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Antialiased soft clipping using a polynomial approximation of the integrated bandlimited ramp function

Fabián Esqueda$^{(a)}$**, Stefan Bilbao**$^{(b)}$**, Vesa Välimäki**$^{(c)}$

[a] Aalto University, Dept. of Signal Processing and Acoustics, Espoo, Finland, fabian.esqueda@aalto.fi
[b] Acoustics and Audio Group, University of Edinburgh, United Kingdom, s.bilbao@ed.ac.uk
[c] Aalto University, Dept. of Signal Processing and Acoustics, Espoo, Finland, vesa.valimaki@aalto.fi

Abstract:

An efficient method for aliasing reduction under soft clipping using a piecewise polynomial is presented. Soft clipping is commonly used to model the saturating behavior of electronic musical systems such as guitar amplifiers and voltage-controlled filters used in subtractive synthesis. Saturations introduce high levels of harmonic distortion and, as such, are a major source of aliasing distortion which can lead to severe audible disturbances. The high level of aliasing distortion introduced by piecewise soft clippers can be mostly attributed to the discontinuities they introduce in the second and higher derivatives of the signal. The proposed method works by quasi-bandlimiting these discontinuities using a correction function defined as the integral of the bandlimited ramp (BLAMP) function. Due to the high computational costs of evaluating the analytic form of the integrated BLAMP function at every clipping point, a polynomial approximation is proposed instead. This approximation can be used to correct four samples, two on each side of every clipping point. Performance tests using sinusoidal signals show that the proposed method successfully attenuates aliasing components, particularly at low frequencies, by up to 30 dB with minimal computational costs.

Keywords: acoustic signal processing, antialiasing, fractional delay, interpolation

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1 Introduction

In digital audio signal processing, saturating nonlinear functions are commonly used to model the overdriven response of analog musical systems. Examples of analog music systems that exhibit this saturating behavior include production tools such as guitar amplifiers and distortion units [1], dynamic range compressors and limiters [2, 3], and voltage controlled filters like those used in analog synthesizers [4]. These saturations introduce harmonic distortion which can be a major source of aliasing in the digital domain if untreated. Aliasing can cause severe audible disturbances such as beating and inharmonicity; however, if aliases are sufficiently attenuated they become inaudible and their effects can be neglected [5]. Until recently, the only available methods to prevent aliasing have been oversampling [2] and the harmonic mixer [6]. However, these techniques are computationally costly [2].

Previous work on the topic of antialiasing has focused mostly on synthesis of alias-free oscillator waveforms [4, 7, 8, 9]. These waveforms and, in some cases, their derivatives contain discontinuities which require infinite bandwidth and are major sources of aliasing. To reduce aliasing distortion, Brandt proposed the use of a bandlimited step (BLEP) function [7] to quasi-bandlimit the region around signal discontinuities. This approach was further expanded by Välimäki et al. [9] who proposed the more efficient polynomial approximation of the BLEP function, or polyBLEP. Huovilainen proposed the use of bandlimited correction functions to synthesize signals with discontinuities in their first derivative using a bandlimited ramp (BLAMP) function [4]. Recent work has elaborated the use of the BLAMP function and its polynomial form (polyBLAMP) to reduce the aliasing caused by the discontinuities in the first derivative introduced by hard clipping [10, 11]. This idea was later extended to suppress aliasing caused by half- and full-wave rectifiers [12].

In this study we consider the case of a soft-clipping algorithm used to implement guitar distortion. This algorithm is defined by a piecewise function that consists of a cubic polynomial connected to a constant (see Figure 1). This algorithm has a discontinuity in its second derivative at the point where the two subfunctions are joined. This discontinuity is passed on to the input signal after soft clipping. A new correction function, derived from the integral of the BLAMP function, is proposed to reduce the aliasing caused by this type of discontinuity.

This study is organized as follows. Section 2 discusses soft clipping. Sections 3 and 4 provide an overview on the derivation of the integrated BLAMP function and its approximated polynomial form, respectively. Section 5 evaluates the performance of the proposed algorithm and presents some test results. Finally, Section 6 concludes this work.

2 Soft Clipping

In this work, we consider the case of a discrete-time memoryless nonlinearity of the form

$$y[n] = c(x[n]), \tag{1}$$

where n is the sample index, x is the input signal (assumed to be bandlimited), and c is a nonlinear mapping function. As an example, we consider the piecewise nonlinearity

$$c(x) = \begin{cases} \frac{3x}{2} - \frac{x^3}{2} & \text{for } |x| < 1 \\ \text{sgn}(x) & \text{otherwise} \end{cases}, \tag{2}$$

where sgn$(\cdot)$ is the sign function. This waveshaper was used as part of an early commercial digital guitar processing unit [13]. Figure 1 shows the input–output relationship of (2). As shown in this plot, this function limits input values that exceed ± 1. However, signal values below this threshold are also processed by the algorithm. This is the fundamental difference between soft and hard clipping. In the latter, signal values below the clipping threshold are left unchanged.

Now, assuming the input signal x is bounded between $[-1, 1]$, an arbitrary clipping threshold $L \in (0, 1]$ can be implemented by scaling the input signal by $1/L$ prior to clipping and by L after clipping. Figure 2(a) shows the waveform of a 1410-Hz sinewave processed by (2) with $L = 0.5$. This and the rest of the examples in this work were implemented using a standard audio sampling rate $f_s = 44.1$ kHz. Evaluating the first derivative of this signal with respect to time results in the waveform shown in Figure 2(b). The sharp edges or corners that appear in the transition between clipped and non-clipped samples translate into discontinuities in the higher derivatives of the signal, shown in Figure 2(c).

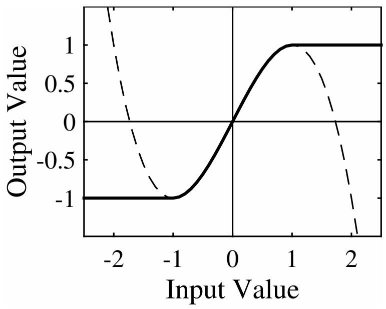

Figure 1: Input-output relationship of the piecewise saturating function (2) (solid line) and the small-signal polynomial nonlinearity (dashed line).

3 The Integrated Bandlimited Ramp Function

The antialiasing technique proposed in this study focuses on waveshaping functions that introduce discontinuities in the second derivative of a signal (like the one described in the previous section) but not in the actual waveform or its first derivative. As mentioned before, these nonlinearities will most likely also introduce discontinuities in further derivatives; however, these are not considered in this study.

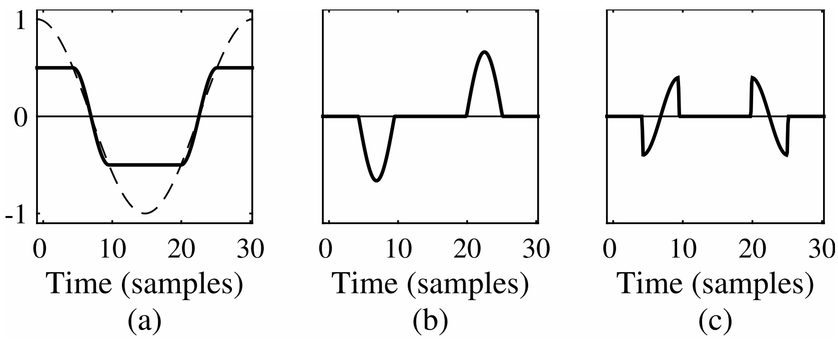

Figure 2: (a) Waveform of a 1410-Hz sinusoid with $L = 0.5$ clipping and (b) its first and (c) second derivatives with respect to time. Y-axis not drawn to scale.

A correction function that can help quasi-bandlimit a discontinuity in the second derivative can be derived from the iterative integration of the bandlimited impulse function, i.e. the sinc function [8]. In the continuous-time domain, an impulse has a flat and infinite magnitude spectrum. Its bandlimited form is defined by the inverse Fourier transform of the ideal brickwall lowpass filter as

$$h^{(0)}(t) = f_{\mathrm{s}}\mathrm{sinc}(f_{\mathrm{s}}t), \tag{3}$$

where t is time, $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$ is the cardinal sine function and f_{s} is the sampling frequency that will determine the bandwidth of the impulse.

Evaluating the indefinite integral of this function yields the bandlimited form of the unit step, or BLEP function, which is used to reduce aliasing in discontinuous signals such as the sawtooth oscillator [7, 9]. The closed form expression for the BLEP function [9] is given by

$$h^{(1)}(t) = \frac{1}{2} + \frac{1}{\pi}\mathrm{Si}(\pi f_{\mathrm{s}}t), \tag{4}$$

where the superscript $^{(1)}$ denotes it is the first integral of the bandlimited impulse and $\mathrm{Si}(x)$ is the sine integral, defined as $\mathrm{Si}(x) = \int_0^x \frac{\sin(t)}{t}\mathrm{d}t$. Computing the integral of (4) will result in the BLAMP function

$$\begin{aligned} h^{(2)}(t) &= t\left[\frac{1}{2} + \frac{1}{\pi}\mathrm{Si}(\pi f_{\mathrm{s}}t)\right] + \frac{\cos(\pi f_{\mathrm{s}}t)}{\pi^2 f_{\mathrm{s}}} && (5) \\ &= t\left[h^{(1)}(t)\right] + \frac{\cos(\pi f_{\mathrm{s}}t)}{\pi^2 f_{\mathrm{s}}} && (6) \end{aligned}$$

which can be used to quasi-bandlimit discontinuities in the first derivative of a signal [4, 10, 11, 12].

Further integration of the BLAMP function will produce the closed form expression for a bandlimited parabolic ramp, referred as the integrated BLAMP function in this study. This function is defined as

$$h^{(3)}(t) = \frac{t^2}{2}\left[\frac{1}{2}+\frac{1}{\pi}\mathrm{Si}(\pi f_\mathrm{s}t)\right]+t\frac{\cos(\pi f_\mathrm{s}t)}{2\pi^2 f_\mathrm{s}}+\frac{\sin(\pi f_\mathrm{s}t)}{2\pi^3 f_\mathrm{s}^2} \quad (7)$$

$$= \frac{t}{2}\left[h^{(2)}(t)\right]+\frac{\sin(\pi f_\mathrm{s}t)}{2\pi^3 f_\mathrm{s}^2}. \quad (8)$$

From (6) and (8) a pattern begins to emerge. Further bandlimited integrals will depend on the product of the previous function with a polynomial plus a new sinusoidal component.

The trivial (i.e. non-bandlimited) counterpart of (7) is given by

$$p(t) = \begin{cases} 0 & \text{when } t < 0 \\ \frac{t^2}{2} & \text{otherwise} \end{cases}, \quad (9)$$

which is a parabolic function that starts rising at $t = 0$ and is derived from the integral of the ramp function. Subtracting (9) from (7) yields the integrated BLAMP residual function shown in Fig. 3.

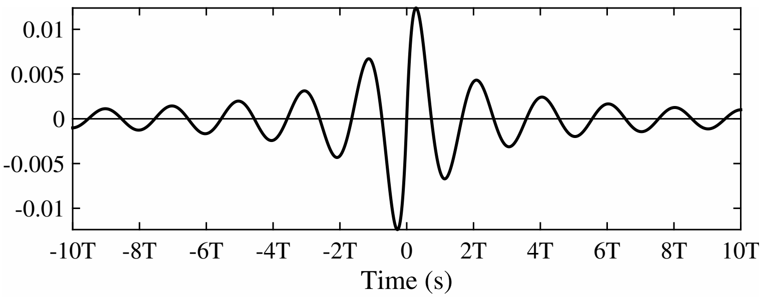

Figure 3: Integrated BLAMP residual function computed from the difference between (9) and (7). Parameter T is the inverse of the sampling rate f_s or sampling period.

This residual function can be used to quasi-bandlimit discontinuities in the second derivative of any signal by centering it around the location where each discontinuity appears, evaluating it at the nearest sampling points, scaling it by the magnitude of such discontinuity and adding it to the signal. The magnitude of the discontinuity is given by the value of the second derivative of the signal, or curvature, at the point where this discontinuity occurs. For the case of signals processed by (2), discontinuities will occur at the junction between non-clipping and clipping points, i.e. when $|x[n]| = 1$.

4 Polynomial approximation of the integrated BLAMP function

Due to the high computational costs associated with evaluating (7) and the artifacts introduced by its truncation, this work proposes the use of a polynomial approximation instead. Following the work of [9], a Lagrangian approximation of the integrated BLAMP function is derived by the iterative integration of the Lagrangian interpolating basis function, which approximates the sinc function.

The proposed polynomial correction function can correct four samples, two on each side of every clipping point, (recall that the discontinuities in the second derivative are introduced at this point). This four-point function is derived by approximating the bandlimited impulse (3) as a piecewise polynomial using Lagrangian interpolation and integrating this expression three times. The trivial parabolic waveform (9) is once again subtracted from the resulting polynomial.

First, we consider that, in practice, the exact sample points at which the input signal will enter or leave the saturating part of (2) (i.e. when $|x[n]| = 1$) will most likely not coincide with the sampling intervals of the system and must be estimated. In the four-point case, the process of centering the correction function around a set of four samples can be seen as equivalent to delaying it by $D = D_{\text{int}} + d$ samples from the second last sample before the clipping point, where $D_{\text{int}} = 1$, and $d \in [0,1)$ is the fractional delay.

Table 1 shows the polynomial expressions for the four-point Lagrangian polyBLAMP and its integral, both in terms of D, and the integrated polyBLAMP residual, expressed simply in terms of d, i.e. the fractional part of the clipping point. Expressing the polynomials in this manner facilitates their implementation in a DSP environment, where only the value of d is needed to sample the integrated polyBLAMP residual at the 4 neighboring sample points.

Span	Second integral: 4-point polyBLAMP
$[-2T,-T]$	$D^5/120 - D^4/24 + D^3/18 - D/24 + 7/360$
$[-T,0]$	$-D^5/40 + 2D^4/12 - 3D^3/12 + D/6 - 7/90$
$[0,T]$	$D^5/40 - 5D^4/24 + 3D^3/6 - 7D/24 + 7/72$
$[T,2T]$	$-D^5/120 + D^4/12 - 11D^3/36 + D^2/2 + 2D/3 + 2/45$
Span	Third integral: 4-point integrated polyBLAMP
$[-2T,-T]$	$D^5/120 - D^4/24 + D^3/12 - D^2/12 + D/24 - 1/120$
$[-T,0]$	$-D^6/240 + D^5/30 - D^4/16 + D^2/12 - 7D/90 + 1/45$
$[0,T]$	$D^6/240 - D^5/24 + D^4/8 - 7D^2/48 + 7D/72 - 7/180$
$[T,2T]$	$-D^6/720 + D^5/60 - 11D^4/144 + D^3/6 + D^2/3 + 2D/45 + 1/45$
Span	Four-point integrated polyBLAMP residual
$[-2T,T]$	$d^6/720 - d^4/144$
$[-T,0]$	$-d^6/240 + d^5/120 + d^4/24 - d^2/48 - 7d/360 - 1/180$
$[0,T]$	$d^6/240 - d^5/60 - d^4/48 + d^3/6 - d^2/4 + 11d/90$
$[T,2T]$	$-d^6/720 + d^5/120 - d^4/72 + d^2/48 - 7d/360 + 1/180$

Table 1: Coefficients for the four-point Lagrangian BLAMP and integrated BLAMP polynomial approximation and its residual ($1 \leq D < 2$ and $0 \leq d < 1$).

Figure 4 shows the waveforms for the analytical integrated BLAMP and its residual function along with their polynomial equivalents. These waveforms are plotted for the time interval $[-2T,2T]$, showing the polynomial form of the residual [Fig. 4(d)] tends to zero on the limits of this interval. Therefore, adding this residual function at every clipping point will not introduce further discontinuities. This is not the case for the analytical form [Fig. 4(c)], which requires a window function prior to its use to avoid introducing new discontinuities.

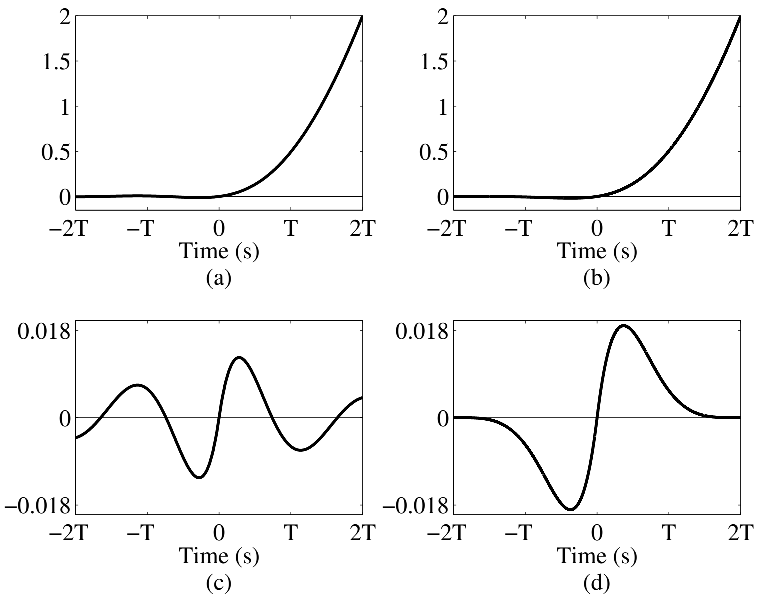

Figure 4: Waveform of the (a) integrated BLAMP function, (b) its polynomial approximation, (c) integrated BLAMP residual and (d) its polynomial equivalent.

Now, implementing this correction function on a given signal requires estimating the exact clipping points, (i.e. the value of d), and the value of the signals curvature at that point. The fractional clipping point d can be estimated by fitting a polynomial to the scaled unclipped signal $x[n]/L$ at the four sample points being corrected and solving its intersection with ± 1 using Newton-Raphson's method, for example. On the other hand, the curvature of the clipped signal at the clipping point will determine the magnitude and polarity of the discontinuity introduced and is used to scale the polynomial correction function. From (1) and switching back to the continuous time, we can define the second derivative of a clipped signal $y(t)$ with respect to time as

$$\frac{d^2y}{dt^2} = \frac{d^2c}{dx^2}\left(\frac{dx}{dt}\right)^2 + \frac{dc}{dx}\frac{d^2x}{dt^2}. \tag{10}$$

Since the slope of the clipped signal is zero at the clipping point $x(t) = \pm 1$, the second term in the derivative can be eliminated, resulting in

$$\left.\frac{d^2y}{dt^2}\right|_{x=\pm 1} = \frac{d^2c}{dx^2}\left(\frac{dx}{dt}\right)^2. \tag{11}$$

From (11) we can better appreciate the relationship between the input signal $x(t)$ and the discontinuity introduced in the second derivative of $y(t)$. When clipping point occurs at signals with small slope values (consider e.g. at the tips of a sinusoidal waveform), the curvature at that point will be very small. This means the discontinuity will also be fairly small and the aliases introduced may be negligible. At low clipping thresholds, the nonlinearity will cause clipping at signal portions with relatively large slope values. This will cause the curvature at the clipping points to be higher than for large clipping thresholds, introducing significant aliasing. Therefore, the curvature value can be estimated using (11), where the value of the slope of the input signal at the clipping point is obtained as a by-product of Newton-Raphson's method.

5 Results and Discussion

The proposed correction function was tested using a 1410-Hz sinusoid soft-clipped with a clipping threshold $L = 0.1$. Figure 5(a) shows the magnitude spectrum of the trivially-clipped signal, showing the high levels of audible aliasing distortion it exhibits. For instance, the level of the most prominent aliasing below the fundamental (highlighted by the horizontal dotted line), at 390 Hz, is -39 dB. Figure 5(b) shows the same signal processed using the approach described in [10] (i.e. hard clipping and polyBLAMP correction prior to soft clipping). This approach manages to reduce the overall level of aliasing distortion. The 390-Hz alias, in particular, has been attenuated by 10 dB. Finally, Figure 5(c) shows the signal after processing with the polynomial integrated BLAMP function shown in Figure 4(d). This time, the aliasing reduction has improved at low frequencies, where is needed the most, and the spurious 390-Hz component has been attenuated by 30 dB with respect to the trivial implementation.

Overall, the signal-to-noise (SNR) ratio of the trivially-clipped signal in Figure 5(a) was increased from 23 dB to 29 dB by using the proposed approach. In this case, the SNR was considered as the ratio between harmonics and aliasing components. For the case of the pre-processing approach, where the signal is hard-clipped and antialiased before being fed to the soft clipper [shown in Figure 5(b)], SNR was improved to 31 dB. This number is actually higher, than that of the proposed method. However, from a perceptual point of view the attenuation of the low-frequency aliases is more meaningful. Comparing Figures 5(b) and (b) clearly show the proposed method exhibits superior suppression of low-frequency aliasing components.

The test example hereby discussed was implemented using Matlab. For a 1-second signal, the total execution time of the proposed method was of approx. 20 ms. A secondary test, using oversampling a factor 2 was also performed. The resulting signal exhibited higher aliasing distortion, particularly at low frequencies, than that of the proposed method and took a total time of 50 ms to compute under similar circumstances. Moreover, the main advantage of using a correction function to quasi-bandlimit discontinuities is that redundant operations are avoided. For instance, for signal values that do not exceed the clipping threshold, no additional operations are required.

6 Conclusions

A novel polynomial correction function, aimed at reducing aliasing caused by the introduction of discontinuities in the second derivative of a signal, has been presented. The proposed correction function is designed from a Lagrangian approximation of the integrated BLAMP function.

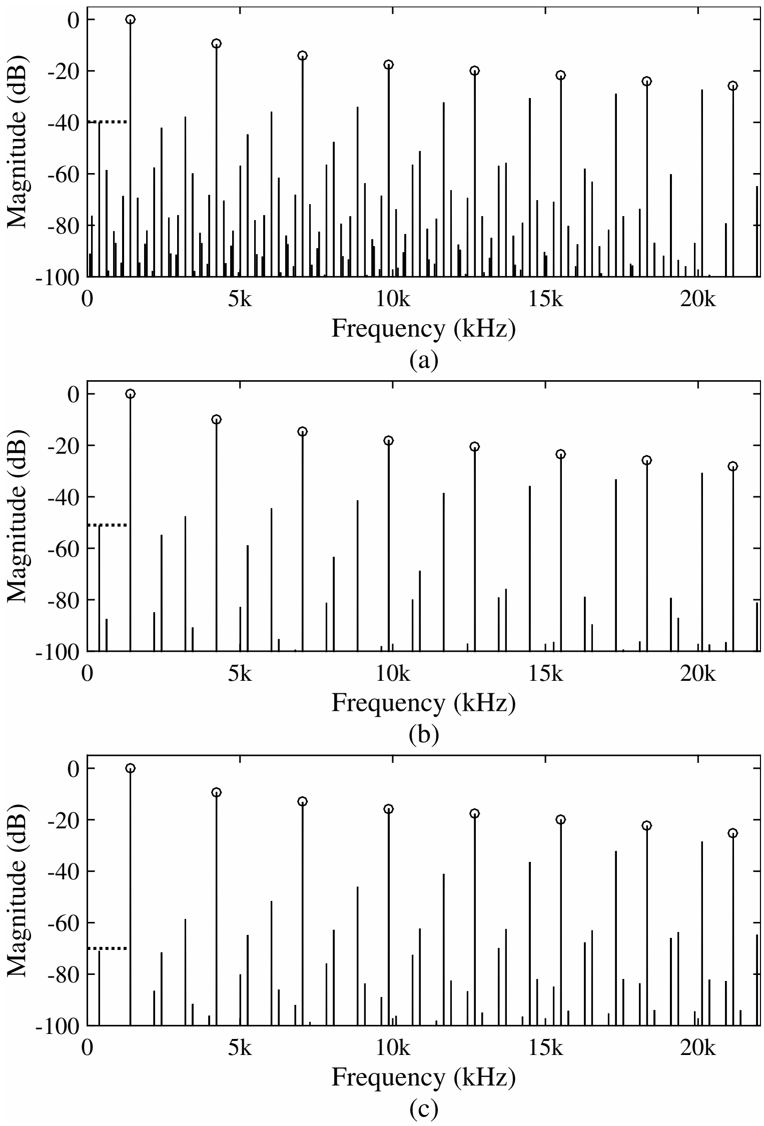

Figure 5: Magnitude spectra of a 1410-Hz sinusoid with $L = 0.1$ clipped [cf. Figure 2(a)] (a) trivially, (b) using an antialiased hard-clipper followed by a third-order polynomial waveshaper [10], and (c) using the polyBLAMP method proposed in this paper. The harmonic components are highlighted with a circle while the rest of the components are aliases. The level of the loudest aliased component below the fundamental is indicated with a horizontal dotted line in each case.

A piecewise polynomial soft clipper, commonly used to implement guitar distortion, was used as an example algorithm and the method was tested using sinusoidal inputs. Results obtained showed improved signal quality specially at low frequencies, where aliasing components below the fundamental were attenuated by up to 30 dB. Due to its efficiency and low latency, this method can be implemented in real-time and incorporated as part of a digital musical system.

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