A meshless optimised mesh-smoothing framework

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Pavanakumar Mohanamuraly* and Jens D. Mueller†

School of Engineering and Material Science,

Queen Mary University of London,

E1 4NS, London, United Kingdom

In this work, we present a least-squares vertex-based mesh quality metric for optimised mesh smoothing, which is inspired from meshless methods. The proposed mesh quality metric requires only point cloud information and does not require mesh topology information. In addition, it is scale, rotational, and translation invariant making it suitable for highly stretched boundary layer type meshes. The proposed quality metric is implemented in the Mesquite mesh optimisation framework.[3] The mesh optimisation is combined with a robust mesh deformation algorithm based on inverse distance weighting[1] for aerodynamic shape optimisation problem. Mesh untangling can be performed using the same mesh optimisation framework by choosing an appropriate quality metric. We show that combining mesh deformation with optimized mesh smoothing can increase the number of design iterations without requiring re-meshing and allows more room for large shape deformation.

I. Introduction

Aerodynamic shape optimisation process typically involves deforming a volume mesh according to a prescribed surface deformation. Robust mesh deformation algorithms based on linear elasticity guarantee deformed meshes with non-negative volumes. But it requires solving a partial differential equation, which is expensive and can have problems with full convergence. The mesh deformation based on inverse distance weighting is a direct interpolation procedure involving simple matrix multiplication. It is found to be quite robust and preserves the quality of the mesh. Mesh deformation methods in general yield meshes of lower quality than the original undeformed mesh. This limits (i) the number of optimisation steps one can run without re-meshing and (ii) the quality of the numerical results. Optimised mesh-smoothing is typically done after the volume mesh deformation to alleviate these problems. Mesh deformation methods like spring analogy or inverse distance weighting can lead to tangled meshes. Mesh smoothing is typically employed to untangle the mesh. An inexpensive mesh smoothing method is the Laplace smoothing[2], which moves the free vertex to the geometric centre of its incident vertices. But the Laplace smoothing does not guarantee improvement in element quality. In reference [2], the authors alleviate this problem by selectively applying the Laplace smoothing to mesh nodes giving rise to the smart Laplacian method.

Guaranteed improvement in mesh quality is obtained using optimisation-based[2-3] approach to mesh smoothing. In this approach, a quality metric such as element angle, skewness, aspect-ratio, etc is chosen as an objective function. The mesh nodes are perturbed to achieve optimal values of the chosen objective function. In practice, the link between an element based objective function and mesh node is not immediate. One has to resort to non-differentiable functions like *min/max* to translate the element measure to the node, which can cause stalling of convergence. We seek a mesh quality metric directly based on nodes and independent of element information, which we present in the next section.

II. Meshless node-based mesh quality metric

Let $\mathbf{x}_i \in \mathcal{T}(\mathbb{R}^N)$ is a vertex in a triangulation $\mathcal{T}(\mathbb{R}^N)$ and $\mathbf{x}_j \in \mathcal{C}(\mathbf{x}_i)$ be the edge neighbours of vertex $\mathbf{x}_i$ connected by the edge vector $\mathbf{r}_{ij} = \mathbf{x}_j - \mathbf{x}_i$. Note that the indices run as $j = 1, ..., m$ and $i = 1, ..., n$. Where, m is the degree of the incident edges to vertex i and n is the total number of vertices in the

*Marie Curie Fellow, School of Engineering and Material Science, Queen Mary University of London

†Senior Lecturer, School of Engineering and Material Science, Queen Mary University of London

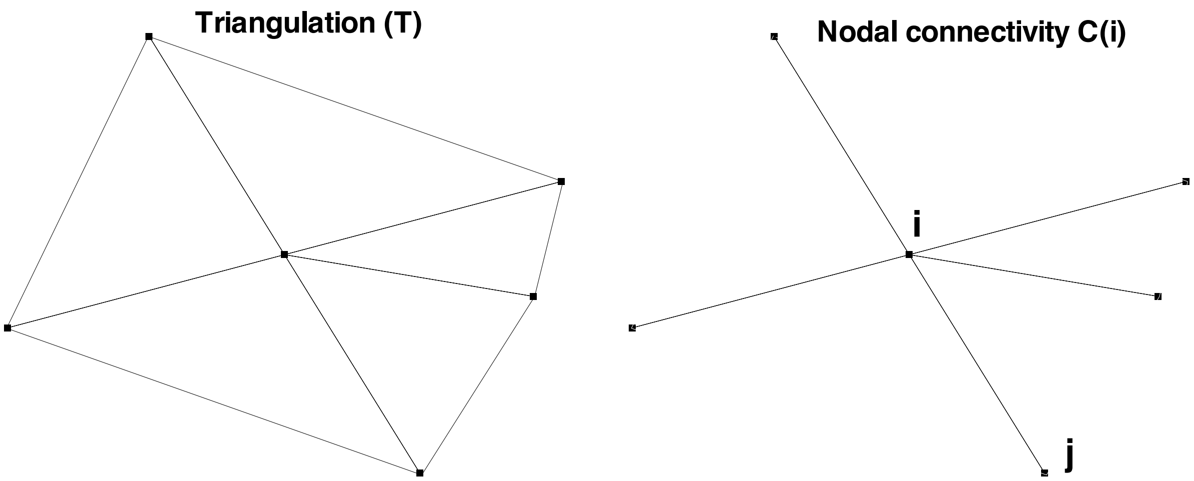

Figure 1. Mesh triangulation and nodal edge neighbour connectivity (solid line)

triangulation $\mathcal{T}(\mathbb{R}^N)$. Let us try to fit a first order polynomial over the neighbourhood of point i, which has the following form,

$$f - f_i = a(x - x_i) + b(y - y_i) + c(z - z_i) = \mathbf{X}\boldsymbol{a} \tag{1}$$

Only three unknowns are required to exactly determine the coefficients $\boldsymbol{\xi}$. But the number of neighbours of the point i are chosen such that, they are more than three, leading to an over-determined system.

$$\mathbf{X}\boldsymbol{a} = \boldsymbol{\Delta}\mathbf{f} \tag{2}$$

Note that we use the notation $\mathcal{C}(i) : \mathbf{x}_j \in \{\mathbf{x}_{j_1}, \mathbf{x}_{j_2}, ..., \mathbf{x}_{j_m}\}$ and $\mathbf{x} = \{x, y, z\}$ in $\mathbb{R}^3$. If we introduce a weighting function $\mathbf{W} = diag\,[w_1 \quad w_2 \quad ... \quad w_m]$ into the equation and convert to the normal form,

$$\boldsymbol{a} = \left(\mathbf{X}^T\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{W}\boldsymbol{\Delta}\mathbf{f} = \mathbf{M}\boldsymbol{\Delta}\mathbf{f} \tag{3}$$

$$\mathbf{M} = \begin{bmatrix} a_1 & a_2 & ... & a_M \\ b_1 & b_2 & ... & b_M \\ c_1 & c_2 & ... & c_M \end{bmatrix} \tag{4}$$

The matrix $\mathbf{M}$ has a special geometric interpretation. It's column vectors point along the direction of the edges forming the node stencil. For a bad-connectivity they start deviating from the edge vector. Praveen[4] observed that the column vectors of $\mathbf{M}$ deviate more from the actual edge vector, when the edges subtended are highly skewed or have small angles. Ideally a good stencil should have both the column vectors of $\mathbf{M}$ and edge vectors parallel to each other. Any deviation from this parallelism signifies a degradation of mesh quality. We define a mesh metric $\delta_p = \sum_j \delta_{p_j}$ to measure the mesh quality at a node, where δ_{p_j} is given in equation 5. For an ideal mesh distribution around a node $\delta_p = 0$ or a positive quantity $(\delta_p > 0)$ for any deviation from the ideal.

$$\delta_{p_j} = |\mathbf{a_j}| - \mathbf{a_j} \cdot \frac{(\mathbf{x_j} - \mathbf{x_i})}{|\mathbf{x_j} - \mathbf{x_i}|} \tag{5}$$

The proposed metric is implemented in Mesquite framework, which comes packaged with a variety of optimisation algorithms. In this work we present results from the steepest descent algorithm.

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