Online Appendix to "Consumption Risk and the Cross-Section of Government Bond Returns"

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This Online Appendix includes complementary notes and results related to the paper "Consumption Risk and the Cross Section of Government Bond Returns". It should be read in conjunction with that paper. It is divided into three parts.

The first part, A, includes a detailed derivation of stochastic discount factor, Euler equation and pricing factors from VAR model.

Part B of the appendix contains the results related to the assessment of the robustness of our conclusions in the main paper. For clarity, we divide these into subsections. In part B1 we report the estimation results related to a joint pricing a cross-section of 10 Fama Maturity Portfolios and equity portfolios. In sections B2 and B3 we provide the empirical results for two alternate sets of test assets: 7 Fixed Term Indices and 5 Fama-Bliss Discount Bonds. Part B4 contains the estimation results for 10 Fama Maturity portfolios when an alternative measure of consumption is used, in the spirit of Piazzesi and Schneider (2007). In section B5 we present the results for 10 Fama Maturity Portfolios during a period 1982–2011. In the last subsection, B6, we report the estimation results of an extended version of our model with an additional pricing factor, the volatility of the innovations to expectations in future consumption growth.

Finally, part C of the appendix includes a detailed description of the panel of macroeconomic and financial variables that are used in factor analysis.

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Appendix A

A1: Derivation of stochastic discount factor

The aim of this appendix is to show a step-by-step derivation of eq.(4) in the main paper, given Epstein–Zin utility function and process for consumption growth.

Let's assume Epstein-Zin preferences that are represented by time t utility of representative consumer of the following form:

$$V_t = \left\{(1-\beta)C_t^{1-\rho} + \beta\left[E_t\left(V_{t+1}^{1-\gamma}\right)\right]^{\frac{1-\rho}{1-\gamma}}\right\}^{\frac{1}{1-\rho}} \tag{1}$$

where γ is the coefficient of risk aversion and $\frac{1}{\rho}$ is the elasticity of intertemporal substitution ($EIS = \frac{1}{\rho}$). Let's denote as $R_r(V_{t+1}) \equiv [E_t(V_{t+1}^{1-\gamma})]^{\frac{1}{1-\gamma}}$. $R_r(V_{t+1})$ represents a risk adjustment to continuation value at time $t+1$. Then we can rewrite (1) as:

$$V_t = \left\{(1-\beta)C_t^{1-\rho} + \beta\left[R_r(V_{t+1})\right]^{1-\rho}\right\}^{\frac{1}{1-\rho}}$$

Stochastic Discount Factor (SDF) M_{t+1} is defined as the Intertemporal Marginal Rate of Substitution (IMRS):

$$M_{t+1} = \frac{\partial V_t/\partial C_{t+1}}{\partial V_t/\partial C_t}$$

and for Epstein-Zin preferences it is the following:

$$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left[\frac{V_{t+1}}{R_t(V_{t+1})}\right]^{\rho-\gamma}$$

We consider in our paper a special case of Epstein-Zin preferences, i.e. when the $EIS = 1$. In this case the Epstein–Zin utility function collapses to

$$\begin{aligned} V_t &= C_t^{1-\beta}[R_t(V_{t+1})]^{\beta} \qquad (2)\\ &= C_t^{1-\beta}\left[E_t\left(V_{t+1}^{1-\gamma}\right)\right]^{\frac{\beta}{1-\gamma}} \end{aligned}$$

and the SDF is defined as

$$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left[\frac{V_{t+1}}{R_t(V_{t+1})}\right]^{1-\gamma} \tag{3}$$

The SDF is a function of consumption growth and and an unobservable element $\frac{V_{t+1}}{R_t(V_{t+1})}$ which depends on future utility V_{t+1}.

One way to find a closed form solution for the above SDF, is to assume a tractable process for consumption growth, and solve for the unobservable element. Specifically, we assume that consumption growth follows some $MA(\infty)$ process of the following general form:

$$\Delta c_{t+1} = \mu_c + \alpha(L)\omega_{t+1} \tag{4}$$

where $\alpha(L)$ is a lag polynomial operator defined as $\alpha(L) = \sum_{s=0}^{\infty}\alpha_s L^s$ and ω_{t+1} is *iid* standard normal process ($\omega_{t+1} \sim iid\ N(0,1)$). Such a specification is quite broad and allows for a wide range of possible models of consumption path. We can rewrite then eq.(4) as:

$$\begin{aligned}\Delta c_{t+1} &= \mu_c + \sum_{s=0}^{\infty}\alpha_s L^s \omega_{t+1} \\ &= \mu_c + \sum_{s=0}^{\infty}\alpha_s \omega_{t+1-s} \\ &= \mu_c + \alpha_0\omega_{t+1} + \alpha_1\omega_t + \alpha_2\omega_{t-1} + \alpha_3\omega_{t-2} + \alpha_4\omega_{t-3} + ...\end{aligned}$$

From the above consumption growth process we can derive the change in expectations for consumption growth over future period s and link it to the current shock to consumption growth ω_{t+1}:

$$E_{t+1}(\Delta c_{t+1+s}) - E_t(\Delta c_{t+1+s}) = \alpha_s\omega_{t+1} \quad \forall s = 0, 1, 2, ... \tag{5}$$

In order to find a closed form solution to $\frac{V_{t+1}}{R_t(V_{t+1})}$, which is linked to future consumption via recursion in (2) we first scale the continuation value V_t in eq.(2) by consumption C_t:

$$\frac{V_t}{C_t} = \left[R_t\left(\frac{V_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)\right]^{\beta} \tag{6}$$

Then, denoting $\upsilon_t \equiv \log(\frac{V_t}{C_t})$ and $c_t \equiv \log(C_t)$, we can rewrite the above equation as:

$$\begin{aligned}\upsilon_t &= \beta \log R_t(e^{\upsilon_{t+1}+\Delta c_{t+1}}) \\ \upsilon_t &= \frac{\beta}{1-\gamma}\log E_t[e^{(1-\gamma)(\upsilon_{t+1}+\Delta c_{t+1})}]\end{aligned} \tag{7}$$

where $\Delta c_{t+1} = c_{t+1} - c_t$.

Assuming Epstein-Zin preferences, given in eq.(2) and rewritten in eq.(7), and consumption growth process, given in eq.(4), the closed form solution for υ_t is the following:

$$\upsilon_t = \mu_\upsilon + v(L)\omega_t \tag{8}$$

where

$$\mu_\upsilon = \frac{\beta}{1-\beta}\left[\mu_c + \frac{1-\gamma}{2}\alpha(\beta)^2\right]$$

and

$$v(L) = \beta\frac{\alpha(L)-\alpha(\beta)}{L-\beta} = \beta[\alpha(L)-\alpha(\beta)]L^{-1}\sum_{s=0}^{\infty}\beta^s L^{-s}$$

The solution is of a simple "guess and try" form: first we guess the form a solution and then verify it and find the parameters μ_υ and $v(L)$ by substituting the solution into eq.(7). It turns out that $v(L)$ is the solution to the following forecasting problem:

$$v(L)\omega_t = \sum_{s=0}^{\infty}\beta^s E_t(\Delta c_{t+1+s} - \mu_c)$$

We can now introduce the solution to υ_t, given in eq.(8), and consumption growth process, expressed in eq.(4), into the SDF in eq.(3) and find a tractable form of SDF. But first let's rewrite the SDF as:

$$
\begin{aligned}
M_{t+1} &= \beta \left(\frac{C_{t+1}}{C_t}\right)^{-1} \left[\frac{V_{t+1}}{R_t(V_{t+1})}\right]^{1-\gamma} \\
&= \beta \left(\frac{C_{t+1}}{C_t}\right)^{-1} \left[\frac{\frac{V_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}}{R_t\left(\frac{V_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)}\right]^{1-\gamma}
\end{aligned}
$$

and compute the log of SDF, denoting $m_{t+1} \equiv \log(M_{t+1})$:

$$
\begin{aligned}
m_{t+1} &= \log\beta - \Delta c_{t+1} + (1-\gamma)\log\left[\frac{\frac{V_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}}{R_t\left(\frac{V_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)}\right] \qquad (9) \\
&= \log\beta - \Delta c_{t+1} + (1-\gamma)[\upsilon_{t+1} + \Delta c_{t+1} - \log R_t(e^{\upsilon_{t+1}+\Delta c_{t+1}})] \\
&= \log\beta - \Delta c_{t+1} + (1-\gamma)(\upsilon_{t+1} + \Delta c_{t+1}) - \log E_t[e^{(1-\gamma)(\upsilon_{t+1}+\Delta c_{t+1})}]
\end{aligned}
$$

Let's now substitute the solution to υ_t, given in eq.(8), and consumption growth process, given in eq.(4), into the relevant parts of the log SDF given in the above equation.

Specifically, we can write the $(1-\gamma)(\upsilon_{t+1} + \Delta c_{t+1})$ element in eq.(9) as:

$$
\begin{aligned}
(1-\gamma)(\upsilon_{t+1} + \Delta c_{t+1}) &= (1-\gamma)[\mu_\upsilon + v(L)\omega_{t+1} + \mu_c + \alpha(L)\omega_{t+1}] \qquad (10) \\
&= (1-\gamma)(\mu_\upsilon + \mu_c) + (1-\gamma)[v(L) + \alpha(L)]\omega_{t+1} \\
&= (1-\gamma)(\mu_\upsilon + \mu_c) + (1-\gamma)A(L)\omega_t
\end{aligned}
$$

where we denote $A(L) \equiv \frac{v(L)+\alpha(L)}{L}$ and we use the following property of lag operator: $v(L)\omega_{t+1} = \frac{v(L)}{L}\omega_t$ and $\alpha(L)\omega_{t+1} = \frac{\alpha(L)}{L}\omega_t$.

We can also write the $\log E_t[e^{(1-\gamma)(\upsilon_{t+1}+\Delta c_{t+1})}]$ element in eq.(9) as:

$$
\begin{aligned}
\log E_t[e^{(1-\gamma)(\upsilon_{t+1}+\Delta c_{t+1})}] &= \log E_t[e^{(1-\gamma)[\mu_\upsilon + v(L)\omega_{t+1} + \mu_c + \alpha(L)\omega_{t+1}]}] \qquad (11) \\
&= \log E_t[e^{(1-\gamma)(\mu_\upsilon+\mu_c)} e^{(1-\gamma)[v(L)+\alpha(L)]\omega_{t+1}}] \\
&= \log E_t[e^{(1-\gamma)(\mu_\upsilon+\mu_c)} e^{(1-\gamma)A(L)\omega_t}] \\
&= (1-\gamma)(\mu_\upsilon + \mu_c) + \log E_t[e^{(1-\gamma)A(L)\omega_t}]
\end{aligned}
$$

Note that $A(L)\omega_t$ is a function of all past shocks up to time $t+1$. Given the assumption of $\omega_t \sim iid$ $N(0,1)$ and the property that $E(e^{\omega_t}) = e^{E(\omega_t)+\frac{1}{2}Var(\omega_t)}$, we can express the above as:

$$
\begin{aligned}
\log E_t[e^{(1-\gamma)(\upsilon_{t+1}+\Delta c_{t+1})}] &= (1-\gamma)(\mu_\upsilon + \mu_c) + \log\left[e^{E_t[(1-\gamma)A(L)\omega_t] + \frac{1}{2}Var_t[(1-\gamma)A(L)\omega_t]}\right] \\
&= (1-\gamma)(\mu_\upsilon + \mu_c) + E_t[(1-\gamma)A(L)\omega_t] + \frac{1}{2}Var_t[(1-\gamma)A(L)\omega_t] \qquad (12)
\end{aligned}
$$

Substituting (10) and (12) into the log of SDF in eq.(9), we obtain:

$$
\begin{aligned}
m_{t+1} &= \log\beta - \Delta c_{t+1} + (1-\gamma)A(L)\omega_t - E_t[(1-\gamma)A(L)\omega_t] - \frac{1}{2}Var_t[(1-\gamma)A(L)\omega_t] \\
&= \log\beta - \Delta c_{t+1} + (1-\gamma)\{A(L)\omega_t - E_t[A(L)\omega_t]\} - \frac{(1-\gamma)^2}{2}Var_t[A(L)\omega_t]
\end{aligned} \tag{13}
$$

We show below, in Proof 1, that

$$A(L)\omega_t - E_t[A(L)\omega_t] = \alpha(\beta)\omega_{t+1} \tag{14}$$

$$Var_t[A(L)\omega_t] = \alpha(\beta)^2 \tag{15}$$

Substituting the above into log SDF in eq.(13) we get:

$$
\begin{aligned}
m_{t+1} &= \log\beta - \Delta c_{t+1} + (1-\gamma)[\alpha(\beta)\omega_{t+1}] - \frac{(1-\gamma)^2}{2}\alpha(\beta)^2 \\
&= \log\beta - \Delta c_{t+1} + (1-\gamma)\sum_{s=0}^{\infty}\alpha_s\beta^s\omega_{t+1} - \frac{(1-\gamma)^2}{2}\left(\sum_{s=0}^{\infty}\alpha_s\beta^s\right)^2
\end{aligned} \tag{16}
$$

where $\alpha(\beta) = \sum_{s=0}^{\infty}\alpha_s\beta^s$. Using (5), we can rewrite the above as:

$$
\begin{aligned}
m_{t+1} &= \log\beta - \Delta c_{t+1} + (1-\gamma)[\alpha(\beta)\omega_{t+1}] - \frac{(1-\gamma)^2}{2}\alpha(\beta)^2 \\
&= \log\beta - \Delta c_{t+1} + (1-\gamma)\sum_{s=0}^{\infty}\beta^s(E_{t+1} - E_t)(\Delta c_{t+1+s}) \\
&\quad - \frac{(1-\gamma)^2}{2}Var_t\left[\sum_{s=0}^{\infty}\beta^s(E_{t+1} - E_t)(\Delta c_{t+1+s})\right]
\end{aligned} \tag{17}
$$

This is equation (5) in our paper.

Proof 1. Our aim is to prove equations (14) and (15). From the properties of conditional expectation function and the fact that $\omega_t \sim iid\ N(0,1)$ we can write:

$$E_t[A(L)\omega_t] = A(L)_+\omega_t$$

where the term $[A(L)]_+$ denotes the nonnegative degrees of the lag polynomial operator $A(L)$. Intuitively, given that all the shocks ω_t have mean zero, the conditional expectation of some linear function of past and future shocks will be a function of past shocks only (expressed by positive degrees of lag polynomial operator) as the future shocks (expressed by negative degrees of lag polynomial operator) will have zero expectation.

Additionally, we use the following other two properties of lag polynomial operators:

$$\frac{v(L) + \alpha(L)}{L} = \frac{v(L)}{L} + \frac{\alpha(L)}{L}$$

$$\left[\frac{v(L)}{L} + \frac{\alpha(L)}{L}\right]_+ = \left[\frac{v(L)}{L}\right]_+ + \left[\frac{\alpha(L)}{L}\right]_+$$

We can write then (14) as:

$$
\begin{aligned}
A(L)\omega_t - E_t[A(L)\omega_t] &= A(L)\omega_t - A(L)_+\omega_t \\
&= [A(L) - A(L)_+]\omega_t \\
&= \left\{ \frac{v(L)+\alpha(L)}{L} - \left[\frac{v(L)+\alpha(L)}{L}\right]_+ \right\} \omega_t \\
&= \left\{ \frac{\alpha(L)}{L} - \left[\frac{\alpha(L)}{L}\right]_+ + \frac{v(L)}{L} - \left[\frac{v(L)}{L}\right]_+ \right\} \omega_t
\end{aligned} \tag{18}
$$

Let's find first $\frac{\alpha(L)}{L} - \left[\frac{\alpha(L)}{L}\right]_+$ and then $\frac{v(L)}{L} - \left[\frac{v(L)}{L}\right]_+$.

(1)

To find $\frac{\alpha(L)}{L} - \left[\frac{\alpha(L)}{L}\right]_+$ we take:

$$
\begin{aligned}
\frac{\alpha(L)}{L} &= \frac{\alpha_0 + \alpha_1 L + \alpha_2 L^2 + \alpha_3 L^3 + ...}{L} \\
&= \alpha_0 L^{-1} + \alpha_1 + \alpha_2 L + \alpha_3 L^2 + ... \\
[\frac{\alpha(L)}{L}]_+ &= [\alpha_0 L^{-1} + \alpha_1 + \alpha_2 L + \alpha_3 L^2 + ...] \\
&= \alpha_1 + \alpha_2 L + \alpha_3 L^2 + ...
\end{aligned}
$$

So

$$
\frac{\alpha(L)}{L} - \left[\frac{\alpha(L)}{L}\right]_+ = \alpha_0 L^{-1} \tag{19}
$$

(2)

Finding $\frac{v(L)}{L} - \left[\frac{v(L)}{L}\right]_+$ is a bit more complex. From the solution to v_t in eq.(4) we know that $v(L) = \beta\frac{\alpha(L)-\alpha(\beta)}{L-\beta} = \beta[\alpha(L) - \alpha(\beta)]L^{-1}\sum_{s=0}^{\infty}\beta^s L^{-s}$ so

$$
\begin{aligned}
\frac{v(L)}{L} - \left[\frac{v(L)}{L}\right]_+ &= \beta\alpha(L)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s} - \beta\alpha(\beta)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s} \\
&\quad - \left[\beta\alpha(L)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s} - \beta\alpha(\beta)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s}\right]_+ \\
&= \left\{\beta\alpha(L)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s} - \left[\beta\alpha(L)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s}\right]_+\right\} \\
&\quad - \left\{\beta\alpha(\beta)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s} - \left[\beta\alpha(\beta)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s}\right]_+\right\}
\end{aligned} \tag{20}
$$

We can also write $\beta\alpha(L)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s}$ as:

$$
\beta\alpha(L)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s} = \beta(\alpha_0 + \alpha_1 L + \alpha_2 L^2 + \alpha_3 L^3 + \alpha_4 L^4 + ...)L^{-2}(1 + \beta L^{-1} + \beta^2 L^{-2} + \beta^3 L^{-3} + ...)
$$

So

$$\beta\alpha(L)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s} - \left[\beta\alpha(L)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s}\right]_+ = \beta(\alpha_0 L^{-2} + \alpha_1 L^{-1}) \tag{21}$$
$$+ \beta^2(\alpha_0 L^{-3} + \alpha_1 L^{-2} + \alpha_2 L^{-1})$$
$$+ \beta^3(\alpha_0 L^{-4} + \alpha_1 L^{-3} + \alpha_2 L^{-2} + \alpha_3 L^{-1})$$
$$+ \beta^4(\alpha_0 L^{-5} + \alpha_1 L^{-4} + \alpha_2 L^{-3} + \alpha_3 L^{-2} + \alpha_4 L^{-1})$$
$$+ .....$$

We can also write $\beta\alpha(\beta)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s}$ as:

$$\beta\alpha(\beta)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s} = \beta(\alpha_0 + \alpha_1\beta + \alpha_2\beta^2 + \alpha_3\beta^3 + \alpha_4\beta^4 + ...)L^{-2}(1 + \beta L^{-1} + \beta^2 L^{-2} + \beta^3 L^{-3} + ...)$$

and

$$\beta\alpha(\beta)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s} - \left[\beta\alpha(\beta)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s}\right]_+ = \beta\alpha(\beta)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s}$$

since $\left[\beta\alpha(\beta)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s}\right]_+ = 0$. So

$$\beta\alpha(\beta)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s} - \left[\beta\alpha(\beta)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s}\right]_+ = \beta(\alpha_0 L^{-2} + \alpha_1\beta L^{-2} + \alpha_2\beta^2 L^{-2} + \alpha_3\beta^3 L^{-2} + \alpha_4\beta^4 L^{-2} + ...) \tag{22}$$
$$= \beta(\alpha_0 L^{-2} + \alpha_1\beta L^{-2} + \alpha_2\beta^2 L^{-2} + \alpha_3\beta^3 L^{-2} + \alpha_4\beta^4 L^{-2} + ...)$$
$$+ \beta^2(\alpha_0 L^{-3} + \alpha_1\beta L^{-3} + \alpha_2\beta^2 L^{-3} + \alpha_3\beta^3 L^{-3} + \alpha_4\beta^4 L^{-3} + ...)$$
$$+ \beta^3(\alpha_0 L^{-4} + \alpha_1\beta L^{-4} + \alpha_2\beta^2 L^{-4} + \alpha_3\beta^3 L^{-4} + \alpha_4\beta^4 L^{-4} + ...)$$
$$+ \beta^4(\alpha_0 L^{-5} + \alpha_1\beta L^{-5} + \alpha_2\beta^2 L^{-5} + \alpha_3\beta^3 L^{-5} + \alpha_4\beta^4 L^{-5} + ...)$$
$$+ ...$$

As stated in (20), we can compute $\frac{v(L)}{L} - \left[\frac{v(L)}{L}\right]_+$ as the difference between (21) and (22). It is the following:

$$\frac{v(L)}{L} - \left[\frac{v(L)}{L}\right]_+ = \left\{\beta\alpha(L)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s} - \left[\beta\alpha(L)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s}\right]_+\right\} \tag{23}$$
$$- \left\{\beta\alpha(\beta)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s} - \left[\beta\alpha(\beta)L^{-2}\sum_{s=0}^{\infty}\beta^s L^{-s}\right]_+\right\}$$
$$= \alpha_1\beta L^{-1} + \alpha_2\beta^2 L^{-1} + \alpha_3\beta^3 L^{-1} + \alpha_4\beta^4 L^{-1} + ...$$
$$= [\alpha(\beta) - \alpha_0]L^{-1}$$

We can now come back to eq.(18) and substitute (19) and (23) into this equation:

$$\begin{aligned} A(L)\omega_t - E_t[A(L)\omega_t] &= \left\{ \frac{\alpha(L)}{L} - \left[\frac{\alpha(L)}{L}\right]_+ + \frac{v(L)}{L} - \left[\frac{v(L)}{L}\right]_+ \right\} \omega_t \\ &= \left\{\alpha_0 L^{-1} + [\alpha(\beta) - \alpha_0]L^{-1}\right\} \omega_t \\ &= \alpha(\beta)L^{-1}\omega_t \\ &= \alpha(\beta)\omega_{t+1} \end{aligned}$$

This ends the proof of (14).

Since $A(L)\omega_t - E_t[A(L)\omega_t] = \alpha(\beta)\omega_{t+1}$, we can now find $Var_t[A(L)\omega_t]$:

$$\begin{aligned} Var_t[A(L)\omega_t] &= E_t\{A(L)\omega_t - E_t[A(L)\omega_t]\}^2 \\ &= E_t[\alpha(\beta)\omega_{t+1}]^2 \\ &= \alpha(\beta)^2 E_t[\omega_{t+1}]^2 \\ &= \alpha(\beta)^2 V_t(\omega_{t+1}) \\ &= \alpha(\beta)^2 \end{aligned}$$

since $V_t(\omega_{t+1}) = 1$. This ends the proof of (15). ■

B2: Derivation of Euler equation

We start with basic Euler equation, which is the following:

$$E_t(M_{t+1}R^i_{t+1}) = 1$$

where M_{t+1} is an SDF and R^i_{t+1} is a gross return on asset i.

A common assumption in asset pricing literature is the joint conditional log-normality of asset returns and SDF, that allows to write the above equation in an equivalent log form:

$$E_t(r^i_{t+1}) + \frac{1}{2}V_t(r^i_{t+1}) + E_t(m_{t+1}) + \frac{1}{2}V_t(m_{t+1}) + Cov_t(r^i_{t+1}, m_{t+1}) = 0 \tag{24}$$

where $r^i_{t+1} \equiv \log(R^i_{t+1})$ and $m_{t+1} \equiv \log(M_{t+1})$. The implication of Euler equation for a risk free rate is the following:

$$r^f_{t+1} = -\left[E_t(m_{t+1}) + \frac{1}{2}V_t(m_{t+1})\right]$$

Note that at time t risk free rate is fixed so $r^f_{t+1} \equiv E_t(r^f_{t+1})$. Rewriting then (24), we get:

$$E_t(r^i_{t+1} - r^f_{t+1}) + \frac{1}{2}V_t(r^i_{t+1}) + Cov_t(r^i_{t+1}, m_{t+1}) = 0$$

Assuming as well that the joint conditional distribution of SDF and asset returns is homoscedastic, the log Euler equation becomes:

$$E_t(r^i_{t+1}) + \frac{1}{2}\sigma^2_i + E_t(m_{t+1}) + \frac{1}{2}\sigma^2_m + \sigma_{im} = 0 \tag{25}$$

where $\sigma_i^2 \equiv V\left[r_{t+1}^i - E_t(r_{t+1}^i)\right]$ denotes the unconditional variance of innovations to log asset i return, $\sigma_m^2 \equiv V\left[m_{t+1} - E_t(m_{t+1})\right]$ denotes the unconditional variance of innovations to log SDF and $\sigma_{im} \equiv Cov\left[r_{t+1}^i - E_t(r_{t+1}^i), m_{t+1} - E_t(m_{t+1})\right]$ denotes the unconditional covariance of innovations to log asset i return and log SDF. Special case of the above equation for risk free rate will yields:

$$r_{t+1}^f = -\left[E_t(m_{t+1}) + \frac{1}{2}\sigma_m^2\right]$$

So the Euler equation will become the following:

$$E_t(r_{t+1}^i - r_{t+1}^f) + \frac{1}{2}\sigma_i^2 + \sigma_{im} = 0 \tag{26}$$

The risk premia will be constant, though the expected asset returns will vary according to variation in risk free rate as below:

$$\begin{aligned} E_t(r_{t+1}^i) &= -E_t(m_{t+1}) - \frac{1}{2}\sigma_m^2 - \frac{1}{2}\sigma_i^2 - \sigma_{im} \\ &= r_{t+1}^f - \frac{1}{2}\sigma_i^2 - \sigma_{im} \end{aligned}$$

From the law of total covariance we can write:

$$\begin{aligned} Cov(m_{t+1}, r_{t+1}^i) &= E[Cov_t(m_{t+1}, r_{t+1}^i)] + Cov[E_t(m_{t+1}), E_t(r_{t+1}^i)] \\ &= E[\sigma_{im}] + Cov[E_t(m_{t+1}), -E_t(m_{t+1})] \\ &= \sigma_{im} - Var(r_{t+1}^f) \end{aligned} \tag{27}$$

and

$$\begin{aligned} Cov(m_{t+1}, r_{t+1}^f) &= E[Cov_t(m_{t+1}, r_{t+1}^f)] + Cov[E_t(m_{t+1}), E_t(r_{t+1}^f)] \\ &= E[0] + Cov[E_t(m_{t+1}), -E_t(m_{t+1})] \\ &= -Var(r_{t+1}^f) \end{aligned} \tag{28}$$

Then from (27) and (28) we get:

$$\begin{aligned} Cov(m_{t+1}, r_{t+1}^i - r_{t+1}^f) &= Cov(m_{t+1}, r_{t+1}^i) - Cov(m_{t+1}, r_{t+1}^f) \\ &= \sigma_{im} - Var(r_{t+1}^f) - [-Var(r_{t+1}^f)] \\ &= \sigma_{im} \end{aligned} \tag{29}$$

Also, from the law of total variance we can write:

$$\begin{aligned} Var(r_{t+1}^i) &= E[Var_t(r_{t+1}^i)] + Var[E_t(r_{t+1}^i)] \\ &= E[\sigma_i^2] + Var[-E_t(m_{t+1})] \\ &= \sigma_i^2 + Var(r_{t+1}^f) \end{aligned}$$

This implies that

$$\sigma_i^2 = Var(r_{t+1}^i) - Var(r_{t+1}^f) \tag{30}$$

Introducing (29) and (30) into Euler equation in (26), we get the following:

$$E_t(r_{t+1}^i - r_{t+1}^f) + \frac{1}{2}\sigma_i^2 + \sigma_{im} = 0$$

$$E_t(r_{t+1}^i - r_{t+1}^f) + \frac{1}{2}[Var(r_{t+1}^i) - Var(r_{t+1}^f)] + Cov(m_{t+1}, r_{t+1}^i - r_{t+1}^f) = 0$$

$$E_t(r_{t+1}^i - r_{t+1}^f) + \frac{1}{2}Var(r_{t+1}^i) - \frac{1}{2}Var(r_{t+1}^f) = -Cov(m_{t+1}, r_{t+1}^i - r_{t+1}^f)$$

which is equation (6) in our paper.

A3: Derivation of pricing factors from VAR model

Let's consider a general VAR(1) model of the following form:

$$\mathbf{Z}_{t+1} = \mathbf{A}\mathbf{Z}_t + \eta_{t+1}$$

In our application, vector $\mathbf{Z}_t$ has log consumption growth Δc_t as its first element. The other elements in this vector, denoted as $\mathbf{x}_t$, are state variables. We assume then the following joint dynamics for consumption growth Δc_t and state variables in $\mathbf{x}_t$:

$$\begin{bmatrix} \Delta c_{t+1} \\ \mathbf{x}_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \Delta c_t \\ \mathbf{x}_t \end{bmatrix} + \begin{bmatrix} \eta_{c,t+1} \\ \eta_{x,t+1} \end{bmatrix}$$

Our aim is to extract the innovation to expectations in future consumption growth, defined as:

$$\varepsilon_{c,t+1} = \sum_{j=0}^{\infty} \beta^j (E_{t+1} - E_t)\Delta c_{t+1+j} \tag{31}$$

given the VAR(1) model for dynamics of consumption growth.

Let's first define ε_{t+1} as

$$\varepsilon_{t+1} = \sum_{j=0}^{\infty} \beta^j (E_{t+1} - E_t)\mathbf{Z}_{t+1+j} \tag{32}$$

Given above, $\varepsilon_{c,t+1}$ is simply the first element of vector ε_{t+1} that can be written as:

$$\varepsilon_{c,t+1} = \mathbf{e}_1'\varepsilon_{t+1} \tag{33}$$

where $\mathbf{e}_1$ is a vector with the first element equal to 1 and all others equal to zero, of a relevant size.

For any $j = 0, 1, 2, ...$ we can write

$$E_{t+1}(\mathbf{Z}_{t+1+j}) - E_t(\mathbf{Z}_{t+1+j}) = \mathbf{A}^j \eta_{t+1}$$

Introducing the above into eq.(32), we get:

$$
\begin{aligned}
\varepsilon_{t+1} &= \sum_{j=0}^{\infty} \beta^j (E_{t+1} - E_t) \mathbf{Z}_{t+1+j} \\
&= \left(\sum_{j=0}^{\infty} \beta^j \mathbf{A}^j \right) \eta_{t+1} \\
&= \left[\mathbf{I} + (\beta \mathbf{A})^1 + (\beta \mathbf{A})^2 + (\beta \mathbf{A})^3 + ... \right] \eta_{t+1} \\
&= (\mathbf{I} + \beta \mathbf{A})^{-1} \eta_{t+1}
\end{aligned}
$$

So the innovation to expectations in future consumption growth, $\varepsilon_{c,t+1}$, equals

$$\varepsilon_{c,t+1} = \mathbf{e}_1' (\mathbf{I} + \beta \mathbf{A})^{-1} \eta_{t+1}$$

The above is equation (10) in our paper.

Appendix B

B1: Estimation results for pricing a cross-section of jointly 10 Fama Maturity Portfolios and equity portfolios.

Table B1.1: GMM estimation of Euler equation with linear SDF for Fama Maturity Portfolios and SP500

Panel A: W=I	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H_0:$b_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	63.168	77.828		78.828			RMSE	0.039
pv HAC(0)	[0.269]	[0.014]	[0.273]	[0.013]	18.571	[0.029]	MAE	0.031
pv HAC(6)	[0.300]	[0.048]	[0.303]	[0.047]	33.579	[0.000]	R^2	0.990
pv HAC(auto)	[0.294]	[0.027]	[0.297]	[0.026]	17.559	[0.040]	$\bar{R}^2$	0.988
Panel B: W=Var(R^e)$^{-1}$	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H_0:$b_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	89.181	58.937		59.937			RMSE	0.241
pv HAC(0)	[0.112]	[0.017]	[0.115]	[0.016]	18.930	[0.025]	MAE	0.228
pv HAC(6)	[0.167]	[0.027]	[0.169]	[0.025]	33.140	[0.000]	R^2	0.660
pv HAC(auto)	[0.136]	[0.014]	[0.139]	[0.013]	17.286	[0.044]	$\bar{R}^2$	0.584

Notes: Table B1.1 presents the results of the GMM estimation of Euler equation with linear SDF with weighting matrix $W = I$ (in Panel A) and $W = Var(R^e)^{-1}$ (in Panel B) for a set of test assets that includes 10 Fama Maturity Portfolios and a market portfolio, represented by S&P500 Index. We report the estimates of coefficients b along with p-values for individul significance, related to the following types of standard errors: Newey-West with 0 and 6 lags and with auto-lag selection (HAC). We also report p-values related to testing the theoretical restriction of $H_0 : b_{\Delta c} = 1$ and the estimate of risk aversion parameter $\hat{\gamma}$ along with the p-values related to its statistical significance. We present as well the J_T statistics, used for testing a joint zero pricing errors hypothesis along with relevant p-values. Finally, we report the goodness-of-fit statistics like R^2, adjusted-R^2 ($\bar{R}^2$), RMSE and MAE (in % per quarter). All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

Table B1.2: GMM estimation of Euler equation with linear SDF for Fama Maturity Portfolios and size portfolios

Panel A: W=I	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H$_0$:b$_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	82.177	83.617		84.617			RMSE	0.109
pv HAC(0)	[0.199]	[0.008]	[0.202]	[0.008]	28.921	[0.049]	MAE	0.086
pv HAC(6)	[0.218]	[0.033]	[0.221]	[0.032]	73.794	[0.000]	R^2	0.988
pv HAC(auto)	[0.218]	[0.033]	[0.221]	[0.032]	16.664	[0.546]	$\bar{R}^2$	0.987
Panel B:								
W=Var(R^e)$^{-1}$	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H$_0$:b$_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	62.756	56.446		57.446			RMSE	0.588
pv HAC(0)	[0.111]	[0.006]	[0.115]	[0.006]	32.629	[0.018]	MAE	0.495
pv HAC(6)	[0.180]	[0.020]	[0.184]	[0.019]	88.401	[0.000]	R^2	0.679
pv HAC(auto)	[0.151]	[0.008]	[0.155]	[0.007]	27.671	[0.067]	$\bar{R}^2$	0.643

Notes: Table B1.2 presents the results of the GMM estimation of Euler equation with linear SDF with weighting matrix $W = I$ (in Panel A) and $W = Var(R^e)^{-1}$ (in Panel B) for a set of test assets that includes 10 Fama Maturity Portfolios and 10 equity portfolios sorted on size. We report the estimates of coefficients b along with p-values for individul significance, related to the following types of standard errors: Newey-West with 0 and 6 lags and with auto-lag selection (HAC). We also report p-values related to testing the theoretical restriction of $H_0 : b_{\Delta c} = 1$ and the estimate of risk aversion parameter $\hat{\gamma}$ along with the p-values related to its statistical significance. We present as well the J_T statistics, used for testing a joint zero pricing errors hypothesis along with relevant p-values. Finally, we report the goodness-of-fit statistics like R^2, adjusted-R^2 ($\bar{\text{R}}^2$), RMSE and MAE (in % per quarter). All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

Table B1.3: GMM estimation of Euler equation with linear SDF for Fama Maturity Portfolios and Book-to-Market portfolios

Panel A: W=I	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H_0:$b_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	97.496	83.931		84.931			RMSE	0.186
pv HAC(0)	[0.180]	[0.010]	[0.182]	[0.009]	21.618	[0.249]	MAE	0.115
pv HAC(6)	[0.214]	[0.038]	[0.217]	[0.036]	57.829	[0.000]	R^2	0.958
pv HAC(auto)	[0.214]	[0.032]	[0.216]	[0.031]	16.747	[0.540]	$\bar{R}^2$	0.954
Panel B: W=Var(R^e)$^{-1}$	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H_0:$b_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	100.703	53.553		54.553			RMSE	0.479
pv HAC(0)	[0.072]	[0.020]	[0.074]	[0.018]	24.346	[0.143]	MAE	0.427
pv HAC(6)	[0.127]	[0.018]	[0.130]	[0.016]	66.800	[0.000]	R^2	0.728
pv HAC(auto)	[0.127]	[0.032]	[0.130]	[0.029]	10.258	[0.923]	$\bar{R}^2$	0.698

Notes: Table B1.3 presents the results of the GMM estimation of Euler equation with linear SDF with weighting matrix $W = I$ (in Panel A) and $W = Var(R^e)^{-1}$ (in Panel B) for a set of test assets that includes 10 Fama Maturity Portfolios and 10 equity portfolios sorted on book-to-market values. We report the estimates of coefficients b along with p-values for individul significance, related to the following types of standard errors: Newey-West with 0 and 6 lags and with auto-lag selection (HAC). We also report p-values related to testing the theoretical restriction of $H_0 : b_{\Delta c} = 1$ and the estimate of risk aversion parameter $\hat{\gamma}$ along with the p-values related to its statistical significance. We present as well the J_T statistics, used for testing a joint zero pricing errors hypothesis along with relevant p-values. Finally, we report the goodness-of-fit statistics like R^2, adjusted-R^2 ($\bar{R}^2$), RMSE and MAE (in % per quarter). All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

Table B1.4: GMM estimation of Euler equation with linear SDF for Fama Maturity Portfolios and 25 Fama-French portfolios

Panel A: W=I	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H_0:$b_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	83.535	90.397		91.397			RMSE	0.518
pv HAC(0)	[0.181]	[0.005]	[0.184]	[0.005]	78.678	[0.000]	MAE	0.322
pv HAC(6)	[0.209]	[0.030]	[0.211]	[0.028]	595.589	[0.000]	R^2	0.798
pv HAC(auto)	[0.199]	[0.009]	[0.202]	[0.008]	56.008	[0.007]	$\bar{R}^2$	0.786
Panel B:								
W=Var(R^e)$^{-1}$	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H_0:$b_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	30.270	57.034		58.034			RMSE	1.179
pv HAC(0)	[0.239]	[0.001]	[0.246]	[0.001]	83.327	[0.000]	MAE	0.957
pv HAC(6)	[0.325]	[0.023]	[0.331]	[0.022]	568.559	[0.000]	R^2	-0.041
pv HAC(auto)	[0.273]	[0.003]	[0.279]	[0.003]	56.892	[0.006]	$\bar{R}^2$	-0.104

Notes: Table B1.4 presents the results of the GMM estimation of Euler equation with linear SDF with weighting matrix $W = I$ (in Panel A) and $W = Var(R^e)^{-1}$ (in Panel B) for a set of test assets that includes 10 Fama Maturity Portfolios and 25 Fama-French equity portfolios sorted on size and book-to-market. We report the estimates of coefficients b along with p-values for individul significance, related to the following types of standard errors: Newey-West with 0 and 6 lags and with auto-lag selection (HAC). We also report p-values related to testing the theoretical restriction of $H_0 : b_{\Delta c} = 1$ and the estimate of risk aversion parameter $\hat{\gamma}$ along with the p-values related to its statistical significance. We present as well the J_T statistics, used for testing a joint zero pricing errors hypothesis along with relevant p-values. Finally, we report the goodness-of-fit statistics like R^2, adjusted-R^2 ($\bar{R}^2$), RMSE and MAE (in % per quarter). All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

B2: Estimation results for 7 Fixed Term Indices

Table B2.1: Summary statistics of excess returns for Fixed Term Indices

	FI1	FI2	FI5	FI7	FI10	FI20	FI30
Mean	1.263	1.707	2.711	3.369	3.298	4.516	4.421
St dev	2.155	3.608	6.565	7.802	9.044	12.604	14.811
Sharpe Ratio	0.586	0.473	0.412	0.431	0.364	0.358	0.298
Skewness	1.888	1.078	0.481	0.414	0.554	0.677	1.113
Kurtosis	12.179	6.436	1.900	0.907	0.206	1.662	3.378
Minimum	-14.787	-23.239	-37.557	-42.788	-38.424	-70.428	-66.845
Maximum	28.177	40.880	56.304	55.253	56.456	86.434	134.248
ρ_1	-0.123	-0.097	-0.075	-0.047	0.018	-0.066	-0.045
ρ_2	-0.004	-0.010	-0.023	-0.005	-0.071	-0.059	-0.106
ρ_3	0.168*	0.149	0.109	0.091	0.098	0.070	0.050
ρ_4	-0.013	0.042	0.051	0.009	-0.002	-0.006	-0.035

Notes: Table B2.1 presents summary statistics for excess returns (annualized % excess returns) on Fixed Term Indices, distinguished by 7 different lengths of maturity, over 30-day Treasury bill rates. The quarterly holding period returns on the Fixed Term Indices are computed using monthly returns obtained from the CRSP US Treasury Database. Quarterly T-bill rates are obtained from the CRSP US Treasury Database as well. Fixed Term Indices represent portfolios of fully taxable, non-callable, and non-flower bonds with 7 different maturities: 1, 2, 5, 7, 10, 20, 30 years (FI1 for1-year bonds, FI2 for 2-year bonds, FI5 for 5-year bonds, FI7 for 7-year bonds, FI10 for 10-year bonds, FI20 for 20-year bonds and FI30 for 30-year bonds). The data span the period 1975Q1–2011Q4. Also reported in this table are autocorrelations up to 4 quarters: those marked with asterisk are significant, exceeding the 95% confidence interval of $\pm 1.96/\sqrt{T} \approx \pm 0.161$.

Table B2.2: GMM estimation of Euler equation with linear SDF for Fixed Term Indices

Panel A: W=I	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H$_0$:b$_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	-131.760	34.633		35.633			RMSE	0.056
pv HAC(0)	[0.149]	[0.173]	[0.148]	[0.167]	11.504	[0.042]	MAE	0.044
pv HAC(6)	[0.102]	[0.189]	[0.100]	[0.183]	20.229	[0.001]	R^2	0.962
pv HAC(auto)	[0.126]	[0.184]	[0.124]	[0.178]	11.715	[0.038]	$\bar{R}^2$	0.946
Panel B: W=Var(R^e)$^{-1}$	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H$_0$:b$_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	-53.255	52.650		53.650			RMSE	0.064
pv HAC(0)	[0.294]	[0.026]	[0.291]	[0.025]	11.426	[0.043]	MAE	0.046
pv HAC(6)	[0.309]	[0.083]	[0.305]	[0.080]	22.863	[0.000]	R^2	0.950
pv HAC(auto)	[0.301]	[0.063]	[0.298]	[0.061]	13.052	[0.022]	$\bar{R}^2$	0.930

Notes: Table B2.2 presents the results of the GMM estimation of Euler equation with linear SDF with weighting matrix $W = I$ (in Panel A) and $W = Var(R^e)^{-1}$ (in Panel B) for 7 Fixed Term Indices. We report the estimates of coefficients b along with p-values for individul significance, related to the following types of standard errors: Newey-West with 0 and 6 lags and with auto-lag selection (HAC). We also report p-values related to testing the theoretical restriction of $H_0 : b_{\Delta c} = 1$ and the estimate of risk aversion parameter $\hat{\gamma}$ along with the p-values related to its statistical significance. We present as well the J_T statistics, used for testing a joint zero pricing errors hypothesis along with relevant p-values. Finally, we report the goodness-of-fit statistics like R^2, adjusted-R^2 ($\bar{\text{R}}^2$), RMSE and MAE (in % per quarter). All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

Table B2.3: Fama-MacBeth regressions for Fixed Term Indices

Panel A	$\lambda_{\Delta c}$	λ_{ε}	χ^2	p-value		
estimates	-0.274	0.163			RMSE	0.056
pv OLS	[0.131]	[0.155]	14.783	[0.011]	MAE	0.044
pv Shanken	[0.162]	[0.187]	11.156	[0.048]	R^2	0.962
pv HAC(0)	[0.150]	[0.167]	11.504	[0.042]	$\bar{R}^2$	0.946
pv HAC(6)	[0.116]	[0.153]	20.229	[0.001]		
pv HAC(auto)	[0.128]	[0.158]	11.452	[0.043]		

Panel B	*const*	$\lambda_{\Delta c}$	λ_{ε}	χ^2	p-value		
estimates	0.080	-0.242	0.151			RMSE	0.046
pv OLS	[0.340]	[0.153]	[0.205]	6.239	[0.181	MAE	0.040
pv Shanken	[0.356]	[0.178]	[0.230]	4.972	[0.290]	R^2	0.973
pv HAC(0)	[0.315]	[0.163]	[0.203]	5.380	[0.250]	$\bar{R}^2$	0.960
pv HAC(6)	[0.299]	[0.131]	[0.176]	7.613	[0.106]	H_0:R^2=1	[0.419]
pv HAC(auto)	[0.310]	[0.144]	[0.187]	6.047	[0.195]	H_0:R^2=0	[0.057]

Notes: Table B2.3 presents the estimation results of cross-sectional regression using the second stage of Fama-MacBeth methodology without constant (in Panel A) and with constant (in Panel B) for 7 Fixed Term Indices. We report the estimates of factor risk prices λ along with p-values for individul significance, related to the following types of standard errors: OLS, Shanken-corrected, Newey-West with 0 and 6 lags and with auto-lag selection (HAC). The values of λ are multiplied by 100 for convenience. We also report the χ^2 statistics, used for testing a joint zero pricing errors hypothesis with relevant p-values. Finally, we report the goodness-of-fit statistics like R^2, adjusted-R^2 ($\bar{R}^2$), RMSE and MAE (in % per quarter) and p-value related to testing the following nulls: H_0:R^2=1 and H_0:R^2=0. All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

Table B2.4: Covariances of excess returns on Fixed Term Indices with risk factors

Panel A	FI1	FI2	FI5	FI7	FI10	FI20	FI30
$cov(R^e, \Delta c)$	-0.076	-0.140	-0.244	-0.291	-0.267	-0.342	-0.415
pv HAC(0)	[0.166]	[0.123]	[0.099]	[0.083]	[0.111]	[0.157]	[0.165]
pv HAC(6)	[0.100]	[0.057]	[0.033]	[0.020]	[0.053]	[0.051]	[0.055]
pv HAC(auto)	[0.096]	[0.051]	[0.025]	[0.014]	[0.043]	[0.037]	[0.039]
		Wald stat	p-value			Wald stat	p-value
		(joint eq of cov)				(joint sign of cov)	
pv HAC(0)		5.135	[0.526]			5.171	[0.638]
pv HAC(6)		6.851	[0.334]			8.165	[0.318]
pv HAC(auto)		7.418	[0.283]			8.750	[0.271]
$corr(R^e, \Delta c)$	-0.154	-0.170	-0.163	-0.163	-0.129	-0.119	-0.123

Panel B	FI1	FI2	FI5	FI7	FI10	FI20	FI30
$cov(R^e, \varepsilon_c)$	0.458	0.693	1.048	1.109	1.271	1.863	1.915
pv HAC(0)	[0.003]	[0.002]	[0.002]	[0.002]	[0.000]	[0.001]	[0.000]
pv HAC(6)	[0.035]	[0.035]	[0.029]	[0.027]	[0.012]	[0.018]	[0.013]
pv HAC(auto)	[0.057]	[0.057]	[0.048]	[0.045]	[0.026]	[0.030]	[0.023]
		Wald stat	p-value			Wald stat	p-value
		(joint eq of cov)				(joint sign of cov)	
pv HAC(0)		18.150	[0.005]			18.194	[0.011]
pv HAC(6)		18.958	[0.004]			18.971	[0.008]
pv HAC(auto)		18.864	[0.004]			18.993	[0.008]
$corr(R^e, \varepsilon_c)$	0.618	0.558	0.464	0.413	0.409	0.430	0.376

Notes: Table B2.4 presents the estimates of the covariances of excess returns on 7 Fixed Term Indices with consumption growth $cov(R^e, \Delta c)$ (in Panel A) and with innovations to expectations in future consumption growth $cov(R^e, \varepsilon_c)$ (in Panel B) along with p-values for individual significance, related to the following types of standard errors: OLS, Shanken-corrected, Newey-West with 0 and 6 lags and with auto-lag selection (HAC). Covariances are multiplied by 10,000 for convenience. We also report the Wald statistics with relevant p-values to test joint equality of covariances (joint eq of cov) and joint significance of covariances (joint sign of cov). Finally, we report as well the correlation coefficients. All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

B3: Estimation results for 5 Fama-Bliss Discount Bonds

Table B3.1: Summary statistics of excess returns for Fama-Bliss Discount Bonds

	FB1	FB2	FB3	FB4	FB5
Mean	0.745	1.718	2.195	2.662	2.849
St dev	1.835	3.649	5.119	6.436	7.613
Sharpe Ratio	0.406	0.470	0.428	0.413	0.374
Skewness	1.840	1.002	0.504	0.351	0.290
Kurtosis	12.409	6.563	3.325	1.656	1.598
Minimum	-12.390	-25.132	-33.722	-37.497	-45.922
Maximum	23.959	40.669	46.687	48.625	56.358
ρ_1	-0.175*	-0.108	-0.074	-0.065	-0.058
ρ_2	-0.010	0.005	0.027	0.020	0.001
ρ_3	0.162*	0.164*	0.156	0.156	0.116
ρ_4	-0.047	0.045	0.038	0.046	0.056

Notes: Table B3.1 presents summary statistics for excess returns (annualized % excess returns) on Fama-Bliss Discount Bonds, over 30-day Treasury bill rates. The quarterly holding period returns on the Fama-Bliss Discount Bonds are computed using monthly yields obtained from the CRSP US Treasury Database. The details of the computations are provided in the Online Appendix. Quarterly T-bill rates are obtained from the CRSP US Treasury Database as well. Fama-Bliss Discount Bonds represent zero-coupon bonds with 5 different maturities: 1, 2, 3, 4, 5, years (FB1 for 1-year zero-coupon bond, FB2 for 2-year zero-coupon bond, FB3 for 3-year zero-coupon bond, FB4 for 4-year zero-coupon bond, FB5 for 5-year zero-coupon bond). The data span the period 1975Q1–2011Q4. Also reported in this table are autocorrelations up to 4 quarters: those marked with asterisk are significant, exceeding the 95% confidence interval of $\pm 1.96/\sqrt{T} \approx \pm 0.161$.

Table B3.2: GMM estimation of Euler equation with linear SDF for Fama-Bliss Discount Bonds

Panel A: W=I	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H_0:$b_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	-57.407	48.278		49.278			RMSE	0.020
pv HAC(0)	[0.409]	[0.152]	[0.407]	[0.148]	3.874	[0.275]	MAE	0.014
pv HAC(6)	[0.397]	[0.198]	[0.395]	[0.193]	4.068	[0.254]	R^2	0.988
pv HAC(auto)	[0.401]	[0.181]	[0.399]	[0.177]	3.412	[0.332]	$\bar{R}^2$	0.980
Panel B: W=Var(R^e)$^{-1}$	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H_0:$b_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	103.271	63.157		64.157			RMSE	0.150
pv HAC(0)	[0.140]	[0.044]	[0.142]	[0.043]	2.549	[0.466]	MAE	0.133
pv HAC(6)	[0.166]	[0.051]	[0.168]	[0.049]	2.696	[0.440]	R^2	0.361
pv HAC(auto)	[0.149]	[0.039]	[0.151]	[0.037]	2.844	[0.416]	$\bar{R}^2$	-0.064

Notes: Table B3.2 presents the results of the GMM estimation of Euler equation with linear SDF with weighting matrix $W = I$ (in Panel A) and $W = Var(R^e)^{-1}$ (in Panel B) for 5 Fama-Bliss Discount Bonds. We report the estimates of coefficients b along with p-values for individul significance, related to the following types of standard errors: Newey-West with 0 and 6 lags and with auto-lag selection (HAC). We also report p-values related to testing the theoretical restriction of $H_0 : b_{\Delta c} = 1$ and the estimate of risk aversion parameter $\hat{\gamma}$ along with the p-values related to its statistical significance. We present as well the J_T statistics, used for testing a joint zero pricing errors hypothesis along with relevant p-values. Finally, we report the goodness-of-fit statistics like R^2, adjusted-R^2 ($\bar{R}^2$), RMSE and MAE (in % per quarter). All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

Table B3.3: Fama-MacBeth regressions for Fama-Bliss Discount Bonds

Panel A		$\lambda_{\Delta c}$	λ_{ε}	χ^2	p-value		
estimates		-0.119	0.228			RMSE	0.020
pv OLS		[0.411]	[0.139]	4.841	[0.183]	MAE	0.014
pv Shanken		[0.416]	[0.151]	4.313	[0.229]	R^2	0.988
pv HAC(0)		[0.409]	[0.134]	3.874	[0.275]	$\bar{R}^2$	0.980
pv HAC(6)		[0.396]	[0.168]	4.068	[0.254]		
pv HAC(auto)		[0.389]	[0.169]	3.612	[0.306]		

Panel B	*const*	$\lambda_{\Delta c}$	λ_{ε}	χ^2	p-value		
estimates	-0.059	0.037	0.326			RMSE	0.010
pv OLS	[0.261]	[0.455]	[0.039]	0.448	[0.799]	MAE	0.009
pv Shanken	[0.282]	[0.460]	[0.050]	0.356	[0.836]	R^2	0.996
pv HAC(0)	[0.278]	[0.458]	[0.040]	0.336	[0.845]	$\bar{R}^2$	0.993
pv HAC(6)	[0.260]	[0.457]	[0.082]	0.798	[0.670]	H_0:R^2=1	[0.602]
pv HAC(auto)	[0.260]	[0.457]	[0.090]	0.851	[0.653]	H_0:R^2=0	[0.072]

Notes: Table B3.3 presents the estimation results of cross-sectional regression using the second stage of Fama-MacBeth methodology without constant (in Panel A) and with constant (in Panel B) for 5 Fama-Bliss Discount Bonds. We report the estimates of factor risk prices λ along with p-values for individul significance, related to the following types of standard errors: OLS, Shanken-corrected, Newey-West with 0 and 6 lags and with auto-lag selection (HAC). The values of λ are multiplied by 100 for convenience. We also report the χ^2 statistics, used for testing a joint zero pricing errors hypothesis with relevant p-values. Finally, we report the goodness-of-fit statistics like R^2, adjusted-R^2 ($\bar{R}^2$), RMSE and MAE (in % per quarter) and p-value related to testing the following nulls: H_0:R^2=1 and H_0:R^2=0. All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

Table B3.4: Covariances of excess returns on Fama-Bliss Discount Bonds with risk factors

Panel A	FB1	FB2	FB3	FB4	FB5
$cov(R^e, \Delta c)$	-0.075	-0.122	-0.179	-0.218	-0.237
pv HAC(0)	[0.134]	[0.156]	[0.116]	[0.100]	[0.121]
pv HAC(6)	[0.074]	[0.082]	[0.044]	[0.031]	[0.048]
pv HAC(auto)	[0.052]	[0.056]	[0.024]	[0.014]	[0.026]
	Wald stat	p-value		Wald stat	p-value
	(joint eq of cov)			(joint sign of cov)	
pv HAC(0)	5.321	[0.255]		11.995	[0.034]
pv HAC(6)	6.722	[0.151]		13.303	[0.020]
pv HAC(auto)	8.608	[0.071]		15.939	[0.007]
$corr(R^e, \Delta c)$	-0.179	-0.147	-0.153	-0.148	-0.136

Panel B	FB1	FB2	FB3	FB4	FB5
$cov(R^e, \varepsilon_c)$	0.384	0.737	0.912	1.089	1.204
pv HAC(0)	[0.003]	[0.001]	[0.002]	[0.001]	[0.001]
pv HAC(6)	[0.037]	[0.031]	[0.030]	[0.024]	[0.026]
pv HAC(auto)	[0.037]	[0.031]	[0.030]	[0.024]	[0.026]
	Wald stat	p-value		Wald stat	p-value
	(joint eq of cov)			(joint sign of cov)	
pv HAC(0)	17.142	[0.001]		17.701	[0.003]
pv HAC(6)	10.125	[0.038]		10.125	[0.071]
pv HAC(auto)	10.125	[0.038]		10.125	[0.071]
$corr(R^e, \varepsilon_c)$	0.609	0.587	0.518	0.492	0.460

Notes: Table B3.4 presents the estimates of the covariances of excess returns on 5 Fama-Bliss Discount Bonds with consumption growth $cov(R^e, \Delta c)$ (in Panel A) and with innovations to expectations in future consumption growth $cov(R^e, \varepsilon_c)$ (in Panel B) along with p-values for individual significance, related to the following types of standard errors: OLS, Shanken-corrected, Newey-West with 0 and 6 lags and with auto-lag selection (HAC). Covariances are multiplied by 10,000 for convenience. We also report the Wald statistics with relevant p-values to test joint equality of covariances (joint eq of cov) and joint significance of covariances (joint sign of cov). Finally, we report as well the correlation coefficients. All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

B4: Estimation results for 10 Fama Maturity Portfolios using an alternative measure of consumption

Table B4.1: GMM estimation of Euler equation with linear SDF for Fama Maturity Portfolios

Panel A: W=I	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H_0:$b_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	-66.973	49.406		50.406			RMSE	0.022
pv HAC(0)	[0.294]	[0.082]	[0.291]	[0.079]	26.180	[0.000]	MAE	0.018
pv HAC(6)	[0.290]	[0.100]	[0.288]	[0.096]	39.801	[0.000]	R^2	0.988
pv HAC(auto)	[0.291]	[0.100]	[0.288]	[0.096]	19.136	[0.014]	$\bar{R}^2$	0.985
Panel B: W=Var(R^e)$^{-1}$	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H_0:$b_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	54.860	49.492		50.492			RMSE	0.253
pv HAC(0)	[0.204]	[0.023]	[0.208]	[0.022]	21.877	[0.005]	MAE	0.241
pv HAC(6)	[0.246]	[0.050]	[0.250]	[0.047]	36.417	[0.000]	R^2	-0.500
pv HAC(auto)	[0.208]	[0.026]	[0.213]	[0.025]	18.832	[0.015]	$\bar{R}^2$	-0.875

Notes: Table B4.1 presents the results of the GMM estimation of Euler equation with linear SDF with weighting matrix $W = I$ (in Panel A) and $W = Var(R^e)^{-1}$ (in Panel B) for10 Fama Maturity Portfolios. We report the estimates of coefficients b along with p-values for individul significance, related to the following types of standard errors: Newey-West with 0 and 6 lags and with auto-lag selection (HAC). We also report p-values related to testing the theoretical restriction of $H_0 : b_{\Delta c} = 1$ and the estimate of risk aversion parameter $\hat{\gamma}$ along with the p-values related to its statistical significance. We present as well the J_T statistics, used for testing a joint zero pricing errors hypothesis along with relevant p-values. Finally, we report the goodness-of-fit statistics like R^2, adjusted-R^2 ($\bar{R}^2$), RMSE and MAE (in % per quarter). All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

Table B4.2: Fama-MacBeth regressions for Fama Maturity Portfolios

Panel A	$\lambda_{\Delta c}$	λ_{ε}	χ^2	p-value		
estimates	-0.139	0.246			RMSE	0.022
pv OLS	[0.308]	[0.095]	38.415	[0.000]	MAE	0.018
pv Shanken	[0.319]	[0.109]	33.632	[0.000]	R^2	0.988
pv HAC(0)	[0.294]	[0.076]	26.180	[0.000]	$\bar{R}^2$	0.985
pv HAC(6)	[0.296]	[0.057]	39.801	[0.000]		
pv HAC(auto)	[0.297]	[0.052]	12.531	[0.129]		

Panel B	*const*	$\lambda_{\Delta c}$	λ_{ε}	χ^2	p-value		
estimates	0.029	-0.118	0.243			RMSE	0.020
pv OLS	[0.428]	[0.283]	[0.116]	22.040	[0.002]	MAE	0.017
pv Shanken	[0.432]	[0.294]	[0.130]	19.595	[0.006]	R^2	0.990
pv HAC(0)	[0.420]	[0.276]	[0.098]	16.622	[0.020]	$\bar{R}^2$	0.987
pv HAC(6)	[0.408]	[0.286]	[0.070]	25.441	[0.000]	H_0:R^2=1	[0.192]
pv HAC(auto)	[0.405]	[0.289]	[0.064]	10.775	[0.148]	H_0:R^2=0	[0.026]

Notes: Table B4.2 presents the estimation results of cross-sectional regression using the second stage of Fama-MacBeth methodology without constant (in Panel A) and with constant (in Panel B) for 10 Fama Maturity Portfolios. We report the estimates of factor risk prices λ along with p-values for individul significance, related to the following types of standard errors: OLS, Shanken-corrected, Newey-West with 0 and 6 lags and with auto-lag selection (HAC). The values of λ are multiplied by 100 for convenience. We also report the χ^2 statistics, used for testing a joint zero pricing errors hypothesis with relevant p-values. Finally, we report the goodness-of-fit statistics like R^2, adjusted-R^2 ($\bar{R}^2$), RMSE and MAE (in % per quarter) and p-value related to testing the following nulls: H_0:R^2=1 and H_0:R^2=0. All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

Table B4.3: Covariances of excess returns on Fama Maturity Portfolios with risk factors

Panel A	FMP1	FMP2	FMP3	FMP4	FMP5	FMP6	FMP7	FMP8	FMP9	FMP10
$cov(R^e, \Delta c)$	-0.083	-0.117	-0.146	-0.172	-0.181	-0.205	-0.229	-0.261	-0.249	-0.317
pv HAC(0)	[0.169]	[0.133]	[0.131]	[0.111]	[0.100]	[0.098]	[0.083]	[0.089]	[0.119]	[0.155]
pv HAC(6)	[0.101]	[0.067]	[0.061]	[0.044]	[0.038]	[0.038]	[0.025]	[0.027]	[0.041]	[0.054]
pv HAC(auto)	[0.060]	[0.032]	[0.026]	[0.016]	[0.014]	[0.012]	[0.007]	[0.006]	[0.014]	[0.014]
		Wald stat (joint eq of cov)			p-value		Wald stat (joint sign of cov)			p-value
pv HAC(0)				13.563	[0.138]				13.691	[0.187]
pv HAC(6)				15.950	[0.067]				16.378	[0.089]
pv HAC(auto)				34.944	[0.000]				35.482	[0.000]
$corr(R^e, \Delta c)$	-0.147	-0.160	-0.160	-0.167	-0.160	-0.163	-0.170	-0.176	-0.142	-0.117

Panel B	FMP1	FMP2	FMP3	FMP4	FMP5	FMP6	FMP7	FMP8	FMP9	FMP10
$cov(R^e, \varepsilon_c)$	0.526	0.648	0.773	0.849	0.876	0.963	0.968	1.082	1.265	1.836
pv HAC(0)	[0.003]	[0.002]	[0.002]	[0.002]	[0.001]	[0.001]	[0.001]	[0.002]	[0.001]	[0.000]
pv HAC(6)	[0.033]	[0.033]	[0.034]	[0.033]	[0.029]	[0.029]	[0.028]	[0.031]	[0.024]	[0.016]
pv HAC(auto)	[0.044]	[0.044]	[0.044]	[0.043]	[0.039]	[0.039]	[0.038]	[0.040]	[0.033]	[0.021]
		Wald stat (joint eq of cov)			p-value		Wald stat (joint sign of cov)			p-value
pv HAC(0)				30.608	[0.000]				31.463	[0.000]
pv HAC(6)				24.569	[0.003]				24.754	[0.005]
pv HAC(auto)				21.723	[0.009]				21.925	[0.015]
$corr(R^e, \varepsilon_c)$	0.596	0.572	0.549	0.530	0.500	0.495	0.465	0.470	0.466	0.437

Notes: Table B4.3 presents the estimates of the covariances of excess returns on 10 Fama Maturity Portfolios with consumption growth $cov(R^e, \Delta c)$ (in Panel A) and with innovations to expectations in future consumption growth $cov(R^e, \varepsilon_c)$ (in Panel B) along with p-values for individual significance, related to the following types of standard errors: OLS, Shanken-corrected, Newey-West with 0 and 6 lags and with auto-lag selection (HAC). Covariances are multiplied by 10,000 for convenience. We also report the Wald statistics with relevant p-values to test joint equality of covariances (joint eq of cov) and joint significance of covariances (joint sign of cov). Finally, we report as well the correlation coefficients. All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

B5: Estimation results for 10 Fama Maturity Portfolios for a period 1982–2011

Table B5.1: GMM estimation of Euler equation with linear SDF for Fama Maturity Portfolios

Panel A: W=I	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H_0:$b_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	-46.564	64.166		65.166			RMSE	0.026
pv HAC(0)	[0.370]	[0.079]	[0.368]	[0.076]	26.587	[0.000]	MAE	0.022
pv HAC(6)	[0.371]	[0.079]	[0.369]	[0.076]	39.938	[0.000]	R^2	0.989
pv HAC(auto)	[0.369]	[0.082]	[0.367]	[0.079]	17.619	[0.024]	$\bar{R}^2$	0.987
Panel B: W=Var(R^e)$^{-1}$	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	H_0:$b_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	82.523	61.783		62.783			RMSE	0.329
pv HAC(0)	[0.115]	[0.017]	[0.117]	[0.016]	22.120	[0.004]	MAE	0.309
pv HAC(6)	[0.172]	[0.035]	[0.175]	[0.033]	33.474	[0.000]	R^2	-0.562
pv HAC(auto)	[0.133]	[0.015]	[0.135]	[0.014]	17.596	[0.024]	$\bar{R}^2$	-0.952

Notes: Table B5.1 presents the results of the GMM estimation of Euler equation with linear SDF with weighting matrix $W = I$ (in Panel A) and $W = Var(R^e)^{-1}$ (in Panel B) for10 Fama Maturity Portfolios. We report the estimates of coefficients b along with p-values for individul significance, related to the following types of standard errors: Newey-West with 0 and 6 lags and with auto-lag selection (HAC). We also report p-values related to testing the theoretical restriction of $H_0 : b_{\Delta c} = 1$ and the estimate of risk aversion parameter $\hat{\gamma}$ along with the p-values related to its statistical significance. We present as well the J_T statistics, used for testing a joint zero pricing errors hypothesis along with relevant p-values. Finally, we report the goodness-of-fit statistics like R^2, adjusted-R^2 ($\bar{R}^2$), RMSE and MAE (in % per quarter). All the p-values are reported in square brackets. The data span the period 1982Q1–2011Q4.

Table B5.2: Fama-MacBeth regressions for Fama Maturity Portfolios

Panel A		$\lambda_{\Delta c}$	λ_{ε}	χ^2	p-value		
estimates		-0.094	0.294			RMSE	0.026
pv OLS		[0.375]	[0.081]	44.465	[0.000]	MAE	0.022
pv Shanken		[0.385]	[0.099]	37.482	[0.000]	R^2	0.989
pv HAC(0)		[0.371]	[0.070]	26.587	[0.000]	$\bar{R}^2$	0.987
pv HAC(6)		[0.374]	[0.034]	39.938	[0.000]		
pv HAC(auto)		[0.369]	[0.069]	24.917	[0.001]		

Panel B	*const*	$\lambda_{\Delta c}$	λ_{ε}	χ^2	p-value		
estimates	-0.025	-0.113	0.295			RMSE	0.025
pv OLS	[0.446]	[0.298]	[0.091]	27.046	[0.000]	MAE	0.020
pv Shanken	[0.451]	[0.314]	[0.110]	22.590	[0.002]	R^2	0.990
pv HAC(0)	[0.442]	[0.297]	[0.077]	17.421	[0.014]	$\bar{R}^2$	0.988
pv HAC(6)	[0.433]	[0.313]	[0.039]	25.843	[0.000]	H_0:R^2=1	[0.186]
pv HAC(auto)	[0.443]	[0.295]	[0.076]	17.767	[0.013]	H_0:R^2=0	[0.006]

Notes: Table B5.2 presents the estimation results of cross-sectional regression using the second stage of Fama-MacBeth methodology without constant (in Panel A) and with constant (in Panel B) for 10 Fama Maturity Portfolios. We report the estimates of factor risk prices λ along with p-values for individual significance, related to the following types of standard errors: OLS, Shanken-corrected, Newey-West with 0 and 6 lags and with auto-lag selection (HAC). The values of λ are multiplied by 100 for convenience. We also report the χ^2 statistics, used for testing a joint zero pricing errors hypothesis with relevant p-values. Finally, we report the goodness-of-fit statistics like R^2, adjusted-R^2 ($\bar{R}^2$), RMSE and MAE (in % per quarter) and p-value related to testing the following nulls: H_0:R^2=1 and H_0:R^2=0. All the p-values are reported in square brackets. The data span the period 1982Q1–2011Q4.

Table B5.3: Covariances of excess returns on Fama Maturity Portfolios with risk factors

Panel A	FMP1	FMP2	FMP3	FMP4	FMP5	FMP6	FMP7	FMP8	FMP9	FMP10
$cov(R^e, \Delta c)$	-0.100	-0.137	-0.168	-0.199	-0.203	-0.231	-0.259	-0.294	-0.275	-0.362
pv HAC(0)	[0.157]	[0.125]	[0.127]	[0.107]	[0.101]	[0.098]	[0.081]	[0.089]	[0.122]	[0.150]
pv HAC(6)	[0.093]	[0.063]	[0.060]	[0.041]	[0.039]	[0.038]	[0.024]	[0.028]	[0.044]	[0.048]
pv HAC(auto)	[0.122]	[0.094]	[0.095]	[0.076]	[0.074]	[0.072]	[0.059]	[0.065]	[0.095]	[0.119]
		Wald stat (joint eq of cov)			p-value		Wald stat (joint sign of cov)			p-value
pv HAC(0)				19.339	[0.022]				19.983	[0.029]
pv HAC(6)				19.900	[0.018]				21.350	[0.018]
pv HAC(auto)				18.194	[0.032]				18.357	[0.049]
$corr(R^e, \Delta c)$	-0.174	-0.185	-0.182	-0.189	-0.177	-0.180	-0.188	-0.193	-0.153	-0.129

Panel B	FMP1	FMP2	FMP3	FMP4	FMP5	FMP6	FMP7	FMP8	FMP9	FMP10
$cov(R^e, \varepsilon_c)$	0.541	0.660	0.789	0.865	0.894	0.984	0.983	1.108	1.289	1.842
pv HAC(0)	[0.006]	[0.005]	[0.006]	[0.005]	[0.004]	[0.003]	[0.004]	[0.005]	[0.003]	[0.002]
pv HAC(6)	[0.047]	[0.046]	[0.047]	[0.046]	[0.040]	[0.040]	[0.040]	[0.042]	[0.034]	[0.024]
pv HAC(auto)	[0.017]	[0.016]	[0.018]	[0.017]	[0.013]	[0.013]	[0.013]	[0.017]	[0.011]	[0.008]
		Wald stat (joint eq of cov)			p-value		Wald stat (joint sign of cov)			p-value
pv HAC(0)				28.589	[0.000]				28.609	[0.001]
pv HAC(6)				19.002	[0.025]				19.161	[0.038]
pv HAC(auto)				25.319	[0.002]				25.328	[0.004]
$corr(R^e, \varepsilon_c)$	0.628	0.596	0.571	0.548	0.518	0.511	0.476	0.485	0.479	0.439

Notes: Table B5.3 presents the estimates of the covariances of excess returns on 10 Fama Maturity Portfolios with consumption growth $cov(R^e, \Delta c)$ (in Panel A) and with innovations to expectations in future consumption growth $cov(R^e, \varepsilon_c)$ (in Panel B) along with p-values for individual significance, related to the following types of standard errors: OLS, Shanken-corrected, Newey-West with 0 and 6 lags and with auto-lag selection (HAC). Covariances are multiplied by 10,000 for convenience. We also report the Wald statistics with relevant p-values to test joint equality of covariances (joint eq of cov) and joint significance of covariances (joint sign of cov). Finally, we report as well the correlation coefficients. All the p-values are reported in square brackets. The data span the period 1982Q1–2011Q4.

B6: Estimation results for 10 Fama Maturity Portfolios using a three-factor model with the volatility of the innovations to expectations in future consumption growth as a third factor

Table B6.1: GMM estimation of Euler equation with linear SDF for Fama Maturity Portfolios

Panel A: W=I	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	$\hat{b}_{var(\varepsilon)}$	H$_0$:b$_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	-83.055	65.364	-1833.660		66.364			RMSE	0.022
pv HAC(0)	[0.195]	[0.092]	[0.383]	[0.192]	[0.090]	27.922	[0.000]	MAE	0.018
pv HAC(6)	[0.193]	[0.088]	[0.366]	[0.191]	[0.085]	26.856	[0.000]	R^2	0.992
pv HAC(auto)	[0.191]	[0.076]	[0.315]	[0.188]	[0.073]	5.651	[0.580]	$\bar{R}^2$	0.989
Panel B: W=Var(R^e)$^{-1}$	$\hat{b}_{\Delta c}$	$\hat{b}_{\varepsilon}$	$\hat{b}_{var(\varepsilon)}$	H$_0$:b$_{\Delta c}$=1	$\hat{\gamma}$	J_T stat	p-value		
estimates	96.969	52.659	1663.826		53.659			RMSE	0.370
pv HAC(0)	[0.121]	[0.050]	[0.270]	[0.123]	[0.048]	23.172	[0.001]	MAE	0.343
pv HAC(6)	[0.178]	[0.089]	[0.264]	[0.180]	[0.085]	36.133	[0.000]	R^2	-0.976
pv HAC(auto)	[0.141]	[0.052]	[0.246]	[0.143]	[0.050]	18.311	[0.010]	$\bar{R}^2$	-1.822

Notes: Table B6.1 presents the results of the GMM estimation of Euler equation with linear SDF with weighting matrix $W = I$ (in Panel A) and $W = Var(R^e)^{-1}$ (in Panel B) for10 Fama Maturity Portfolios. We report the estimates of coefficients b along with p-values for individul significance, related to the following types of standard errors: Newey-West with 0 and 6 lags and with auto-lag selection (HAC). We also report p-values related to testing the theoretical restriction of $H_0 : b_{\Delta c} = 1$ and the estimate of risk aversion parameter $\hat{\gamma}$ along with the p-values related to its statistical significance. We present as well the J_T statistics, used for testing a joint zero pricing errors hypothesis along with relevant p-values. Finally, we report the goodness-of-fit statistics like R^2, adjusted-R^2 ($\bar{\text{R}}^2$), RMSE and MAE (in % per quarter). All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

Table B6.2: Fama-MacBeth regressions for Fama Maturity Portfolios

Panel A		$\lambda_{\Delta c}$	λ_{ε}	$\lambda_{var(\varepsilon)}$	χ^2	p-value		
estimates		-0.169	0.299	-0.002			RMSE	0.022
pv OLS		[0.205]	[0.098]	[0.401]	43.730	0.000	MAE	0.018
pv Shanken		[0.230]	[0.122]	[0.411]	35.054	0.000	R^2	0.992
pv HAC(0)		[0.198]	[0.087]	[0.386]	27.922	0.000	$\bar{R}^2$	0.989
pv HAC(6)		[0.200]	[0.047]	[0.373]	26.856	0.000		
pv HAC(auto)		[0.195]	[0.082]	[0.387]	23.170	0.001		

Panel B	*const*	$\lambda_{\Delta c}$	λ_{ε}	$\lambda_{var(\varepsilon)}$	χ^2	p-value		
estimates	0.134	-0.118	0.204	-0.006			RMSE	0.016
pv OLS	[0.165]	[0.302]	[0.210]	[0.087]	26.982	[0.000]	MAE	0.010
pv Shanken	[0.226]	[0.345]	[0.265]	[0.140]	15.567	[0.016]	R^2	0.996
pv HAC(0)	[0.179]	[0.273]	[0.214]	[0.139]	12.820	[0.045]	$\bar{R}^2$	0.994
pv HAC(6)	[0.207]	[0.268]	[0.186]	[0.044]	17.860	[0.006]		
pv HAC(auto)	[0.195]	[0.276]	[0.214]	[0.112]	12.790	[0.046]		

Notes: Table B6.2 presents the estimation results of cross-sectional regression using the second stage of Fama-MacBeth methodology without constant (in Panel A) and with constant (in Panel B) for 10 Fama Maturity Portfolios. We report the estimates of factor risk prices λ along with p-values for individual significance, related to the following types of standard errors: OLS, Shanken-corrected, Newey-West with 0 and 6 lags and with auto-lag selection (HAC). The values of λ are multiplied by 100 for convenience. We also report the χ^2 statistics, used for testing a joint zero pricing errors hypothesis with relevant p-values. Finally, we report the goodness-of-fit statistics like R^2, adjusted-R^2 ($\bar{R}^2$), RMSE and MAE (in % per quarter). All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

Table B6.3: Covariances of excess returns on Fama Maturity Portfolios with volatility of innovations to expectations in future consumption growth

	FMP1	FMP2	FMP3	FMP4	FMP5	FMP6	FMP7	FMP8	FMP9	FMP10
$cov(R^e, var(\varepsilon_c))$	0.003	0.004	0.004	0.005	0.004	0.005	0.005	0.006	0.006	0.007
pv HAC(0)	[0.238]	[0.246]	[0.253]	[0.258]	[0.258]	[0.263]	[0.272]	[0.253]	[0.283]	[0.308]
pv HAC(6)	[0.089]	[0.093]	[0.097]	[0.101]	[0.106]	[0.112]	[0.121]	[0.106]	[0.140]	[0.174]
pv HAC(auto)	[0.181]	[0.188]	[0.195]	[0.198]	[0.198]	[0.206]	[0.214]	[0.195]	[0.227]	[0.254]
		Wald stat (joint eq of cov)			p-value		Wald stat (joint sign of cov)			p-value
pv HAC(0)				4.379	[0.884]				5.962	[0.818]
pv HAC(6)				4.554	[0.871]				5.066	[0.886]
pv HAC(auto)				5.210	[0.815]				6.301	[0.789]
$corr(R^e, var(\varepsilon_c))$	0.258	0.227	0.216	0.199	0.175	0.168	0.151	0.179	0.139	0.104

Notes: Table B6.3 presents the estimates of the covariances of excess returns on 10 Fama Maturity Portfolios with volatility of innovations to expectations in future consumption growth $cov(R^e, var(\varepsilon_c))$ along with p-values for individual significance, related to the following types of standard errors: OLS, Shanken-corrected, Newey-West with 0 and 6 lags and with auto-lag selection (HAC). Covariances are multiplied by 10,000 for convenience. We also report the Wald statistics with relevant p-values to test joint equality of covariances (joint eq of cov) and joint significance of covariances (joint sign of cov). Finally, we report as well the correlation between excess returns and consumption growth at the bottom of the table. All the p-values are reported in square brackets. The data span the period 1975Q1–2011Q4.

Appendix C

This Appendix contains a detailed description of the panel of 125 macroeconomic and financial variables used in factor analysis to extract common factors. The series are obtained from the Global Insights Basic Economics database, unless a different source is listed in parentheses or a series is computed by authors (AC), and are grouped into different cathegories. Each variable is accompanied by a series number, its label or mnemonic, transformation code (in square brackets) and a brief description. The transformation codes are as follows: 1 – no transformation, 2 – first difference in levels, 3 – logarithm, 4 – first difference of logarithm, 5 – second difference of logarithm. The data is sampled at a quarterly frequency and span the period from 1960 to 2011.

Real Output and Income

1. YPR – [4] Personal Income (AR, Bil. Chain 2000$)
2. A0M051 – [4] Personal Income Less Transfer Payments (AR, Bil. Chain 2000$)
3. CONS_R – [4] Real Consumption, A0M224/GMDC (or PI031/GMDC) (Source: AC)
4. U0M083 – [2] Univ.of Michigan Index of Consumer Expectations
5. IPS10 – [4] Industrial Production Index: Total (2002=100, SA)
6. IPS11 – [4] Industrial Production Index: Products, Total (2002=100, SA)
7. IPS299 – [4] Industrial Production Index: Final Products (2002=100, SA)
8. IPS12 – [4] Industrial Production Index: Consumer Goods (2002=100, SA)
9. IPS13 – [4] Industrial Production Index: Durable Consumer Goods (2002=100, SA)
10. IPS18 – [4] Industrial Production Index: Nondurable Consumer Goods (2002=100, SA)
11. IPS25 – [4] Industrial Production Index: Business Equipment (2002=100, SA)
12. IPS32 – [4] Industrial Production Index: Materials (2002=100, SA)
13. IPS34 – [4] Industrial Production Index: Durable Goods Materials (2002=100, SA)
14. IPS38 – [4] Industrial Production Index: Nondurable Goods Materials (2002=100, SA)
15. IPS43 – [4] Industrial Production Index: Manufacturing (Sic, 2002=100, SA)
16. IPS307 – [4] Industrial Production Index: Residential Utilities (2002=100, SA)
17. IPS306 – [4] Industrial Production Index: Fuels (2002=100, SA)
18. PMP – [1] NAPM Production Index (Percent)
19. UTL11 – [2] Capacity Utilization – Manufacturing (Sic, SA)

Employment and Labor Market

20. LHEM – [4] Civilian Labor Force: Employed, Total (Thous., SA)
21. LHNAG – [4] Civilian Labor Force: Employed, Nonagricultural Industries (Thous., SA)
22. LHUR – [2] Unemployment Rate: All Workers, 16 Years & Over (Percent, SA)
23. LHU680 – [2] Unemployment by Duration: Average Duration in Weeks (SA)
24. LHU5 – [4] Unemployment by Duration: Persons Unemployed less than 5 weeks (Thous., SA)
25. LHU14 – [4] Unemployment by Duration: Persons Unemployed 5 to 14 weeks (Thous., SA)
26. LHU15 – [4] Unemployment by Duration: Persons Unemployed 15 weeks+ (Thous., SA)
27. LHU26 – [4] Unemployment by Duration: Persons Unemployed 15 to 26 weeks (Thous., SA)
28. LHU27 – [4] Unemployment by Duration: Persons Unemployed 27 weeks+ (Thous, SA)
29. LUINC – [4] Average Weekly Initial Claims, Unemployment Insurance (Thous., SA)
30. CES002 – [4] Employees on Nonfarm Payrolls: Total Private (Thous., SA)
31. CES003 – [4] Employees on Nonfarm Payrolls: Goods Producing (Thous., SA)
32. CES006 – [4] Employees on Nonfarm Payrolls: Mining (Thous., SA)
33. CES011 – [4] Employees on Nonfarm Payrolls: Construction (Thous., SA)

34. CES015 – [4] Employees on Nonfarm Payrolls: Manufacturing (Thous., SA)
35. CES017 – [4] Employees on Nonfarm Payrolls: Durable Goods (Thous., SA)
36. CES033 – [4] Employees on Nonfarm Payrolls: Nondurable Goods (Thous., SA)
37. CES046 – [4] Employees on Nonfarm Payrolls: Service Providing (Thous., SA)
38. CES048 – [4] Employees on Nonfarm Payrolls: Trade, Transportation, and Utilities (Thous., SA)
39. CES049 – [4] Employees on Nonfarm Payrolls: Wholesale Trade (Thous., SA)
40. CES053 – [4] Employees on Nonfarm Payrolls: Retail Trade (Thous., SA)
41. CES088 – [4] Employees on Nonfarm Payrolls: Financial Activities (Thous., SA)
42. CES140 – [4] Employees on Nonfarm Payrolls: Government (Thous., SA)
43. CES151 – [1] Avg. Wkly Hrs. of Prod or Nonsup Workers on Private Nonfarm Payrolls: Goods-Producing Hrs (SA)
44. CES155 – [2] Avg. Wkly Hrs. of Prod or Nonsup Workers on Private Nonfarm Payrolls: Mfg., Overtime Hrs (SA)
45. A0M001 – [1] Average weekly hours: Mfg. (SA)
46. PMEMP – [1] NAPM Employment Index (Percent)
47. CES275 – [5] Avg. Hourly Earnings of Prod or Nonsupervisory Workers on Private Nonfarm Payrolls: Goods-Producing (Current $, SA)
48. CES277 – [5] Avg. Hourly Earnings of Prod or Nonsupervisory Workers on Private Nonfarm Payrolls: Construction (Current $, SA)
49. CES278 – [5] Avg. Hourly Earnings of Prod or Nonsupervisory Workers on Private Nonfarm Payrolls: Manufacturing (Current $, SA)

Housing Market

50. HSFR – [3] Housing Starts: Nonfarm (1947-58); Total Farm & Nonfarm (1959-) (Thous., SAAR)
51. HSNE – [3] Housing Starts: Northeast (Thous.U.) S.A.
52. HSMW – [3] Housing Starts: Midwest (Thous.U.) S.A.
53. HSSOU – [3] Housing Starts: South (Thous.U.) S.A.
54. HSWST – [3] Housing Starts: West (Thous.U.) S.A.
55. HSBR – [3] Housing Authorized: Total New Private Housing Units (Thous., SAAR)
56. HSBNE – [3] Houses Authorized by Building Permits: Northeast (Thou.U.) S.A
57. HSBMW – [3] Houses Authorized by Building Permits: Midwest (Thou.U.) S.A.
58. HSBSOU – [3] Houses Authorized by Building Permits: South (Thou.U.) S.A.
59. HSBWST – [3] Houses Authorized by Building Permits: West (Thou.U.) S.A.

Orders and Inventories

60. PMI – [1] Purchasing Managers' Index (SA)
61. PMNO – [1] NAPM New Orders Index (Percent)
62. PMDEL – [1] NAPM Vendor Deliveries Index (Percent)
63. PMNV – [1] NAPM Inventories Index (Percent)
64. A1M008 – [4] Mfrs' New Orders, Consumer Goods and Materials (Mil. Chain 1982$, SA)
65. A0M007 – [4] Mfrs' New Orders, Durable Goods Industries (Mil. Chain 2000$, SA)
66. A0M027 – [4] Mfrs' New Orders, Nondefense Capital Goods Industries (Mil. Chain 1982$, SA)
67. A1M092 – [4] Mfrs' Unfilled Orders, Durable Goods Indus. (Bil. Chain 1996$, SA)
68. INVM&T00C – [4] Manufacturing and Trade Inventories (Bil. Chain 2000$, SA)
69. RISMAT – [2] Ratio, Real Inventories to Sales for Manufacturing and Trade Industries (SAAR)
70. MTQ – [4] Manufacturing and Trade Sales (Bil. Chain 1996$, SA)
71. A0M059 – [4] Retail Stores Sales (Mil. Chain 2000$, SA)

Money and Credit Markets

72. FM1 – [5] Money Stock: M1 (Curr, Trav.Cks, Dem Dep, Other Ck's able Dep, Bil$, SA)
73. FM2 – [5] Money Stock: M2 (M1+O'Nite Rps, Euro$,G/P&B/D Mmmfs&Sav&Sm Time Dep, Bil$, SA)
74. FM2_R – [4] Money Supply: Real M2, FM2/GMDC (Source: AC)
75. FMFBA – [5] Monetary Base, Adjusted for Reserve Requirement Changes (Mil$, SA)
76. FMRRA – [5] Depository Inst Reserves: Total, Adjusted for Reserve Requirement Changes (Mil$, SA)
77. FMRNBA – [5] Depository Inst Reserves: Non-borrowed, Adjusted for Reserve Requirement Changes (Mil$, SA)
78. FCLNBW – [5] Commercial & Industrial Loans Outstanding (Mil$, BCD72)
79. CCIPY – [2] Ratio, Consumer Installment Credit to Personal Income (Percent, SA)

Stock Indices

80. FSPCOM – [4] S&P's Common Stock Price Index: Composite (1941-43=10)
81. FSPIN – [4] S&P's Common Stock Price Index: Industrials (1941-43=10)
82. SPCOM-DY – [2] S&P's Composite Common Stock: Dividend Yield (Percent PA) (Source: Datastream)
83. SPCOM-PE – [4] S&P's Composite Common Stock: Price-Earnings Ratio (Percent, NSA) (Source: Datastream)

Interest Rates and Bond Yields

84. FYFF – [2] Interest Rate: Federal Funds (Effective) (Percent PA, NSA)
85. CP90 – [2] Commercial Paper Rate (Source: AC based on Federal Reserve Board of Governors data on 3M Commercial Paper Rate)
86. FYGM3 – [2] Interest Rate: U.S. Treasury Bills, Sec Mkt., 3-Month (Percent PA, NSA)
87. FYGM6 – [2] Interest Rate: U.S. Treasury Bills, Sec Mkt., 6-Month (Percent PA, NSA)
88. FYGT1 – [2] Interest Rate: U.S. Treasury Const Maturities, 1-Year (Percent PA, NSA)
89. FYGT5 – [2] Interest Rate: U.S. Treasury Const Maturities, 5-Year (Percent PA, NSA)
90. FYGT10 – [2] Interest Rate: U.S. Treasury Const Maturities, 10-Year (Percent PA, NSA)
91. FYAAAC – [2] Bond Yield: Moody's AAA Corporate (Percent PA)
92. FYBAAC – [2] Bond Yield: Moody's BAA Corporate (Percent PA)
93. SCP90 – [1] Spread CP90 – FYFF (Source: AC)
94. SFYGM3 – [1] Spread FYGM3 – FYFF (Source: AC)
95. SFYGM6 – [1] Spread FYGM6 – FYFF (Source: AC)
96. SFYGT1 – [1] Spread FYGT1 – FYFF (Source: AC)
97. SFYGT5 – [1] Spread FYGT5 – FYFF (Source: AC)
98. SFYGT10 – [1] Spread FYGT10 – FYFF (Source: AC)
99. SFYAAAC – [1] Spread FYAAAC – FYFF (Source: AC)
100. SFYBAAC – [1] Spread FYBAAC – FYFF (Source: AC)

Exchange Rates

101. EXRUS – [4] Foreign Exchange Rate: United States; Effective (Index)
102. EXRSW – [4] Foreign Exchange Rate: Switzerland (Swiss Franc per U.S.$)
103. EXRJAN – [4] Foreign Exchange Rate: Japan (Yen per U.S.$)
104. EXRUK – [4] Foreign Exchange Rate: United Kingdom (Cents per Pound)

105. EXRCAN – [4] Foreign Exchange Rate: Canada (Canadian $ per U.S.$)

Prices

106. PWFSA – [5] Producer Price Index: Finished Goods (1982=100, SA)
107. PWFCSA – [5] Producer Price Index: Finished Consumer Goods (1982=100, SA)
108. PWIMSA – [5] Producer Price Index: Intermediate Material Supplies & Components (1982=100, SA)
109. PWCMSA – [5] Producer Price Index: Crude Materials (1982=100, SA)
110. PSCCOM – [5] Spot Market Price Index: Bls&Crb: all Commodities(1967=100)
111. PMCP – [1] NAPM Commodity Prices Index (Percent)
112. PUNEW – [5] CPI-U: All Items (82-84=100, SA)
113. PU83 – [5] CPI-U: Apparel & Upkeep (82-84=100, SA)
114. PU84 – [5] CPI-U: Transportation (82-84=100, SA)
115. PU85 – [5] CPI-U: Medical Care (82-84=100, SA)
116. PUC – [5] CPI-U: Commodities (82-84=100, SA)
117. PUCD – [5] CPI-U: Durables (82-84=100, SA)
118. PUS – [5] CPI-U: Services (82-84=100, SA)
119. PUXF – [5] CPI-U: All Items Less Food (82-84=100, SA)
120. PUXHS – [5] CPI-U: All Items Less Shelter (82-84=100, SA)
121. PUXM – [5] CPI-U: All Items Less Medical Care (82-84=100, SA)
122. GMDC – [5] PCE, Implicit Price Deflator: PCE (2005=100) (Source: Bureau of Labor Statistics)
123. GMDCD – [5] PCE, Implicit Price Deflator: Durables (2005=100) (Source: Bureau of Labor Statistics)
124. GMDCN – [5] PCE, Implicit Price Deflator: Nondurables (2005=100) (Source: Bureau of Labor Statistics)
125. GMDCS – [5] PCE, Implicit Price Deflator: Services (2005=100) (Source: Bureau of Labor Statistics)