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Spatial transformations: from fundamentals to applications

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Introduction



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This paper forms the introduction to this themed issue of the *Philosophical Transactions of the Royal Society A* on 'Spatial Transformations', arising from the Royal Society Scientific Discussion Meeting held in January 2015. The paper begins with a review of the concepts and history of spatial transformations, followed by a discussion of the contributions from the papers in this themed issue. A summary of the advantages and current limitations of spatial transformations concludes the paper, with the key challenges identified at the Scientific Discussion Meeting also given.

1. Introduction

Recent years have seen a wealth of media interest in such topics as '*invisibility cloaks*' and '*perfect lenses*'. 'Cloaking', in particular, has captured the imagination of the media and public, no doubt reflecting a deep fascination with illusions and invisibility that can be traced throughout human history. Cloaking devices, and many more, designed using a process of *spatial transformations*, have now moved from the realm of fiction to scientific fact, albeit with severe restrictions.

Within this themed issue, we wish to address the following questions:

- What are spatial transformations?
- Why are spatial transformations important and what advantages do they bring?

- What applications have been conceived to date for spatial transformations?
- What limitations are currently seen to restrict the usefulness of spatial transformations?
- How might the field of spatial transformations develop?

The papers in this themed issue arise from the Royal Society Discussion Meeting “Spatial Transformations: from fundamentals to applications”, held at Chicheley Hall on 26–27 January 2015. We begin with a brief overview of spatial transformations in the light of these questions, before summarising the papers in this themed issue in the next section.

Spatial transformations refer to changes to coordinate systems that provide a new approach to controlling the way a wave, such as an electromagnetic or acoustic wave, propagates, by defining the spatial variation of material parameters. Typically, waves will travel in straight lines in a homogeneous, isotropic medium in Euclidean space, usually imagined as rays following the grid lines in the Cartesian coordinate system. Transforming the coordinate system results in associated transformations of the waves; this can be imagined as rays now following curved grid lines. By taking advantage of the *form-invariance* of the underlying wave equations for the system, a medium can be derived which is equivalent to the transformation in terms of its effect on the waves, due to the spatially varying properties.

Form invariance is the property where the governing equations, such as the Maxwell equations for electromagnetics, remain in the same form in any coordinate system. It is this that enables a transformation to be interpreted as a material, via constitutive equations like

$$D = \varepsilon E \quad (1.1a)$$

$$B = \mu H \quad (1.1b)$$

for electromagnetics.

The last decade or so has seen a rapid growth in the use of spatial transformations across a number of engineering and physics disciplines, particularly those relating to electromagnetics. This interest in spatial transformations stems from two papers for the electromagnetic regime, by Leonhardt [1] and by Pendry, Schurig and Smith [2], published in the same issue of *Science* in 2006. The concept itself stems (historically) from work on general relativity and was understood as early as the 1920s, as evidenced by Tamm, who first pointed out that a curved space-time is equivalent to an electromagnetic material [3,4]. Dolin used this concept to obtain an ‘invisible material’ in a paper published in 1961 (the first on ‘*transformation optics*’); though invisible, there was no cloaked region [5]. Post’s *Formal Structure of Electromagnetics*, first published in 1962, covered form invariance in considerable depth [6]. Transformations were used in computational electromagnetics during the 1990s, particularly in enabling curved geometries to be solved using the Finite-Difference Time-Domain method, which ordinarily uses a Cartesian grid (hence, introduces errors when modelling a curved geometry) [7–11]. Transformations have also been used in other fields within physics, such as hydromechanics, where they have been used [12, see ref. 12 therein] to solve problems since the late 1890s! It is also worth noting that the concepts can also be applied in certain scenarios where form-invariance does not hold for the governing equations, such as for elastic solids, if certain other restrictions are applied [12].

Despite this long history, the modern application to the control of waves is unprecedented and builds on three main developments:

- the (re-)discovery of *metamaterials* in the mid- to late 1990s;
- the development of nano-composite materials and advanced fabrication methods (such as additive manufacturing);
- the increased speed and power of computation.

Metamaterials are typically defined as materials that derive their properties as much from their geometrical structure as from the chemistry of their constituent materials. They go hand-in-hand with spatial transformations, because they can be engineered to provide material properties

not readily available in ordinary materials. This makes the realisation of a wider range of transformation media possible, assuming the metamaterial unit cell (or element) is smaller than the wavelength being considered, so that *homogenisation* can be used. Homogenisation is the averaging of local properties to derive effective properties, which requires features to be much smaller than a wavelength in dimension; the concept can be applied in all regimes (electromagnetics, acoustics, etc.). Metamaterials can also include structures where the unit cell is not smaller than a wavelength, as in photonic or phononic crystals. Such metamaterials cannot be represented using macroscopic quantities derived through homogenisation. Whilst spatial transformations do not apply directly to such structures, some techniques can apply to both types of metamaterial (e.g., [13]).

Developments in nano-composite materials have also widened the range of materials available for implementing transformation media. The requirement for continuous spatial variation in the material properties is usually approximated via discretised versions; advanced fabrication techniques now allow a far greater control of materials as a function of position, and at a smaller scale. This means that homogenisation, typically used for most transformation media, can be applied at ever-higher frequencies. Similarly, the advances in computational capability (both in terms of hardware and also software algorithms) have supported the modelling requirements for verification and optimisation of devices based on spatial transformations.

One of the key attractions and benefits of the spatial transformation approach is the ability to separate device performance from the geometry, through use of a suitably-graded (spatially varied) material. For example, using a graded material to achieve a required electromagnetic response allows the geometry to be optimised for other reasons, such as aerodynamics (for example, a lens antenna in an aeroplane or automobile body). It should be noted that the spatial transformations approach differs from the earlier and complementary work on *graded index* (GRIN) materials, as it offers a means of controlling both refractive index and impedance; GRIN materials only control the refractive index. Other benefits include the ability to produce functionalities not previously realisable, including from transformations relying on ‘unusual’ material properties (such as relative permittivities and permeabilities less than unity, or even with negative values, in electromagnetics).

The main limitations at this point revolve around the realisation of such transformation media. For example, despite much progress in this area, achieving permittivity responses less than unity can be difficult, if not impossible, to realise without sacrificing some measure of performance, such as efficiency or bandwidth. This trade-off is particularly true in electromagnetics, although encountered to some degree in other fields. Even transformations that retain ‘ordinary’ materials (e.g., permittivities and permeabilities greater than unity) can require extremely high values of material properties that are challenging to realise. Despite this, the promise of spatial transformations is for unprecedented degrees of control over wave phenomena, with subsequent impact on device design and even the creation of new classes of device. The papers in this themed issue demonstrate both the progress being made and the challenges remaining, as discussed in the next sections.

2. Papers in this Themed Issue

The first article in this issue, by Jiang et al., serves as an introduction to spatial transformations as they apply to electromagnetics, from radio frequencies to optics [14]. Beginning by reviewing the mathematical basis for this *transformation electromagnetics* or *transformation optics*, they then apply it to a concrete example of a three-dimensional (3D) far-field focussing lens. Next, they demonstrate a series of modifications to the basic transformation approach, beginning with complex coordinate transformations. In these transformations, one or more of the coordinates are no longer purely real; the advantage provided is greater control of both phase (as in ordinary real transformations) and amplitude, as the control of intensity density provided through control of the path of the wave is supplemented by deliberately including loss or gain in the transformation medium. The trade-offs involved between performance and material requirements are then

explored through the use of two simplifying transformations (quasi-conformal and linear, respectively). A tunable or reconfigurable transformation design is then described, followed by an application of quasi-conformal transformations to photonic integrated circuit design. Finally, the paper discusses the use of graphene in two-dimensional (2D) implementations.

Mitra and Zhou, in the second paper, provide a counterpoint to the rest of the issue [15]. In this paper, the transformation electromagnetics algorithm is re-phrased in terms of the generalised scattering matrix formulation, and then applied to a number of practical problems. Their objective is to investigate the trade-offs between performance and material requirements when the transformation algorithm is relaxed in various ways, as seems most appropriate to a given problem. They suggest that the full transformation, with challenging material requirements, may not always be required in practical problems, when clear performance requirements are available. This further emphasises the current challenge regarding the implementation of spatial transformation solutions with existing materials and metamaterials.

Advanced applications of spatial transformations, for electromagnetic and other regimes, are the focus of the next two papers. In the paper by Ginis and Tassin, three applications are discussed that go beyond the 'ordinary' application of transformation optics to control the trajectory of light [16]. First, they consider a deeply sub-wavelength cavity to confine light, where "... *the concept of trajectory ceases to have any meaning...*" [16]. Second, the design of transformation media for the manipulation of Cherenkov radiation, emitted when a charged particle travels with a velocity greater than the speed of light in a given medium and potentially useful for particle physics, is explored. Finally, they investigate the use of transformation concepts to control optical forces, rather than fields.

In the paper by Kadic et al., the concept of 'cloaking' is investigated in a number of regimes: optics, thermodynamics and mechanics [12]. The authors begin with a review of spatial transformations and cloaking, and then compare the different regimes discussed in the paper. They proceed with a discussion of optical cloaking, explaining that the ideal 'omni-directional broadband free-space cloak' is impossible to achieve in practice, and discussing some of the limitations of cloaks when one or more of these ideal properties is relaxed. Particular emphasis is made of the 'carpet cloak' in this regard.

Next, thermodynamic cloaking is discussed, where many of the restrictions faced in transformation optics can be ignored. A key point is made that spatial transformation concepts can be applied to thermodynamics, even though waves are not a solution to the heat conduction equation. Experimental results for a thermal cloak are provided. A parallel is then drawn between heat diffusion and the propagation of light through diffuse media, which can also be described (at least approximately) by a diffusion equation. Experiments on optical cloaking in such media are described.

Mechanical waves are also not subject in practice to the same restrictions as faced in transformation optics. However, the elasticity tensors of ordinary media are such that, in general, the elasto-dynamic equations are not form-invariant under coordinate transformations. Fortunately, there are exceptions, such as acoustics in fluids or gases. Work on cloaking in this area is reviewed. Mechanical surface waves are also an exception; experiments in this area, which could (in principle) scale up for seismic waves, are also discussed, including mechanical metamaterials to realise the required behaviour.

The next paper, by Silveirinha et al., reviews their work in two topics. The first topic continues the discussion of advanced applications of spatial transformations by describing their work on '*transformation electronics*', which is the application of spatial transformations and metamaterials to semiconductor electronics [17]. They begin with an exploration of the possibility of applying spatial transformation concepts to the effective mass of an electron, and suggest three examples to show why this concept is of interest [17]. First, engineering the effective mass of electrons may improve speed and response times in electronic and optical systems, by increasing mobility and conductivity. Second, the relative permittivity of a material with free electrons is affected by the effective mass of charge carriers, including electrons; one possible result of this is being able to

obtain nearly perfect conducting materials (readily available at microwave frequencies) at optical frequencies. Third, electronic materials with small effective mass of electrons could possibly lead to materials with enhanced non-linearities, potentially suitable for diodes at higher frequencies than currently available (including optical).

They continue by reviewing *epsilon-near-zero* (ENZ) metamaterials and materials with highly anisotropic permittivity, such as those obtained from stacks of thin layers with alternating positive and negative permittivity. The ENZ concept is then applied to the effective mass of semiconductor materials, with opposite signs of effective mass of electrons and holes. The resulting effective mass of the semiconductor superlattice can have extremely anisotropic behaviour, including effective masses near zero and extremely high positive values. The resulting potential enhancements in conductivity are described, with extensions to graphene electronics noted, amongst other possible applications of the spatial transformations paradigm [17].

In the remainder of this paper, the authors turn to methods for implementing spatial transformation designs, with a discussion of the ‘*digital metamaterials*’ concept [17]. In this approach to realising metamaterials with the required properties, an analogy is drawn between how digital bytes are made from bits on the one hand, and the use of two different metamaterial ‘bits’ (a ‘one’ and a ‘zero’) to achieve a metamaterial ‘byte’ with properties formed from the average of the bits. To achieve this, one metamaterial must have a positive property (i.e., permittivity or permeability) and the other must have a negative property. Each bit in a metamaterial byte is essentially a unit cell (hence, sub-wavelength in dimension) of the particular metamaterial. The metamaterial byte must also be sub-wavelength, for homogenisation (averaging) of multiple bytes to be valid. Some examples are given, focussing on the effective permittivity of a ‘core-shell’ model. The authors demonstrate that, by controlling the ratio of the core radius a to the shell radius b , the effective permittivity can be designed to:

- have a value lying between the values of the permittivities of the ‘bits’, when the shell is metal (negative permittivity) and the core is dielectric (positive permittivity);
- have a value outside the range defined by the permittivities of the ‘bits’, when the core is metal and the shell is dielectric.

The next two papers continue the theme of the implementation of spatially-varying material properties generated by the transformation optics algorithm. Ways of using periodic structures to implement the spatially-varying material properties are the focus of the paper by Rumpf et al. They describe a numerical method to “... *spatially vary any periodic structure while minimizing deformations to the unit cells that would weaken or destroy the electromagnetic properties*” [13]. The method can also be used for devices that cannot be designed using spatial transformations, such as those based on photonic crystals. Three examples are provided to illustrate the method. First, a spatially-varying photonic crystal, made with ordinary materials of low refractive index, is described, where light propagates around a very sharp, tight bend. Second, they report a multi-mode waveguide that maintains isolation between modes, even at bends. Finally, control of the near field around electric components is demonstrated using spatially-varying anisotropic materials, improving isolation (hence, electromagnetic compatibility).

Grant et al. provide an alternative view to metamaterials on the realisation of transformation media, focussing on engineered composite materials [18]. In contrast to the metamaterial approach, this uses “... *spatial distributions of electrical and magnetic properties using arrangements of different materials of differing inherent electrical and magnetic response.*” Using such materials can avoid some of the issues common to resonant metamaterial implementations (e.g., narrow-band performance), and can offer flexibility for practical applications.

They begin by exploring the issues surrounding the practical realisation of transformation optics design, identifying two key challenges: first, the need for a large ‘palette’ of electromagnetic materials covering a broad range of permittivity and permeability values in a controllable manner; second, the need to spatially vary these electromagnetic materials in a controlled manner, preferably using inexpensive and scalable production methods [18]. The authors proceed by

discussing a number of composite materials, focussing on polymer-based nano- and micro-composites with dielectric, ferrite and superconducting nano-particles. Advanced issues, such as introducing controllable anisotropy, are also discussed. A number of manufacturing approaches (spray deposition, extrusion, casting and 3D printing) are then described for producing these composite materials and devices that utilise them. The relative advantages and limitations for these “*materials-process combinations*” [18] are discussed, to highlight what is achievable in practice and what challenges still remain.

Next, we have a group of papers dealing with metasurfaces and two dimensional spatial transformations. Martini et al. focus on metasurfaces for surface waves (i.e., waves tangential to the metasurface). The development and analysis of metasurfaces is discussed first, with an emphasis on the use of an equivalent surface impedance for describing the metasurface [19]. A number of unit cells are described, together with ways of controlling the effective metasurface properties by modifying the unit cells. Metasurface transformations are defined in the latter part of the paper, which also provides a number of examples of such devices designed with spatial transformations and implemented with the metasurfaces discussed earlier in the paper.

The second paper in this group, by Tretyakov, complements the first by reviewing metasurfaces that transform waves propagating through, rather than along (tangential to), the metasurface [20]. Beginning with a discussion of what a metasurface is, he highlights the usefulness of such a 2D electromagnetic object in the context of Huygens’ equivalence principle. This states that “... *the electromagnetic fields created by arbitrary sources in an arbitrary volume V can be found as the fields created by equivalent currents on the volume surface S* ” [20]. Equivalently, the fields outside a volume can be controlled by either the fields within that volume or by the surface currents surrounding it. Hence, metasurfaces offer an alternative to 3D metamaterials as a thin, lightweight means of controlling fields. After a historical review of the field, he proceeds by examining homogenisation models for metasurfaces. A basic classification of metasurfaces is then discussed, summarised as:

- electrically *or* magnetically polarizable metasurfaces;
- electrically *and* magnetically polarizable metasurfaces;
- general bi-anisotropic metasurfaces.

After a brief discussion of metasurfaces for surface waves, he provides some examples of field-transforming metasurfaces, before concluding with some of the remaining questions about metasurfaces and transformations of these types.

Estakhri et al. [21] continue with the themes raised by Tretyakov, focussing on the optical regime and the use of nanoparticles to create the metasurface. Beginning with a review of metamaterials and metasurfaces, the authors then discuss the equivalence principle and the restrictions encountered in the optical regime, where magnetic responses are negligible. They then present a design methodology for utilising ‘*surface engineering*’ to transform an incident wave into an arbitrary scattering profile. For practical reasons, they use examples based on reflecting surfaces, rather than transmitting surfaces. After describing the methodology, the realisation of the required metasurface at optical frequencies is briefly discussed. Three examples are then provided: a unidirectional carpet cloak; an ultra-thin polarization beam splitter to convert and separate a circularly polarised wave into its linear components; and a method for improving the absorption properties of thin film solar cells.

Finally, the paper by Wright and Matsuda turns from electromagnetic surface waves to acoustic surface waves [22]. They begin by describing experiments to measure the acoustic dispersion relation for phononic crystals, using Fourier analysis of ultra-fast time-domain imaging of the acoustic field on the surface of such crystals. Determining the dispersion relation is a necessary first step to creating spatially-varying structures. They then give examples, first for one-dimensional phononic crystals and then for two-dimensional crystals. Phononic-crystal waveguides are then described, with extensions to imaging of the acoustic field in k -space shown to offer new insights.

We conclude this themed issue with two papers, by Leonhardt and Smolyaninov et al., that demonstrate the versatility of spatial transformations by returning to cosmology. As discussed in section 1, the use of spatial transformations as seen in transformation optics was inspired, at least in part, by the mathematics of general relativity. Leonhardt demonstrates how spatial transformations can enable the design of analogues of cosmological objects, describing a means of investigating the quantum physics of the event horizon of black holes [23]. First, he reviews the event horizon and Hawking radiation, before showing the difficulties in measuring Hawking radiation from cosmological sources. He then describes a number of laboratory experiments that seek to use analogues to overcome such problems. Such experiments, conducted in optics, fluid mechanics or using ultra-cold atoms, are described and the experimental challenges remaining discussed. Smolyaninov et al. continue by describing a ferro-fluid metamaterial and the defects that can be observed, discussing how such defects can be considered analogues of such cosmological objects as magnetic monopoles and cosmic strings, as well as the space-time cloak [24].

3. Summary

The brief review we have given, in section 1 above, combined with the papers in this themed issue, demonstrate the wealth of potential in the spatial transformations approach, from the relatively well-established *transformation optics* to new proposals for “*transformation electronics*” [17], such that it is now possible to speak of “*transformation physics*”. The basic approach of spatial transformations has been described, together with numerous advanced techniques, such as complex coordinate transformations [14], and advanced applications, such as manipulation of optical forces [16].

One of the key advantages of spatial transformations is that it provides an elegant framework for solving inverse problems. However, it only provides a partial solution, in that it prescribes a material that will produce a desired wave behaviour, but does not specify how to fabricate such a material, or even whether such a material is even possible. This emphasises the close links between the fields of spatial transformations and metamaterials and also nano-composite materials. The papers from Silveirinha et al. [17], Rumpf et al. [13] and Grant et al. [18] all contribute different perspectives on the fabrication of materials for transformation electromagnetics.

The question of realisation of spatial transformation designs appears elsewhere in this issue, particularly in the papers by Jiang et al. [14] and Mittra and Zhou [15]. The choice of transformation affects the required material properties and trade-offs can be made against various performance figures, such as conformal, quasi-conformal and linear transformations. Sometimes, as suggested by [15], the application requirements are such that the spatial transformation approach may not be optimal or even necessary to achieve the required performance. However, it is also clear, as demonstrated by Jiang et al., Ginis and Tassin, and Kadic et al., that the spatial transformation paradigm enables the design of devices with functionalities that could not have been achieved previously [12,14,16].

Special sub-categories of spatial transformations and metamaterials exist when dealing with essentially 2D transformations. On the one hand, a 2D device can be designed to perform a variety of transformations on a wave propagating in 3D space, as thoroughly described by Tretyakov and Estakhri et al. [20,21]. On the other, 2D waves (such as waves propagating on a surface) can be controlled by metasurfaces to perform a variety of functions, including beam splitting, wavefront transformations and conversion between surface (2D) and space (3D) waves, described by Martini et al. [19]. The design of metasurfaces to achieve such functionality is a similar challenge to designing 3D metamaterials, in many respects, but the methodologies described in this issue demonstrate convincingly how this may be achieved at both radio and optical frequencies [19–21].

The discussions held during the Royal Society Scientific Meeting reflected the general feeling, also expressed in the papers of this themed issue, that the advent of spatial transformations has been of major importance and enables many new directions of research, as well as the ability to

design for functions never before possible. The key challenges and directions for the future of spatial transformations were felt to be:

- the realisation of materials and metamaterials with required properties (including, but not limited to, magnitude, anisotropy, loss, and dispersion);
- the ability to control the spatial distribution of such materials;
- the ability to select the optimal transformation for a given design;
- the application of spatial transformations in the quantum regime.

We commend these papers to you and trust you will find them interesting and informative.

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