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Errors in Estimating Ω_Λ due to the Fluid Approximation

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The matter content of the Universe is strongly inhomogeneous on small scales. Motivated by this fact, we consider a model of the Universe that has regularly spaced discrete masses, rather than a continuous fluid. The optical properties of such space-times can differ considerably from the continuous fluid case, even if the 'average' dynamics are the same. We show that these differences have consequences for cosmological parameter estimation, and that fitting to recent supernovae observations gives a correction to the inferred value of Ω_Λ of $\sim 10\%$.

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In the standard approach to relativistic cosmology it is usual to assume that the cosmological principle implies that the Universe is permeated by a set of continuous perfect fluids. Examples of these are photons, baryons, dark matter and dark energy. Yet when we observe the Universe, there is clear evidence for discreteness. Matter is accumulated in stars and galaxies which arrange themselves in clusters, filaments and walls. These structures occupy a small volume of space, the rest of which is almost completely devoid of electromagnetically interacting matter. It is therefore reasonable to ask if the fluid approximation is a good representation of the real Universe, and, if not, what are the corrections due to the discretization we observe.

In [1] (henceforth, paper I) we considered just such a model of the Universe: The matter content was taken to be discrete islands of mass, rather than the usual continuous fluid. This approach, which we dubbed 'Archipelagian Cosmology', was based on the Lattice Universe model of Lindquist and Wheeler [2], a construction analogous to that of Wigner and Seitz in electromagnetism [3]. The principal result of the study was that even if the large-scale dynamics of a universe with discretized matter content approach those of a Friedmann-Robertson-Walker (FRW) universe, this does not mean that the optical properties will. Observers in a universe filled with discrete objects can measure different redshifts and luminosity distances to distant astrophysical objects, even if the 'average' expansion is identical to FRW.

Of course, paper I is not the first study of the optical properties of an inhomogeneous universe. Previous attempts have been made in the context of Swiss Cheese cosmologies [4, 5, 6, 7, 8, 9], as well as with linear perturbations to FRW geometry [10, 11, 12, 13, 14]. The difference in paper I is that the model used does not rely on an FRW background (neither as an embedding structure for the inhomogeneities, nor as a background to be perturbed around). Instead, the model is a bottom up construction that is not FRW at any point in space-time, yet behaves dynamically like FRW on large scales. The model does not, therefore, rely on any ill-defined concepts such as 'average' geometry, and is free from ambiguities associated with the scale of smoothing. Hence, we expect this approach to provide us with new insights, and, in particular, new ways to test concepts such as the fluid approximation without the constraint of being tied to FRW geometry.

In paper I we considered the simple case of Milky Way-sized spherically symmetric masses arranged on a regular lattice with a spacing of $\sim$ 1Mpc, and with critical density. We found that in the absence of a Cosmological Constant the deceleration parameter measured by an observer in such a space-time would be $q_0 \simeq 8/7$, rather than the usual value of $1/2$ that would be measured in a universe filled with a perfect fluid of dust (i.e. an Einstein-de Sitter universe). This result is due to two separate effects. Firstly, the null geodesics that connect observers and sources pass through the empty regions between masses, rather than through a continuous energy density. This means that the focusing due to the notional fluid of an FRW cosmology is absent [15, 16]. Secondly, the redshift experienced by the photons is due to the effect of anisotropic expansion integrated along their trajectories. This does not, in general, correspond to the same redshift that would be experienced by a photon in the 'average' space-time. Analytic expressions for these quantities are given in paper I, together with the results of numerical simulations.

In this paper we wish to include the effect of a Cosmological Constant. In some sense, this is a very straightforward extension of the cosmology that was considered in paper I. Crucially, however, it will allow us to assess the influence of the effects we have uncovered on cosmological parameter estimation, as well as provide us with a test of the validity of our approach. Huge resources are being invested into gathering ever more observational data, and it is critical to the success of the missions involved in this that we understand to the highest degree possible the relationships between redshifts, luminosity distances and the expansion of the Universe. Any attempts at precision parameter estimation [17], or, for example, reconstructing the equation of state of Dark Energy [18], will be highly dependent on such considerations.

Let us consider a critical density cosmology where we have a lattice of cells with Schwarzschild-de Sitter geometry. The line-element inside each cell is then given

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by

$$ds^2 = -\left(1-\frac{2m}{r}-\frac{\Lambda}{3}r^2\right)d\tau^2 -2\sqrt{\frac{2m}{r}+\frac{\Lambda}{3}r^2}d\tau dr + dr^2 + r^2 d\Omega^2. \quad (1)$$

Here the time coordinate τ is the proper time of a freely falling observer with 4-velocity

$$u^a = \left(1; \sqrt{\frac{2m}{r}+\frac{\Lambda r^2}{3}}, 0, 0\right). \quad (2)$$

The relationship between τ and the Schwarzschild time coordinate t is given in Appendix A of paper I. The reader is also referred to Section 3 of paper I for an explanation of why τ corresponds to 'cosmological time'.

Now, the principal difference between the Lindquist-Wheeler lattice construction in General Relativity (GR) [2], and Wigner-Seitz construction in electromagnetism [3] (aside from the non-linearity of the field equations in GR) is that in the case of the former the lattice itself becomes dynamical. In the case of a universe with critical density, this leads to cosmological evolution governed by the equation

$$\frac{\dot{r}^2}{r^2} = \frac{2m}{r^3}+\frac{\Lambda}{3}, \quad (3)$$

where the over-dot here denotes differentiation with respect to τ. If we replace r by the scale factor a, then this is clearly just the Friedmann equation, with the usual solution

$$a^3(\tau) = \frac{6m}{\Lambda L_0^3}\sinh^2\left[\frac{\sqrt{3\Lambda}}{2}\tau\right], \quad (4)$$

where L_0 is the lattice spacing today (so that $a(\tau_0)=1$). The large-scale dynamics of this model are therefore identical to an FRW model with $8\pi\rho = 2m/(L_0 a)^3$. However, as discussed above, this does not necessarily mean that the optical properties are also the same.

We know from our study of the $\Omega_\Lambda = 0$ case that these corrections can be considerable. However, we also know that as $\Omega_\Lambda \to 1$ they should vanish. This is due to the space-time approaching de Sitter space in this limit. The influence of the mass at the centre of each cell then becomes negligible, as the space-time is dominated by Λ. Of course, the energy density associated with Λ is constant everywhere, and so we should expect any effects due to the discreteness of m to disappear. Indeed, we find just such a convergence to occur in the results below, and we consider this to be an important verification of the model we are using (analogous to Shockley's 'empty lattice test' with a constant potential [19], that is used to justify the Wigner-Seitz approximation).

Now, to begin our study of redshifts and distance measures in this space-time, let us consider a bundle of null rays that connect an observer with an unobscured source. The null geodesic equations can then be integrated to give

$$B = \left(1-\frac{2m}{r}-\frac{\Lambda}{3}r^2\right)\dot{\tau} + \sqrt{\frac{2m}{r}+\frac{\Lambda}{3}r^2}\,\dot{r} \quad (5)$$

$$\dot{r}^2 = B^2 - \frac{J^2}{r^2}\left(1-\frac{2m}{r}-\frac{\Lambda}{3}r^2\right) \quad (6)$$

$$\dot{\theta}^2 = \frac{J^2}{r^4} - \frac{J_\phi^2}{r^4\sin^2\theta} \quad (7)$$

$$\dot{\phi} = \frac{J_\phi}{r^2\sin^2\theta}, \quad (8)$$

where B, J and J_ϕ are constants, and over-dots here correspond to derivatives with respect to the affine parameter, λ. These equations allow us to propagate null geodesics away from our observer, and can be connected between lattice cells using the methods discussed in paper I (with $2m/r$ replaced by $2m/r+\Lambda r^2/3$, wherever it occurs in the matching conditions).

The redshift between emission and observation is given, as always, by the ratio of frequencies at these two events. For source and observer both moving with 4-velocity u^a, as given in (2), this is

$$1+z = \frac{(-u^a k_a)|_e}{(-u^b k_b)|_o} = \frac{\dot{\tau}|_e}{\dot{\tau}|_o}, \quad (9)$$

where k^a is the 4-vector tangent to the null geodesic, and subscripts e and o denote quantities at emission and observation, respectively. The null geodesics equations, and redshift, can now be calculated numerically.

For the $\Omega_\Lambda = 0$ case we obtained in paper I the analytic approximation

$$1+z \simeq (1+z_{FRW})^{\langle\gamma\rangle}, \quad (10)$$

where $\langle\gamma\rangle = 0.7$. However, we know that in the limit $\Omega_\Lambda \to 1$ that we should find $\langle\gamma\rangle \to 1$, as the space-time approaches de Sitter space. We therefore expect that we should find $0.7 \lesssim \langle\gamma\rangle \lesssim 1$, for $0<\Omega_\Lambda<1$.

Our numerical simulations verify this expectation, and we find that excellent fits can be achieved in the range $0<z<2$ by allowing $\langle\gamma\rangle$ to be a running function of z_{FRW}, with the form

$$\langle\gamma\rangle = A + B z_{FRW}. \quad (11)$$

Performing a least squares fit to the output of our numerical code we find best fit values of A and B for various different values of Ω_Λ. These are displayed in Fig. 1, along with the fitted functions $A = 0.69+0.29\Omega_\Lambda$ and $B = 0.0021-0.057\Omega_\Lambda+0.055\Omega_\Lambda^{12}$. It can be seen that $\langle\gamma\rangle$ approaches a constant as both $\Omega_\Lambda \to 0$ (as found in paper I), and as $\Omega_\Lambda \to 1$ (as the de Sitter limit is approached).

To go further, and obtain expressions for angular diameter, and luminosity distance in these models, we must integrate the Sachs optical equations [20] along the

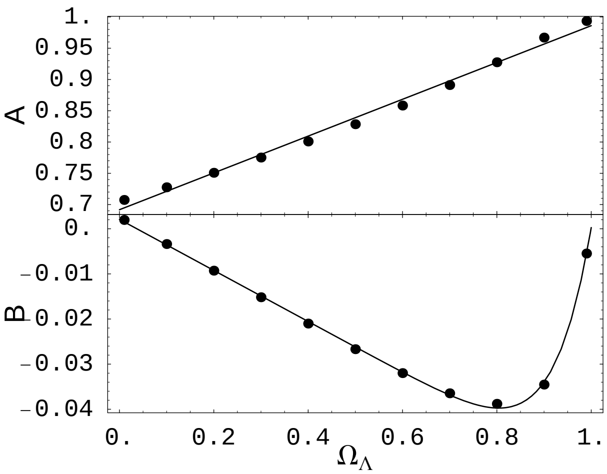

FIG. 1: Dots show the best fit values of A and B from Eq. (11) for various values of Ω_Λ. The curves in the upper and lower plots are the fitted functions $0.69+0.29\Omega_\Lambda$ and $0.0021-0.057\Omega_\Lambda+0.055\Omega_\Lambda^{12}$, respectively.

geodesics that are solutions of (5)-(8). Without rotation, these equations are

$$\frac{d\tilde{\theta}}{d\lambda}+\tilde{\theta}^2+\sigma^*\sigma \;=\; -\frac{1}{2}R_{ab}k^ak^b \qquad (12)$$

$$\frac{d\sigma}{d\lambda}+2\sigma\tilde{\theta} \;=\; C_{abcd}(t^*)^ak^b(t^*)^ck^d\equiv C, \qquad (13)$$

where $\tilde{\theta}$ and σ are the expansion and complex shear scalars, respectively. The C_{abcd} is Weyl's tensor, R_{ab} is the Ricci tensor, and t^a is a vector that is orthogonal to k^a, null, and has a magnitude of 1 (i.e. $t^ak_a=0$, $t^at_a=0$ and $t^a(t^*)_a=1$). The initial conditions for integrating these equations are $\sigma|_o=0$, $r_A|_o=0$ and $dr_A/d\lambda|_o=$ constant.

Once the expansion scalar is known, then the angular diameter distance is given by the integral

$$r_A\propto \exp\left\{\int_e^o \tilde{\theta}d\lambda\right\}, \qquad (14)$$

and the luminosity distance is given by Etherington's theorem [21] as $r_L=(1+z)^2r_A$. For the geometry specified by (1) we then find that the driving terms in (12) and (13) are given by

$$R_{ab}k^ak^b=0 \quad \text{and} \quad C=\frac{3mJ^2}{r^5}e^{i\Psi}, \qquad (15)$$

where Ψ is a constant, specifying the complex phase. These equations are independent of Ω_Λ, and so their solution is very similar to the $\Omega_\Lambda=0$ case (up to the effect Ω_Λ has on the trajectories $r=r(\lambda)$).

By numerically integrating (12) and (13) we find that the shear is typically unimportant at $z\lesssim 1$, unless a trajectory happens to pass very close to a central mass. At higher redshifts, however, the shear can accumulate and become more important, eventually causing the divergence of the optical scalars along some trajectories. This corresponds to caustics occurring in the beams.

For the present purposes, at $z\lesssim 1$, it is sufficient to neglect the shear, and set $\sigma=0$. The solution to Eqs. (12) and (14) is then given simply by $r_A\propto\lambda$, and so we find that $r_L\propto(1+z)^2\lambda$. We now need to find the relationship between λ and z. Using the expression for redshift, (10), together with the Friedmann equation, (3), we can write

$$\begin{aligned}1+z \;&=\; \frac{\dot{\tau}_e}{\dot{\tau}_o}\simeq\left(\frac{a_o}{a_e}\right)^{\langle\gamma\rangle} \qquad (16)\\ &=\; \frac{1}{a_o\dot{\tau}_o}\frac{da_e}{d\lambda}\frac{1}{\sqrt{\Omega_mH_o^2\frac{a_0}{a_e}+\Omega_\Lambda H_o^2\left(\frac{a_e}{a_o}\right)^2}},\end{aligned}$$

where the usual expressions for Ω_m and Ω_Λ have been used. In general, this equation needs to be integrated numerically to find a solution for $\lambda(z)$. However, we find that by treating $\langle\gamma\rangle$ as a quasi-static variable we can obtain analytic results that are accurate to better than 1%. In this case, the solution to Eq. (16) is

$$\lambda\simeq-\frac{2}{\dot{\tau}_oH_o(3+2\langle\gamma\rangle)\sqrt{\Omega_m}}\left[\frac{f(z)}{(1+z)^{\frac{3+2\langle\gamma\rangle}{2\langle\gamma\rangle}}}-f(0)\right],$$

where

$$f(z)=\,{}_2F_1\left(\frac{3+2\langle\gamma\rangle}{6},\frac{1}{2};\frac{9+2\langle\gamma\rangle}{6};-\frac{\Omega_\Lambda}{\Omega_m}(1+z)^{-\frac{3}{\langle\gamma\rangle}}\right),$$

and ${}_2F_1(a,b;c;d)$ is the hypergeometric function [25]. We then have that, in the absence of shear, the luminosity distance is well approximated as a function of z by

$$r_L\propto(1+z)^2\left[\frac{f(z)}{(1+z)^{\frac{3+2\langle\gamma\rangle}{2\langle\gamma\rangle}}}-f(0)\right]. \qquad (17)$$

We will now fit our model to the recent supernova observations, and obtain an estimate for the difference in the best fit value of Ω_Λ due to using a discretized matter content, rather than a perfect fluid. This will be done using the first-year Supernovae Legacy Survey (SNLS) data, consisting of 115 supernovae [22], calibrated with the Spectral Adaptive Light-curve Template (SALT) fitter [23]. Assuming a critical density, both FRW and our model have 5 free parameters: Ω_Λ and the 4 'nuisance' parameters required to calibrate the supernova data. These are the absolute magnitude, the intrinsic error and the color and stretch parameters used in the process of light curve fitting, $\{M_0,\sigma_{int},\alpha,\beta\}$.

We show the best fits for both our model, and a spatially flat continuous fluid FRW cosmology in Fig. 2. The fits are very similar with a difference in log likelihood of just $\Delta\ln\mathcal{L}=0.37$, in favor of the FRW model. In fact, it can be seen from the figure that the distance moduli of these two best fit models are virtually identical, with the two solid lines only actually becoming distinguishable by eye at $z\gtrsim 0.8$. It seems clear that a lot more data will be needed before there is any hope of distinguishing these two curves observationally.

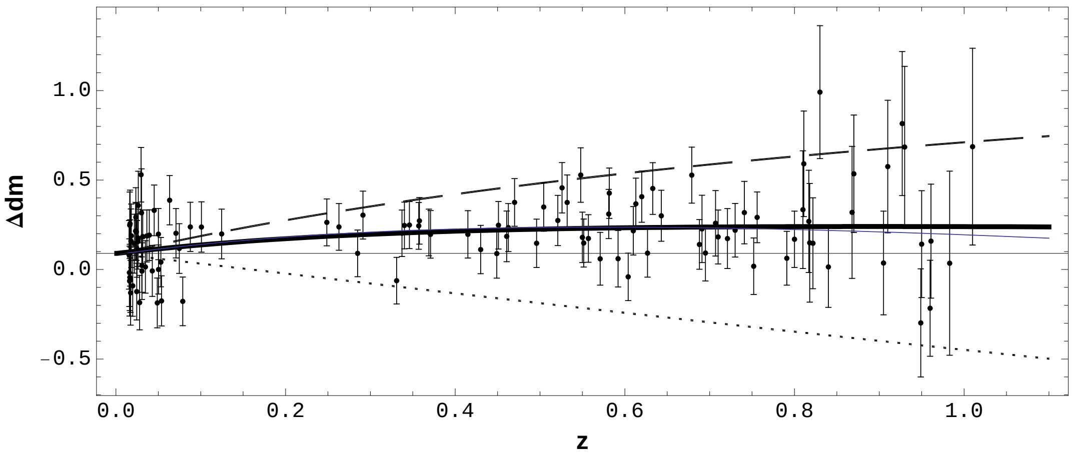

FIG. 2: The best fit critical density models with discrete matter (thick line) and continuous fluid (thin line) sources. The data is the 115 high and low redshift supernovae from SNLS [22], fitted to the discrete matter model with the SALT light curve fitter [23]. Einstein-de Sitter (dotted line) and de Sitter (dashed line) models are also shown, for reference. The distance modulus, Δdm, is the magnitude of the source, minus the magnitude it would have at the same z in an empty, open Milne universe.

The interesting point, and the main result of this paper, is that these two best fit models correspond to quite different values of Ω_Λ. For the more usual perfect fluid FRW cosmology we recover the standard result $\Omega_\Lambda^{FRW} = 0.74 \pm 0.04$. For the model with a matter content composed of discrete masses, however, we obtain $\Omega_\Lambda = 0.66 \pm 0.04$. The best fit values of the nuisance parameters are very similar in each case, as would be expected from the similarity of the two distance moduli.

One may initially react by observing that this difference in Ω_Λ is only at the level of 2σ, and as such is not very significant. Our point, however, is that irrespective of how good the data eventually becomes, there will still be a difference in Ω_Λ of $\sim 10\%$ between these two models. This difference is not due to any new physics, exotic matter content, or unexpectedly large structures of voids in the universe, but only to the fact that we have treated the matter content of the Universe in our model as being discrete, rather than continuous.

Inevitably the reader will be concerned with the generality of this result, and whether it should apply to the real Universe, or only to our simple model. We strongly suspect that the real Universe, with its complicated networks of voids, filaments, walls and nodes [24] will not behave exactly as our model does, and that there will be new behaviour beyond that which we have uncovered here. In future work we hope to make progress in generalizing our results to more realistic situations. However, we believe that there is good reason to expect some of the essential features of our study to hold in the more general case.

Firstly, the photons we observe on Earth have not reached us by travelling through a continuous fluid of critical density; they have travelled through mostly empty space. As such, they should not experience the focusing that such a fluid would produce. Photons experience the geometry of space-time through which they have passed, and *not* the global average. Secondly, the expansion that photons experience is also not the global average, it is the integrated effect of the expansion which is strongly anisotropic at any given point along their trajectory (as long as $\Omega_m > 0$). For further discussion of these points, and discussion on the appropriateness of using an approximate solution of Einstein's equations, we refer the reader to paper I.

Of course, one will also be concerned about systematic errors, and the validity of the assumptions that have gone into this model. In future publications we intend to establish the validity of these assumptions through detailed investigation, but for now we already good reason to suspect that the approximations that have been made are good. In paper I it was shown that the approximate nature of the boundary conditions between lattice cells is insensitive to the details of the matching conditions, lending credence to the idea of an average tangency between neighbouring space-like surfaces. Furthermore, we also obtained in paper I analytic approximations that are in good agreement with our numerical results. These allow some physical insight into the source of the effects we have uncovered, and further their legitimacy. Beyond this, the analogy with the highly successful Wigner-Seitz construction and the recovery of the familiar optics of de Sitter space in the appropriate limit is also encouraging.

In summary, we find that the fluid approximation in cosmology, while appearing innocuous, can introduce considerable errors in interpreting cosmological data. Using a simple model of the Universe, with discrete masses arranged on a regular lattice, we have shown that even if the average dynamics of the Universe are unchanged, the error introduced in the estimation of Ω_Λ due to different optical properties can be of the order of 10%. Such effects will need to be understood and accounted for if we are to attempt precision cosmology.

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