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Exploring General Gauge Mediation

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We explore various aspects of General Gauge Mediation (GGM). We present a reformulation of the correlation functions used in GGM, and further elucidate their IR and UV properties. Additionally we clarify the issue of UV sensitivity in the calculation of the soft masses in the MSSM, highlighting the role of the supertrace over the messenger spectrum. Finally, we present weakly coupled messenger models which fully cover the parameter space of GGM. These examples demonstrate that the full parameter space of GGM is physical and realizable. Thus it should be considered a valid basis for future phenomenological explorations of gauge mediation.

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1. Introduction

Low-energy supersymmetry, in its minimal incarnation as the MSSM, is probably the most attractive candidate for physics beyond the Standard Model, since it solves the hierarchy problem and predicts gauge coupling unification. However, the MSSM has one major drawback, namely, its immense parameter space. Soft SUSY-breaking introduces $\mathcal{O}(100)$ new parameters compared to the SM. These parameters are highly constrained by stringent experimental limits on flavor-changing neutral currents and CP violation. A conservative ansatz for the parameter space which is automatically consistent with flavor and CP is known as "soft SUSY-breaking universality" (see [1] for a nice review). Here there are five flavor-diagonal sfermion masses, three real gaugino masses, three flavor-diagonal A-terms, and three independent real Higgs mass parameters, for a total of 14 real parameters in all. If one accepts the hypothesis of universality, then the theoretical challenge is to construct models of SUSY-breaking and mediation that automatically produce universal patterns of soft parameters without fine tuning.

Gauge mediation [2-12], or the idea that SUSY-breaking is communicated to the MSSM via the SM gauge interactions, is a promising partial solution to this challenge.[1] Since the gauge interactions are flavor blind, the soft masses obtained through gauge mediation are automatically flavor universal. However, the absence of CP phases is less automatic in gauge mediation. Also, the Higgs μ and B_μ parameters are not generated in pure gauge mediation, so one typically assumes that additional interactions are present to produce these (for a recent discussion of this see [14]).

Recently in [15], gauge mediation was given a general, model-independent definition: *in the limit that the MSSM gauge couplings $\alpha_i \to 0$, the theory decouples into the MSSM and a separate hidden sector that breaks SUSY*. It follows then that the SM gauge group must be part of a weakly-gauged global symmetry G of the hidden sector. By studying a small set of current-current correlators of G, it was shown that all the dependence of the soft masses on the hidden sector could be encapsulated by three real parameters that determine the sfermion masses, and three complex parameters that determine the gaugino masses. This framework was called "General Gauge Mediation" (GGM) in [15]; for more recent work on GGM, see [16-21]. In this paper we will further develop several aspects of GGM and explore its properties and its parameter space.

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[1] For a review of gauge mediation from both the model building and phenomenological point of view see [13].

The definition of GGM must be augmented with several phenomenological and consistency requirements, which we will now review. First, the fact that the gaugino masses are complex in general gauge mediation (GGM) implies that GGM does not solve the SUSY CP problem. So additional mechanisms (such as an R-symmetry as in [22], or having the hidden sector be CP invariant) must be invoked to explain why the gaugino masses are real.[2] For the rest of the paper, wherever it is relevant, we will *assume* that such a mechanism is at work and only consider CP invariant theories, so that the parameter space of GGM spans $\mathbb{R}^6$. With this assumption, the GGM parameter space comprises a much smaller, but still sizeable subspace of the full "universal" soft mass ansatz.

Additionally, as in [15], we will impose a $\mathbb{Z}_2$ symmetry, called "messenger parity," on our hidden sector. In the context of messengers this is typically defined as an interchange symmetry of the messengers combined with $V \to -V$ [6,23]. More generally, messenger parity can be defined in terms of the gauge current and its supersymmetric partners, without explicit reference to messengers [15]. This symmetry does not have to be imposed, but it is typically a phenomenological necessity: messenger parity prevents dangerous hypercharge D-terms (which could lead to tachyonic sleptons) from being generated in the hidden sector.

Messenger parity has various other consequences, including one on the sum rules of GGM. The fact that the five flavor-diagonal sfermion masses $(m_Q^2, m_U^2, m_D^2, m_L^2, m_E^2)$ are determined in terms of three real numbers implies that they must satisfy two sum rules [15]:

$$\begin{aligned}
\operatorname{Tr} Y m^2 &\propto m_Q^2 - 2m_U^2 + m_D^2 - m_L^2 + m_E^2 = 0 \\
\operatorname{Tr} (B - L)m^2 &\propto 2m_Q^2 - m_U^2 - m_D^2 - 2m_L^2 + m_E^2 = 0.
\end{aligned} \tag{1.1}$$

These sum rules are valid at the characteristic scale M of the gauge mediated model, and they are preserved by the (one-loop) running of the soft masses in the MSSM. There could in principle be violations to these sum rules arising at higher order in the SM gauge couplings, coming from 3-point functions in the hidden sector. We will show in section 2 that in fact these threshold contributions satisfy the sum rules if one imposes messenger parity on the hidden sector. Additionally, the leading log contributions at all higher orders also satisfy the sum rules. Therefore there are no contributions at any relevant order in the hidden sector which would violate the sum rules and they truly are predictions of GGM.

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[2] Of course, one can have non-zero phases in this framework as long as they are consistent with the experimental bounds. For convenience though, we will only concentrate on CP invariant hidden sectors.

In [15], it was shown that the GGM parameter space is the most general that can be populated by models of gauge mediation. However, this left open the important question of whether models existed that could actually span this space. For instance there may have been additional relations or inequalities satisfied by the parameters that were not manifest from the analysis of the current-current correlators. Or it could have been that for some regions of the GGM parameter space there was simply no field theory that could populate it. Indeed, a quick survey of existing models of gauge mediation (e.g. the original models of "minimal gauge mediation"[10,11]) would suggest that this could be the case, as these models clearly do not cover the parameter space. These models are based on a set of weakly coupled "messengers," chiral superfields, Φ^i, that transform under a real representation of the SM gauge group and couple to a field that has a SUSY breaking F-component. This can be expressed as having a generic supersymmetric mass term for the messengers

$$W = M_{ij}\Phi^i\Phi^j \tag{1.2}$$

and a SUSY-breaking mass term of the form

$$V \supset f_{ij}\phi^i\phi^j + c.c. \tag{1.3}$$

In [16] it was shown that in the context of such models, the right number (6) of parameters in GGM could be realized. However, in their models the full space of GGM was not actually spanned.

In this paper we further explore the model building possibilities in the context of weakly coupled messengers and show that there *are* models that span the GGM parameter space. This is because there can be additional contributions to the MSSM soft masses from gauge mediation in addition to those of the form (1.3), namely "diagonal-type"[24,25] messenger masses of the form

$$V \supset \xi_{ij}\phi^i\phi^{\dagger j} \tag{1.4}$$

Such terms typically arise from D-term breaking, but they can also arise from strong hidden sector dynamics (such as in [26]) where the distinction between F-term and D-term breaking is not obvious.

Using both (1.3) and (1.4), we demonstrate that there exist weakly coupled messenger models which span the space of GGM. Thus there can be no additional relations for the soft SUSY breaking parameters beyond (1.1).

The outline of the paper is as follows. First, in section 2 we present a reformulation of GGM that does not rely upon superspace and that leads to extremely compact formulas for the gaugino and sfermion soft masses. Using this formalism we will demonstrate both the UV and IR finiteness of the soft masses in GGM. We will then discuss in section 3 the dependence on the various mass scales that can enter the correlation functions. We will further elaborate on the issues of UV sensitivity for SUSY breaking parameters, clearing up some confusion in the existing literature regarding the interpretation of a nonzero messenger supertrace. Finally, in section 4 we present a simple explicit model involving weakly-coupled messengers that spans the entire six-dimensional parameter space of GGM. This model should be viewed merely as an "existence proof" that the entire GGM parameter space can be realized and that there are no additional hidden relations between the parameters that are not obvious from the general formulation. In light of this we believe that future phenomenological studies of gauge mediation should not restrict themselves to the parameterization of minimal gauge mediation (for example see [27]), but instead should explore the entire parameter space of GGM. This should in principle open up new avenues for possible experimental/phenomenological studies that have not yet been explored (for recent work in this direction, see [21]). We finish by collecting a few technical results in two appendices. In Appendix A we will review the role of the supertrace in models with messenger fields. We demonstrate that certain classes of models always generate a particular sign for the supertrace in an effective field theory. In appendix B we collect some general results for the correlation functions of models with arbitrary numbers of messengers.

2. General Gauge Mediation: A New and Improved Formulation

2.1. Review and reformulation

In this section we wish to review the basic features of GGM. Along the way, we will reformulate and streamline various aspects of it. This will lead to various new physical insights, including a direct proof of the finiteness of the sfermion soft masses in GGM.

To begin, let us describe the setup. Consider a renormalizable hidden sector[3] which is characterized by the scale M and where supersymmetry is broken spontaneously. Suppose that this hidden sector has a global symmetry group $G \supset G_{SM} = SU(3) \times SU(2) \times U(1)$

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[3] We will consider non-UV-complete scenarios in later sections.

that is weakly gauged. Suppose further that the only coupling to the visible sector occurs through the SM gauge interactions (so the hidden and visible sectors decouple in the $g_{SM} \to 0$ limit). We will refer to this setup as general gauge mediation, and we are interested in the visible-sector soft masses that arise. As shown in [15], all of the information in the soft masses is encoded in two-point functions of the current superfield of the symmetry group G.

To avoid writing all the gauge theory factors, we will assume for simplicity that $G = U(1)$ in this subsection. Recall now the definition of of the current superfield $\mathcal{J}$

$$D^2 \mathcal{J} = 0 \tag{2.1}$$

which leads in components to

$$\mathcal{J} = J + i\theta j - i\overline{\theta}\overline{j} - \theta\sigma^\mu\overline{\theta}j_\mu + \frac{1}{2}\theta^2\overline{\theta}\overline{\sigma}^\mu\partial_\mu j - \frac{1}{2}\overline{\theta}^2\theta\sigma^\mu\partial_\mu\overline{j} - \frac{1}{4}\theta^2\overline{\theta}^2\Box J \tag{2.2}$$

with $\partial^\mu j_\mu = 0$.

The use of superspace is not essential. Without it, we can replace the definition of the current superfield $\mathcal{J}$ (2.1) as follows. We study the hermitian operator J which satisfies

$$\{Q_\alpha, [Q_\beta, J]\} = 0 \tag{2.3}$$

where Q_α are the supercharges, which satisfy the SUSY algebra

$$\{Q_\alpha, \overline{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu. \tag{2.4}$$

Then, we can define

$$\begin{aligned}
j_\alpha &\equiv -i[Q_\alpha, J] \\
j_{\dot\alpha} &\equiv i[\overline{Q}_{\dot\alpha}, J] \\
j_\mu &\equiv -\frac{1}{4}\overline{\sigma}_\mu^{\dot\alpha\alpha}\left(\{\overline{Q}_{\dot\alpha}, [Q_\alpha, J]\} - \{Q_\alpha, [\overline{Q}_{\dot\alpha}, J]\}\right),
\end{aligned} \tag{2.5}$$

and derive the current conservation by applying two supercharges to this definition of j_μ and using the SUSY algebra (2.4).

The relation between the original presentation in superspace with (2.1) and this one is similar to the relation between the definition of chiral superfields in terms of $\overline{D}\Phi = 0$

and the definition of chiral operators (the first component of Φ) as $[\overline{Q}, \phi] = 0$.[4] As we will now show, (2.3) proves to be extremely useful when computing current-current correlation functions.

The correlators of interest are the nonzero current-current two-point functions

$$
\begin{aligned}
\langle J(x)J(0)\rangle &= \frac{1}{x^4}C_0(x^2M^2) \\
\langle j_\alpha(x)\overline{j}_{\dot\alpha}(0)\rangle &= -i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\left(\frac{1}{x^4}C_{1/2}(x^2M^2)\right) \\
\langle j_\mu(x)j_\nu(0)\rangle &= (\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\left(\frac{1}{x^4}C_1(x^2M^2)\right) \\
\langle j_\alpha(x)j_\beta(0)\rangle &= \epsilon_{\alpha\beta}\frac{1}{x^5}B(x^2M^2)
\end{aligned}
\tag{2.6}
$$

or in momentum space,

$$
\begin{aligned}
\langle J(p)J(-p)\rangle &= \widetilde{C}_0(p^2/M^2) \\
\langle j_\alpha(p)\overline{j}_{\dot\alpha}(-p)\rangle &= -\sigma^\mu_{\alpha\dot\alpha}p_\mu\widetilde{C}_{1/2}(p^2/M^2) \\
\langle j_\mu(p)j_\nu(-p)\rangle &= -(p^2\eta_{\mu\nu} - p_\mu p_\nu)\widetilde{C}_1(p^2/M^2) \\
\langle j_\alpha(p)j_\beta(-p)\rangle &= \epsilon_{\alpha\beta}M\widetilde{B}(p^2/M^2)
\end{aligned}
\tag{2.7}
$$

where now a factor of $(2\pi)^4\delta^{(4)}(0)$ is understood.

These two-point functions encode the mediation of SUSY breaking to the MSSM gaugino and sfermion soft-masses at leading order in the gauge coupling g. Specifically, the gaugino masses are given by

$$
M_{gaugino} = g^2 M\widetilde{B}(0).
\tag{2.8}
$$

while the sfermion soft mass-squareds are given by

$$
m^2_{sfermion} = g^4Y^2A
\tag{2.9}
$$

where Y is the $U(1)$ charge of the sfermion and A is the following linear combination of correlators integrated over momentum:

$$
\begin{aligned}
A &\equiv -\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2}\left(3\widetilde{C}_1(p^2/M^2) - 4\widetilde{C}_{1/2}(p^2/M^2) + \widetilde{C}_0(p^2/M^2)\right) \\
&= -\frac{M^2}{16\pi^2}\int dy\left(3\widetilde{C}_1(y) - 4\widetilde{C}_{1/2}(y) + \widetilde{C}_0(y)\right)
\end{aligned}
\tag{2.10}
$$

---

[4] We will not pursue it here, but it would be interesting to consider correlators of J's defined by (2.3) along with any number of supercharges, in the case when SUSY is unbroken. Perhaps there could be an interesting mathematical structure analogous to operators in the chiral ring.

Using (2.3) and (2.5), one easily finds that formula for the gaugino mass can be rewritten as

$$M_{gaugino} = -\frac{1}{4}g^2 \int d^4x \, \langle Q^2(J(x)J(0)) \rangle \tag{2.11}$$

where we use the notation

$$Q^2(\ldots) = Q^\alpha Q_\alpha(\ldots) \equiv \{Q^\alpha, [Q_\alpha, (\ldots)]\}. \tag{2.12}$$

Indeed, according to (2.3)(2.5), $Q^2(J(x)J(0)) = 2[Q^\alpha, J(x)][Q_\alpha, J(0)] = -2j^\alpha(x)j_\alpha(0)$. Similar reasoning shows that the action of four supercharges on $J(x)J(0)$ yields

$$\langle \overline{Q}^2(Q^2(J(x)J(0))) \rangle = -8\partial^2(C_0(x) - 4C_{1/2}(x) + 3C_1(x)) \tag{2.13}$$

and so the formula for the sfermion mass can be rewritten as

$$m^2_{sfermion} = -\frac{1}{128\pi^2}g^4Y^2 \int d^4x \, \log(x^2 M^2)\langle \overline{Q}^2(Q^2(J(x)J(0))) \rangle \tag{2.14}$$

Note that the order of the four supercharges is not essential – a different ordering of Q and $\overline{Q}$ leads to terms that vanish after using the SUSY algebra and momentum conservation. Note also that the scale M appearing in (2.14) is arbitrary (i.e. the dependence on M drops out), since according to (2.13) the integrand $\langle \overline{Q}^2(Q^2(J(x)J(0))) \rangle$ is a total derivative. (The short distance behavior of the correlator, to be discussed below, guarantees that there is no surface term.)

Let us make some brief comments on the results (2.11), (2.14). In [15] it was shown using the SUSY algebra that when SUSY is unbroken, $B = 0$ and $C_0 = C_{1/2} = C_1$. Hence the gaugino and sfermion masses vanish in the SUSY limit, as they must. Writing the gaugino and sfermion masses as multiple commutators, as we have done here, makes this fact obvious.

It is well known that when supersymmetry is broken at a scale F and the dynamics is characterized by the scale $M \gg \sqrt{F}$, we can effectively describe the soft terms in an expansion in $\frac{F}{M^2}$ using spurions. Then the gaugino masses arise as an F-term and the sfermion masses as a D-term. The expressions (2.11) and (2.14) generalize this result to the more generic situation of $F \sim M^2$. The small $\frac{F}{M^2}$ limit can be obtained by realizing that in (2.11) the two Qs lead to one factor of F and in (2.14) the four Qs lead to $|F|^2$.

Another interesting feature of the formula (2.14) is that all the information at large momentum is contained within the OPE of J with itself. This observation has immediate

implications about the convergence of the momentum integral in (2.10) and (2.14). In [15] an indirect proof of the convergence of these integrals was given using the fact that otherwise there would be no supersymmetric counterterm that could cancel a divergence in this integral. Here we can easily give a direct proof which is intrinsic to the properties of the hidden sector. The most singular term in the OPE $J(x)J(0)$ is associated with the identity operator. Since this is annihilated by the action of the supercharges in (2.14), to get a nonzero result we must use an operator with $\Delta > 0$. Its coefficient is $x^{-4+\Delta}$ and therefore the integral (2.14) converges at small x.

Finally, let us examine the low momentum behavior of the integral in (2.10). We can exclude any zero-momentum divergences in these integrals by invoking messenger parity $\mathcal{J} \to -\mathcal{J}$. On general grounds, any such zero-momentum poles in the current two point functions in (2.7) must be due to massless intermediate one-particle states:

$$\langle \mathcal{J}(x)\mathcal{J}(0)\rangle = \langle 0|\mathcal{J}(x)|\lambda\rangle\langle\lambda|\mathcal{J}(0)|0\rangle + \dots \tag{2.15}$$

Assuming that the only massless particles in the spectrum are due to spontaneously broken symmetries (bosonic or fermionic), and that messenger parity commutes with all the symmetries of the theory, it follows that the one-point functions on the RHS of (2.15) must vanish. Therefore massless modes can never contribute zero-momentum poles to the current two point function, and the integral (2.10) must always converge at $p = 0$.

2.2. Generalization to the MSSM

Finally, let us briefly generalize the discussion from our $G = U(1)$ toy model to the MSSM, where $G = SU(3) \times SU(2) \times U(1)$. We will label the gauge group factors $U(1)$, $SU(2)$ and $SU(3)$ by $k = 1, 2, 3$ respectively. Then are three complex numbers $B_k \equiv \widetilde{B}_k(0)$ and three real numbers A_k which determine the gaugino and sfermion soft masses. They are defined as above, using the current supermultiplet of the respective gauge group. The soft masses are given to leading order in the α by

$$M_k = g_k^2 M B_k, \qquad m_f^2 = \sum_{k=1}^{3} g_k^4 c_2(f, k) A_k \tag{2.16}$$

$f = Q, U, D, L, E$ labels the matter representations of the MSSM, and $c_2(f, k)$ is the quadratic Casimir of f with respect to the gauge group k.

Since the five sfermion masses are determined by three real numbers, they must satisfy two sum rules. These take the form [15]:

$$m_Q^2 - 2m_U^2 + m_D^2 - m_L^2 + m_E^2 = 0$$
$$2m_Q^2 - m_U^2 - m_D^2 - 2m_L^2 + m_E^2 = 0.$$

(2.17)

From (2.16), it is clear that these sum rules are valid at $\mathcal{O}(\alpha^2)$. However, we can further demonstrate that they are valid at $\mathcal{O}(\alpha^3)$ and to leading-log order for any α, meaning that the sum rules must be satisfied to very high accuracy.

First, it was already shown in [15] that the sum rules are preserved by the MSSM RGEs (neglecting contributions from the Higgs sector proportional to the Yukawa interactions). This takes care of the leading-log corrections. Second, we can consider the $\mathcal{O}(\alpha^3)$ corrections coming from the hidden sector. These arise from various current three-point functions in the hidden sector. It is easy to see that gauge invariance allows only five three-point functions: $SU(3)^3$, $SU(2)^3$, $U(1)^3$, $SU(3)^2 U(1)$, $SU(2)^2 U(1)$. If one imposes messenger parity (which sends $V_Y \rightarrow -V_Y$), this eliminates the mixed three-point functions and the $U(1)^3$, leaving us with only the $SU(3)^3$ and $SU(2)^3$ three point functions. These represent additional contributions to the parameters A_2 and A_3. Their presence does not spoil the sum rules, which only rely on the fact that there are three A's and not that they only receive contributions at a given order in α.

3. Sensitivity to UV physics

3.1. General remarks

In the previous section, we restricted our analysis to renormalizable, UV-complete hidden sectors. However, it is often the case that our understanding of the hidden sector is incomplete, that we have only an effective description of it at low energies. In this section we would like to make some general comments about the dependence of the MSSM soft-breaking terms on unknown UV physics. This will have immediate applications in the next section, when we wish to use incomplete messenger-spurion models of gauge mediation to cover the parameter space of GGM. With our understanding of the (in)sensitivity of gauge mediation to UV physics, we will be sure that the models we study in the next section are indeed calculating correctly the MSSM soft masses.

We will begin with a more abstract discussion of UV sensitivity in a theory with spontaneously broken SUSY. Then in the next subsection we will give an example to

illustrate some of our general comments. The reader may find it useful to reread the general discussion after having gone through the example calculation in the next subsection.

Consider a hidden sector consisting of an effective field theory valid below a UV cutoff scale Λ (which could be e.g. the Planck scale, or some UV scale), with SUSY spontaneously broken at a scale $\sqrt{F}$. As long as $\sqrt{F} \ll \Lambda$, all the soft terms are calculable in terms of the effective theory. The reason is that at energies much larger than $\sqrt{F}$ supersymmetry is restored and all the supersymmetry breaking contributions arise at energies of order $\sqrt{F}$ or smaller.

Now suppose the hidden sector is a messenger model of gauge mediation. Such models are weakly coupled truncations of a more complete theory valid above the scale Λ. They are fully specified by the set of messenger quantum numbers and the set of messenger masses given in (1.2), (1.3), (1.4). In this scheme, the soft parameters are calculable in terms of the messenger mass matrices. Let us denote the scale of the messenger sector by M. Clearly, when we study these models, we are implicitly taking the limit $\Lambda \to \infty$ with M fixed.

Typically one considers the messenger scale M and the SUSY-breaking scale $\sqrt{F}$ to be of the same order. In this case there is no problem and the soft terms are indeed unambiguously calculable, insensitive to the physics above the UV cutoff Λ. However, it is often the case that the messengers at the scale M receive supersymmetry breaking mass splittings which are much smaller than $\frac{F}{M}$. Then, we might want to reconsider the $\Lambda \to \infty$ limit in such a way that the messenger mass splittings are kept finite.

For example, imagine that these mass splittings are or order $\frac{F}{\Lambda}$. Then, the proper decoupling limit is $\Lambda, \sqrt{F} \to \infty$ with fixed $\frac{F}{\Lambda}$ and M. In this case the soft-breaking terms may not be calculable. A simple way to see that is to add to the theory additional messengers with mass of order Λ and supersymmetry breaking mass splittings of order $\frac{F}{\Lambda}$. These messengers contribute to gaugino masses and sfermion mass-squareds additional terms of order $\frac{F}{\Lambda}$ and $(\frac{F}{\Lambda})^2$ respectively. We can view these additional contributions as *finite local counterterms* for gaugino masses and sfermion masses which are determined by the details of the high energy theory.

From the point of view of the effective theory, such counterterms are ambiguous, controlled by the choice of UV completion above the scale Λ. It is important to note, however, that any such ambiguity must necessarily arise *only at leading order* in the SUSY breaking parameter F, since higher-order contributions from the UV states are necessarily suppressed by additional powers of $\frac{F}{\Lambda^2}$ (which goes to zero as $\Lambda \to \infty$).

The sensitivity to the UV is particularly dramatic when the supertrace of the messenger spectrum is nonzero [24,28]. In this case the necessary counterterms include a logarithmically divergent sfermion mass. (See Appendix B for an explicit proof of this fact.) We stress that this divergence is a symptom of the problem, but the problem might arise even if the supertrace vanishes.

We conclude by roughly summarizing the foregoing discussion: if the messenger splittings are parametrically smaller than F/M, the soft-breaking terms in the MSSM are not calculable without further UV input.

3.2. Example

Let us now illustrate these general points with a simple example. To that end, consider the messenger theory with superpotential

$$W_{\text{eff}} = M\phi_1\widetilde{\phi}_1 \tag{3.1}$$

and Kähler potential

$$K_{\text{eff}} = |X|^2 + |\widetilde{\phi}_1|^2 + \left(1 + \left|\frac{X}{\Lambda}\right|^2 + ...\right)|\phi_1|^2 \tag{3.2}$$

where the ellipsis contains higher dimensional operators and X is a SUSY breaking field with

$$\langle X \rangle = M' + \theta^2 F \tag{3.3}$$

It will be convenient to introduce the following notation:

$$x \equiv \frac{M'}{\Lambda}, \qquad y = \frac{F}{M\Lambda} \tag{3.4}$$

As described above, we consider the limit $\Lambda \to \infty$ with x and y and the low energy mass parameter M held fixed.

By the general arguments above, we expect that the soft parameters computed in this effective theory are sensitive to large corrections from states at the scale Λ where the description of the physics given by (3.1) and (3.2) breaks down. Moreover, we expect that such corrections only enter in at leading order in the SUSY-breaking parameter F. We will now explicitly show that this is indeed the case.

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Using our messenger GGM formalism developed in Appendix B, or equivalently in this case using the explicit formulas from [24], we find the low energy soft parameters to be

$$B_{\text{eff}} = \frac{Mx}{48\pi^2(1+x^2)^2}\left(6(1+x^2)y + (2+x^2)y^3\right) + \mathcal{O}(y^5) \tag{3.5}$$

and

$$A_{\text{eff}} = \frac{M^2}{64\pi^4(1+x^2)^2}\left(\left(\log\left(\frac{\Lambda_{\text{cutoff}}^2}{M^2}\right) - 2 + x^2 + 2\log(1+x^2)\right)y^2 + \frac{x^2(6+x^2)}{36(1+x^2)}y^4\right) + \mathcal{O}(y^6). \tag{3.6}$$

Note that while B_{eff} is finite, A_{eff} is logarithmically divergent with the UV cutoff Λ_{cutoff}. The appearance of this divergence which multiplies the supertrace in the low energy effective theory

$$\text{STr}\mathcal{M}_{\text{IR}}^2 = -\frac{2M^2y^2}{(1+x^2)^2} \tag{3.7}$$

reminds us that our theory must be UV completed. Note, however, that even though the gaugino mass parameter is finite, it too will be sensitive to the UV physics as we will see below.

We can regulate the divergence in (3.6) by embedding the IR theory in a renormalizable UV theory with the following superpotential

$$W = X\phi_1\widetilde{\phi}_2 + M\phi_1\widetilde{\phi}_1 + \Lambda\phi_2\widetilde{\phi}_2 \tag{3.8}$$

and a canonical Kähler potential.[5] Integrating out the heavy fields (with mass Λ) ϕ_2, $\widetilde{\phi}_2$, we readily derive the effective low energy Lagrangian (3.1), (3.2).[6]

The contribution of the messengers in our full theory (3.8) to the soft SUSY breaking masses in the MSSM is manifestly finite. Let's compare it to the calculation in the low energy theory (3.5), (3.6).

Again, using our messenger GGM formulas we find the following soft parameters

$$B_{\text{full}} = \frac{Mx}{48\pi^2(1+x^2)^2}(2+x^2)y^3 + \mathcal{O}(y^5) \tag{3.9}$$

---

[5] Some authors (see e.g.[24]) regularize the theory using dimensional reduction with "ϵ-scalars." We prefer to replace the unphysical ϵ-scalars with physical heavy fields as in (3.8).

[6] In this regularization, we see that the negative sign of the supertrace in (3.7) corresponds precisely to what we expect from the general results on integrating out massive chiral matter in Appendix A.

and

$$A_{\text{full}} = \frac{M^2}{64\pi^4(1+x^2)^2}\left(\left(\log\left(\frac{\Lambda^2}{M^2}\right) + 2x^2 + 2\log(1+x^2)\right)y^2 + \frac{x^2(6+x^2)}{36(1+x^2)}y^4\right) + \mathcal{O}(y^6) \tag{3.10}$$

We see that B_{eff} and B_{full} differ *only* at leading order in y, with the counterterm given by[7]

$$\delta B = \frac{M}{8\pi^2}\left(\frac{x}{1+x^2}\right)y \tag{3.11}$$

For the particular UV definition we have chosen, we can understand this term as arising from the rescaling anomaly in the recanonicalization of the IR Kähler potential. Notice, however, that if we had added messengers to the UV theory that did not couple to the light messengers, they would have also contributed at order y to the counterterm in (3.11). These contributions cannot be captured by the rescaling anomaly.

Similarly, the difference between A_{full} and A_{eff} is also only at leading order in the SUSY breaking. However, here it includes an infinite counterterm:

$$\delta A = \frac{M^2}{64\pi^4(1+x^2)^2}(\log(\Lambda^2/\Lambda^2_{\text{cutoff}}) + x^2 + \log(1+x^2))y^2 . \tag{3.12}$$

Again, adding messengers in the UV decoupled from the IR has the effect of generating additional corrections at leading order in the SUSY breaking.

With a sharp set of criteria for defining calculable gauge mediation models in hand, we will now explore the covering of the GGM parameter space in the next section. In particular, when using messenger models we will specialize to the case of vanishing supertrace and $\frac{F}{\Lambda} \to 0$.

4. Covering the General Gauge Mediation Parameter Space

4.1. The general setup

In this section we will demonstrate, using a general model with messengers, that the entire parameter space of GGM can be covered by a calculable weakly coupled field theory.

Consider a theory with N chiral messengers Φ^i, $\widetilde{\Phi}^i$, $i = 1, \ldots, N$ transforming in some vector-like representation $\mathbf{R} \oplus \overline{\mathbf{R}}$ of a gauge group G (which will later be identified with

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[7] One can check that the full expressions for both B and A in the effective and the full theories agree at *all* higher orders in y and not just at the next-to-leading order we have written down in our expressions above.

the SM gauge group). The messenger spectrum determines the GGM soft masses, so we will focus on that. The most general messenger spectrum is of the form

$$V_{\text{mass terms}} = (\widetilde{\psi}^T \mathcal{M}_F \psi + c.c.) + \begin{pmatrix} \phi \\ \phi^* \end{pmatrix}^\dagger \mathcal{M}_B^2 \begin{pmatrix} \phi \\ \phi^* \end{pmatrix} \tag{4.1}$$

with

$$\mathcal{M}_B^2 \equiv \begin{pmatrix} \mathcal{M}_F^\dagger \mathcal{M}_F + \xi & F \\ F^\dagger & \mathcal{M}_F \mathcal{M}_F^\dagger + \widetilde{\xi} \end{pmatrix} \tag{4.2}$$

Here $\mathcal{M}_F$, ξ, $\widetilde{\xi}$ and F are all $N \times N$ matrices. We take $\mathcal{M}_F$ to be diagonal with real, positive entries without loss of generality. ξ and $\widetilde{\xi}$ are Hermitian; and F is complex. The off-diagonal parameters F can arise from "F-term breaking" e.g. from a superpotential coupling to spurion field. The diagonal parameters ξ can arise from "D-term breaking" e.g. from FI-U(1) terms. More generally, the general spectrum shown in (4.1) can arise from complicated non-Abelian dynamics such as in [26].

We will impose the following restrictions on the messenger spectrum, motivated by phenomenology and overall consistency:

1. In order to avoid the SUSY CP problem, we require all the mass parameters to be real

$$\xi = \xi^*, \qquad \widetilde{\xi} = \widetilde{\xi}^*, \qquad F = F^* . \tag{4.3}$$

2. In order to guarantee that no dangerous FI-term for hypercharge is generated, we impose invariance under messenger parity [6,23][8]

$$\Phi^i \leftrightarrow \widetilde{\Phi}^i. \tag{4.4}$$

This restricts the parameters to satisfy

$$\xi = \widetilde{\xi}, \qquad F = F^T . \tag{4.5}$$

3. Since we want our theory to be calculable and insensitive to UV physics, we require vanishing messenger mass-squared supertrace. This translates to

$$\text{Tr}\,\xi = 0 \tag{4.6}$$

---

[8] Actually, the authors of [23] considered another action for this symmetry which maps chiral superfields to anti-chiral superfields. Such a symmetry does not commute with the Lorentz symmetry. However, if we also impose CP symmetry, our choice is equivalent to theirs.

4. In the case where $G = SU(3) \times SU(2) \times U(1)$, we want the gauge couplings to unify. This restricts the messengers to be in complete $SU(5)$ representations. Furthermore, we limit the number of representations such that the theory remains perturbative.

5. The messengers must be non-tachyonic for consistency of the model. So this puts upper limits on the magnitudes of the entries in ξ and F.

Finally, we note that if the messengers are in a reducible representation

$$\mathbf{R} = \bigoplus_R (n_R \times R) \tag{4.7}$$

then the messenger mass matrices must be block-diagonal. Each block couples the messengers with the same R. Consequently, all of the statements above hold for each R separately, and the leading-order in α contributions from each R to the soft masses are additive.

4.2. Covering the GGM parameter space of a toy $U(1)$ visible sector

In this subsection we will consider a simplified theory with only $G = U(1)$ symmetry and messengers with charges ± 1. This example is instructive because the detailed representation theory of the messengers does not play an important role in this case. It will also be useful in the next subsection when we consider the full $G = SU(3) \times SU(2) \times U(1)$ case.

Here there is only one A parameter and only one B parameter and covering the parameter space means finding a theory that covers the range

$$\kappa = \frac{A}{|B|^2} \in (0, \infty). \tag{4.8}$$

Notice that $\kappa \to 0$ corresponds to the limit of either a very massive gaugino or vanishing sfermion mass, while $\kappa \to \infty$ corresponds to either a very massive scalar or vanishing gaugino mass.

Let us first ask if we can cover (4.8) with a single messenger pair and, at the same time, obey the microscopic constraints on our messenger sector described in the previous subsection. To answer this question, note that the most general single messenger model allowed by messenger parity and vanishing supertrace is of the form

$$\mathcal{M}_F = M, \qquad \mathcal{M}_B^2 = \begin{pmatrix} M^2 & F \\ F & M^2 \end{pmatrix}. \tag{4.9}$$

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i.e. only minimal gauge mediation is allowed. This model has two parameters, M and F, and spans a two-dimensional subspace of the full A and B parameter space. However, an explicit calculation shows [29] that this subspace is not the full GGM parameter space and that in fact

$$\kappa \in (.37, 1) \tag{4.10}$$

where the upper bound for κ is obtained in the limit of small SUSY breaking and the lower bound arises because the messengers cannot be tachyonic.

Next, we try a system with two messengers. Since we are only interested in giving an existence proof of (4.8), we will not consider the most general possible two-messenger mass matrix satisfying the conditions above. Instead, we consider the following special mass matrix

$$\mathcal{M}_F = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \tag{4.11}$$

and

$$\mathcal{M}_B^2 = \begin{pmatrix} M_1^2 + D & 0 & F_1 & 0 \\ 0 & M_2^2 - D & 0 & F_2 \\ F_1 & 0 & M_1^2 + D & 0 \\ 0 & F_2 & 0 & M_2^2 - D \end{pmatrix}. \tag{4.12}$$

This model could arise, e.g. from a simple MGM-like setup with the messengers charged under an additional $U(1)'$ gauge group with a nonzero FI D-term.

With the added assumption

$$F_1, F_2, D \ll M_{1,2}^2 . \tag{4.13}$$

we can use the techniques of wavefunction renormalization [30,22] to compute the A and B parameters

$$B = \frac{1}{8\pi^2} \left(\frac{F_1}{M_1} + \frac{F_2}{M_2} \right) + \mathcal{O}(F^3, DF) \tag{4.14}$$

and

$$A = A_F + A_\xi$$
$$A_F = \frac{1}{64\pi^4} \left(\frac{F_1^2}{M_1^2} + \frac{F_2^2}{M_2^2} \right) + \mathcal{O}(F^4, DF^2) \tag{4.15}$$
$$A_\xi = \frac{D}{32\pi^4} \log(M_1^2/M_2^2) + \mathcal{O}(DF^2).$$

From these expressions, it is straightforward to see that this example in fact covers the range

$$\kappa \in (-\infty, \infty). \tag{4.16}$$

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First, for $D = 0$ we can set $\frac{F_1}{M_1} \approx -\frac{F_2}{M_2}$ such that B is very small while A is finite. This leads to arbitrarily large $|\kappa|$. However, setting $D = 0$ prevents us from making $|\kappa|$ arbitrarily small. For that, we use nonzero D to set

$$A_\xi < 0 \tag{4.17}$$

such that $A = A_F + A_\xi$ is arbitrarily small with fixed B.

We conclude that this example covers the full parameter space of GGM for a $U(1)$ visible sector.

4.3. Covering the MSSM GGM parameter space

Let us now generalize the discussion of the previous section to the physically relevant case of $G = SU(3) \times SU(2) \times U(1)$. We will see that, when properly analyzed, this case reduces to the $U(1)$ case considered in the previous subsection.

We would like to find weakly-coupled messenger theories that cover the full GGM parameter space of the MSSM, namely the six parameters A_k, $B_k \in \mathbb{R}^+$, where $k = 1, 2, 3$ labels $U(1)$, $SU(2)$ and $SU(3)$, respectively. A first analysis of this subject was presented by Carpenter, Dine, Festuccia and Mason in [16]. We will extend their analysis, by demanding not only the right number of parameters, but that the entire parameter space can be covered.

As noted above around equation (4.7), the messenger mass matrices are block diagonal with respect to different irreps R, and the contribution from messengers of different irreps are additive. It follows then that

$$A_k = \sum_R N_{k,R} A_R \ , \qquad B_k = \sum_R N_{k,R} B_R \tag{4.18}$$

where the sum is over the different messenger irreps, and $N_{k,R}$ are the total Dynkin indices of the irrep R with respect to the gauge group k. Notice how the dependence on the gauge group is trivial and factors out completely. The functions A_R and B_R are universal in the sense that they depend only on the mass parameters of the messengers with representation R. In fact, they are identical to what one would compute for n_R $U(1)$ messengers with charges ± 1.

Since we are interested in models that are compatible with unification, we should consider messengers in complete representations of $SU(5)$. The smallest $SU(5)$ representations **5** and **10** can be decomposed under the usual matter representations of the MSSM as

$$\overline{\mathbf{5}} = D \oplus L \,, \qquad \mathbf{10} = Q \oplus U \oplus E. \tag{4.19}$$

So we will restrict our attention to $R = Q, U, D, L, E$. Just for reference, the Dynkin indices for these representations are

$$
\begin{aligned}
&N_{1,Q} = \frac{1}{10}, \ N_{1,U} = \frac{4}{5}, \ N_{1,E} = \frac{3}{5}, \ N_{1,D} = \frac{1}{5}, \ N_{1,L} = \frac{3}{10} \\
&N_{2,Q} = \frac{3}{2}, \ N_{2,L} = \frac{1}{2} \\
&N_{3,Q} = 1, \ N_{3,U} = \frac{1}{2}, \ N_{3,D} = \frac{1}{2}
\end{aligned}
\tag{4.20}
$$

where in the first line we have used the standard GUT normalization for the $U(1)_Y$ charge.

The expressions (4.18) immediately lead to a necessary condition on the messenger content, in order for the model to cover the full parameter space: we need messengers transforming in at least three different irreps. Otherwise, we do not have three linearly independent functions A_R and three linearly independent functions B_R.

This means that any number of messengers in $\mathbf{5} \oplus \overline{\mathbf{5}}$ cannot cover the parameter space (they have only two values of $R = D, L$). Next we can attempt to use messengers in a single copy of $\mathbf{10} \oplus \overline{\mathbf{10}}$. Here we have three values of $R = Q, U, E$ and therefore three linearly independent constants. However, the result (4.10) in the $U(1)$ toy example discussion shows that these constants are bounded, $.37 < \kappa_R \equiv \frac{A_R}{|B_R|^2} < 1$. In particular, we cannot make the gauginos arbitrarily heavy compared to the scalars.

As in the $U(1)$ example, we can avoid this difficulty by having at least two copies of the representations and then using D-type supersymmetry breaking. We are therefore led to the following simplest possible models

$$2 \times (\mathbf{10} \oplus \overline{\mathbf{10}}) \qquad \text{or} \qquad 2 \times (\mathbf{5} \oplus \overline{\mathbf{5}}) \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \,. \tag{4.21}$$

The latter is more "minimal" since it has slightly smaller total Dynkin index (and thus contributes slightly less to the MSSM gauge coupling beta functions). However, the former is easier to analyze, since we can now build a theory that is three copies of the two-messenger models discussed in the previous section, one for each irrep in the **10**. The small SUSY breaking result (4.16) is then enough to show that we can in fact cover the

parameter range. This is true even if we take universal fermion mass for each $\mathbf{10} \oplus \overline{\mathbf{10}}$ factor, so we can cover the parameter space without introducing supersymmetric GUT-breaking splittings in the messenger sector. This shows that covering the parameter space is compatible with unification, up to possible threshold corrections coming from the SUSY-splittings.

The analysis of a theory with messenger content $2 \times (\mathbf{5} \oplus \overline{\mathbf{5}}) \oplus \mathbf{10} \oplus \overline{\mathbf{10}}$ is slightly different since the $\mathbf{10} \oplus \overline{\mathbf{10}}$ representations must have pure F-type breaking. In particular, the Q, U, and E type messengers must satisfy (4.10) and so

$$0.37 < \kappa_R < 1 \quad \text{for } R = Q, U, E \tag{4.22}$$

Substituting (4.22) into (4.18), we find six equations for seven non-compact variables ($A(D)$, $A(L)$, $B(D)$, $B(L)$, $B(Q)$, $B(U)$, and $B(E)$) and three compact variables ($\kappa_{Q,U,E}$). However, it is not completely obvious that a real solution exists, because the substitution is quadratic in $B(Q)$, $B(U)$ and $B(E)$. One can check that this is always possible if we take $\kappa_Q > \kappa_E, \kappa_U$. Note that this takes us outside the small SUSY-breaking limit (where $\kappa = 1$) for the E and the U messengers.

These results show that there cannot be any additional field theoretic restrictions on the GGM parameter space. Another consequence of this result is the following. Assume that all the soft terms are measured someday, and our two sum rules (1.1) are satisfied. Then, we can derive the six numbers A_k, B_k and try to match them with a more microscopic theory. Our result here shows that whatever these numbers are, we'll be able to obtain them from weakly coupled messengers. In fact, we'll be able to do it in more than one way. This implies that the gaugino and sfermion masses alone will not be enough to distinguish between different gauge mediation scenarios. More input, such as the messenger scale or the SUSY-breaking scale (equivalently, the gravitino mass), will be needed in order to break this degeneracy.

Appendix A. General results on the effective supertrace

In this appendix we analyze the effect of integrating out massive modes at tree-level in a renormalizable theory. In particular, we will be interested in the supertrace over the spectrum of the low-energy effective theory. We will assume that the low-energy theory is described by a non-linear sigma model without gauge interactions. Then the supertrace over the light modes is given by the following general formula [31,32]:

$$\mathrm{STr}\mathcal{M}^2 = 2R_{c\bar{k}}g^{\bar{k}a}g^{\bar{b}c}W_a W_{\bar{b}}^* \tag{A.1}$$

where the indices run over the chiral superfields Φ^a comprising the low-energy effective theory; $g^{\bar{a}b}$ is the inverse Kähler metric; $R_{a\bar{b}}$ is the Ricci tensor associated with the Kähler metric, and W is the effective superpotential.

We will show that integrating out massive chiral matter results in a negative semi-definite Ricci tensor, so $\mathrm{STr}\mathcal{M}^2 \leq 0$ in this case. We then show that integrating out massive vector fields results in an indefinite Ricci tensor and correspondingly a supertrace of indefinite sign.

A.1. Integrating out massive chiral matter

Consider the most general renormalizable theory of heavy chiral superfields H^A coupled to light chiral superfields ℓ^a. This must have the form (we take the Kähler potential to be canonical)

$$W = \frac{1}{2}\lambda_{Abc}H^A\ell^b\ell^c + \frac{1}{2}M_{AB}H^A H^B + \frac{1}{2}m_{ab}\ell^a\ell^b + ... \tag{A.2}$$

where the ellipsis contains unimportant marginal and higher dimensional couplings, and $m \ll M$. Integrating out the heavy fields yields the following equation of motion

$$H^A = -\frac{1}{2}(M^{-1})^{AB}\ell^T\lambda_B\ell + ... \tag{A.3}$$

Substituting this into (A.2) we obtain the effective superpotential

$$W_{\mathrm{eff}} = \frac{1}{2}m_{ab}\ell^a\ell^b + \mathcal{O}(\ell^4) \tag{A.4}$$

We also find the following effective Kähler potential

$$K_{\mathrm{eff}} = \ell^\dagger\ell + \frac{1}{4}\sum_A \left|(M^{-1})^{AB}\ell^T\lambda_B\ell\right|^2 ... \tag{A.5}$$

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It follows that the Ricci tensor of the effective Kähler metric

$$R_{a\bar{b}} = -\partial_a(g^{\bar{c}d}g_{d\bar{b},\bar{c}}) \tag{A.6}$$

is at $\ell = 0$

$$R_{a\bar{b}} = -\delta^{\bar{c}d}g_{a\bar{b},\bar{c}d} = -\sum_A \left((M^{-1}\lambda)^A(M^{-1}\lambda)^{\dagger\bar{A}}\right)_{a\bar{b}} \tag{A.7}$$

This is a sum over negative semi-definite matrices, so it is also negative semi-definite. It then follows from (A.1) that the effective supertrace over the light fields is non-positive. One application of this result is to gauge mediation models of the type discussed in section 3, where the H^A fields are heavy messengers and the ℓ^a are light messengers and SUSY breaking fields.

A.2. Integrating out massive vector superfields

Next we consider what happens when one classically integrates out massive vector superfields. Here it turns out that the Ricci tensor of the effective Kähler metric is indefinite and therefore the supertrace over the light spectrum is also of indefinite sign.

The setup is as in [26]; we will review it here. Consider a gauge theory with matter chiral superfields Φ^a transforming under gauge group G (not necessarily simple), where $a = 1, \ldots, N$ denotes the collective set of gauge and flavor indices. Suppose that the Φ^a acquire supersymmetric vevs ϕ_0 which Higgs the entire gauge group. These vevs must lie along the D-flat moduli space $\mathcal{M}$ defined by the equations:

$$\phi_0^\dagger T^I \phi_0 = 0 \tag{A.8}$$

where T^I are the generators of G. Now consider the fluctuations around this point in moduli space:

$$\Phi = \phi_0 + \delta\Phi \tag{A.9}$$

We are interested in the effective Kähler potential for these fluctuations induced by integrating out the massive vector supermultiplets of G. In what follows we will work in the unitary gauge discussed in [26]

$$\phi_0^\dagger T^I \delta\Phi = 0 \tag{A.10}$$

which guarantees that the fluctuations lie within $\mathcal{M}$. It will be convenient to perform a unitary transformation so that $\delta\Phi^{a=1,\ldots,N-\dim G}$ satisfy (A.10) and the other elements of $\delta\Phi$ are in the orthogonal subspace.

Now according to [26], the effective Kähler potential is given by

$$K_{\text{eff}} = \delta\Phi^\dagger \delta\Phi - \frac{1}{2}(\delta\Phi^\dagger T^I \delta\Phi) h_{IJ}^{-1} (\delta\Phi^\dagger T^J \delta\Phi) + \mathcal{O}(\delta\Phi^6) \tag{A.11}$$

where h^{IJ} is the matrix

$$h^{IJ} = \frac{1}{2}\Phi^\dagger \{T^I, T^J\}\Phi \tag{A.12}$$

(Note the analogy with the previous subsection: h_{IJ}^{-1} is analogous to $M^{-1\dagger}M^{-1}$ and $T_{\overline{b}a}^I$ is analogous to λ_{Abc}. The only difference is in the type of the indices, which dictates how they are contracted.) As in the previous subsection, we can compute the Ricci tensor at leading order in the fluctuations. However, we must be careful not to differentiate with respect to all the fluctuations $\delta\Phi^a$, but only those which satisfy the gauge condition (A.10). In our convenient basis, these are simply the $a = 1, \ldots, N - \dim G$ entries of $\delta\Phi^a$. So the metric is simply

$$g_{a\overline{b}} = \delta_{a\overline{b}} - (\delta\Phi^\dagger T^I)_a h_{IJ}^{-1} (T^J \delta\Phi)_{\overline{b}} - (T^I)_{\overline{b}a} h_{IJ}^{-1} (\delta\Phi^\dagger T^J \delta\Phi) + \mathcal{O}(\delta\Phi^4) \tag{A.13}$$

with $a, b = 1, \ldots, N - \dim G$. Therefore, the Ricci tensor at $\delta\Phi = 0$ is:

$$R_{a\overline{b}} = -\delta^{c\overline{d}} g_{a\overline{b},c\overline{d}} = (T^I)^c{}_a h_{IJ}^{-1} (T^J)_{\overline{b}c} + (T^I)_{\overline{b}a} h_{IJ}^{-1} \text{Tr}\,' T^J \tag{A.14}$$

Here the sum is only over indices in the range $1, \ldots, N - \dim G$, and $\text{Tr}\,'$ refers to the restricted trace over the subspace of fluctuations satisfying (A.10). Even though the full trace of T^J must vanish due to the anomaly condition, the restricted trace need not vanish since the gauge symmetry is spontaneously broken. This is important, because while the first term in (A.14) enjoys definiteness properties, the second term obviously does not. Thus there is no reason to expect the Ricci tensor to have any definiteness property. Indeed, it is straightforward to construct simple examples where $R_{a\overline{b}}$ has both positive and negative eigenvalues.[9] Therefore we conclude in this case that the effective supertrace can have either sign.

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[9] For instance, consider a $U(1)$ gauge theory with fields $\Phi^{1,2,3,4}$ having charges $q_1 = +1$, $q_2 = -1$, $q_3 = +q$ and $q_4 = -q$ with $q \neq \pm 1$. The D-flat moduli space is characterized by $\phi_0 = (\Phi^1, \Phi^2, \Phi^3, \Phi^4)$ with Φ^i satisfying the equation

$$|\Phi^1|^2 - |\Phi^2|^2 + q(|\Phi^3|^2 - |\Phi^4|^2) = 0 \tag{A.15}$$

Going to a point on this moduli space, we can impose the gauge fixing condition (A.10) by solving for $\delta\Phi^4$. Substituting back into the Kähler potential (A.11) gives the effective Kähler potential for $\delta\Phi^{1,2,3}$. From this one can compute the Ricci tensor at $\delta\Phi = 0$ using $R_{a\overline{b}} = -\delta^{c\overline{d}} g_{a\overline{b},c\overline{d}}$. Then by varying ϕ_0 and q it is easy to find places where $R_{a\overline{b}}$ has both positive and negative eigenvalues.

Appendix B. General results on multiple messenger models

In this appendix we write down the GGM correlation functions for a general messenger theory. We then explicitly show that a messenger sector with non-vanishing supertrace generates contributions to the scalar mass-squareds that are logarithmically divergent and proportional to the supertrace.

As in section 4, let us restrict ourselves to the case that the messengers are charged under a $U(1)$ gauge group with mass terms

$$V \supset \xi_{ij}\phi_i\phi_j^* + \widetilde{\xi}_{ij}\widetilde{\phi}_i\widetilde{\phi}_j^* + |M_i|^2(\phi_i\phi_i^* + \widetilde{\phi}_i\widetilde{\phi}_i^*) + f_{ij}\phi_i\widetilde{\phi}_j + f_{ij}^*\phi_i^*\widetilde{\phi}_j^* + M_i\psi_i\widetilde{\psi}_i + M_i^*\overline{\psi}_i\overline{\widetilde{\psi}}_i \quad \text{(B.1)}$$

and $i = 1, ..., N$. Again, taking the ϕ_i and $\widetilde{\phi}_i$ to have $U(1)$ charge $+1$ and -1 respectively, we find

$$
\begin{aligned}
J(x) &= \phi_i^*\phi_i - \widetilde{\phi}_i^*\widetilde{\phi}_i \\
j_\alpha(x) &= -\sqrt{2}i(\phi_i^*\psi_{i\alpha} - \widetilde{\phi}_i^*\widetilde{\psi}_{i\alpha}) \\
\overline{j}_{\dot{\alpha}}(x) &= \sqrt{2}i(\phi_i\overline{\psi}_{i\dot{\alpha}} - \widetilde{\phi}_i\overline{\widetilde{\psi}}_{i\dot{\alpha}}) \\
j_\mu(x) &= i(\phi_i\partial_\mu\phi_i^* - \phi_i^*\partial_\mu\phi_i - \widetilde{\phi}_i\partial_\mu\widetilde{\phi}_i^* + \widetilde{\phi}_i^*\partial_\mu\widetilde{\phi}_i) + \psi_i\sigma_\mu\psi_i - \widetilde{\psi}_i\sigma_\mu\widetilde{\psi}_i
\end{aligned}
\quad \text{(B.2)}
$$

where we have implicitly summed over i.

Let us now write the various current two-point functions. To perform the calculation, it will be convenient to change basis from the gauge eigenstates appearing in (B.2) to the mass eigenstates via the following expressions

$$\phi_i = R_{ia} \cdot \varphi_a, \quad \widetilde{\phi}_i^* = R_{(i+N)a} \cdot \varphi_a \quad \text{(B.3)}$$

where $i = 1, ..., N$, $a = 1, ..., 2N$, and R is a $2N \times 2N$ unitary matrix. Let us also denote the bosonic (fermionic) mass eigenvalues by μ_a (M_i). Inserting (B.3) into (B.2), and performing the contractions to evaluate the correlators, we find

$$C_0(p) = \left(\overline{R}_{ia}R_{ib} - \overline{R}_{(j+N)a}R_{(j+N)b}\right)\left(\overline{R}_{kb}R_{ka} - \overline{R}_{(l+N)b}R_{(l+N)a}\right)I(p, \mu_a, \mu_b)$$

$$C_{1/2}(p) = \frac{p^2 + \mu_a^2 - M_i^2}{p^2}\left(R_{ia}\overline{R}_{ia} + R_{(i+N)a}\overline{R}_{(i+N)a}\right)I(p, \mu_a, M_i) + \frac{1}{p^2}\left(J(\mu_a) - 2J(M_i)\right)$$

$$C_1(p) = \frac{1}{3p^2}\left((p^2 + 4\mu_a^2)I(p, \mu_a, \mu_a) + 4(p^2 - 2M_i^2)I(p, M_i, M_i) + 4J(\mu_a) - 8J(M_i) + \frac{\mu_a^2 - 2M_i^2}{4\pi^2}\right)$$

$$B = -4M_i R_{ia}\overline{R}_{(i+N)a}I(0, M_i, \mu_a)$$

$$\text{(B.4)}$$

where all indices are summed, and we define

$$
I(p, m_1, m_2) = \int \frac{d^4q}{(2\pi)^4} \frac{1}{((p+q)^2 + m_1^2)(q^2 + m_2^2)}
$$

$$
= \frac{1}{16\pi^2} \left(\log \frac{\Lambda_q^2}{p^2} + 1 \right) + \frac{1}{16\pi^2 p^2} \left(m_1^2 \log \frac{m_1^2}{p^2} + m_2^2 \log \frac{m_2^2}{p^2} - m_1^2 - m_2^2 \right)
$$

$$
+ \mathcal{O}\left(\frac{1}{p^4}, \frac{\log p^2}{p^4} \right)
$$

$$
J(m) = \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 + m^2} = \frac{\Lambda_q^2}{16\pi^2} + \frac{m^2}{16\pi^2} \log \frac{m^2}{\Lambda_q^2}
$$

(B.5)

where Λ_q is a momentum cutoff for the q integral. Simple consistency checks of the expressions in (B.4) are the following. As follows from supersymmetry, they all have the same asymptotic behavior, $\frac{N}{8\pi^2} \log \frac{\Lambda^2}{p^2}$, at large p. Also, since there are no massless particles in the loop, they are finite as $p \to 0$.[10]

Let us now show that a non-vanishing messenger supertrace necessarily generates a logarithmically divergent scalar counterterm. Recall first the expression (2.10) for the A parameter

$$
A \equiv - \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \left(3C_1(p) - 4C_{1/2}(p) + C_0(p) \right)
$$

(B.6)

Using (B.5), (B.4) and focussing on the $\mathcal{O}(1/p^2)$ terms (one can check that the $\mathcal{O}(p^0, \log p)$ terms, and hence the dependence on Λ_q, always vanish in (B.6)), we find

$$
\delta A = -\frac{1}{64\pi^4} \left(\text{Tr}\mu^2 - 2\text{Tr}M^2 \right) \log \Lambda^2 = -\frac{1}{128\pi^4} \text{Str}\,\mathcal{M}^2 \cdot \log \Lambda^2
$$

(B.7)

where Λ is the cutoff of the p integral in (B.6).

In this example we took the gauge group to be $U(1)$ and took all the messengers to have charge ± 1. More generally, one obtains a charge-weighted supertrace, or to be precise

$$
\delta A = -\frac{1}{128\pi^4} \sum_R \text{Str}\, N_R \mathcal{M}_R^2 \cdot \log \Lambda^2
$$

(B.8)

where the supertrace is taken over the subset of messengers transforming in irrep R and N_R is the Dynkin index of irrep R.

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