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Goldstinos, Supercurrents and Metastable SUSY Breaking in $\mathcal{N} = 2$ Supersymmetric Gauge Theories

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We construct an $\mathcal{N} = 2$ supersymmetric generalization of the $\mathcal{N} = 1$ supercurrent formalism of Komargodski and Seiberg (KS) and use it to show that $\mathcal{N} = 2$ theories with linear superconformal anomalies cannot break SUSY under certain broad assumptions. This result suggests that there are no metastable SUSY breaking vacua in a large class of theories that includes $\mathcal{N} = 2$ Super Yang-Mills (SYM).

May 2010

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1. Introduction

It is well-known that $\mathcal{N} = 1$ supersymmetric QCD (SQCD) has metastable supersymmetry (SUSY) breaking vacua [1]. However, the situation for $\mathcal{N} = 2$ SQCD is less clear. Indeed, while Seiberg and Witten found an exact description of the quantum moduli space of SUSY vacua in $\mathcal{N} = 2$ SQCD [2][3], one expects, in particular, that higher derivative corrections would be important in any (metastable) SUSY breaking vacua. In this paper, we give strong evidence that $\mathcal{N} = 2$ SYM and its cousins do not have metastable SUSY breaking vacua.

Our proof is based on generalizing the results of a recent work by Komargodski and Seiberg [4] (see also [5][6]). In that paper, the authors showed that the superconformal anomaly multiplet defined in the UV flows, under the renormalization group, to a constrained chiral superfield in the IR that contains the goldstino as a component.

In what follows, we will use the RG evolution of the $\mathcal{N} = 2$ linear superconformal anomaly multiplet to show that as long as an $\mathcal{N} = 2$ QFT has such a multiplet, it cannot contain SUSY breaking vacua that admit a weakly coupled description in terms of goldstinos and goldstone bosons.[2] We explicitly show that $\mathcal{N} = 2$ SYM falls into this class of theories and therefore conclude that it cannot contain weakly coupled SUSY breaking vacua. In fact, this proof generalizes to arbitrary gauge group and matter content as long as the theory possesses an (at most spontaneously broken) $SU(2)_R$ symmetry.[3] We will describe more explicitly some of these extensions after our detailed discussion of $\mathcal{N} = 2$ SYM.

It would also be interesting to extend our analysis to theories that explicitly break $SU(2)_R$. We know that some theories in this class *do* admit SUSY breaking vacua since they contain $\mathcal{N} = 2$ D-terms which also explicitly break $SU(2)_R$ (this set of theories includes modifications of the original $\mathcal{N} = 2$ SUSY breaking theory studied by Fayet [8], as well as models of partial supersymmetry breaking [9]). These theories do not posses linear superconformal anomaly multiplets.

In the next section we briefly review the KS formalism and then quickly proceed to generalize it to the case of interest. We end with some open questions.

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[2] It would be interesting to better understand the relation of this statement to the no-go theorems recently discussed in [7].

[3] We would like to emphasize that it is not important whether the theory admits additional superconformal anomaly multiplets. Indeed, as long as the theory has a linear superconformal anomaly multiplet, we are free to study its RG flow. In doing so, we will find the contradictions described below.

2. The KS formalism

The starting point of the KS formalism is the supercurrent multiplet constructed by Ferrara and Zumino (FZ) [10]. In a superconformal theory, the FZ multiplet is described by a real superfield, $\mathcal{J}_{\alpha\dot\alpha}$, subject to the constraint

$$D^\alpha \mathcal{J}_{\alpha\dot\alpha} = 0. \tag{2.1}$$

This multiplet contains as its lowest component a conserved $U(1)_R$ current, j^μ, while the θ and $\theta\bar\theta$ components correspond to the supercurrent, S^μ_α, and the stress-tensor, $T_{\mu\nu}$, respectively. One can describe the breaking of superconformal symmetry in a relatively general way[4] by modifying (2.1) as follows:

$$D^\alpha \mathcal{J}_{\alpha\dot\alpha} = \overline{D}_{\dot\alpha}\overline{X}, \tag{2.2}$$

where X is a chiral superfield, i.e.

$$\overline{D}_{\dot\alpha}X = 0. \tag{2.3}$$

After solving the equation (2.2) in components one finds

$$X = x + \sqrt{2}\theta^\alpha \left(\frac{\sqrt{2}}{3}\sigma^\mu_{\alpha\dot\alpha}\overline{S}^{\dot\alpha}_\mu\right) + \theta^2\left(\frac{2}{3}T + i\partial_\mu j^\mu\right), \tag{2.4}$$

where the θ component of X is just the spin-half component of the supercurrent, T is the trace of the stress tensor, and $\partial_\mu j^\mu$ is the divergence of the $U(1)_R$ current.

The remarkable insight of Komargodski and Seiberg was to realize that in theories of SUSY breaking, the operator X, defined in the UV, flows to a well-defined operator, X_{NL}, in the deep IR that contains the goldstino. For a generic theory, the scalar partner of the goldstino will acquire a mass. The simplest state such an operator can then create is the two goldstino state. Since supercharge (anti)commutators are well-defined even when SUSY is spontaneously broken, the components of X must satisfy chiral commutation relations along the full RG flow. It is then easy to see that the unique form of X, up to an uninteresting constant, is [4]

$$X_{NL} = \frac{G^2}{2F} + \sqrt{2}\theta G + \theta^2 F, \tag{2.5}$$

where G_α is the goldstino. Additionally, it follows from (2.5) that X_{NL} satisfies the nilpotent relation

$$X_{NL}^2 = 0. \tag{2.6}$$

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[4] For various caveats that will not be important to us below, see [6] and [11].

3. Generalizing the KS formalism to $\mathcal{N} = 2$

Let us now generalize the above construction to $\mathcal{N} = 2$ SUSY. While there is no unique generalization of the previous discussion to $\mathcal{N} = 2$, we will find that a simple generalization, due originally to Sohnius [12] (see also the more recent discussion in [13]), suffices for our purposes. In particular, we will see that the conservation equation (2.2) follows from a projection to $\mathcal{N} = 1$ superspace and that the multiplet we consider consists of the two supercurrents, the stress tensor, and the four currents of the $U(2)_R$ $\mathcal{N} = 2$ R-symmetry (along with various other operators required by extended SUSY). Of the four R-currents in the supercurrent multiplet, three of them, corresponding to $SU(2)_R \subset U(2)_R$, will be conserved while the fourth, corresponding to the $\mathcal{N} = 1$ superconformal R-current, will not.[5] The reason we choose such a multiplet is simple: it is the precise multiplet structure one finds in $\mathcal{N} = 2$ SYM [14][15][16].

3.1. $\mathcal{N} = 2$ SCFT warmup

We begin our analysis by considering the $\mathcal{N} = 2$ analog of (2.1). It turns out that the supercurrents of an $\mathcal{N} = 2$ superconformal theory can be packaged into a real dimension two superfield, $\mathcal{J}$, that satisfies

$$D^{\langle ij \rangle} \mathcal{J} = \overline{D}^{\langle ij \rangle} \mathcal{J} = 0, \tag{3.1}$$

where $i = 1, 2$ is an index in the fundamental of $SU(2)_R$ and we place $SU(2)_R$ indices inside angular brackets to distinguish them from powers of operators.[6] The independent components of $\mathcal{J}$ are then

	$SU(2)_R$	Dim	
J	**1**	2	
$J_\alpha^{\langle i \rangle}$	**2**	5/2	
$J_{\alpha\beta}$	**1**	3	
$J_{\langle j \rangle \mu}^{\langle i \rangle}$	**3**	3	(3.2)
J_μ	**1**	3	
$J_{\mu\alpha}^{\langle i \rangle}$	**2**	7/2	
$T_{\mu\nu}$	**1**	4	

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[5] One can also consider a multiplet in which all four $U(2)_R$ currents are conserved [14]. This is a simple generalization of the $\mathcal{N} = 1$ case in which the theory has an exact $U(1)_R$ symmetry.

[6] In (3.1) we define the $SU(2)_R$ spin one differential operator, $D^{\langle ij \rangle}$, to be $D^{\langle ij \rangle} = D^{\langle ji \rangle} \equiv D^{\langle j \rangle \alpha} D_\alpha^{\langle i \rangle}$.

where we have omitted the hermitian conjugates $\overline{J}_{\langle i \rangle \dot{\alpha}}$, $\overline{J}_{\dot{\alpha}\dot{\beta}}$, and $\overline{J}_{\langle i \rangle \mu \dot{\alpha}}$.[7]

<div align="center">Reformulation in terms of $\mathcal{N} = 1$ superspace</div>

For our discussion below, we will not need the full $\mathcal{N} = 2$ superspace. In fact, it will be convenient to consider the above operators in an explicit $\mathcal{N} = 1$ basis and to define rotations into the remaining half of $\mathcal{N} = 2$ superspace via the action of the second set of supercharges. Therefore, let us consider the following $\mathcal{N} = 1$ projections of $\mathcal{J}^{\langle ij \rangle}$

$$\widehat{\mathcal{J}} \equiv \mathcal{J}|, \quad \mathcal{J}_\alpha \equiv (D_\alpha^{\langle 2 \rangle} \mathcal{J})|, \quad \mathcal{J}_{\alpha\dot\alpha} = \left(-\frac{1}{3} \left[D_\alpha^{\langle 1 \rangle}, \overline{D}_{\langle 1 \rangle \dot\alpha} \right] + \left[D_\alpha^{\langle 2 \rangle}, \overline{D}_{\langle 2 \rangle \dot\alpha} \right] \right) \mathcal{J}|, \qquad (3.3)$$

where the vertical line "|" denotes the origin of the $(\theta_{\langle 2 \rangle}, \overline{\theta}^{\langle 2 \rangle})$ superspace. These multiplets obey the following $\mathcal{N} = 1$ constraints descending from the $\mathcal{N} = 2$ constraint in (3.1)

$$\overline{D}^2 \widehat{\mathcal{J}} = D^2 \widehat{\mathcal{J}} = 0, \quad D^\alpha \mathcal{J}_\alpha = 0, \quad \overline{D}^2 \mathcal{J}_\alpha = 0, \quad \overline{D}^{\dot\alpha} \mathcal{J}_{\alpha\dot\alpha} = 0, \qquad (3.4)$$

where the third constraint follows from the first by applying a variation with respect to the second SUSY. Note that the first equation implies that $\widehat{\mathcal{J}}$ is a conserved current multiplet. The second and third equations imply that $\mathcal{J}_\alpha$ is a conserved current multiplet as well. As we will see momentarily, the conserved currents sitting in the $\theta\overline{\theta}$ components $\mathcal{J}_\alpha$ and $\widehat{\mathcal{J}}$ are just the second supercurrent and the R-current corresponding to the R-symmetry that leaves the explicit $\mathcal{N} = 1$ superspace invariant.[8]

We can expand the above multiplets in $\mathcal{N} = 1$ superspace as follows

$$\widehat{\mathcal{J}} = J + \theta^\alpha J_\alpha^{\langle 1 \rangle} + \overline{\theta}_{\dot\alpha} \overline{J}_{\langle 1 \rangle}^{\dot\alpha} + \theta\sigma^\mu\overline{\theta}(-\frac{1}{2}J_\mu + J_{\langle 1 \rangle \mu}^{\langle 1 \rangle}) + \mathcal{O}(\theta^2\overline{\theta}, \overline{\theta}^2\theta)$$

$$\mathcal{J}_\alpha = J_\alpha^{\langle 2 \rangle} + \theta^\beta J_{\beta\alpha} + \sigma_{\alpha\dot\alpha}^\mu \overline{\theta}^{\dot\alpha} J_{\langle 1 \rangle \mu}^{\langle 2 \rangle} + \theta\sigma^\mu\overline{\theta}(-J_{\mu\alpha}^{\langle 2 \rangle} + \frac{2}{3}i\sigma_{\mu\nu\alpha}{}^\beta \partial^\nu J_\beta^{\langle 2 \rangle}) + \mathcal{O}(\theta^2\overline{\theta}, \overline{\theta}^2\theta)$$

$$\mathcal{J}_\mu = \frac{1}{3}J_\mu + \frac{4}{3}J_{\langle 1 \rangle \mu}^{\langle 1 \rangle} + \theta^\alpha J_{\mu\alpha}^{\langle 1 \rangle} + \overline{\theta}_{\dot\alpha} J_{\langle 1 \rangle \mu}^{\dot\alpha} + \theta\sigma^\nu\overline{\theta} \left(2T_{\nu\mu} - \frac{1}{4}\epsilon_{\nu\mu\rho\sigma}\partial^{[\rho} j^{\sigma]} \right) + \mathcal{O}(\theta^2\overline{\theta}, \overline{\theta}^2\theta)$$

$$(3.5)$$

where the higher-order terms depend on the lower components.[9] From the expansions in (3.5) and the conservation equations in (3.4), we easily verify the following component

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[7] In our conventions, $J_{\langle j \rangle \mu}^{\langle i \rangle} \equiv \frac{1}{2}\sigma^{a\langle i \rangle}_{\langle j \rangle} J_{a\mu}$.

[8] Since the $\mathcal{N} = 1$ supercurrent does not sit in $\widehat{\mathcal{J}}$, the only consistent R-current that can sit in $\widehat{\mathcal{J}}$ is the one we have described.

[9] Note that as in [4], the lowest component of $\mathcal{J}_{\alpha\dot\alpha}$ is just the superconformal R-current of the manifest $\mathcal{N} = 1$ supersymmetry. Also, in the last equation of (3.5) we have defined $\mathcal{J}_\mu = \frac{1}{4}\overline{\sigma}^{\mu\dot\alpha\alpha} \mathcal{J}_{\alpha\dot\alpha}$.

<div align="center">4</div>

conservation equations and trace identities

$$\partial^\mu J_\mu^{\langle ij\rangle} = 0, \quad \partial^\mu J_\mu = 0, \quad \partial^\mu J_{\mu\alpha}^{\langle i\rangle} = 0, \quad \overline{\sigma}^{\mu\dot\alpha\alpha} J_{\mu\alpha}^{\langle i\rangle} = 0, \quad \partial^\mu T_{\mu\nu} = 0, \quad T = 0. \qquad (3.6)$$

In fact, one can check that $J_{\langle j\rangle\mu}^{\langle i\rangle}$ and J_μ are just the conserved superconformal R-currents, $J_{\mu\alpha}^{\langle i\rangle}$ are the conserved supercurrents, and $T_{\mu\nu}$ is the conserved and traceless stress-tensor.[10]

3.2. The $\mathcal{N} = 2$ linear superconformal anomaly multiplet

To eventually make contact with SUSY breaking, we must generalize (2.2) by adding an appropriate representation of $\mathcal{N} = 2$ SUSY to the RHS of (3.1). For our purposes it suffices to introduce the linear anomaly multiplet originally considered by Sohnius in [12] and so we take

$$D^{\langle ij\rangle}\mathcal{J} = 3\mathcal{L}^{\langle ij\rangle}, \qquad (3.7)$$

with $\mathcal{L}^{\langle ij\rangle}$ being a real $SU(2)_R$ spin one $\mathcal{N} = 2$ linear superfield, i.e. a multiplet satisfying the following conditions

$$(\mathcal{L}^{\langle ij\rangle})^\dagger = \epsilon_{\langle ik\rangle}\epsilon_{\langle jl\rangle}\mathcal{L}^{\langle kl\rangle}, \quad \mathcal{L}^{\langle ij\rangle} = \mathcal{L}^{\langle ji\rangle}, \quad D_\alpha^{(\langle i}\mathcal{L}^{\langle jk\rangle)} = \overline{D}_{\dot\alpha}^{(\langle i}\mathcal{L}^{\langle jk\rangle)} = 0, \qquad (3.8)$$

where "(...)" denotes total symmetrization of the included indices. The independent components of $\mathcal{L}^{\langle ij\rangle}$ are then

	$SU(2)_R$	Dim	
$L^{\langle ij\rangle}$	**3**	3	
$L_\alpha^{\langle i\rangle}$	**2**	7/2	(3.9)
L_0	**1**	4	
L_μ	**1**	4	

Let us stress once more that, as mentioned in the introduction, the theories we study have a conserved (possibly spontaneously broken) $SU(2)_R \subset U(2)_R$ symmetry. Indeed, as we will see below, any solution to (3.7) necessarily satisfies

$$\partial_\mu J_{\langle j\rangle}^{\langle i\rangle\mu} = 0. \qquad (3.10)$$

The intuitive reason for this is that, as we can see from (3.9), $\mathcal{L}^{\langle ij\rangle}$ lacks a dimension four $SU(2)_R$ scalar triplet operator.

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[10] As a result, the $\mathcal{J}$ multiplet has twenty-four bosonic and twenty-four fermionic components.

The $\mathcal{N}=2$ linear anomaly multiplet in $\mathcal{N}=1$ superspace

Momentarily, it will prove useful to consider the organization of the fields in $\mathcal{L}^{\langle ij \rangle}$ under the manifest $\mathcal{N}=1$ SUSY of the previous subsection, and so we define

$$X \equiv \mathcal{L}^{\langle 22 \rangle}|, \quad L \equiv i\mathcal{L}^{\langle 12 \rangle}| \tag{3.11}$$

Notice that the reality condition in (3.8) implies that X is complex while L is real.[11] Furthermore, the $\mathcal{N}=2$ current conservation equation in (3.8) descends to the following equations in $\mathcal{N}=1$ superspace

$$\overline{D}_{\dot{\alpha}} X = 0, \quad D^2 L = \overline{D}^2 L = 0. \tag{3.12}$$

As a result, we see that X is just a chiral superfield while L is an $\mathcal{N}=1$ linear superfield. Therefore, the full anomaly multiplet has eight bosonic and eight fermionic components with the following superfield expansion

$$X = L^{\langle 22 \rangle} + \theta^\alpha L_\alpha^{\langle 2 \rangle} + \theta^2 L_0 + i\theta\sigma^\mu\overline{\theta}\partial_\mu L^{\langle 22 \rangle} + \mathcal{O}(\theta^2\overline{\theta})$$

$$L = iL^{\langle 12 \rangle} + \frac{i}{2}\theta^\alpha L_\alpha^{\langle 1 \rangle} - \frac{i}{2}\overline{\theta}_{\dot{\alpha}}\overline{L}_{\langle 1 \rangle}^{\dot{\alpha}} + i\theta\sigma^\mu\overline{\theta}L_\mu + \mathcal{O}(\theta^2\overline{\theta}, \overline{\theta}^2\theta) \tag{3.13}$$

where L^μ is the conserved dimension four central charge current.

Analyzing (3.7) in $\mathcal{N}=1$ superspace, we find the following equations

$$\overline{D}^2 \widehat{\mathcal{J}} = 3X, \quad D^\alpha \mathcal{J}_\alpha = -3iL, \quad \overline{D}^2 \mathcal{J}_\alpha = 0, \quad \overline{D}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X \tag{3.14}$$

The third equation in (3.14) follows from the first equation because X is *anti-chiral* with respect to the second SUSY. The fourth equation defines $\mathcal{J}_{\alpha\dot{\alpha}}$ as the FZ multiplet of the explicitly realized $\mathcal{N}=1$ SUSY.

Solving the second, third, and fourth equations in (3.14) we find

$$\mathcal{J}_\alpha = J_\alpha^{\langle 2 \rangle} + \theta^\beta\left(J_{\beta\alpha} - \frac{3}{2}i\epsilon_{\beta\alpha}\ell\right) + \sigma_{\alpha\dot{\alpha}}^\mu\overline{\theta}^{\dot{\alpha}}J_{\langle 1\rangle\mu}^{\langle 2 \rangle} + \theta\sigma^\mu\overline{\theta}(-J_{\mu\alpha}^{\langle 2 \rangle} - \frac{1}{2}(\sigma_\mu\overline{\sigma}^\rho J_\rho^{\langle 2 \rangle})_\alpha$$

$$+ \frac{2}{3}i\sigma_{\mu\nu\alpha}^{\ \ \ \beta}\partial^\nu J_\beta^{\langle 2 \rangle}) + 2\theta^2\sigma_{\alpha\dot{\alpha}}^\mu\overline{J}_{\langle 2\rangle\mu}^{\dot{\alpha}} + \theta^2\overline{\theta}_{\dot{\alpha}}\left[\frac{3}{2}\sigma_\alpha^{\mu\dot{\alpha}}\left(-\frac{1}{2}\partial_\mu\ell + i\ell_\mu\right) + \frac{i}{2}\sigma_\beta^{\mu\dot{\alpha}}\partial_\mu J_\alpha^{\ \beta}\right] + \mathcal{O}(\overline{\theta}^2\theta)$$

$$\mathcal{J}_\mu = j_\mu^{\mathcal{N}=1} + \theta^\alpha\left(J_{\mu\alpha}^{\langle 1 \rangle} + \frac{1}{3}(\sigma_\mu\overline{\sigma}^\rho J_\rho^{\langle 1 \rangle})_\alpha\right) + \overline{\theta}_{\dot{\alpha}}\left(\overline{J}_{\langle 1\rangle\mu}^{\dot{\alpha}} + \frac{1}{3}\epsilon^{\dot{\alpha}\dot{\beta}}(\overline{J}_{\langle 1\rangle\rho}\overline{\sigma}^\rho\sigma_\mu)_{\dot{\beta}}\right)$$

$$+ \theta\sigma^\nu\overline{\theta}\left(2T_{\nu\mu} - \frac{2}{3}\eta_{\mu\nu}T - \frac{1}{4}\epsilon_{\nu\mu\rho\sigma}\partial^{[\rho}j_{\mathcal{N}=1}^{\sigma]}\right) + \frac{i}{2}\theta^2\partial_\mu\overline{x} - \frac{i}{2}\overline{\theta}^2\partial_\mu x + \mathcal{O}(\theta^2\overline{\theta}, \overline{\theta}^2\theta) \tag{3.15}$$

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[11] The definitions in (3.11) and the conservation equation in (3.8) imply that $\left[Q_\alpha^{\langle 2 \rangle}, X\right] = 0$, $\left[\overline{Q}_{\langle 2\rangle\dot{\alpha}}, X\right] = -2i\overline{D}_{\dot{\alpha}}L$, $\left[Q_\alpha^{\langle 2 \rangle}, \overline{X}\right] = 2iD_\alpha L$, $\left[\overline{Q}_{\langle 2\rangle\dot{\alpha}}, \overline{X}\right] = 0$, $\left[\overline{Q}_{\langle 2\rangle\dot{\alpha}}, L\right] = \frac{i}{2}\overline{D}_{\dot{\alpha}}\overline{X}$, and $\left[Q_\alpha^{\langle 2 \rangle}, L\right] = -\frac{i}{2}D_\alpha X$.

with X and L expressed in terms of these components as follows[12]

$$X = x + \theta^\alpha \left(\frac{2}{3} \sigma^\mu_{\alpha\dot\alpha} \overline{J}^{\dot\alpha}_{\langle 1\rangle\mu} \right) + \theta^2 \left(\frac{2}{3} T + \frac{i}{3} \partial_\mu J^\mu \right) + \mathcal{O}(\theta\overline\theta)$$

$$L = \ell - \frac{i}{2} \theta^\alpha \left(\frac{2}{3} \sigma^\mu_{\alpha\dot\alpha} \overline{J}^{\dot\alpha}_{\langle 2\rangle\mu} \right) + \frac{i}{2} \overline\theta_{\dot\alpha} \left(\frac{2}{3} \sigma^{\mu\dot\alpha}_\alpha J^{\langle 2\rangle\alpha}_\mu \right) + \theta\sigma^\mu\overline\theta\ell_\mu + \mathcal{O}(\theta^2\overline\theta, \overline\theta^2\theta). \tag{3.16}$$

In writing the F component of X, we have used the fact that our solution in (3.15) preserves $SU(2)_R$ $(\partial_\mu J^{\langle i\rangle\mu}_{\langle j\rangle} = 0)$ to rewrite

$$\partial_\mu j^\mu_{\mathcal{N}=1} = \frac{1}{3} \partial_\mu J^\mu. \tag{3.17}$$

Furthermore, note that, as claimed above, the conserved current in L corresponds to the central charge current since (3.15) yields

$$\left\{ Q_\alpha, J^{\langle 2\rangle\mu}_\beta \right\} = 3i\epsilon_{\alpha\beta}\ell^\mu + \left(2i\sigma^{\mu\nu}_{\alpha\gamma}\partial_\nu J^\gamma_\beta - \frac{2}{3} i\sigma^{\mu\nu}_{\beta\gamma}\partial_\nu J^\gamma_\alpha - 2\sigma^{\mu\nu}_{\beta\alpha}\partial_\nu\ell \right), \tag{3.18}$$

where the terms in parenthesis are Schwinger terms.

Of course, the solution we have derived in (3.15) is not unique. As stressed in [5][4], there is still the freedom to choose improvement terms. In our case, these improvement terms fill out a reduced $\mathcal{N} = 2$ chiral multiplet, W, i.e., a chiral multiplet satisfying the $\mathcal{N} = 2$ generalization of the Bianchi identity,

$$D^{\langle ij\rangle}W = \overline{D}^{\langle ij\rangle}\overline{W}. \tag{3.19}$$

Indeed, it is easy to check that for any solution $\mathcal{J}, \mathcal{L}^{\langle ij\rangle}$ to (3.7), the pair

$$\mathcal{J} - \frac{3}{2}(W + \overline{W}), \quad \mathcal{L}^{\langle ij\rangle} - \frac{1}{2}D^{\langle ij\rangle}W, \tag{3.20}$$

also satisfies the same equations with the component currents shifted by the following improvement terms

$$\delta J^{\langle j\rangle}_{\langle i\rangle\mu} = 0, \quad \delta\ell^\mu = 2\sqrt{2}\partial_\nu F^{\mu\nu}, \quad \delta J^{\langle i\rangle}_{\mu\alpha} = 2i\sigma_{\mu\nu\alpha}{}^\beta\partial^\nu\lambda^{\langle i\rangle}_\beta,$$

$$\delta T_{\mu\nu} = -(\partial_\nu\partial_\mu - \eta_{\mu\nu}\partial^2)(\phi + \overline\phi), \tag{3.21}$$

where $\lambda^{\langle i\rangle}$ are the two Weyl fermions in W, $\phi = W|_{\theta_{\langle i\rangle}, \overline\theta^{\langle i\rangle}}$ is the lowest component of the multiplet, and $F^{\mu\nu}$ is the anti-symmetric tensor in $W_\alpha \equiv \{Q^2_\alpha, W\}|_{\theta_{\langle 2\rangle}, \overline\theta^{\langle 2\rangle}}$.[13]

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[12] In (3.15) we have defined $j^{\mathcal{N}=1}_\mu \equiv \frac{1}{3}J_\mu + \frac{4}{3}J^{\langle 1\rangle}_{\langle 1\rangle\mu}$ and $\ell_\mu = iL_\mu$.

[13] The terms given in (3.21) are improvement terms as long as the fields on the RHS vanish sufficiently quickly as we go to infinity.

3.3. Spontaneous SUSY breaking

We would now like to study the solutions to (3.7) as we flow along the RG of a theory that spontaneously breaks the $\mathcal{N} = 2$ SUSY completely. Deep in the IR, the spin half projections of the two supercurrents should flow to the two independent goldstinos

$$S_\alpha^{\mu\langle i\rangle} = \sqrt{2} f^{\langle i\rangle} \sigma_{\alpha\dot\alpha}^\mu \overline{G}^{\langle i\rangle\dot\alpha} + ... \tag{3.22}$$

where we can take the $f^{\langle i\rangle}$ to be real and positive without loss of generality. Therefore, from (3.16) we see that in the IR X and L contain the two goldstinos as their $\mathcal{O}(\theta)$ components

$$X_{IR} = x + \sqrt{2}\theta^\alpha G_{\langle 1\rangle\alpha} + \theta^2 F + \mathcal{O}(\theta\bar\theta)$$

$$L_{IR} = \ell - \frac{i}{\sqrt{2}}\theta^\alpha G_{\langle 2\rangle\alpha} + \frac{i}{\sqrt{2}}\bar\theta_{\dot\alpha}\overline{G}^{\langle 2\rangle\dot\alpha} + \theta\sigma^\mu\bar\theta\ell_\mu + \mathcal{O}(\theta^2\bar\theta, \bar\theta^2\theta). \tag{3.23}$$

where we have rescaled the superfields by $3/8f$ and redefined the goldstino fields by taking[14]

$$G_{\langle i\rangle} \to \frac{f}{f^{\langle i\rangle}} G_{\langle i\rangle}. \tag{3.24}$$

It will be useful to remind ourselves again of the $SU(2)_R$ transformation properties of the various components in (3.23). For convenience, we translate table (3.9) into the language of (3.23)

	$SU(2)_R$	Dim
$(X, -iL)$	**3**	1
$G_{\langle i\rangle\alpha}$	**2**	3/2
F	**1**	2
ℓ_μ	**1**	2

(3.25)

Note that the decay constant sitting in F is an $SU(2)_R$ singlet.

Let us now proceed with our analysis of the IR form of the anomaly multiplet. On general grounds, we expect that unless the bosonic superpartners of the goldstinos in X_{IR} and L_{IR} correspond to goldstone bosons of broken symmetries, they will acquire a mass.

Keeping these facts in mind, first consider the case of unbroken $SU(2)_R$. At zeroth order in an expansion in derivatives, we find that the $\mathcal{N} = 2$ SUSY algebra for the linear multiplet (see Appendix A) fixes the low energy superfields as follows

$$X_{IR} = \frac{G_{\langle 1\rangle}G_{\langle 1\rangle}}{2F} + \frac{\overline{G}^{\langle 2\rangle}\overline{G}^{\langle 2\rangle}}{2\overline{F}} + \sqrt{2}\theta G_{\langle 1\rangle} + \theta^2 F + \mathcal{O}(\partial^\mu),$$

$$L_{IR} = -i\frac{G_{\langle 1\rangle}G_{\langle 2\rangle}}{2F} + i\frac{\overline{G}^{\langle 1\rangle}\overline{G}^{\langle 2\rangle}}{2\overline{F}} - \frac{i}{\sqrt{2}}\theta G_{\langle 2\rangle} + \frac{i}{\sqrt{2}}\bar\theta\overline{G}^{\langle 2\rangle} + \mathcal{O}(\partial^\mu). \tag{3.26}$$

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[14] We give the resulting component SUSY transformations in Appendix A.

This solution satisfies the zero momentum (i.e., zero derivative) $\mathcal{N} = 2$ SUSY algebra. However, the explicit solution in (3.26) does not satisfy the full $\mathcal{N} = 2$ SUSY algebra. Indeed, consider imposing the $\mathcal{N} = 1$ chirality condition, $[\overline{\xi Q}^{\langle 1 \rangle}, x_{IR}] = 0$. We then must demand that at the one derivative level

$$x_{IR} = \frac{G_{\langle 1 \rangle} G_{\langle 1 \rangle}}{2F} + \frac{\overline{G}^{\langle 2 \rangle} \overline{G}^{\langle 2 \rangle}}{2\overline{F}} - \frac{i}{2|F|^2} \left(G_{\langle 1 \rangle} \sigma^\mu \overline{G}^{\langle 1 \rangle} + G_{\langle 2 \rangle} \sigma^\mu \overline{G}^{\langle 2 \rangle} \right) \partial_\mu \left(\frac{\overline{G}^{\langle 2 \rangle} \overline{G}^{\langle 2 \rangle}}{\overline{F}} \right)$$
$$+ g_{(0)\dot{\alpha}}^\mu (G_{\langle i \rangle}, \overline{G}^{\langle 2 \rangle}) \partial_\mu \left(\frac{\overline{G}^{\langle 1 \rangle \dot{\alpha}}}{\overline{F}} \right) + g_{(1)}(G_{\langle i \rangle}, \overline{G}^{\langle 2 \rangle}) + \mathcal{O}(\partial^2),$$

$$(3.27)$$

where the terms in $g_{(0)\dot{\alpha}}^\mu$ have zero derivatives and are independent of $\overline{G}^{\langle 1 \rangle}$. Similarly, the terms in $g_{(1)}$ have one derivative and are independent of $\overline{G}^{\langle 1 \rangle}$.

On the other hand, imposing the anti-chirality condition with respect to the second SUSY, $\left[\xi Q_{\langle 2 \rangle}, x_{IR} \right] = 0$, we find that

$$x_{IR} = \frac{G_{\langle 1 \rangle} G_{\langle 1 \rangle}}{2F} + \frac{\overline{G}^{\langle 2 \rangle} \overline{G}^{\langle 2 \rangle}}{2\overline{F}} + \frac{i}{2|F|^2} \left(G_{\langle 1 \rangle} \sigma^\mu \overline{G}^{\langle 1 \rangle} + G_{\langle 2 \rangle} \sigma^\mu \overline{G}^{\langle 2 \rangle} \right) \partial_\mu \left(\frac{G_{\langle 1 \rangle} G_{\langle 1 \rangle}}{F} \right)$$
$$+ \widetilde{g}_{(0)\mu}^\alpha (G_{\langle 1 \rangle}, \overline{G}^{\langle i \rangle}) \partial^\mu \left(\frac{G_{\langle 2 \rangle \alpha}}{F} \right) + \widetilde{g}_{(1)}(G_{\langle 1 \rangle}, \overline{G}^{\langle i \rangle}) + \mathcal{O}(\partial^2),$$

$$(3.28)$$

where the terms in $\widetilde{g}_{(0)\mu}^\alpha$ have zero derivatives and are independent of $G_{\langle 2 \rangle}$. Similarly, the terms in $\widetilde{g}_{(1)}$ have one derivative and are independent of $G_{\langle 2 \rangle}$. Clearly, (3.27) and (3.28) are not mutually compatible and so we conclude that the goldstinos must have scalar superpartners in the deep IR. Since the SUSY breaking theory was assumed to be interacting, this is unnatural and represents a contradiction.

We should note that in writing (3.27) and (3.28), we have assumed that the spin one operator in L_{IR} is composite, i.e.

$$\ell_{IR}^\mu = \partial_\nu \left(\frac{1}{F} G_{\langle 1 \rangle} \sigma^{\mu\nu} G_{\langle 2 \rangle} + \frac{1}{\overline{F}} \overline{G}^{\langle 2 \rangle} \overline{\sigma}^{\nu\mu} \overline{G}^{\langle 1 \rangle} \right) + \mathcal{O}(\partial^2). \qquad (3.29)$$

One may worry that it is possible to have ℓ^μ non-composite in the IR if, for instance, the central charge symmetry is spontaneously broken or if ℓ_{IR}^μ is given by the field strength of an abelian gauge field of an unbroken gauge symmetry. While this situation is certainly possible, it does not affect our above reasoning. Indeed, we would find that (3.27) and (3.28) are essentially the same up to terms proportional to the difference of the right and left hand sides of (3.29), and so our logic is not affected.

It follows from the above analysis that if the goldstinos of $\mathcal{N} = 2$ spontaneous SUSY breaking belong to a linear superfield of the type $\mathcal{L}^{\langle ij \rangle}$, or equivalently (L, X), there are no constraints compatible with $\mathcal{N} = 2$ SUSY and $SU(2)_R$ that can eliminate all remaining components. In principle, this should be possible to show directly using $\mathcal{N} = 2$ superfields which we do not use here.[15] The next step would be to weaken our requirement on the IR theory, by including other massless degrees of freedom besides the goldstinos and ℓ^μ_{IR}. However, as we show below, in that case, although weaker constraints might be compatible with $\mathcal{N} = 2$ SUSY, the symmetries of the low energy theory turn out to be different from those in the UV.

For instance, the previous reasoning does not necessarily go through when the $SU(2)_R$ symmetry is spontaneously broken. Indeed, in such a case, the lowest components of X_{IR} and L_{IR} can contain the R-axions of the broken $SU(2)_R$ since their dimensions and quantum numbers are the same.

However, it is still useful to ask whether we can construct a consistent low energy effective action using X_{IR} and L_{IR} (with the R-axions now in the lowest components). In order to make this question meaningful, we will assume that whatever strong dynamics takes place in the theory can be integrated out and what is left in the deep IR is a weakly coupled theory of goldstinos and Goldstone bosons. In such a case, we expect

$$\mathcal{L}_{IR} = \int d^4\theta \left(X_{IR} \overline{X}_{IR} - 2L^2_{IR} \right) + \left(\int d^2\theta f X_{IR} + c.c. \right) + ... \qquad (3.30)$$

where the ellipses contain weak interactions and kinetic terms for the other light fields.

In order for (3.30) to constitute a consistent low energy description of our original UV theory, it must be the case that the superconformal anomaly multiplet for (3.30) matches the (RG evolved) superconformal anomaly multiplet for the UV theory. In other words, the anomaly multiplet for (3.30) should just be the pair (X_{IR}, L_{IR}) up to possible field redefinitions. However, this is not what happens. The heuristic reason for this difference is that the action for the linear multiplet has a different set of symmetries than the original theory. In particular, there is no (non-trivial) conserved central charge current.

To make our discussion more precise, consider the $\mathcal{N} = 2$ supercurrent for the theory in (3.30)

$$\mathcal{J} = 2 \left(\mathcal{L}^{\langle 11 \rangle} \mathcal{L}^{\langle 22 \rangle} - \mathcal{L}^{\langle 12 \rangle} \mathcal{L}^{\langle 12 \rangle} \right). \qquad (3.31)$$

---

[15] A possible starting point would be to use the off-shell formulation of the $\mathcal{N} = 2$ linear superfield used in [17] which admits also a constraint with one non-linear SUSY.

From this expression, we then deduce the following component conservation equations

$$\overline{D}^2\widehat{\mathcal{J}} = 8fX_{IR} + 2\overline{D}^2(L_{IR}^2),$$

$$D^\alpha\mathcal{J}_\alpha = 2iD^\alpha L_{IR}D_\alpha X_{IR} - 8ifL_{IR},$$

$$\overline{D}^2\mathcal{J}_\alpha = -2i\overline{D}^2\left(L_{IR}D_\alpha X_{IR}\right),$$

$$\overline{D}^{\dot\alpha}\mathcal{J}_{\alpha\dot\alpha} = \frac{8}{3}D_\alpha\left(fX_{IR} - \overline{D}_{\dot\alpha}L_{IR}\overline{D}^{\dot\alpha}L_{IR}\right) + 2\overline{D}^2D_\alpha\left(L_{IR}^2\right) + 2\overline{D}_{\dot\alpha}D_\alpha X_{IR}\overline{D}^{\dot\alpha}\overline{X}_{IR}.$$

$$(3.32)$$

Notice that this set of equations cannot be brought to the form given in (3.14) with $(X, L) \to (X_{IR}, L_{IR})$. Indeed, suppose we try to shift $\widehat{\mathcal{J}}$ so that it satisfies the first conservation equation in (3.14). After performing this transformation, we would still find that $D^\alpha\mathcal{J}_\alpha$ is not a real linear superfield. This mismatch is a symptom of the fact that the symmetry structure of the IR theory is not compatible with the symmetry structure of the original theory.

4. Metastable vacua in $\mathcal{N} = 2$ SYM

In the previous section, we saw that any $\mathcal{N} = 2$ theory with a linear superconformal anomaly cannot break SUSY under the assumption of a weakly coupled description in the deep IR. As we will now see, one interesting consequence of the above proof is that if this assumption holds for $\mathcal{N} = 2$ SYM, it cannot contain metastable SUSY breaking vacua.

To understand this statement, we begin by noting that in the asymptotically free regime, dimensional analysis and $U(2)_R$ covariance tell us that, up to an overall constant, the SYM supercurrent conservation equation should be (see also the discussions in [16][14][15])

$$D^{\langle ij\rangle}\mathcal{J} = \mathcal{L}_{\mathbf{C},\mathrm{SYM}}^{\langle ij\rangle} = \frac{c}{2}\overline{D}^{\langle ij\rangle}\mathrm{tr}\overline{W}^2 \tag{4.1}$$

where $c = 8\pi i\beta$ is the one-loop beta function [18] and W is the SYM field strength superfield. Notice that the anomaly satisfies

$$D_\alpha^{(\langle i\rangle}\mathcal{L}_{\mathbf{CSYM}}^{\langle jk\rangle)} = \overline{D}_{\dot\alpha}^{(\langle i\rangle}\mathcal{L}_{\mathbf{CSYM}}^{\langle jk\rangle)} = 0 \tag{4.2}$$

and so it is an $\mathcal{N} = 2$ linear superfield. However, unlike the linear superfield described in the previous section, this superfield is complex (hence the subscript "**C**") and so

$$(\mathcal{L}_{\mathbf{CSYM}}^{\langle ij\rangle})^\dagger \neq \epsilon_{\langle ik\rangle}\epsilon_{\langle jl\rangle}\mathcal{L}_{\mathbf{CSYM}}^{\langle kl\rangle} \tag{4.3}$$

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In particular, (4.1) and (4.3) imply that $\mathcal{J}$ embeds all of the conserved currents considered in the previous section plus an additional central charge current (i.e., the central charge current is now complex).

However, as mentioned by the authors in [13], (4.1) can be brought to the form (3.7) by the local gauge invariant shift $\mathcal{J} \to \mathcal{J} - \frac{c}{2}\mathrm{tr}W^2 + \frac{c}{2}\mathrm{tr}\overline{W}^2$. In particular, we find

$$D^{\langle ij \rangle}\mathcal{J} = \frac{c}{2}(\overline{D}^{\langle ij \rangle}\mathrm{tr}\overline{W}^2 - D^{\langle ij \rangle}\mathrm{tr}W^2) \tag{4.4}$$

Therefore, it follows from the results of the previous section that $\mathcal{N} = 2$ SYM has no metastable vacua as long as the theory has a weakly coupled description in the IR. Alternatively, one can use arguments analogous to those given in the previous section for the complex anomaly multiplet in (4.1) and reach the same conclusions.

4.1. Beyond $\mathcal{N} = 2$ SYM

We can now conjecture a generalization of this argument. Indeed, notice that the particular $\mathcal{N} = 2$ representation appearing on the RHS of (4.4) is fixed by the symmetries of the theory and the requirement that the equation be well-defined. Therefore, it seems plausible that a whole host of theories like $\mathcal{N} = 2$ SQCD (and various quiver generalizations) with arbitrary gauge group and number of flavors also cannot have weakly-coupled metastable vacua since these theories have supercurrent multiplets with the same conserved symmetries (conserved $SU(2)_R$ and central charge currents) as $\mathcal{N} = 2$ SYM.

More precisely, consider defining a $\mathcal{J}$ as in (4.1) for an arbitrary non-abelian gauge group (possibly a product gauge group consisting of many non-abelian factors) and adding matter hypermultiplets $Q_i, \widetilde{Q}^i$ transforming in an arbitrary representation of the gauge group so that the theory is still asymptotically free. Let us also assume that we do not deform the theory in the UV by introducing explicit $SU(2)_R$ breaking (note that we cannot include an FI term). Then, it is easy to see that $\mathcal{J}$ in (4.1) should be deformed as follows

$$\mathcal{J} \to \mathcal{J} + \frac{1}{2}\sum_i \left(Q_i\overline{Q}^i + \overline{\widetilde{Q}_i}\widetilde{Q}^i\right). \tag{4.5}$$

Therefore, the theory admits a linear superconformal anomaly and cannot posses weakly coupled SUSY breaking vacua.

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5. Conclusions

We have shown that theories with an $\mathcal{N} = 2$ linear superconformal anomaly cannot break SUSY as long as they admit a free description in the IR. This fact strongly suggests that a large class of theories that includes $\mathcal{N} = 2$ SYM cannot have metastable SUSY breaking vacua.

An interesting future direction would be to study generalizations of the ansatz in (3.7) to include anomalies that describe the breaking of more symmetries (like the $SU(2)_R$ symmetries). For example, one could imagine considering theories in which

$$D^{\langle ij \rangle} \mathcal{J} = 3 \mathcal{L}^{\langle ij \rangle}, \tag{5.1}$$

with real but non-conserved $\mathcal{L}^{ij}$, i.e.

$$D_\alpha^{(\langle i} \mathcal{L}^{\langle jk \rangle)} = D_{\alpha \langle l \rangle} \mathcal{T}^{\langle lijk \rangle}, \quad D_\alpha^{(\langle i} \mathcal{T}^{\langle jklm \rangle)} = \overline{D}_{\dot{\alpha}}^{(\langle i} \overline{\mathcal{T}}^{\langle jklm \rangle)} = 0 \tag{5.2}$$

where $\mathcal{T}^{\langle ijkl \rangle}$ is a real $SU(2)_R$ spin two field.[16]

---

[16] The pair $(\mathcal{L}^{\langle ij \rangle}, \mathcal{T}^{\langle ijlm \rangle})$ define the so-called "relaxed hypermultiplet" [19].

Appendix A. Component field commutators

The variations of the component fields in the $\mathcal{N} = 2$ linear anomaly multiplet, $\mathcal{L}_{IR}^{ij}$, with respect to the first SUSY are

$$
\begin{aligned}
&\left[\xi Q^{\langle 1 \rangle}, x_{IR}\right] = \sqrt{2}\xi G_{\langle 1 \rangle}, \quad \left[\overline{\xi Q}_{\langle 1 \rangle}, x_{IR}\right] = 0 \\
&\left[\xi Q^{\langle 1 \rangle}, G_{\langle 1 \rangle \alpha}\right] = \sqrt{2}\xi_\alpha F, \quad \left[\overline{\xi Q}_{\langle 1 \rangle}, G_{\langle 1 \rangle \alpha}\right] = i\sqrt{2}\sigma_{\alpha\dot{\alpha}}^\mu \overline{\xi}^{\dot{\alpha}} \partial_\mu x_{IR} \\
&\left[\xi Q^{\langle 1 \rangle}, F\right] = 0, \quad \left[\overline{\xi Q}_{\langle 1 \rangle}, F\right] = i\sqrt{2}\overline{\xi}_{\dot{\alpha}}\overline{\sigma}^{\mu\dot{\alpha}\alpha}\partial_\mu G_{\langle 1 \rangle \alpha} \\
&\left[\xi Q^{\langle 1 \rangle}, \overline{x}_{IR}\right] = 0, \quad \left[\overline{\xi Q}_{\langle 1 \rangle}, \overline{x}_{IR}\right] = \sqrt{2}\overline{\xi G}^{\langle 1 \rangle} \\
&\left[\xi Q^{\langle 1 \rangle}, \overline{G}_{\dot{\alpha}}^{\langle 1 \rangle}\right] = -i\sqrt{2}\xi^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu \overline{x}_{IR}, \quad \left[\overline{\xi Q}_{\langle 1 \rangle}, \overline{G}_{\dot{\alpha}}^{\langle 1 \rangle}\right] = \sqrt{2}\overline{\xi}_{\dot{\alpha}} F \\
&\left[\xi Q^{\langle 1 \rangle}, \overline{F}\right] = i\sqrt{2}\xi^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu \overline{G}^{\langle 1 \rangle \dot{\alpha}}, \quad \left[\overline{\xi Q}_{\langle 1 \rangle}, \overline{F}\right] = 0 \\
&\left[\xi Q^{\langle 1 \rangle}, \ell_{IR}\right] = -\frac{i}{\sqrt{2}}\xi G_{\langle 2 \rangle}, \quad \left[\overline{\xi Q}_{\langle 1 \rangle}, \ell_{IR}\right] = \frac{i}{\sqrt{2}}\overline{\xi G}^{\langle 2 \rangle} \\
&\left[\xi Q^{\langle 1 \rangle}, G_{\langle 2 \rangle \alpha}\right] = 0, \quad \left[\overline{\xi Q}_{\langle 1 \rangle}, G_{\langle 2 \rangle \alpha}\right] = i\sqrt{2}\sigma_{\alpha\dot{\alpha}}^\mu \overline{\xi}^{\dot{\alpha}}(\ell_{IR\mu} + i\partial_\mu \ell_{IR}) \\
&\left[\xi Q^{\langle 1 \rangle}, \overline{G}_{\dot{\alpha}}^{\langle 2 \rangle}\right] = -i\sqrt{2}\xi^\alpha \sigma_{\alpha\dot{\alpha}}^\mu (\ell_{IR\mu} - i\partial_\mu \ell_{IR}), \quad \left[\overline{\xi Q}_{\langle 1 \rangle}, \overline{G}_{\dot{\alpha}}^{\langle 2 \rangle}\right] = 0 \\
&\left[\xi Q^{\langle 1 \rangle}, \ell_{IR}^\mu\right] = \sqrt{2}\xi \sigma^{\mu\nu}\partial_\nu G_{\langle 2 \rangle}, \quad \left[\overline{\xi Q}_{\langle 1 \rangle}, \ell_{IR}^\mu\right] = -\sqrt{2}\partial_\nu \overline{G}^{\langle 2 \rangle}\overline{\sigma}^{\mu\nu}\overline{\xi}
\end{aligned}
\tag{A.1}
$$

The variations of the component fields with respect to the second SUSY are

$$
\begin{aligned}
&\left[\xi Q^{\langle 2 \rangle}, x_{IR}\right] = 0, \quad \left[\overline{\xi Q}_{\langle 2 \rangle}, x_{IR}\right] = \sqrt{2}\overline{\xi G}^{\langle 2 \rangle} \\
&\left[\xi Q^{\langle 2 \rangle}, G_{\langle 1 \rangle \alpha}\right] = 0, \quad \left[\overline{\xi Q}_{\langle 2 \rangle}, G_{\langle 1 \rangle \alpha}\right] = -i\sqrt{2}\sigma_{\alpha\dot{\alpha}}^\mu \overline{\xi}^{\dot{\alpha}}(\ell_{IR\mu} - i\partial_\mu \ell_{IR}) \\
&\left[\xi Q^{\langle 2 \rangle}, F\right] = 0, \quad \left[\overline{\xi Q}_{\langle 2 \rangle}, F\right] = i\sqrt{2}\overline{\xi}\overline{\sigma}^\mu \partial_\mu G_{\langle 2 \rangle} \\
&\left[\xi Q^{\langle 2 \rangle}, \overline{x}_{IR}\right] = \sqrt{2}\xi G_{\langle 2 \rangle}, \quad \left[\overline{\xi Q}_{\langle 2 \rangle}, \overline{x}_{IR}\right] = 0 \\
&\left[\xi Q^{\langle 2 \rangle}, \overline{G}_{\dot{\alpha}}^{\langle 1 \rangle}\right] = i\sqrt{2}\xi^\alpha \sigma_{\alpha\dot{\alpha}}^\mu (\ell_{IR\mu} + i\partial_\mu \ell_{IR}), \quad \left[\overline{\xi Q}_{\langle 2 \rangle}, \overline{G}_{\dot{\alpha}}^{\langle 1 \rangle}\right] = 0 \\
&\left[\xi Q^{\langle 2 \rangle}, \overline{F}\right] = i\sqrt{2}\xi \sigma^\mu \partial_\mu \overline{G}_{\langle 2 \rangle}, \quad \left[\overline{\xi Q}_{\langle 2 \rangle}, \overline{F}\right] = 0 \\
&\left[\xi Q^{\langle 2 \rangle}, \ell_{IR}\right] = -\frac{i}{\sqrt{2}}\xi G_{\langle 1 \rangle}, \quad \left[\overline{\xi Q}_{\langle 2 \rangle}, \ell_{IR}\right] = \frac{i}{\sqrt{2}}\overline{\xi G}^{\langle 1 \rangle} \\
&\left[\xi Q^{\langle 2 \rangle}, G_{\langle 2 \rangle \alpha}\right] = \sqrt{2}\xi_\alpha F, \quad \left[\overline{\xi Q}_{\langle 2 \rangle}, G_{\langle 2 \rangle \alpha}\right] = i\sqrt{2}\sigma_{\alpha\dot{\alpha}}^\mu \overline{\xi}^{\dot{\alpha}} \partial_\mu \overline{x}_{IR} \\
&\left[\xi Q^{\langle 2 \rangle}, \overline{G}_{\dot{\alpha}}^{\langle 2 \rangle}\right] = -i\sqrt{2}\xi^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu x_{IR}, \quad \left[\overline{\xi Q}_{\langle 2 \rangle}, \overline{G}_{\dot{\alpha}}^{\langle 2 \rangle}\right] = \sqrt{2}\overline{\xi}_{\dot{\alpha}} F \\
&\left[\xi Q^{\langle 2 \rangle}, \ell_{IR}^\mu\right] = -\sqrt{2}\xi \sigma^{\mu\nu}\partial_\nu G_{\langle 1 \rangle}, \quad \left[\overline{\xi Q}_{\langle 2 \rangle}, \ell_{IR}^\mu\right] = \sqrt{2}\partial_\nu \overline{G}^{\langle 1 \rangle}\overline{\sigma}^{\mu\nu}\overline{\xi}
\end{aligned}
\tag{A.2}
$$

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At zero momentum (i.e., no derivatives), the algebra simplifies considerably, and the only non-vanishing commutators are

$$\left[\xi Q^{\langle 1\rangle}, x_{IR}\right] = \sqrt{2}\xi G_{\langle 1\rangle}, \quad \left[\xi Q^{\langle 1\rangle}, G_{\langle 1\rangle\alpha}\right] = \sqrt{2}\xi_\alpha F, \quad \left[\overline{\xi Q}_{\langle 1\rangle}, \overline{x}_{IR}\right] = \sqrt{2}\overline{\xi G}^{\langle 1\rangle}$$
$$\left[\overline{\xi Q}_{\langle 1\rangle}, \overline{G}_{\dot\alpha}^{\langle 1\rangle}\right] = \sqrt{2}\overline{\xi}_{\dot\alpha}\overline{F}, \quad \left[\xi Q^{\langle 1\rangle}, \ell_{IR}\right] = -\frac{i}{\sqrt{2}}\xi G_{\langle 2\rangle}, \quad \left[\overline{\xi Q}_{\langle 1\rangle}, \ell_{IR}\right] = \frac{i}{\sqrt{2}}\overline{\xi G}^{\langle 2\rangle} \tag{A.3}$$

and

$$\left[\overline{\xi Q}_{\langle 2\rangle}, x_{IR}\right] = \sqrt{2}\overline{\xi G}^{\langle 2\rangle}, \quad \left[\xi Q^{\langle 2\rangle}, \overline{x}_{IR}\right] = \sqrt{2}\xi G_{\langle 2\rangle}, \quad \left[\xi Q^{\langle 2\rangle}, \ell_{IR}\right] = -\frac{i}{\sqrt{2}}\xi G_{\langle 1\rangle}$$
$$\left[\overline{\xi Q}_{\langle 2\rangle}, \ell_{IR}\right] = \frac{i}{\sqrt{2}}\overline{\xi G}^{\langle 1\rangle}, \quad \left[\xi Q^{\langle 2\rangle}, G_{\langle 2\rangle\alpha}\right] = \sqrt{2}\xi_\alpha F, \quad \left[\overline{\xi Q}_{\langle 2\rangle}, \overline{G}_{\dot\alpha}^{\langle 2\rangle}\right] = \sqrt{2}\overline{\xi}_{\dot\alpha}\overline{F} \tag{A.4}$$

Appendix B. Some simple examples

In this appendix we will consider some simple examples to illustrate the above discussion. To that end, consider the following $\mathcal{N} = 2$ $U(1)$ gauge theory with $\mathcal{N} = 2$ field strength $W = (\Phi, W_\alpha)$

$$\mathcal{L} = \int d^2\theta_1 d^2\theta_2 \ \mathcal{F}(W) + h.c. \tag{B.1}$$

where

$$\mathcal{F}(W) = \frac{1}{4}W^2 + ... \tag{B.2}$$

is the $\mathcal{N} = 2$ prepotential and the ellipses contain possible higher-order interactions. Clearly, if the higher-order interactions are non-vanishing, the theory has a superconformal anomaly.

We know that this example must fall within the class of theories we have considered because it appears, for example, in the low energy theory on the Coulomb branch of $SU(2)$ SYM [2].[17] To see this fact more explicitly, we can construct the $\mathcal{N} = 2$ supercurrent multiplet and anomaly by starting with the operator

$$\widehat{J} = -\Phi\partial_{\overline{\Phi}}\overline{\mathcal{F}} - \overline{\Phi}\partial_\Phi\mathcal{F} - \left(\widetilde{\mathcal{F}} + \overline{\widetilde{\mathcal{F}}}\right). \tag{B.3}$$

---

[17] Note also that the expression in (B.1) makes it clear that the theory is $SU(2)_R$ invariant for arbitrary $\mathcal{F}$. Indeed, $\mathcal{F}$ is itself $SU(2)_R$ invariant and so too is the $\mathcal{N} = 2$ chiral integration measure.

The first two terms in (B.3) are (minus) the Kähler potential for Φ that descends from (B.1),(B.2), and the two terms in the parenthesis are local chiral and antichiral shifts. We define the chiral function $\widetilde{\mathcal{F}}$, up to an uninteresting constant, as follows

$$\partial_\Phi\widetilde{\mathcal{F}} \equiv \partial_\Phi\mathcal{F} - \Phi\partial_\Phi^2\mathcal{F}. \tag{B.4}$$

From (B.3) we construct the full supercurrent multiplet by the action of the second set of supercharges. The crucial point is that the anomaly multiplet is just

$$X = -\frac{1}{3}\left(\left(\partial_\Phi\mathcal{F} - \Phi\partial_\Phi^2\mathcal{F}\right)\overline{D}^2\overline{\Phi} + 2\Phi\partial_\Phi^3\mathcal{F}\,W^2 + \overline{D}^2\overline{\widetilde{\mathcal{F}}}\right)$$

$$L = \frac{\sqrt{2}}{3}\left(\overline{D}_{\dot\alpha}\left[\overline{W}^{\dot\alpha}\left(\partial_{\overline{\Phi}}\overline{\mathcal{F}} - \overline{\Phi}\partial_{\overline{\Phi}}^2\overline{\mathcal{F}}\right)\right] + D^\alpha\left[W_\alpha\left(\partial_\Phi\mathcal{F} - \Phi\partial_\Phi^2\mathcal{F}\right)\right]\right) \tag{B.5}$$

Clearly X is chiral and L is real and linear. Therefore, this example is in our class of theories, and it is easy to see directly that SUSY cannot be broken in this case. Finally, notice that for a trivial prepotential the theory is superconformal and $X = L = 0$.

Massive hypermultiplets

For our next example, let us consider a simple generalization of the original example in [12] and examine a theory of free massive hypermultiplets

$$\mathcal{L} = \int d^4\theta\left(\overline{Q}^i Q_i + \widetilde{Q}^i\overline{\widetilde{Q}}_i\right) + \left(\int d^2\theta\frac{1}{\sqrt{2}}M_i Q_i\widetilde{Q}^i + h.c.\right). \tag{B.6}$$

Without loss of generality we can take the masses to be real. The hypermultiplets are doublets under $SU(2)_R$ and have charge -1 under the R-symmetry that leaves the manifest $\mathcal{N} = 1$ superspace invariant. Therefore, we can construct the following $\widehat{J}$ operator for the theory

$$\widehat{J} = \frac{1}{2}\sum_i\left(Q_i\overline{Q}^i + \overline{\widetilde{Q}}_i\widetilde{Q}^i\right). \tag{B.7}$$

From (B.7) and the action of the second SUSY it follows that the anomaly multiplet is

$$X = \frac{2\sqrt{2}}{3}\sum_i M_i Q_i\widetilde{Q}^i,$$

$$L = -\frac{\sqrt{2}}{3}\sum_i M_i\left(Q_i\overline{Q}^i - \overline{\widetilde{Q}}_i\widetilde{Q}^i\right). \tag{B.8}$$

This theory is therefore in the class we have considered in this paper, and it is straight forward to see directly that SUSY is not broken. As a final note, we see that in the limit of vanishing masses, the theory is superconformal and $X = L = 0$.

We can also consider coupling the massive hypermultiplets to an $\mathcal{N} = 2$ $U(1)$ gauge multiplet

$$
\begin{aligned}
\mathcal{L} = \int d^4\theta \left(\overline{\Phi}\Phi + \sum_i \left(\overline{Q}^i e^{n_i V} Q_i + \widetilde{Q}^i e^{-n_i V} \overline{\widetilde{Q}}_i \right) \right) \\
+ \left(\int d^2\theta \left(\frac{1}{4} W^2 + \frac{n_i}{\sqrt{2}} (\Phi + M_i) Q_i \widetilde{Q}^i \right) + h.c. \right),
\end{aligned}
\tag{B.9}
$$

where we have taken the prepotential to be trivial for simplicity. Using the fact that the free superconformal R-charge of Φ under the $U(1)_R$ symmetry that leaves the $\mathcal{N} = 1$ superspace invariant is $+2$ we can construct the $\widehat{J}$ operator

$$
\widehat{J} = -\overline{\Phi}\Phi + \frac{1}{2} \sum_i \left(Q_i \overline{Q}^i + \overline{\widetilde{Q}}_i \widetilde{Q}^i \right).
\tag{B.10}
$$

It then follows that the (classical) anomaly multiplet is just

$$
\begin{aligned}
X_{cl} &= \frac{2\sqrt{2}}{3} \sum_i (M_i - M_1) Q_i \widetilde{Q}^i, \\
L_{cl} &= -\frac{\sqrt{2}}{3} \sum_i (M_i - M_1) \left(Q_i \overline{Q}^i - \overline{\widetilde{Q}}_i \widetilde{Q}^i \right).
\end{aligned}
\tag{B.11}
$$

Without loss of generality, we have shifted the definition of Φ to $\Phi + M_1$ so as to absorb $M_1 \neq 0$ into the vacuum expectation value (VEV) of Φ. By examining (B.10) we see that this corresponds to an improvement transformation since we are only considering a trivial prepotential for Φ (in the quantum theory, with a non-trivial prepotential, this is no longer true, but we consider the classical case for simplicity). Since the VEV of Φ breaks the superconformal symmetry spontaneously, it does not contribute to the classical anomaly.

This example is also clearly in our class of theories, and it is easy to check directly that SUSY is not broken.

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