H11	0.404	0.072	0.000	0.264	0.545
	H12	0.421	0.069	0.000	0.285	0.557
	H13	0.350	0.067	0.000	0.218	0.482
	H14	0.294	0.065	0.000	0.166	0.423
	H15	0.261	0.064	0.000	0.136	0.387
	H16	0.322	0.062	0.000	0.201	0.443
	H17	0.432	0.059	0.000	0.316	0.548
	H18	0.592	0.057	0.000	0.481	0.704
	H19	0.575	0.053	0.000	0.471	0.679
	H20	0.425	0.048	0.000	0.331	0.519
	H21	0.321	0.040	0.000	0.242	0.400
	H22	0.206	0.030	0.000	0.147	0.265
	H23	0.126	0.017	0.000	0.092	0.160
	mon	0.055	0.009	0.000	0.037	0.072
	tue	0.081	0.012	0.000	0.058	0.105
	wed	0.093	0.013	0.000	0.067	0.119
	thu	0.079	0.013	0.000	0.053	0.104
	fri	0.045	0.012	0.000	0.021	0.068
	gbhol	-0.034	0.015	0.022	-0.064	-0.005
	jan	0.124	0.041	0.003	0.042	0.205
	feb	0.151	0.062	0.015	0.029	0.273
mar	0.167	0.069	0.015	0.032	0.302	
apr	0.140	0.079	0.075	-0.014	0.295	
constant	3.479	0.061	0.000	3.360	3.599	
<b>ARMA Equation</b>	AR1	0.947	0.004	0.000	0.940	0.954
	AR2	-0.092	0.003	0.000	-0.099	-0.085
	AR23	0.031	0.004	0.000	0.023	0.039
	AR24	0.375	0.005	0.000	0.365	0.385
	AR25	-0.360	0.003	0.000	-0.367	-0.354
	AR48	0.025	0.003	0.000	0.020	0.030
	AR72	0.023	0.003	0.000	0.018	0.028
	AR96	0.009	0.003	0.001	0.004	0.014
	AR120	0.008	0.003	0.006	0.002	0.013
	AR144	0.014	0.003	0.000	0.009	0.020
	AR167	0.038	0.004	0.000	0.030	0.047
	AR168	0.219	0.006	0.000	0.208	0.230
	AR169	-0.240	0.003	0.000	-0.246	-0.233
	constant	0.010	0.000	0.000	0.010	0.010

Figure 7: GB ARMA regression results

No. of observations 27,792      Dependent variable: NO(t)  
 Log likelihood 44,215

	Coefficient	Standard error	P value	95% confidence interval	
<b>Structural Equation</b>					
H2	-0.023	0.007	0.001	-0.036	-0.009
H3	-0.044	0.010	0.000	-0.063	-0.025
H4	-0.060	0.012	0.000	-0.083	-0.038
H5	-0.058	0.013	0.000	-0.084	-0.033
H6	-0.033	0.013	0.014	-0.058	-0.007
H8	0.030	0.013	0.022	0.004	0.056
H9	0.063	0.013	0.000	0.037	0.088
H10	0.067	0.013	0.000	0.041	0.092
H11	0.069	0.014	0.000	0.041	0.097
H12	0.065	0.017	0.000	0.032	0.098
H13	0.058	0.019	0.002	0.021	0.094
H14	0.049	0.019	0.010	0.012	0.086
H15	0.044	0.018	0.017	0.008	0.080
H16	0.040	0.017	0.022	0.006	0.074
H17	0.045	0.017	0.006	0.013	0.078
H18	0.054	0.016	0.001	0.023	0.086
H19	0.055	0.015	0.000	0.025	0.084
H20	0.048	0.014	0.001	0.021	0.076
H21	0.040	0.012	0.001	0.017	0.063
H22	0.035	0.009	0.000	0.018	0.053
H23	0.024	0.005	0.000	0.014	0.033
mon	0.012	0.004	0.001	0.005	0.018
tue	0.019	0.004	0.000	0.011	0.027
wed	0.024	0.004	0.000	0.016	0.032
thu	0.013	0.004	0.001	0.005	0.021
sat	0.010	0.002	0.000	0.005	0.014
may	-0.189	0.086	0.027	-0.356	-0.021
jun	-0.192	0.085	0.024	-0.360	-0.025
jul	-0.192	0.081	0.017	-0.351	-0.034
normed	0.005	0.000	0.000	0.005	0.006
break	-0.194	0.068	0.005	-0.327	-0.060
res	0.006	0.002	0.002	0.002	0.010
constant	3.210	0.233	0.000	2.754	3.666
<b>ARMA Equation</b>					
AR1	1.134	0.001	0.000	1.132	1.136
AR2	-0.249	0.002	0.000	-0.253	-0.246
AR3	-0.021	0.003	0.000	-0.027	-0.016
AR4	0.010	0.003	0.000	0.005	0.015
AR6	-0.015	0.003	0.000	-0.021	-0.009
AR7	0.042	0.003	0.000	0.035	0.048
AR9	0.028	0.004	0.000	0.021	0.036
AR10	-0.030	0.005	0.000	-0.039	-0.021
AR11	0.020	0.005	0.000	0.010	0.029
AR12	0.009	0.005	0.062	0.000	0.019
AR13	-0.010	0.003	0.002	-0.017	-0.004
AR14	0.023	0.004	0.000	0.016	0.030
AR16	-0.013	0.004	0.002	-0.021	-0.005
AR17	0.009	0.004	0.026	0.001	0.017
AR20	0.014	0.003	0.000	0.009	0.019
AR21	0.049	0.004	0.000	0.042	0.056
AR22	0.028	0.004	0.000	0.020	0.036
AR23	-0.060	0.003	0.000	-0.066	-0.054
AR24	0.120	0.003	0.000	0.114	0.126
AR25	-0.104	0.003	0.000	-0.110	-0.098
AR26	0.025	0.004	0.000	0.017	0.034
AR27	-0.017	0.003	0.000	-0.024	-0.010
AR47	0.012	0.001	0.000	0.009	0.014
AR49	-0.021	0.001	0.000	-0.024	-0.018
AR72	0.028	0.003	0.000	0.022	0.034
AR73	-0.025	0.003	0.000	-0.031	-0.019
AR95	0.009	0.002	0.000	0.006	0.012
AR119	-0.011	0.005	0.024	-0.020	-0.001
AR120	0.027	0.006	0.000	0.015	0.040
AR121	-0.026	0.004	0.000	-0.033	-0.018
AR144	0.035	0.003	0.000	0.030	0.040
AR145	-0.026	0.003	0.000	-0.032	-0.021
AR167	0.036	0.003	0.000	0.031	0.041
AR168	0.073	0.003	0.000	0.067	0.080
AR169	-0.104	0.002	0.000	-0.108	-0.100
MA50	0.063	0.002	0.000	0.059	0.066
MA51	0.050	0.002	0.000	0.045	0.055
MA52	-0.006	0.003	0.035	-0.012	0.000
MA169	0.043	0.003	0.000	0.037	0.048
constant	0.002	0.000	0.000	0.002	0.002

Figure 8: NO ARMA regression results

Although the effect of unexpected changes in fuel and European Union Emission Trading Sys-

tem (EU ETS) prices on electricity prices are classed as stochastic factors, for the purposes of scenario building and to give context to the simulations presented in Section 4.2, these effects are examined here. Daily as opposed to hourly data is used in regressions since gas, coal and EU ETS prices are quoted on a daily basis<sup>6</sup>. For the purposes of this analysis, hourly electricity prices are weighted by the system load for the corresponding hour and aggregated into daily prices.

The relationship between daily fuel prices and average daily electricity prices is determined by several factors. Since the price of electricity is set by the marginal plant, the cost to a marginal plant of converting a given type of fuel into electricity is one such factor. Another factor is whether the marginal generation technology is the same across peak and off-peak hours. Also, since varying fuel prices can change the merit order of generating plants, the degree of substitutability between different types of generation technology and the relationship between the prices of alternative fuels are other such factors. If coal becomes prohibitively expensive relative to gas and it is possible to satisfy all demand using gas fired generation, above a certain price threshold, the relationship between the price of electricity and the price of coal would be non-existent. However, if there is no spare capacity in the system, expensive coal would always be the marginal generation technology. In that case, coal and electricity prices could be expected to change proportionally<sup>7</sup>.

In the short-run, significant deviations can take place from the relationship described above. This can quite clearly be seen in Figure 9. It plots the average daily NL electricity price against daily NL gas, NL coal and EU ETS prices, all of which are indexed at the beginning of the sample period. While the NL electricity price tracks the gas price quite closely in the medium term, it is considerably more volatile even on a daily level, which masks a great deal of intra-day volatility in the electricity price. The same may be observed in Figure 10 for the average daily GB electricity price, though changes in the price of electricity track changes in GB coal and EU ETS prices as well as the price of gas.

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<sup>6</sup>Closing daily prices from the day-ahead market are used in each case and are sourced from Bloomberg. The APX gas price and the EEX coal price are used for NL. NBP gas and coal prices are used for GB

<sup>7</sup>All these factors may be characterised as medium term. Over a longer time horizon, investment makes different generation technologies almost perfect substitutes. However, investment takes time and investors need to be convinced that changes in fuel prices are permanent if the pattern of generation investment is to be influenced by those changes. Given the time span of the data set used in this paper, the long-run relationship between electricity prices and fuel prices is outside of the scope of our analysis

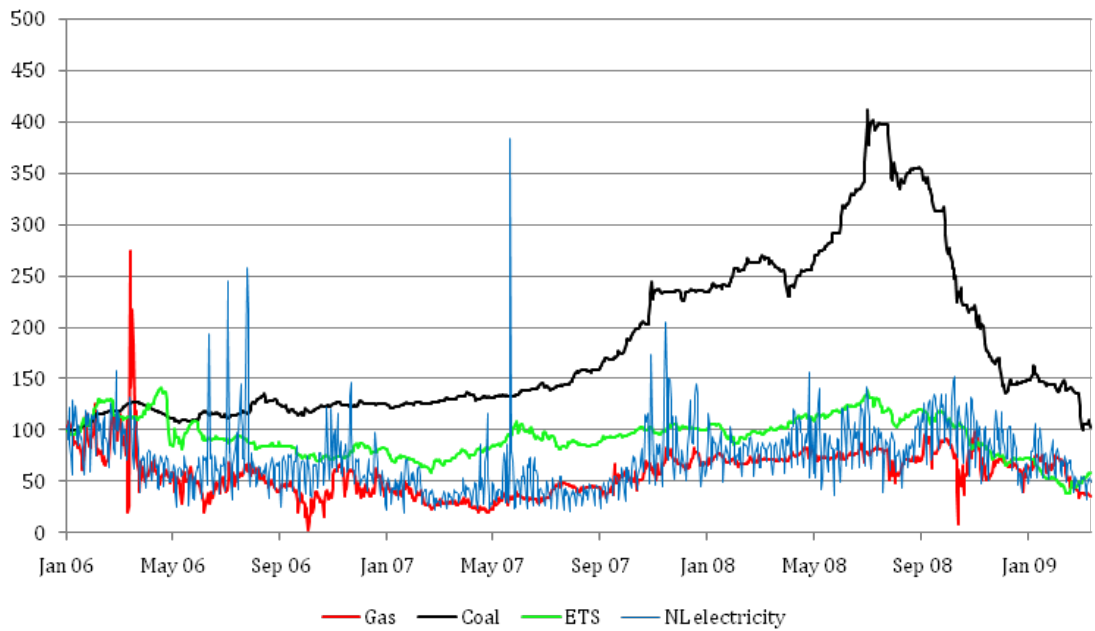


Figure 9: Daily NL electricity and fuel prices

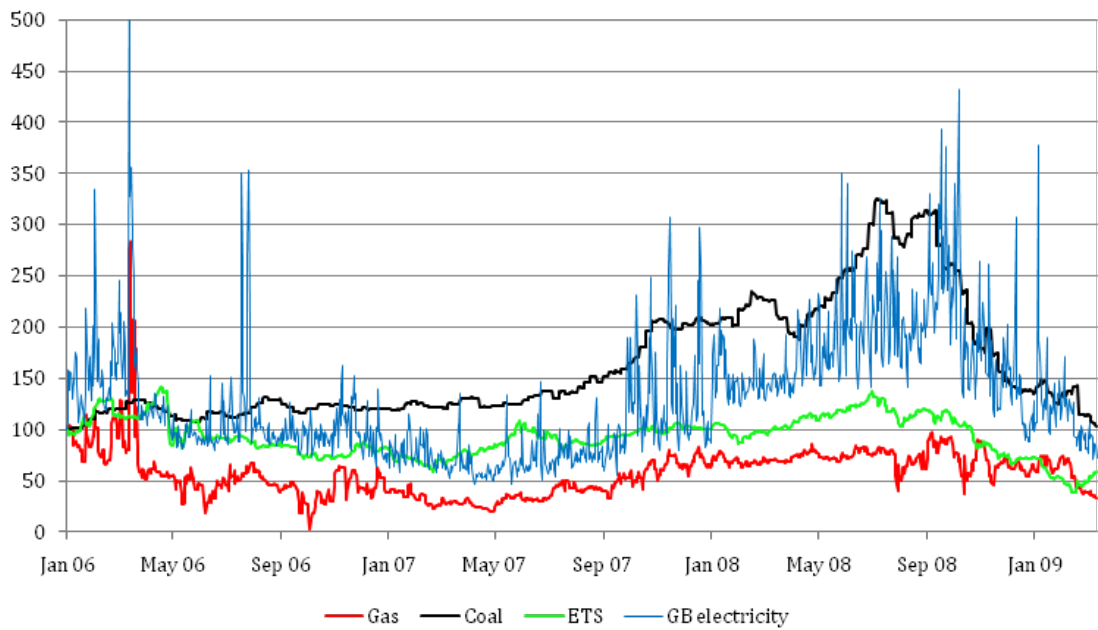


Figure 10: Daily GB electricity and fuel prices

The appropriate econometric technique would need to estimate the medium-term equilibrium relationship between fuel prices and electricity prices. This cannot be achieved using a standard regression methodology because it only captures contemporaneous co-movement in relevant prices. An alternative is available in the form of the Vector Error Correction Model (VECM), as proposed in Engle and Granger (1987). Among related applied papers which employ this model, Brown and Yucel (2008) use it to estimate the long-run relationship between crude oil and natural gas prices in the US. The VECM allows a relationship between two or more variables that may themselves be non-stationary to be estimated consistently. Depending on the extent of autocorrelation specified in the model, it allows for prolonged departures from the equilibrium relationship. The model is specified by the following equations:

$$P_{E,t} = \alpha + \beta P_{F,t} + \epsilon_t$$

$$\Delta P_{E,t} = a + \gamma(CI_{t-1}) + \sum_{i=1}^n b_i \Delta P_{E,t-i} + \sum_{i=1}^n c_i \Delta P_{F,t-i} + \mu_t$$

$$\Delta P_{F,t} = d + \delta(CI_{t-1}) + \sum_{i=1}^n e_i \Delta P_{F,t-i} + \sum_{i=1}^n f_i \Delta P_{E,t-i} + \nu_t$$

The first equation represents the cointegrating relationship.  $P_{E,t}$  and  $P_{F,t}$  represent log electricity and fuel prices in period  $t$  respectively and corresponding  $\Delta$  represent their first differences.  $CI_t = \epsilon_t$  is the period  $t$  equilibrium error in the cointegrating relationship as specified in the first equation.

The first step is to determine whether the relevant variables are stationary. This is done using the Elliott-Rothenberg-Stock efficient test under the null hypothesis of a unit root. This test is similar to the Augmented Dickey-Fuller test but is adjusted for heteroskedastic errors and allows for a trend in a stationary time series. The test is applied at lag 1 and also at an optimal lag chosen by the Ng-Perron sequential t-test. Table 1 gives the test results. The highlighted test statistics indicate that the corresponding variable is not stationary, i.e. the null hypothesis of a unit root cannot be rejected at the 95% confidence level.

Variable	Test statistic at lag 1	Ng-Perron optimal lag	Test statistic at optimal lag	5% critical value at lag 1	5% critical value at optimal lag
NL electricity	-9.834	21	-1.612	-2.855	-2.829
GB electricity	-7.279	20	-1.772	-2.855	-2.830
NL gas	-4.165	22	-1.480	-2.855	-2.828
GB gas	-3.356	21	-1.573	-2.855	-2.829
NL coal	0.796	19	-0.460	-2.855	-2.832
GB coal	0.317	21	-0.349	-2.855	-2.829
EU ETS	-1.748	16	-1.756	-2.855	-2.836

Table 1: Unit root tests

For both sets of coal prices and the EU ETS price, the results are clear-cut. The null hypothesis of a unit root cannot be rejected. However, for both sets of electricity and gas prices, the results are less clear-cut. The null hypothesis of a unit root is rejected at lag 1, but cannot be rejected at the corresponding optimal lag chosen by the Ng-Perron sequential test. This means that electricity and gas prices are likely to lie on the borderline between a unit root and a stationary but strongly autocorrelated time series.

Error Correction Models are able to estimate the relationship between two or more non-stationary variables consistently only if those variables are cointegrated, which means that the relationship between them is actually stationary<sup>8</sup>. Likewise, they would give consistent estimates of the relationship between two or more strongly autocorrelated but stationary variables. This is shown in Keele and De Boef (2004), who derive the Error Correction Model from the Autoregressive Distributed Lag (ADL) model as in Bannerjee (1993) and Davidson and MacKinnon (1993) to prove this result. To check where the VECM model may be used with NL and GB data, the next step is to apply the Johansen test for cointegration in a pairwise fashion. Failure to reject the null hypothesis of a cointegrating relationship would indicate that the relationship between two variables is not stationary. Among other reasons, this could happen if one of the variables is stationary and the other one is not since the relationship between a stationary variable and a non-stationary one is non-stationary by construction.

Table 2 lists the results of the Johansen test for pairwise cointegrating relationships. The table gives Johansen test statistics and the 5% critical value in brackets for each relevant pairwise relationship<sup>9</sup>. The highlighted statistics are those where the 5% critical value for a cointegrating relationship has been exceeded.

	NL electricity	GB electricity
NL gas	6.51 (3.76)	N/a
NL coal	1.52 (3.76)	N/a
EU ETS	3.56 (3.76)	3.34 (3.76)
GB gas	N/a	5.48 (3.76)
GB coal	N/a	1.66 (3.76)

Table 2: Johansen test

Results of the test suggest that the relationship between log electricity and log gas prices can be consistently estimated for both NL and NO. However, estimates of the relationship between log

<sup>8</sup>See Chapter 19 in Hamilton, J.D. (1994), "Time Series Analysis," *Princeton University Press*

<sup>9</sup>All tests are carried out using lags of up to order 13. This number was chosen on the basis of the Akaike Information Criterion (AIC)

electricity prices and log coal prices or log EU ETS prices derived from an Error Correction Model may not be consistent. Analysis of these relationships would only be possible after differencing one or both variables to make them stationary. Unfortunately, much of the information would be lost in the process of differencing, rendering the subsequent analysis meaningless.

A simple test for the presence of error correcting behaviour, as suggested in Keele and De Boef, is to estimate an ADL model<sup>10</sup>, given by

$$P_{E,t} = \alpha_0 + \sum_{i=1}^n \alpha_i P_{E,t-i} + \sum_{j=0}^m \beta_j P_{E,t-j} + \epsilon_t$$

and then test the joint significance of coefficients of the lagged electricity price and current and lagged gas prices. A positive result would indicate the presence of error correcting behaviour. For both NL and GB regressions, testing for error correcting behaviour in the relationship between electricity and gas prices in NL and GB gives a positive result.<sup>11</sup>

The next step is to estimate pairwise cointegrating relationships between electricity prices and gas prices in GB and NL. The full results of this exercise may be seen in Figures 11 and 12.

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<sup>10</sup>Estimates from the ADL model are consistent as long as the residuals in that regression are iid

<sup>11</sup>Test conducted at the 95% confidence level. Various configurations of the ADL model yield this result, the most parsimonious being one that uses a first order lag of the electricity price and a seventh order lag of the gas price.

		No. of observations	1,152			
		Log likelihood	727.1			
		AIC	-1.156			
		Coefficient	Standard error	P value	95% confidence interval	
<b>Cointegrating equation</b>	Dependent variable: NL(t)					
	NLG (t)	0.902	0.102	0.000	0.703	1.102
	Constant	0.403	-	-	-	-
<b>Equation 1</b>	Dependent variable: $\Delta$ NL(t)					
	CI (t-1)	0.092	0.031	0.003	0.032	0.153
	$\Delta$ NL(t-1)	0.298	0.041	0.000	0.218	0.378
	$\Delta$ NL(t-2)	0.376	0.041	0.000	0.296	0.456
	$\Delta$ NL(t-3)	0.245	0.041	0.000	0.164	0.327
	$\Delta$ NL(t-4)	0.317	0.041	0.000	0.236	0.398
	$\Delta$ NL(t-5)	0.291	0.042	0.000	0.210	0.373
	$\Delta$ NL(t-6)	0.212	0.042	0.000	0.130	0.294
	$\Delta$ NL(t-7)	-0.116	0.042	0.006	-0.198	-0.033
	$\Delta$ NL(t-9)	0.083	0.041	0.042	0.003	0.163
	$\Delta$ NL(t-10)	0.118	0.039	0.002	0.042	0.194
	$\Delta$ NL(t-11)	0.110	0.036	0.002	0.039	0.182
	$\Delta$ NL(t-12)	0.156	0.035	0.000	0.088	0.225
	$\Delta$ NL(t-13)	0.125	0.032	0.000	0.063	0.188
	$\Delta$ NL(t-14)	-0.162	0.030	0.000	-0.220	-0.104
$\Delta$ NLG(t-11)	0.087	0.044	0.045	0.002	0.173	
Constant	0.002	0.006	0.743	0.010	0.014	
<b>Equation 2</b>	Dependent variable: $\Delta$ NLG(t)					
	CI (t-1)	-0.097	0.024	0.000	-0.144	-0.050
	$\Delta$ NL(t-1)	0.067	0.032	0.035	0.005	0.130
	$\Delta$ NLG(t-1)	0.306	0.035	0.000	0.238	0.374
	$\Delta$ NLG(t-2)	0.295	0.036	0.000	0.226	0.365
	$\Delta$ NLG(t-4)	0.110	0.036	0.002	0.039	0.181
	$\Delta$ NLG(t-5)	0.080	0.036	0.026	0.009	0.150
	$\Delta$ NLG(t-6)	0.082	0.036	0.023	0.011	0.152
	$\Delta$ NLG(t-8)	0.072	0.035	0.041	0.003	0.141
	$\Delta$ NLG(t-9)	0.071	0.035	0.042	0.002	0.140
	$\Delta$ NLG(t-10)	0.103	0.035	0.003	0.036	0.171
Constant	0.002	0.005	0.690	0.007	0.011	

Figure 11: Results: NL Vector Error Correction Model



		No. of observations	1,154			
		Log likelihood	1116.0			
		AIC	-1.842			
		<b>Coefficient</b>	<b>Standard error</b>	<b>P value</b>	<b>95% confidence interval</b>	
<b>Cointegrating equation</b>	Dependent variable: GB(t)					
	GBG (t)	1.083	0.097	0.000	0.892	1.273
	Constant	-0.266	-	-	-	-
<b>Equation 1</b>	Dependent variable: ΔGB(t)					
	CI (t-1)	0.084	0.027	0.002	0.031	0.137
	ΔGB(t-1)	0.326	0.038	0.000	0.253	0.400
	ΔGB(t-2)	0.305	0.039	0.000	0.229	0.380
	ΔGB(t-3)	0.273	0.039	0.000	0.197	0.350
	ΔGB(t-4)	0.255	0.039	0.000	0.178	0.332
	ΔGB(t-5)	0.256	0.040	0.000	0.178	0.334
	ΔGB(t-6)	0.162	0.040	0.000	0.083	0.241
	ΔGB(t-8)	0.064	0.038	0.096	0.011	0.139
	ΔGB(t-9)	0.101	0.037	0.006	0.029	0.173
	ΔGB(t-10)	0.114	0.035	0.001	0.046	0.183
	ΔGB(t-11)	0.071	0.033	0.031	0.007	0.135
	ΔGB(t-12)	0.174	0.030	0.000	0.115	0.233
	Constant	0.001	0.005	0.819	0.009	0.012
<b>Equation 2</b>	Dependent variable: ΔGBG(t)					
	CI (t-1)	-0.071	0.019	0.000	-0.107	-0.034
	ΔGB(t-8)	0.056	0.026	0.032	0.005	0.108
	ΔGBG(t-1)	0.191	0.034	0.000	0.124	0.257
	ΔGBG(t-2)	0.089	0.034	0.009	0.022	0.157
	ΔGBG(t-3)	0.079	0.034	0.021	0.012	0.147
	ΔGBG(t-7)	0.100	0.033	0.002	0.036	0.165
	ΔGBG(t-8)	0.094	0.033	0.004	0.030	0.158
Constant	0.001	0.004	0.691	0.006	0.009	

Figure 12: Results: GB Vector Error Correction Model

We are interested in the medium-term co-movement between electricity prices and gas prices. This is given by the two cointegrating equations as follows:

$$NL_t = 0.40 + 0.90NLG_t + \epsilon_t$$

$$GB_t = -0.27 + 1.08GBG_t + \epsilon_t.$$

Prices in the cointegrating equation are in logs, hence  $\beta = 1$  would imply that the two prices change proportionally with one another in the long run. Whilst all of the above parameters

are significant at the 95% confidence level, they should be interpreted carefully. No definitive statements about the direction of causality can be made without further tests because prices are driven by both supply and demand. Demand for electricity and prices of alternative fuels are all determinants of demand for a given type of fuel. At the same time, fuel prices are determinants of supply of electricity.

On the basis of the estimates derived by the VECM model, it is possible to say with some confidence that, both in the GB and NL, electricity and gas prices move roughly in line with one another in the medium term. What can be surmised from Figures 9 and 10 is that the NL electricity price tracks the NL gas price very closely and does not appear to be influenced by the price of coal. On the other hand, the GB electricity price takes more prolonged departures from equilibrium with the electricity price<sup>12</sup> and does appear to be influenced by the price of coal, though the latter assertion cannot be tested econometrically.

### 3.3 Bootstrapping stochastic components of prices

Estimates of deterministic properties of electricity prices can be made using the ARMA regression methodology as described in Section 3.2. It is more difficult to establish the distributional properties of the stochastic elements of electricity prices, represented by  $\mu_t$  in the ARMA model set out in Section 3.2. The reason is that autocorrelation in  $\mu_t$  produces an irregular distribution unless the sample size is extremely large and way beyond the available sample size.

Residuals from the ARMA model, represented by  $\epsilon_t$ , are serially uncorrelated, have a regular shape and can be approximated by a parametric distribution. However, it is not good enough to simulate  $\epsilon_t$  and then apply the serial correlation structure implied by the fitted ARMA model to produce  $\mu_t$ . Fundamentally, it would not help to answer the question of what distribution the future stochastic price shocks would likely be drawn from. This methodology would effectively only produce one observation since the serial correlation structure is only applied once. It is possible to approximate the true underlying distribution of residuals by repeating this process many times for randomly selected  $\epsilon_t$ , but given that experiments require altering the parameters of the simulated distribution, the computational burden of applying this methodology several times and in several experiments would be overwhelming.

A computationally viable alternative lies in using a variation on the bootstrap methodology. Bootstrapping, which was first introduced by Efron (1979), is a way of artificially generating a

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<sup>12</sup>This observation is confirmed by plotting the autocorrelation functions for  $\epsilon_t$ , the residual in the cointegrating equation between electricity and gas prices in NL and GB, which may be seen in Appendix B. For GB, the residuals show a greater degree of autocorrelation, which would correspond to more prolonged departures from the equilibrium relationship between GB electricity and GB gas prices

population out of a sample by repeated random sampling with replacement from an empirical distribution of the observed data. The bootstrap is asymptotically consistent as long as the underlying population is i.i.d., the observed data is independently sampled from the population and samples taken from the empirical distribution of the observed data are independent. This methodology is most frequently used to generate confidence intervals for statistical inference, but it is easily extended to generating a sample distribution for the purposes of simulation.

Efron's standard bootstrap would fail for dependent data because it ignores the order of observations in resampling. More specifically, it is inappropriate for the simulations undertaken in this paper because the stochastic determinants of electricity prices are serially correlated, thus violating the i.i.d. assumption. However, dependence is a ubiquitous feature of time series models and solutions to this problem exist. There are two common approaches to bootstrapping dependent data. The first approach involves modelling serially uncorrelated disturbances together with a model of dependence. Bose (1988) demonstrates that, in the context of an autoregressive model, bootstrapping the serially uncorrelated disturbances and applying the model of dependence produces the same level of correction as a standard bootstrap for i.i.d. data. However, this methodology is sensitive to model specification and thus does not possess the non-parametric property of the standard bootstrap. An alternative approach known as the block bootstrap was pioneered by Kunsch (1989). It involves resampling blocks of sequential observations from the original sample, thus preserving the serial correlation within those blocks but destroying it between blocks. Careful selection of block size, which would depend on the nature of serial correlation and the sample size, should ensure that enough of the essential features of the original sample are preserved in the process. This approach is non-parametric but risks neglecting some key features of the original sample if the block size chosen is too small, that choice being rather ad hoc.

A combination of the two approaches detailed above involves approximating an infinite dimensional non-parametric model by a sequence of finite-dimensional parametric models. This is known as the method of sieves, as detailed in Buhlmann (1997) and also described in Politis (2003). The method of sieves is adopted in this paper. To begin with, an autoregressive process is fitted to the dependent data. In the context of this paper, the dependent data is represented by the serially correlated residuals  $\mu_t$  in the ARMA model of electricity prices. Define the order of the autoregressive process as  $p(n)$ , where  $n$  is the size of the entire sample of dependent data. The order  $p(n)$  must be increasing with sample size  $n$  such that  $p(n) \rightarrow \infty$  as  $n \rightarrow \infty$  with  $p(n) = o(n)$ . This implies that  $p(n)/n \rightarrow 0$  as  $n \rightarrow \infty$ . Buhlmann (1997) suggests that selecting the autoregressive model on the basis of the AIC satisfies this condition.

Thereafter, resampling is based on the autoregressive process fitted to the serially correlated residuals  $\mu_t$  as follows. Residuals  $\epsilon_t$  from the autoregressive process are given by

$$\epsilon_t = \sum_{j=0}^{p(n)} \phi_j (\mu_{t-j} - \bar{\mu})$$

where  $\phi_0 = 1$  and  $t = p + 1, \dots, n$ . This means that the first  $p$  residual values from the fitted regression are thrown away. The remaining residuals are centred such that

$$\hat{\epsilon}_t = \epsilon_t - (n - p)^{-1} \sum_{t=p+1}^n \epsilon_t.$$

Resampling is done from the empirical distribution of  $\hat{\epsilon}_t$ , defining the bootstrap sample  $\mu_t^b$  by the recursion

$$\tilde{\epsilon}_t = \sum_{j=0}^{p(n)} \phi_j (\mu_{t-j}^b - \bar{\mu}),$$

where  $\tilde{\epsilon}_t$  is sampled with replacement from  $\hat{\epsilon}_t$ . In other words, resampling is carried out by repeatedly applying the fitted autoregressive model to blocks of randomly sampled i.i.d. ARMA residuals  $\tilde{\epsilon}_t$  to produce samples characterised by autocorrelation as specified in the fitted autoregressive model. For every such block, the initial value is assumed to be equal to  $\tilde{\epsilon}_0$ . Buhlmann finds that this methodology yields a conditionally stationary bootstrap sample  $\mu_t^b$  and, unlike the block bootstrap, does not exhibit artifacts in the dependence structure due to the dependence between different blocks being neglected. By comparing the method of sieves against the block bootstrap, Buhlmann (2002) finds that that the former has better finite sample properties than the latter when dependence in the underlying sample is sufficiently weak.

Buhlmann (1997) shows that the sieve bootstrap yields consistent estimates of the first two moments of the sample mean under relatively weak conditions on the stationarity of the process that generated the sample. Under slightly stronger assumptions, the sieve bootstrap also yields consistent properties of the full distribution from which the sample is drawn. Full details of the relevant conditions are given in the paper and are not reproduced here.

The method of sieves works well in the context of this paper because it minimises the computational burden of the bootstrap. Adopting a relatively large block size of 2,190 periods, or one quarter of a calendar year, ensures that the properties of price series due to dependence are captured adequately. The bootstrap involves randomly sampling with replacement from ARMA model residuals for each price series and applying the estimated serial correlation 2,190 times to blocks consisting of 2,190 observations each, producing a total of 4,796,100 observations. The resulting histograms are plotted below.

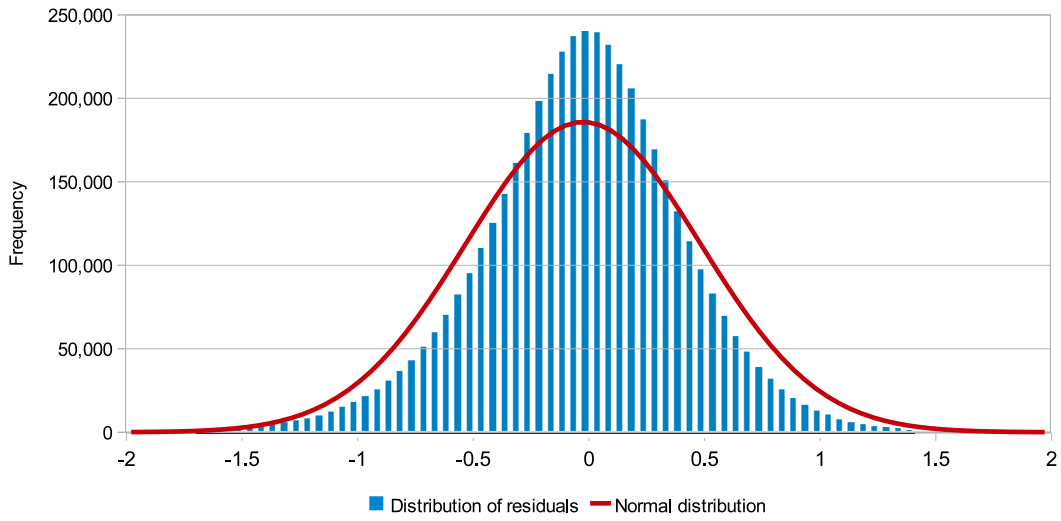


Figure 13: Bootstrap residuals for log NL price

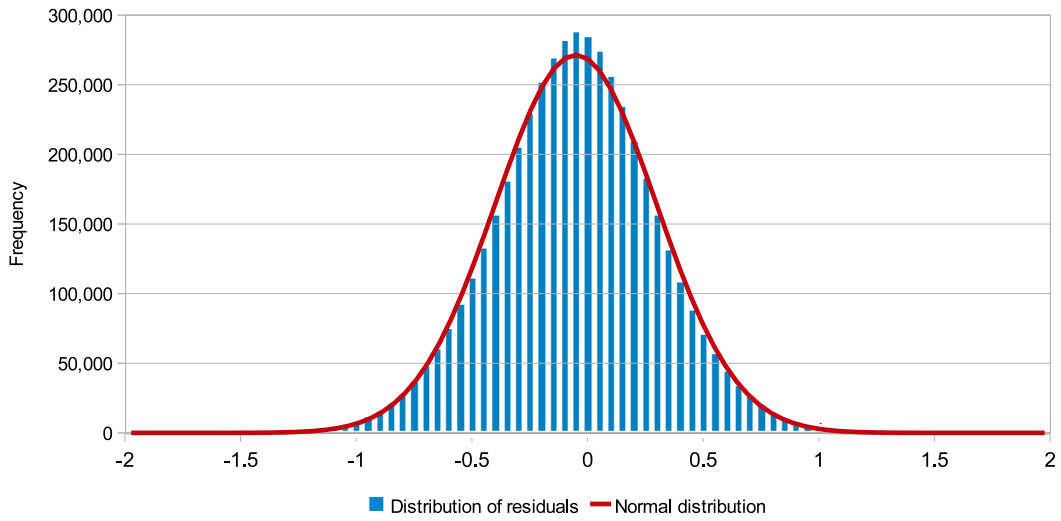


Figure 14: Bootstrap residuals for log GB price

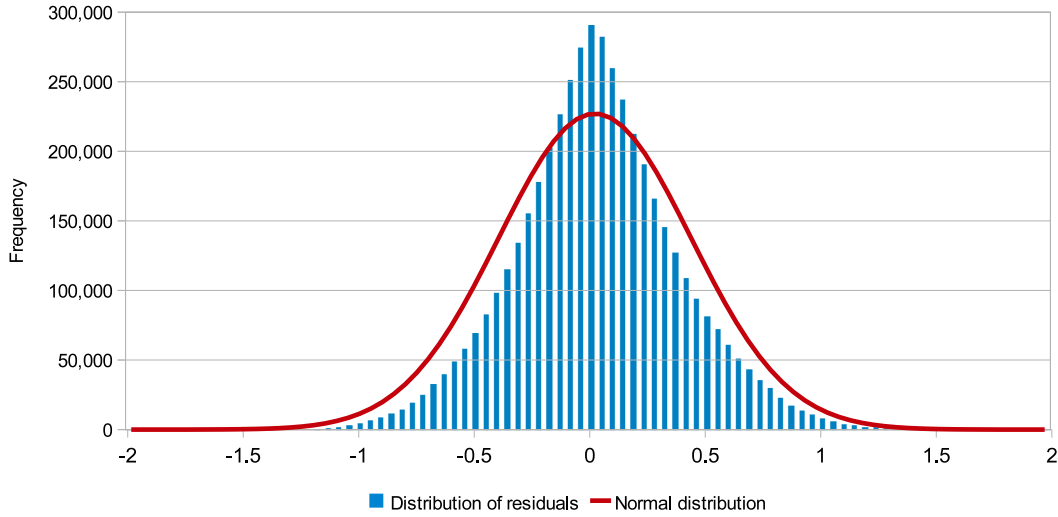


Figure 15: Bootstrap residuals for log NO price

It can be seen by inspection that all three histograms are symmetric and have a regular shape<sup>13</sup>. The distribution of stochastic shocks to the GB price as estimated by bootstrapping is very well approximated by the normal distribution, displaying only marginally more kurtosis<sup>14</sup>. The estimated distributions of stochastic shocks to the NL and NO prices are characterised by a considerably greater level of kurtosis. Although they both have regular shapes, they are not well approximated by the normal distribution.

### 3.4 Price simulation

#### 3.4.1 Calibrate distributions of residuals

For the purposes of simulating the stochastic component of GB prices, the normal distribution is employed as per the results of bootstrapping in Section 3.3. Since the properties of the normal

<sup>13</sup>The graphs are centered around zero, though the bootstrapped residuals  $\mu_t^b$  may have a mean that is trivially different from zero. For simulation purposes, the mean of bootstrapped residuals is calibrated to zero simply by subtracting the actual mean of the entire sample of bootstrapped residuals from every observation and then adjusting the value of the constant for the simulated price accordingly

<sup>14</sup>Kurtosis is a measure of “peakiness” of a probability distribution. Higher kurtosis indicates that more of the variance is accounted for by large infrequent deviations from the mean. More formally, kurtosis is given by  $(E[(X - E[X])^4]) / \sigma^4$ , where  $X$  is a real valued random variable and  $\sigma$  is the standard deviation

distribution are defined by its first two moments, the stochastic component of GB prices can be modelled by specifying the mean and variance of the distribution of bootstrapped residuals  $\mu_t^b$  from the ARMA model of the log GB price. In order to model the distributional properties of the stochastic components of NO and NL prices, a more complex procedure is required. The distributions of bootstrapped residuals  $\mu_t^b$  from ARMA models of log NL and NO prices are plotted in Figures 13 and 15 respectively. Although each distribution appears to have a regular shape and is symmetric, they both display a high level of kurtosis not characteristic of the normal distribution. The Weibull distribution possesses the property of having a very long tail and a sharp peak for some parameter values<sup>15</sup>. However, it is defined over the positive domain only. Nevertheless, it is possible to use this distribution to model positive and negative values separately. For negative values, its mirror image can be created simply by multiplying all values by  $-1$ .

We will create a symmetric composite distribution from two Weibull distributions, modelling the positive and negative halves of the composite distribution separately. The first step in the calibration process is to take absolute values of all observations of bootstrapped residuals  $\mu_t^b$ . This produces an asymmetric distribution which takes values over the positive domain only. The second step is to calibrate the Weibull distribution to the resulting sample by working out the Weibull parameters that maximise its log likelihood functions given the sample data. The log likelihood function of the Weibull distribution is given by

$$l_W = T(\ln(k) - k\ln(\lambda)) + (k-1) \sum_{t=1}^T \ln(x_t) - \sum_{t=1}^T \left(\frac{x_t}{\lambda}\right)^k,$$

where  $k > 0$  is the shape parameter and  $\lambda > 0$  is the scale parameter. The constant is not specified because it drops out when the function is maximised. The maximisation is carried out numerically in Matlab and distribution parameters that represent the best fit are generated.

Denote by  $X$  the random variable characterised by the composite distribution and by  $X^+$  and  $X^-$  the positive and negative components of  $X$  respectively.  $X^+$  will have a Weibull distribution and  $X^-$  will have a 'negative' Weibull distribution, being a mirror image of the distribution of  $X^+$  around the mean. It follows that

$$X = \Pr(X \leq 0)X^- + \Pr(X > 0)X^+.$$

The cumulative distribution function (cdf) for  $X$  is then given by

$$\Pr(X \leq x) = \Pr(X \leq 0) \cdot \Pr(X^- \leq x) + \Pr(X > 0) \cdot \Pr(X^+ \leq x),$$

where  $\Pr(X^- \leq x)$  and  $\Pr(X^+ \leq x)$  are cdfs for positive and negative components of  $X$  respectively. This can be summarised as

$$\Pr(X \leq x) = \begin{cases} \Pr(X \leq 0) \cdot \Pr(X^- \leq x) & \text{if } x \leq 0 \\ \Pr(X \leq 0) + \Pr(X > 0) \cdot \Pr(X^+ \leq x) & \text{if } x > 0 \end{cases}.$$

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<sup>15</sup>The exponential distribution can also display those properties, but it is in fact a special case of the Weibull distribution

Since the composite distribution models regression residuals, it will have a zero mean by construction. Therefore  $\Pr(X \leq 0) = \frac{1}{2}$  and the above expressions can be simplified to

$$\Pr(X \leq x) = \begin{cases} \frac{1}{2} \Pr(X^- \leq x) & \text{if } x \leq 0 \\ \frac{1}{2} + \frac{1}{2} \Pr(X^+ \leq x) & \text{if } x > 0 \end{cases}.$$

The positive side of the composite distribution is modelled as a Weibull, hence  $\Pr(X^+ \leq x)$  is given by the cdf of the Weibull, i.e.

$$\Pr(X^+ \leq x) = 1 - \exp\left\{-\left(\frac{x}{\lambda}\right)^k\right\} \quad \text{if } x > 0.$$

Since the negative side of the composite distribution is modelled as a negative Weibull,  $\Pr(X^- \leq x)$  is given by one minus the cdf of the Weibull, taking the absolute value of  $x$ , i.e.

$$\Pr(X^- \leq x) = \exp\left\{-\left(\frac{|x|}{\lambda}\right)^k\right\} \quad \text{if } x \leq 0.$$

This can be easily seen by looking at the two extremes of the possible values of  $x$  on the negative side of the composite distribution. When  $x$  is a large negative number and given that the parameters  $\lambda$  and  $k$  of the Weibull distribution are always positive,  $|x|$  is a large positive number and  $\Pr(X^- \leq x)$  as defined above is close to zero. When  $x$  is a small negative number,  $|x|$  is a small positive number and  $\Pr(X^- \leq x)$  is close to one. Hence, for the composite distribution

$$\Pr(X \leq x) = \begin{cases} \frac{1}{2} \exp\left\{-\left(\frac{|x|}{\lambda}\right)^k\right\} & \text{if } x \leq 0 \\ 1 - \frac{1}{2} \exp\left\{-\left(\frac{x}{\lambda}\right)^k\right\} & \text{if } x > 0 \end{cases}.$$

A check on the suitability of the calibration process, as suggested in Nelson (1982), is to test whether the bootstrapped residuals conform to the calibrated composite Weibull in the NL and NO cases or the Normal representation in the GB case. Suppose that a random variable  $X$  has a continuous probability distribution  $f$  and its associated cdf is given by  $F$ . Applying the probability integral transform

$$U = F(X)$$

generates a variable  $U$  which is uniformly distributed<sup>16</sup>. This means that for the symmetric composite distribution centered around zero that is modelled here,  $\Pr(X \leq x)$  is also a uniformly distributed random variable and

$$\Pr(X \leq x) \in [0, 1/2] \quad \text{for } x \leq 0$$

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<sup>16</sup>See Dodge, Y. (2003), *The Oxford Dictionary of Statistical Terms*, Oxford University Press



$$\Pr(X \leq x) \in (1/2, 1] \text{ for } x > 0.$$

If the calibration of bootstrapped residuals  $\mu_i^b$  is accurate, applying the probability integral transform with the calibrated distributional parameters to the bootstrapped data will produce uniformly distributed data.

The test is based on the property of the cdf of a uniform distribution, which is simply a 45° line. The probability integral transform is applied separately for the positive and negative sides of the distribution of bootstrapped residuals modelled by the composite Weibull distribution and in a single step for bootstrapped GB price residuals. The resulting observations are then sorted in ascending order and graphed against a 45° line. If the resulting graph does not differ significantly from a straight 45° line, this indicates that the calibration process successfully mimics the properties of the data.

The results of the test for each set of bootstrapped residuals may be seen in Appendix C. In each case, the calibration process appears to successfully mimic the properties of the underlying data.

### 3.4.2 Obtain simulated prices

For the purposes of simulating interconnectors, it is important to be able to specify dependence between stochastic shocks in the connected markets. Such dependence could arise from factors such as shocks caused by variable wind output in neighbouring markets, which are likely to be correlated. The dependence between any two random variables is often described by the linear correlation coefficient. However, linear correlation is not preserved when a non-linear transformation is applied to one or both of those random variables. This is too restrictive as simulating stochastic price shocks will involve applying the probability integral transform, set out in Section 3.4.1, which is a non-linear transformation. A viable alternative to linear correlation exists in rank correlation. Rank correlation is preserved under any monotone transformation<sup>17</sup> and is invariant to the choice of marginal distribution.

Spearman's rank correlation coefficient, denoted by  $\rho$ , is a non-parametric measure of statistical dependence between two variables. It assesses how well the relationship between two random variables  $X_i$  and  $Y_i$  can be described using a monotonic function. It is calculated by converting raw scores  $X_i$  and  $Y_i$  to ranks  $x_i$  and  $y_i$ . If no two ranks are tied<sup>18</sup>, Spearman's rank correlation

<sup>17</sup>The probability integral transform and its inverse are monotone transformations

<sup>18</sup>Tied ranks do not occur in the simulations carried out in this paper since they are zero probability events with continuous distributions

coefficient is given by

$$\rho = 1 - \frac{6 \sum_{i=1}^n (x_i - y_i)^2}{n(n^2 - 1)}.$$

When simulating stochastic price shocks, the dependence structure is specified using Spearman's rank correlation. The methodology set out in this section refers to simulating two sets of stochastic price shocks, but is easily generalised to simulating any number of interdependent random variables. Whilst random draws from a multivariate normal distribution can be generated directly by many appropriate software packages, most cannot generate random draws from a multivariate Weibull distribution<sup>19</sup> or specify a dependence structure between random draws from different distributions. However, in the context of this paper, both would be required in order to accurately simulate the effect of connecting NO, NL and GB markets. This is achieved as set out below.

Suppose that we would like to simulate the stochastic shocks for NO and GB prices. The former is characterised by a composite distribution as set out in Section 3.4.1 and the latter is characterised by a normal distribution with mean  $\mu_1$  and standard deviation  $\sigma_1$ . Suppose further that the Spearman rank covariance factor for the two sets of stochastic price shocks is given by  $\rho$ .

The first step is to simulate a sample from a bivariate normal distribution with mean  $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$  and covariance matrix  $\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$ . Next, apply the probability integral transform  $U = F(X)$ , where  $X$  is the bivariate normal random variable generated earlier,  $U$  is the bivariate uniform random variable resulting from the transformation and  $F(\cdot)$  is the cdf of a multivariate normal with mean  $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$  and covariance matrix  $\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$ .  $U$  will have the same rank dependence structure that was present in  $X$ .

The next step is to generate two sets of stochastic price shocks with the required distributional properties and dependence structure. From Section 3.4.1, supposing that a random variable  $X$  has a continuous probability distribution  $f$  and its associated cdf is given by  $F$ , applying the probability integral transform

$$U = F(X)$$

generates a variable  $U$  which is uniformly distributed. It follows that applying the inverse of the above transformation

$$X = F^{-1}(U)$$

to a uniformly distributed random variable generates a random variable that is distributed according to  $f$ . If a random variable follows some parametric distribution, this process allows the

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<sup>19</sup>This is the case with Matlab 7, which is used here

distribution of that random variable to be transformed into any other parametric distribution as long as both of their respective cumulative distribution functions are known.

Denote by  $U_1$  the marginal uniform associated with the GB stochastic price shocks. Applying the inverse probability integral transform,  $X_1 = F^{-1}(U_1)$ , where  $F^{-1}$  is the inverse cdf of a univariate normal with associated mean of  $\mu_1$  and standard deviation of  $\sigma_1$ , gives marginal normal  $X_1$  with parameters  $\mu_1$  and  $\sigma_1$ . Denote by  $U_2$  the marginal uniform associated with NO stochastic price shocks. As derived in Section 3.4.1,

$$\Pr(X \leq x_2) = \begin{cases} \frac{1}{2} \exp\left\{-\left(\frac{|x_2|}{\lambda}\right)^k\right\} & \text{if } x_2 \leq 0 \\ 1 - \frac{1}{2} \exp\left\{-\left(\frac{x_2}{\lambda}\right)^k\right\} & \text{if } x_2 > 0 \end{cases}.$$

Denote by  $x_2^-$  the observations from the negative side of the composite distribution and by  $x_2^+$  the observations from the positive side. For the negative side

$$u_2 = \frac{1}{2} \exp\left\{-\left(\frac{|x_2^-|}{\lambda}\right)^k\right\}.$$

Hence applying the inverse probability integral transform to all observations in  $u_2$  that satisfy  $u_2 \leq \frac{1}{2}$ , which means solving the above expression for  $|x_2^-|$ , yields

$$|x_2^-| = \lambda (-\ln(2u_2))^{1/k}.$$

Since  $x_2^- \leq 0$ ,

$$x_2^- = -\lambda (-\ln(2u_2))^{1/k}.$$

It is easy to show that the pre-set rank dependence structure is preserved when all observations of  $|x_2^-|$  are multiplied by  $-1$  to obtain  $x_2^-$ . Recall that, for the negative side of the composite distribution,  $u_2 \in [0, 1/2]$ . When  $u_2$  is close to zero and given that the parameters  $\lambda$  and  $k$  of the Weibull distribution are always positive,  $\lambda (-\ln(2u_2))^{1/k}$  is a large positive number and multiplying by  $-1$  makes it a large negative number. Thus multiplying by  $-1$  turns one of the lowest values in the distribution of  $U_2$  into one of the lowest values in the distribution of  $X_2^-$ . Equally, when  $u_2$  is close to  $1/2$ ,  $\lambda (-\ln(2u_2))^{1/k}$  is close to zero and multiplying by  $-1$  does not change this. Thus multiplying by  $-1$  turns one of the highest values in in the distribution of  $U_2$  into one of the highest values in in the distribution of  $X_2^-$ . Hence the rank dependence structure of  $u_2$  is preserved under this transformation.

For the positive side of the composite distribution

$$u_2 = 1 - \frac{1}{2} \exp\left\{-\left(\frac{x_2^+}{\lambda}\right)^k\right\}.$$

Applying the inverse probability integral transform to all observations in  $u_2$  that satisfy  $u_2 > \frac{1}{2}$  yields

$$x_2^+ = \lambda (-\ln(2(1 - u_2)))^{1/k}.$$

Finally, putting all  $x_2^-$  and  $x_2^+$  together produces a sample with the required distributional properties and dependence structure.

As an additional check on the accuracy of the calibration process set out in Section 3.4.1, the results of this exercise can be seen below. In each case, the blue bars represent the distribution of randomly sampled bootstrap residuals and the red line represents the distribution of simulated stochastic price shocks that are calibrated as described in Section 3.4.1<sup>20</sup>. For all three sets of data, the properties of the bootstrapped residuals, including fat tails, are closely replicated by the calibrated samples.

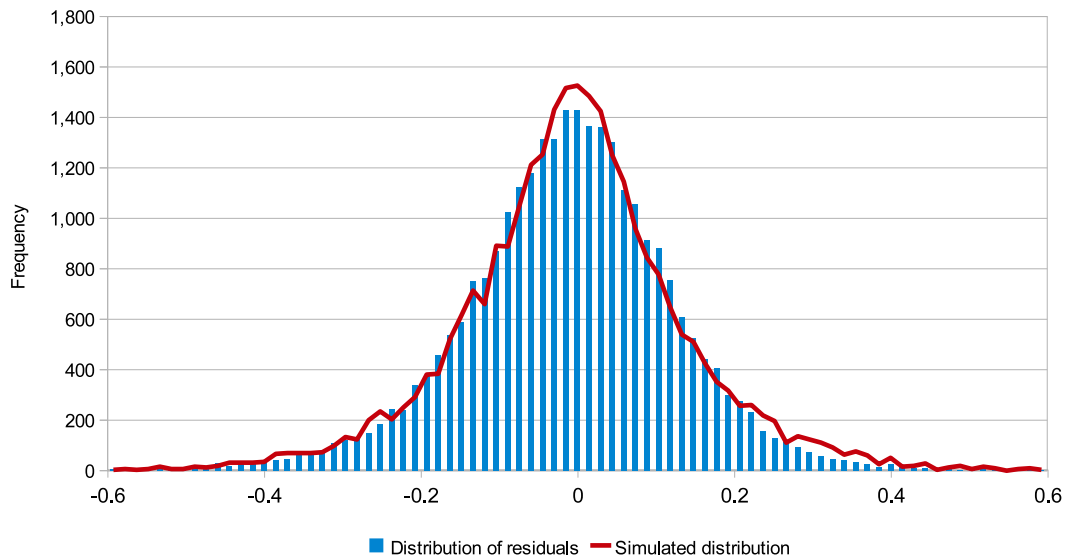


Figure 16: Calibrated residuals for log NL price

<sup>20</sup>In each case, 27,792 separate observations were generated

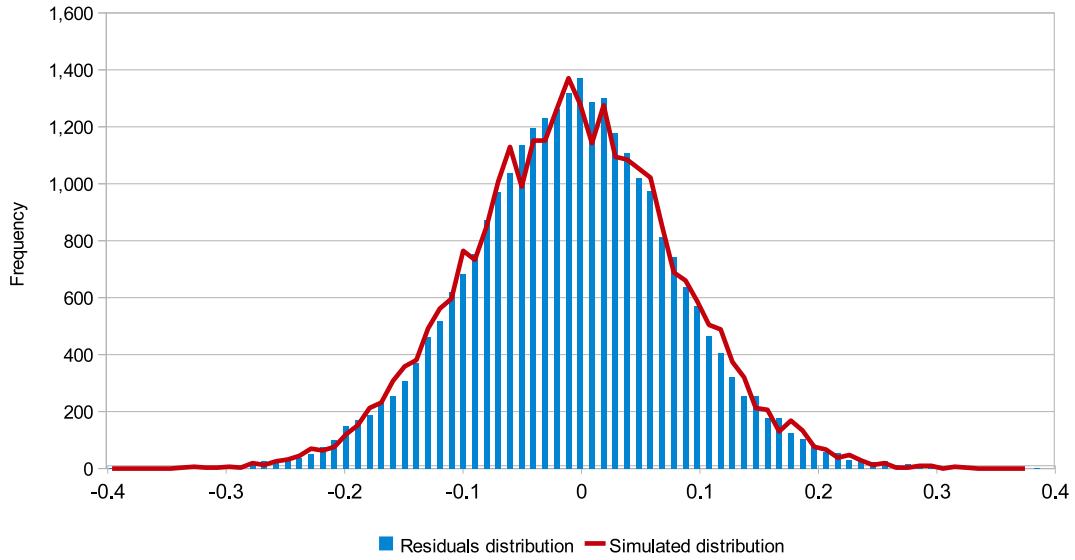


Figure 17: Calibrated residuals for log GB price

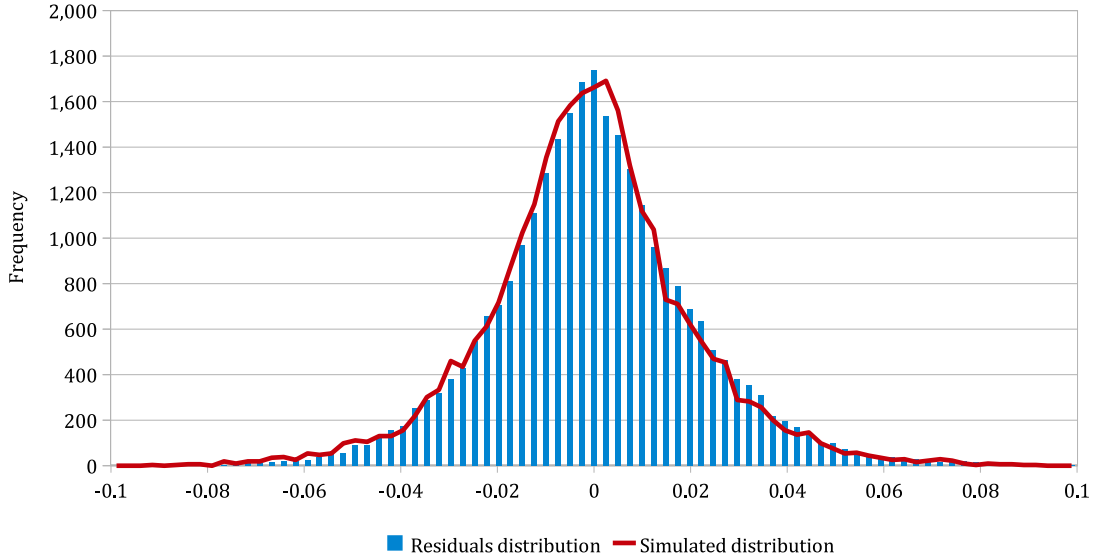


Figure 18: Calibrated residuals for log NO price

The next step in simulating electricity prices is to generate the deterministic component of

prices. This is carried out in the same way as generating predicted values from a regression. Given the vector of estimated coefficients  $b$  from the ARMA model of log electricity prices and the matrix of deterministic factors  $X$ , the predictable component of log electricity price  $y^*$  is simply given by

$$y^*_{t \times 1} = X_{t \times n} \cdot b_{n \times 1}.$$

The deterministic properties of prices can be manipulated by changing the individual values in vector  $b$ . The final step is to add up the vectors of the stochastic components of log electricity prices generated in Section 3.4.2 and the deterministic components of log electricity prices generated here and take the exponent of the resulting vector to obtain the required simulated prices.

### 3.5 Market integration

The effect of connecting two or more markets on the level and volatility of prices in those markets, as well as the revenues from one or both interconnectors, is simulated using an algorithm that mimics an efficient market coupling mechanism. For each hourly period, it calculates the market equilibrium solution and optimum flows over interconnectors under the assumption that transmission constraints are not binding<sup>21</sup>. It then checks if the solution violates any of those constraints, and if it does, calculates separate equilibria for markets that are constrained from one another. This calculation is carried out period by period, with equilibrium prices and revenues calculated for each period.

For the purposes of this exercise, unless stated otherwise, it is assumed that if three markets are connected, they are connected in line in the order of GB-NL-NO. The price effect of interconnectors is calibrated to the econometric estimates obtained in Parail (2009). In the case of GB, the price effect of BritNed on the GB price is assumed to be equivalent to the price effect of NorNed on the NL price after adjusting for market size and interconnector size. The effect of BritNed on the NL price is as for NorNed after adjusting for interconnector size. The algorithms assume that power always flows from the low price region to the higher price region. They also assume that transmission losses are zero, which, in the case of DC links, is not far from the truth. Interconnector availability is assumed to be 100% unless stated otherwise. This means that the interconnector revenues estimated in this exercise represent an upper bound on what is feasible in reality. The two algorithms, and the corresponding derivations, are set out in full in Appendix D.

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<sup>21</sup>The transmission constraints for NorNed and BritNed in the base case are 700MW and 1,000MW respectively

## 4 Applications

### 4.1 Comparison of historic and simulated prices

This section firstly attempts to answer the question of what would have happened if the NorNed interconnector had been operational since 1 Jan 2006, the beginning of the sample period, and how would this situation have changed if BritNed came online at the same time. It does so in the specific context of NorNed and BritNed, taking the properties of the historic NL, NO and GB electricity prices as given. This exercise is then repeated using simulated prices. These prices have stochastic and deterministic properties that are identical to historic prices except that the stochastic element is obtained from bootstrapped regression residuals. This gives a measure of the bias introduced into forecasts by using historic price data.

Differences between historic and bootstrapped price samples occur because of the presence of strong autocorrelation in stochastic shocks affecting prices. Taking an example relating to fuel prices, imagine that the historic sample of electricity prices is characterised by an unexpected shock to gas prices that made them much higher and more volatile than usual for a significant proportion of the sample period, feeding directly into electricity prices. In the context of European electricity markets, a short gas dispute between the Ukraine and Russia could exemplify this kind of situation. Autocorrelation would mean that the shock persists for a long time, affecting the mean level and the volatility of electricity prices for the entire sample period. However, bootstrapping would unravel the effect of shock persistence, thus producing the underlying distribution of shocks to electricity prices that has a lower mean and is less volatile than the corresponding distribution for the historic sample.

Since NorNed had actually been operational for a part of the sample period, the first step is to unravel its effect by subtracting the estimated effect of NorNed on historic NL and NO electricity prices. This is done on the basis of the effect of NorNed on prices in those two markets as estimated in Parail (2009). Once this step is complete, we have three historic stand-alone hourly price series spanning the period between 01 Jan 2006 and 03 Mar 2009.

The next step involves applying the algorithms that simulate the effect of either connecting NL and NO via NorNed or connecting NL with both NO and the GB via NorNed and BritNed. The results are compared in terms of the mean levels and variances of prices in the connected markets, as well as revenue generated by the interconnectors. The precise method for calculating post-connection prices and interconnector revenues is derived in Appendix D. Interconnector revenue is given in units of € per MW per hour and is thus calculated simply as the difference in post-connection electricity prices between the connected markets. The nominal capacities for NorNed and BritNed are 700MW and 1,000MW respectively. The sensitivity table also shows the effect of increasing the capacity of both interconnectors by factors of 2, 4 and 8 respectively. This

is done under the assumption that the price effect of interconnectors increases proportionally with their size.

The top half of the table shows the result of connecting NL and NO only, while the bottom half shows the result of connecting NL, NO and GB. Table columns differ in the size of interconnectors simulated. The top half of the second column, for example, shows the effect of connecting NL and NO with a 700MW cable, while the bottom half of that column shows the effect of connecting NL and NO with a 700MW cable and connecting the GB and NL with a 1,000MW cable.

		<b>NorNed capacity</b>				
		<b>0</b>	<b>700MW</b>	<b>1,400MW</b>	<b>2,800MW</b>	<b>5,600MW</b>
<b>NorNed</b>	Mean NL price (€/MWh)	56.4	55.1	54.1	52.2	49.0
	Variance of NL price (€/MWh)	1,497	1,384	1,297	1,133	856
	Mean NO price (€/MWh)	38.1	38.9	39.5	40.5	42.1
	Variance of NO price (€/MWh)	245	250	253	268	311
	NorNed revenue (€/MW/hour)	0.0	21.9	19.3	14.8	8.2
		<b>BritNed capacity</b>				
		<b>0</b>	<b>1,000MW</b>	<b>2,000MW</b>	<b>4,000MW</b>	<b>8,000MW</b>
<b>NorNed + BritNed</b>	Mean NL price (€/MWh)	56.4	55.2	54.5	53.6	53.0
	Variance of NL price (€/MWh)	1,497	1,319	1,189	997	794
	Mean NO price (€/MWh)	38.1	39.0	39.6	40.7	42.9
	Variance of NO price (€/MWh)	245	249	252	265	314
	Mean GB price (€/MWh)	61.4	61.1	60.8	60.0	58.5
	Variance of GB price (€/MWh)	1,556	1,521	1,485	1,413	1,272
	NorNed revenue (€/MW/hour)	0.0	21.8	19.3	15.6	11.1
BritNed revenue (€/MW/hour)	0.0	16.7	14.4	10.9	6.8	

Figure 19: Simulation results using historic data

A quick glance at the figures would reveal that, without exception and regardless of the capacity of NorNed and BritNed, interconnector revenues per MW per hour are greater than the difference between the average prices in the connected markets. This is due to variations in electricity prices around the mean, which represent additional opportunities for interconnectors to generate revenue. As will be demonstrated in Section 4.2.6, it is possible for an interconnector to generate substantial amounts of revenue without any difference in the mean prices of the markets it connects.

As expected, connecting different markets results in absolute price convergence as well a reduction in price volatility in the market where that volatility is initially greater. These effects are increasing in the size of connections. In absolute terms, the price decrease in the higher price region is greater than the price increase in the lower price region. This happens for two



reasons. The first reason is that the effect of an interconnector on prices is given by a percentage change<sup>22</sup>. A given percentage change will have a greater absolute effect when applied to larger values and corresponds to flows over an interconnector having a greater effect in absolute terms on prices in the higher priced region. The second reason is related to the first and is partly driven by the shape of the price distribution, which has a long tail on the right, making positive price shocks larger than negative price shocks in absolute terms. This can be seen in Appendix A. When arbitraging stochastic price differences, the interconnector will, on average, have a greater absolute effect on positive shocks than on negative shocks, thus driving average prices down in both markets, all other factors being equal.

Connecting two markets with volatile prices appears to reduce price volatility in both markets. This is the case when GB is connected to NL. However, when price volatility is considerably lower in one of those markets, as is the case with NO, volatility actually increases in that market. Price changes can be exported indirectly via another market. This is demonstrated by the result that the NO price is always higher when GB is connected to NL than when there is no GB-NL link. The picture is more complex with respect to volatility. Connecting NL to GB appears to lower NO price volatility for all but very large possible BritNed capacities, which reflects the Mean Variance Portfolio effect of price shock diversification leading to lower “portfolio” variance. However, for very large BritNed capacity, volatility is exported from GB to NO indirectly via NL. This is because the NL price becomes increasingly aligned with the GB price due to the GB market’s larger size as the capacity of BritNed increases, which means that it becomes less aligned with the NO price and thus becomes an exporter of GB price volatility.

Revenue does not increase proportionally with interconnector capacity due to the fact that arbitrage decreases price differences. The effect is very minor for lower interconnector capacities around 1,000MW and severe at higher capacities of 3,000 - 4,000MW for markets of the size of NO and NL. Interestingly, for capacities upwards of 1,000MW, the effect on NorNed of connecting GB and NL is positive. This is because the GB price is higher on average and more volatile than the NL price. Hence, when GB and NL are connected with a high capacity cable, GB exports its higher and more volatile prices to NL. Since prices in NO are lower and less volatile, this effect increases the revenue from NorNed.

Figure 20 compares the results obtained with historic prices to those obtained using simulated prices with stochastic properties obtained from the bootstrapping exercise.

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<sup>22</sup>As estimated in Parail (2009)

		Actual historic		Bootstrap	
		0	700MW	0	700MW
<b>NorNed capacity</b>					
NorNed	Mean NL price (€/MWh)	56.4	55.1	61.7	60.5
	Variance of NL price (€/MWh)	1,497	1,384	1,986	1,868
	Mean NO price (€/MWh)	38.1	38.9	36.7	37.4
	Variance of NO price (€/MWh)	245	250	369	365
	NorNed revenue (€/MW/hour)	0.0	21.9	0.0	28.5
<b>BritNed capacity</b>					
NorNed + BritNed	Mean NL price (€/MWh)	56.4	55.2	61.7	59.2
	Variance of NL price (€/MWh)	1,497	1,319	1,986	1,703
	Mean NO price (€/MWh)	38.1	39.0	36.7	37.4
	Variance of NO price (€/MWh)	245	249	369	363
	Mean GB price (€/MWh)	61.4	61.1	49.3	49.4
	Variance of GB price (€/MWh)	1,556	1,521	466	467
	NorNed revenue (€/MW/hour)	0.0	21.8	0.0	27.2
	BritNed revenue (€/MW/hour)	0.0	16.7	0.0	18.9

Figure 20: Comparison of historic and bootstrap data analysis

Note first of all the properties of prices with bootstrapped stochastic components as compared to historic prices before the effects of any interconnectors are simulated. Both the mean and the variance of the NL price are underestimated by using historic data. The same is true for the variance of the NO price. The opposite is true for the GB electricity price, where using historic data greatly overestimates both the price mean and the variance.

Once the effects of NorNed and BritNed are simulated, the predicted revenue of the NorNed interconnector is underestimated by around a quarter when using historic price data. This is not surprising given that historic data underestimates the average price difference between NL and NO as well as price volatility in both markets. BritNed revenue is only underestimated by around 10% when using historic data. Abstracting from the complexities that serial correlation introduces into the distribution of historic prices, using historic prices underestimates the average price difference between GB and NL as well as the variance of prices in the latter. On the other hand, it greatly overestimates the variance of GB prices. Since the last of those biases has the opposite effect on BritNed revenue from the first two, it is not surprising that the resulting bias on the forecast of BritNed revenue is relatively small.

Although bootstrapping stochastic shocks to electricity prices provides an unbiased long run forecast of future stochastic price shocks, in the short run, the actual outcome can differ significantly from that forecast due to persistence in those shocks. In the case of NorNed, the year

when it became operational was characterised by unusually high reservoir levels in NO and high electricity prices in NL. Since reservoir levels are mostly a cumulative function of precipitation, they persisted for a long period of time, depressing electricity prices in NO and allowing NorNed to generate much more revenue than anticipated.

To put the differences in predicted interconnector revenues based on historical and simulated data into perspective, some estimate of the costs associated with each interconnector is required. The estimates provided here are crude and for demonstration purposes only. The total cost of the NorNed interconnector is estimated to be €600m<sup>23</sup>. Assume that the associated running costs are negligible and that outages between 6 May 2008 and 3 Mar 2009, which ran at approximately 8%, are representative of its entire period of operation. Assume further that the interconnector has a useful life of 30 years<sup>24</sup>. Given these assumptions, the Internal Rate of Return (IRR) for NorNed based on the revenue estimates using historic data is 23%<sup>25</sup>. The IRR for NorNed based on simulated prices, keeping all other assumptions constant, is 29%. For BritNed, scaling the cost of NorNed proportionally by capacity and cable length, the total cost of the project should be €384m. Making the same assumptions as for NorNed, the IRR for BritNed is 42% years based on historic prices and 48% years based on simulated prices.

Whilst the above analysis of the investment case for the two interconnectors is based on a number of simplifying assumptions, given the extremely high IRR estimated here, it is consistent with the interpretation that the business case for both interconnectors, and especially BritNed, is very strong. Indeed, there is some evidence that the estimates of NorNed profits made in the planning stage were pessimistic. The forecast annual profits for NorNed were €64m, whereas the total profits in only the first two months of operation amounted to €50m.

## 4.2 Simulated prices

### 4.2.1 Introduction

This section takes advantage of the flexibility offered by the model in order to examine the main determinants of interconnector revenues, as well as the drivers behind the effect of interconnectors on price levels and volatility in the connected markets. For each factor, the effect of which is examined in a simulation, the analysis mimics the properties of historic prices in the GB, NO and NL in one of the presented scenarios, deriving the stochastic properties of prices

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<sup>23</sup>Source: Power Engineering International (PenWell Corp.)

<sup>24</sup>This assumption complements the assumption of zero operational and maintenance costs as the useful life of the interconnector could be significantly extended given appropriate maintenance

<sup>25</sup>Based on the scenario where BritNed is also operational

from bootstrapped residuals, and then makes changes to that factor incrementally in the other scenarios to determine the effect of those changes.

The factors examined in this section include differences in mean prices between connected markets, differences in the average daily pattern of prices, volatility of stochastic price shocks and correlation of stochastic shocks in connected markets. Changes in those factors are put into the context of their possible drivers. Changes in mean electricity prices could be caused by changes in fuel prices. This is explored in Section 4.2.3 using estimates of the long-run relationship between fuel prices and electricity prices obtained in Section 3.2. Changes in volatility of electricity prices could be caused by changes in penetration of wind generation in the overall generation mix. This is explored in Section 4.2.4 using estimates from Green and Vasilakos (2009). Likewise, correlation in stochastic shocks in the connected markets could be caused by consistent wind patterns and geographic proximity of those markets. This factor is explored in Section 4.2.5. Finally, Section 4.2.6 examines the question of whether an interconnector between two extremely similar market can generate substantial revenue from arbitrage.

As before, the response of market prices to electricity flows across interconnectors is calibrated to the estimates obtained in Parail (2009) and adjusted for market and interconnector size. Each simulation generates 8,760 hourly observations, being equal to the number of hours in a non-leap year. Where needs dictate, this exercise abstracts away from the example of NL, GB and NO, referring instead to markets A, B and C where market B is in the middle and can be connected to A alone or to A and C. In these instances, unless otherwise stated, the size of both interconnectors is assumed to be 1,000MW and the size of markets A, B and C to be the same as that of NL. The effects of interconnectors on prices are calibrated accordingly.

#### **4.2.2 Daily price pattern**

The average daily price pattern is derived by calculating the average predicted price for each given hour across all days in our data sample. This is done using the estimated coefficients of dummy variables that represent different hours of the day taken from the regression described in Section 3.2<sup>26</sup>. It therefore indicates the consistent tendency for prices to differ in peak and off-peak hours over a 24 hour period. This section focuses on the differing average daily price patterns in NO on the one hand, and in the GB and NL on the other<sup>27</sup>. It tests the extent to which interconnector revenue is driven by any such consistent differences and also whether those differences influence the extent of price stabilisation in the connected markets. The idea

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<sup>26</sup>To avoid perfect multicollinearity, there are only 23 such dummy variables, with the 11pm to midnight hour dropping into the constant term of the regression

<sup>27</sup>See Figure 1 for details

behind this is that NorNed revenue and price stabilisation in NL caused by NorNed may be driven to a large extent by the smooth average daily price pattern in NO.

The sensitivity of key outputs of the model to changes in the average daily price pattern in GB and NL is given in Figure 21 below. The format in which results are presented differs slightly from Figure 20. For mean and variance of NL, NO and GB electricity prices, rather than giving actual mean and variance figures for each simulated scenario, the table gives the percentage change in those parameters resulting from connecting NL and NO markets in the top half of the table and from connecting NL, NO and GB markets in the bottom half of the table. This format for presenting simulation results is used for the remaining simulations.

The first column in Figure 21 gives interconnector revenues and the change in mean and variance of prices when, for each of the three markets, the stochastic properties of prices are calibrated to corresponding bootstrapped residuals with stochastic shock rank correlations calculated from historic residuals and the deterministic properties of prices are calibrated to the corresponding regression results obtained in Section 3.2 as described in the paragraph above. The subsequent columns give those outputs under scenarios where the average daily price pattern in NL, GB or both is flat, meaning that there is no difference in expected prices across different hours of any given day. All other properties and model parameters remain the same throughout the exercise.

A flat average daily price pattern is derived by setting the coefficients of the dummy variables for different hours of the day to zero and calibrating the constant term of the regression from Section 3.2 such that simulating the relevant price series as described in Section 3.4 using these coefficients results in the same average price across all hours and all days of the sample as would simulating the same price series using the actual estimates obtained in Section 3.2. The result will be a price series with the same stochastic and deterministic properties as the baseline price series but with no consistent price difference across different hours of the day.

<b>Daily price profile</b>		<b>Base</b>	<b>NL flat</b>	<b>GB flat</b>	<b>NL + GB flat</b>
<b>NorNed</b>	% change in mean NL price	-2.0%	-2.2%	-2.0%	-2.2%
	% change in NL price variance	-5.9%	-5.6%	-5.9%	-5.6%
	% change in mean NO price	1.8%	2.5%	1.8%	2.5%
	% change in NO price variance	-0.4%	-2.3%	-0.4%	-2.3%
	NorNed revenue (€/MW/hour)	28.8	25.6	28.8	25.6
<b>NorNed + BritNed</b>	% change in mean NL price	-4.1%	-4.1%	-4.1%	-4.4%
	% change in NL price variance	-14.2%	-13.7%	-15.0%	-14.8%
	% change in mean NO price	1.7%	2.5%	1.8%	2.5%
	% change in NO price variance	-1.2%	-2.8%	-1.2%	-2.9%
	% change in mean GB price	0.3%	0.2%	0.2%	0.4%
	% change in GB price variance	0.5%	-1.4%	0.6%	0.3%
	NorNed revenue (€/MW/hour)	27.4	24.4	27.3	24.3
BritNed revenue (€/MW/hour)	19.3	18.3	22.7	15.6	

Figure 21: Daily price profile

The first thing to notice is that eliminating consistent daily price variation in NL only reduces NorNed revenue by 11%, suggesting that a consistent difference in average daily prices and price differences resulting from stochastic price shocks play a more important role in determining NorNed revenue. Also worth noting is the result that the effect of NorNed on price volatility in NL is virtually the same with or without consistent price variation across hours. Less surprising effects of this change are slightly greater convergence of average prices in the two markets connected by NorNed and a significantly greater reduction in price volatility in NO.

Adding BritNed into the equation produces one additional surprising result. With a flat daily profile of GB prices, BritNed revenue is higher compared to the baseline case. However, with a flat daily profile of NL prices, BritNed revenue is slightly lower than in the baseline case. One would expect a flat price profile in one of the two countries to increase BritNed revenue in both cases because that would generate significant consistent price differences where there were only small differences before, both markets being characterised by large differences between peak and off-peak prices. This intuition turns out to be wrong because any such consistent differences also interact with the difference in average prices. Figure 22 demonstrates this using plots of average price differences by hour between the GB and NL from simulated data.

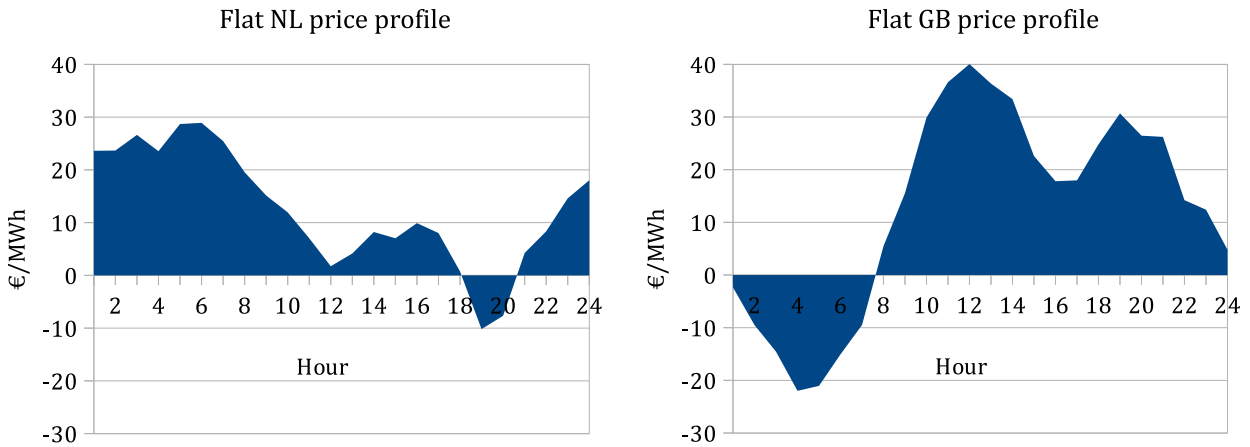


Figure 22: Simulated NL - GB price difference curves for a typical year

The average absolute price difference between the GB and NL across all hours is 50% higher when the GB has a flat price profile than when NL has a flat price profile. The equivalent difference is virtually the same in the baseline scenario as in the scenario where NL has a flat price profile.

Overall, the results indicate that consistent differences in the daily price profile can create arbitrage opportunities for interconnectors, though their role may not be as important as those of stochastic price differences and consistent differences in average prices. Simple intuition may turn out to be wrong when trying to predict interconnector revenue on the basis of differing average daily price profiles, necessitating the use of modelling techniques. Some intuitive results are obtained with regard to the effect of the price profile in one market on price volatility in the connected market. Connecting to a market with a less variable daily price profile is more effective in reducing price volatility after controlling for other factors. However, due to the stochastic properties of prices, connecting a market with a flat daily price profile to one characterised by large differences between peak and off-peak prices may still reduce price variance in the former.

The one hour price difference between the GB and NL turns out to be insignificant in determining BritNed revenue. Modelling GB and NL prices as if they are in the same time zone only reduces BritNed revenue by 0.1% compared to the baseline scenario, keeping all other factors constant, and it makes no visible difference to any other outputs of the model.

### 4.2.3 Differences in average prices

This section deals with the extent to which interconnector revenues are driven by differences in average prices and whether such differences can influence the effect of interconnectors on price levels and volatility in the connected markets. Figure 23 compares the change in key model outputs in the baseline case against scenarios where the average price in NO, GB or both is as high as the average price in NL, leaving all other factors unchanged. In the baseline scenario, average NL, NO and GB prices before the effect of interconnectors is simulated are €62/MWh, €37/MWh and €49/MWh respectively.

<b>Mean price</b>		<b>Base</b>	<b>NO=NL</b>	<b>GB=NL</b>	<b>NO=GB=NL</b>
<b>NorNed</b>	% change in mean NL price	-2.0%	-0.7%	-2.0%	-0.7%
	% change in NL price variance	-5.9%	-6.6%	-5.9%	-6.6%
	% change in mean NO price	1.8%	-0.8%	1.8%	-0.8%
	% change in NO price variance	-0.4%	-2.3%	-0.4%	-2.3%
	NorNed revenue (€/MW/hour)	28.8	23.4	28.8	23.4
<b>NorNed + BritNed</b>	% change in mean NL price	-4.1%	-2.8%	-2.7%	-1.5%
	% change in NL price variance	-14.2%	-14.7%	-14.3%	-14.9%
	% change in mean NO price	1.7%	-0.9%	1.8%	-0.8%
	% change in NO price variance	-1.2%	-2.8%	-1.0%	-2.9%
	% change in mean GB price	0.3%	0.3%	-0.2%	-0.2%
	% change in GB price variance	0.5%	0.5%	0.3%	0.2%
	NorNed revenue (€/MW/hour)	27.4	22.5	28.0	22.4
	BritNed revenue (€/MW/hour)	19.3	19.6	18.8	18.6

Figure 23: Average price

For NorNed, simulation results show that connecting NO and NL markets when they are characterised by the same average price but different stochastic and deterministic properties results in more even sharing of the benefits of interconnection for the consumers in the two markets. The average price and price volatility are reduced in both markets. The effect on price volatility is unsurprising, being the direct result of arbitrage. The effect on average prices is due to the inherent asymmetry of positive and negative shocks to electricity prices as discussed in Section 4.1. Removing a difference in average prices between the two markets reduces NorNed revenue by approximately 19%.

The effect of BritNed on average GB and NL prices is largely the same as for NorNed, remembering that the GB is a much larger market and is thus less affected by the interconnector. Note,



however, that the effect of BritNed on GB price volatility is positive. This shows that volatility can also be exported from a volatile NL market to a less volatile GB market<sup>28</sup>. Factors that make such outcomes more likely are examined in Sections 4.2.4 and 4.2.5. Removing the difference in average prices between the GB and NL markets makes very little difference to BritNed revenue, suggesting that this difference on its own is not a significant driver of interconnector revenue in this case.

To put these results into context, one of the factors that can produce a consistent difference in average prices between two markets is a change in fuel prices. Utilising the results of the analysis in Section 3.2, a difference between the average NO and NL electricity prices would be canceled out by a 40% fall in the gas price<sup>29</sup>. For the period covered by our data set, the average price of gas in the NL market was €57. The proportion of days for which the NL gas price was less than 60% of the average during that period was around 15%, which gives a rough measure of the probability of average NL and NO electricity prices becoming aligned due to a fall in the gas price.

Since the relationship between gas and electricity prices in GB and NL is estimated to be approximately the same, and gas prices in the two markets are highly correlated, a change in the price of coal is the only factor related to fuel prices that could potentially change the difference in average electricity prices between the two markets. However, as the simulation results presented in Figure 23 demonstrate, the difference in average prices plays only a small role in determining BritNed profits. Therefore changes in the price of coal are unlikely to have a significant effect on BritNed profits.

#### 4.2.4 Variance of stochastic price shocks

This section deals with the effect of changes in the variance of stochastic price shocks in one of the connected markets on the key outputs of the model. The free variable in this experiment is the variance of the stochastic component of the NL electricity price<sup>30</sup>. In the first column, this variance is set to zero, meaning that the only variation present in the NL price is due to deterministic factors such as the time of day or the day of the week. In the second column, this variance is set at a half of the estimated variance of bootstrapped residuals. In the third and fourth

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<sup>28</sup>Although GB prices are more volatile than NL prices in the historic sample, the opposite is true for prices generated from bootstrapped residuals, which are used here

<sup>29</sup>Assuming that NL electricity and gas prices change proportionally, a 40% fall would reduce the NL electricity price from €62/MWh to €37/MWh

<sup>30</sup>More precisely, it is the variance of stochastic shocks to the log electricity price. Hence, even setting aside the complication of adding up the stochastic and deterministic components of prices, an increase in the variance of the log stochastic component of prices increases the variance of nominal prices, but not in a linear way

column, the variance is set at the baseline level and twice of the baseline level respectively. All other factors remain unchanged throughout the experiment.

		<b>NL stochastic shock variance</b>	<b>0.0x Base</b>	<b>0.5x Base</b>	<b>Base</b>	<b>2.0x Base</b>
<b>NorNed</b>	% change in mean NL price		-1.8%	-1.9%	-2.0%	-2.1%
	% change in NL price variance		-7.9%	-6.3%	-5.9%	-5.5%
	% change in mean NO price		1.4%	1.7%	1.8%	1.8%
	% change in NO price variance		-8.7%	-3.4%	-0.4%	2.0%
	NorNed revenue (€/MW/hour)		22.6	24.6	28.8	38.8
<b>NorNed + BritNed</b>	% change in mean NL price		-3.0%	-3.7%	-4.1%	-4.7%
	% change in NL price variance		-14.3%	-14.4%	-14.2%	-13.2%
	% change in mean NO price		1.4%	1.7%	1.7%	1.7%
	% change in NO price variance		-8.8%	-3.8%	-1.2%	1.7%
	% change in mean GB price		0.0%	0.2%	0.3%	0.3%
	% change in GB price variance		-3.2%	-0.5%	0.5%	1.3%
	NorNed revenue (€/MW/hour)		21.9	23.6	27.4	36.9
BritNed revenue (€/MW/hour)		14.2	14.9	19.3	29.7	

Figure 24: Variance of stochastic shocks

As expected, increasing the variance of stochastic price shocks in NL increases the revenue from both interconnectors. It is perhaps surprising that reducing the variance to zero reduces BritNed revenue by only a quarter and NorNed revenue by less than that as compared to the baseline level. However, it must be remembered that the variance of stochastic shocks in GB and NO remains at the baseline level in all scenarios, hence interconnectors can still arbitrage stochastic shocks originating from those markets in addition to any deterministic price differences.

At higher stochastic shock variance levels, the effect of interconnectors on mean prices in the connected regions increases. This is due to the fact that the distribution of prices acquires an even longer tail on the right side of the distribution at higher variance levels. Greater price volatility in NL means that it is more likely to be exported to its neighbouring markets through the interconnector. The ability of either one or both interconnectors to dampen price volatility in NL decreases as the volatility level in that market increases, indicating that, for a given capacity level, the ability of an interconnector to arbitrage stochastic price shocks is limited.

The level of stochastic price volatility can vary greatly between different markets. This is demonstrated clearly in the case of the three markets considered here and needs to be properly accounted for in any model of merchant interconnector revenues. However, this volatility may

also change with time in a way that is to some extent predictable. The most obvious change that is likely to lead to greater stochastic price volatility is an increase in wind penetration in the overall generation mix. It would be very helpful to have an estimate of how greater wind penetration is likely to affect interconnector revenues.

The relationship between wind penetration and electricity price volatility has received surprisingly little attention in the applied economic literature. Studies of the effects of wind power have focused almost exclusively on system stability in the presence of fluctuating wind output, leaving electricity prices out of the equation. More specifically, as far as the author is aware, at the time of writing, no econometric study had been undertaken to estimate the effect of wind penetration on electricity prices. One paper that tackles this question from an applied economic angle is Green and Vasilakos (2009). They use historic wind profiles for thirty onshore and offshore regions across Great Britain for the period between 1994 and 2005 by matching them on an hourly basis to actual hourly demand using supply function equilibria given the existing generation base and assuming varying degrees of wind penetration<sup>31</sup>.

The average price variance over all hours between 1994 and 2005 as estimated by the model in Green and Vasilakos increases by 14% when the amount of wind capacity in the overall generation mix increases from 20GW to 30GW, keeping all other factors constant. An increase of 10GW in wind capacity represents approximately a 10% increase in total GB generation capacity, though this figure does not take into account the intermittent nature of wind generation. In the context of the NL generation base, 1.5GW of wind plant would add approximately 10% to existing installed generation capacity. Referring back to Figure 24, a 14% increase in overall NL price variance is roughly equivalent to a 7% increase in the variance of stochastic shocks to the log NL price compared to the base level<sup>32</sup>. This change would be expected to increase NorNed and BritNed revenues by approximately 2.4% and 3.8% respectively<sup>33</sup>.

Although the estimated effect of increasing wind penetration on interconnector revenues does not appear to be big, it must be remembered that it is based on static analysis which assumes that existing thermal generation capacity remains constant as wind capacity increases. This is unlikely to happen in the medium and long term as greater wind capacity will push average prices down. The model put forward by Green and Vasilakos predicts that an increase in GB wind capacity from 20GW to 30GW would push the average GB electricity price down by around 5%. Hence an increase in wind penetration is likely to induce exit of thermal generators from the market and more of the remaining thermal generators being restricted to operating in the hours when prices are high because their marginal cost will be considerably higher than for

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<sup>31</sup>The model is estimated for different degrees of market concentration. For the results presented here, the scenario with six competing generators is used

<sup>32</sup>Derived from the NL price data used in the simulation

<sup>33</sup>This calculation assumes that the effect of stochastic log price volatility on interconnector revenues is locally linear

wind generators in windy periods. The dynamic effect of greater wind capacity would therefore increase price volatility even further because peak electricity prices, or prices when wind power is scarce, would have to rise substantially to compensate the remaining thermal generators for their capital investment, which will be spread over much fewer hours of operation. Therefore the static effect of increasing wind capacity represents a lower bound on the resulting increase in electricity price volatility.

#### 4.2.5 Correlation of stochastic price shocks

This section tests the role of correlation in stochastic price shocks between different markets in determining interconnector revenues and the effect of interconnectors on the level and volatility of prices in the connected markets. The free variable in this case is Spearman's rank correlation coefficient defined over the stochastic element of all three sets of electricity prices. It varies between zero correlation in the first column and 0.75 in the last column. As before, all other model and price parameters remain the same throughout the experiment. Since there is no baseline case, it is worthwhile to keep in mind the actual observed rank correlations for the stochastic elements of the three price series. These are given as follows:  $\text{Corr}(\text{NL}, \text{GB})=0.52$ ;  $\text{Corr}(\text{NL}, \text{NO})=0.39$ ;  $\text{Corr}(\text{GB}, \text{NO})=0.26$ .

		<b>Spearman's rank correlation</b>	<b>0.00</b>	<b>0.25</b>	<b>0.50</b>	<b>0.75</b>
<b>NorNed</b>	% change in mean NL price		-1.9%	-2.0%	-2.1%	-2.2%
	% change in NL price variance		-6.1%	-6.0%	-6.0%	-5.9%
	% change in mean NO price		1.0%	1.5%	1.9%	2.4%
	% change in NO price variance		-7.7%	-3.4%	0.9%	5.6%
	NorNed revenue (€/MW/hour)		32.4	29.7	28.2	26.1
<b>NorNed + BritNed</b>	% change in mean NL price		-3.7%	-3.9%	-4.2%	-4.5%
	% change in NL price variance		-14.7%	-14.3%	-14.2%	-14.6%
	% change in mean NO price		1.1%	1.5%	1.9%	2.4%
	% change in NO price variance		-7.9%	-3.7%	0.6%	5.3%
	% change in mean GB price		0.0%	0.1%	0.3%	0.5%
	% change in GB price variance		-2.1%	-1.0%	0.3%	2.6%
	NorNed revenue (€/MW/hour)		31.2	28.4	26.8	24.4
BritNed revenue (€/MW/hour)		25.2	22.1	19.5	15.9	

Figure 25: Correlation of stochastic shocks

The first thing to note is that NorNed revenue is resilient to increases in rank correlation of

stochastic price shocks in the two markets, suggesting that consistent differences in average prices and differences in the daily and seasonal patterns of prices drive a significant proportion of NorNed revenue. This is not, however, the case with BritNed. Increasing the rank correlation of stochastic price shocks in the GB and NL from 0 to 0.75 reduces BritNed revenue by 37%, which suggests that stochastic price shocks are a more significant driver of revenue in the case of BritNed than NorNed.

The other noteworthy result is that increasing rank correlation in stochastic price shocks influences the effect of interconnectors on price volatility differently for more and less volatile markets. For NL, which is by far the most volatile of the three, the effect of the two interconnectors in reducing price volatility is stable for a wide range of rank correlations. This is not the case for GB and NO, both of which are characterised by much lower price variance. Volatility in those markets is reduced at low correlation factors and increased at high correlation factors as a result of being connected to NL.

The underlying reason for this difference is the long right tail of the distribution of electricity prices. Because of the long tail, large and infrequent positive shocks account for the bulk of price variance in a volatile market, which in this case is NL. A high rank correlation between stochastic shocks in a volatile and a relatively stable market means that the largest positive price shock in the former is likely to coincide with the largest positive shock in the latter. However, because the magnitude of the largest positive shock in a volatile market is likely to be considerably greater, an interconnector can still arbitrage this shock despite the high rank correlation.

As discussed in Section 4.2.4, wind generators can make a significant contribution to electricity price volatility. The results presented here imply that interconnectors are likely to be less effective at generating revenue and reducing price volatility caused by variable wind output if wind patterns are highly correlated in the connected markets. They also imply that the benefits of interconnection in terms of reduced price volatility and lower mean prices can be much less evenly shared between the connected markets if stochastic price shocks in those markets caused by factors such as variable wind output are highly correlated.

#### **4.2.6 Connections between similar markets**

So far, the analysis has focused on connecting the NL, NO and GB markets. These markets display significant differences in deterministic properties of the price curve, properties of the distribution of stochastic price shocks and market size, which make the logic of connecting these markets compelling. However, this does not help to answer the question of whether connecting similar markets can be a sensible proposition both in terms of sufficient interconnector revenue that would cover the costs and also benefits to consumers in terms of lower and more stable

electricity prices. This section addresses exactly this type of question by abstracting from the specific example of the setup dealt with so far.

The following example refers to markets A, B and C, where market B is in the middle and can be connected to A alone or to A and C. The size of both interconnectors is assumed to be 1,000MW and the sizes of markets A, B and C are assumed to be the same as that of NL. The stochastic and deterministic properties of prices in all three markets are calibrated to those of NL in the first two columns and to those of GB in the last two columns. The rank correlation between the stochastic shocks of any two markets is set at 0.5.

The first and third column of Figure 26 show the results of simulations calibrated as described above, where the only source of price differences between markets comes from stochastic price shocks that do not coincide. This represents a test of the benefits of connecting markets that are very similar in all aspects, including their generation mix. The second and fourth column of the table correspond to the first and third columns except that they also incorporate a 20% difference in mean prices between markets such that  $0.8 \cdot \text{mean}(p_A) = \text{mean}(p_B) = 0.8 \cdot \text{mean}(p_C)$ .

		NL price properties		GB price properties	
		No	Yes	No	Yes
		<b>Mean price difference</b>			
Interconnector 1	% change in mean price in A	-0.7%	-2.1%	-0.5%	-2.0%
	% change in price variance in A	-4.3%	-4.6%	-8.0%	-7.9%
	% change in mean price in B	-0.6%	1.0%	-0.5%	1.2%
	% change in price variance in B	-5.3%	-4.8%	-5.1%	-4.2%
	Interconnector 1 revenue (€/MW/hour)	16.5	19.8	10.1	12.3
Interconnectors 1 + 2	% change in mean price in A	-0.7%	-2.0%	-0.6%	-1.9%
	% change in price variance in A	-4.4%	-4.7%	-8.2%	-8.3%
	% change in mean price in B	-1.1%	1.9%	-0.9%	2.2%
	% change in price variance in B	-9.7%	-8.7%	-10.6%	-9.4%
	% change in mean price in C	-0.6%	-2.0%	-0.5%	-1.9%
	% change in price variance in C	-4.9%	-5.1%	-6.8%	-6.3%
	Interconnector 1 revenue (€/MW/hour)	16.1	19.0	9.7	11.7
Interconnector 2 revenue (€/MW/hour)	16.2	18.9	9.6	11.6	

Figure 26: Connecting similar markets

Interconnector revenues are lower when the stochastic and deterministic properties of prices correspond to those of GB. This is as expected because the variance of the stochastic component of prices is much lower in GB than in NL. Also, because the average price is lower in GB, given the shape of the distribution of electricity prices, this implies fewer large positive price shocks

which make a noticeable contribution to interconnector revenues. Importantly though, interconnector revenues can still be generated despite no differences in average prices, deterministic properties of prices and market sizes and a significant correlation in stochastic price shocks that is calibrated to the correlation in such shocks between the GB and NL. Adding scenarios where market B is connected to either one or two identical markets with 20% higher average prices confirms that arbitrage on stochastic price differences is a greater contributor to interconnector revenues than arbitrage on differences in mean prices.

Note that interconnectors reduce average prices and price variance in the connected markets even when they are built between identical markets with strongly correlated stochastic shocks. Given the different properties of prices in the GB and NL, it is not clear that the effect of interconnectors in terms of reducing average prices and price volatility in the connected markets is different for those two calibrations if the connected markets have identical properties. Adding a difference in average prices between markets does not appear to impact the ability of interconnectors to arbitrage stochastic price differences. However, due to the shape of the distribution of prices, it shifts reductions in price variance slightly in the favour of higher priced markets. Also, with a difference in average prices, the interconnector reduces average price differences between markets as expected.

## 5 Conclusion

This paper sets out a widely applicable procedure that generates an unbiased long-run estimate of revenues of a proposed interconnector on the basis of limited historic price data. It also provides an estimate of the effect of that interconnector on the level and volatility of prices in the connected markets. The paper improves on the methodologies used in existing empirical literature on interconnectors in a number of ways. It acknowledges that electricity prices in less regulated markets are often very volatile and interconnector revenues can be generated from this volatility.

Electricity price volatility is modelled explicitly, separating prices into their stochastic and deterministic components and bootstrapping the stochastic components in order to eliminate the biases in historic data caused by serial correlation in stochastic price shocks. By proposing an algorithm that mimics market coupling, it also models the effect of interconnectors on prices in the connected markets and the feedback of that effect into interconnector revenues. The effect of flows over an interconnector on prices in the connected markets is calibrated to an econometric estimate of this effect obtained in Parail (2009).

The simulation methodology set out in this paper allows the stochastic and deterministic properties of prices, as well as most model parameters, to be varied freely. This flexibility is used

to gauge the relative importance of the various drivers of interconnector revenues. It is found that, generally, stochastic properties of prices play a more important role in determining interconnector revenues than consistent deterministic differences and that it is possible for interconnectors to generate considerable revenues without any consistent price differences between the connected markets.

Given the estimated relationship between fuel prices and electricity prices, it is also found that medium-term changes<sup>34</sup> in fuel prices are not generally a big driver of interconnector revenues. However, exceptions to this may occur in cases similar to NorNed, where the relevant fuel price plays no role in determining the electricity price in one of the connected markets.

Increasing penetration of wind power in the connected markets, given the results taken from Green and Vasilakos (2009), can increase interconnector revenues even if the interconnector is built between neighbouring countries where price shocks due to varying wind output are correlated. However, more empirical research is needed into the long-term relationship between wind penetration and electricity price volatility as currently available research is simulation based and is only able to establish a lower bound on the extent of that relationship. The results also suggest that interconnectors can play an important part in reducing electricity price volatility due to varying wind output. However, this effect is diminished when price shocks in the connected markets are correlated.

Since one of the algorithms put forward in this paper permits the simulation of three markets being connected in line, as expected, it is found that, generally, different interconnectors have a negative effect on each other's revenues, though given the capacities of the interconnectors relative to the sizes of the connected markets considered here, this effect is found to be small.

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<sup>34</sup>Where medium term denotes a period less than that in which a country's generation mix can alter significantly



# A Price distributions

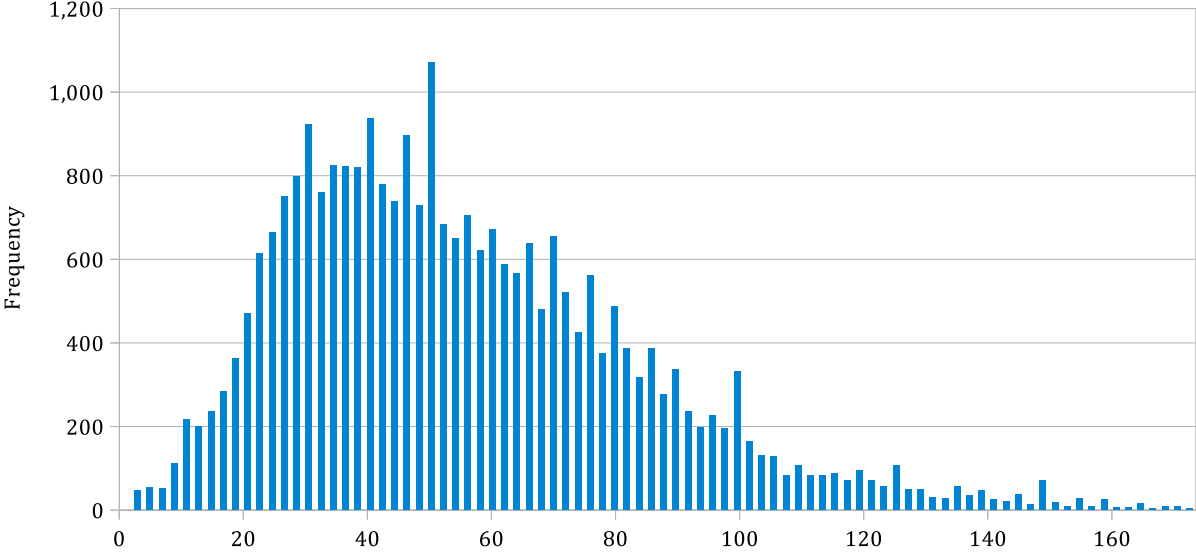


Figure 27: Distribution of NL price

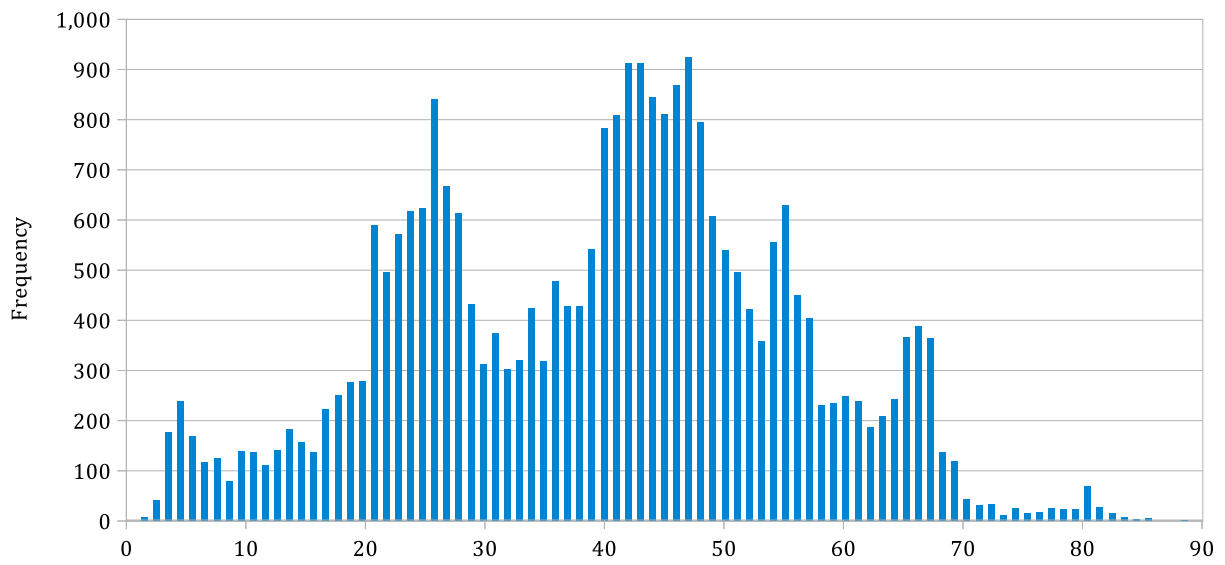


Figure 28: Distribution of NO price

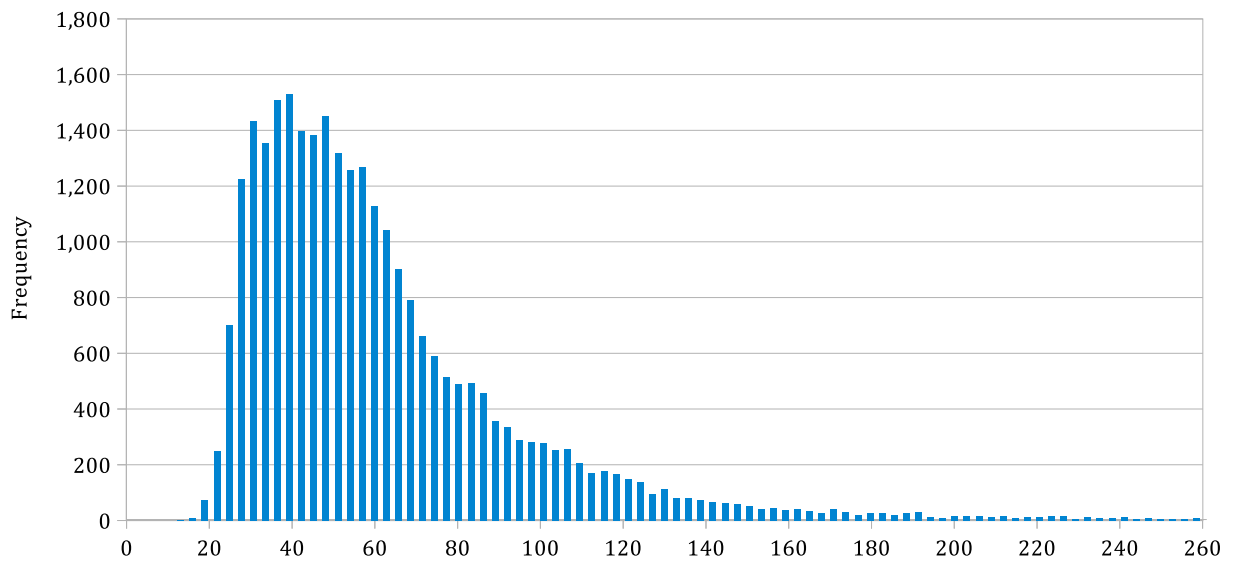


Figure 29: Distribution of GB price

# B Autocorrelation functions of cointegrating equation residuals

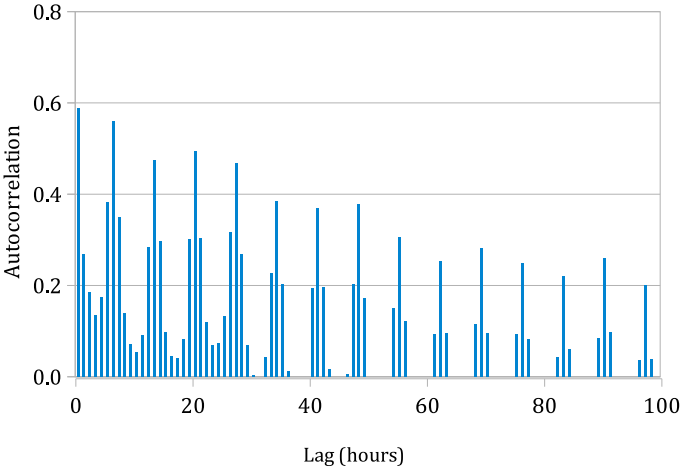


Figure 30: Autocorrelation of NL cointegrating equation residuals

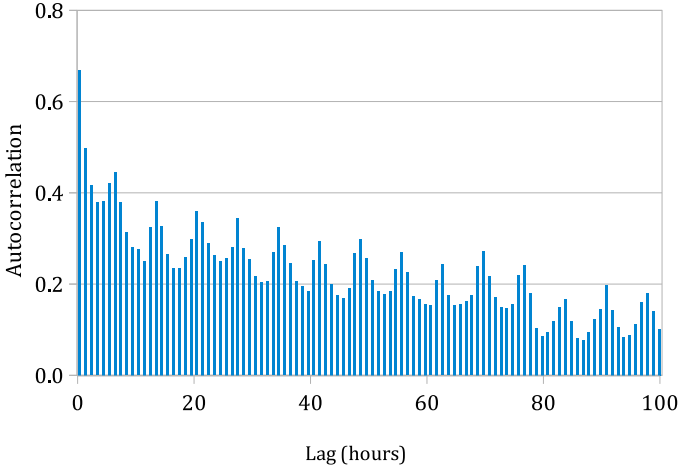


Figure 31: Autocorrelation of GB cointegrating equation residuals

### C Test of calibration accuracy

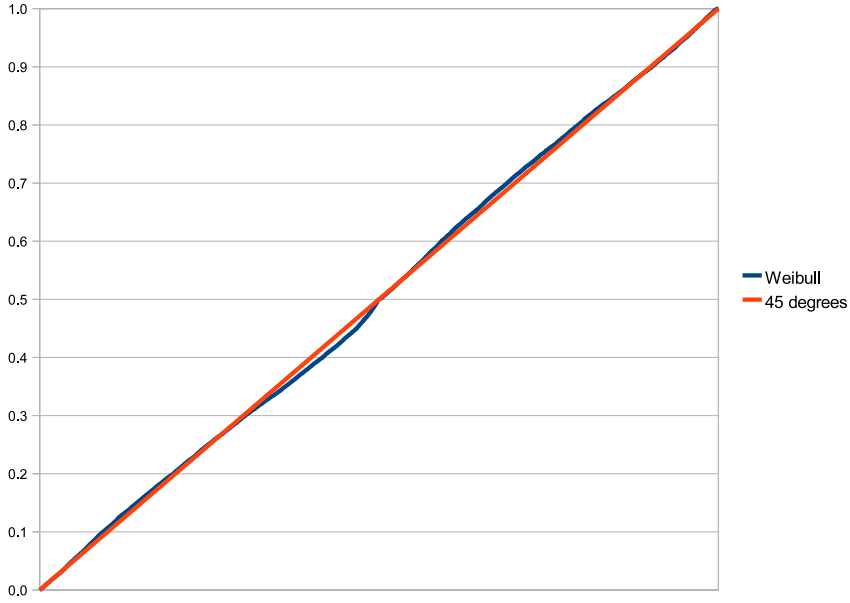


Figure 32: Test of calibrated log NL price residuals

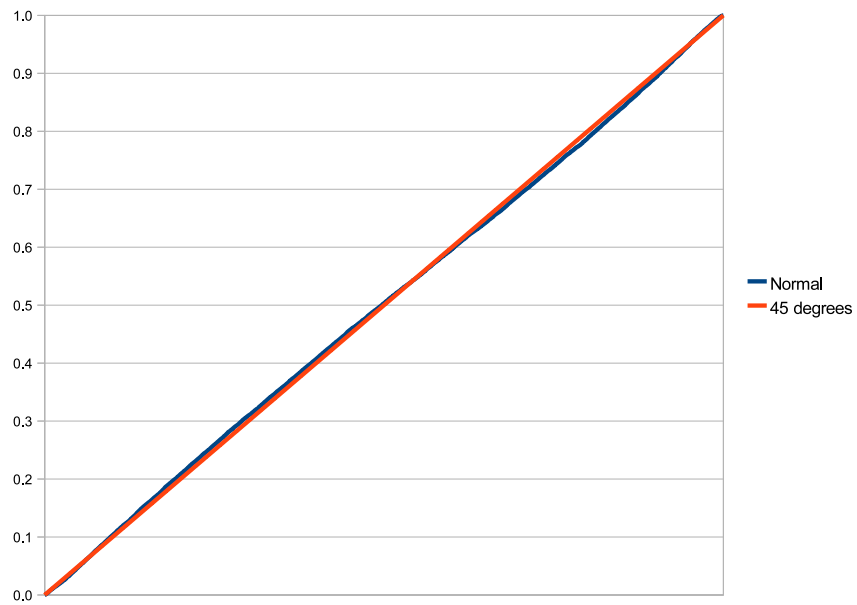


Figure 33: Test of calibrated log GB price residuals

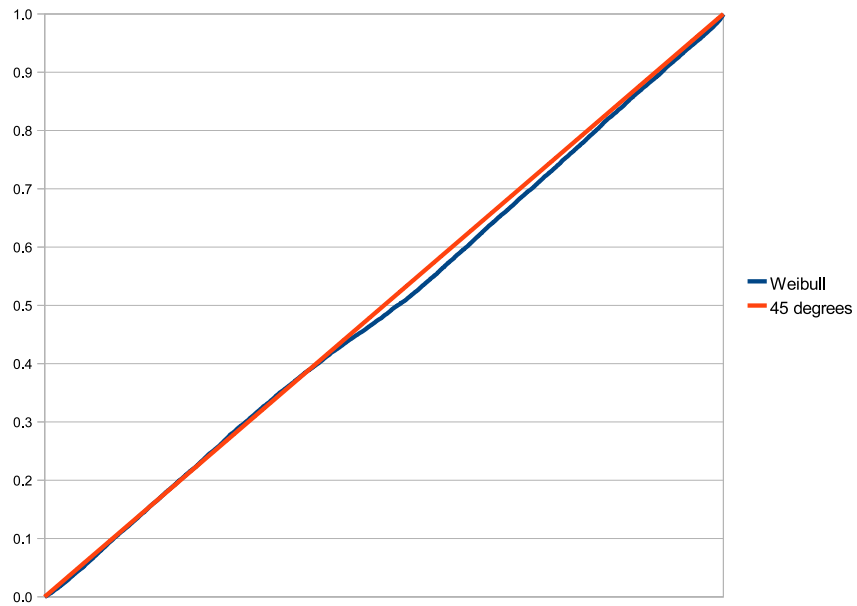


Figure 34: Test of calibrated log NO price residuals

## D Market integration algorithms

### D.1 2 market algorithm

#### Annotation:

1.  $p(t,2)$  represent prices in NL and NO respectively before any trading over the interconnectors
2.  $I1$  represents the NorNed interconnector, i.e. connection between markets 1 and 2
3.  $a$  and  $b$  represent elasticity parameters in markets 1 and 2 respectively such that 1MW of imports from an interconnector into market 1 reduces the electricity price in that market by  $a\%$
4.  $x$  represents the proportion of  $I1$  used to transmit electricity in equilibrium, where negative numbers represent exports from market 1 (NL) to market 2 (NO).

#### Scenarios:

i) Markets 1 and 2 are coupled.

The first equation defines price equality between markets 1 and 2

$$p(t,1)(1 - aI1x) = p(t,2)(1 + bI1x).$$

The equilibrium usage of  $I1$  is given by

$$x = \frac{p(t,1) - p(t,2)}{I1(ap(t,1) + bp(t,2))}.$$

The equilibrium price in market 1 and 2 is given by

$$result(t, \alpha) = p(t,2) \left( 1 + b \frac{p(t,1) - p(t,2)}{ap(t,1) + bp(t,2)} \right)$$

where  $\alpha = \{1,2\}$ . Equilibrium revenue from  $I1$  is zero. The condition for this equilibrium holding is as follows

$$abs \left( \frac{p(t,1) - p(t,2)}{I1(ap(t,1) + bp(t,2))} \right) \leq 1$$

ii) The resulting price in market 1 is higher than in market 2. Prices in the two markets are as follows

$$result(t,1) = p(t,1)(1 - aI1)$$

$$result(t,2) = p(t,2)(1 + bI1)$$

Equilibrium revenue from  $I1$  is given by

$$result(t,3) = I1 (p(t,1)(1 - aI1) - p(t,2)(1 + bI1))$$

The condition for this equilibrium holding is as follows

$$p(t,1)(1 - aI1) > p(t,2)(1 + bI1)$$

iii) The resulting price in market 1 is lower than in market 2. Prices in the two markets are as follows

$$result(t,1) = p(t,1)(1 + aI1)$$

$$result(t,2) = p(t,2)(1 - bI1)$$

Equilibrium revenue from  $I1$  is given by

$$result(t,3) = I1 (p(t,2)(1 - bI1) - p(t,1)(1 + aI1))$$

The condition for this equilibrium holding is as follows

$$p(t,1)(1 + aI1) < p(t,2)(1 - bI1)$$

## D.2 3 market algorithm

### Annotation:

1.  $p(t,1)$ ,  $p(t,2)$  and  $p(t,3)$  represent prices in NL, NO and the GB respectively before any trading over the interconnectors
2.  $I1$  and  $I2$  represent NorNed and BritNed interconnectors respectively, i.e. connections between markets 1 and 2 and between markets 1 and 3 respectively
3.  $a$ ,  $b$  and  $c$  represent elasticity parameters in markets 1, 2 and 3 respectively such that 1MW of imports from an interconnector into market 1 reduces the electricity price in that market by  $a\%$
4.  $x$  and  $y$  represent the proportions of  $I1$  and  $I2$  used to transmit electricity in equilibrium, where negative numbers represent exports from market 1 (NL) to neighbouring markets.

### Scenarios:

i) All three markets are coupled

The first two equations define price equality between the three markets.

$$\begin{cases} p(t,1)(1 - a(I1x + I2y)) = p(t,2)(1 + bI1x) \\ p(t,1)(1 - a(I1x + I2y)) = p(t,3)(1 + cI2y) \end{cases}$$

The equilibrium usage of interconnectors is given by

$$x = \frac{ap(t,1)p(t,3) + cp(t,1)p(t,3) - ap(t,1)p(t,2) - cp(t,2)p(t,3)}{I1(abp(t,1)p(t,2) + acp(t,1)p(t,3) + bcp(t,2)p(t,3))}$$

$$y = \frac{ap(t,1)p(t,2) + bp(t,1)p(t,2) - ap(t,1)p(t,3) - bp(t,2)p(t,3)}{I2(abp(t,1)p(t,2) + acp(t,1)p(t,3) + bcp(t,2)p(t,3))}.$$

This gives an equilibrium price in all three markets of

$$result(t, \alpha) = p(t,3) \left( 1 + c \frac{ap(t,1)p(t,2) + bp(t,1)p(t,2) - ap(t,1)p(t,3) - bp(t,2)p(t,3)}{abp(t,1)p(t,2) + acp(t,1)p(t,3) + bcp(t,2)p(t,3)} \right),$$



where  $\alpha = \{1, 2, 3\}$ . Equilibrium revenues from both interconnectors are zero, hence

$$result(t, \beta) = 0,$$

where  $\beta = \{4, 5\}$ . The conditions for this equilibrium holding are as follows

$$abs\left(\frac{ap(t, 1)p(t, 3) + cp(t, 1)p(t, 3) - ap(t, 1)p(t, 2) - cp(t, 2)p(t, 3)}{I1(abp(t, 1)p(t, 2) + acp(t, 1)p(t, 3) + bcp(t, 2)p(t, 3))}\right) \leq 1,$$

$$abs\left(\frac{ap(t, 1)p(t, 2) + bp(t, 1)p(t, 2) - ap(t, 1)p(t, 3) - bp(t, 2)p(t, 3)}{I2(abp(t, 1)p(t, 2) + acp(t, 1)p(t, 3) + bcp(t, 2)p(t, 3))}\right) \leq 1.$$

ii) Markets 1 and 2 are coupled.  $I2$  is constrained with imports into market 1.

The first equation defines price equality between markets 1 and 2, given that  $I2$  is constrained with imports into market 1

$$p(t, 1)(1 - a(I1x + I2)) = p(t, 2)(1 + bI1x).$$

The equilibrium usage of  $I1$  is given by

$$x = \frac{p(t, 1)(1 - aI2) - p(t, 2)}{I1(ap(t, 1) + bp(t, 2))}.$$

The equilibrium price in market 1 and 2 is given by

$$result(t, \alpha) = p(t, 2) \left(1 + b \frac{p(t, 1)(1 - aI2) - p(t, 2)}{ap(t, 1) + bp(t, 2)}\right)$$

where  $\alpha = \{1, 2\}$ . The equilibrium price in market 3 is given by

$$result(t, 3) = p(t, 3)(1 + cI2).$$

Equilibrium revenues from  $I1$  and  $I2$  are given respectively as follows

$$result(t, 4) = 0$$

$$result(t, 5) = I2 \left( p(t, 2) \left(1 + b \frac{p(t, 1)(1 - aI2) - p(t, 2)}{ap(t, 1) + bp(t, 2)}\right) - p(t, 3)(1 + cI2) \right).$$

The conditions for this equilibrium holding are as follows

$$abs\left(\frac{p(t, 1)(1 - aI2) - p(t, 2)}{I1(ap(t, 1) + bp(t, 2))}\right) \leq 1$$

$$p(t,3)(1+cI2) < p(t,2) \left( 1 + b \frac{p(t,1)(1-aI2) - p(t,2)}{ap(t,1) + bp(t,2)} \right).$$

iii) Markets 1 and 2 are coupled.  $I2$  is constrained with exports out of market 1.

The first equation defines price equality between markets 1 and 2, given that  $I2$  is constrained with exports out of market 1

$$p(t,1)(1 - a(I1x - I2)) = p(t,2)(1 + bI1x).$$

The equilibrium usage of  $I1$  is given by

$$x = \frac{p(t,1)(1 + aI2) - p(t,2)}{I1(ap(t,1) + bp(t,2))}.$$

The equilibrium price in market 1 and 2 is given by

$$result(t, \alpha) = p(t,2) \left( 1 + b \frac{p(t,1)(1 + aI2) - p(t,2)}{ap(t,1) + bp(t,2)} \right)$$

where  $\alpha = \{1,2\}$ . The equilibrium price in market 3 is given by

$$result(t,3) = p(t,3)(1 - cI2).$$

Equilibrium revenues from  $I1$  and  $I2$  are given respectively as follows

$$result(t,4) = 0$$

$$result(t,5) = I2 \left( p(t,3)(1 - cI2) - p(t,2) \left( 1 + b \frac{p(t,1)(1 + aI2) - p(t,2)}{ap(t,1) + bp(t,2)} \right) \right).$$

The conditions for this equilibrium holding are as follows

$$abs \left( \frac{p(t,1)(1 + aI2) - p(t,2)}{I1(ap(t,1) + bp(t,2))} \right) \leq 1$$

$$p(t,3)(1 - cI2) > p(t,2) \left( 1 + b \frac{p(t,1)(1 + aI2) - p(t,2)}{ap(t,1) + bp(t,2)} \right).$$

iv) Markets 1 and 3 are coupled.  $I1$  is constrained with imports into market 1.

The first equation defines price equality between markets 1 and 3, given that  $I1$  is constrained with imports into market 1

$$p(t,1)(1 - a(I2y + I1)) = p(t,3)(1 + cI2y).$$

The equilibrium usage of  $I2$  is given by

$$y = \frac{p(t,1)(1 - aI1) - p(t,3)}{I2(ap(t,1) + cp(t,3))}.$$

The equilibrium price in markets 1 and 3 is given by

$$result(t, \alpha) = p(t,3) \left( 1 + c \frac{p(t,1)(1 - aI1) - p(t,3)}{ap(t,1) + cp(t,3)} \right)$$

where  $\alpha = \{1,3\}$ . The equilibrium price in market 2 is given by

$$result(t,2) = p(t,2)(1 + bI1).$$

Equilibrium revenues from  $I1$  and  $I2$  are given respectively as follows

$$result(t,4) = I1 \left( p(t,3) \left( 1 + c \frac{p(t,1)(1 - aI1) - p(t,3)}{ap(t,1) + cp(t,3)} \right) - p(t,2)(1 + bI1) \right)$$

$$result(t,5) = 0.$$

The conditions for this equilibrium holding are as follows

$$abs \left( \frac{p(t,1)(1 - aI1) - p(t,3)}{I2(ap(t,1) + cp(t,3))} \right) \leq 1$$

$$p(t,2)(1 + bI1) < p(t,3) \left( 1 + c \frac{p(t,1)(1 - aI1) - p(t,3)}{ap(t,1) + cp(t,3)} \right).$$

v) Markets 1 and 3 are coupled.  $I1$  is constrained with exports out of market 1.

The first equation defines price equality between markets 1 and 3, given that  $I1$  is constrained with exports out of market 1

$$p(t,1)(1 - a(I2y - I1)) = p(t,3)(1 + cI2y).$$

The equilibrium usage of  $I2$  is given by

$$y = \frac{p(t,1)(1 + aI1) - p(t,3)}{I2(ap(t,1) + cp(t,3))}.$$

The equilibrium price in market 1 and 3 is given by

$$result(t, \alpha) = p(t,3) \left( 1 + c \frac{p(t,1)(1 + aI1) - p(t,3)}{ap(t,1) + cp(t,3)} \right)$$

where  $\alpha = \{1, 3\}$ . The equilibrium price in market 2 is given by

$$result(t,2) = p(t,2)(1 - bI1).$$

Equilibrium revenues from  $I1$  and  $I2$  are given respectively as follows

$$result(t,4) = I1 \left( p(t,2)(1 - bI1) - p(t,3) \left( 1 + c \frac{p(t,1)(1 + aI1) - p(t,3)}{ap(t,1) + cp(t,3)} \right) \right)$$

$$result(t,5) = 0.$$

The conditions for this equilibrium holding are as follows

$$abs \left( \frac{p(t,1)(1 + aI1) - p(t,3)}{I2(ap(t,1) + cp(t,3))} \right) \leq 1$$

$$p(t,2)(1 - bI1) > p(t,3) \left( 1 + c \frac{p(t,1)(1 + aI1) - p(t,3)}{ap(t,1) + cp(t,3)} \right).$$

vi) The resulting price in market 1 is higher than in markets 2 and 3.

Prices in the three markets are as follows

$$result(t,1) = p(t,1)(1 - a(I1 + I2))$$

$$result(t,2) = p(t,2)(1 + bI1)$$

$$result(t,3) = p(t,3)(1 + cI2)$$

Equilibrium revenues from  $I1$  and  $I2$  are given respectively as follows

$$result(t,4) = I1 (p(t,1)(1 - a(I1 + I2)) - p(t,2)(1 + bI1))$$

$$result(t,5) = I2(p(t,1)(1 - a(I1 + I2)) - p(t,3)(1 + cI2))$$

The conditions for this equilibrium holding are as follows

$$p(t,1)(1 - a(I1 + I2)) > p(t,2)(1 + bI1)$$

$$p(t,1)(1 - a(I1 + I2)) > p(t,3)(1 + cI2)$$

vii) The resulting price in market 1 is lower than in markets 2 and 3.

Prices in the three markets are as follows

$$result(t,1) = p(t,1)(1 + a(I1 + I2))$$

$$result(t,2) = p(t,2)(1 - bI1)$$

$$result(t,3) = p(t,3)(1 - cI2)$$

Equilibrium revenues from  $I1$  and  $I2$  are given respectively as follows

$$result(t,4) = I1(p(t,2)(1 - bI1) - p(t,1)(1 + a(I1 + I2)))$$

$$result(t,5) = I2(p(t,3)(1 - cI2) - p(t,1)(1 + a(I1 + I2)))$$

The conditions for this equilibrium holding are as follows

$$p(t,1)(1 + a(I1 + I2)) < p(t,2)(1 - bI1)$$

$$p(t,1)(1 + a(I1 + I2)) < p(t,3)(1 - cI2)$$

ix) The resulting price in market 1 is higher than in market 2 but lower than in market 3.

Prices in the three markets are as follows

$$result(t,1) = p(t,1)(1 - a(I1 - I2))$$

$$result(t,2) = p(t,2)(1 + bI1)$$

$$result(t,3) = p(t,3)(1 - cI2)$$

Equilibrium revenues from  $I1$  and  $I2$  are given respectively as follows

$$result(t,4) = I1(p(t,1)(1 - a(I1 - I2)) - p(t,2)(1 + bI1))$$

$$result(t,5) = I2(p(t,3)(1 - cI2) - p(t,1)(1 - a(I1 - I2)))$$

The conditions for this equilibrium holding are as follows

$$p(t,1)(1 - a(I1 - I2)) > p(t,2)(1 + bI1)$$

$$p(t,1)(1 - a(I1 - I2)) < p(t,3)(1 - cI2)$$

ix) The resulting price in market 1 is lower than in market 2 but higher than in market 3.

Prices in the three markets are as follows

$$result(t,1) = p(t,1)(1 - a(I2 - I1))$$

$$result(t,2) = p(t,2)(1 - bI1)$$

$$result(t,3) = p(t,3)(1 + cI2)$$

Equilibrium revenues from  $I1$  and  $I2$  are given respectively as follows

$$result(t,4) = I1(p(t,2)(1 - bI1) - p(t,1)(1 - a(I2 - I1)))$$

$$result(t,5) = I2(p(t,1)(1 - a(I2 - I1)) - p(t,3)(1 + cI2))$$

The conditions for this equilibrium holding are as follows

$$p(t,1)(1 - a(I2 - I1)) < p(t,2)(1 - bI1)$$

$$p(t,1)(1 - a(I2 - I1)) > p(t,3)(1 + cI2)$$

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