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A novel consolidation-based representative volume element for granular materials and its application for the characterization of the mechanical response of sand during impact loading

F.De Cola[a], A.Pellegrino[a]*, E.Barbieri[b],D.Penumadu[c] and N.Petrinic[a]

[a]Department of Engineering Science, University of Oxford, Oxford OX1 3PJ, United Kingdom
[b]School of Engineering and Material Science, Queen Mary University of London, E1 4NS, London, United Kingdom
[b]Fred Peebles Professor and JIAM Chair of Excellence, CEE Department, University of Tennessee, Knoxville, TN 37936, USA

Abstract

A new procedure for the determination of the smallest Representative Volume Element (RVE) of granular media is proposed in the present investigation. The procedure is based on the simulation of consolidated granular assemblies using the Discrete Element Method (DEM). The existence of a lower limit for the dimensions of specimens used in the high-strain rate experiments on granular materials as a function of their consolidation state is demonstrated. The repeatability of the experimental results presented demonstrates the validity of the proposed method for the determination of the RVE. The results obtained show clearly the influence of chemical/physical composition, grain shape, initial consolidation state and type of confinement on the measured mechanical response.

Peer-review under responsibility of the Scientific Committee of ECF21.

Keywords: Sand; High Strain Rate; Dynamics, Computational Mechanics

* Corresponding author. Tel.: +441865-613452;
E-mail address: antonio.pellegrino@eng.ox.ac.uk

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1. Introduction

The capability of predicting the effects of impact and penetration of external objects on granular materials and of modeling the mechanical behaviour of sand depends predominantly on the ability to determine the stress–strain characteristics of granular media under consideration at high strain rates. The evaluation of the strain rate dependent mechanical response of a wide range of natural occurring and advanced material relies largely on Split Hopkinson Pressure Bar (SHPB) experiments.

Granular materials are highly heterogeneous and their mechanical response is strongly affected by several factors such as: initial consolidation state, particles morphology, moisture content, type of confinement etc. (Georgiannou et al., (1990), Leroueil et al, (1990), Omidvar et al., (2012)). For that reason, the interpretation of the mechanical behaviour of this class of materials needs careful judgement in order to provide data for calibration and validation of large scale numerical simulations. However, particulate assemblies exhibit a substantially homogeneous behaviour at certain large scales. At such macroscopic scales, it is possible to use continuum mechanics approaches for both experimental determination of mechanical properties and modeling purposes. This consideration poses the need of defining the smallest Representative Volume Element for the evaluation of the rate dependent behaviour of sand. Measurements conducted on samples of size equal or larger than the RVE produce consistent and repeatable results, representative of the bulk material. On the contrary, evaluations carried out on granular samples of volume smaller than the RVE fluctuate, leading to uncertainty in the experimental results. The determination of the RVE of granular materials is generally based on physical and geometrical properties such as particle size, density, porosity and void ratio (Graham, S. & Yang, N. (2003)) or on mechanical properties such as elastic and shear modulus (Evesque, P. & Adjemian, F. (2002), Ren, Z. Y. & Zheng, Q. S. (2002), Stroeven, M. et al. (2004)).

The results of studies presented in this paper address the need to define the relationship between the RVE and the sample size used in uniaxial compression experiments at elevated strain rate, in order to achieve dynamic equilibrium conditions and reduce the experimental scatter due to discontinuities in the material. It is common practice in experimental mechanics to define the dimensions of samples using trial and error approaches or, in case of SHPB experiments, reducing the aspect ratio L/D of specimens to very low values for the achievement of higher strain rates and better dynamic equilibrium conditions. However, it is shown here that it is not appropriate to reduce the dimensions of the sample below certain size arbitrarily. The approach presented herein proves the existence of a lower limit for the length of samples used in uniaxial compression dynamic experiments. This limit is posed by the measure of the RVE, based on the initial void ratio 'e' of the sand under investigation:

$$e = \frac{V_v}{V_s} \tag{1}$$

where V_v is the volume of voids and V_s the volume of solids within the samples. This parameter is a measure of the compaction within the sample and it is related to the porosity 'n' by the following:

$$n = \frac{V_v}{V_{tot}} \tag{2}$$

$$e = \frac{n}{1-n} \tag{3}$$

Sand assemblies characterized by different void ratios yield discrepant mechanical responses. The adoption of a RVE, based on this parameter, allows for the reliable characterization of the rate dependent behaviour of particulate materials. Using this concept, experimental results are generated, demonstrating the importance of hereby proposed methodology.

The effect of the grain shape on the dynamic compressive mechanical response of sand was assessed by conducting experiments on several types of sand characterized by grains of different morphology, physical and chemical composition. Quartz sand assemblies composed of quasi-spherical grains, sub-angular and polyhedral grains have been investigated. Furthermore, series of experiments were carried out at high strain rate in order to characterize the response of amorphous Etnean volcanic ashes collected from the South East Flank of the Volcano during the paroxysm of December 2014. Two types of confinements were designed in order to characterize the granular assemblies under uniaxial strain and uniaxial stress loading conditions. A stiff titanium alloy cylindrical confinement was used to reproduce uniaxial strain loading conditions whilst a compliant latex one was used to reproduce uniaxial stress conditions. Sand samples of mass correspondent to the representative volume element were defined by means of a high precision weighing scale and subsequently compacted using a specimen filling procedure established to obtain experimentally fixed and reproducible void ratios.

Nomenclature

e	void ratio
n	porosity
V_v	volume of voids
V_s	volume of solids
V_{tot}	total volume available
d	dimension of the fictitious volume
D	diameter of the real container
L	length of the sample in the loading direction
n_p	number of particles in the assembly
$\varepsilon_i(t)$	history of the incident strain signal
$\varepsilon_r(t)$	history of the reflected strain signal
$\varepsilon_t(t)$	history of the transmitted strain signal
A_0, A_s	cross sectional area of bars and specimen
E_0, C_0	Young's modulus and elastic wave speed of the bars radius of

2. Computation of the void ratio within the sample

The definition of the RVE takes into account the variation of relative density within the granular assembly. A series of three dimensional simulations were conducted using an in-house developed software (DEST – Discrete Elements Simulation Tools) in order to simulate the statistically representative distributions of sand grains which were expected to be generated during specimen preparation and predict their consolidation states within the samples volume. The boundary conditions imposed by the lateral walls of the confinement influence the distribution of grains in the sand assembly. Their distribution appears ordered in proximity of the walls whilst particles located far from the walls are disposed in a casual manner. Consequently particles within a distance of 2-4 equivalent particle diameters have shown not to be statistically representative of the consolidation of the granular media (Blumenfeld et al. (2005), Landry et al. (2003), Man et al. (2005)). The hereby proposed numerical procedure allows for the prediction of the relative packing density obtained during diverse deposition processes and is applicable to different granular materials.

Rewriting equation (1) it is possible to express the void ratio as a function of the total volume V_{tot} of a generic container and the volume V_s of the particles, or fraction of particles, enclosed in it.

$$e = \frac{V_v}{V_s} = \frac{V_{tot} - V_s}{V_s} = \frac{V_{tot}}{V_s} - 1 \tag{4}$$

The proposed numerical procedure computes the volume V_{tot} of a sequence of virtual boxes of dimension (d) smaller than the dimension (D) of the real sample as a whole (Fig 1) and the volume V_s of solids enclosed in them. The volume of fraction of particles is taken into account and it is determined using different geometry relations dependent on the relative position between particles and boundary (De Cola et al., (2016)). By incrementing simultaneously the dimensions of these virtual volumes starting from the geometrical center of the sample, it is possible to define how the void ratio varies across the specimen.

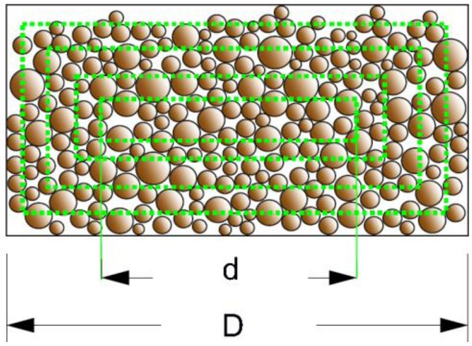

Fig. 1. Schematic representation of the variation of all the dimensions of the fictitious volume

A typical chart showing the variation of the computed void ratio as a function of the dimension "*d*" of the aforementioned virtual volumes is drawn in Fig. 2. The presence of lateral walls and of the free surface affects the distribution of the void ratio within each sample. The variation of the void ratio within each granular assembly was assessed by calculating the volume of voids and solids inside fictitious cylinders of progressively increasing dimensions "*d*".

Very small cylinders yield values of void ratio approaching zero because their dimension is smaller (or comparable) to the dimension of the single particle ($d/D \rightarrow 0$ in Fig. 2). Conversely, large cylinders yield to larger void ratios because the calculations are affected by both the free surface and cylindrical wall boundaries ($d/D \rightarrow 1$ in Fig. 2). Void ratios calculated using cylinders of dimension *d/D* comprised between 0.2 and 0.8 are representative of the average compaction state within the granular assembly (Fig. 2), for specific sands and specimen diameters. This is simply because, in this range, as shown, the relevant calculations are not affected by the boundary conditions.

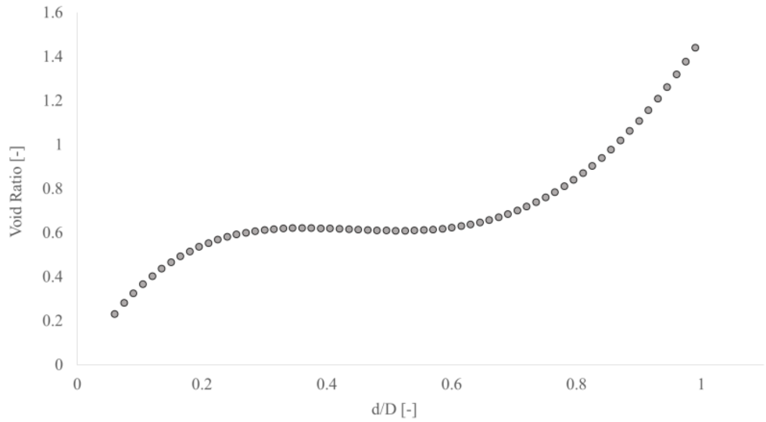

Fig. 2. Qualitative distribution of the void ratio within granular material, varying the dimension (*d*) of the imaginary volume with respect to the real container (*D*)

3. Method for the estimation of the smallest RVE for uniaxial compression experiments at high strain rates

This section illustrates the method for the evaluation of the smallest RVE for granular materials with respect to the characterization of their mechanical response during uniaxial compression experiments at high strain rates. The method is based on numerically evaluating the convergence of the void ratio in the portion of the sample not affected by boundary effects. It is summarized by the flow chart in Fig. 3 and discussed in detail below.

The process starts with the definition of the diameter of the sample, equal to the diameter of the SHPB bars (*D* = 20 mm in our case), determined on the base of considerations on mechanical impedance, inertia of the samples and expected signal to noise ratio (ASM, (2000)) (Box A of the flow chart).

Once the diameter is fixed, only the thickness of the sample along the direction of loading (*L*) needs to be defined. The first iteration of the process starts choosing an exploratory number of particles (n_p). A number $n = 3$ of DEM simulations is carried out. Every simulation with equal n_p corresponds to a different arrangement of particles, in analogy with repeated samples preparation in experiments (Box B in the flow chart). The particles arrangement within the sample, its length *L* and its slenderness (*L/D*) are defined.

The results of the simulations are then analyzed to evaluate the distribution of the void ratio within the samples (Box C in the flow chart). The variation of the void ratio is clearly affected by the boundary conditions of lateral walls and free surface. The analysis is carried out calculating the volume of voids and solids inside virtual cylinders of progressively increasing dimensions *d*.

The ratio *d/D* is determined for every numerically generated sample. It is worth emphasizing that these charts are used exclusively for convergence study (Box *D* in the flow chart). In case the graphs obtained for the *n* samples show repeatability and the plateau area is wide and horizontal (Fig. 4 (b)) then the dimension *L* averaged from the *n* simulations determines the thickness of the specimen corresponding to the RVE. On the contrary, if the charts show significant scatter or the plateau region is inclined (Fig. 4 (a)), the number of particles n_p, and consequently the specimen length *L*, has to be increased.

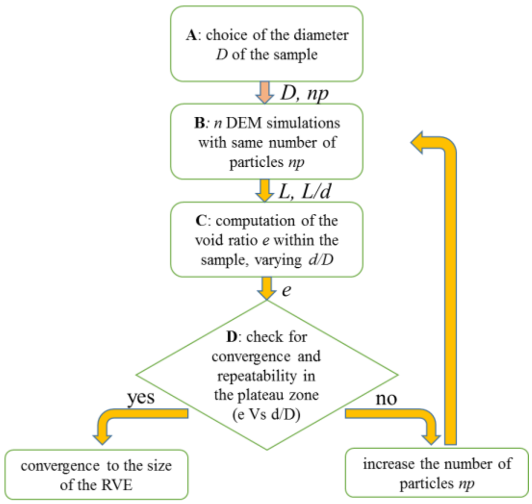

Fig. 3. Iterative algorithm for the calculation of the RVE

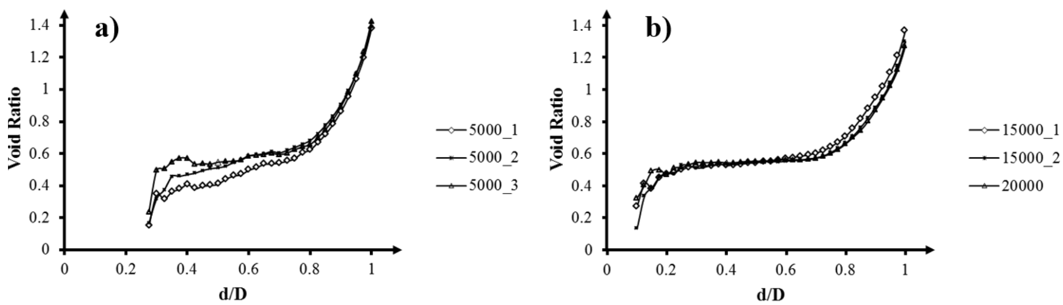

Fig. 4. a) Void ratio within three different numerical samples containing 5,000 particles; b) void ratio within 15,000 particles and 20,000 particles virtual samples (De Cola et al. (2016))

4. Experimental results and discussion

The mechanical response at high strain rate of three types of sand, characterized by different morphology, was assessed conducting a series of SHPB experiments on samples with dimensions defined following the previous numerical procedure. The Split Hopkinson Pressure Bar apparatus utilized for the experiment was comprised of two instrumented titanium bars of length and diameter equal to 2.7 m and 20 mm respectively and by a low pressure compressed air system. During the experiments, the striker, accelerated by the motion of a piston driven by compressed air, impacts the incident bar, generating an elastic stress wave of duration and amplitude dependent on length and velocity of the striker respectively. The stress pulse produced propagates along the incident bar until reaching the sample, interposed between the two instrumented bars. The mechanical impedance mismatch between the sample and the bars causes the incident pulse to be partially reflected back to the input bar. The remaining portion of the stress pulse is transmitted through the output bar.

The one dimensional analysis of the recorded strain gauges signals allows for the computation of strain rate, strain and stress histories in the specimen as follows (Kolsky, (1949)):

$$\dot{\varepsilon}(t) = \frac{C_0}{l}[\varepsilon_i(t) - \varepsilon_r(t) - \varepsilon_t(t)] \quad (5)$$

$$\varepsilon = \int_0^T \dot{\varepsilon}(t)dt = \frac{C_0}{L_s}\int_0^T [\varepsilon_i(t) - \varepsilon_r(t) - \varepsilon_t(t)]dt \quad (6)$$

$$\sigma(t) = \frac{A_0}{2A_s}E_0[\varepsilon_i(t) + \varepsilon_r(t) + \varepsilon_t(t)] \quad (7)$$

Where l is the length of the sample, $\varepsilon_i(t)$, $\varepsilon_r(t)$ and $\varepsilon_t(t)$ are the incident, reflected and transmitted strain histories respectively, A_0 and A_s are the cross section areas of bars and specimen, while E_0 and c_0 are the Young's modulus and the elastic wave propagation velocity of the material of the loading bars.

Three types of sand investigated were crystalline silica sands and one was amorphous Etnean volcanic sand collected right after the paroxysm of December 2014. Specifically, the three silica sands were: the Euroquartz Siligran – trocken 0.125-0.71 mm characterized by sub-rounded grains, Schlingmeier Quartzsand S0.4-S0.8 characterized by sub-angular grains, and the Q-Rok sand characterized by polyhedral grains. Etnean volcanic sand was, instead, characterized by angular porous grains and by the presence of numerous glassy vesicles.

Dense samples were prepared following an unambiguous specific procedure. The prescribed amount of sand, corresponding to the RVE determined numerically using the computational procedure detailed above, was quantified using precision weighing scales. This quantity of sand was divided in three equal parts and the following procedure was reiterated for each of the portions. Initially the sand was deposited into the container using a funnel. After that, the container was patted five times in four antipodal points. Then, the sand was further compressed dropping a weight of fixed mass (340 g) from a precise height for 10 times until the final required dimension of the specimen was obtained.

The mechanical response at high rates of strain of the dense dry samples of Q-Rok, Euroquartz Siligran - trocken 0.125-0.71 mm, Schlingmeier Quartzsand S0.4-S0.8 and Etnean sand enclosed on rigid confinements is compared in Fig. 5. The strain rates achieved were in the range of 1.5 10^3 s^{-1} for all tested samples. The dimensions of the samples were determined according to the procedure for the determination of the RVE described in section 3. Specimen size, mass and void ratios are reported in Table 1.

Table 1. Summary of specimen dimensions and corresponding void ratios for dense samples

sand	*consolidation state*	*mass (g)*	*specimen diameter (mm)*	*specimen length (mm)*	*void ratio*
Q-Rok	dense	3	20	6	0.69
Schlingmeier	dense	3.2	20	6	0.56
Euroquartz	dense	3.2	20	6	0.56
Etnean	dense	2.9	20	6.8	1.11

The repeatability of the results shown in Fig. 5 demonstrates the validity of the proposed method. The mechanical response of granular materials is very sensitive to the initial conditions within the samples. Specimens composed of the same material but characterized by a not uniform consolidation state would yield inconsistent mechanical responses. Hence the low experimental scatter obtained for each type of sand provides evidence of the uniformity of the void ratio within each sample and between sand assemblies of the same type.

The response of each type of sand at the dense state is characterized by the typical approximately poly-linear diagram (Hagerty et al., (1993)). At small stresses, the shear forces acting between grains do not exceed the static friction. The particles deform without any macroscopic sliding generating an apparent elastic portion in the stress strain curve. Higher loads cause the particles to roll and slide, rearranging themselves to produce a denser state. The transition between these two phases of the stress versus strain curve takes place at strains in the region of 1%, 2-3% and 7-10% for Schlingmeier, Q Rok and Etnean sand respectively. Successively the relative movement between the grains becomes more and more difficult until lock-up conditions occur. These conditions arise at strains of order 10%, 17% and 35% for Schlingmeier, Q Rok and Etnean sand. Additional compression causes the crushing of particles with further reduction of the void ratio and a supplementary increment of stiffness.

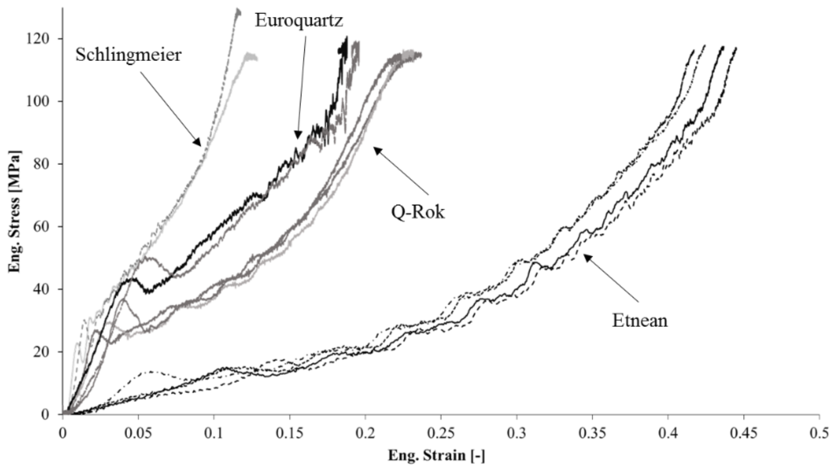

Fig. 5. Comparison of the mechanical behaviour at high strain rate of dry dense Schlingmeier quartzsand, Euroquartz, Q-Rok and Etnean sand assemblies enclosed in rigid confinements.

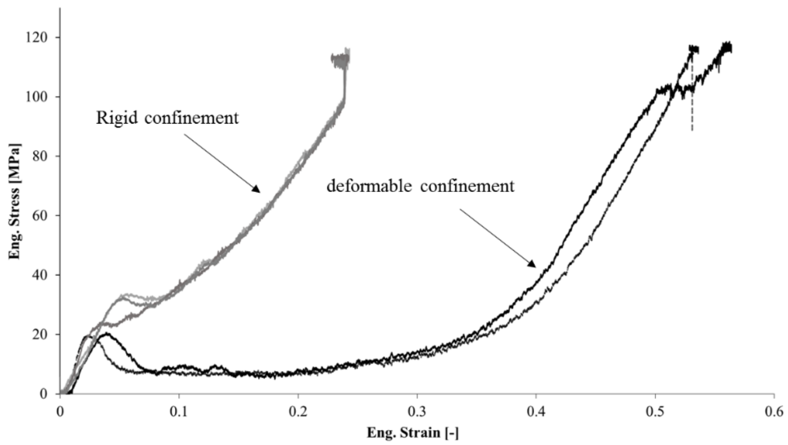

Fig. 6. Comparison of the mechanical behaviour at high strain rate of dry loose Euroquartz sand enclosed in rigid and deformable (latex) confinements.

Fig. 6 illustrates the mechanical behaviour of dry loose Euroquartz sand at high strain rate. The characteristics of this sand obtained when using metallic and latex confinements are substantially identical up to strains in the area of 1.5-2%. Higher loads cause the latex confinement to expand radially with the response becoming a plateau up to nominal strains of about 30%. Further compression causes the particles to comminute causing the response to become steeper. Also in this case the repeatability of the results shown corroborates the efficacy of the proposed method.

5. Conclusions

A numerical procedure for the determination of the smallest Representative Volume Element for experimental characterization of the mechanical response of granular materials to uniaxial compression at high strain rates was developed. The procedure, based on the initial void ratio, was applied to determine the dimensions of specimens to be employed on the Split Hopkinson Bar apparatus. The mechanical behaviour at high strain rate of a number of dry sands characterized by different grain morphologies was assessed and compared. The reproducibility of the experimental results obtained demonstrates the effectiveness of the proposed method in both quasi uniaxial strain and quasi uniaxial stress loading conditions. Forthcoming research comprises the development of an analogous procedure for the mechanical characterization of wet sand subjected to impact loading.

6. Acknowledgements

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