



## Constraints on the Variation of $G$ from Primordial Nucleosynthesis

Clifton, T; Barrow, JD; Scherrer, RJ

© 2005 American Physical Society

This is a pre-copyedited, author-produced PDF of an article accepted for publication in Physical Review D following peer review. The version of record is available <http://journals.aps.org/prd/abstract/10.1103/PhysRevD.71.123526>

For additional information about this publication click this link.

<http://qmro.qmul.ac.uk/xmlui/handle/123456789/18066>

Information about this research object was correct at the time of download; we occasionally make corrections to records, please therefore check the published record when citing. For more information contact [scholarlycommunications@qmul.ac.uk](mailto:scholarlycommunications@qmul.ac.uk)

# Constraints on the Variation of $G$ from Primordial Nucleosynthesis

Timothy Clifton\* and John D. Barrow†

*DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, UK*

Robert J. Scherrer‡

*Department of Physics and Astronomy, Vanderbilt University,  
6301 Stevenson Center, Nashville, TN, 37235, USA*

(Dated: February 2, 2008)

We study here the effect of a varying  $G$  on the evolution of the early Universe and, in particular, on primordial nucleosynthesis. This variation of  $G$  is modelled using the Brans–Dicke theory as well as a more general class of scalar–tensor theories. Modified nucleosynthesis codes are used to investigate this effect and the results obtained are used to constrain the parameters of the theories. We extend previous studies of primordial nucleosynthesis in scalar–tensor theories by including effects which can cause a slow variation of  $G$  during radiation domination and by including a late–time accelerating phase to the Universe’s history. We include a brief discussion on the epoch of matter–radiation equality in Brans–Dicke theory, which is also of interest for determining the positions of the cosmic microwave background power–spectrum peaks.

PACS numbers: 04.50.+h, 26.35.+c, 98.80.Ft, 98.80.Jk

## I. INTRODUCTION

The possibility of the physical “constants” taking different values at different times in the Universe’s history has recently received much attention with the apparent observation that the fine structure constant had a different value in the distant past [1]. These observations, if correctly interpreted [2], motivate further study of the effects of any variation on other physical constants. The scalar–tensor (ST) theories of gravity provide us with a self-consistent way of modelling a possible variation in Newton’s constant,  $G$ . In these theories the metric tensor field of general relativity (GR) is no longer the only field mediating the gravitational interaction; there is an additional scalar component. Test particles in these theories still follow geodesics of the metric but the manner in which the metric is generated from the sources is altered by the presence of the scalar field. Such theories were first considered by Jordan in 1949 [5] and later developed by Brans and Dicke in 1961 [6] for the case of a constant coupling parameter. The more general case of a dynamic coupling was considered by Bergmann [7], Nordtvedt [8] and Wagoner [9] in the 1960s and 70s. Solution–generating procedures for these theories were found by Barrow and Mimoso [10], Barrow and Parsons [11] and Barrow [12].

Using these theories we investigate the earliest well understood physical process, primordial nucleosynthesis. Previous studies on this subject have been carried out by a number of authors. In particular, the Brans–Dicke (BD) theory has been especially well studied in this con-

text by, for example, Casas, García–Bellido and Quiros [13], [14] and Serna, Dominguez–Tenreiro and Yepes [15]. The more general class of ST theories has also been well studied, most notably by Serna and Alimi [16], Santiago, Kalligas and Wagoner [17] and Damour and Pichon [18]. More recently, a number of studies have been performed investigating the effect of a nonminimally coupled quintessence field on primordial nucleosynthesis (see e.g. [19],[20]). Although these studies are very detailed they all make the simplifying assumption of a constant  $G$  during the radiation–dominated phase of the Universe’s history (with the exception of [15], who numerically investigate the effect of an early scalar–dominated phase on the BD theory, and [18] who use the idea of a “kick” on the scalar field during electron–positron annihilation). We relax this assumption and investigate the effects of entering the radiation–dominated phase with a nonconstant  $G$ . The constraints we impose upon the variation of  $G$  during primordial nucleosynthesis are then used to constrain the parameters of the theory. In carrying out this study we consider the more general class of ST theories, paying particular attention to the BD theory.

## II. SCALAR–TENSOR GRAVITY

The ST theories of gravity are described by the action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left( \phi R + \frac{\omega(\phi)}{\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + 16\pi L_m \right). \quad (1)$$

Here,  $\phi$  is the scalar field,  $R = R^\mu{}_\mu$  is the Ricci scalar and  $L_m$  is the Lagrangian describing matter fields in the space–time. The action above can be extremized with

\*Electronic address: T.Clifton@damtp.cam.ac.uk

†Electronic address: J.D.Barrow@damtp.cam.ac.uk

‡Electronic address: Robert.J.Scherrer@Vanderbilt.Edu

respect to  $g_{\mu\nu}$  to give the field equations

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + \frac{1}{\phi}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\nu}g^{\rho\sigma})\phi_{;\rho\sigma} + \frac{\omega(\phi)}{\phi^2}(g^{\mu\rho}g^{\nu\sigma} - \frac{1}{2}g^{\mu\nu}g^{\rho\sigma})\phi_{;\rho}\phi_{;\sigma} = -\frac{8\pi}{\phi}T^{\mu\nu} \quad (2)$$

and with respect to  $\phi$  to give scalar-field propagation equation

$$\square\phi = \frac{1}{2\omega(\phi) + 3}(8\pi T - \omega'(\phi)g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}) \quad (3)$$

where a prime denotes differentiation with respect to  $\phi$ ,  $T^{\mu\nu}$  is the energy-momentum tensor of the matter fields, defined in the normal way, and  $T = T^\mu{}_\mu$ . In the limit  $\omega \rightarrow \infty$  and  $\omega'/\omega^3 \rightarrow 0$  the ST theories reduce to GR.

A useful property of these theories is that they can be conformally transformed to a frame where the field equations take the same form as in GR, with a nonminimally coupled, massless scalar field. We refer to these two frames as the Jordan (J) frame and the Einstein (E) frame, respectively. Under the conformal transformation  $g_{\mu\nu} = A^2(\phi)\bar{g}_{\mu\nu}$ , where  $A^2(\phi) = 1/\phi$ , the Einstein-Hilbert action for GR containing a scalar field,  $\psi$ , and other matter fields,  $\Psi_m$ ,

$$S = \int d^4x \sqrt{-\bar{g}} \left( \frac{1}{16\pi} \bar{R} + \frac{1}{2} \bar{g}^{\mu\nu} \psi_{;\mu} \psi_{;\nu} \right) + S_m[\Psi_m, A^2 \bar{g}_{\mu\nu}]. \quad (4)$$

is equivalent to the ST action (1), if we make the definitions  $\Gamma \equiv \ln A(\phi)$ ,  $\sqrt{4\pi}\alpha = \partial\Gamma/\partial\psi$  and the identification  $\alpha^{-2} = 2\omega(\phi) + 3$ . Here, symbols with bars refer to quantities in the E frame and symbols without bars refer to quantities in the J frame.

By extremizing the action (4) with respect to  $\bar{g}_{\mu\nu}$  we get the E-frame field equations

$$\bar{R}_{\mu\nu} = -8\pi(\bar{T}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{T} + \psi_{;\mu}\psi_{;\nu})$$

and by extremizing with respect to  $\psi$  we get the E frame propagation equation

$$\square\psi = -\sqrt{4\pi}\alpha\bar{T},$$

where we have defined the energy-momentum tensor in the E frame with respect to  $\bar{g}_{\mu\nu}$  so that  $\bar{T}^{\mu\nu} = A^6 T^{\mu\nu}$ .

It is worth noting that whilst the J-frame energy-momentum tensor is always covariantly conserved,  $T^{\mu\nu}{}_{;\nu} = 0$ , its counterpart in the E frame is not,  $\bar{T}^{\mu\nu}{}_{;\nu} = \sqrt{4\pi}\alpha\bar{T}\psi^{;\mu}$ . By insisting that in a physical frame the energy-momentum of the matter fields should always be conserved we are forced to identify the J frame as the physical one. Any results derived in the E frame must be transformed appropriately, if they are to be considered as physical.

### III. COSMOLOGICAL SOLUTIONS

#### A. Brans-Dicke theory, $\omega = \text{constant}$

Consider the homogeneous and isotropic space-times described by the J-frame Friedmann-Robertson-Walker (FRW) line-element

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (5)$$

where  $k = 0, \pm 1$  is the curvature parameter.

Substituting (5) into (2) and (3) gives, for a perfect fluid, the J-frame Friedmann equations

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\omega\dot{\phi}^2}{2\phi^2} + 2\frac{\dot{a}\dot{\phi}}{a\phi} + \frac{\ddot{\phi}}{\phi} = -\frac{8\pi}{\phi}p - \frac{k}{a^2}, \quad (6)$$

$$\frac{8\pi}{3\phi}\rho = \left(\frac{\dot{a}}{a}\right)^2 + \frac{\dot{a}\dot{\phi}}{a\phi} - \frac{\omega\dot{\phi}^2}{6\phi^2} + \frac{k}{a^2}, \quad (7)$$

and

$$\frac{\ddot{\phi}}{\phi} = \frac{8\pi}{\phi} \frac{(\rho - 3p)}{(2\omega + 3)} - 3\frac{\dot{a}\dot{\phi}}{a\phi}, \quad (8)$$

where overdots denote differentiation with respect to  $t$ . From  $T^{\mu\nu}{}_{;\nu} = 0$  we also obtain the perfect fluid conservation equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0.$$

#### 1. Radiation domination

We consider first the case of a flat radiation-dominated universe. Assuming the equation of state  $p = \frac{1}{3}\rho$  and defining the conformal time variable,  $\eta$ , by  $a d\eta \equiv dt$ , the equations (6), (7) and (8) integrate to give [21]

$$\frac{a'(\eta)}{a(\eta)} = \frac{\eta + \eta_2}{\eta^2 + (2\eta_2 + 3\eta_1)\eta + \eta_2^2 + 3\eta_1\eta_2 - \frac{3}{2}\omega\eta_1^2} \quad (9)$$

$$\frac{\phi'(\eta)}{\phi(\eta)} = \frac{3\eta_1}{\eta^2 + (2\eta_2 + 3\eta_1)\eta + \eta_2^2 + 3\eta_1\eta_2 - \frac{3}{2}\omega\eta_1^2} \quad (10)$$

where  $\eta_1$  and  $\eta_2$  are integration constants.

For  $\omega > -3/2$  the solutions to these equations are

$$a(\eta) = a_1(\eta + \eta_+)^{\frac{1}{2} + \frac{1}{2\sqrt{1+\frac{2}{3}\omega}}} (\eta + \eta_-)^{\frac{1}{2} - \frac{1}{2\sqrt{1+\frac{2}{3}\omega}}} \quad (11)$$

$$\phi(\eta) = \phi_1(\eta + \eta_+)^{-\frac{1}{2\sqrt{1+\frac{2}{3}\omega}}} (\eta + \eta_-)^{\frac{1}{2\sqrt{1+\frac{2}{3}\omega}}} \quad (12)$$

where  $\eta_{\pm} = \eta_2 + \frac{3}{2}\eta_1 \pm \frac{3}{2}\eta_1\sqrt{1 + \frac{2}{3}\omega}$ ,  $a_1$  and  $\phi_1$  are integration constants, and  $8\pi\rho_{r0}/3\phi_1 a_1^2 = 1$  (subscript 0 indicates a quantity measured at the present day and we rescale so that  $a_0 = 1$ , throughout).

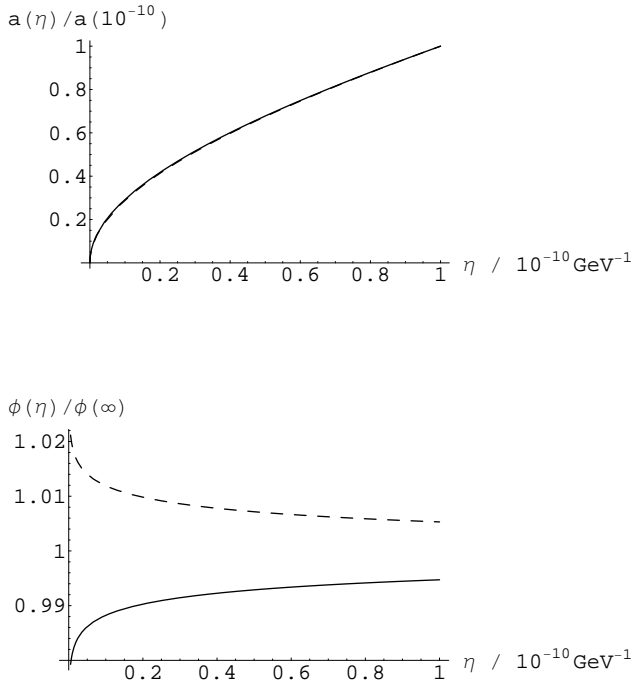


FIG. 1: The evolution of  $a$  and  $\phi$  as functions of  $\eta$  for  $\omega > -\frac{3}{2}$  with  $|\eta_1| = 10^{-12} \text{GeV}^{-1}$ ,  $\omega = 40000$ , and  $\eta_2$  set so that  $a(0) = 0$ . The solid line corresponds to  $\eta_1 > 0$  and the dashed line to  $\eta_1 < 0$ .

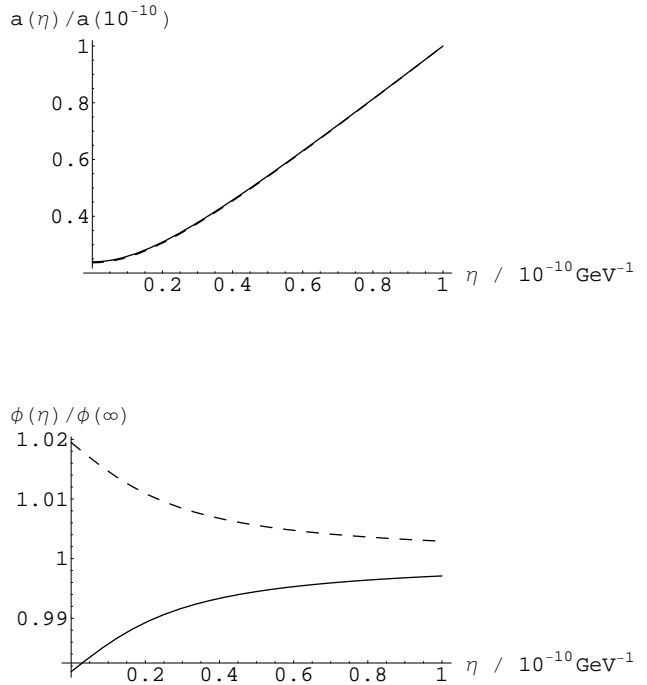


FIG. 2: The evolution of  $a$  and  $\phi$  as functions of  $\eta$  for  $\omega < -\frac{3}{2}$  with  $|\eta_1| = 10^{-13} \text{GeV}^{-1}$ ,  $\omega = 40000$ , and  $\eta_2$  set so that  $a'(0) = 0$ . The solid line corresponds to  $\eta_1 > 0$  and the dashed line to  $\eta_1 < 0$ .

For  $\omega < -3/2$  we find

$$a(\eta) = a_1 \exp\left(\frac{-\pi}{2\sqrt{\frac{2}{3}|\omega|-1}}\right) \sqrt{(\eta + \eta_-)^2 + \eta_+^2} \times \exp\left(\frac{-1}{\sqrt{\frac{2}{3}|\omega|-1}} \tan^{-1} \frac{\eta + \eta_-}{\eta_+}\right), \quad (13)$$

$$\phi(\eta) = \phi_1 \exp\left(\frac{\pi}{\sqrt{\frac{2}{3}|\omega|-1}}\right) \times \exp\left(\frac{2}{\sqrt{\frac{2}{3}|\omega|-1}} \tan^{-1} \frac{\eta + \eta_-}{\eta_+}\right), \quad (14)$$

where  $\eta_+ = \frac{3}{2}\eta_1\sqrt{\frac{2}{3}|\omega|-1}$ ,  $\eta_- = \eta_2 + \frac{3}{2}\eta_1$  and  $8\pi\rho_{r0}/3\phi_1 a_1^2 = 1$ .

In these solutions  $\eta_1$  determines the evolution of the scalar field during radiation domination, and is a physically interesting quantity;  $\eta_2$  sets the origin of the conformal time coordinate. The evolution of  $a$  and  $\phi$ , for

$\omega > -3/2$  and  $\omega < -3/2$ , is shown in figures 1 and 2, respectively. A value of  $\omega = 40000$  is chosen, in agreement with recent solar system observations [22].

For  $\omega > -3/2$  ( $\omega < -3/2$ ) we see that the scale factor here undergoes an initial period of rapid (slow) expansion and at late times is attracted towards the solution  $a(\eta) \propto \eta$ , or  $a(t) \propto t^{\frac{1}{2}}$ . Similarly,  $\phi$  can be seen to be changing rapidly at early times and slowly at late times. We attribute these two different behaviours, at early and at late times, to periods of free scalar-field domination and radiation domination, respectively. In fact, setting  $\eta_1 = 0$  in (9) and (10), we remove the scalar-dominated period and gain the power-law exact solutions  $a(\eta) \propto \eta$  and  $\phi = \text{constant}$ . These are the ‘‘Machian’’ solutions usually considered in the literature. Allowing  $\eta_1$  to be nonzero we will have a nonconstant  $\phi$ , and hence  $G$ , during primordial nucleosynthesis. If  $\rho_{r0} = 0$  is chosen then these solutions become vacuum ones that are driven by the  $\phi$  field. For  $k = 0$  these solutions do not have GR counterparts.

We see from (14), and figure 2, that for  $\omega < -3/2$  the initial singularity is avoided; for a more detailed discussion of this effect see [23].

## 2. Matter domination

When considering a matter-dominated universe we could proceed as above and determine a set of general solutions to (6), (7) and (8) that at early times are described by free scalar-field domination and at late times by matter domination (see [21]), but for our purposes this is unnecessary. For a realistic universe we require a period of radiation domination during which primordial nucleosynthesis can occur. If the scalar-field-dominated period of the Universe's history were to impinge upon the usual matter-dominated period then we would effectively lose the radiation-dominated era. For this reason it is sufficient to ignore the free scalar component of the general solution and consider only the Machian component. This is equivalent to imposing the condition  $\dot{\phi}a^3 \rightarrow 0$  as  $a \rightarrow 0$ . With this additional constraint the solutions to (6), (7) and (8), for  $k = 0$  and  $p = 0$ , are given by [24]

$$a(t) = a_* t^{\frac{2+2\omega}{4+3\omega}} \quad \text{and} \quad \phi(t) = \phi_* t^{\frac{2}{4+3\omega}} \quad (15)$$

where

$$\frac{8\pi}{3\phi_*} \frac{\rho_{m0}}{a_*^3} = \frac{2(3+2\omega)}{3(4+3\omega)}$$

and  $a_*$  and  $\phi_*$  are constants. These solutions can be seen to approach their GR counterparts as  $\omega \rightarrow \infty$ .

## 3. Vacuum domination

Similarly, for a vacuum ( $p = -\rho$ ) dominated period of expansion we can impose the condition  $\dot{\phi}a^3 \rightarrow 0$  as  $a \rightarrow 0$  to get the power law exact solutions [24]

$$a(t) = a_{\dagger} t^{\frac{1}{2}+\omega} \quad \text{and} \quad \phi(t) = \phi_{\dagger} t^2 \quad (16)$$

where

$$\frac{8\pi}{3\phi_{\dagger}} \rho_{\Lambda 0} = \frac{1}{12}(3+2\omega)(5+6\omega).$$

and  $a_{\dagger}$  and  $\phi_{\dagger}$  are constants. Note that this vacuum stress does not produce a de Sitter metric as in GR.

### B. Dynamically-coupled theories, $\omega = \omega(\phi)$

In order to evaluate more general scalar-tensor theories it is convenient to work in the E frame. In this frame, substituting the FRW line-element into the field equations gives

$$\left(\frac{\dot{\bar{a}}}{\bar{a}}\right)^2 = \frac{8\pi}{3}(\bar{\rho} + \frac{1}{2}\dot{\psi}^2) - \frac{k}{\bar{a}^2}, \quad (17)$$

$$\frac{\ddot{\bar{a}}}{\bar{a}} = -\frac{4\pi}{3}(\bar{\rho} + 3\bar{p} + 2\dot{\psi}^2) \quad (18)$$

and

$$\ddot{\psi} + 3\frac{\dot{\bar{a}}}{\bar{a}}\dot{\psi} = -\sqrt{4\pi}\alpha(\bar{\rho} - 3\bar{p}) \quad (19)$$

where over-dots here denote differentiation with respect to  $\bar{t}$  and  $\bar{a}(\bar{t}) = A(\phi)a(t)$  is the scale-factor in the E frame. Defining  $N = \ln(\bar{a}/\bar{a}_0)$  Damour and Nordtvedt [25] write (17), (18) and (19) as

$$\frac{2(1-\epsilon)}{(3-4\pi\dot{\psi}^2)}\psi'' + (2-\gamma-\frac{4}{3}\epsilon)\psi' = -(4-3\gamma)\frac{\alpha}{\sqrt{4\pi}}, \quad (20)$$

where  $\epsilon = 3k/8\pi\bar{\rho}\bar{a}^2$ ,  $\bar{p} = (\gamma-1)\bar{\rho}$  and  $'$  denotes differentiation with respect to  $N$ .

Now we consider a coupling parameter of the form

$$\Gamma(\psi) = \Gamma(\psi_{\infty}) + \alpha_0(\psi(\bar{t}) - \psi_{\infty}) + \frac{1}{2}\beta(\psi(\bar{t}) - \psi_{\infty})^2$$

so that

$$\alpha(\bar{t}) = \alpha_0 + \frac{\beta}{\sqrt{4\pi}}(\psi(\bar{t}) - \psi_{\infty}). \quad (21)$$

This is of the same form as chosen by Santiago, Kalligas and Wagoner [17] and Damour and Pichon [18]. Santiago, Kalligas and Wagoner arrive at this form of  $\alpha$  by assuming the evolution of the Universe has been close to the GR solutions throughout the period from primordial nucleosynthesis to the present. They therefore consider themselves justified in performing a Taylor expansion about the asymptotic value of  $\psi$  and discarding terms of second order or higher. Damour and Pichon consider  $\Gamma$  to be a potential down which  $\psi$  runs. They assume that a particle near a minimum of a potential experiences a generically parabolic form for that potential. This leads them to consider a quadratic form for  $\Gamma$  which gives, on differentiation,  $\alpha$  as above. These two lines of reasoning are, of course, equivalent as the parabolic form of a particle near its minimum of potential can be found using a Taylor series. Theories with this form of  $\alpha$  belong to the attractor class which approach GR at late times, if we impose the additional condition  $\alpha_0 = 0$ .

### 1. Radiation domination

For the case of a radiation-dominated flat universe, the general solutions of (17), (18) and (19) with the choice (21), are, for  $\omega > -3/2$ :

$$\bar{a}^2(\eta) = \frac{8\pi\bar{\rho}_{r0}}{3}(\eta - \eta_2)(\eta - \eta_2 + |\eta_1|)$$

$$\psi - \psi_1 = \sqrt{\frac{3}{16\pi}} \frac{\eta_1}{|\eta_1|} \ln\left(\frac{\eta - \eta_2}{\eta - \eta_2 + |\eta_1|}\right),$$

and, for  $\omega < -3/2$ ,

$$\bar{a}^2(\eta) = \frac{2\pi\bar{\rho}_{r0}}{3}|\eta_1|^2 + \frac{8\pi\bar{\rho}_{r0}}{3}(\eta - \eta_2)^2$$

$$\psi - \psi_1 = -\sqrt{\frac{3\pi}{16}} \frac{\eta_1}{|\eta_1|} + \sqrt{\frac{3}{4\pi}} \frac{\eta_1}{|\eta_1|} \tan^{-1}\left(\frac{2(\eta - \eta_2)}{|\eta_1|}\right).$$

Here,  $\eta_1$  determines the evolution of the scalar field,  $\psi_1$  is the value it approaches asymptotically and  $\eta_2$  sets the origin of the conformal time coordinate defined by  $\bar{a}(\eta)d\eta = d\bar{t}$ . For  $\omega > -3/2$ ,  $\eta_1 \in \mathcal{R}$  such that  $\psi \in \mathcal{R}$  whilst for  $\omega < -3/2$ ,  $\eta_1 \in \mathcal{I}$  such that  $\psi \in \mathcal{I}$  and  $\omega \in \mathcal{R}$ . If the idea of an imaginary scalar field seems strange the reader is reminded that the E frame is considered to be unphysical: all observable quantities measured in the physical J frame remain real at all times.

These solutions in the J frame are

$$\phi(\eta) = \exp(-\beta(\psi(\eta) - \psi_\infty)^2) \quad (22)$$

and

$$a^2(\eta) = \frac{\bar{a}^2(\eta)}{\phi}. \quad (23)$$

They exhibit the same features as their BD counterparts: at early times there is a period of free-scalar-field domination, and at late times they approach  $a \propto t^{\frac{1}{2}}$  and  $\phi = \text{constant}$ .

### 2. Matter domination

Solutions during the matter-dominated era are difficult to find because the energy-momentum tensor for the matter field is not conserved. However, it is possible to obtain an evolution equation for  $\psi$ . Using (20), we get

$$\frac{2(1-\epsilon)}{(3-4\pi\psi'^2)}\psi'' + (1-\frac{4}{3}\epsilon)\psi' = -\frac{\beta}{4\pi}(\psi - \psi_\infty). \quad (24)$$

For a flat universe  $\epsilon = 0$  and we can solve (24) by making the simplifying assumption  $\psi'^2 \ll 3$ . This gives the solution

$$\begin{aligned} \psi(N) - \psi_\infty = & A e^{-\frac{3}{4}(1+\sqrt{1-\frac{2\beta}{3\pi}})N} \\ & + B e^{-\frac{3}{4}(1-\sqrt{1-\frac{2\beta}{3\pi}})N} \end{aligned} \quad (25)$$

where  $A$  and  $B$  are constants of integration.

### 3. Vacuum domination

Similarly, for the case of a vacuum-dominated universe the evolution of  $\psi$  can be approximated using (20) to obtain

$$\begin{aligned} \psi(N) - \psi_\infty = & C e^{-\frac{3}{2}(1+\sqrt{1-\frac{2\beta}{3\pi}})N} \\ & + D e^{-\frac{3}{2}(1-\sqrt{1-\frac{2\beta}{3\pi}})N} \end{aligned} \quad (26)$$

where  $C$  and  $D$  are constants of integration.

## IV. PRIMORDIAL NUCLEOSYNTHESIS

### A. Modelling the form of $G(t)$

If primordial nucleosynthesis were to occur during the scalar-dominated period then the very different expansion rate would have disastrous consequences for the light-element abundances (see e.g. [15]). Therefore we limit our study to times at which the scale factor can be approximated by a form that is close to  $a(\eta) \propto \eta$ , and so primordial nucleosynthesis can safely be described as occurring during radiation domination. Performing a power-series expansion of the solutions (11), (12), (13) and (14) in  $\eta_1/(\eta + \eta_2)$  we find

$$a(\eta) = a_1(\eta + \eta_2 + 3\eta_1) + O(\eta_1^2) \quad (27)$$

$$\phi(\eta) = \phi_1 \left( 1 - \frac{3\eta_1}{\eta + \eta_2} \right) + O(\eta_1^2) \quad (28)$$

for both  $\omega > -3/2$  and  $\omega < -3/2$ . We can then set the origin of the  $\eta$  coordinate such that  $a(0) = 0$  with the choice  $\eta_2 = -3\eta_1$ . The solutions (27) and (28) then become, in terms of the time coordinate  $t$ ,

$$\begin{aligned} a(t) &= a_1 t^{\frac{1}{2}} + O(\eta_1^2) \\ \phi(t) &= \phi_1 \left( 1 + \frac{a_2}{a(t)} \right) + O(\eta_1^2), \end{aligned}$$

where  $a_2 = -3\eta_1 a_1$  and the origin of  $t$  has been chosen to coincide with the origin of  $\eta$ .

Similarly, expanding the solutions (22) and (23) in  $\eta_1/(\eta + \eta_2)$  we find that, for both  $\omega > -3/2$  and  $\omega < -3/2$ ,

$$\begin{aligned} a(t) &= a_1 t^{\frac{1}{2}} + O(\eta_1^2) \\ \phi(t) &= \phi_1 \left( 1 + \frac{a_2}{a(t)} \right) + O(\eta_1^2) \end{aligned}$$

where

$$\begin{aligned} a_1 &= \sqrt{8\pi\bar{\rho}_{r0}/3} \exp(\beta(\psi_1 - \psi_\infty)^2/2) \\ a_2 &= \sqrt{2\bar{\rho}_{r0}\beta}\eta_1(\psi_1 - \psi_\infty) \exp(\beta(\psi_1 - \psi_\infty)^2/2) \\ \phi_1 &= \exp(-\beta(\psi_1 - \psi_\infty)^2) \end{aligned}$$

and  $\eta_2$  has been set so that  $a(0) = 0$ .

In the limit  $\dot{\phi} \rightarrow 0$  we can see from equations (7) that the standard GR Friedmann equations are recovered, with a different value of Newton's constant given by

$$G(t) = \frac{1}{\phi(t)} = G_1 \frac{a(t)}{a(t) + a_2} \quad (29)$$

where  $G_1 = 1/\phi_1$ . We conclude that the solutions found above correspond to a situation that can be described using the GR Friedmann equations with a different, and adiabatically changing, value of  $G$ . This is just the situation considered in recent work by Bambi, Giannotti and Villante [26], but we now have an explicit form for the evolution of  $G(t)$  derived from ST gravity theory.

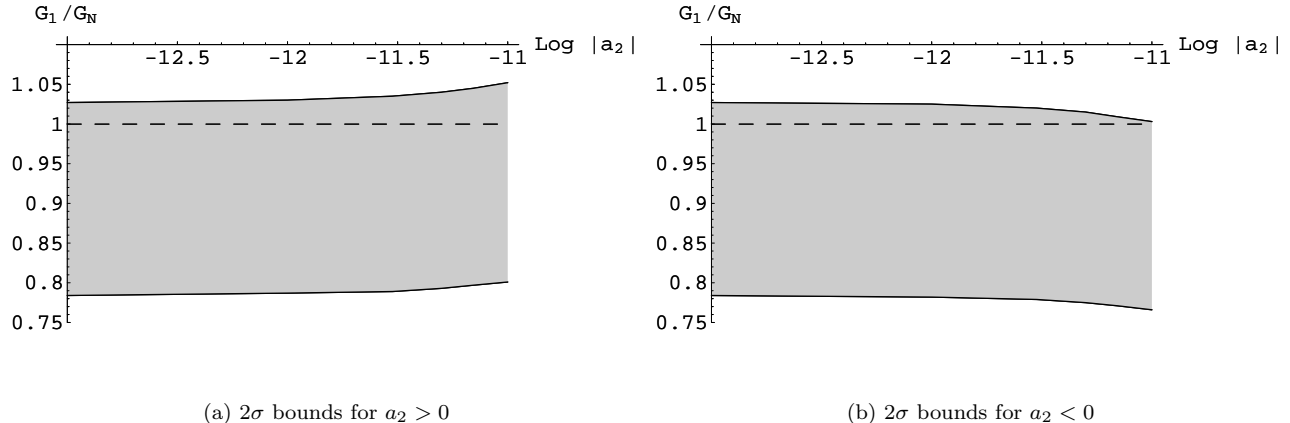


FIG. 3: The upper and lower bounds on  $G_1/G_N$  as a function of  $a_2$  are shown by the solid lines. The shaded regions correspond to the allowed parameter space and the dashed lines show  $G_1 = G_N$ .

### B. The effect of a non-constant $G$

The time at which weak interactions freeze out in the early Universe is determined by equality between the rate of the relevant weak interactions and the Hubble rate. When the weak interaction rate is the greater then the ratio of neutrons to protons tracks its equilibrium value,  $n/p = \exp(-(m_n - m_p)/T)$  where  $m_n$  and  $m_p$  are the neutron and proton masses. If, however, the Hubble rate is greater than the weak-interaction rate then the ratio of neutrons to protons is effectively “frozen-in”, and  $\beta$  decay is the only weak process that still operates with any efficiency. This will be the case until the onset of deuterium formation, at which time the neutrons become bound and  $\beta$  decay ceases. The onset of deuterium formation is primarily determined by the photon to baryon ratio,  $\eta_\gamma$ , which inhibits the formation of deuterium nuclei until the critical temperature for photodissociation is past. As the vast majority of neutrons end up in  ${}^4\text{He}$  the primordial abundance of this element is influenced most significantly by the number of neutrons at the onset of deuterium formation, which is most sensitive to the temperature of weak-interaction freeze-out, and hence the Hubble rate, and so  $G$ , at this time. Conversely, the primordial abundances of the other light elements are most sensitive to the temperature of deuterium formation, and hence  $\eta_\gamma$ , when nuclear reactions occur and the light elements form. (See [26] for a more detailed discussion of these points).

Using the simple forms of  $a(t)$  and  $G(t)$  derived above we use a modified version of the Kawano code [27] to investigate the effect of this variation of  $G$  on primordial nucleosynthesis directly. We use the deuterium abundance estimated by Kirkman et al. [28]

$$\log\left(\frac{D}{H}\right) = -4.556 \pm 0.064 \quad \text{to } 1\sigma, \quad (30)$$

and the  ${}^4\text{He}$  abundance estimated by Barger et al. [29]

$$Y_P = 0.238 \pm 0.005 \quad \text{to } 1\sigma, \quad (31)$$

to create a parameter space in  $(\eta_\gamma, G_1, a_2)$ . The three-dimensional 95%  $\chi^2$  confidence region is projected into the  $G_1, a_2$  plane to give figure 3, where three species of light neutrinos and a neutron mean lifetime of  $\tau = 885.7$  seconds have been assumed.

The apparent disfavoring of the value of  $G_1$  much greater than  $G_N$  in figure 3 is due to the observational limits we have adopted for  $Y_P$  (equation (31)), which suggest a helium abundance that is already uncomfortably low compared to the theoretical prediction for  $Y_P$  derived from the baryon density corresponding to equation 30. We expect these constraints to be updated by future observations (see e.g. [30]); this will require a corresponding update of any work, such as this, that seeks to use these constraints to impose limits on physical processes occurring during primordial nucleosynthesis.

These limits on the parameters  $G_1$  and  $a_2$  can be used to construct plots showing the explicit evolution of  $G(t)$  for various limiting combinations of the two parameters, this is done in figure 4. It is interesting to note that in both of these plots the lines corresponding to different values of  $a_2$  all appear to cross at approximately the same point,  $\log a \sim -9.4$ . This confirms our earlier discussion, and the results of [26], that the  ${}^4\text{He}$  abundance is mostly only sensitive to the value of  $G$  at the time when the weak interactions freeze out. In reality, this freeze-out happens over a finite time interval, but from figure 4 we see that it is a good approximation to consider it happening instantaneously - where the lines cross. To a reasonable accuracy one could then take  $G$  through-out primordial nucleosynthesis to be its value during the freeze-out process.

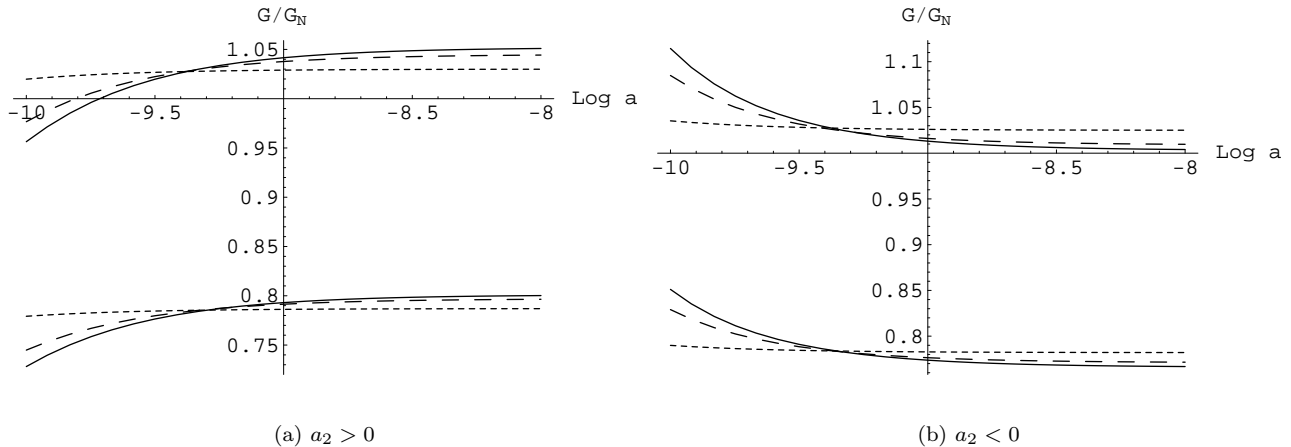


FIG. 4: These plots show the explicit evolution of  $G$  for some limiting values of  $G_1$ , as taken from figure 3, for various values of  $a_2$ . Solid lines correspond to  $|a_2| = 10^{-11}$ , dashed lines correspond to  $|a_2| = 10^{-11.155}$  and dotted lines correspond to  $|a_2| = 10^{-12}$ .

## V. CONSTRAINING THE THEORY

Using the results in the previous section it is possible to constrain the underlying ST gravitational theory. This is done separately for the BD theory and for the more general ST theories. For each theory we consider the case of a universe containing matter and radiation only and then a universe containing matter, radiation and a non-zero vacuum energy, with equation of state  $p = -\rho$ .

### A. Brans–Dicke theory

#### 1. Universe containing matter and radiation

Using equations (15) we can write the ratio of  $G$  at matter–radiation equality,  $G_{eq}$ , to its present–day laboratory value,  $G_N$ , as

$$\begin{aligned} \frac{G_{eq}}{G_N} &= \frac{(2\omega + 3) \phi_N}{(2\omega + 4) \phi_{eq}} = \frac{(2\omega + 3)}{(2\omega + 4)} \left( \frac{a_0}{a_{eq}} \right)^{1/(\omega+1)} \\ &\equiv \frac{(2\omega + 3)}{(2\omega + 4)} (1 + z_{eq})^{1/(\omega+1)} \end{aligned} \quad (32)$$

where we have used the expression for  $G$  in the weak–field limit [31]

$$G(\phi) = \frac{(2\omega + 4) 1}{(2\omega + 3) \phi}, \quad (33)$$

for  $G_N$ , whilst using the expression  $G(\phi) = 1/\phi$  for  $G_{eq}$ , as in (29). In reality, the laboratory value of  $G$  today is not equal to that in the background cosmology [32], although we take it to be so here for simplicity.

We now proceed by calculating  $1 + z_{eq}$  in the Brans–Dicke cosmology, following Liddle, Mazumdar and Barrow [33]. As  $T^{\mu\nu}_{;\nu} = 0$  in the J frame we have that

$\rho_r \propto a^{-4}$  and  $\rho_m \propto a^{-3}$ . For a universe containing matter and radiation only this gives

$$1 + z_{eq} = \frac{a_0}{a_{eq}} = \frac{\rho_{m0}}{\rho_{r0}} \frac{\rho_{req}}{\rho_{meq}} = \frac{\rho_{m0}}{\rho_{r0}}$$

as well as the usual relation  $T \propto a^{-1}$ , when entropy increase is neglected.

Photons and neutrinos both contribute to the value of  $\rho_{r0}$ , the present–day energy–density of radiation. From  $T_{\gamma 0} = 2.728 \pm 0.004 K$  [34], we get  $\rho_{\gamma 0} = 4.66 \times 10^{-34} g \text{ cm}^{-3}$  and using the well known result  $T_{\nu 0} = (4/11)^{1/3} T_{\gamma 0}$ , and assuming three families of light neutrinos, we also have  $\rho_{\nu 0} = 0.68 \rho_{\gamma 0}$ . This gives the total present–day radiation density as  $\rho_{r0} = 7.84 \times 10^{-34} g \text{ cm}^{-3}$ . Now, recalling our assumption of spatial flatness and (7), we can write

$$\rho_{tot0} = \rho_{m0} + \rho_{r0} = \frac{3H_0^2}{8\pi G_N} \frac{(4 + 3\omega)(4 + 2\omega)}{6(1 + \omega)^2}.$$

For  $G_N = 6.673 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$ ,  $H_0 = 100 h \text{ kms}^{-1} \text{ Mpc}^{-1}$ , and the value of  $\rho_{r0}$  above, we have

$$\begin{aligned} 1 + z_{eq} &= 2.39 \times 10^4 h^2 \frac{(4 + 3\omega)(4 + 2\omega)}{6(1 + \omega)^2} - 1 \\ &= 2.39 \times 10^4 h^2 \left( 1 + \frac{4}{3\omega} \right) + O(\omega^{-2}). \end{aligned}$$

This correction to  $1 + z_{eq}$  has direct observational consequences in the power spectrum of cosmic microwave background (CMB) perturbations. After  $1 + z_{eq}$  the sub-horizon scale perturbations, that were previously effectively frozen, are allowed to grow. Changing the value of  $1 + z_{eq}$  therefore causes a shift in the power–spectrum peaks, which is potentially observable (see [33], [35], [36] [37], [38] and [39] for a more detailed discussion of the effect of a varying  $G$  on CMB formation). For our purposes



this modified expression for  $1 + z_{eq}$  can then be substituted into (32) to give an equation in terms of  $\omega$ ,  $G_{eq}$  and  $h$ . Assuming a value of  $h = 0.7$  and that  $G_{eq} = G_1$ , as appears a very good approximation for the models above, we can use our bounds on  $G_1$  in terms of  $a_2$  to create an allowed parameter space in the  $(\omega, a_2)$  plane. This is shown in figure 5.

We remind the reader that the a large  $\omega$  corresponds to a slowly varying Machian component of  $\phi(1+z)$ , as can be seen directly from equations (15), (16) and the late-time limits of (11), (12), (13) and (14). The parameter  $a_2$  determines the evolution of the free component of  $\phi$ , as can be seen from the early time limits of (11), (12), (13) and (14). In the limits  $\omega \rightarrow \infty$  and  $a_2 \rightarrow 0$  the Machian and free components of  $\phi(1+z)$  both become constant, respectively, resulting in a constant  $G(1+z)$ . The constraints imposed upon  $\omega$  and  $a_2$  in figure 5 therefore correspond to constraints upon the evolution of  $G(1+z)$  in this theory, valid both during primordial nucleosynthesis and at other cosmological epochs.

## 2. Universe containing matter, radiation and a nonzero vacuum energy

A more realistic constraint would involve taking into account a late-time period of vacuum domination; so as well as  $\rho_r \propto a^{-4}$  and  $\rho_m \propto a^{-3}$  we now also have  $\rho_\Lambda = \text{constant}$  in the J frame. Hence,

$$\begin{aligned} \frac{G_{eq1}}{G_N} &= \frac{(2\omega + 3)}{(2\omega + 4)} \frac{\phi_0}{\phi_{eq1}} = \frac{(2\omega + 3)}{(2\omega + 4)} \frac{\phi_{eq2}}{\phi_{eq1}} \frac{\phi_0}{\phi_{eq2}} \\ &= \frac{(2\omega + 3)}{(2\omega + 4)} (1 + z_{eq1})^{\frac{1}{(1+\omega)}} (1 + z_{eq2})^{-\frac{1}{(1+\omega)(1+2\omega)}} \end{aligned}$$

where we have defined the redshift of matter-radiation equality,  $z_{eq1}$ , and the redshift of matter-vacuum equality,  $z_{eq2}$ , as

$$1 + z_{eq1} = \frac{\rho_{m0}}{\rho_{r0}} \quad \text{and} \quad 1 + z_{eq2} = \left( \frac{\rho_{\Lambda 0}}{\rho_{m0}} \right)^{\frac{1}{3}}.$$

As above, we still have  $\rho_{r0} = 7.84 \times 10^{-34} g \text{ cm}^{-3}$  but now our assumption of spatial flatness gives

$$\rho_{tot0} = \rho_{\Lambda 0} + \rho_{m0} + \rho_{r0} = \frac{3H_0^2}{8\pi G_N} \frac{(4+3\omega)(4+2\omega)}{6(1+\omega)^2}.$$

or, using  $G_N = 6.673 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$  and  $H_0 = 100h \text{ kms}^{-1}\text{Mpc}^{-1}$ ,

$$1 + z_{eq1} = \frac{\left( 2.39 \times 10^4 h^2 \frac{(4+3\omega)(4+2\omega)}{6(1+\omega)^2} - 1 \right)}{\left( 1 + \frac{\rho_{\Lambda 0}}{\rho_{m0}} \right)}.$$

Taking the value  $\rho_{\Lambda 0}/\rho_{m0} = 2.7$ , consistent with WMAP observations [40], we get the results shown in 5.

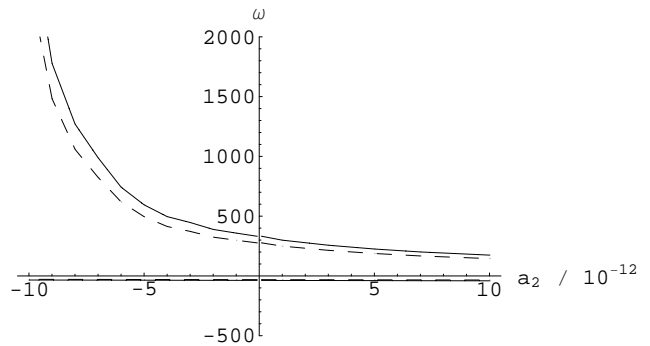


FIG. 5: The allowed parameter space in this plot is the region above the higher solid line and below the lower solid line, with  $h = 0.7$  to  $2\sigma$  for a universe containing matter and radiation only. The dashed lines show the corresponding result for a universe with a non-zero vacuum energy.

## B. Dynamically-coupled theories

### 1. Constraints on $\omega(\phi)$ at matter-radiation equality

Recalling that  $A^2 = 1/\phi$  and  $\alpha^{-2} = 2\omega + 3$  allows us to re-write (33) as

$$G = A^2(1 + \alpha^2),$$

the ratio  $G_{eq}/G_N$  can then be expressed in terms of  $A$  and  $\alpha$  as

$$\frac{G_{eq}}{G_N} = \frac{A_{eq}}{A_0 \sqrt{1 + \alpha_0^2}}.$$

Assuming that  $\ln A_0$  and  $\ln(1 + \alpha_0)$  are negligible compared to  $\ln A_{eq}$  (i.e. the Universe is close to GR today, [22]) we can write

$$\begin{aligned} \ln \left( \frac{G_{eq}}{G_N} \right) &\simeq \ln A_{eq} = \frac{1}{2} \beta (\psi_{eq} - \psi_\infty)^2 \\ &= \frac{2\pi}{(2\omega_{eq} + 3)} \frac{1}{\beta}. \end{aligned}$$

This allows us to constrain  $\omega_{eq}$  in terms of  $\beta$  and  $a_2$ , as shown in figure 6. Constraints imposed upon  $\omega$  and  $a_2$  have the same consequences for the evolution of  $G$  as previously discussed. The parameter  $\beta$  controls the evolution of  $\omega$ , as can be seen from (21). In the limit  $\beta \rightarrow 0$  it can be seen that  $\omega$  becomes constant and this class of ST theories becomes indistinguishable from the BD theory. Constraints upon  $\beta$  therefore correspond to constraints on the allowed variation of  $\omega$  and hence  $G$ .

The parameter  $\beta$  is taken to be small here so that the “kick” on the scalar field during electron-positron annihilation can be neglected. These effects have been explored by Damour and Pichon in [18], for the case  $a_2 = 0$ , and are expected to have the same result in this more general scenario; we will not repeat their analysis of this effect here.

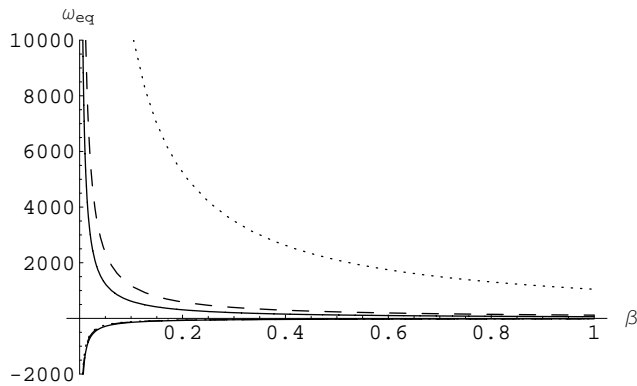


FIG. 6: The allowed parameter space in this graph is the region above the line in the region  $\omega > 0$  and below the line in the region  $\omega < 0$ , to  $2\sigma$ . The solid lines correspond to  $a_2 = 10^{-11}$  the dashed lines to  $a_2 = 10^{-13}$  and the dotted lines to  $a_2 = -10^{-11}$ .  $\omega_{eq}$  is the value of  $\omega$  at matter–radiation equality and  $\beta$  is a parameter of the theory, defined in (21).

## 2. Constraints on $\omega_0$ for a universe containing matter and radiation

The scalar field can be evolved forward in time from the time of matter–radiation equality to the present, using equation (25). The two arbitrary constants in this expression can be fixed using our limiting values of  $G_{eq}$  derived above and by assuming that the evolution of the scalar field has effectively ceased by this time, i.e.  $\psi'_{eq} = 0$ , as is the case for the models considered here.

In order to gain a quantitative limit on  $\psi_1 - \psi_\infty$ , and hence on  $\omega_0$ , it is necessary to calculate  $N_{eq}$ . From the definition of  $N$ , we can write

$$\begin{aligned} N_{eq} &= -\ln(1 + z_{eq}) - \ln A_{eq} + \ln A_0 \\ &\simeq -\ln(1 + z_{eq}) - \frac{1}{2}\beta(\psi_{eq} - \psi_\infty)^2 \end{aligned}$$

where  $z_{eq}$  is the red–shift at  $t_{eq}$  and we have assumed, as before, the term  $\ln A_0$  to be negligible.

It now remains to determine  $1 + z_{eq} = \rho_{m0}/\rho_{r0}$  for the case  $\omega = \omega(\phi)$ . If we now assume  $\omega_0$  to be moderately large, and recall our assumption of spatial flatness, we can write

$$\rho_{tot0} = \rho_{m0} + \rho_{r0} = \frac{3H_0^2}{8\pi G_N}.$$

For  $\rho_{r0} = 7.84 \times 10^{-34} g cm^{-3}$  this gives  $\rho_{m0} \simeq 1.87 \times 10^{-29} h^2 g cm^{-3}$ . We have  $1 + z_{eq} \simeq 2.4 \times 10^4 h^2$  and so finally obtain, for  $h = 0.7$ ,

$$N_{eq} \simeq -9.37 - \frac{1}{2}\beta(\psi_{eq} - \psi_\infty)^2.$$

Now, evolving  $\psi - \psi_\infty$  from  $N_{eq}$  to  $N_0 = 0$  and using

$$\omega = \frac{2\pi}{\beta^2(\psi_0 - \psi_\infty)^2} - \frac{3}{2} \quad (34)$$

we obtain the results shown in figure 7.

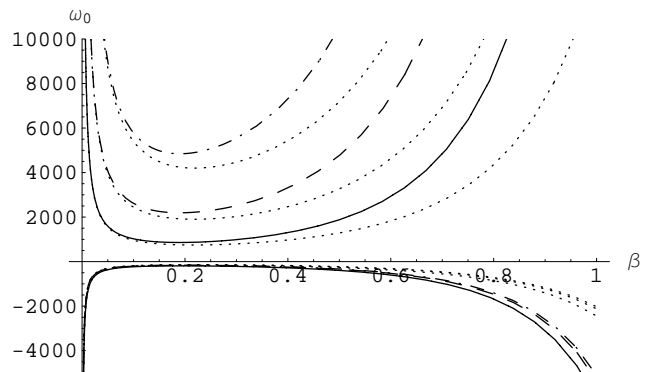


FIG. 7: The allowed parameter space in this graph is the region above the line in the region  $\omega > 0$  and below the line in the region  $\omega < 0$ , to  $2\sigma$  for a universe containing matter and radiation only. The solid lines correspond to  $a_2 = 10^{-11}$ , the dashed lines to  $a_2 = -10^{-11.523}$ , and the dot-dashed lines to  $a_2 = -10^{-11.155}$ . The dotted lines show the corresponding results for a universe with a non-zero vacuum energy.

## 3. Constraints on $\omega_0$ for a universe containing matter, radiation and nonzero vacuum energy

We can repeat the previous analysis for the more realistic case of a universe with a period of late–time vacuum domination. We now need the time of matter–radiation equality,  $N_{eq1}$ , and the time of matter–vacuum equality,  $N_{eq2}$ .

Assuming spatial flatness, a large  $\omega_0$ ,  $h = 0.7$ ,  $\rho_{\Lambda 0}/\rho_{m0} = 2.7$ , and  $\rho_{r0}$  as above gives

$$N_{eq1} \simeq -8.06 - \frac{1}{2}\beta(\psi_{eq1} - \psi_\infty)^2,$$

and

$$N_{eq2} \simeq -0.33 - \frac{1}{2}\beta(\psi_{eq2} - \psi_\infty)^2.$$

We can now evolve  $\psi(N) - \psi_\infty$  from  $N_{eq1}$  to  $N_{eq2}$  using (25) and the same boundary conditions as before. (In finding  $N_{eq2}$  we used an iterative method to evaluate  $\psi_{eq2} - \psi_\infty$ ). We then use (26) to evolve the field from  $N_{eq2}$  to  $N_0$ ; the constants  $C$  and  $D$  in (26) are set by matching  $\psi(N) - \psi_\infty$  and its first derivative with (25) at  $N_{eq2}$ . Finally, we can calculate  $\omega_0$  using (34). The results of this procedure are shown in figure 7.

## VI. DISCUSSION

Using the framework provided by ST theories of gravity we have investigated the effect of a time–varying  $G$  during primordial nucleosynthesis. We determined the effect on primordial nucleosynthesis numerically, using a modified version of the Kawano code [27], and constrained the parameters of the underlying theory using

these results. Our results are consistent with the interpretation that the abundance of  ${}^4\text{He}$  is primarily only sensitive to the value of  $G$  at the time when weak interactions freeze out, for the class of ST models studied.

Using our numerically determined constraints on the evolution of  $G$ , we imposed the  $2\sigma$  upper and lower bounds shown in figure 5 on the BD parameter  $\omega$ . For a constant  $G$  (i.e.  $a_2 = 0$ ) we get the bounds  $\omega \gtrsim 332$  or  $\omega \lesssim -37$ , for a universe containing matter and radiation only, and the bounds  $\omega \gtrsim 277$  or  $\omega \lesssim -31$ , for a universe containing matter, radiation and a nonzero vacuum energy. As the parameter  $a_2$  is increased, the strength of the upper bound decreases whilst that of the lower bound increases; the opposite behaviour occurs as  $a_2$  is decreased. This is because the strength of the bound on  $\omega$  is essentially due to the allowed value of  $G$  at matter–radiation equality. If primordial nucleosynthesis allows this value to be vastly different from  $G_N$  then a significant evolution of  $G$  during matter domination, and hence a low  $|\omega|$ , is permitted. A value of  $G_{eq}$  close to  $G_N$  means that only a very slow variation of  $G$ , and hence large  $|\omega|$  is permitted. As  $a_2 > 0$  corresponds to an increasing  $G$  during primordial nucleosynthesis this corresponds to a higher value of  $G_{eq}$  and hence a tighter upper bound on  $\omega$  and a looser lower bound (as  $G$  decreases during matter domination for  $\omega > -3/2$  and increases for  $\omega < -3/2$ );  $a_2 < 0$  corresponds to  $G$  decreasing during radiation domination, and so has the opposite effect.

The more stringent effect of a nonzero value of  $a_2$  on the upper bounds is due to the current observational determinations of the  ${}^4\text{He}$  abundance [29] disfavoring  $G > G_N$  during primordial nucleosynthesis (see e.g. [41]).

The interpretation of the constraints on the more general ST theories is a little more complicated, due to the increased complexity of the theories. The bounds on  $\omega(\phi)$  at matter–radiation equality, shown in figure 6, are seen to be stronger for smaller  $\beta$  and weaker for larger for  $\beta$  (assuming  $\beta$  to be small enough to safely ignore the ef-

fect of the  $e^-e^+$  kick analysed by [18]). This should be expected as  $\omega \sim \beta^{-2}$  in the models we are studying. We see from the constraints on  $\omega$  at the present day, shown in figure 7, that the bounds on  $\omega$  become tighter as  $\beta$  gets very small and large, with an apparent minimum in the bound at  $\beta \sim 0.2$ . The tight bounds for very small  $\beta$  are due to the tight bounds on  $\omega_{eq}$  for small  $\beta$ . The tight bounds at large  $\beta$  are due to the attraction towards GR at late times that occurs for this class of ST theories (see e.g. [25]). This attractor mechanism is more efficient for larger values of  $\beta$ , as can be seen from (20), and at late times so  $\omega$  is drawn to a larger value for a larger  $\beta$ .

The inclusion of a late–time vacuum–dominated stage of the Universe’s evolution is to weaken slightly the bounds that can be placed on  $\omega$  at the present day, as can be seen from figures 5 and 7. This weakening of the bounds is due to a shortening of the matter–dominated period of the Universe’s history which is essentially the only period, after the effects of the free scalar–dominated phase become negligible, during which  $G$  evolves.

We find that the constraints that can be imposed upon the present day value of  $\omega$  from primordial nucleosynthesis are, for most of the allowed parameter space, considerably weaker than those obtained from observations within the solar system. To date, the tightest constraint upon  $\omega_0$  are imposed by Bertotti, Iess and Tortora [22] who find  $|\omega_0| \gtrsim 40000$ , to  $2\sigma$ . This constraint is obtained from observations of the Shapiro delay of radio signals from the Cassini spacecraft as it passes behind the Sun. We consider the constraints imposed upon  $\omega$  here to be complementary to these results as they probe different length and time scales, as well as different epochs of the Universe’s history.

## ACKNOWLEDGEMENTS

T.C. is supported by PPARC.

- 
- [1] J. K. Webb et al, Phys. Rev Lett. **82**, 884 (1999); M. T. Murphy et al, MNRAS **327**, 1208 (2001); J. K. Webb et al, Phys. Rev. Lett. **87** 91301 (2001); M. T. Murphy, J. K. Webb and V. V. Flambaum, MNRAS **345**, 609 (2003).
  - [2] It has been suggested (see e.g. [3]) that these observations could be due to a different isotopic magnesium abundance. However, it has also been shown that matching the isotopic abundances so as to emulate the apparently observed variation in alpha also creates an over abundance of nitrogen [4].
  - [3] T. Ashenfelter, G. J. Mathews and K. A. Olive, Phys. Rev. Lett. **92**, 41102 (2004).
  - [4] Y. Fenner, B. Gibson and M Murphy, MNRAS **358**, 468 (2005).
  - [5] P. Jordan, Nature **164**, 637 (1949).
  - [6] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).
  - [7] P. G. Bergmann, Int. J. Theo. Phys. **1**, 25 (1968).
  - [8] K. Nordtvedt, Astrophys. J. **161**, 1059 (1970).
  - [9] R. V. Wagoner, Phys. Rev. D **1**, 3209 (1970).
  - [10] J. D. Barrow and J. P. Mimoso, Phys. Rev. D **50**, 3746 (1994).
  - [11] J. D. Barrow and P. Parsons, Phys. Rev. D **55**, 1906 (1997).
  - [12] J. D. Barrow, Phys. Rev. D **47**, 5329 (1992).
  - [13] J. A. Casas, J. Garcia–Bellido and M. Quiros, Mod. Phys. Lett. A **7**, 447 (1992).
  - [14] J. A. Casas, J. Garcia–Bellido and M. Quiros, Phys. Lett. B **278**, 94 (1992).
  - [15] A. Serna, R. Dominguez–Tenreiro and G. Yepes, Astrophys. J. **391**, 433 (1992).
  - [16] A. Serna and J. M. Alimi, Phys. Rev. D. **53**, 3087 (1996).
  - [17] D. I. Santiago, D. Kalligas and R. V. Wagoner, Phys. Rev. D **56**, 7627 (1997).
  - [18] T. Damour and B. Pichon, Phys. Rev. D **59**, 123502

- (1999).
- [19] X. Chen, R. J. Scherrer and Gary Steigman, *Phys. Rev. D* **63**, 123504 (2001).
- [20] V. Pettorino, C. Baccigalupi and G. Mangano, *J. Cosmol. Astropart. Phys.* **01** 014 (2004).
- [21] L. E. Gurevich, A. M. Finkelstein and V. A. Ruban, *Astrophys. and Sp. Sci.* **22**, 231 (1973).
- [22] B. Bertotti, L. Iess and P. Tortora, *Nature* **425**, 374 (2003).
- [23] J. D. Barrow, D. Kimberly and J. Magueijo, *Class. Quant. Grav.* **21**, 4289 (2004).
- [24] H. Nariai, *Prog. Theo. Phys.* **40**, 49 (1968).
- [25] T. Damour and K. Nordtvedt, *Phys. Rev. D* **48**, 3436 (1993).
- [26] C. Bambi, M. Giannotti and F. L. Villante, *astro-ph/0503502* [*Phys. Rev. D* (to be published)].
- [27] L. Kawano, *Fermilab-pub-92/04-A*, 1992.
- [28] D. Kirkman, D. Tytler, N. Suzuki, J. M. O'Meara and D. Lubin, *Astrophys. J. Suppl.* **149**, 1 (2003).
- [29] V. Barger, J. P. Kneller, H. S. Lee, D. Marfatia and G. Steigman, *Phys. Lett. B* **566**, 8 (2003).
- [30] R. Trotta and S. H. Hansen, *Phys. Rev. D* **69**, 023509 (2004).
- [31] C. M. Will, 1993, *Theory and Experiment in Gravitational Physics, 2nd edn.* (Cambridge University Press, Cambridge).
- [32] T. Clifton, D. F. Mota and J. D. Barrow, *MNRAS* **358**, 601 (2005).
- [33] A. R. Liddle, A. Mazumdar and J. D. Barrow, *Phys. Rev. D* **58**, 027302 (1998).
- [34] D. J. Fixen et al., *Astrophys. J.* **473**, 576 (1996).
- [35] X. Chen and M. Kamionkowski, *Phys. Rev. D* **60**, 104036 (1999).
- [36] R. Nagata, T. Chiba and N. Sugiyama, *Phys. Rev. D* **66**, 103510 (2002).
- [37] O. Zahn and M. Zaldarriaga, *Phys. Rev. D* **67**, 063002 (2003).
- [38] R. Nagata, T. Chiba and N. Sugiyama, *Phys. Rev. D* **69**, 083512 (2004).
- [39] V. Acquaviva, C. Baccigalupi, S. M. Leach, A. R. Liddle and F. Perrotta, *Phys. Rev. D* **71**, 104025 (2004).
- [40] C. L. Bennett et al., [WMAP collaboration], *Astrophys. J. Supplement Series*, **148**, 1 (2003).
- [41] J. D. Barrow and R. J. Scherrer, *Phys. Rev. D* **70**, 103515 (2004).