A METHODOLOGY FOR THE ECONOMIC EVALUATION OF POWER STORAGE TECHNOLOGIES IN THE UK MARKET

by

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Abstract

In this thesis, we present a methodology for assessing the economic impact of power storage technologies. The methodology is founded on classical approaches to the optimal stopping of stochastic processes. Power storage is regarded as a complement to the intermittent output of renewable energy generators, and is important in contributing to the reduction of carbon intensive power generation. Therefore, the recommendations to study the future economic storage assessment have been increased. Our aim is to present a methodology suitable for use by policy makers that is simple to maintain, adapt to different technologies and is easy to interpret.

The thesis start by giving an overview of the UK power market and an introduction to storage technologies in Chapter 2. Chapter 3 summarize the mathematical tools, that the methodology is based on, more precisely the discretionary stopping theory based on dynamic programming techniques. An algorithm to assess the storage is presented in Chapter 4, where the storage problem is formulated as an entry, exit problem, which allow the investigation of different optimal strategies to fill and empty a storage facility. An analysis of power demand, and an approximation of power prices through the merit order curve of the UK power market presented in Chapter 5. Based on a theoretical study, the methodology is applied to a Compressed Air Energy Storage (CAES) in Chapter 6. Chapter 7 present an empirical study that applied the methodology directly on the observed data, this approach is shown to have benefits over current techniques and is able to value, by identifying a viable optimal operational strategy for a CAES operating in the UK market.

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Declaration Statement

The Research Thesis Submission form is attached.

Contents

Chapter 1

Introduction

1.1 Motivation

UK has to meet two main European targets: 15% of their total energy consumption (transport, heat and electricity) should come from renewables by 2020; and 80% reduction of carbon emission by 2050. That means that the proportion of electricity generation from renewable energy should increase to about 30% by 2020. Moreover, around a fifth of the current generation plants will be closed over the coming decade, and the electricity demand may double by 2050. Initiated in 2010, the Government's Electricity Market Reform (EMR) has three main objectives: the decarbonisation of electricity generation, keep the lights on and keep the cost of electricity to consumers down. EMR aims to transform the UK electricity sector into one in which low-carbon generation (wind and nuclear) can compete with conventional, fossil-fuel generation, providing a cleaner and more sustainable energy mix (DECC[1]).

Consequently, renewable energy generations from sources such as wind, solar and tidal, are expected to have a significant increases in the energy generation system. These resources are naturally controlled and have variability and uncertainty related to their outputs. For the electricity market this means that matching power demand with supply provided by high proportion of intermittent renewable resources pose new challenges to those tasked with balancing supply and demand. This is an economic problem of matching supply and demand with different sources, when one

[1]Department of Energy and Climate Change.

part of the supply, wind is unpredictable, while other parts, such as nuclear, "must run"[2].

The UK power market under "The British Electricity Trading and Transmission Arrangements (BETTA)" disadvantage renewable energy in particular wind energy, because of its intermittent nature:

> "BETTA, favours generators and suppliers who are capable of: Guaranteeing specific levels of generation or supply in advance, and guaranteeing agreed flexibility in output/demand at short notice. This becomes problematic and disadvantageous when you consider that many renewables; in particular wind energy; produce inherently intermittent and difficult to accurately predict levels of energy generation. Therefore, the BETTA currently disadvantages many renewable technologies and will continue to do so, unless an appropriate storage/buffering mechanism can be established, or the arrangements are revised to better support the uptake of renewable generation technologies"[1].

One of the proposed solutions to tackle this problem is power storage which can be used as liquidity reserve; this will work by absorbing excess power at low demand and release it other time of shortfall. Power storage is not new to the power system, has been part of the system for a long time. However, the renaissance of power storage in the last years is due to the breakthroughs in storage technologies and increasing variable renewable resources.

1.2 Thesis contribution to knowledge

Currently, the focus from a number of organizations including researchers, investors and most importantly policy makers is the economics of electricity storage. In the assessment of the economics of electricity storage three main areas are of interest: the methodology used, the profitability of the storage and the impact of regulation on the storage.

[2]Nuclear power are not flexible plants, they cannot turn on and off efficiently, which means they must-run 24/7 to supply the base-load demand.

Adopting methodologies to assess the economics of energy storage means developing and using models. For investors, it is profitable to invest in developing detailed and complex, models of specific plants to address the specific issues of optimally managing a single implementation of a power storage technology. Policy makers, in contrast to investors, would like to employ models that are less complex, not only for a specific plant but rather adaptable to different technologies, easy to maintain and straightforward to interpret. These features are not always given in most of the existing studies on power storage.

The profitability of power storage technologies may rely on revenue from several streams, and studies of business models which are yet to be proven. These streams include arbitrage, providing liquidity reserve during low wind periods, and ensuring the reliability of power supply. Therefore, policy-makers need to be aware and informed of these in details, and not only take a narrow view of what the technology can offer.

There are challenges associated with the market and regulatory issues. Currently in the UK the storage is not explicitly recognized as an asset class or activity. There is the absence of a well-defined position for storage and so is treated as a generation, however putting storage within this scope is questionable and unclear [4]. This emphasise that storage should be explicitly acknowledged by policy makers and be given a specific regulatory framework.

We aim from this thesis to provide insights on the three areas discussed above, that are required for further research as stated in Zucker at al. [119]. The methodology we presented here is based on advanced mathematics; however we are able to give an algorithm that straightforward, can be implemented; only require few data and can be easily interpreted by policy makers. The benefit of the methodology is in studying different services that power storage could provide to the grid. Further, we consider the efficiency parameter of the storage facility as a variable input to the model, as more studies are required on the impact of techno-economic parameters on the storage business case. Finally we believe that this methodology will support the need for grid fees or market mechanism to improve the viability of power storage in the UK market.

1.3 Background

1.3.1 Storage models

The assessment of the economics of electricity storage has been studied by many researchers, where different models have been designed. There are two main categories of models regarding the assessment of the economics of power storage; the engineering models and the system models, where many researchers adopt the engineering approach. This approach is based on techno-economic analysis, and requires less data to study as opposite to the system models. The engineering models are used mostly when it comes to studying the optimization and control of a specific storage facility, and usually address the problem from an investor's point of view in a given regulatory framework (Zucker et al. [119]). On the other hand, system models are concerned with an entire energy system and try to find feasible and least cost solutions under constraints such as carbon emission targets. As there are different classifications of models proposed for storage, there are also different terms related to the way the problem is formulated and different mathematical techniques and concepts applied to solving the problems.

The model may assume linearity or non-linearity either in the objective function or in the constraints. It can also be deterministic or stochastic. Stochastic models are used when the future is uncertain and some factors are uncontrollable or governed by nature. When operators have to take decisions and pre-defined strategy and policy without a perfect view of the future, and then stochastic modelling can provide such strategy. There are two main steps in stochastic modelling, first an optimization is carried out to provide strategies at all future states of the system, and the second step is to apply these strategies to a given scenario. Deterministic models directly provide decisions, without need to define strategy [119]. Techniques such as linear programming, genetic algorithms and stochastic programming, with dynamic programming most used method to solve stochastic models by dividing them into simpler problems.

Numerous papers worked on storage optimization, ranging from ones that assume the storage operator is a price taker[3], e.g. Denholm et al. [43], He et al. [65], Bathurst and Strbac [12]. Other papers take a non-deterministic approach by allowing for uncertainty on the price level e.g Mokrian and Stephen [93], Xi and Sioshanisi [116], Keles et al. [79], Grünewald [62]. Hybrid storage studies also exist where authors extend the previous studies and consider an optimization control with one more stochastic variable e.g. wind as in Howell [69], where partial differential equations are utilised to model the behaviour and control of a store. Barton and Infield [11] use a spectral analysis of electrical power system with large fractions of wind and solar, Garcia Gonzalez et al. [54] provide a two-stage stochastic programming problem with market prices and wind generation as two random parameters.

The problem of the optimal operation of a power storage facility has long been of interest to economists and management scientists, for example [85], [81], [57], [95], [58]. Despite the recent proliferation in papers addressing the optimal operation of power storage facilities in engineering, there has been relatively little interest in the contemporary economics literature, notable exceptions include [67], [68], [36], [109] and [50]. Many of these studies do not address the broader policy questions around choosing the best portfolio of storage technologies that deliver a resilient and low cost power system. However these approaches are considered the first step in research endeavour, in order to evaluate the need for system modelling analysis. There are number of authors who have provide system models that address policy issues, Nyamdash and Denny ([97], [98]), Grünewald et al. [63], Wilson et al. [115].

1.3.2 Optimal stopping models

The theory of optimal stopping is an important field of stochastic control. It has the objective of finding stopping times of the underlying stochastic processes, with the aim of maximizing an expected payoff or minimizing an expected cost. There are two main approaches of solving such optimal stopping problems, the martingale

[3]The storage's size is not big enough to impact the market price.

approach and the Markovian approach. The similarities and distinctions of the two approaches reveal how they describe the probabilistic evolution of stochastic processes which underlies the optimal stopping problem. In the martingale approach, the probabilistic structure of the process $X = (X_t)_{t\geq 0}$ is determined by its finite dimensional distributions which generate the corresponding probability distribution. The martingale technique is based on concepts in the theory of martingale, more precisely the "Snell envelop" concept that is to look for the smallest supermartingale that dominates the gain function. In the Markovian approach the underlying process is determined by a family of transition functions leading to a Markovian family of transition probabilities. This approach has proven to be an effective tool in optimal stopping problems due to the powerful tools provided by the theory of Markov processes and that how it has the name of Markovian approach.

The theory of stopping problems has many applications and has attracted the interest of many researchers. These include Shiryav [39], El-Karaoui [49], Salminen [106] and Oksendal and Reikvam [99]. Stopping problems and its connections to free-boundary problems has been discussed in the book by Peskir and Shiryaev [100]. Using the notion of concavity, a characterization of excessive function for a general one-dimensional diffusion was obtained by Dayanik and Karatzas [40]. This characterisation gives an explicit solution to the stopping problem in some suitably generalized sense. Alvarez [8] gives an explicit solution to class of impulse control problems, using a combination of the classical theory of diffusions, stochastic calculus, and ordinary non-linear programming to verify a general condition for the existence and uniqueness of an optimal impulse control for controlled diffusion that are considered linear, regular, and time homogeneous. Inspired by tools to treat general parking problem, Beible and Lerche [[14], [15]] described a simple solution method to determine optimal strategies based on martingale techniques. In the absence of assumptions on regularity of the value function, the viscosity solution approach for optimal stopping was presented in the paper by Pham [101].

The theory of optimal stochastic optimal control involving sequential switching decisions is a generalization of optimal stopping theory that plays a crucial role in stochastic control field. The increase interest in the study of optimal switching problems (also called starting and stopping problems) is due to its application in economy and finance, and in real options that are concern with the optimal timing of investment decision. The incorporation of switching problems into the study of real options was started by Brennan and Schwartz [22], Dixit[44] and Dixit and Pindyck [44]. For example, in the case of resource extraction, the switching problem is that an operator observe the price of natural resource and would like to optimize his/her profit by managing an extraction facility in an optimal way. Depending on the price fluctuations, he/she has to choose when to start/stop extracting this resource. Therefore the problem is to find an optimal switching policy and a corresponding value function. Several papers undertake research on optimal switching problems, Brekke and Oksendal [21] studied optimal switching problems in the case of geometric Brownian motion for the underlying state variable and special case for the value function, Duckworth and Zervos [46] and Zervos [118] solved the variational inequality associated with the optimal switching problem using a verification theorem, Johnson and Zervos[76] have presented an explicit solution to sequential switching problems assuming non-smoothness for running payoff and switching costs, Bayraktar and Egami [13] employed dynamic programming principle and assume excessive characterization of the value function to solve one-dimensional optimal switching problem. A relation between singular and switching problems was studies by Guo and Tomecek [64].

1.3.3 Power price modelling

In the 1970's the electricity industry, across the globe, was monopolistic, hence the concern of economists was in identifying 'fair' prices ([96], [59]), while the concern of engineers running power generation was in balancing load with supply. When electricity markets became liberalised through the 1990's, generating companies sought to maximise profits, meaning prices became important, in an environment defined by some external regulator, and regulatory policy becomes an issue. Therefore, the deregulation of electricity market has increased the interest in modelling prices in these markets and has become important topic of research in academia and industry.

Electricity spot prices exhibit seasonality (at daily, weekly and annual time scales), sudden spikes and very high volatility, all can be traced to non-storability of electricity and matching supply and demand at all times. During periods when demand for electricity is high and there is little spare generation, prices rise. Conversely, during periods of oversupply, that is when demand is low and there is a large excess of generation, prices fall, because the technology is more profitable if kept running. Unexpected features in the electricity market is when enough generators are actually willing to pay to generate, and this causes the possibility for the price of electricity to become negative, because is more profitable to pay consumers to consume electricity. Negative prices in electricity market may also occur from subsidies of renewable energy for example a wind plant may reduce its sale price to become negative in order to receive a subsidy of a greater value to cover the negative price. Although negative prices have not occurred in the UK power market, they have been observed in other markets like Germany, the Netherlands and the Nordic region. The issue in the UK market is what is called "constraint payments", that is the payment from National Grid to generators in particular wind farms to reduce their outputs at a certain times. While all generators prevented from selling their outputs gets constraint payment, there is a fundamental difference between payments made to conventional plant and those to wind farms. In the case of oil, coal and gas, the generators need to save in fuel so National Grid gets rebate from these generators. In the case of wind where there is no fuel cost, if they are called to reduce outputs they loss the type of subsidy they have like Renewable Obligation Certificate (ROC) and the Climate Change Levy Exemption Certificates (LECs). The constraint payments by National Grid to wind farms started in 2010, and what has been observed is that wind plants charge a lot more than the subsidy they lost. For example, the average price paid to Scottish wind farms to reduce output in 2011 was £220 per MWh, whereas the lost subsidy is approximately £55 per MWh. However, the payment to conventional plants was only £34 MWh to reduce outputs in 2011. This cost of balancing the electricity system is actually going to be paid by the electricity consumers [4].

All these features of electricity prices suggest that the techniques already developed for financial markets could not apply directly. Instead some other factors need to be taken into account. In the literature of modelling electricity prices there are two main categories. The category of models that captures some of the prices drivers, such as fuel prices, capacity, carbon emission and power demand, are called structural models. The structural approach in electricity prices modelling is based on the economic principle that supply and demand must match all the time, for modelling the power price a function of supply and demand factors is presented. In this models a dynamic is chosen for the factors instead of choosing directly a dynamic for the prices. The early work in this category is due to Barlow [10]. His model consider a supply curve combined with a mean reverting process for the power demand as the state variable. The same approach has been considered by Skantze et al. [108], Eydeland and Geman [51], while Caretea and Villaplana [30] considered two state variables, power and capacity as a stochastic processes merged with an exponentially shaped supply curve. Other studies that takes into account fuel prices includes Pirrong and Jermayyan [102], Coulon and Howison [71], Aïd et al. [5]. The cost of carbon emission has also been considered for example in Howison and Schwarz [70] and Coulon [34], and a multi-fuel structural approach has been proposed by Carmona, et al. [27]. An extension of these models and a survey on electricity price modelling can be found in Carmona and Coulon [26].

The second category are those that are called reduced-form models. These models give an analytic formula that facilitate pricing techniques by proposing models directly to electricity spot price dynamics. An early model in this category for spot prices is due to Lucia and Schwartz [87], Schwartz and Smith [107]. They capture the long and short-term dynamics by a two factor diffusion model which allow them to make use of the Black-Scholes formula for option pricing because of the log-normality assumption for spot and forward prices. However, the spikes lead authors to use jump diffusion processes instead, to capture the jumps for example Cartea and Figueroa [29], Kluge [80], and a closed-form formula for option pricing

[4] http://www.ref.org.uk/energy-data/notes-on-wind-farm-constraint-payments.

was obtained by employing affine jump model framework as in Deng [42] and Culot et al. [38]. Regime switching models that extract mean reversion from spikes were also suggested by De Jong and Huisman [41] and Weron et al. [114]. An approach by Geman and Roncoroni [56] captures the trajectorial properties of electricity prices by allowing jumps to be downwards when prices are below some threshold.

1.4 Objectives

This thesis provides a methodology to assess the economics of storage in the UK power market. We focus on understanding the value of storage to a stochastic net demand, which is the total power demand minus wind supply.

One of the objectives is to investigate how a storage technology could operate optimally in a power market market, where the decision maker would like to know the optimal time to buy power from the market and store it, then the optimal time to sell this power and make a profit. This problem is considered as buy low sell high problem in which can be characterised in general by entry and exit problems. The ultimate aim is to provide a tool that can inform government energy policy by identifying the right mix of storage facilities to support an increase penetration of wind energy into the power system. The methodology we presenting in this thesis, is based on a stochastic control model with the following inputs: net demand (total demand minus wind supply) as an Itô diffusion X_t; the "merit order curve" (MOC), as function that maps net demand into prices, and the technical characteristics of the storage facilities. While MOC is discontinuous, the payoff of a storage facility will be a time integral of demand passing through the MOC which will be a C^1 function. Johnson and Zervos[76], Lamberton and Zervos [84], enable the solution of stopping problems without the payoff function being "smooth" (C^2) and so enable analysis based on identifying payoff functions based on the MOC. The outputs of the stochastic control model will be switching boundaries that defines when to fill, and when to empty power from storage facility based on the net demand.

1.5 Thesis outline

An outline of this thesis is as follow. Chapter 2 gives a brief history on the UK power market, how does it work under BETTA and EMR, energy mix and an overview on electricity storage technologies.

Chapter 3 will present the core theory we base our analysis on, a summary of the main assumptions and results on recent published work on discretionary optimal stopping problems.

Chapter 4 describes the methodology we adopt to assess the economic impact of storage technology. We consider three main optimal strategies, an irreversible case, when one allows one use of a full storage facility and want to know the optimal time to discharge. The second case is where one have an empty facility, that need to be charged before it can be discharged and not used again. The final case is where one can use the facility many times and would like to know the optimal strategy to use the facility sequentially.

In Chapter 5, we have two main sections, section 5.1 deals with the net demand data, which is the total demand minus wind supply, where we calibrated an OrnsteinUhlenbeck and Feller processes to the historical net demand data. While section 5.2, describe the merit order curve and how it is used to approximate the electricity prices in the UK power market.

A detailed case study, where we applied the methodology to a real mechanical storage technology, a compressed air energy storage CAES is presented in Chapter 6. We investigate number of possible utilisation of this technology given the UK market data. Two main methods are considered in employing the methodology, one based on a theoretical process assumption for the net demand with estimated parameters (Chapter 6), and the other one is based on an empirical study using the net demand data directly (Chapter 7). Finally, conclusions with further extensions are presented in Chapter 8.

Chapter 2

The UK's power market

The UK electricity market has changed significantly since 1990, the year for privatising electricity supply industry in England and Wales. In 1990, the electricity industry was built on four sectors: generation, transmission, distribution and supply. Transmission and distribution sectors are both natural monopolies. In the wholesale market, the Pool was reformed after the privatisation to bring more competition into the generation and supply sectors. However, two large generation companies were manipulating the Pool and causing market power problems in the generation sector. This led the regulators to introduce the new electricity trading arrangement (NETA) in England and Wales in 2001, which replaced the Pool with a voluntary bilateral market and power exchange. To have a single, integrated, competitive market in the whole of Great Britain, in 2005, NETA was extended to Scotland following the British Electricity Trading and Transmission Arrangement (BETTA). Under BETTA arrangement, generation and supply companies chose to merge, as well as electricity supply, water and gas, resulting in a decentralised generation sector. Therefore the competition improved in the generation and supply sectors and so the wholesale electricity market started operating more like a commodity market. A detailed discussion on the competition of the UK electricity market after the privatisation can be found in the paper by Green and Newbery [60].

In 2010, the UK government introduced the Market Electricity Reform (EMR). The EMR is set to ensure three main gaols: affordability, security of supply and decarbonisation. Under this new reform, the investment in low carbon energy will increase, and will help to diversify the UK energy supply. A high level of wind energy is expected to be part of the UK energy mix. As this resource is considered to be intermittent, this will cause problems to the regulators that need to balance supply and demand. Therefore, electricity storage should be part of the puzzle, as this might solve the issue of wind variability.

2.1 British electricity trading and transmission arrangements (BETTA)

The UK power market under BETTA arrangement based on bilateral trading between generators, willing to sell electricity at price they would like to receive, and suppliers, willing to buy electricity to meet their needs at price they would like to pay, with the final price made by negotiation or exchange trading, that is an open and competitive wholesale market. Also, organisations without any generation to sell or any physical demand for electricity, like Banks for example, are allowed to trade in the wholesale market, and usually called Non-Physical-Traders (NPT). The NPT will buy electricity from generators with a negotiated price, and sell it to a supplier, aiming to make a profit as market makers.

Electricity is traded in half an hour blocks, and in real time. While demand might differ from supply, the responsibility of BETTA arrangements is to guarantee demand meets supply; second by second. The role of BETTA is also settling finance imbalances in when supply does not match demand including the following: NPT may buy more or less electricity than they have sold; generators may produce more or less electricity than they have sold; and suppliers (through customer demand) may physically consume more or less electricity than they have purchased. The National Grid Company (NGC), called the system operator, is the organisation that ensures supply and demand is controlled and balanced. The volume of electricity being traded must be notified to NGC, and the role of the NGC is to predict any surplus or deficit in the actual generation and supply from that predicted, and then the generation or supply can be modified accordingly, utilising the Balancing

Mechanism. This maintains security of supply, keeping the overall system in balance in real time and it is governed by the Balancing and Settlement Code (BSC), which is managed by a company sub-contracted by NGC called ELEXON.

Figure 2.1 gives an overview of the UK electricity market under BETTA. Trading is allowed between generators, suppliers and traders before the "gate closure" this is 1 hour ahead the actual delivery of each half-hour chunk. The trading could be either through forwards and future market contracts that could have a time-scale ranging from years to 24 hours ahead the half-hour in subject, or through short-term contracts markets (short-term power exchange). These contracts ranges usually from 1 hour to 24 hours before delivery, and are bilateral deals at the price registered on the power exchange. In close to real time, there is an opportunity for generators

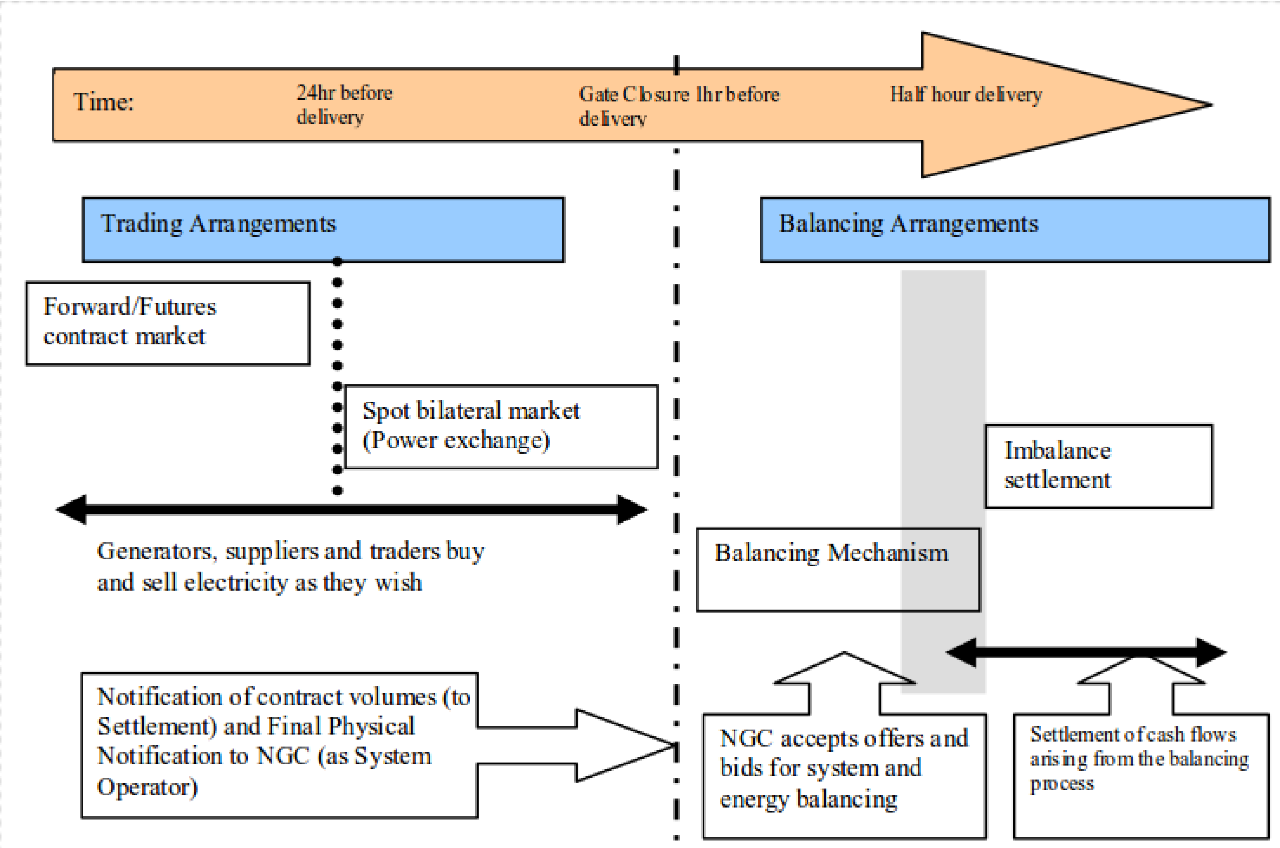


Figure 2.1: Overview of the electricity market under BETTA [37].

and suppliers to participate in the balancing mechanism and gain additional revenue by submitting bids and offers to the NGC after the "gate closure". Where bids are proposals to reduce generation or increase demand, and offers are proposals to increase generation or reduce demand.

However, accepting bids and offers available may not be sufficient to balance the system. It is therefore the responsibility of the NGC to balance the system in real time using other balancing options such as reserve generation service, where it has contracts with generators that could ramp up quickly. Here a storage facility will be the most suitable technology for this service. Currently pumped hydro storage and reserve hydro-electric power stations are already used by the system. Then, Elexon will charge generators and suppliers if they were subjected to any imbalances in their own contracted positions causing the overall imbalance of the transmission system, this process is called "cash out". Much research on the value of storage focuses on balancing (e.g. Black and Strbac[17]).

2.2 Electricity market reform (EMR)

Currently, the Electricity Market Reform (EMR) is still in development. There are three main objectives the UK government want to achieve through the EMR plan. The first is to secure electricity supply, as the demand is expected to double by 2050, reliable and diverse electricity supply to "keep the lights on" will be needed. The second is the de-carbonisation of electricity where there is a need to ensure sufficient investment in low carbon technologies to meet the 2020 target (15% from renewable energy) and 2050 target (80% of greenhouse emissions). The last objective is affordability, which is minimizing the cost for consumers and tax payers, while delivering the investment needed. The key elements for EMR plan to achieve these objectives are Feed in Tariffs with Contract for Difference (FiT CfDs) and capacity Market. FiT CfDs are made in order to support the low carbon investments and are defined by DECC as follow:

"A Contract for Difference (CFD) is a private law contract between a low carbon electricity generator and the Low Carbon Contracts Company (LCCC), a government-owned company. A generator party to a CFD is paid the difference between the 'strike price' - a price for electricity reflecting the cost of investing in a particular low carbon technology-and the 'reference price'-a measure of the average market price for electricity in the GB market. It gives greater certainty and stability of revenues to

electricity generators by reducing their exposure to volatile wholesale prices, whilst protecting consumers from paying for higher support costs when electricity prices are high."

While, capacity market is set to deliver generation adequacy, the main aim is to support security of electricity supply by offering capacity payments to those who commit to deliver energy when it is most needed. This is additional predictable revenue for generators and non-generators that can provide reliable capacity. Fig 2.2

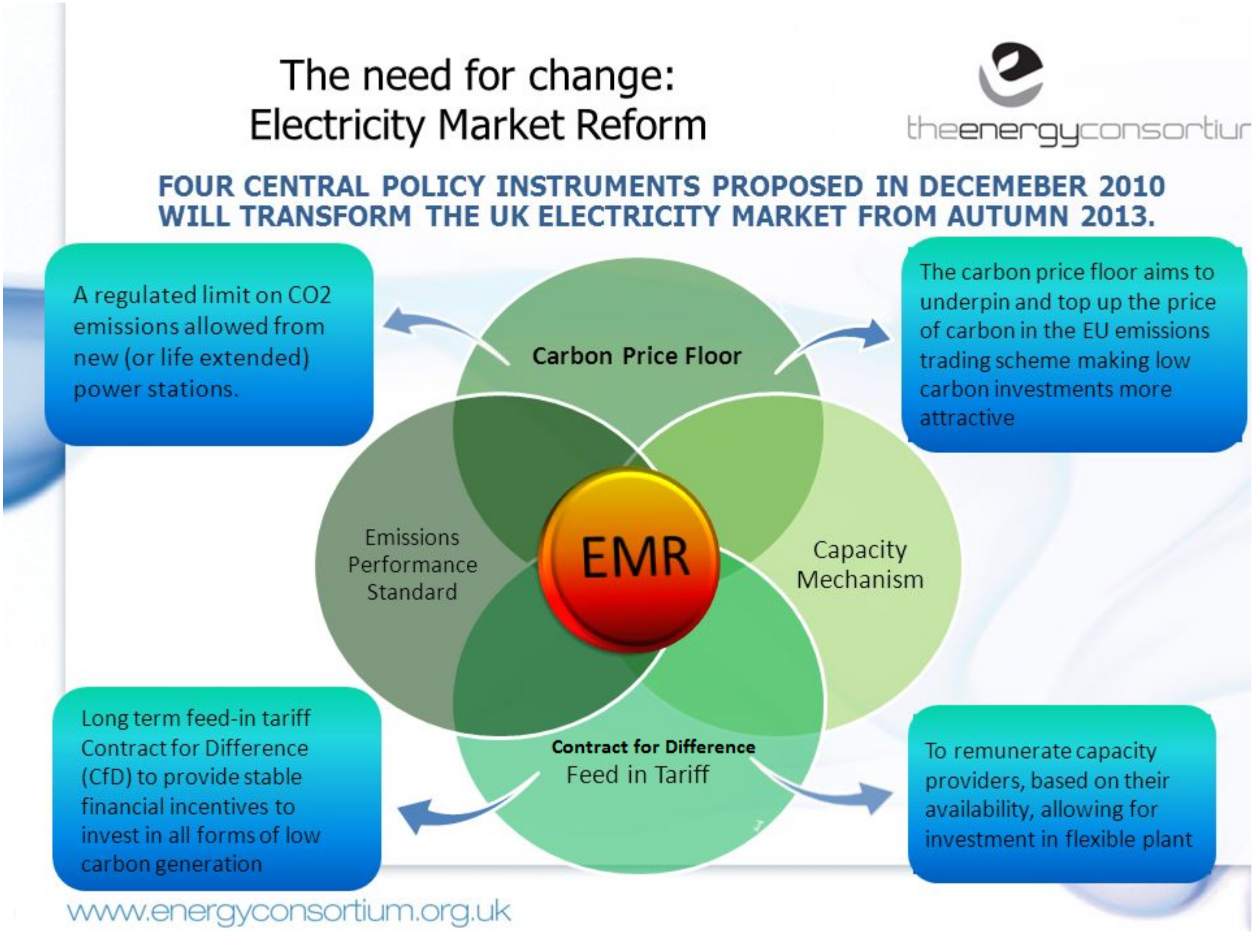


Figure 2.2: Overview of the Electricity Market Reform (EMR) [55].

gives an overview on different elements of the EMR, with some mechanisms already implemented under BETTA, like Carbon Price Support and Emission performance Standard. While Contracts for Difference and Capacity Market are new elements expected to shape electricity prices in the future.

2.3 Electricity generation mix

Currently, the UK's electricity production relies on burning fossil fuels, mainly, coal and natural gas. Coal energy is considered to be cheap, reliable source and a secure supply for decades. However, it has a high level of greenhouse gas emissions like carbon dioxide (CO2) that impact the environment. With the decarbonisation policy that the EMR is aiming for in the future, the UK will have to look for alternative low carbon energy sources rather than relying on coal. Natural gas is considered to be flexible, reliable source of energy. However, importation of gas has increased as the UK's North Sea reserves have dropped, resulting volatile electricity prices that change in response to events outside the UK. There are around 19 gas fired Combined Cycle Gas Turbines operating (CCGT) in the UK, with 30 GW installed capacity and emitting around 365g CO2 per kWh. But there is an increase interest in investing in Carbon Capture Storage (CCS) as part of CCGT plant to help in decarbonisation of the energy market. Open Cycle Gas Turbine (OCGT) which are less efficient than CCGT, is used only when demand is very high once or twice a year.

Nuclear power is a source that is affordable, low carbon and is able to secure a diverse energy supply in the UK. There are nine plants with 16 operational nuclear reactors currently producing electricity, but most of these nuclear power stations are planned to close by 2023 and so new nuclear power stations are needed to help the UK meeting the target of reducing 80% greenhouse gas emissions by 2050 and secure its energy supply. The low flexibility of nuclear sources, it has been managed by pumped hydro storage that absorb the surplus of nuclear power generation during low demand during night time to use it for the demand peak during the day. Therefore, increasing nuclear in the power system would also mean increasing storage technologies to absorb the extra energy from nuclear generation.

Renewable sources are natural energy sources used to produce electricity. The natural fuel sources are mainly wind, marine (wave and tidal), hydro and solar. In 2014, the installed capacity of renewable electricity reached 24.2 GW. Wind energy is the most important renewable source in the UK, since the country is considered to be well placed for such energy comparing to other countries in Europe. With around 12000 MW (8084 MW onshore and 4049 MW offshore) installed capacity, wind energy is contributing significantly in the UK electricity supply. In 2014, wind generation reach 15% of total electricity, assuring electricity to a quarter of homes in Britain [2].

Marine sources are dominated by tidal and waves, with UK considered to be the global leader in this industry, having 10 MW of wave and tidal stream devices already tested in their waters. Wave and tidal energy are going to have significant help with decarbonising the UK energy supply, reduce their dependence on imported fossil fuels as well as providing secure supply. Surprisingly, the installed capacity of solar power hits 5 GW in 2014, says Paul Barwell, chief executive of the Solar Trade Association. "This milestone achievement is testament to the hard work of Britains several thousand solar businesses, almost all of them small and medium sized companies, are at the forefront of real solar transformation as the technologies steadily becoming one of the cheapest sources of clean, home-grown power", he said.

Hydroelectric power is the energy derived from flowing water. This can be from rivers or dam, where water flows from a high-level reservoir down to a lower reservoir and generational electricity. Hydro power contributes with 1.5% in the UK electricity supply. All these renewable technologies are low carbon, have very low operating costs so need more investments.

However, there are some challenges that need to be tackled. Wind does not blow all the time so that make it a variable resource, waves also depend on the wind this make it variable too. While tidal and solar are predictable but not controllable, they may not necessarily occur when electricity is needed, for example the power generated by night-time tides need to be stored, and storage needed when it is cloudy. Therefore, to the UK to meet its target of 30% of their electricity should come from renewable by 2020, new electricity storage technologies should be integrated into the power system, pumping water up the hill it is not enough (Hydroelectric with pump).

Figure 2.3 gives the electricity generation mix in years 2013 and 2014. The coal share decreased from 36.4% in 2013 to 29.1% in 2014, while gas has been increased from 26.9% to 30.2%. A significant increase was observed in energy production from renewables form 14.9% to 19.2% and very small change in nuclear with a decrease from 19.7% to 19%.

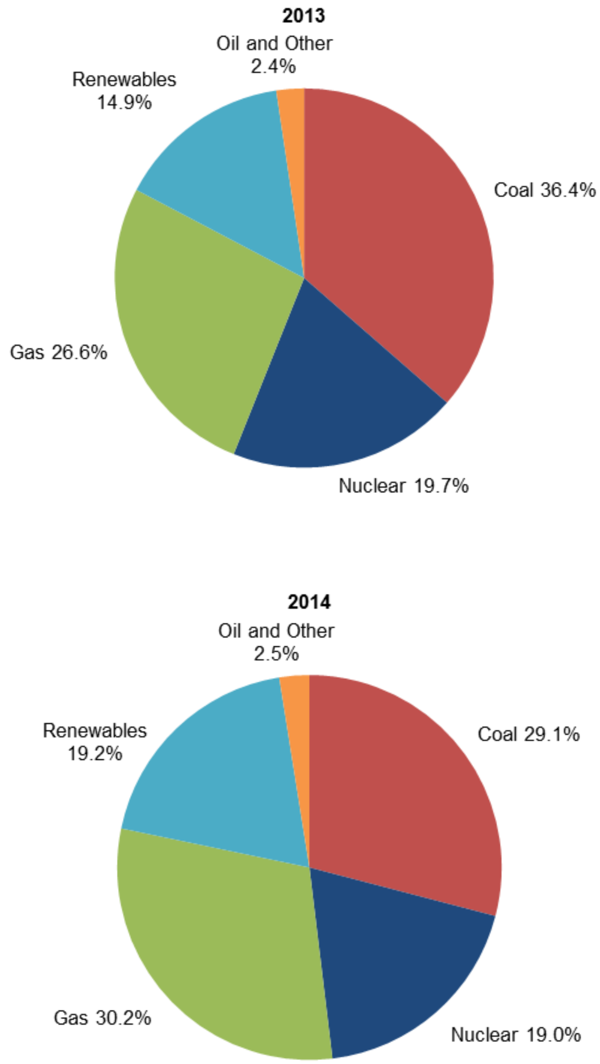


Figure 2.3: Electricity generation mix for 2013-2014 in the UK (DECC).

2.4 Electricity demand and wind supply

The demand side in an electricity market is called "load", which can be defined as the sum of all electricity needed in a market at a given time. The load change from hour to hour, day to day and season to season and there is an underlying pattern, called the "load shape", that enables demand to be forecast with a large degree of accuracy.

The UK's electricity demand like many other countries shows strong seasonality patterns. Example of load shape for a day in September shown in Figure 2.4 demonstrates the changes of electricity load during the day, determined by human activities. There is low demand at night when peoples are sleeping, then visible increase in demand in the morning, when people wake-up and begin using kettles, toasters and power showers. The increases in demand and stabilize, around 9am when offices, shops, organizations start to open and the electrical equipment such as computers are utilized. A second increase in demand will occur between 4 am and 6 am when children and workers return home and start using electrical equipment, lighting, watching television and cooking dinner. Demand then decreases as people begin to sleep.

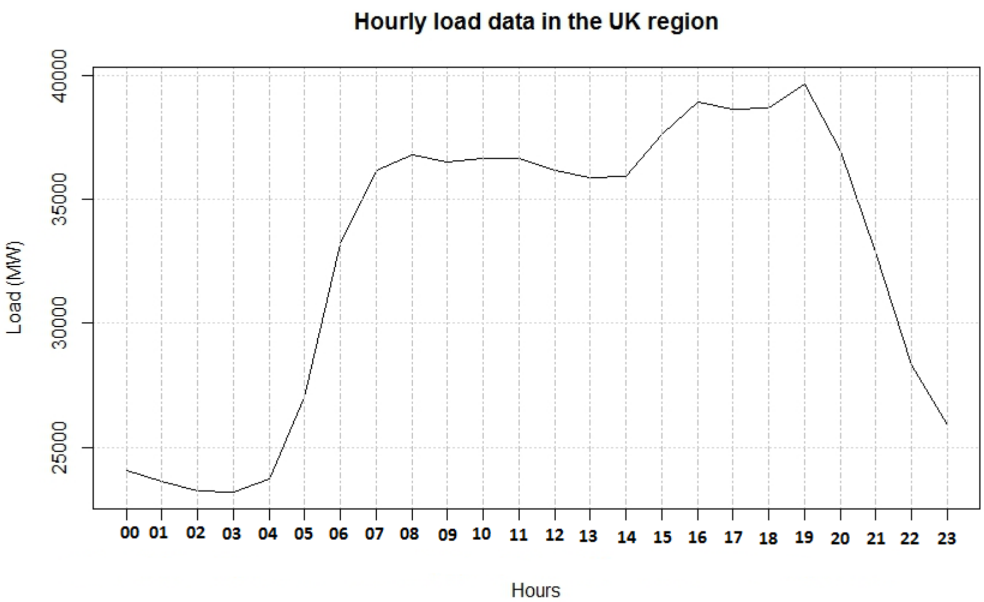


Figure 2.4: The path of load for the day 8-10-2014 in the UK (data from National Grid website).

The weekly patterns are shown in Figure 2.5 as load drops on weekends (Saturday-Sundays). Over the year in Figure 2.6, we see load high in the winter (apart from the Christmas break) and low in the summer.

Now consider a system with high penetration of wind energy, the problem with

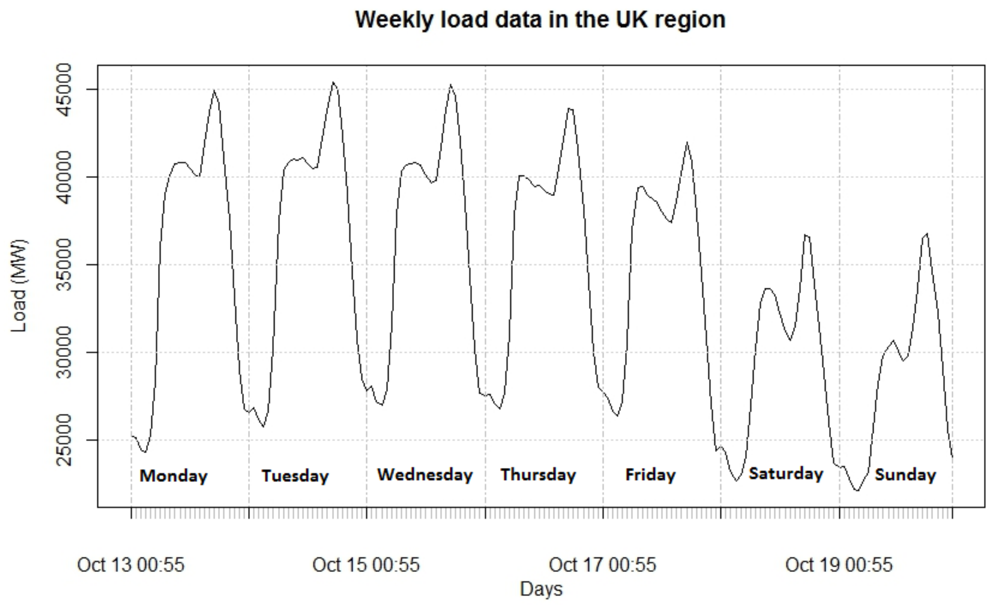


Figure 2.5: The path of load for the week 13 to 19 October 2014 in the UK (data from National Grid website).

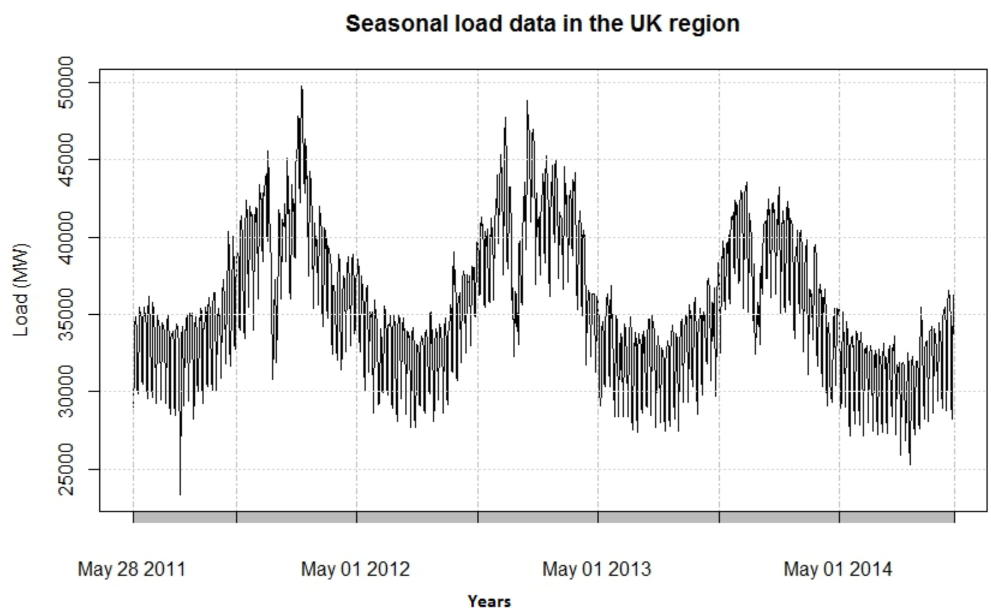


Figure 2.6: The path of daily load for 2011-2014 in the UK (data from National Grid website).

wind supply is the unpredictability, this will introduce an uncertainty in balancing the system. Analysis of integrating a high level of wind power into the power system discussed in [47], shows that when subtracting a 30 GW of variable wind generation from the total demand at each hour, the load histogram shows a normal distribution;

this is clearly observed in Fig 2.7. This result can be explained by the Central Limit Theorem states that the distribution of the sum of a large number of independent random variables, each with finite mean and variance, will approach normality: this signifies that the load shape will disappear, making the load prediction harder. Fig 2.7 (e) also shows that residual demand with high wind level could be negative.

These observations, motivates our study to assess the value of electricity storage in the UK market, for that we denote the wind supply by W_t and total demand by D_t , then we model the net demand (Total demand minus the wind supply) which will be given a stochastic process X_t i.e.

$$X_t := D_t - W_t \tag{2.1}$$

and the units of X_t are MW.

2.5 Electricity energy storage (EES)

From section 2.3 we can observe that UK electricity system is in process of decarbonisation. While sections 2.4 shows that significant penetrations of unpredictable wind, will results uncertainty and variability on the power system. This is a sign for the need of flexible generation to manage the variability of intermittent generation and make the system stable. Interconnection, to other countries is one simple solution; however its attractions are somewhat limited. Demand response to manage the demand side is also suggested. But the solution that getting the most interest is energy storage. In this context, electricity energy storage has the ability to provide a source of flexibility in the UK electricity mix in the future.

2.5.1 Classification of EES technologies

Energy storage encompasses many different technologies with different characteristics. Figure 2.8 show different storage technologies by their power rating, discharge time, category and their suitable grid-scale applications.

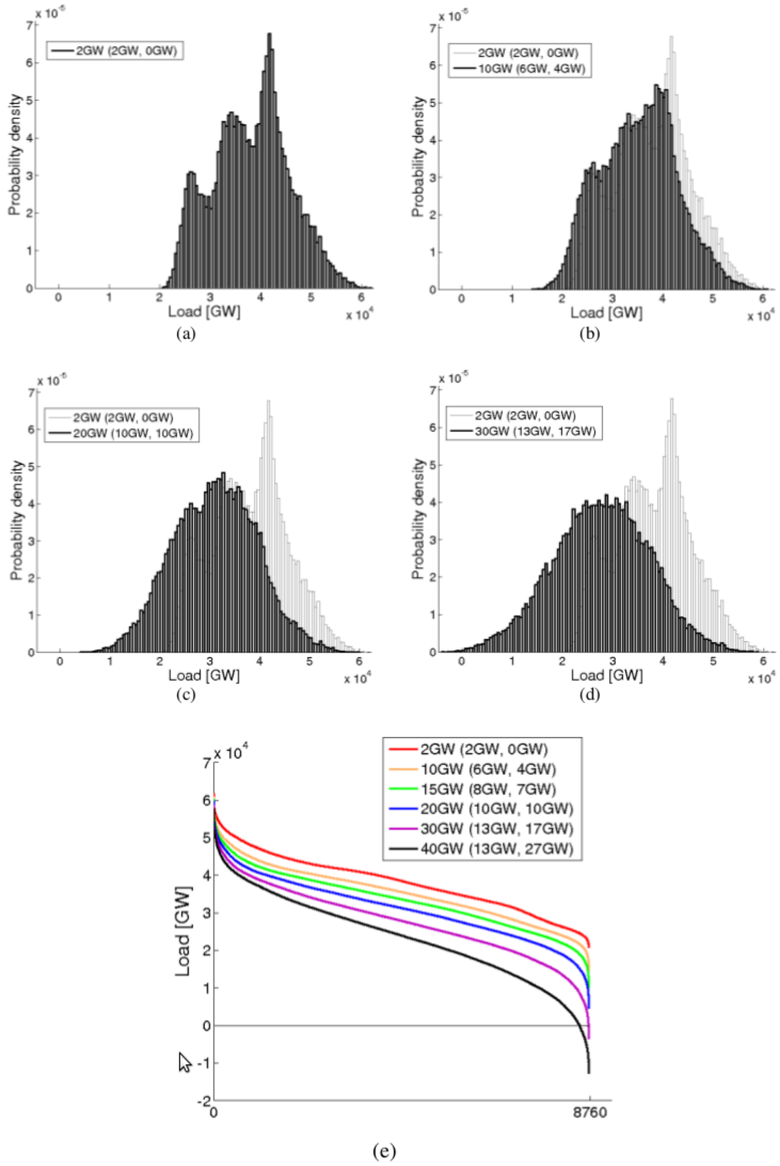


Figure 2.7: Result of increasing installed wind capacity from (a) 2 GW to (b) 10 GW to (c) 20 GW to (d) 30 GW on residual load histograms. Also shown in (e) are some exam- ple residual. Numbers in brackets indicate volume of onshore and offshore capacity respectively [48, p. 169].

Mechanical

Three forms are known for mechanical energy: gravitational potential, elastic potential, and kinetic. All these forms can turn back to electricity. Three mechanical energy technologies are:

Pumped Hydro Storage (PHS): this technology uses gravitational potential based on the idea of pumping the water up into a reservoir when electricity prices are low and then use this reservoir to generate electricity when prices are high. PHS

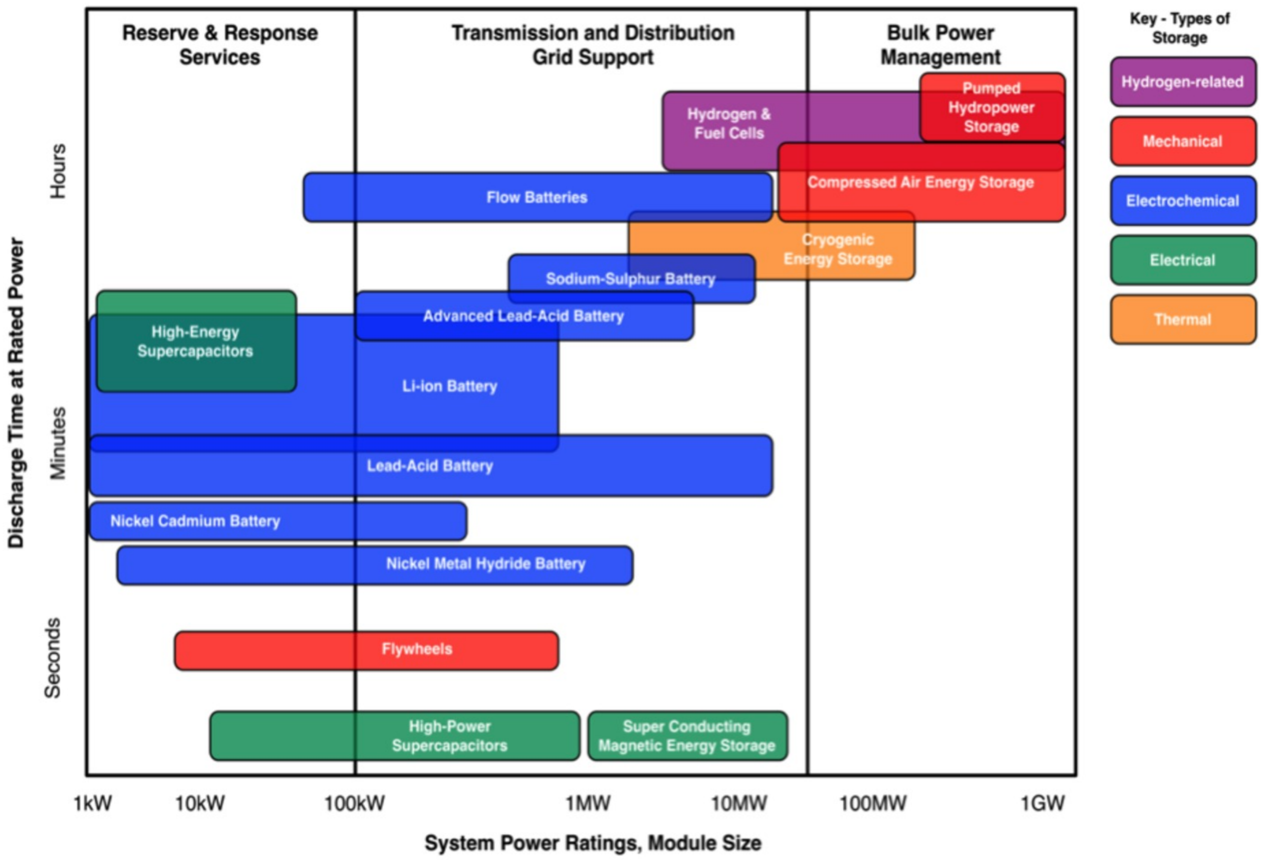


Figure 2.8: Classification of EES options with respect to their power rating and discharge duration capacities [104].

is a mature, large-scale technology characterised by the long storage period, high efficiency (70 – 87%), fast response times and relatively low operating costs with a typical rating between 100 MW and 3000 MW which is the highest rating among all the other EES technologies. In the world there are over 104 GW of PHS in operation with about 44 GW in Europe [31]. In the UK PHS contributes to 99% of the country's electricity storage with 2800 MW installed capacity. However the technology found it difficult to compete for investment in UK since privatisation. There are geological restriction that limits PHS for further deployment, however there are 100 MW PHS site already planned for 2018 [31].

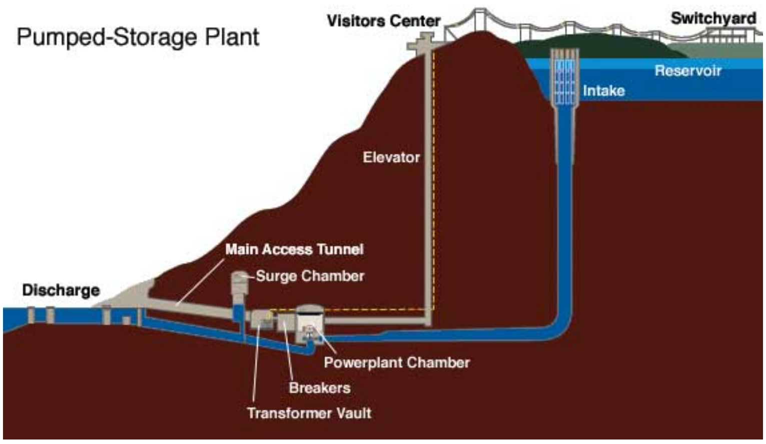


Figure 2.9: Pumped Hydro Storage (PHS) plant [31].

Compressed Air Energy Storage (CAES): this technology uses elastic potential. The air is compressed into a cavern during low demand, then to produce electricity in high demand the compressed air is heated and expanded through high pressure turbine, the air is then mixed with fuel and combusted with the exhaust expanded through a low pressure turbine, and the two turbines are connected to a generator [31]. CAES are characterised by low capital cost, efficiency around $(42 - 54\%)$, relatively long storage period, fast response times and a rating between 50 MW and 300 MW, this rating still higher than other technologies except for PHS. In the

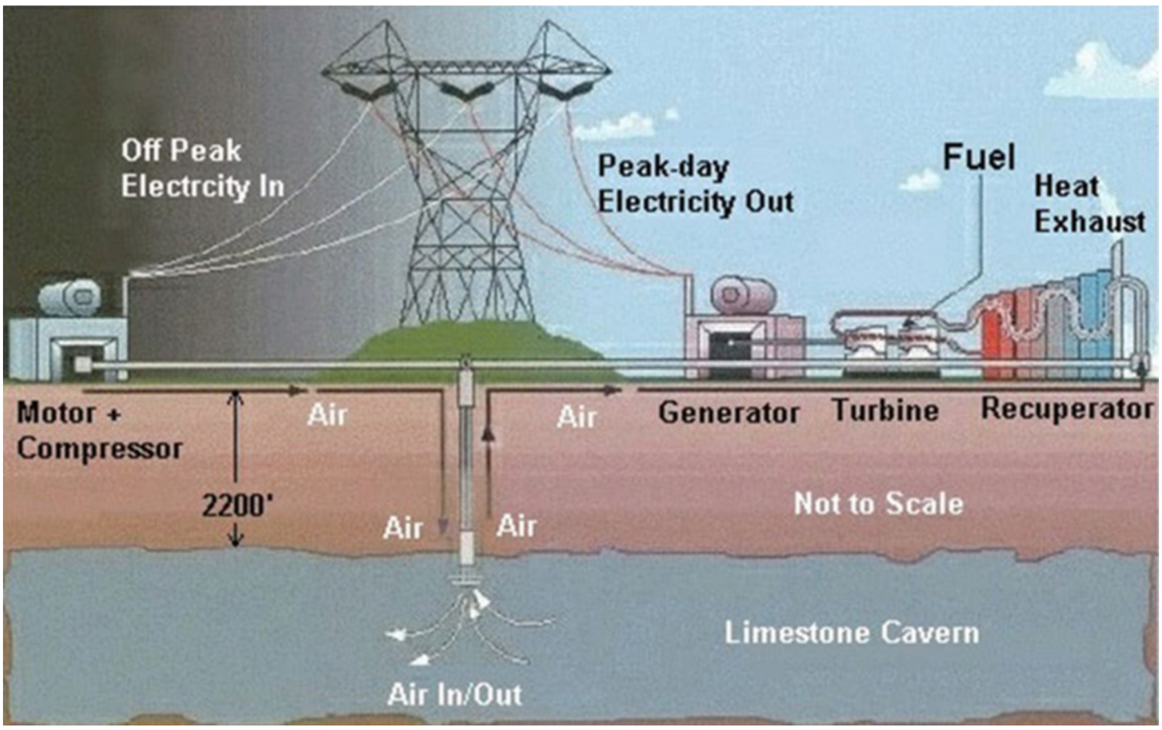


Figure 2.10: Compressed Air Energy Storage System (CAES)[31].

world there are two facilities currently installed of this type: a 290 MW in Huntorf, Germany built in 1978 and 110 MW in Alabama, USA built in 1991. There are other CAES being planned in USA such as Norton, Ohio Project (9×300 MW), Texas project (4×135 MW), Iowa Project(200 MW) and some others in Japan (Chunbu Electric Project) and South Africa (Eskom Project)[31].

Flywheels: this technology uses rotational kinetic energy that has been stored using electric energy as input. It works by accelerating a rotor in the minimum friction losses possible, and then electricity is produced, the spinning force can derive a device like a turbine and produce electricity. This technology is also able to absorb energy from intermittent energy sources, then deliver frequency regulation to the grid. Then installed capacity is a 20MW technology in New York, which help with balancing supply and demand and keep the frequency constant in the system when demand exceed supply and vice-versa [31].

Electrochemical

Battery storage includes different technologies; vary from different chemicals such as lead-acid, nickel-cadmium, nickel-metal, hydride, and different characteristics such as energy density, costs, efficiency and life time.

Lead acid batteries: is old type of batteries, in the charge state the battery consists of electrodes of lead metal and lead oxide in an electrolyte of about 37% sulphuric acid. In the discharge state the electrodes turn into lead sulphate and the electrolyte loses its dissolved sulphuric acid and becomes primarily water. These types of batteries have a low cost ($\$300 - 600$/kWh), high reliability and efficiency ($70 - 90\%$). It is a very good storage utility for power quality. However has a limitation for energy management due to its short life time and low energy density. Although, there exist some applications of large-scale lead acid batteries such as the 8.5 MWh/1 h system in Berlin, the 4 MWh/1 h system in Madrid and the largest one in California with a power rating of 10 MW for 4 h [31].

Nickel cadmium batteries (NiCd): these type of batteries exist more than 100 years ago, contain nickel hydroxide positive electrode plate, a cadmium hydroxide negative electrode plate, a separator and an alkaline electrolyte. NiCd batteries are characterised by high energy density (50 – 75 Wh/kg), efficiency (60 – 90%), very low maintenance requirements and low cycle life. However, the manufacturing process of NiCd is very expensive (£1000/kWh). Nevertheless, the NiCd system has its application in the "world's largest batteries", at Golden valley, Fairbanks, Alaska with a rating of 27 MW for 15 min, 40 MW for 7 min [31].

Sodium sulphur (NaS): NaS batteries consist of liquid sodium (Na) as the negative electrode and liquid sulphur (S) as the positive separated by a solid beta alumina ceramic electrolyte. The NaS batteries are characterised by life cycle of 2500, typical energy in the range of 150 – 240, and density 150 – 230 W/kg and cells efficiency of 75 – 90%. The technology has been demonstrated at over 190 sites in Japan, with 270 MW of stored energy suitable for 8 hours of daily peak shaving installed. There are some applications of the NaS in the USA from the American Electric Power that launched the first demonstration of the technology in Ohio with capacity 1.2 MW [31].

Vehicle-to-Grid(V2G): this technology can be defined as a system that have a bi-directional charges between vehicle and the electricity grid. The battery of the vehicle could be charged from the grid, and then the electricity should flows in the other direction when the grids need electricity. This could give a value to the utilities of up to $4,000 per year per car. A project in Bronholm Danish Island called EDISON has been launched to investigate how a large fleet of electric vehicles (EVs) can be integrated to supports the electric grid and benefit both the individual car owners and society [16]. Bornholm, the Danish island has been selected as simulation scenario for EDISON project because it represents a small grid with the option of operating in island mode[1] and with a high wind power penetration [16].

Thermal

Thermal energy storage is not only about heating, energy could also be stored by cooling materials.

Cryogenic energy storage: is a system that can use the surplus electricity from

[1]When not connected to Sweden via the HVDC connection.

the grid to liquefy air, and form a cryogenic liquid that can be stored in vessel. To deliver electricity back to the grid at peak times, an expansion process could be held by vaporizing the cryogenic liquid into a gas to drive a turbine. Currently, this technology already employed in the UK started from 2011 and operated by Scottish & Southern Energy (SSE), with 60% efficiency. However the technology could reach 70% efficiency if it is located next to conventional plants where excess heat could be recycled.

Hydrogen related

Hydrogen and Fuel Cells: fuel cell is a technology that combines oxygen with hydrogen using highly efficient electrochemical process to produce electricity, heat and water. This technology is able to produce amounts of electric power, ranges from small cells the size of a car battery to larger industrial size cells, which could power entire cities in the future. The UK already has buses that are using hydrogen fuel cells with a possibility that hydrogen fuel cell cars will soon become commercially viable in the UK [3].

Electrical

Super Conducting magnetic energy storage (SMES): is a system that store energy in superconducting coil as a magnetic field created by the flow of direct current. The coil can be discharged by converting the magnetic energy into electricity and released back to a connected power system. The SMES is characterised by strong power density, almost "infinite" number of charge/discharge, cycles and a very high efficiency superior to 95%.

Supercapacitor (SC): is a technology that can store energy electrostatically on the surface of the material, without any chemical reactions. Supercapacitors are also called electrochemical capacitors contain two electrodes separated by electrolyte solution. The electrodes are often made from porous carbon, and because the surface area of activated carbons is very high, and since the distance between the plates is very small, very large capacitances and stored energy are possible using Supercapacitors. This technology is characterised by a higher power density, it can be charged quickly and do not lose its storage characteristic over time [92]. Supercapacitors can accept and deliver charge much faster than batteries, and can afford more charge/discharge cycles than rechargeable batteries. There are number of companies that develop supercapacitors includes, SAFT (France), NESS(Korea), ESMA(Russie), PowerCache(Maxwell, USA), PowerSystem (Japan), etc. [31].

2.5.2 The role of EES

Electricity storage is diverse, with different technologies as we detailed in the previous section. The diversity across the range of electricity storage gives it the ability to be integrated at different levels of the electricity system and providing different applications.

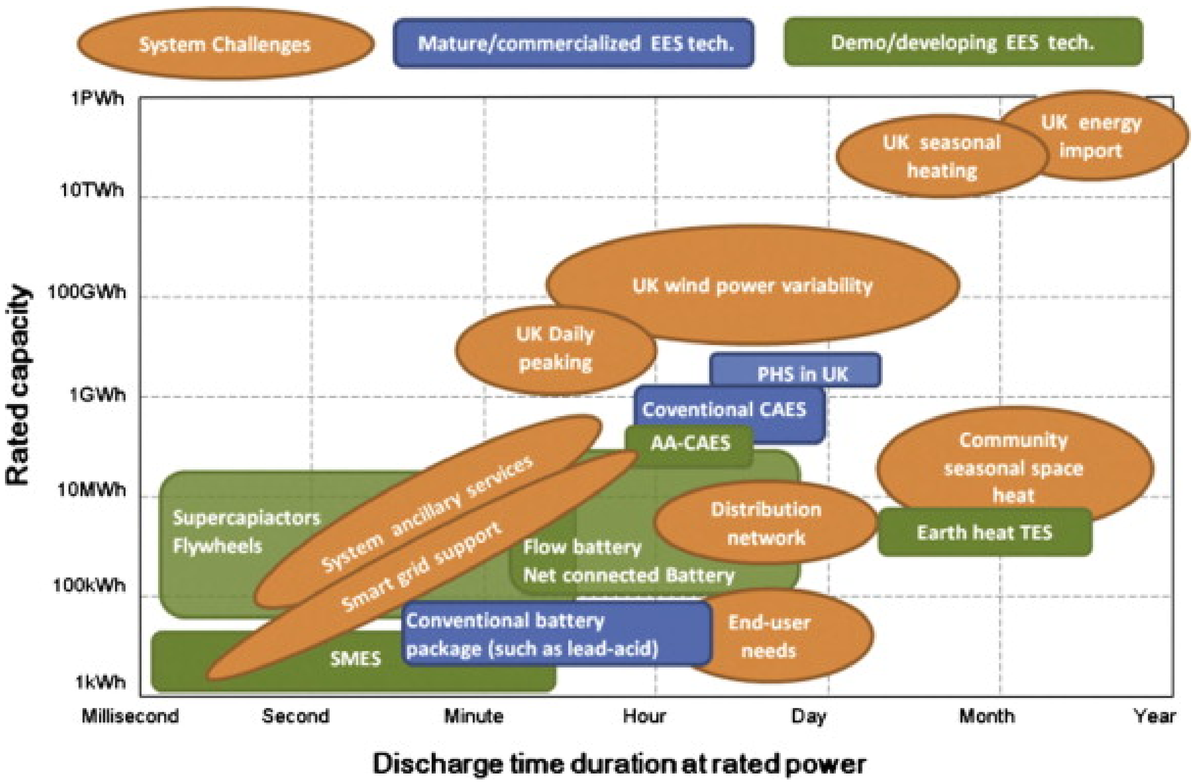


Figure 2.11: Electrical energy storage technologies with challenges to the UK energy systems [89].

There are broad of functions at which EES could be applied depending on the different challenges facing the future energy system in the UK at different time scales, months, days, hours and minutes. Fig 2.11, demonstrate various EES technologies with potentials to address the challenges faced by the UK energy systems [89]. The variability of supply/demand of energy could be managed by providing effective frequency regulation, balancing or load levelling services over a period such as a day, large scale EES technologies should be able to do the job. Fig 2.12 shows, the role of a large scale EES with base load, intermediate and peak generation at different hours of the day, and the potential to reduce the requirements for expensive and less clean peaking generation, that are usually used to meet the peak demand. In this thesis, we are mainly interested in the challenges caused by integrating high

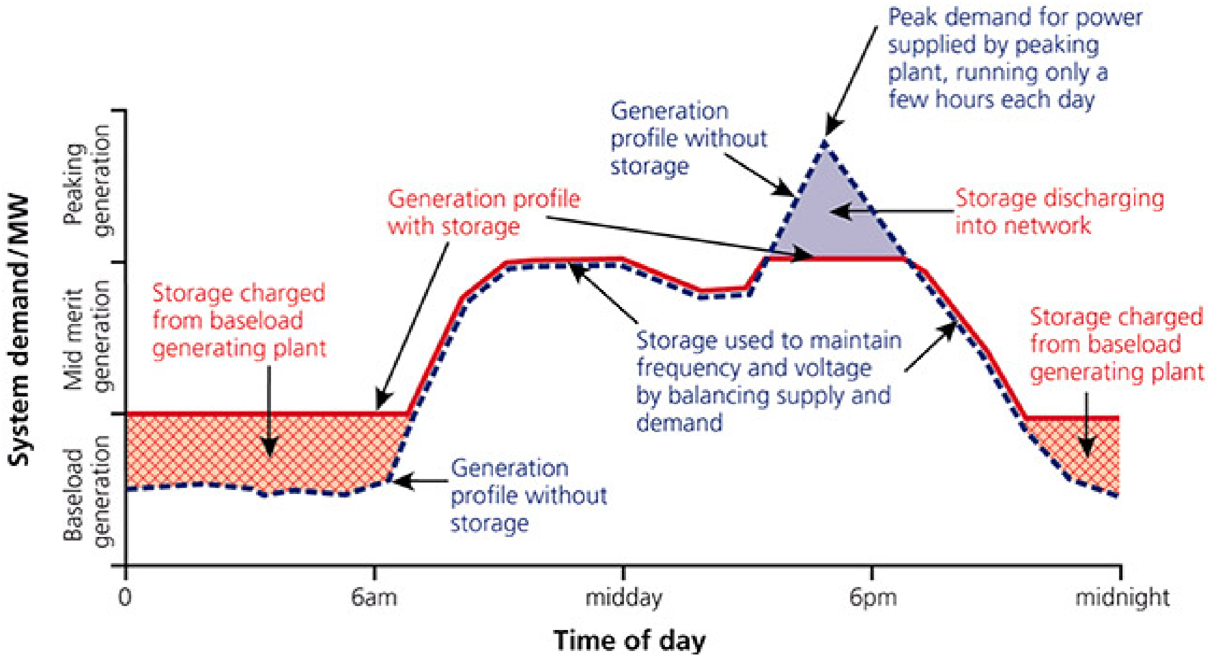


Figure 2.12: Load profile of a large-capacity energy storage system [6].

level of wind energy into the system.

The potential role of EES to play as a source of flexibility is well recognized in the UK and Europe. However there are barriers to further development and deployment of storage to be part of power system. Challenges associated with the technologies, selection of appropriate EES technology have been considered to be important challenge. It is expected that the future deployment of EES will favour a subset of dominant technologies; however it is not yet clear with certainty which those will be. Also, there are challenges associated with strategic aspects; means there is a need for developing a holistic approach to storage, where a link between technical, regulatory, market and policy should be addressed.

Chapter 3

Discretionary stopping problems

The theory of discretionary stopping has been widely employed in finance and economics. This theory is applied, for example, when one need to determine the optimal time to take an action in a stochastic environment, such as buying or selling an asset in a market, or operating a manufacturing facility in response to demand. This provides the basis of our analysis of the power storage problem.

In this chapter, we review the results in Johnson and Zervos [76], Johnson [74] and Lamberton and Zervos [84], that forms the basis of our solution. The main work in Johnson [74], is to develop the theory of discretionary stopping in order to understand under what condition, there exist an optimal strategy in the interval $\mathcal{I}$ for the case where payoff function is C^1 but not necessarily C^2, this is done within a framework based on dynamic programming techniques employing variational inequalities. A verification theorem under assumptions 3.1.1, 3.1.2 and 3.1.4, to ensure that the value function v defined in 3.3 is associated with w the solution to HJB equation 3.5 was proved in Johnson and Zervos [76], with a complete theory to include the case of inaccessible or absorbing boundaries was developed in Lamberton and Zervos [84]. These results give a methodology to obtain an explicit solution to infinite time horizon optimal stopping problems involving general one-dimensional Itô diffusion.

3.1 Approach to solve discretionary stopping problems

3.1.1 Notation

To start let us assume a filtered probability space $\left(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\geq 0}, \mathbb{P}\right)$ satisfies the usual conditions and carrying a standard one-dimensional $(\mathcal{F}_t)$ Brownian motion, W. We denote by $\mathcal{I}$ a given open interval with left end point $-\infty \leq \alpha$ and right endpoint $\beta \leq \infty$, and by $\mathcal{B}(\mathcal{I})$ the Borel σ-algebra on $\mathcal{I}$. Throughout this chapter we consider signed measures of σ-finite total variation, and a refer to them simply as "measures". Given a μ on $(\mathcal{I}, \mathcal{B}(\mathcal{I}))$, the unique positive measures on $(\mathcal{I}, \mathcal{B}(\mathcal{I}))$ resulting from Random decomposition of μ are denoted μ^+ and μ^-, and we have $\mu = \mu^+ - \mu^-$ and $\mid \mu \mid = \mu^+ + \mu^-$, where $\mid \mu \mid$ is the total variation of μ. Also, we say that measure μ on a measurable space $\left(\overline{\mathcal{I}}, \mathcal{B}\left(\overline{\mathcal{I}}\right)\right)$, where $\overline{\mathcal{I}} \subseteq \mathcal{I}$ and $\mathcal{B}\left(\overline{\mathcal{I}}\right)$ is the Borel σ-algebra on $\overline{\mathcal{I}}$, is non-atomic if $\mu(\{c\}) = 0$, for all $c \in \overline{\mathcal{I}}$.

3.1.2 The Itô diffusion and optimization problem

We consider a stochastic economy driven by a one-dimensional, time homogeneous Markov process given by the following stochastic differential equation (SDE),

$$dX_t = b(X_t)\,dt + \sigma(X_t)\,dW_t,\ X_0 = x \in \mathcal{I} = \,]\alpha, \beta[\,, \tag{3.1}$$

where the functions $b, \sigma : \mathcal{I} \to \mathbb{R}$ are $\mathcal{B}(\mathcal{I})$-measurable satisfies the following two assumptions.

Assumption 3.1.1.

$$\sigma^2(x) > 0,\ \forall x \in \mathcal{I},$$
$$\int_{\underline{\alpha}}^{\overline{\beta}} \frac{1+\mid b(s)\mid}{\sigma^2(s)} ds < \infty \ \ \textit{and} \ \ \sup_{s\in[\underline{\alpha},\overline{\beta}]} \sigma^2(s) < \infty, \ \ \textit{for all} \ \ \alpha < \underline{\alpha} < \overline{\beta} < \beta.$$

Assumption 3.1.2. *The solution of the SDE 3.1 is non-explosive.*

Assumption 3.1.1 gives sufficient conditions for the SDE 3.1 to have a weak solution $\mathbb{S}_x$ that is unique in the sense of probability law up to a possible explosion time, for all initial condition $x \in \mathcal{I}$. In particular this assumption implies that for a given $c \in \mathcal{I}$, the scale function p_c given by

$$p_c(x) = \int_c^x \exp\left(-2\int_c^s \frac{b(u)}{\sigma^2(u)} du\right) ds, \text{ for } x \in \mathcal{I}, \tag{3.2}$$

and the speed measure m given by

$$m(dx) = \frac{2}{\sigma^2(x)\, p'(x)\, dx},$$

which characterise the Itô diffusion 3.1, are well defined. Assumption 3.1.2 make sure that the hitting time of the boundary $\{\alpha, \beta\}$ of the interval $\mathcal{I}$ is infinite with probability 1 i.e. the boundaries α and β are inaccessible (see Karatzas and Shreve [77]).

We adopt a weak formulation of the optimal stopping problem that we study. In particular, we allow for a stopping strategy to depend, in principle, on the underlying diffusion's initial condition $x > 0$. Then a stopping strategy is given by the following definition.

Definition 3.1.3. *Given an initial condition $x > 0$, a pair $(\mathbb{S}_x, \tau)$ is called a stopping strategy if $\mathbb{S}_x = (\Omega, \mathcal{F}, \mathcal{F}_t, X, W)$ is weak solution to 3.1 and τ is an $\mathcal{F}_t$-stopping-time. Then the set of all stopping strategies is denoted by $\mathcal{A}_x$.*

The performance criterion associated with each stopping strategy is given as follow,

$$J(\mathbb{S}_x, \tau) = \mathbb{E}_x\left[e^{-\Lambda_\tau} g(X_\tau)\, 1_{\{\tau<\infty\}}\right],$$

where g is the pay-off function and Λ_t the discounted factor given by

$$\Lambda_t = \int_0^t r(X_t)\, ds.$$

The objective of the optimal stopping problem is to maximise the performance criterion $J(\mathbb{S}_x, \tau)$ over all stopping strategies $(\mathbb{S}_x, \tau)$. Therefore, the value function

v is given by

$$v(x) = \sup_{(\mathbb{S}_x,\tau)\in\mathcal{A}_x} J(\mathbb{S}_x,\tau), \text{ for } x \in \mathcal{I}. \tag{3.3}$$

The discounted factor satisfies the following assumption:

Assumption 3.1.4. *The function $r : \mathcal{I} \to]0,\infty[$ is $\mathcal{B}(\mathcal{I})$-measurable and locally bounded, there exists $r_0 > 0$ such that $r(x) > r_0$, for all $x \in \mathcal{I}$ and*

$$\int_{\underline{\alpha}}^{\overline{\beta}} \frac{r(s)}{\sigma^2(s)} ds < \infty, \; \textit{for all}\; \alpha < \underline{\alpha} < \overline{\beta} < \beta. \tag{3.4}$$

Assumption 3.1.4 is needed for the problem 3.3 to be well defined. Intuitively, to solve this stopping problem the dynamic programming is employed. If we consider at time zero, the cases either to wait or stop. If we wait for a time Δt and then continue optimally, we expect that the value function v should satisfy the following inequality

$$v(1,x) \geq \mathbb{E}_x\left[e^{-r\Delta t} v(X_{\Delta t})\right],$$

using Itô formula, dividing by Δt and taking the limit $\Delta t \downarrow 0$, gives

$$\frac{1}{2}\sigma^2(x) v_{xx}(x) + b(x) v_x(x) - rv(x) \leq 0.$$

Alternatively, we can stop, and so we expect that

$$v(x) \geq g(x).$$

Therefore, we expect the value function v to be associated with w solution to the Hamilton-Jacobi-Bellman (HJB) equation,

$$\max\left\{\frac{1}{2}\sigma^2(x) w''(x) + b(x) w'(x) - r(x) w(x), g(x) - w(x)\right\} = 0, \; x \in \mathcal{I}. \tag{3.5}$$

The interval $\mathcal{I}$ can be split into two regions, the stopping region $\mathcal{D} \subseteq \mathcal{I}$ and the continuation region $\mathcal{C} = \mathcal{I} \setminus \mathcal{D}$. For values in $\mathcal{D}$ we require that

$$g(x) - w(x) = 0 \text{ and } \frac{1}{2}\sigma^2(x) w''(x) + b(x) w'(x) - r(x) w(x) \leq 0,$$

while for all values in $\mathcal{C}$, we have

$$g(x) - w(x) \leq 0 \text{ and } \frac{1}{2}\sigma^2(x) w''(x) + b(x) w'(x) - r(x) w(x) = 0.$$

3.1.3 The solution to the associated ODE

Assumptions 3.1.1, 3.1.2 and 3.1.4 ensure the existence of a general solution the following ODE,

$$\mathcal{L}f(x) := \frac{1}{2}\sigma^2(x) f''(x) + b(x) f'(x) - r(x) f(x) = 0, \; x \in \mathcal{I}, \tag{3.6}$$

and is given by

$$f(x) = A\phi(x) + B\psi(x), \tag{3.7}$$

for some constant $A, \; B \in \mathbb{R}$. These solutions have some important properties that we will list here. The functions ϕ and ψ are C^1, their first derivatives are absolutely continuous with $\phi > 0$ (resp. $\psi > 0$) is strictly decreasing (resp. increasing) and for any point $x_1 < x_2$ in $\mathcal{I}$ and a weak solution $\mathbb{S}_{x_1}$, $\mathbb{S}_{x_2}$ to 3.1 we have

$$\frac{\phi(x_2)}{\phi(x_1)} = \mathbb{E}_{x_2}\left[e^{-\Lambda_{\tau_{x_1}}}\right] \text{ and } \frac{\psi(x_1)}{\psi(x_2)} = \mathbb{E}_{x_1}\left[e^{-\Lambda_{\tau_{x_2}}}\right], \tag{3.8}$$

where τ_y is the first hitting time of $\{z\}$, $z \in \mathcal{I}$ and is defined by

$$\tau_y = \inf\{t \geq 0 \,|\, X_t = y\},$$

and

$$\lim_{x\downarrow\alpha}\phi(x)=\lim_{x\uparrow\beta}\psi(x)=\infty.$$

The two functions ψ and ϕ are unique, modulo multiplicative constants, and the scale function p_c defined by 3.2, satisfies the following

$$p_c^{'}(x)=\frac{\phi(x)\,\psi^{'}(x)-\phi^{'}(x)\,\psi(x)}{\phi(c)\,\psi^{'}(c)-\phi^{'}(c)\,\psi(c)}=\frac{\mathcal{W}(x)}{\mathcal{W}(c)}\text{ for all }x,c\in\mathcal{I}, \tag{3.9}$$

where $\mathcal{W}>0$ is the Wronskian of the functions ϕ and ψ. These results are well known in the probability literature, including Feller [53], Breiman [20], Itô and McKean [72], Rogers and Williams [105], Karlin and Taylor [78] and Borodin and Salminen [18].

The previous results are related to classical solution to the ODE 3.6, where the function g is assumed to be C^2. However many of the problems arise in economy and finance violate this assumption. Here it is worth recalling results that has been proved in Johnson and Zervos [75], and Lamberton and Zervos [84], which is associated solution to the ODE 3.6, where the pay-off g is C^1, but not necessarily C^2. The works employ probabilistic methods and study the solutions to the non-homogeneous ODE,

$$\mathcal{L}R_\mu+\mu=0, \tag{3.10}$$

with μ is a measure on $(\mathcal{I},\mathcal{B}(\mathcal{I}))$ and the measure-value operator $\mathcal{L}$ is given by

$$\mathcal{L}g(dx)=\frac{1}{2}\sigma^2(x)\,g^{''}(dx)+b(x)\,g_-^{'}(x)\,dx-r(x)\,g(x)\,dx,$$

on the space of all the functions $g:\mathcal{I}\to\mathbb{R}$ that are the difference of two convex functions. Here we recall that the function g is the difference of two convex function if and only if its left-hand side derivative $g_-^{'}$ exists, is of finite variation, and its second distributional derivative is a measure, which is denoted by $g^{''}(dx)$. In this case the Lebesgue decomposition is given by

$$g^{''}(dx) = g^{''}_{ac}(x)\,dx + g^{''}_{s}(dx)\,, \tag{3.11}$$

where, $g^{''}_{ac}(x)\,dx$ is absolutely continuous with respect of the Lebesgue measure and $g^{''}_{s}(dx)$ is mutually singular with the Lebesgue measure.

Definition 3.1.5. *The set of all measures μ on $(\mathcal{I}, \mathcal{B}(\mathcal{I}))$ such that*

$$\int_{\alpha}^{\gamma} \Psi(s)\,|\mu|\,(ds) + \int_{\gamma}^{\beta} \Phi(s)\,|\mu|\,(ds) < \infty,\ \forall\,\gamma \in \mathcal{I},$$

with

$$\Phi(x) = \frac{2\phi(x)}{\sigma^2(x)\mathcal{W}(x)},\ \Psi(x) = \frac{2\psi(x)}{\sigma^2(x)\mathcal{W}(x)},$$

is denoted by $\mathcal{I}_{\phi/\psi}$ and called space of (ϕ,ψ)-integrable measures.

The continuous process A^{μ} is defined by

$$A^{\mu} = \int_{\alpha}^{\beta} \frac{L_t^y}{\sigma^2(y)}\mu(dy)\,, \tag{3.12}$$

where L^y is the local-time process of X at $y \in \mathcal{I}$. The total variation process $|A^{\mu}|$ of A^{μ} is given by $|A^{\mu}| = A^{|\mu|}$, if μ positive measure and L^y an increasing process $\forall y \in \mathcal{I}$, then A^{μ} is an increasing process, and because A^{μ} has continuous paths we have

$$\int_{0}^{\infty} 1_{\Gamma}(t)\,dA_t^{|\mu|} = 0,\ \text{for all countable sets}\ \Gamma \subset \mathcal{I}.$$

Given this the $r(.)$-potential of this process is given by

$$R_{\mu}(x) = \mathbb{E}_x\left[\int_{0}^{\infty} e^{-\Lambda_t} dA_t^{|\mu|}\right]. \tag{3.13}$$

A measure μ belong to $\mathcal{I}_{\phi/\psi}$ if and only if

$$R_\mu(x) = \mathbb{E}_x\left[\int_0^\infty e^{-\Lambda_t} dA_t^{|\mu|}\right] < \infty,\ \forall x \in \mathcal{I},$$

in which case the function R_μ admit the analytical representation

$$R_\mu(x) = \phi(x)\int_{]\alpha,x[} \Psi(s)\,\mu(ds) + \psi(x)\int_{[x,\beta[} \Phi(s)\,\mu(ds) \tag{3.14}$$

$$= \phi(x)\int_{]\alpha,x]} \Psi(s)\,\mu(ds) + \psi(x)\int_{]x,\beta[} \Phi(s)\,\mu(ds), \tag{3.15}$$

and satisfies the ODE 3.10, and also

$$\lim_{x\downarrow\alpha} \frac{|R_\mu(x)|}{\phi(x)} = \lim_{x\uparrow\beta} \frac{|R_\mu(x)|}{\psi(x)} = 0.$$

Now if $-\mathcal{L}R_\mu = \mu$, we can see that, if $R_{-\mathcal{L}R_\mu}$ is defined by 3.26 and 3.14 with $-\mathcal{L}R_\mu$ in place of μ, then we have

$$R_{-\mathcal{L}R_\mu} = R_\mu. \tag{3.16}$$

Given $\mathcal{F}_t$ stopping time ϱ,

$$\mathbb{E}_x\left[e^{-\Lambda_\varrho}\left|R_\mu(X_\varrho)\right| 1_{\{\varrho<\infty\}}\right] < \infty.$$

The function R_μ satisfies Dynkin's formula, i.e. given any $\mathcal{F}_t$-stopping time $\varrho_1 < \varrho_2$,

$$\mathbb{E}_x\left[e^{-\Lambda_{\varrho_2}} R_\mu(X_{\varrho_2}) 1_{\{\varrho_2<\infty\}}\right] = \mathbb{E}_x\left[e^{-\Lambda_{\varrho_1}} R_\mu(X_{\varrho_1}) 1_{\{\varrho_1<\infty\}}\right] - \mathbb{E}_x\left[\int_{\varrho_1}^{\varrho_2} e^{-\Lambda_t} dA_t^\mu\right]. \tag{3.17}$$

In addition we have the strong transversality condition, i.e. given a sequence of $\mathcal{F}_t$-stopping times ϱ_n such that $\lim_{n\to\infty}\varrho_n = \infty$ (see [76])

$$\lim_{n\to\infty} \mathbb{E}_x\left[e^{-\Lambda_{\varrho_n}}\left|R_\mu(X_{\varrho_n})\right| 1_{\{\varrho_n<\infty\}}\right] = 0. \tag{3.18}$$

Remark 3.1.6. *We recall that the stopping time ρ can take an infinite value ($\rho =$*

∞). In this case we should make an agreement about the value of $e^{-\Lambda_\rho} R_\mu(X_\rho)$ when $\rho = \infty$. For the optimal stopping problem to be well defined we restrict $e^{-\Lambda_\rho} R_\mu(X_\rho)$ to finite stopping times and when $\lim_{n\to\infty} \rho_n = \infty$ we assume the strong universality condition 3.18. If $\rho_n = \infty$ for $n \geq n_0$, $n_0 < \infty$, then we can not employ the Markovian property of the process X_t defined in eq 3.1, and therefore integrals in 3.19-3.22 will not be well defined. The condition of strong transversality has a natural economic interpretation as it reflects the idea that one should expect the present value of any asset at the end of time should be zero, given that nobody can benefit by holding the asset after the end of time.

Furthermore, we have

$$\phi(x)(R_\mu)'_+(x) - \phi'(x) R_\mu(x) = -\mathcal{W}(x) \int_{]x,\beta[} \Phi(s) \mathcal{L} R_\mu(ds), \tag{3.19}$$

$$\phi(x)(R_\mu)'_-(x) - \phi'(x) R_\mu(x) = -\mathcal{W}(x) \int_{]x,\beta[} \Phi(s) \mathcal{L} R_\mu(ds), \tag{3.20}$$

$$\psi(x)(R_\mu)'_+(x) - \psi'(x) R_\mu(x) = \mathcal{W}(x) \int_{]\alpha,x]} \Psi(s) \mathcal{L} R_\mu(ds), \tag{3.21}$$

$$\psi(x)(R_\mu)'_+(x) - \psi'(x) R_\mu(x) = \mathcal{W}(x) \int_{]\alpha,x]} \Psi(s) \mathcal{L} R_\mu(ds). \tag{3.22}$$

We recall that $\mathcal{W}$ is the Wronskian of the functions ϕ, ψ defined by 3.9. The following assumption on the pay-off g is to ensure that the stopping problem is well posed.

Assumption 3.1.7. *The function $g : \mathcal{I} \to \mathbb{R}$ is the difference of two convex functions, and the measure $\mathcal{L}g$ is (ϕ, ψ)-integrable. In addition*

$$\lim_{x\downarrow\alpha} \frac{|\, g(x)\,|}{\phi(x)} = \lim_{x\uparrow\beta} \frac{|\, g(x)\,|}{\psi(x)} = 0, \tag{3.23}$$

and the limits $\lim_{x\uparrow\beta} g(x)/\phi(x)$ and $\lim_{x\downarrow\alpha} g(x)/\psi(x)$ exist in $[-\infty, \infty]$.

The necessary and sufficient conditions for $\mathcal{L}g$ to be (ϕ, ψ)-integrable together with the proof are given in [[84], Theorem 7]. Under Assumptions 3.1.1-3.1.7 it has been proved in [76] and [84] that the pay-off function g can be given by the analytical expression 3.14 and satisfies the equations 3.19-3.22. Also given a function f that is C^1, and having in mind the following calculation

$$\frac{d_\pm}{dx}\left(\frac{g(x)}{f(x)}\right)=\frac{g'_\pm(x)\,f(x)-g(x)\,f'(x)}{f^2(x)}, \tag{3.24}$$

we can see that 3.19-3.20 related to the slop of the function g/ϕ, while 3.21-3.22 related to the slop of g/ψ. We rely on the work by Lamberton and Zervos [84], where it has been proved that under Assumptions 3.1.1-3.1.7, the value function in v associated with the stopping problem and presented by 3.3, is of the form

$$v(x)=\begin{cases}A_j\phi(x)+B_j\psi(x), & \text{if } x\in\mathcal{C}_j\subseteq\mathcal{C}\\ g(x), & \text{if } x\in\mathcal{D},\end{cases} \tag{3.25}$$

where $\mathcal{D}$ is a closed set represent the stopping region, the continuation or waiting region is given by $\mathcal{C}=\mathcal{I}\setminus\mathcal{D}$ and $A_j,\ B_j\geq 0$ are specific to each open interval $\mathcal{C}_j$ of the partition that makes up $\mathcal{C}$. In addition, v is the unique solution to the variational inequality

$$\max\left\{\mathcal{L}v(dx),\ g(x)-v(x)\right\}=0,\ x\in\mathcal{I}, \tag{3.26}$$

in the sense of Definition 3.1.8 bellow, that satisfy the boundary condition

$$\lim_{x\downarrow\alpha}\frac{v(x)}{\phi(x)}=\lim_{x\downarrow\alpha}\frac{g(x)}{\phi(x)}=0 \text{ and } \lim_{x\uparrow\beta}\frac{v(x)}{\psi(x)}=\lim_{x\uparrow\beta}\frac{g(x)}{\psi(x)}=0. \tag{3.27}$$

Definition 3.1.8. *A function $v:\mathcal{I}\to\mathbb{R}$ is a solution to the variational inequality 3.26 if $v(x)$ is the difference of two convex functions, $\mathcal{L}v$ is (ϕ,ψ)-integrable,*

$$\textit{the measure } \mathcal{L}v \textit{ does not charge the set } \mathcal{C}=\{x\in\mathcal{I}\,|v(x)>g(x)\}, \tag{3.28}$$

$$-\mathcal{L}v \textit{ is a positive measure on } (\mathcal{I},\mathcal{B}(\mathcal{I})), \tag{3.29}$$

and

$$g(x)\leq v(x)\ \forall x\in\mathcal{I}. \tag{3.30}$$

Remark 3.1.9. *We note that the condition 3.29 guarantees that a left hand stopping boundary will occur for a decreasing payoff, while a right hand stopping boundary will occur for an increasing payoff, [see [74], Remark 3.1].*

Chapter 4

Approach to assess the economics of power storage

This thesis proposes a methodology, based on recent results on the theory of stochastic control, to inform policy makers of the economic impact of different power storage technologies on power system with a significant level of wind generation. Stochastic control theory has been used in the economic and financial study of gas storage, such as Byers [25], Chen and Forsyth [32], Davison et al. [110], Carmona and Ludkovski [28], Weber et al. [52] and Ware [112]. An explanation of why there have been many interest in applying the theory of stochastic control to gas storage, but not to power storage will be because the electricity prices typically regarded as exhibiting jumps unlike gas prices, and the theory of stochastic control with jumps still immature, inhibiting its use in economics. The electricity prices have more complex features than gas prices, mainly because electricity cannot be stored directly.

In this study we assume that the UK power prices are driven by so called "merit order curve" (MOC) or "stack". The MOC ranks different power plants in ascending order based on their bids to deliver a specific quantity. In a competitive market, the price to be paid to all generators for their generated power is the marginal cost of the most expensive plant selected to meet the total demand needed in the system. More details about the merit order curve and electricity prices in the UK will be given in the next Chapter 5. The jumps in electricity prices can be occurred when demand crosses one of the discontinuities in the merit order curve. The implication is that demand can be modelled as a continuous processes and the price process is given by a staircase function of demand process. Therefore, based on these observations, the analysis undertaking centres modelling demand as one-dimensional Itô process, a random process driven by Brownian motion passing through a merit order curve. This approach is also taken in Moriarty [94]. Itô's Lemma, core theory of optimal control of Itô processes, requires pay-off functions to be C^2, however recent mathematical research by Johnson and Zervos [76] and Lamberton and Zervos [84], has worked around this restriction. The approach we take is based on the results reviewed in Chapter 3.

In this chapter, we will investigate three cases of optimal strategies, section 4.2 describe the case of being endowed with a storage facility and need to know the optimal time to empty it (exit problem), section 4.3 describe the case when you have an empty facility need to be filled before you can empty it and make a profit (entry then exit problem). Finally, section 4.4, describe the case when sequential switching between full and empty could be optimal (sequential entry and exit problem).

4.1 The model formulation

Our analysis will be based on answering the question "how one would operate a storage technology participating in a market where the demand that need to be met by conventional plants is stochastic". The main driver in this market is the electricity demand that will be given by a stochastic process X_t. More precisely our stochastic process X_t will denote the net demand needed in the system, this is the total demand minus wind supply. Let $\mathcal{I} =]\alpha, \beta[$ denote an open interval with left endpoint $\alpha \geq -\infty$ and right endpoint $\beta \leq \infty$. We assume that the stochastic process X_t is driven by one-dimensional Itô diffusion.

$$dX_t = b(X_t)dt + \sigma(X_t)dW_t \ , \quad X_0 = x \in \mathcal{I}, \tag{4.1}$$

where W is a standard Brownian motion hosted by a probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$ satisfying the usual conditions, and the functions b and σ satisfies Assumptions

3.1.1, 3.1.2.

Formally, we consider a stochastic system that can be operated in two modes, “full”, or “empty”. The cost of filling/emptying the facility is linear in price, resulting a “bang-bang” control. We consider the control process given by a càdlàg finite variation process $Z \in \{0,1\}$, where $Z = 0$ represent the facility being empty and $Z = 1$ represent the facility being full. The bang bang control assumption is for convenience and can be relaxed. However more complex analysis will be involved, if the cost of filling/emptying are related to the level of charge and Z will have values in $[0,1]$. This can be considered in further research applied to a specific storage technology.

Definition 4.1.1. *Given an initial condition $(z,x) \in \{0,1\} \times \mathcal{I}$, an admissible strategy, is any collection $\mathbb{Z}_{z,x} = (\mathbb{S}_x, Z, T_n)$, such that*

1. *The control process Z is an $\mathbb{F}_t$-adapted, finite variation, càglàd process with value in $\{0,1\}$ and such that $Z_0 = z$ and,*

2. *The sequence is strictly increasing of $\mathcal{F}_t$-stopping times at which the jumps of Z occur, which can be defined recursively by*

$$T_1 = \inf\{t > 0 | Z_t \neq z\} \;\; and \;\; T_{j+1} = \inf\{t > T_j | Z_t \neq Z_{T_j}\}, \; for \, j = 1, 2, \dots \tag{4.2}$$

with the usual convention that $\inf \emptyset = \infty$.

We denote by $\mathcal{A}_{z,x}$ the set of all admissible strategies.

In particular, if $Z_t = 1$ (resp $Z_t = 0$), then the system is in its charged (resp. discharged) operating mode at time t. The jumps of Z occur at the sequence of times T_n when the operator switches the system between its two operating modes. Therefore, if the system is initially in operating mode $z \in \{0,1\}$, the operator's objective is the select a strategy $\mathbb{S}_{x,z}$ that maximize the following performance criterion

$$J(\mathbb{Z}_{x,z}) = \lim_{n\to\infty} \mathbb{E}_x \left[\sum_{j=1}^{n} e^{-rT_j} \left(E(X_{T_j}) 1_{\{\Delta Z_{T_j} = -1\}} - F(X_{T_j}) 1_{\{\Delta Z_{T_j} = 1\}} \right) 1_{\{T_j < \infty\}} \right], \tag{4.3}$$

with E represent the income from emptying the facility, and F is the cost of filling the facility. Accordingly, the value function v that maximize this performance criterion is given as follow

$$v(x,z) = \sup_{\mathbb{Z}_{x,z} \in \mathcal{A}_{x,z}} J\left(\mathbb{S}_{x,z}\right), \text{for } x \in \mathcal{I} \text{ and } z \in \{0,1\}.$$

For the problem to be well-posed it is required that we have the following assumption

Assumption 4.1.2. *The two functions $F, E : \mathcal{I} \to \mathbb{R}$ satisfy Assumption 3.1.7, and*

$$E(x) - F(x) < 0, \ \textit{for all } x \in \mathcal{I}. \tag{4.4}$$

The economical interpretation for the condition 4.4 is to exclude the possibility of making arbitrary high profits by instantaneously filling and emptying the facility.

Therefore, we expect that the value function v is identifies with ω solution to the Hamilton-Jacobi-Bellman (HJB) equation that takes the form of the following variational inequality

$$\begin{aligned} \max\Big\{ &\frac{1}{2}\sigma^2(x) w_{xx}(x,z) + b(x) w_x(x,z) - r(x) w(x,z), \\ & w(x, 1-z) - w(x,z) + zE(x) - (1-z)F(x) \Big\} = 0. \end{aligned} \tag{4.5}$$

4.2 Emptying a one-use time facility

Let us first consider the case where we are endowed with a charged storage facility and we have one opportunity to discharge it. In this case an optimal strategy will involve waiting until the price go up when it is optimal to empty the facility at a payoff $E\left(x\right)$. If there is no chance to re-use the facility, then we looking for a solution w to the HJB Eq 4.5 that satisfies the following,

$$\frac{1}{2}\sigma^2\left(x\right) w^{''}\left(x\right) + b\left(x\right) w^{'}\left(x\right) - r\left(x\right) w\left(x\right) = 0, \ \text{for } x < b,$$

$$E\left(x\right) - w\left(x\right) = 0 \ \text{ for } x \geq b,$$

with a solution given by

$$w\left(x\right)=\begin{cases}B\psi\left(x\right), & \text{if } x<b,\\ E\left(x\right), & \text{if } x\geq b,\end{cases} \tag{4.6}$$

where ϕ and (resp. ψ) are strictly decreasing (resp. increasing) positive functions defined in Section 3.1.3. Now defining B and b will complete the solution. For that the so called "smooth-fit" condition is applied, which requires the value function to be C^1 at the free boundary point b. This yield to the following system of equations

$$B\psi\left(b\right)=E\left(b\right) \text{ and } B\psi^{'}\left(b\right)=E^{'}\left(b\right)$$

solving both equations for B gives,

$$B=\frac{E\left(b\right)}{\psi\left(b\right)}=\frac{E^{'}\left(b\right)}{\psi'\left(b\right)}.$$

If we define a function Q as

$$Q\left(x\right)=E\left(x\right)\psi^{'}\left(x\right)-E^{'}\left(x\right)\psi\left(x\right),$$

then finding B and b is equivalent to solving the following equation

$$Q\left(b\right)=0. \tag{4.7}$$

We first note that $Q\left(b\right)=0$ correspond to a turning point of the function E/ψ, since $\psi\left(x\right)>0$ for all $x\in\mathcal{I}$. The payoff E is increasing and b is the boundary between the continuation region and the stopping region. If we start from the continuation region, the value function can be given as follows

$$v(x)=\sup_b \mathbb{E}_x\left[e^{-\Lambda_{\tau_b}}E(b)\right] \tag{4.8}$$

$$=\sup_b \mathbb{E}_x\left[e^{-\Lambda_{\tau_b}}\right]E(b). \tag{4.9}$$

Since E is increasing and a consequence of 3.8 we have

$$\frac{\psi(x_1)}{\psi(x_2)} = \mathbb{E}_{x_1}\left[e^{-\Lambda_{\tau_{x_2}}}\right], \tag{4.10}$$

and so

$$v(x) = \sup_b \left(\frac{E(b)}{\psi(b)}\right)\psi(x). \tag{4.11}$$

Therefore, the location of the maximum point of E/ψ gives the stopping boundary and the value of the E/ψ at the maximum provides the coefficient B of ψ where waiting is optimal. This case is implemented in Chapter 6, Figure 6.2.

4.3 Filling and emptying a one-use time facility

Now let us consider the case where we start with an empty storage facility, this storage facility needs to be filled. In this case an optimal strategy will involve waiting until the price falls when it is optimal to fill the facility at a cost $F(x)$, which will deliver the value $B\psi(x)$. Then we can associate the value function v with w defined by

$$w(x) = \begin{cases} B\psi(x) - F(x) & , \text{ if } x < a \\ A\phi(x) & , \text{ if } x \geq a \text{ where } a < b, \end{cases} \tag{4.12}$$

where B and b are defined in section 4.2, and a is such that

$$\frac{B\psi(a) - F(a)}{\phi(a)} = \frac{B\psi'(a) - F'(a)}{\phi'(a)}, \tag{4.13}$$

and $A > 0$ given by

$$A = \frac{\frac{E(b)}{\psi(b)}\psi(a) - F(a)}{\phi(a)}. \tag{4.14}$$

4.4 The sequential filling/emptying time facility

Now consider the case when a decision maker can reverse their decisions sequentially, by assuming that switching between the "discharge" and "charge" regimes, depending on the state process is the optimal strategy.

In this case, we would expect that if we start with a full facility we would wait until it is optimal to empty it, at which point we receive the payoff $E(x)$ (if there no chance to re-use the facility). This suggests the value function is

$$w(x,1) = \begin{cases} B\psi(x), & \text{if } x \in \,]\alpha, b] \\ E(x) + A\phi(x), & \text{if } x \in \,]b, \beta[\,. \end{cases}$$

If we start with empty facility we will wait until it is optimal to fill the facility, at a cost of $F(x)$, that will deliver a value $B\psi$ and we have

$$w(x,0) = \begin{cases} B\psi(x) - F(x), & \text{if } x \in \,]\alpha, a] \\ A\phi(x), & \text{if } x \in \,]a, \beta[\,, \end{cases}$$

with A, B constants, and free boundaries a, b, note that we require filling takes place before emptying so that $\alpha < a < b < \beta$ and we have a non-zero value of the empty facility for $x \geq b$, then w the solution to the HJB equation admit the following form

$$w(x,0) = \begin{cases} B\psi(x) - F(x), & \text{if } x \in \,]\alpha, a] \\ A\phi(x) & \text{if } x \in \,]a, \beta[\,, \end{cases} \tag{4.15}$$

$$w(x,1) = \begin{cases} B\psi(x), & \text{if } x \in \,]\alpha, b]\,, \\ E(x) + A\phi(x) & \text{if } x \in \,]b, \beta[\,. \end{cases} \tag{4.16}$$

A sketch of the solutions 4.15 and 4.16 to HJB is shown in Fig 4.1. Therefore we looking for a solution that satisfies the following: as long as the power demand

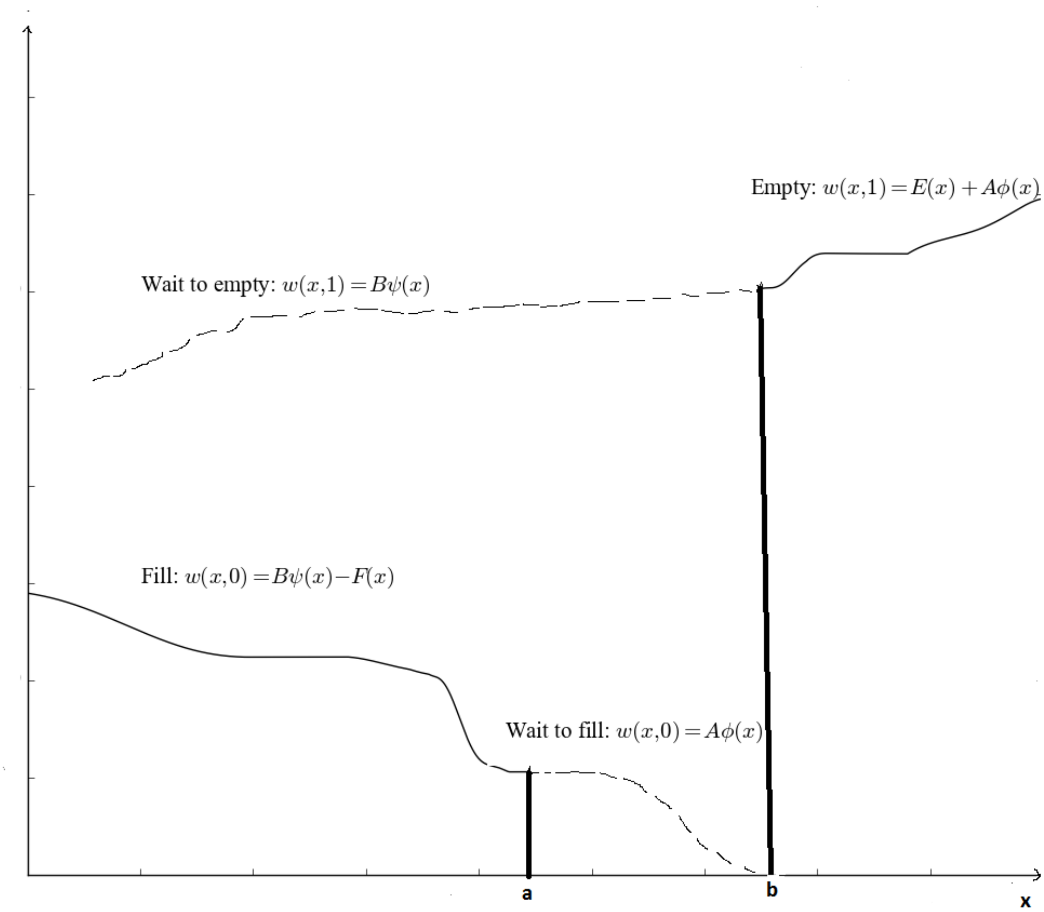


Figure 4.1: A sketch of the value of filling $w(x, 0)$, and the value of emptying $w(x, 1)$. The solid lines represent where filling/emptying take place and the dashed lines represent where waiting to fill/empty is taking a place, the boundaries a and b are the solution to the switching problem.

satisfies $x \leq a$, $w(x, 0) = B\psi(x) - F(x)$, $w(x, 1) = B\psi(x)$, and we have

$$\frac{1}{2}\sigma^2(x)w_{xx}(x, 1) + b(x)w_x(x, 1) - r(x)w(x, 1) = 0, \tag{4.17}$$

$$\frac{1}{2}\sigma^2(x)w_{xx}(x, 0) + b(x)w_x(x, 0) - r(x)w(x, 0) = -\mathcal{L}F(x), \tag{4.18}$$

$$w(x, 1) - F(x) - w(x, 0) = 0 \tag{4.19}$$

$$w(x, 0) + E(x) - w(x, 1) = E\,(x) - F\,(x)\,. \tag{4.20}$$

If the power demand satisfies $a < x < b$, $w(x, 0) = A\phi(x)$, $w(x, 1) = B\psi(x)$, and we have

$$\frac{1}{2}\sigma^2(x)w_{xx}(x,1) + b(x)w_x(x,1) - r(x)w(x,1) = 0, \tag{4.21}$$

$$\frac{1}{2}\sigma^2(x)w_{xx}(x,0) + b(x)w_x(x,0) - r(x)w(x,0) = 0, \tag{4.22}$$

$$w(x,1) - F(x) - w(x,0) = B\psi(x) - F(x) - A\phi(x), \tag{4.23}$$

$$w(x,0) + E(x) - w(x,1) = A\phi(x) + E(x) - B\psi(x). \tag{4.24}$$

If demand satisfies $x \geq b$, and $w(x,0) = A\phi(x)$, $w(x,1) = E(x) + A\phi(x)$, we have

$$\frac{1}{2}\sigma^2(x)w_{xx}(x,1) + b(x)w_x(x,1) - r(x)w(x,1) = \mathcal{L}E(x), \tag{4.25}$$

$$\frac{1}{2}\sigma^2(x)w_{xx}(x,0) + b(x)w_x(x,0) - r(x)w(x,0) = 0, \tag{4.26}$$

$$w(x,1) - F(x) - w(x,0) = E(x) - F(x) \tag{4.27}$$

$$w(x,0) + E(x) - w(x,1) = 0. \tag{4.28}$$

4.4.1 Finding boundaries for a single diffusion

To specify the parameters A, B, a and b let us first assume we can make use of the smooth fit condition of optimal stopping that required the value function to be C^1 in particular on the free boundaries a and b, therefore at the free boundary a we have the following,

$$\begin{cases} B\psi(a) - F(a) = A\phi(a) \\ B\psi'(a) - F'(a) = A\phi'(a). \end{cases}$$

Solving the system for A and B we have,

$$\begin{aligned} \frac{B\psi(a) - F(a)}{\phi(a)}\phi'(a) - \left(B\psi'(a) - F'(a)\right) &= 0 \\ \left(B\psi(a) - F(a)\right)\phi'(a) - \left(B\psi'(a) - F'(a)\right)\phi(a) &= 0 \\ B\left(\psi(a)\phi'(a) - \psi'(a)\phi(a)\right) + F'(a)\phi(a) - F(a)\phi'(a) &= 0 \\ F'(a)\phi(a) - F(a)\phi'(a) &= B\mathcal{W}(a), \end{aligned}$$

using the equations 3.19-3.22 we have

$$\int_{a}^{\beta} \Phi\left(x\right) \mathcal{L}F\left(ds\right) = B,$$

with

$$\Phi(x) = \frac{\phi(x)}{\sigma^2(x)\mathcal{W}(x)}$$

where $\mathcal{W}$ is as defined in 3.9. We also have

$$\begin{aligned}
\frac{A\phi\left(a\right) + F\left(a\right)}{\psi\left(a\right)}\psi^{'}\left(a\right) - \left(A\phi^{'}\left(a\right) + F^{'}\left(a\right)\right) &= 0 \\
\left(A\phi\left(a\right) + F\left(a\right)\right)\psi^{'}\left(a\right) - \left(A\phi^{'}\left(a\right) + F^{'}\left(a\right)\right)\psi\left(a\right) &= 0 \\
A\left(\phi\left(a\right)\psi^{'}\left(a\right) - \phi^{'}\left(a\right)\psi\left(a\right)\right) + \left(F\left(a\right)\psi^{'}\left(a\right) - F^{'}\left(a\right)\psi\left(a\right)\right) &= 0 \\
A\mathcal{W}\left(a\right) = F^{'}\left(a\right)\psi\left(a\right) - F\left(a\right)\psi^{'}\left(a\right),
\end{aligned}$$

using the equations 3.19-3.22 we have

$$\int_{\alpha}^{a} \Psi\left(x\right) \mathcal{L}F\left(ds\right) = A,$$

with

$$\Psi(x) = \frac{\psi(x)}{\sigma^2(x)\mathcal{W}(x)}.$$

While at the free boundary $x = b$ we have

$$\begin{cases} B\psi(b) = E(b) + A\phi(b), \\ B\psi^{'}(b) = E^{'}(b) + A\phi^{'}(b), \end{cases}$$

solving again for A and B we have

$$
\begin{aligned}
\frac{B\psi(b) - E(b)}{\phi(b)}\phi^{'}(b) + E^{'}(b) - B\psi^{'}(b) &= 0 \\
\left(B\psi(b) - E(b)\right)\phi^{'}(b) - \left(B\psi^{'}(b) - E^{'}(b)\right)\phi(b) &= 0 \\
B\left(\psi(b)\phi^{'}(b) - \psi^{'}(b)\phi(b)\right) + E^{'}(b)\phi(b) - E(b)\phi^{'}(b) &= 0 \\
E^{'}(b)\phi(b) - E(b)\phi^{'}(b) &= B\mathcal{W}(b) \\
\int_b^\beta \Phi(s)\mathcal{L}E(ds) &= B,
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{A\phi(b) + E(b)}{\psi(b)}\psi^{'}(b) - E^{'}(b) - A\phi^{'}(b) &= 0 \\
A\left(\phi(b)\psi^{'}(b) - \phi^{'}(b)\psi(b)\right) + E(b)\psi^{'}(b) - E^{'}(b)\psi(b) &= 0 \\
\int_\alpha^b \Psi(s)\mathcal{L}E(ds) &= A.
\end{aligned}
$$

Therefore we have

$$
B = \int_a^\beta \Phi(s)\mathcal{L}F(ds) = \int_b^\beta \Phi(s)\mathcal{L}E(ds), \text{ which need to be } \geq 0, \tag{4.29}
$$

and simultaneously

$$
A = \int_\alpha^a \Psi(s)\mathcal{L}F(ds) = \int_\alpha^b \Psi(s)\mathcal{L}E(ds), \text{ which need to be } \geq 0. \tag{4.30}
$$

In particular if we assume E and F are smooth we have

$$\begin{aligned} E^{'}(x)\,\phi(x) - E(x)\,\phi^{'}(x) &= -\mathcal{W}(x)\int_{x}^{\beta}\Phi(s)\,\mathcal{L}E(s)\,ds \\ &= \frac{d}{dx}\left(\frac{E(x)}{\phi(x)}\right)\phi^{2}(x)\,, \end{aligned} \tag{4.31}$$

$$\begin{aligned} E^{'}(x)\,\psi(x) - E(x)\,\psi^{'}(x) &= \mathcal{W}(x)\int_{\alpha}^{x}\Psi(s)\,\mathcal{L}E(s)\,ds \\ &= \frac{d}{dx}\left(\frac{E(x)}{\psi(x)}\right)\psi^{2}(x)\,, \end{aligned} \tag{4.32}$$

$$\begin{aligned} F^{'}(x)\,\phi(x) - F(x)\,\phi^{'}(x) &= -\mathcal{W}(x)\int_{x}^{\beta}\Phi(s)\,\mathcal{L}F(s)\,ds \\ &= \frac{d}{dx}\left(\frac{F(x)}{\phi(x)}\right)\phi^{2}(x)\,, \end{aligned} \tag{4.33}$$

$$\begin{aligned} F^{'}(x)\,\psi(x) - F(x)\,\psi^{'}(x) &= \mathcal{W}(x)\int_{\alpha}^{x}\Psi(s)\,\mathcal{L}F(s)\,ds \\ &= \frac{d}{dx}\left(\frac{F(x)}{\psi(x)}\right)\psi^{2}(x)\,. \end{aligned} \tag{4.34}$$

Here we note that 4.31-4.32, are related to the slope of the functions E/ϕ and E/ψ. While 4.33-4.34, are related to the slope of the functions F/ϕ and F/ψ.

Therefore the algorithm to find the free boundaries a and b could be identified as follows.

Algorithm 1

1. Identify the interval of points $\mathcal{I}$(i.e. demand is 0-60 GW).

2. For a set of points $\mathcal{A} \subset \mathcal{I}$

   (a) identify the function l^A such that $a < l^A(a)$ and,

$$\frac{\psi^2(a)}{\mathcal{W}(a)}\left[\frac{d}{dx}\left(\frac{F(x)}{\psi(x)}\right)\bigg|_{x=a}\right] = \frac{\psi^2\left(l^A(a)\right)}{\mathcal{W}\left(l^A(a)\right)}\left[\frac{d}{dx}\left(\frac{E(x)}{\psi(x)}\right)\bigg|_{x=l^A(a)}\right].$$

   (b) identify the function l^B such that $a < l^B(a)$ and,

$$\frac{\phi^2(a)}{\mathcal{W}(a)}\left[\frac{d}{dx}\left(\frac{F(x)}{\phi(x)}\right)\bigg|_{x=a}\right] = -\frac{\phi^2\left(l^B(a)\right)}{\mathcal{W}\left(l^B(a)\right)}\left[\frac{d}{dx}\left(\frac{E(x)}{\phi(x)}\right)\bigg|_{x=l^B(a)}\right].$$

3. Find the crossing point of l^A, l^B, which gives the points (a, b).

4. Check that (a, b), A, B satisfy 4.18-4.25.

and we have

$$A = \frac{F(a)\,\psi(b) - E(b)\,\psi(a)}{\psi(a)\,\phi(b) - \psi(b)\,\phi(a)}, \quad B = \frac{F(a)\,\phi(b) - E(b)\,\phi(a)}{\psi(a)\,\phi(b) - \psi(b)\,\phi(a)}.$$

However, the case we have in hand and many of the real economic problems, the functions E, and F are not necessarily C^1, and that the "principle of smooth fit" will not be applied. In this case we can argue as in Johnson and Zervos [76], that we have the following

$$A\phi\,(a) = B\psi\,(a) - F\,(a) \tag{4.35}$$

$$A\phi^{'}\,(a) \leq B\psi^{'}\,(a) - F_{-}^{'}\,(a) \tag{4.36}$$

$$A\phi^{'}\,(a) \geq B\psi^{'}\,(a) - F_{+}^{'}\,(a)\,, \tag{4.37}$$

should hold at the free boundary a, and the inequalities

$$B\psi\,(b) = A\phi\,(b) + E\,(b) \tag{4.38}$$

$$B\psi^{'}\,(b) \leq A\phi^{'}\,(b) + E_{-}^{'}\,(b) \tag{4.39}$$

$$B\psi^{'}\,(b) \geq A\phi^{'}\,(b) + E_{+}^{'}\,(b)\,, \tag{4.40}$$

should hold at the free boundary b. Rearranging the equations 4.59-4.61 and in view of identities 3.19-3.22 we have the following system of inequalities

$$\frac{d_+}{dx}\left(\frac{F\,(x)}{\phi\,(x)}\right)\Bigg|_{x=a}\frac{\phi^2\,(a)}{\mathcal{W}\,(a)} \geq B \geq \frac{d_-}{dx}\left(\frac{F\,(x)}{\phi\,(x)}\right)\Bigg|_{x=a}\frac{\phi^2\,(a)}{\mathcal{W}\,(a)} \tag{4.41}$$

$$\frac{d_+}{dx}\left(\frac{F\,(x)}{\psi\,(x)}\right)\Bigg|_{x=a}\frac{\psi^2\,(a)}{\mathcal{W}\,(a)} \geq A \geq \frac{d_-}{dx}\left(\frac{F\,(x)}{\psi\,(x)}\right)\Bigg|_{x=a}\frac{\psi^2\,(a)}{\mathcal{W}\,(a)}. \tag{4.42}$$

while 4.62-4.64 gives the system of inequalities

$$\frac{d_+}{dx}\left(\frac{E\,(x)}{\phi\,(x)}\right)\Bigg|_{x=b}\frac{\phi^2\,(b)}{\mathcal{W}\,(b)} \leq B \leq \frac{d_-}{dx}\left(\frac{E\,(x)}{\phi\,(x)}\right)\Bigg|_{x=b}\frac{\phi^2\,(b)}{\mathcal{W}\,(b)} \tag{4.43}$$

$$\frac{d_+}{dx}\left(\frac{E\,(x)}{\psi\,(x)}\right)\Bigg|_{x=b}\frac{\psi^2\,(b)}{\mathcal{W}\,(b)} \leq A \leq \frac{d_-}{dx}\left(\frac{E\,(x)}{\psi\,(x)}\right)\Bigg|_{x=b}\frac{\psi^2\,(b)}{\mathcal{W}\,(b)}. \tag{4.44}$$

These imply that the free boundary points $a < b$ should satisfy the system of inequalities

$$Q_{\phi}^{c}\,(a,b) \leq 0 \leq Q_{\phi}^{o}\,(a,b) \tag{4.45}$$

$$Q_{\psi}^{c}\,(a,b) \leq 0 \leq Q_{\psi}^{o}\,(a,b)\,, \tag{4.46}$$

with

$$Q^o_\phi(u,v) = \left[\frac{d_-}{dx}\left(\frac{E(x)}{\phi(x)}\right)\bigg|_{x=v}\frac{\phi^2(v)}{\mathcal{W}(v)}\right] - \left[\frac{d_-}{dx}\left(\frac{F(x)}{\phi(x)}\right)\bigg|_{x=u}\frac{\phi^2(u)}{\mathcal{W}(u)}\right] \tag{4.47}$$

$$Q^c_\phi(u,v) = \left[\frac{d_+}{dx}\left(\frac{E(x)}{\phi(x)}\right)\bigg|_{x=v}\frac{\phi^2(v)}{\mathcal{W}(v)}\right] - \left[\frac{d_+}{dx}\left(\frac{F(x)}{\phi(x)}\right)\bigg|_{x=u}\frac{\phi^2(u)}{\mathcal{W}(u)}\right] \tag{4.48}$$

$$Q^o_\psi(u,v) = \left[\frac{d_-}{dx}\left(\frac{E(x)}{\psi(x)}\right)\bigg|_{x=v}\frac{\psi^2(v)}{\mathcal{W}(v)}\right] - \left[\frac{d_-}{dx}\left(\frac{F(x)}{\psi(x)}\right)\bigg|_{x=u}\frac{\psi^2(u)}{\mathcal{W}(u)}\right] \tag{4.49}$$

$$Q^c_\psi(u,v) = \left[\frac{d_+}{dx}\left(\frac{E(x)}{\psi(x)}\right)\bigg|_{x=v}\frac{\psi^2(v)}{\mathcal{W}(v)}\right] - \left[\frac{d_+}{dx}\left(\frac{F(x)}{\psi(x)}\right)\bigg|_{x=u}\frac{\psi^2(u)}{\mathcal{W}(u)}\right]. \tag{4.50}$$

Therefore, once we identify the cost F of filling the storage facility, and the income E of emptying the facility, an algorithm to solve the optimal stopping problem could be given in the case where E and F are not necessarily C^1.

Algorithm 2

1. Identify the set $\mathcal{A}$, of possible values for the “fill points”, u based on where:

   a. Ensure that 3.29, holds for all $u \in \mathcal{A}$, that is

$$sgn\left(\frac{d^2}{dx^2}\left(\frac{F(x)}{\phi(x)}\right)\bigg|_{x=u}\right) = sgn\left(\frac{d^2}{dx^2}\left(\frac{F(x)}{\psi(x)}\right)\bigg|_{x=u}\right) \geq 0. \quad (4.51)$$

   b. A will be positive, that is

$$\frac{\psi^2(u)}{\mathcal{W}(u)}\left[\frac{d_-}{dx}\left(\frac{F(x)}{\psi(x)}\right)\bigg|_{x=u}\right] > 0. \quad (4.52)$$

   c. B will be positive, that is ,

$$\frac{\phi^2(u)}{\mathcal{W}(u)}\left[\frac{d_-}{dx}\left(\frac{F(x)}{\phi(x)}\right)\bigg|_{x=u}\right] > 0. \quad (4.53)$$

2. Identify the set, $\mathcal{B}$, of possible values for the ’empty points’, v, based on where

   a. Ensure that 3.29, holds for all $v \in \mathcal{A}$, that is

$$sgn\left(\frac{d^2}{dx^2}\left(\frac{E(x)}{\phi(x)}\right)\bigg|_{x=v}\right) = sgn\left(\frac{d^2}{dx^2}\left(\frac{E(x)}{\psi(x)}\right)\bigg|_{x=v}\right) \leq 0. \quad (4.54)$$

   b. A will be positive, that is

$$\frac{\psi^2(v)}{\mathcal{W}(v)}\left[\frac{d_+}{dx}\left(\frac{E(x)}{\psi(x)}\right)\bigg|_{x=v}\right] > 0. \quad (4.55)$$

   c. B will be positive, that is,

$$\frac{\phi^2(v)}{\mathcal{W}(v)}\left[\frac{d_+}{dx}\left(\frac{E(x)}{\phi(x)}\right)\bigg|_{x=v}\right] > 0. \quad (4.56)$$

3. (a) Identify the function $l^A : \mathcal{A} \to \mathcal{B}$ such that $u < l^A(u)$ and

$$Q^c_\psi\left(u, l^A(x)\right) \leq 0 \leq Q^o_\psi\left(u, l^A(x)\right). \tag{4.57}$$

(b) Identify the function $l^B : \mathcal{A} \to \mathcal{B}$ such that $u < l^B(u)$ and

$$Q^c_\phi\left(u, l^B(x)\right) \leq 0 \leq Q^o_\phi\left(u, l^B(x)\right). \tag{4.58}$$

4. If there is a u such that $l^A(u) = l^B(u)$, then we have a candidate pair (a, b) with $a = u$ and $b = l^A(u) = l^B(u)$.

5. Repeat steps [3]-[4] to see if there are different pairs (a, b). Use 4.47-4.50 to identify A and B for each candidate pair (a, b). Choose the one that maximizes the candidate function w. When E and F are not smooth at a or b, the equations 4.47-4.50 will involve inequalities, in this case a continuous fit on the boundaries that are not smooth should be applied.

6. Should check that there are not intervals in the "stopping region" that do not satisfy 3.29 or intervals in the "continuation region" that do not satisfy 3.30. If it happen to have conditions that contradict Definition 3.1.8, then repeat steps [1]-[5] restricting only on the intervals where condition in Definition 3.1.8 are defined (see Example 4.5).

4.4.2 Finding boundaries for two diffusions

Here, we consider a storage facility that works by buying energy and store it when demand is low and prices are cheap, i.e. during night hours. Then selling power back to the grid when demand is high and prices are expensive i.e. during day hours. Here we consider two diffusions for each regime, we empty during day diffusion, then store during night diffusion. In this case we expect the solution w to take the following form

$$w(x,0) = \begin{cases} B\psi_d(x) - F(x) & , \text{ if }]\alpha, a], \\ A\phi_n(x) & , \text{ if }]a, \beta[\end{cases}$$

$$w(x,1) = \begin{cases} B\psi_d(x) & , \text{ if }]\alpha, b], \\ E(x) + A\phi_n(x) & , \text{ if }]b, \beta[, \end{cases}$$

where ψ_d is related to the first hitting time of the day diffusion , and ϕ_n is related to the first hitting time of the night diffusion. As in the single diffusion case, we expect the inequalities

$$A\phi_n(a) = B\psi_d(a) - F(a) \tag{4.59}$$

$$A\phi_n^{'}(a) \leq B\psi_d^{'}(a) - F_{-}^{'}(a) \tag{4.60}$$

$$A\phi_n^{'}(a) \geq B\psi_d^{'}(a) - F_{+}^{'}(a), \tag{4.61}$$

should hold at the free boundary a, and the inequalities

$$B\psi_d(b) = A\phi_n(b) + E(b) \tag{4.62}$$

$$B\psi_d^{'}(b) \leq A\phi_n^{'}(b) + E_{-}^{'}(b) \tag{4.63}$$

$$B\psi_d^{'}(b) \geq A\phi_n^{'}(b) + E_{+}^{'}(b), \tag{4.64}$$

should hold at the free boundary b. And the boundaries for the two diffusions should satisfy

$$Q_{\phi_n}^{c}(a,b) \leq 0 \leq Q_{\phi_n}^{o}(a,b) \tag{4.65}$$

$$Q_{\psi_d}^{c}(a,b) \leq 0 \leq Q_{\psi_d}^{o}(a,b), \tag{4.66}$$

with

$$Q^{o}_{\phi_n}(u,v) = \left[\frac{d_-}{dx}\left(\frac{E(x)}{\phi_n(x)}\right)\bigg|_{x=v}\frac{\phi_n^2(v)}{\mathcal{W}_m(v)}\right] - \left[\frac{d_-}{dx}\left(\frac{F(x)}{\phi_n(x)}\right)\bigg|_{x=u}\frac{\phi_n^2(u)}{\mathcal{W}_m(u)}\right] \quad (4.67)$$

$$Q^{c}_{\phi_n}(u,v) = \left[\frac{d_+}{dx}\left(\frac{E(x)}{\phi_n(x)}\right)\bigg|_{x=v}\frac{\phi_n^2(v)}{\mathcal{W}_m(v)}\right] - \left[\frac{d_+}{dx}\left(\frac{F(x)}{\phi_n(x)}\right)\bigg|_{x=u}\frac{\phi_n^2(u)}{\mathcal{W}_m(u)}\right] \quad (4.68)$$

$$Q^{o}_{\psi_d}(u,v) = \left[\frac{d_-}{dx}\left(\frac{E(x)}{\psi_d(x)}\right)\bigg|_{x=v}\frac{\psi_d^2(v)}{\mathcal{W}_m(v)}\right] - \left[\frac{d_-}{dx}\left(\frac{F(x)}{\psi_d(x)}\right)\bigg|_{x=u}\frac{\psi_d^2(u)}{\mathcal{W}_m(u)}\right] \quad (4.69)$$

$$Q^{c}_{\psi_d}(u,v) = \left[\frac{d_+}{dx}\left(\frac{E(x)}{\psi_d(x)}\right)\bigg|_{x=v}\frac{\psi_d^2(v)}{\mathcal{W}_m(v)}\right] - \left[\frac{d_+}{dx}\left(\frac{F(x)}{\psi_d(x)}\right)\bigg|_{x=u}\frac{\psi_d^2(u)}{\mathcal{W}_m(u)}\right], \quad (4.70)$$

and

$$\mathcal{W}_m = \psi_d^{'}(x)\,\phi_n(x) - \phi_d(x)\,\phi^{'}(x).$$

Therefore, the algorithm to solve Equation 4.65-4.66 and find the free boundaries a and b in the case of the two diffusions case will take the same steps [1]-[6] as the single diffusion case.

4.5 Example

Consider X, a geometric Brownian motion, given by the dynamics,

$$dX_t = bX_t dt + \sigma X_t dW_t \quad (4.71)$$

where, b, σ and r are constant, and the functions ϕ, ψ given by

$$\phi(x) = x^m,\ \psi(x) = x^n,$$

where $m < 0 < n$ are given by

$$\left(\frac{1}{2} - \frac{b}{\sigma^2}\right) \pm \sqrt{\left(\frac{1}{2} - \frac{b}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}. \quad (4.72)$$

In this example, consider $b = 0$, $\sigma = 0.2$, $r = 0.01$, so we have

$$\phi(x) = x^{1/2-\sqrt{3/4}} \text{ and } \psi(x) = x^{1/2+\sqrt{3/4}}.$$

In case we have a payoff function given by

$$g(x) = \max(5 - x, 1)$$

and the operator $\mathcal{L}g$ is given by

$$\mathcal{L}g(x) = \begin{cases} 0.01(x-5) & , \ x \in \left]\alpha, 4\right[, \\ 0.32 & , \ x = 4.0, \\ -0.01 & , \ x \in \left]4, \beta\right[. \end{cases}$$

If we have an interval such that

$$\mathcal{E}^s := \left]x_l, x_r\right[\subset \mathcal{J}, \text{ such that } \mathcal{L}g(\mathcal{E}^s) > 0, \tag{4.73}$$

then there exist a unique pair $a \in \left]\alpha, x_l\right]$ and $b \in \left[x_r, \beta\right[$ solve the optimal stopping problem , where the continuation region $\mathcal{C} = \left]a, b\right[$, and the stopping region is defined by $\mathcal{D} = \left]\alpha, x_l\right] \cup \left[x_r, \beta\right[$. In this example we cannot define such an interval. However, as in Johnson [[74], Remark 3.2] the condition $\mathcal{L}g(\mathcal{E}^s) > 0$ can be relaxed to

$$\int_{\mathcal{E}^s} \Psi(s) \mathcal{L}g(s) > 0, \int_{\mathcal{E}^s} \Phi(s) \mathcal{L}g(s) > 0. \tag{4.74}$$

If we define $u^\star = x_l = 3.95$, from the inequality

$$Q^c_\psi(u^\star, v^\star) \le 0 \le Q^o_\psi(u^\star, v^\star)$$

we can deduce that $v^\star = 166.97$, while if we define $v_\star = x_r = 4.05$, from the inequality,

$$Q^c_\psi (u_\star, v_\star) \leq 0 \leq Q^o_\psi (u_\star, v_\star),$$

we can deduce that $u_\star = 0.06$. We still need to check that

$$Q^c_\phi (u_\star, v_\star) = -23.54 < 0, \; Q^o_\phi (u^\star, v^\star) = 0.157 > 0.$$

Therefore from Lemma A.2 in Johnson [74], there exist a unique (a, b), that can be approximated by $a = 1.34$, $b = 88.6$ and solves the optimal stopping problem, the value function v is given by

$$v(x) = \begin{cases} 5 - x & , \text{ if } x \in \mathcal{D}_1 =]\alpha, 1.34[\,, \\ 4.0732\, x^{1/2-\sqrt{3/4}} + 4.6 \times 10^{-4} x^{1/2+\sqrt{3/4}} & , \text{ if } x \in \mathcal{C} =]1.34, 88.6[\,, \\ 1 & , \text{ if } x \in \mathcal{D}_2 = [88.6, \beta[\,. \end{cases} \tag{4.75}$$

4.6 Notes

1. We note that the problem of filling at night and emptying during the day of storage technology can be considered as a time delay problem. Stochastic systems with time delay have been studied by several authors, for example Kuchler and Platen [82] gives an effective Monte Carlo simulation scheme that converges in a weak sense. While Buckwar [24] studied a numerical solutions of Itô type differential equations and their convergence where the system considered has time delay both in diffusion and drift term, where the stochastic delay differential equations of Itô form is given by

$$dX(t) = b(X(t), X(t-\tau))\,dt + \sigma(X(t), X(t-\tau))\,dW(t), t \in [0, T], \tag{4.76}$$

and $X(t) = \Psi(t)$, $t \in [-\tau, 0]$, with $\tau > 0$ and $\Psi(t)$ is prescribed initial function.

The study of time delay systems is outside the scope of this thesis, however an extension of the method presented here to consider the driver process 4.76 instead of 4.1, it would be a good consideration for future research.

2. The approach we take here to solve the optimal stopping problem is similar to the approach taking in Johnson and Zervos [76], this can be observed by looking at Equations 4.45-4.46, and Equations (129)-(134) in [76]. However, in our analysis we focus on the ratios E/ϕ, F/ψ, E/ψ and F/ϕ rather than the measures $\mathcal{L}E$ and $\mathcal{L}F$ as in the existing literature. This approach was taken by Beibel and Lerche [14] and [15], in this papers they consider the problem of finding a stopping time $\tau^\star$ that maximize

$$v(x) = \sup_{\tau \in \mathcal{T}} \mathbb{E}_x \left[e^{-r\tau} g(X_\tau) 1_{\{\tau < \infty\}} \right], \tag{4.77}$$

where X is Markov process in continuous time. To solve this problem they make use of the martingale property instead of solving a free boundary problem. A crucial point in this paper was finding a positive function h such that $M = \left(e^{-rt} h(X_t)\right)_{t \geq 0}$ is a martingale and the function g/h attains its maximum at x (sufficient conditions for property for continuous local martingales have been given by Protter [103]). Then x is on the optimal stopping set i.e. $v(x) = g(x)$. The question however is whether each point in the optimal stopping set is a maximum point of g/h for an appropriate function h. An efficient way to check that is by ensuring that condition 3.29 in Definition 3.1.8 hold, this was omitted in the papers [14] , [15], however we make a trivial observation that the sign of $\mathcal{L}E$ and $\mathcal{L}F$, which is important part of 3.29 is given by the derivative of the ratio.

Chapter 5

Understanding UK's power prices

In this thesis, we consider a reduced form model with structural features, for electricity prices in the UK power market, where the main driver in this market is considered to be the electricity net demand that is the total power demand minus wind supply. The stochastic assumption for net demand is based on the result by Eager et al.[47] which shows that a high level of wind energy on the system will cause randomness on the need of conventional plants to meet the total demand in the system.

This chapter consist of two main parts. Section 5.1 devoted to the analysis of the UK electricity demand data and section 5.2 contain a construction of the stack model used in this study.

5.1 Analysing UK's electricity demand

This section is devoted to study the UK's electricity demand data, we propose an Ornstein-Uhlenbeck process (OU) and Feller process, to model the data and we make use of maximum likelihood method to estimate the parameters. We also derive the two functions ϕ and ψ we discussed in the previous chapter that are related to the fist hitting times of OU and Feller processes, those two functions play an important role in the methodology we are proposing to study the impact of power storage technologies.

In the previous chapter, we presented an approach to assess the economic impact of power storage technologies. The approach is based on solving an optimal stopping problem where the reward function is not necessarily smooth. The framework, accommodates diffusions with Markovian and time homogeneity property this including: the geometric Brownian motion

$$dX_t = bX_t dt + \sigma X_t dW_t, \;\; \mathcal{I} =]0, \infty[\,, \;\; X_0 = x \in \mathcal{I}, \tag{5.1}$$

the Ornstein-Uhlenbeck (Vasicek) process,

$$dX_t = \kappa\left(\theta - X_t\right) dt + \sigma dW_t, \;\; \mathcal{I} =]-\infty, \infty[\,, \;\; X_0 = x \in \mathcal{I}. \tag{5.2}$$

and the so-called Feller, square-root mean-reverting, or Cox-Ingersoll-Ross process.

$$dX_t = \kappa\left(\theta - X_t\right) dt + \sigma\sqrt{X_t} dW_t, \;\; \mathcal{I} =]0, \infty[\,, \;\; X_0 = x \in \mathcal{I}. \tag{5.3}$$

Where r in Eq 3.5 is constant, the expressions for the general solutions 3.7 to the ODE 3.6, associated with all of these diffusions are all well known. In situations where ϕ and ψ are not known, it is possible to approximate them through simulation, by employing 3.8.

The state variable X_t is assumed to model power demand in this study, or more precisely net demand, that is the total demand minus the production from wind. In other words, X_t is the electricity demand that needs to be met by conventional plants. The obvious choice one may think about is to consider the dynamics of X_t to be given by a Geometric Brownian motion, however part of the analysis is to study the sequential entry and exit for the storage facility, and it has been shown in Alazemi, Johnson and Zervos [90] that it is never optimal to sequentially enter and exit the market in the case where X_t is a Geometric Brownian motion. On the other hand the paper showed that in the case of mean reverting process, it may be optimal to sequentially enter and exit the market depend on the problem data. Therefore, the choice we have between OU process 5.2 and Feller process 5.3. A high level of wind energy in the system may also lead to negative values for net demand.

This lead us to choose Ornstein-Uhlenbeck process 5.2 to model our demand data, since this model revert to the mean level θ with mean reversion speed κ, and could have a negative value. The Feller process 5.3 with the square root in the standard deviation has 0 as a reflecting boundary.

5.1.1 ϕ and ψ for Ornstein-Uhlenbeck process

In chapter 4, we have seen that the solution to the discretionary problem based on knowing the general solution to the following classical ODE

$$\frac{1}{2}\sigma^2(x) f^{''}(x) + b(x) f^{'}(x) - r(x) f(x) = 0, \tag{5.4}$$

which is giving by

$$f(x) = A\psi(x) + B\phi(x).$$

Thus knowing $\phi(x)$ and $\psi(x)$ is crucial. These functions are related to the first hitting time of the process X_t denoted τ_y and given by

$$\tau_y = \inf\{t : X_t = y\},$$

such that for $r > 0$ we have

$$\mathbb{E}_x\left[e^{-r\tau_y}\right] = \begin{cases} \frac{\psi(x)}{\psi(y)}, & x \leq y, \\ \frac{\phi(x)}{\phi(y)}, & x \geq y, \end{cases}$$

this represent the Laplace transform of the density function of the first hitting time of the diffusion X_t, these claims can be found in many references such as Feller [53], Itô and McKean [73], Rogers and Williams [105].

We aim to derive expression for ϕ and ψ for an Ornstein-Uhlenbeck process. An Ornstein-Uhlenbeck process U_t can be defined as a solution to the stochastic differential equation given by

$$dU_t = -\kappa U_t dt + \sigma dW_t,\ U_0 = u. \tag{5.5}$$

with $\sigma > 0$, and $\kappa > 0$. In this study we assume X_t is the power demand net of renewable supply, and κ the rate by which reverts towards the mean θ, in this case we have

$$dX_t = \kappa\left(\theta - X_t\right)dt + \sigma dW_t,\ X_0 = x. \tag{5.6}$$

Let $U_t = X_t - \theta$, we get

$$dU_t = dX_t = -\kappa U_t dt + \sigma dW_t. \tag{5.7}$$

Therefore, the functions ϕ and ψ for the eq in 5.5 are the same for eq in 5.6 with change of variable $u = x - \theta$. The Laplace transform of the density function of the first hitting time for OU in 5.5, is presented by (Alili et al. [7]),

$$\mathbb{E}_u\left(e^{-r\tau_y}\right) = \begin{cases} \dfrac{e^{\kappa u^2/2\sigma^2} D_{-r/\kappa}\left(-u\frac{\sqrt{2\kappa}}{\sigma}\right)}{e^{\kappa y^2/2\sigma^2} D_{-r/\kappa}\left(-y\frac{\sqrt{2\kappa}}{\sigma}\right)} & , u \leq y, \\ \dfrac{e^{\kappa u^2/2\sigma^2} D_{-r/\kappa}\left(u\frac{\sqrt{2\kappa}}{\sigma}\right)}{e^{\kappa y^2/2\sigma^2} D_{-r/\kappa}\left(y\frac{\sqrt{2\kappa}}{\sigma}\right)} & , u \geq y. \end{cases}$$

Also ψ and ϕ appear in the Green's function of the process U_t as follow

$$G\left(u, y\right) = \begin{cases} \dfrac{\psi\left(u\right)\phi\left(y\right)}{\mathcal{W}\left(u\right)} & , u \leq y, \\ \dfrac{\phi\left(u\right)\psi\left(y\right)}{\mathcal{W}\left(u\right)} & , u \geq y. \end{cases}$$

The Green's functions for an Ornstein-Uhlenbeck process indexed by a strictly positive parameter κ with the generator $\mathcal{L}f = \frac{1}{2}\sigma^2 f''\left(u\right) - \kappa u f'\left(u\right)$ is giving by (Borodin and Salminen [19])

$$G(u,y) = \frac{1}{\mathcal{W}(u)} \exp\left(\frac{\kappa u^2}{2\sigma^2}\right) D_{-r/\kappa}\left(u\frac{\sqrt{2\kappa}}{\sigma}\right)$$
$$\times \exp\left(\frac{\kappa y^2}{2\sigma^2}\right) D_{-r/\kappa}\left(-y\frac{\sqrt{2\kappa}}{\sigma}\right), \; u \geq y,$$

and

$$\psi(u) = \exp\left(\frac{\kappa u^2}{2\sigma^2}\right) D_{-r/\kappa}\left(u\frac{\sqrt{2\kappa}}{\sigma}\right)$$
$$\phi(u) = \exp\left(\frac{\kappa u^2}{2\sigma^2}\right) D_{-r/\kappa}\left(-u\frac{\sqrt{2\kappa}}{\sigma}\right),$$

are continuous solutions to generalized differential equation

$$\mathcal{L}f = rf,$$

where D_ν is the parabolic cylinder function with index ν (see Weber [113]). Note that in the case where the function f is not necessarily C^2, the generator is given by $\mathcal{L}f = \frac{1}{2}f''(u) - \kappa u f'_-(u)$. Therefore, the functions ϕ and ψ related to the first hitting times of the OU process in 5.6 are given by the following representation

$$\phi(x) = \exp\left(\frac{\kappa(x-\theta)^2}{2\sigma^2}\right) D_{-r/\kappa}\left(\sqrt{2\kappa}\frac{x-\theta}{\sigma}\right) \tag{5.8}$$
$$\psi(x) = \exp\left(\frac{\kappa(x-\theta)^2}{2\sigma^2}\right) D_{-r/\kappa}\left(-\sqrt{2\kappa}\frac{x-\theta}{\sigma}\right). \tag{5.9}$$

Let,

$$z = \sqrt{\kappa}\frac{x-\theta}{\sigma} \; \textit{and} \; x = \frac{\sigma}{\sqrt{\kappa}}z + \theta, \tag{5.10}$$

so we have,

$$\phi(x) = \exp\left(\frac{z^2}{2}\right) D_{-r/\kappa}\left(\sqrt{2}z\right) \;\; \text{and} \;\; \psi(x) = \exp\left(\frac{z^2}{2}\right) D_{-r/\kappa}\left(-\sqrt{2}z\right),$$

and

$$\phi^{'}(x) = \frac{\sqrt{\kappa}\exp\left(\frac{z^2}{2}\right)}{\sigma}\left[zD_{-r/\kappa}(\sqrt{2}z) + \sqrt{2}D^{'}_{-r/\kappa}(\sqrt{2}z)\right]$$

$$\psi^{'}(x) = \frac{\sqrt{\kappa}\exp\left(\frac{z^2}{2}\right)}{\sigma}\left[zD_{-r/\kappa}(-\sqrt{2}z) - \sqrt{2}D^{'}_{-r/\kappa}(-\sqrt{2}z)\right],$$

also

$$\phi^{''}(x) = \frac{\kappa}{\sigma^2}\exp\left(\frac{z^2}{2}\right)\left[(1+z^2)D_{-r/\kappa}\left(\sqrt{2}z\right) + 2\sqrt{2}zD^{'}_{-r/\kappa}\left(\sqrt{2}z\right) + 2D^{''}_{-r/\kappa}\left(\sqrt{2}z\right)\right]$$
$$\psi^{''}(x) = \frac{\kappa}{\sigma^2}\exp\left(\frac{z^2}{2}\right)\left[\left(1+z^2\right)D_{-r/\kappa}\left(-\sqrt{2}z\right) - 2\sqrt{2}zD^{'}_{-r/\kappa}\left(-\sqrt{2}z\right) + 2D^{''}_{-r/\kappa}\left(-\sqrt{2}z\right)\right].$$

We would like to show that

$$\mathcal{L}\phi(x) = \frac{1}{2}\sigma^2(x)\phi^{''}(x) + b(x)\phi^{'}(x) - r(x)\phi(x) = 0, \tag{5.11}$$

and

$$\mathcal{L}\psi(x) = \frac{1}{2}\sigma^2(x)\psi^{''}(x) + b(x)\psi^{'}(x) - r(x)\psi(x) = 0, \tag{5.12}$$

with

$$\sigma^2(x) = \sigma^2, b(x) = \kappa(\theta - x) = -\sigma\sqrt{\kappa}z, r(x) = r.$$

We can calculate that

$$\mathcal{L}\phi(x) = \exp\left(\frac{z^2}{2}\right)\left[\left(\frac{1}{2} - \frac{z^2}{2} - \frac{r}{\kappa}\right)D_{-r/\kappa}\left(\sqrt{2}z\right) + D^{''}_{-r/\kappa}\left(\sqrt{2}z\right)\right],$$

and

$$\mathcal{L}\psi(x) = \exp\left(\frac{z^2}{2}\right)\left[\left(\frac{1}{2} - \frac{z^2}{2} - \frac{r}{\kappa}\right)D_{-r/\kappa}\left(-\sqrt{2}z\right) + D^{''}_{-r/\kappa}\left(-\sqrt{2}z\right)\right]. \tag{5.13}$$

Using the fact that $D_\nu(y)$ and $D_\nu(-y)$ are linearly independent solutions to the Weber equation [113],

$$\frac{d^2u}{d^2y} + (\nu + \frac{1}{2} - \frac{y^2}{4})u = 0, \tag{5.14}$$

we confirm that $\mathcal{L}\psi = \mathcal{L}\phi = 0$ by using the substitution,

$$\nu = -r/\kappa \quad and \quad y = \sqrt{2}z. \tag{5.15}$$

Now we would like to show that ϕ and ψ are respectively decreasing increasing. For that we make use of the following recurrence relationship of the parabolic cylinder functions

$$\begin{aligned} D_{\nu+1}(z) &= zD_\nu(z) - \nu D_{\nu-1}(z) \\ D'_\nu(z) &= \frac{-z}{2}D_\nu(z) + \nu D_{\nu-1}(z), \end{aligned}$$

so we have

$$\begin{aligned} \phi'(x) &= \frac{\sqrt{\kappa}}{\sigma}\exp\left(\frac{z^2}{2}\right)\left[zD_{-r/\kappa}(\sqrt{2}z) + \sqrt{2}D'_{-r/\kappa}(\sqrt{2}z)\right] \\ &= \frac{\sqrt{\kappa}}{\sigma}\exp\left(\frac{z^2}{2}\right)\left[zD_{-r/\kappa}(\sqrt{2}z) + \sqrt{2}\left[\frac{-\sqrt{2}z}{2}D_{-r/\kappa}\left(\sqrt{2}z\right) - \frac{r}{\kappa}D_{-\frac{r}{\kappa}-1}\left(\sqrt{2}z\right)\right]\right] \\ &= \frac{-r\sqrt{2\kappa}}{\kappa\sigma}\exp\left(\frac{z^2}{2}\right)D_{-\frac{r}{\kappa}-1}\left(\sqrt{2}z\right), \end{aligned}$$

and

$$\begin{aligned} \psi'(x) &= \frac{\sqrt{\kappa}}{\sigma}\exp\left(\frac{z^2}{2}\right)\left[zD_{-r/\kappa}\left(-\sqrt{2}z\right) - \sqrt{2}D'_{-r/\kappa}\left(-\sqrt{2}z\right)\right] \\ &= \frac{\sqrt{\kappa}}{\sigma}\exp\left(\frac{z^2}{2}\right)\left[zD_{-r/\kappa}\left(-\sqrt{2}z\right) - \sqrt{2}\left[\frac{\sqrt{2}z}{2}D_{-r/\kappa}\left(-\sqrt{2}z\right) - \frac{r}{\kappa}D_{-\frac{r}{\kappa}-1}\left(-\sqrt{2}z\right)\right]\right] \\ &= \frac{r\sqrt{2\kappa}}{\kappa\sigma}\exp\left(\frac{z^2}{2}\right)D_{-\frac{r}{\kappa}-1}\left(-\sqrt{2}z\right), \end{aligned}$$

we showed that $\phi'(x) < 0$ and $\psi'(x) > 0$ and that ϕ, ψ decreasing increasing respectively and also are positive. According to Abramowitz and Stegun [91], the

Wronskian of ϕ and ψ is given by

$$\begin{aligned}\mathcal{W}(x) &= \phi(x)\psi^{'}(x) - \phi^{'}(x)\psi(x) \\ &= 2\frac{\sqrt{\pi\kappa}}{\sigma\Gamma\left(\frac{r}{\kappa}\right)}\exp\left(\frac{\kappa(x-\theta)^2}{2\sigma^2}\right).\end{aligned}$$

5.1.2 ϕ and ψ for Feller process (CIR)

Feller process described by the diffusion 5.3, also called the square-root mean reverting process, known in the theory of finance as the model of the short rate in the Cox-Ingersoll-Ross model that is why called CIR as well. The parameters of this diffusion should satisfies the condition $\frac{2\kappa\theta}{\sigma^2} > 1$, for the process to be non-explosive and the hitting time of 0 to be infinite with probability 1. The associated ODE 5.4 for the CIR diffusion is given as follows

$$\frac{1}{2}\sigma^2 x f^{''}(x) + \kappa(\theta - x) f^{'}(x) - rf(x) = 0,\ x > 0. \tag{5.16}$$

Setting $y = \frac{2\kappa x}{\sigma^2}$, and $g(y) = f(x)$, we have

$$yg^{''}(y) + \left(\frac{2\kappa\theta}{\sigma^2} - y\right)g^{'}(y) - \frac{r}{\kappa}g(y) = 0 \tag{5.17}$$

taking, $\gamma = \frac{r}{\kappa}$, $\rho = \frac{2\kappa\theta}{\sigma^2} = 0$, the ODE 6.4 become

$$yg^{''}(y) + (\gamma - y)g^{'}(y) - \rho g(y), \tag{5.18}$$

that is the Kummer's equation, and ψ and ϕ associated to this equation are given as follows

$$\psi(x) = {}_1F_1\left(\frac{r}{\kappa}, \frac{2\kappa\theta}{\sigma^2}; \frac{2\kappa}{\sigma^2}x\right) \text{ and } \phi(x) = U\left(\frac{r}{\kappa}, \frac{2\kappa\theta}{\sigma^2}; \frac{2\kappa}{\sigma^2}x\right), \tag{5.19}$$

where ${}_1F_1(\gamma, \rho, y)$ is the Kummer's confluent hypergeometric function introduced by Kummer [83], and $U(\gamma, \rho, y)$ Tricomi's confluent hypergeometric function introduced by Tricomi [111], these two functions form a linearly independent solutions to 5.18.

Using the following two equations,

$$\frac{d}{dz}\,{}_1F_1\left(a,b,z\right)=\frac{a}{b}\,{}_1F_1\left(a+1,b+1,z\right)$$
$$\frac{d}{dz}U\left(a,b,z\right)=-aU\left(a+1,b+1,z\right),$$

we can calculate the derivatives of ψ and ϕ as follow

$$\psi^{'}\left(x\right)=\frac{r}{\kappa\theta}\,{}_1F_1\left(\frac{r}{\kappa}+1,\frac{2\kappa\theta}{\sigma^2}+1;\frac{2\kappa}{\sigma^2}x\right)>0,$$
$$\phi^{'}\left(x\right)=\frac{-2r}{\sigma^2}U\left(\frac{r}{\kappa}+1,\frac{2\kappa\theta}{\sigma^2}+1;\frac{2\kappa}{\sigma^2}x\right)<0,$$

we conclude that ψ is an increasing function and ϕ a decreasing function. According to Abramowitz and Stegun [91], the Wronskian of ψ and ϕ is given as follow

$$\mathcal{W}\left(x\right)=\frac{\left(\frac{-2\kappa}{\sigma^2}x\right)^{-\frac{2\kappa\theta}{\sigma^2}}}{\Gamma\left(\frac{r}{\kappa}\right)}\exp\left(\frac{2\kappa}{\sigma}x\right).$$

5.1.3 Preparing power demand data

The demand data was taken from the 'Gridwatch' website published by Elexon[1]. The data considered in this study is that of demand net wind supply and it is given at 5-min intervals for the period 1 August 2013 to 31 July 2015. As we mentioned in Chapter 2, the electricity demand in the UK exhibit seasonality in hourly, weekly and yearly time scales. These features make modelling such data hard task, and in this thesis we don't aim to take seasonality in account, because we assume that the seasonality for X_t will disappear with significant wind generation. Therefore, we undertake a methodology that obviates different seasonality patterns in the data. To avoid the daily cycles, we consider two blocks separately (5-min, interval data between 9:00 am to 20:59 pm) and night data (5-min, interval data between 22:00

[1]http://www.gridwatch.templar.co.uk/

pm to 5:59 am) [2]. We only consider day and night data on working days (Monday to Friday), excluding weekends to avoid the weekly cycles. Finally to avert the yearly cycles, the day and night data will be considered for each season in the year, which is common in modelling UK power demand, winter period (November, December, January and February) when demand is high, summer period (May, June, July and August) when demand is low, and shoulder period (March, April, September and October) when demand can be variable year on year. In total we have six data sets that we will fit to a stochastic process in the next section. A graph of the data that we will be using is shown in Fig 5.1. Fig 5.2 shows a profile of 5 min interval of net demand on weekdays (Monday-Friday) calculated by averaging each 5 min interval of each the day and night for the period 1 August 2013-31 July 2015. Some cycles can be observed mainly in day in the winter period and day in the shoulder period. The cycles in the day winter are due to the Christmas break (December), where most of the businesses are shut down and many people are on holiday, so demand drop and then go up in January and February. For the day in the shoulder period, the cycles are due to September and October where the weather start to be colder (high demand) after a warm summer, comparing to March and April where the weather start to be warmer (low demand) after a cold winter. The point about the shoulder months is basically that the winter months are predictably cold, the summer is predictably warm but the shoulder months are more variable. The day in the summer period shows a decrease in demand, which is explained by the fact that the summer in 2015 was very warm. These shows that power demand in the UK is very much depends on the heating.

[2] Note that data does not cover the full 24 hours, this is to resolve the problem of having a negative value for κ when calibrating this data to OU process.

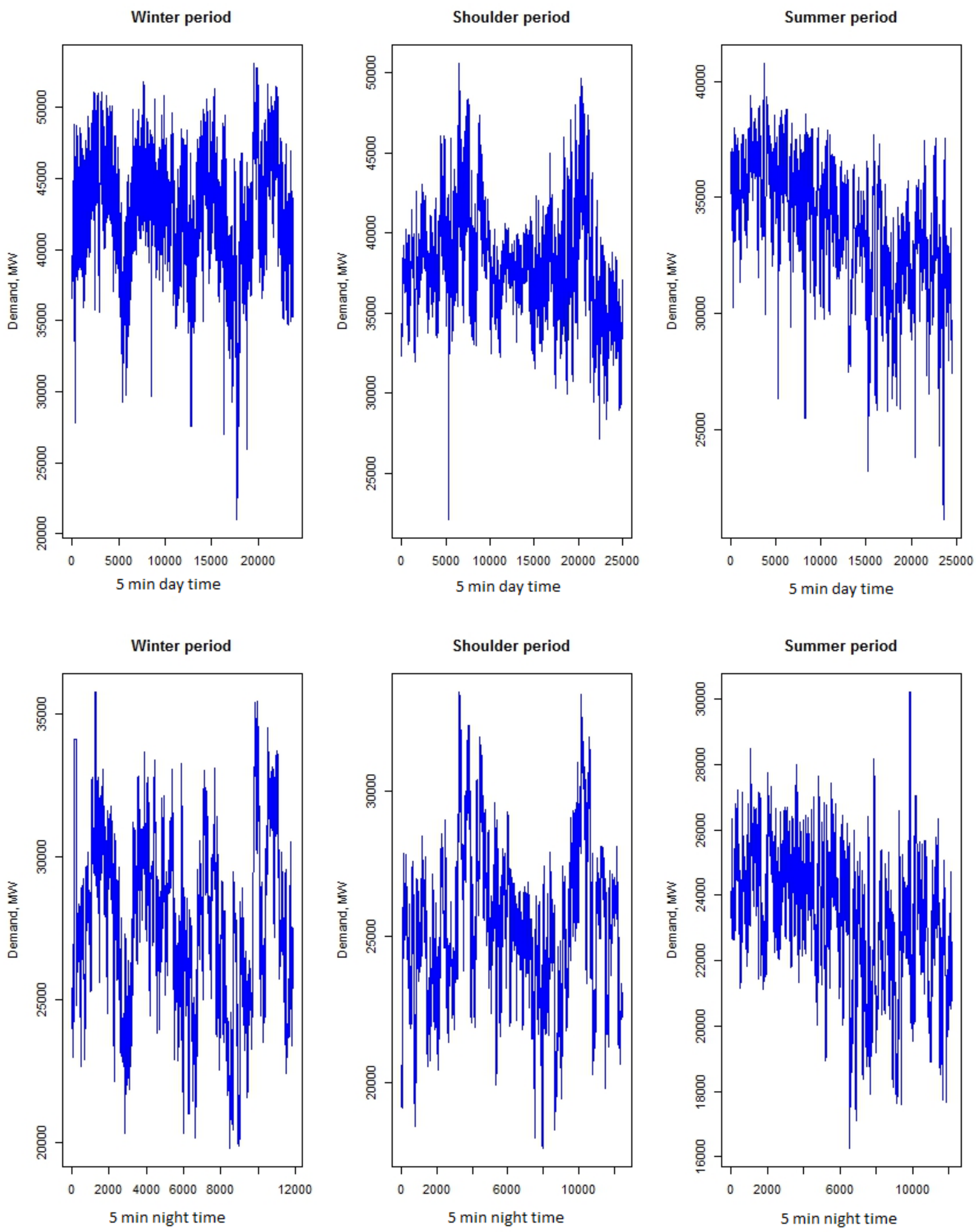
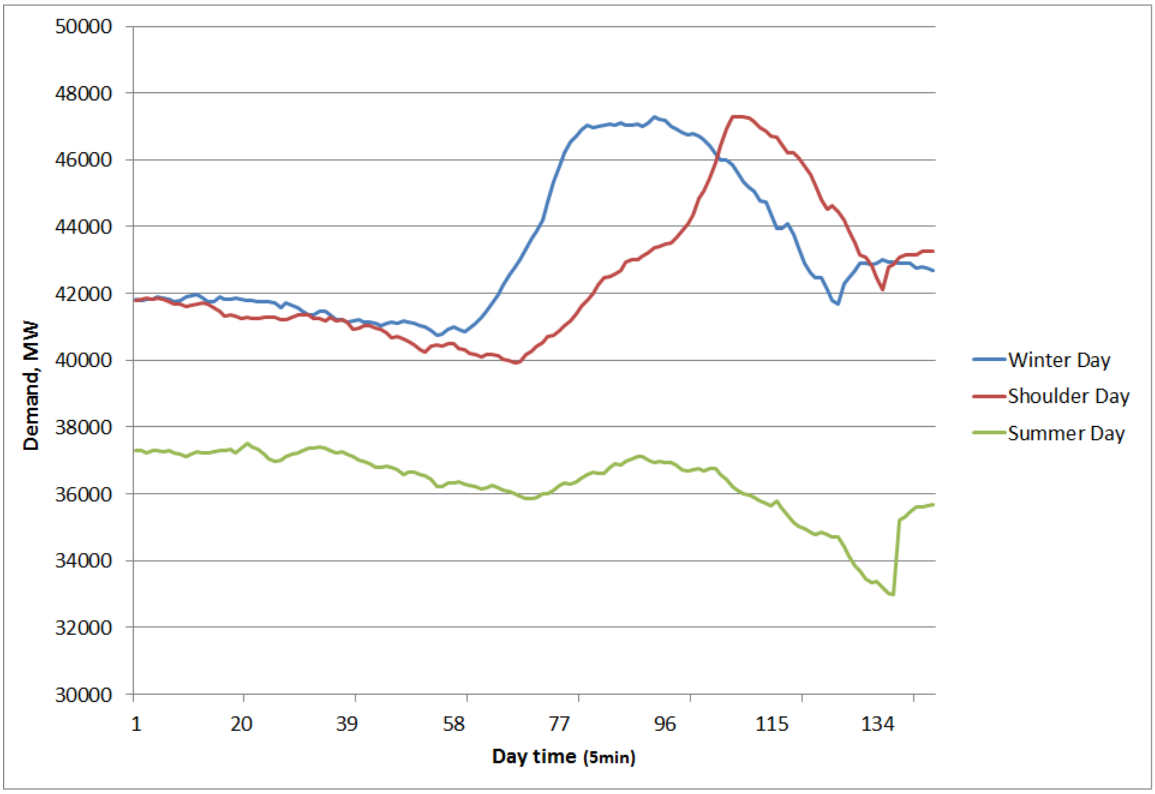


Figure 5.1: Time series of UK net electricity demand for each season, during day time (up) and night time (down) for the period 1 August 2013-31 July 2015.



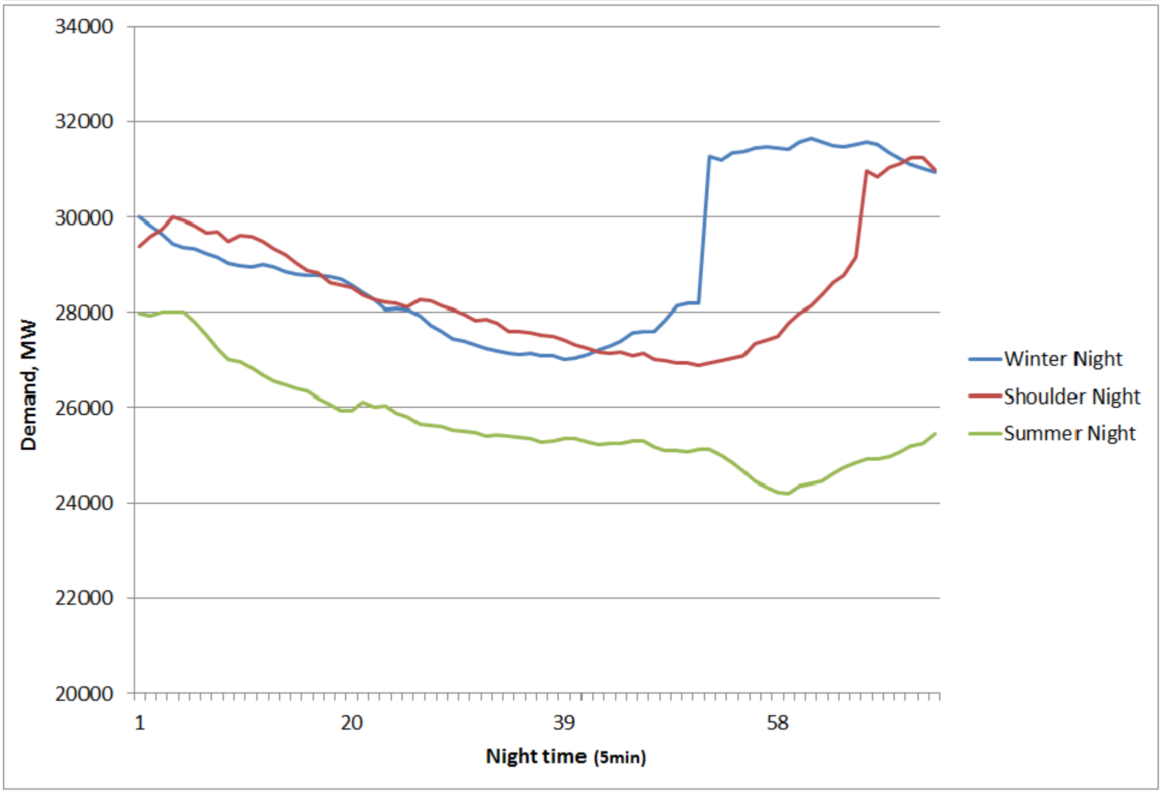


Figure 5.2: Average UK 5-min net demand for weekdays during day hours (up) and night hours (down) for each season.

5.1.4 Calibrating Ornstein-Uhlenbeck process

This section is devoted to describe the maximum likelihood estimation method for Ornstein-Uhlenbeck process. The stochastic differential equation governs the Ornstein-Uhlenbeck process is given by

$$dX_t = \kappa\left(\theta - X_t\right)dt + \sigma dW_t.$$

Consider $e^{\kappa t}X_t$ and applying Itô's lemma, using the product rule, we have

$$\begin{aligned} d\left(e^{\kappa t}X_t\right) &= \kappa e^{\kappa t}dtX_t + e^{\kappa t}dX_t, \\ &= e^{\kappa t}\left(\kappa\left(\theta - X_t\right)dt + \sigma dW_t\right) + \kappa e^{\kappa t}X_t dt \\ &= \kappa\theta e^{\kappa t}dt + \sigma e^{\kappa t}dW_t, \end{aligned}$$

and we have

$$e^{\kappa T}X_T = X_0 + \int_0^T \kappa\theta e^{\kappa t}dt + \int_0^t \sigma e^{\kappa u}dW_u.$$

Therefore,

$$X_T = X_t e^{-\kappa(T-t)} + \theta\left(1 - e^{-\kappa(T-t)}\right) + \sigma\int_t^T e^{-\kappa(T-u)}dW_u. \tag{5.20}$$

This means that X_T is normally distributed with mean and variance respectively given by

$$\mathbb{E}\left[X_T\right] = X_t e^{-\kappa(T-t)} + \theta\left(1 - e^{-\kappa(T-t)}\right).$$

and

$$\mathbb{V}\left[X_T\right] = \mathbb{E}\left[\left(\sigma\int_t^T e^{-\kappa(T-u)}dW_u\right)^2\right]$$

using Itô Isometry,

$$\begin{aligned}
&= \sigma^2 \mathbb{E}\left[\int_t^T \left(e^{-\kappa(T-u)}\right)^2 du\right] \\
&= \sigma^2 \int_t^T e^{-2\kappa(T-u)} du \\
&= \frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa(T-t)}\right),
\end{aligned}$$

and we have

$$X_T \sim \mathcal{N}\left(X_t e^{-\kappa(T-t)} + \theta\left(1 - e^{-\kappa(T-t)}\right), \frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa(T-t)}\right)\right).$$

Maximum Likelihood estimates

If we assume that θ is the long-run mean demand, we need to identify κ and σ. According to Brigo et al. [23] the conditional probability density is given by

$$f\left(X_T \mid X_t; \kappa, \widehat{\sigma}, \theta\right) = \frac{1}{\sqrt{2\pi\widehat{\sigma}^2}} \exp\left\{\frac{X_T - X_t e^{-\kappa(T-t)} - \theta\left(1 - e^{-\kappa(T-t)}\right)^2}{2\widehat{\sigma}^2}\right\},$$

where

$$\widehat{\sigma}^2 = \frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa(T-t)}\right).$$

with the log-likelihood function for estimations from n observations x_t is given by

$$\begin{aligned}
L\left(\kappa, \widehat{\sigma}, \theta\right) &= \sum_{i=1}^{n} \log f\left(x_{t_i} \mid x_{t_{i-1}}; \kappa, \widehat{\sigma}, \theta\right) \\
&= -\frac{n}{2}\log\left(2\pi\right) - n\log\left(\widehat{\sigma}\right) - \frac{1}{2\widehat{\sigma}^2}\sum_{i=1}^{n}\left[x_{t_i} - x_{t_{i-1}} e^{-\kappa\Delta t} - \theta\left(1 - e^{-\kappa\Delta t}\right)\right]^2.
\end{aligned}$$

The Maximum of the log-likelihood can be found at the location where all the partial derivatives are zero: that is,

$$\frac{\partial L}{\partial \theta} = 0, \quad \frac{\partial L}{\partial \kappa} = 0, \quad \frac{\partial L}{\partial \widehat{\sigma}} = 0.$$

Hence,

$$\frac{\partial L(\kappa,\widehat{\sigma},\theta)}{\partial\theta}=\frac{1}{\widehat{\sigma}^2}\sum_{i=1}^{n}\left[x_{t_i}-x_{t_{i-1}}e^{-\kappa\Delta t}-\theta\left(1-e^{-\kappa\Delta t}\right)\right],$$

that implies

$$\theta=\frac{\sum_{i=1}^{n}\left[x_{t_i}-x_{t_{i-1}}e^{-\kappa\Delta t}\right]}{n\left(1-e^{-\kappa\Delta t}\right)} \tag{5.21}$$

and

$$\begin{aligned}\frac{\partial L(\kappa,\widehat{\sigma},\theta)}{\partial\kappa}&=-\frac{1}{\widehat{\sigma}^2}\sum_{i=1}^{n}\left[x_{t_i}-x_{t_{i-1}}e^{-\kappa\Delta t}-\theta\left(1-e^{-\kappa\Delta t}\right)\right]\left(\Delta t x_{t_{i-1}}e^{-\kappa\Delta t}-\Delta t\theta e^{-\kappa\Delta t}\right),\\&=-\frac{\Delta t e^{-\kappa\Delta t}}{\widehat{\sigma}^2}\sum_{i=1}^{n}\left[x_{t_i}-x_{t_{i-1}}e^{-\kappa\Delta t}-\theta\left(1-e^{-\kappa\Delta t}\right)\right]\left(x_{t_{i-1}}-\theta\right)\\&=-\frac{\Delta t e^{-\kappa\Delta t}}{\widehat{\sigma}^2}\sum_{i=1}^{n}\left(x_{t_i}-\theta\right)\left(x_{t_{i-1}}-\theta\right)-e^{-\kappa\Delta t}\left(x_{t_{i-1}}-\theta\right)^2=0,\end{aligned}$$

implying

$$\sum_{i=1}^{n}\left(x_{t_i}-\theta\right)\left(x_{t_{i-1}}-\theta\right)=e^{-\kappa\Delta t}\sum_{i=1}^{n}\left(x_{t_{i-1}}-\theta\right)^2$$

and then

$$\kappa=-\frac{1}{\Delta t}\log\left(\frac{\sum_{i=1}^{n}\left(x_{t_i}-\theta\right)\left(x_{t_{i-1}}-\theta\right)}{\sum_{i=1}^{n}\left(x_{t_{i-1}}-\theta\right)^2}\right).$$

If $\sum_{i=1}^{n}\left(x_{t_i}-\theta\right)\left(x_{t_{i-1}}-\theta\right)\leq 0$, then κ is not defined. Substituting κ into the equation 5.21 of θ gives

$$\theta=\frac{\sum_{i=1}^{n}x_{t_i}\sum_{i=1}^{n}x_{t_{i-1}}^2-\sum_{i=1}^{n}x_{t_{i-1}}\sum_{i=1}^{n}x_{t_{i-1}}x_{t_i}}{n\left(\sum_{i=1}^{n}x_{t_{i-1}}^2-\sum_{i=1}^{n}x_{t_i}x_{t_{i-1}}\right)-\left(\left(\sum_{i=1}^{n}x_{t_{i-1}}\right)^2-\sum_{i=1}^{n}x_{t_{i-1}}\sum_{i=1}^{n}x_{t_i}\right)}.$$

The problem that the maximum likelihood condition has is that the solutions of θ, κ and σ are depending on each other. However since κ and θ are independent of σ, then determining either κ or θ (the slop and intercept of the OLS fit of $x_{t_{i-1}}$ against x_{t_i}) we can deduce σ (the residuals of the OLS fit), and we have

$$\frac{\partial L(\kappa, \widehat{\sigma}, \theta)}{\partial \widehat{\sigma}} = -\frac{n}{\widehat{\sigma}} + \frac{2}{2\widehat{\sigma}^3} \sum_{i=1}^{n} \left[x_{t_i} - x_{t_{i-1}} e^{-\kappa \Delta t} - \theta \left(1 - e^{-\kappa \Delta t}\right)\right]^2 = 0$$

so

$$\begin{aligned} \widehat{\sigma}^2 &= \frac{1}{n} \sum_{i=1}^{n} \left[x_{t_i} - x_{t_{i-1}} e^{-\kappa \Delta t} - \theta \left(1 - e^{-\kappa \Delta t}\right)\right]^2 \\ &= \frac{1}{n} \sum_{i=1}^{n} \left[x_{t_i} - \theta + e^{-\kappa \Delta t} \left(\theta - x_{t_{i-1}}\right)\right]^2. \end{aligned}$$

A normal Q-Q plot for the net demand data sets is shown in Figure 5.3.

5.1.5 Calibrating Feller process

The Feller process with positive parameters κ, θ and σ, and with $2\kappa\theta \geq \sigma^2$ has gamma distribution as marginal density. We consider the vector parameter $\alpha \equiv (\kappa, \theta, \sigma)$ to be estimated using maximum likelihood estimation method. Following Cox et al. [35], given X_t at time t, the density of $X_{t+\Delta t}$ at time $t + \Delta t$ is given by

$$f\left(X_{t+\Delta t} \mid X_t; \alpha, \Delta t\right) = c e^{-u-v} \left(\frac{u}{v}\right)^{\frac{q}{2}} I_q\left(2\sqrt{uv}\right),$$

where

$$\begin{aligned} c &= \frac{2\kappa}{\sigma^2 \left(1 - e^{-\kappa \Delta t}\right)}, \\ u &= c X_t e^{-\kappa \Delta t}, \\ v &= c X_{t+\Delta t}, \\ q &= \frac{2\kappa\theta}{\sigma^2} - 1, \end{aligned}$$

and $I_q\left(2\sqrt{uv}\right)$ is the modified Bessel function of the first kind and of order q. Then the likelihood functions for n observations x_t is given by

$$L(\alpha) = \prod_{i=1}^{n-1} f\left(x_{t_{i+1}} \mid x_{t_i}; \theta, \Delta t\right), \tag{5.22}$$

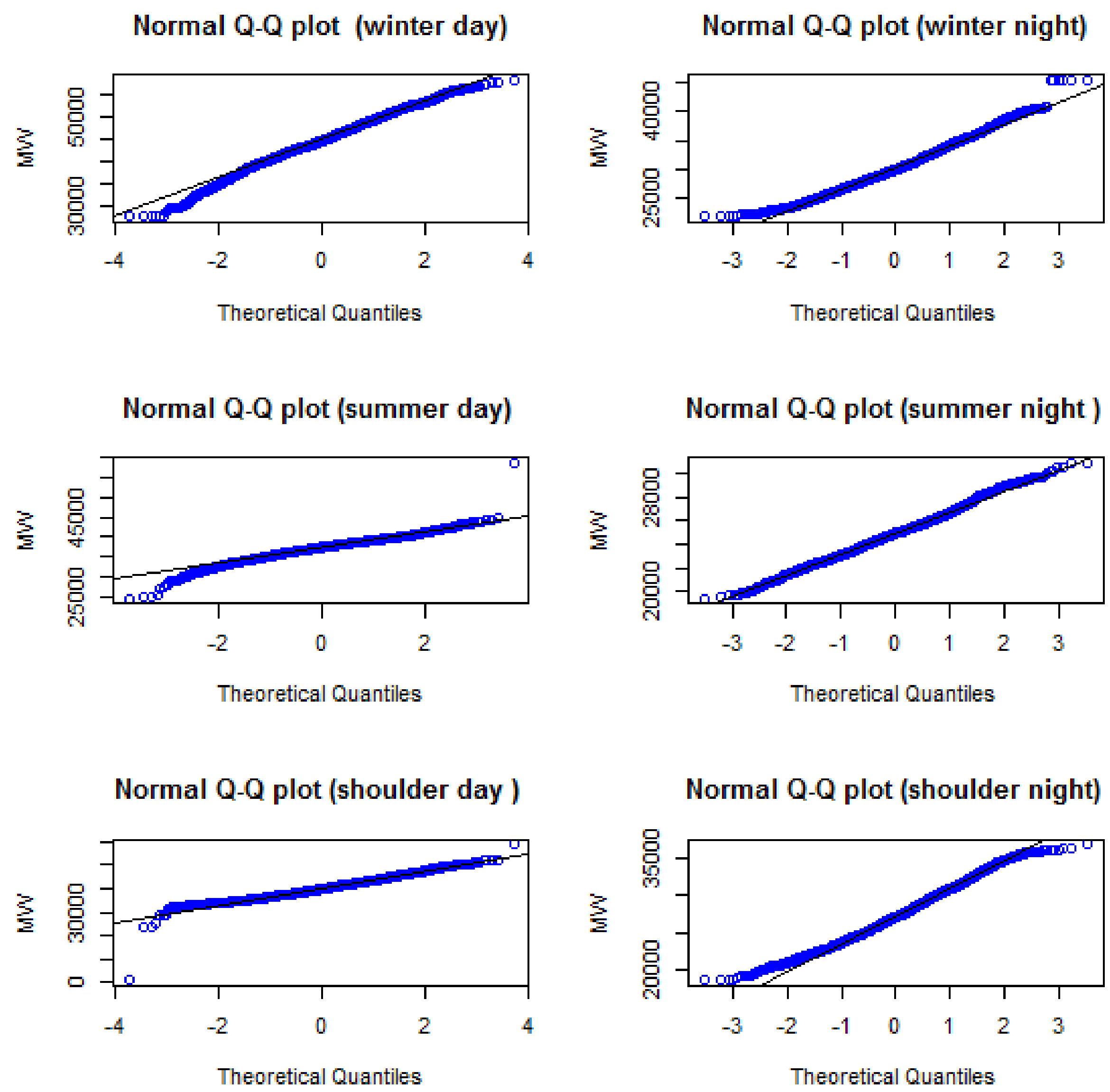


Figure 5.3: Normal Q-Q plots for day/night net demand data for winter, summer and shoulder seasons.

taking the logarithm we have

$$\log L(\alpha) = \sum_{i=1}^{n-1} \log f\left(x_{t_{i+1}} \mid x_{t_i}; \theta, \Delta t\right).$$

Therefore, the log likelihood function of the Feller process is given by

$$\log L(\alpha) = (n-1)\log c + \sum_{i=1}^{n-1}\left[-u_{t_i} - v_{t_{i+1}} + 0.5q\log\left(\frac{v_{t_{i+1}}}{v_{t_i}}\right) + \log\left(I_q\left(2\sqrt{u_{t_i} v_{t_{i+1}}}\right)\right)\right],$$

where, $u_{t_i} = cx_{t_i}e^{-\kappa\Delta t}$ and $v_{t_{i+1}} = cx_{t_{i+1}}$. Maximizing the log-likelihood function over its parameters gives the maximum likelihood estimates $\widehat{\theta}$

$$\widehat{\alpha} \equiv \left(\widehat{\kappa}, \widehat{\theta}, \widehat{\sigma}\right) = \arg\max_{\alpha} \log L\left(\alpha\right).$$

Using the Ordinary Least Squares (OLS) on discretized version of 5.3, we have

$$x_{t+\Delta t} - x_t = \kappa\left(\theta - x_t\right)\Delta t + \sigma\sqrt{x_t}\varepsilon_t, \tag{5.23}$$

where ε_t is normally distributed with zero mean and variance Δt, and the drift initial estimates can be found by minimizing the OLS objective function

$$\left(\widehat{\kappa}, \widehat{\theta}\right) = \arg\min_{\kappa,\theta} \sum_{i=1}^{n-1}\left[\frac{x_{t_{i+1}} - x_{t_i}}{\sqrt{x_{t_i}}} - \frac{\kappa\theta\Delta t}{\sqrt{x_{t_i}}} + \kappa\sqrt{x_{t_i}}\Delta t\right]^2$$

which is solved by as in [35].

$$\widehat{\kappa} = \frac{n^2 - 2n + 1 + \sum_{i=1}^{n-1} x_{t_{i+1}} \sum_{i=1}^{n-1} \frac{1}{x_{t_i}} - \sum_{i=1}^{n-1} x_{t_i} \sum_{i=1}^{n-1} \frac{1}{x_{t_i}} - (n-1)\sum_{i=1}^{n-1} \frac{x_{t_{i+1}}}{x_{t_i}}}{\left(n^2 - 2n + 1 - \sum_{i=1}^{n-1} x_{t_i} \sum_{i=1}^{n-1} \frac{1}{x_{t_i}}\right)\Delta t}$$

$$\widehat{\mu} = \frac{(n-1)\sum_{i=1}^{n-1} x_{t_{i+1}} - \sum_{i=1}^{n-1} \frac{x_{t_{i+1}}}{x_{t_i}} \sum_{i=1}^{n-1} x_{t_i}}{n^2 - 2n + 1 + \sum_{i=1}^{n-1} x_{t_{i+1}} \sum_{i=1}^{n-1} \frac{1}{x_{t_i}} - \sum_{i=1}^{n-1} x_{t_i} \sum_{i=1}^{n-1} \frac{1}{x_{t_i}} - (n-1)\sum_{i=1}^{n-1} \frac{x_{t_{i+1}}}{x_{t_i}}}.$$

The volatility parameter initial estimate $\widehat{\sigma}$ can be found as standard deviation of residuals. The vector $\left(\widehat{\kappa}, \widehat{\theta}, \widehat{\sigma}\right)$ is the initial points for maximizing the log-likelihood equation 5.22. The maximum likelihood estimation is used to calibrate OU process and Feller process to the collected data described in section 5.1.3, and the results are summarised in the Table 5.1 and 5.2. The time interval used is $\Delta t = 5\ min$, and the parameters κ and σ are related to time unit of an hour. Hence, $\Delta t = 1/12$ hour. The mean reversion speed κ is higher during day time comparing to night time. This suggests that the half-life of the process during the day is longer than the half-life of the process during the night. More precisely the half-life of an Ornstein Unlbeck is given by the expression $ln\left(2\right)/\kappa$, and it gives the average time it will take the process to get pulled half-way back to the mean θ. For example, in winter days the half life of the process is $ln\left(2\right)/0.040 = 7.53h$, however in winter night the

Parameters	Day demand (MW)	Night demand (MW)
Winter		
$\widehat{\kappa}$	0.040	0.253
$\widehat{\theta}$	42,748	25,067
$\widehat{\sigma}$	276	223
Shoulder		
$\widehat{\kappa}$	0.08	0.176
$\widehat{\theta}$	35,405	24,930
$\widehat{\sigma}$	461	256
Summer		
$\widehat{\kappa}$	0.080	0.198
$\widehat{\theta}$	30,825	22,829
$\widehat{\sigma}$	335	208

Table 5.1: Calibration of net demand data to OU process.

half life of the process is $ln\,(2)\,/0.253 = 1.19h$. Figure 5.4 shows the function ϕ and ψ in equations 5.8-5.9 over the range of observed demand data. In the rest of this thesis we restrict the analysis on the OU process as this process allow for negative values for net demand opposite to Feller process.

5.2 Merit order curve "stack model"

In a perfect competitive market, the electricity prices are determined by the marginal cost of the last most expensive plant called to meet the predicted demand. A successful model for electricity prices is the one that takes into account the most important factors, demand, fuel prices and carbon emission. To construct a stack model there are two main steps. The first step is to calculate marginal cost of each of the plants providing energy, the second step is to match the merit order curve with demand. The centre of our analysis is modelling demand as one-dimensional Itô process passing through a merit order curve. Similar approach has been also considered by Moriarty [94].

Parameters	Day demand (MW)	Night demand (MW)
Winter		
$\widehat{\kappa}$	0.040	0.253
$\widehat{\theta}$	42,748	25,067
$\widehat{\sigma}$	1.22	0.84
Shoulder		
$\widehat{\kappa}$	0.08	0.176
$\widehat{\theta}$	35,405	24,930
$\widehat{\sigma}$	1.19	1.09
Summer		
$\widehat{\kappa}$	0.080	0.198
$\widehat{\theta}$	30,825	22,829
$\widehat{\sigma}$	0.72	0.84

Table 5.2: Calibration of net demand data to Feller process.

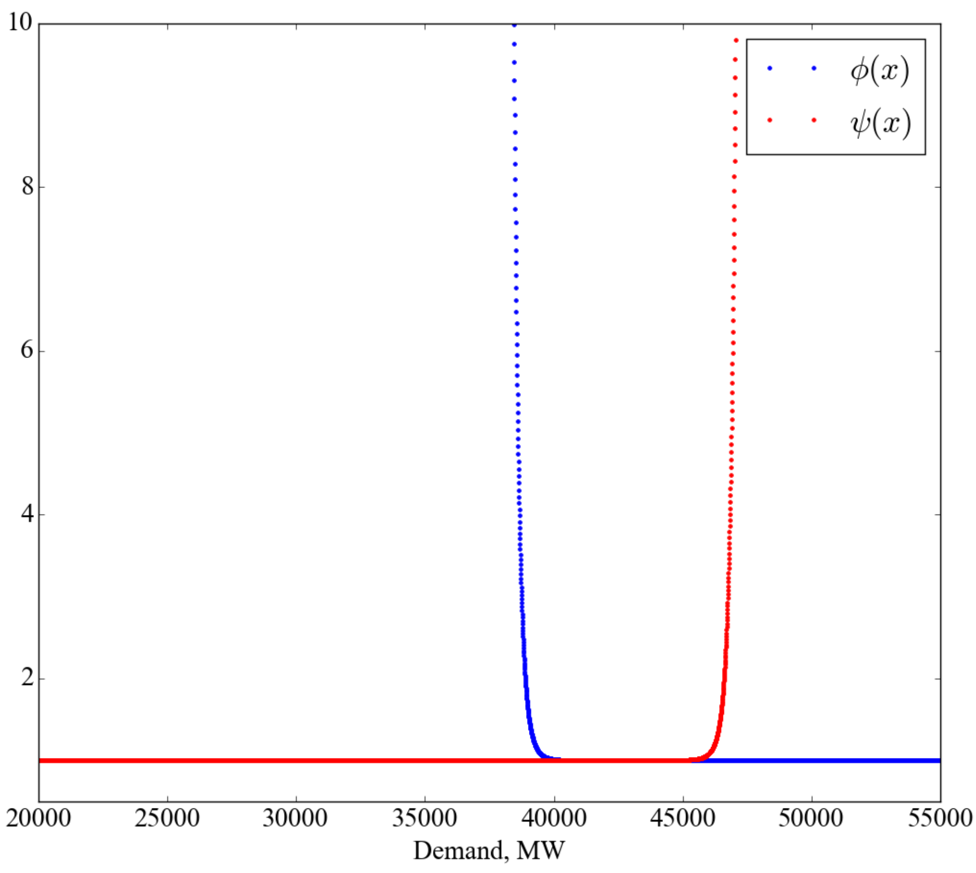


Figure 5.4: The construction of ϕ and ψ based on winter parameters for OU process.

5.2.1 Fuel prices

In order to understand the fundamentals of electricity prices, we need to understand how electricity is generated and the costs associated with each fuels used by different plants. The main long-term factors that affect the power prices are fuel prices. In the UK power sector, coal, nuclear and natural gas are the main sources where electricity is generated from. Coal plants are the largest contributor to the UK energy generation mix, though are restricted in running hours because of the carbon emissions, so tend to run in winter when prices are high. Nuclear plants are run to maximize income, since the cost of fuel is very low, it pays them to sell at any price they can get. Combined Cycle Gas Turbine (CCGT) is gas turbine where the hot exhausts are used to drive a boiler and a steam turbine. These plants are very efficient and flexible makes them available for peak demand and balancing wind supply. Open Cycle Gas Turbines (OCGT), are gas turbines without steam plants to maximize their efficiency. OCGT are cheap to build but expensive to run, so are only used for emergencies in winter, where they can make a profit because of the very high electricity prices. Fig 5.5 shows annual prices for oil, gas, and coal data provided by AF-Mercados EMI and together with annual electricity prices obtain by averaging monthly data available from APX website. Electricity prices shows positive correlation with fuel price from 2010 till 2013, however in 2014 we observe a decrease in electricity prices, but not in fuel prices. This observation might be explained by the new regulations have been introduced in year 2014 to the UK power market. According to APX the year 2014 ended with 70 members, where 5 new members have been welcomed to the UK market, these are Flow Enegy, PowerTra, Petroineos Trading, Stadtwerke Munchen and TrailStone. This means that competition has been increased in 2014 which results a decrease in prices.

5.2.2 Carbon emission

In European countries, electricity prices are also affected by EU Emission Trading System (EU ETS). Introduced in 2005, EU ETS is the fundamental principle of EU policy to combat climate change, with an aim to reduce the industrial greenhouse

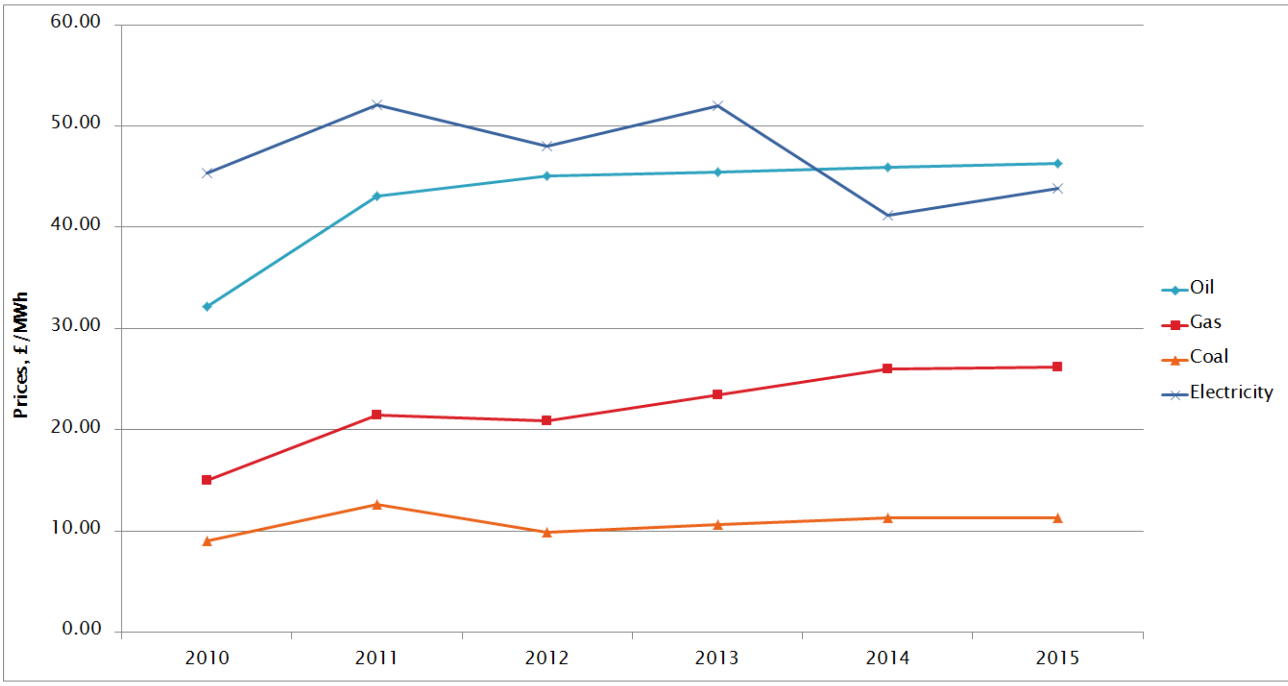


Figure 5.5: Input fuel prices for 2010-2015,

gas emissions with cost efficient way. As a cap-and-trade system, the EU ETS sets an emissions limit on the total emissions allowed by all generators that produce carbon emissions. In order to reach the carbon reduction and renewable targets, the UK governments introduced a carbon price floor (CPF) to increase investments in low-carbon power plants. Introduced in 2013, CPF calculated using the price of CO_2 and the carbon price support (CPS) with a rate per ton of CO_2 produced (see Fig 5.6). The Carbon Price Support (CPS) rates apply to fossil fuels used in electricity generation (gas, solid fuels and liquefied petroleum gas), rates of CPS from 2013 to 2017 are presented in Table 5.3.

Table 5.3: Rate of the carbon price support in $£/tCO_2$

	2013-2014	2014-2015	2015-2016	2016-2017
CPS ($£/tCO_2$)	4.94	9.55	18.08	18.00

5.2.3 Thermal efficiency

Thermal efficiency is also an important factor that should be taken into account in the 'stack' model. This factor is associated with combustion-based power plants

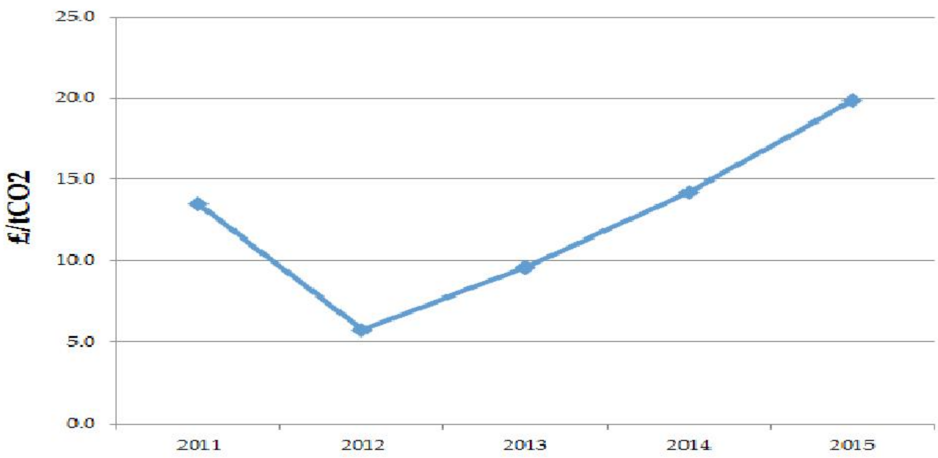


Figure 5.6: The total carbon price including CPS rate 2011-2015.

that burn coal, natural gas, or petroleum to generate electricity. Thermal efficiency measured using heat rate, and heat rate is defined as the amount of energy used by an electrical generator or power plant to generate one kilowatt-hour (kWh) of electricity. Thermal efficiency of combustion-based power plants changes with technology used and type of the fuel burned. Coal plants have a thermal efficiency less than that of CCGT. Table 5.4 shows the average thermal efficiency of different plants based on the year 2012. In practice, thermal efficiency of combustion-based power plants does change with output level as well. Therefore in the calculation of the stack for power plants, variable thermal efficiency has been considered for different capacities. Fig 5.7 shows thermal efficiency for CCGT power plants in the UK delivering different capacities.

Table 5.4: Average thermal efficiency for thermal power plants.

	CCGT	Coal	OCGT	Oil
Thermal efficiency	45.7%	36.5%	28.8%	20%

5.2.4 Short run marginal cost

In economy sector, the Short Run Marginal Cost (SRMC) refers to the cost of producing a small quantity of additional unit of good or service. More specifically

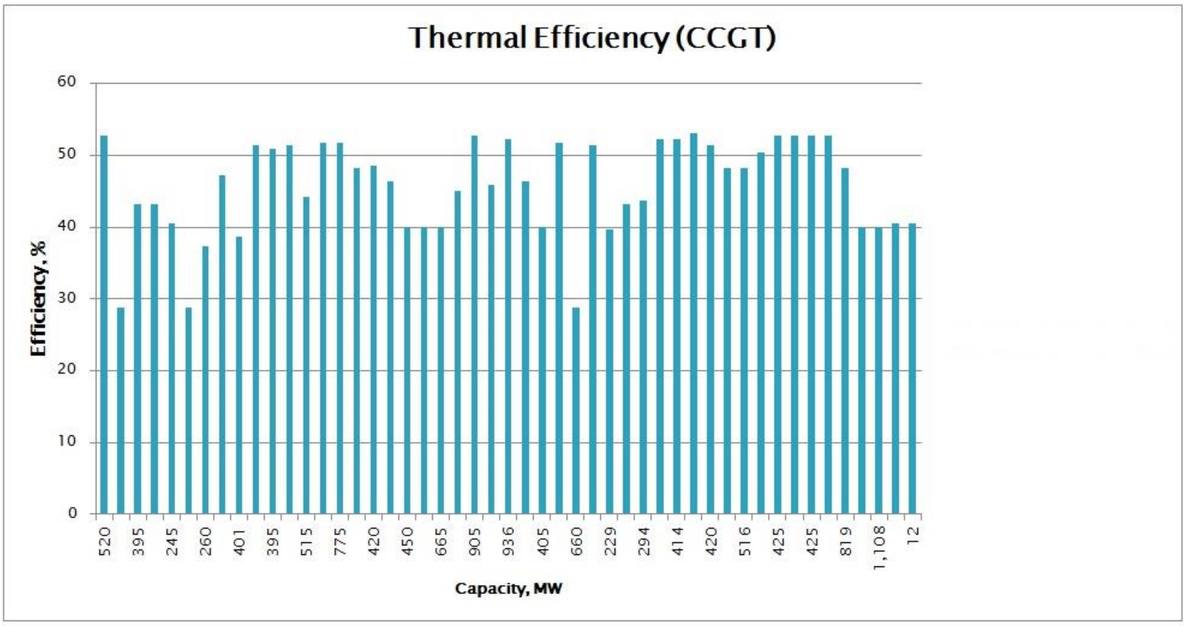


Figure 5.7: Thermal efficiencies of different output level for different CCGT plants. The data obtained from the energy markets consultancy, AF-Mercados EMI (Personal communication).

in power systems, SRMC is the change in total power plant cost from a small output change, and is in unit $£/MWh$. The SRMC for thermal power plants is the cost of the fuel burned to generate one MWh, and it is changing by generation level because of the changes in the thermal efficiency. Wind, solar PV and hydro plants have SRMC of zero which means there is no change in total cost if output is changed by a small amount for a short time.

The main components that construct the SRMC include costs of fuels used by each generator, whether an expensive oil and gas or a cheap coal. Variable operation and maintenance (O&M) expenses, for example periodic inspection, repair of the system component, chemicals necessary for water treatment, limestone consumed for example in coal plant with Flue Gas Desulphurisation. Variable O&M also depend on whether plants are combined with CCS or not, plants with CCS have higher variable costs than the other plants. Fixed costs are not included and cannot be recovered through SRMC. In addition to fuel costs and variable O&M it is important to include the environmental costs for plants with emissions in their SRMC, in the UK that will be the EU ETS cost. Of course plants with combined CCS are not subject to EU ETS cost. Fig 5.8 shows the components of SRMC for different plants in the UK, in this figure fuel costs and variable O&M are averaged across types.

In this study, the short run marginal cost of each plant was calculated considering each unit separately, to take into account the thermal efficiency of each unit depend on its output level. A total of 113 generation plants have been considered, CCGT (60 plants), CHP (11 plants), coal (60 plant), nuclear (12 plants), OCGT (16), oil (2 plants) and hydro (12 plants). The formula used to simulate the SRMC across all plants is given as follow

$$SRMC = \frac{F}{\nu} + \gamma\, E + V,$$

where F $(£/MWh)$ is the fuel cost, ν $(\in [0,1])$ is the unit thermal efficiency, E is the carbon emission price $(£/MWh)$, γ is the carbon intensity per unit (tCO_2/MWh), and V $(£/MWh)$ is the variable operating and maintenance cost.

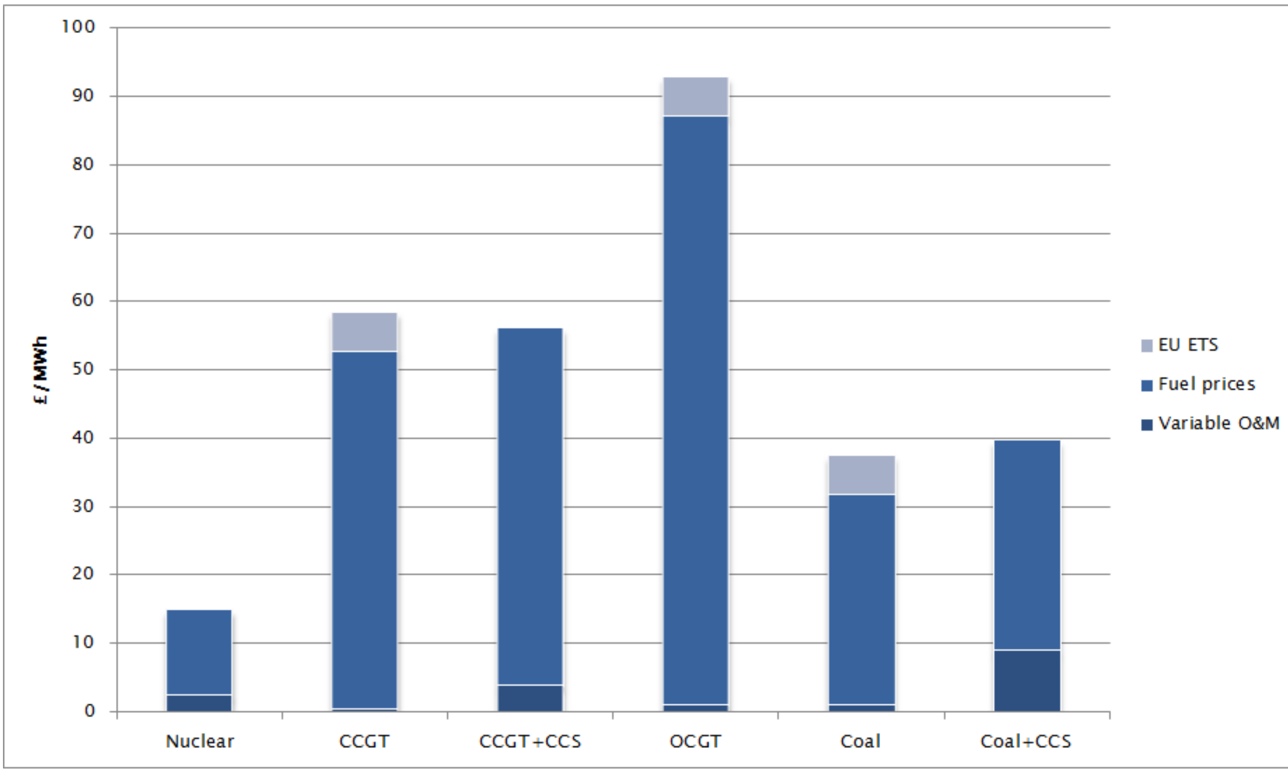


Figure 5.8: Components of SRMC for different technologies in the UK. The data obtained from the energy markets consultancy, AF-Mercados EMI (Personal communication).

5.2.4.1 Merit order curve (MOC)

A perfect competitive market in the classical economic theory assumes that the market price is equal SRMC, which is not realistic. However the assumption of perfect competition is important to understand the foundation of electricity market. Under this assumption the electricity generators are expected to offer their generation at the SRMC. In each trading interval, in the UK is half-hour, generators submit their bids based on the generator SRMC to the system operator (NG), the role of the NG is to order individual units by increasing SRMC constructing the system merit order. The MOC approximation of the UK market is presented in Fig 5.9. The system merit order can be divided to three main categories: based load-units those are plants operating most of the hours with low variable costs (hydro and nuclear), mid-load units plants are flexible but does not most of the hours (coal and CCGT) and peaking plants are those that are used few hours in a year for very high demand in winter (OCGT and oil).

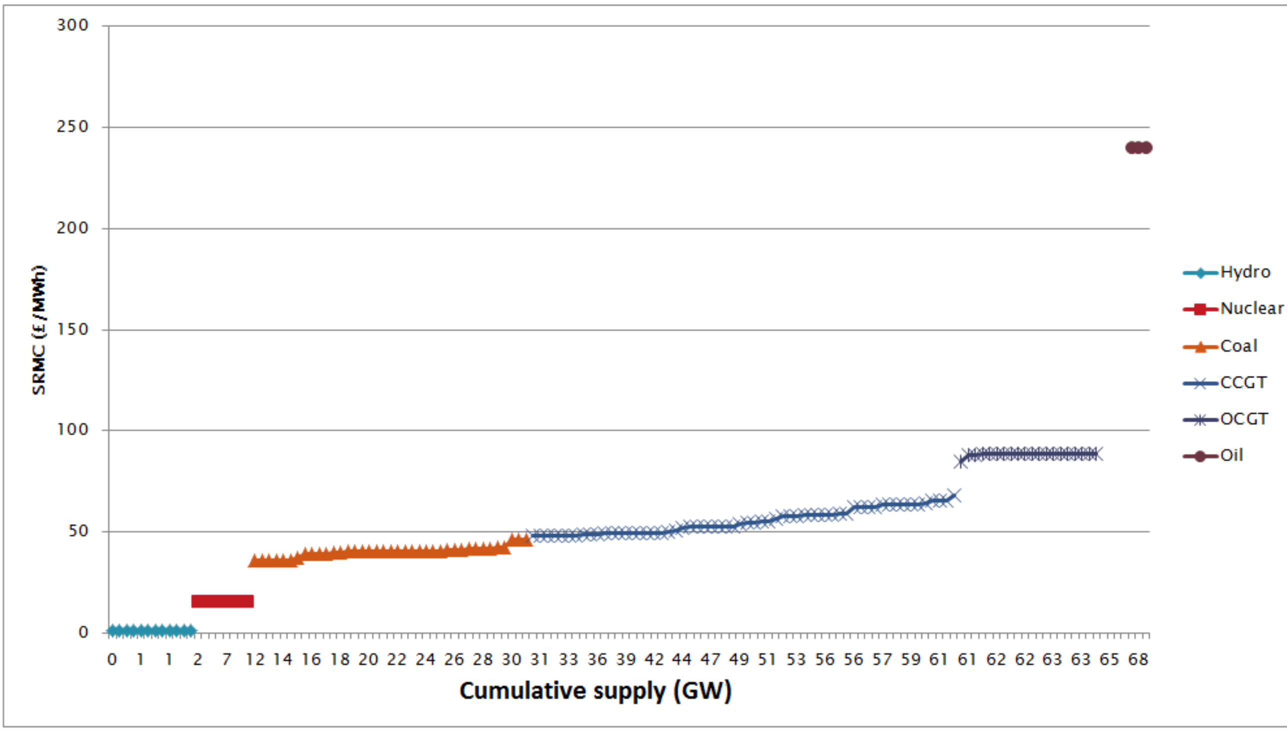


Figure 5.9: The merit order curve approximation for UK market. The data obtained from the energy markets consultancy, AF-Mercados EMI (Personal communication).

5.2.5 Mapping demand to prices

In this thesis we assume that the UK power price is driven by the MOC. The MOC ranks different power generators based on the ascending order of the price the generators bid to deliver a specific quantity of power; it is a monotone increasing non continuous (staircase) function that maps demand to price. The justification of this mapping lay on the Markovian assumption for X_t, where the future prediction of the process based only on the present state and not the past states. In this setting the problem of optimal stopping become a problem of random path in the state space $\mathcal{I}$, where a transition density for a random path of diffusion can be defined and satisfies the Kolmogorov equations.

We were provided with the merit order curve data from the energy markets consultancy, AF-Mercados EMI[3], see Figure 5.9. The relationship between the price paid for power and the merit order curve is not straight forward, but we argue the merit order curve is fundamental to prices, and gives a good approximation of the prices. On this basis we approximate the average price paid to power suppliers by the intersection of load and the MOC.

To emphasise the short-term relationship between demand and prices in the UK, we choose one day 09/10/2015 and we looked at the demand and prices profile that day together with the MOC approximation (see Fig 5.10). The left axes corresponds to the daily power demand in MW, and the right axes correspond to the spot prices from APX in $£/MWh$. The positive correlation between demand and prices are clearly shown, high demand corresponds to high prices and low demand corresponds to low prices. Mainly two peaks are observed in demand and prices, the first one is in the morning when working hours start and the second is in the evening the end of working hours. The merit order curve is derived by ordering the supplier bids according to ascending marginal cost. The intersection of the demand curve with the merit order defines the market clearing price that is the electricity spot market price paid to generators. Fig 5.10 shows APX spot prices for the day on 09/10/2015 in red plot, and the green is the anticipated prices obtained by intersecting the

[3] info.gb@afconsult.com

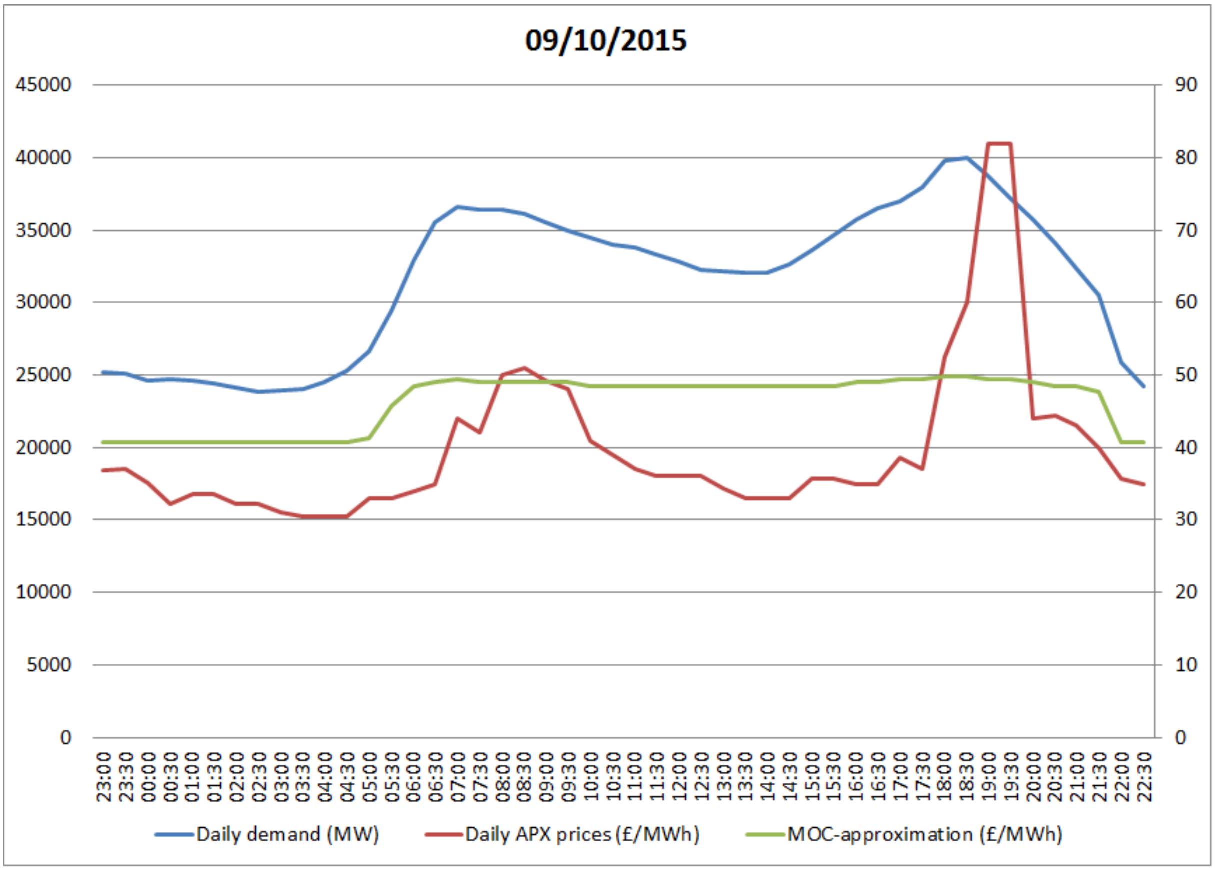


Figure 5.10: Daily demand (National Grid), daily prices (APX) and the merit order curve approximation for 09/10/2015.

demand that day in blue with the merit order curve (Fig 5.9). The APX represent the difference between anticipated and realised demand. The plot indicates that the anticipated demand was close to realised for most of the 24 hour period, as the APX price was bellow the MOC price, but between 17:30 and 20:30 there was mis-match and supply becomes very valuable.

The stack model is complete when the available capacity is equal to the predicted demand. In section 5.1.4 we calibrated an Ornstein-Uhlenbeck process to six data sets presenting electricity demand at night and day during winter, summer and shoulder months. The average mean demand is projected on the MOC shown in Fig 5.11 (day time) and Fig 5.12 (night time where we excluded hydro units). For example in a winter day with an average demand of (42.74 GW) the last unit to complete the stack is a CCGT unit providing a SRMC of (£52.13/MWh) and the system price will be approximately (£52.13/MWh) to be paid for all other units providing a SRMC less than the system price. The generation mix for winter and

	Average demand (GW)	Price (£$/MWh$)	Plant	Type	Owner
Day					
W	42.74	52.13	GreatYarmouth	CCGT	NPower
SH	35.40	48.98	DidcotB5	CCGT	NPower
S	30.82	48.56	Marchwood	CCGT	Marchwood
Night					
W	25.06	41.30	Kings north 1	Coal	E.ON
SH	24.93	41.30	Kings north 2	Coal	E.ON
S	22.82	40.73	Didcot AC1	Coal	NPower

Table 5.5: The averaged demand, the average system price based on the MOC, the plant, the type and owner bases on the data of 2013-2015 for day and night for each season in UK market. The data obtained from the energy markets consultancy, AF-Mercados EMI (Personal communication).

shoulder days consists mainly of nuclear, coal and CCGT plants. However in summer and night hours the generation mix is only nuclear and coal. A complete summary of average hourly prices of different seasons together with the type of plants and owners are presented in Table 6.1.

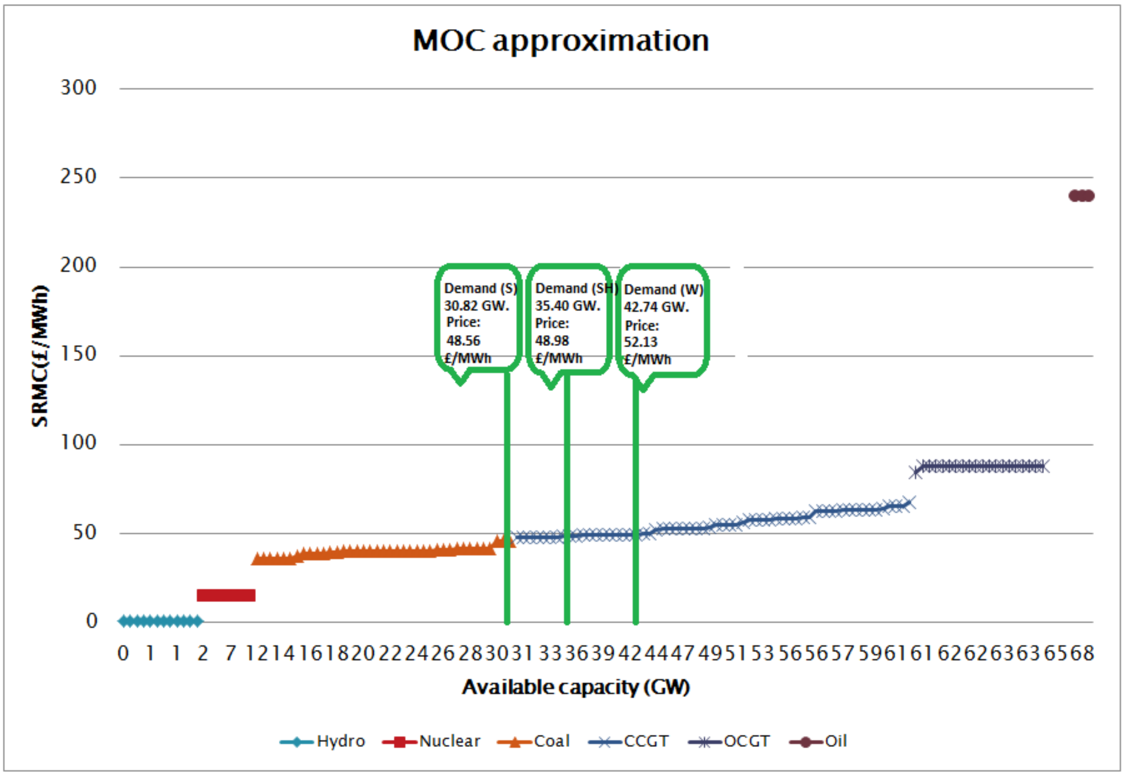


Figure 5.11: The MOC approximation with the projected demand during day hours.

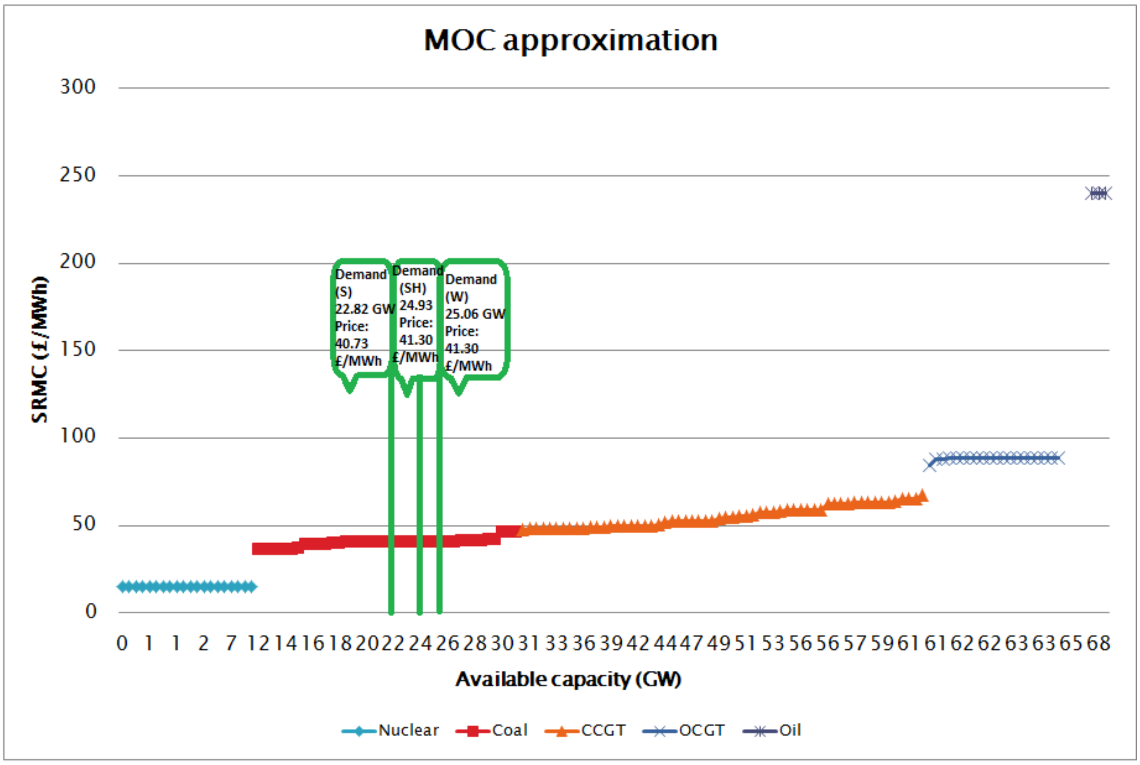


Figure 5.12: The MOC approximation with the projected demand during night hours.

Chapter 6

Case study: Evaluation of Compressed Air Energy Storage in UK's power market

This chapter will present a validation of the methodology we proposed in this thesis to assess the economic impact of storage technologies. To this end we shall consider a live mechanical storage facility, Compressed air energy storage (CAES). The CAES technology was chosen because it is between chemical batteries technology and pumped hydro technology in scale. A more details on the technology are presented in section 2.5.1. This technology has been recognized as to play a role in integrating high level of volatile wind energy production into the power system (Arsie et al. [9]). Number of authors investigate the value of CAES includes, Lund et al. [88], the authors argue the role of CAES in the future energy system based on the Danish market. Drury et al. [45] evaluate the value of CAES in arbitrage and energy reserve markets using deterministic historical prices. Other authors study a combination of CAES with wind farms, such as Greenblatt et al. [61], Denholm et al. [45], Loisel et al. [86] and Hessami et al. [66]. Few studies take into account the stochastic volatility of the power market. Keles et al. [79] used a time-consuming Monte Carlo simulation to evaluate CAES under uncertain electricity prices. Yucekaya [117], take into account the hourly fluctuation of fuel costs and electricity prices and used a mixed integer programming method. In this case study we are considering the methodology discussed in Chapter 4, to evaluate a CAES technology in the UK power market with high level of wind energy by assuming a stochastic residual demand discussed in section 5.1.

The data based on the Huntorf CAES plant in Germany was used in this study. The Huntorf plant is the first CAES in the world, operating successfully for more than 30 years in Germany. The power plant with capacity $Q = 290$ MW could transfer off-peak (night hours) electricity demand to high demand periods (day hours). This technology uses electricity in low demand to derive a compressor and inject air into an underground chamber. During high demand the compressed air is released, heated via combustion with fuel, and passed through an expansion turbine to drive a generator and produce electricity to the grid for $T = 2$h. The assumption we make on the data is that the CAES facility fully charged/discharged over two hours, the efficiency is treated as variable. We using the short-run production cost data for generation in the UK discussed in section 5.2. The discount rate set a constant 5% per annum continuously compounded and there are no "working day" adjustments, this implies that Assumption 3.1.4 is satisfied.

In this chapter we adopt a theoretical framework for the dynamics of the net demand process, that is the first hitting times ϕ and ψ of the mean reverting process 5.2, are assumed to be given by Equations 5.8 and 5.9, and the estimated parameters are summarized in Table 5.1. The results were obtained by implementing **Algorithm 2** presented in Chapter 4. Python 2.7.8, with version for 1.8.1 of numpy and 0.14.0 of scipy was used for numerical result with the following Pseudocode.

Pseudocode

```
# Set up parameters
  #Demand range and time-step
#Storage technology
  #Capacity
  #Fill/empty rate
#Diffusion -- Net demand
#Payoff
for i in Demand_range:
  #Simulate R_h[i]=F[i]
E=F* eta
#Construct phi and psi
if (phi) or (psi) not_well _defined for all i in Demand_range:
  extrapolate (phi) or (psi)

#Construct the Wronskian
#Plot F/phi, F/psi and E/phi, E/psi to give intuition as to the solution
#Construct LopE and LopF
#Identify a_list (candidates for values of a) and b_list (candidates for value of b)  based on Lop E and Lop F
#Ignore points in a_list or b_list that would deliver negative A or B (Using 1(b)-(c) and 2(b)-(c)) in Algorithm 2
#For remaining points in a_list calculate F_phi_A and F_psi_A using 1(a) in Algorithm 2
#For remaining points in b_list calculate E_phi_B and E_psi_B using 2(a) in Algorithm 2

for i in a_list:

  for j in b_list:

    if E_psi_B[j] <= F_psi_A[i] <= E_psi_B[j+1]:
      # There is a pair u,l_A(u); interpolate l_A(u)
      # Store the pair
      break

```

```
for i in a_list:

  for j in b_list:

    if E_phi_B[j] <= F_phi_A[i] <= E_phi_B[j+1]:
      # There is a pair u,l_B(u); interpolate l_B(u)
      # Store the pair
      break

#Search for a crossing point of l_A and l_B; or find if they come close

for each crossing:
  Outputs:
    'New set'
    u, l_A, l_B #a=u, b=average(l_A(u), l_B(u))
    sub(l_A ,l_B) #error
    A,B
    B psi(a)-F(a)
    A phi(a)
    sub(B psi(a)-F(a), A phi(a))# error
    B\psi'(a)-F'(a)
    A\phi'(a)
    sub(B\psi'(a)-F'(a), A\phi'(a))# error

    B\psi(b)
    A\phi(b)+E(b)
    sub(B\psi(b), A\phi(b)+E(b]))# error
    B\psi'(b)
    A\phi'(b)+E(b])
    sub(B\psi'(b), A\phi'(b)+E(b]))# error
```

Listing 6.1: Pseudocode

6.1 Generation of the payoff of emptying and cost of filling functions.

In order to evaluate the CAES technology, the methodology described earlier is employed. We start by generating path that represent electricity demand based on the estimated parameters θ, κ, σ, the electricity prices is a function that takes demand and return the price for one MWh of energy (stack function presented in section 5.2). The energy capacity of CAES is rated by the number of hours of full power output, and the rate is given by $\rho = \frac{T}{Q}$, where $Q = 290$ MW is the capacity of the technology. Then the cost of filling the technology is presented by

$$F(x) = \mathbb{E}_x\left[\int_0^T \rho h(X_s)\, ds\right],\ X_0 = x, \tag{6.1}$$

where X_t is the calibrated OU process govern by the Eq 5.2, h is the merit order curve given in figure 5.11 during day hours that deliver prices per 1 MWh (figure 5.12 during night hours). The expectation in 6.1 can be approximated using Monte Carlo simulation method, based on a numerical integration of the following SDE

$$dX_t = \kappa(\theta - X_t)\, dt + \sigma dW_t,\ X_0 = x. \tag{6.2}$$

The first step is to obtain an approximation $\overline{X}$ of the process X using Euler-Maryama scheme[1] and time step of 5 minutes and for increments in x of 1 MW between 15GW and 65GW. With $\overline{X}^{(n)}(t)$ being independent realization of $\overline{X}(t)$ given for $1 \leq n \leq N$ by

$$\begin{aligned}
&\overline{X}_0^{(n)} = x\\
&\overline{X}^{(n+1)}(t) = \overline{X}^{(n)}(t) + \kappa\left(\theta - \overline{X}^{(n)}(t)\right)\Delta t + \sigma\Delta W_n,
\end{aligned}$$

where, $\Delta t = T/N$, $\Delta W_n = W_{t_{n+1}} - W_{t_n}$, $T = 2h$ and $N = 24$. This allows an

[1] There was no benefit in employing the Milstein scheme.

approximation of the expectation $\mathbb{E}\left[\int_0^T \rho h\left(\overline{X}^{(n)}(t)\,ds\right)\right]$ as follow

$$\mathbb{E}\left[\int_0^T \rho h\left(\overline{X}^{(n)}(t)\,ds\right)\right] \simeq \frac{1}{M}\sum_{m=1}^{M}\left(\int_0^T \rho h\left(\overline{X}^{(n,m)}(t)\right)ds\right),\ m \in \mathbb{N},$$

where M the number of simulations. Therefore the function $F(x)$ in 6.1 is constructed by undertaking a piece-wise least squares polynomial fit of the simulation results, with a constraint that F is increasing. The pay-off of emptying the CAES facility $E(x)$ is then given by

$$E(x) = \eta F(x), \tag{6.3}$$

where the parameter $0 < \eta < 1$ a constant parameter represent the efficiency of the store. The parameter η called the round trip efficiency is a critical factor in the usefulness of a storage technology. For a CAES technology, the level of air leaks on the system depend on the level of system pressure i.e. the higher the system pressure, the more air lost through leaks. Therefore, the choice of the minimum inlet pressure to the expander determines the minimum system pressure, sets the required air compressor discharge pressure, and influence the efficiency of the overall CAES plant operating cycle presented by the parameter η. Setting $\eta < 1$ ensure Assumption 4.1.2 is satisfied. Since the MOC is bounded and F is continuous and differentiable the conditions is Assumption 3.1.7 are satisfied. The results based on the Winter-day parameters are shown in Figure 6.1.

6.2 Intra-filling/emptying for CAES technology

This section will analyse the entry (filling) and exit (emptying) for the CAES technology during a day time in winter. We consider three cases, the first case is where we have a full facility and we look for the exit time, second case will be having an empty facility that need filling then emptying (entry then exit), and the last case is

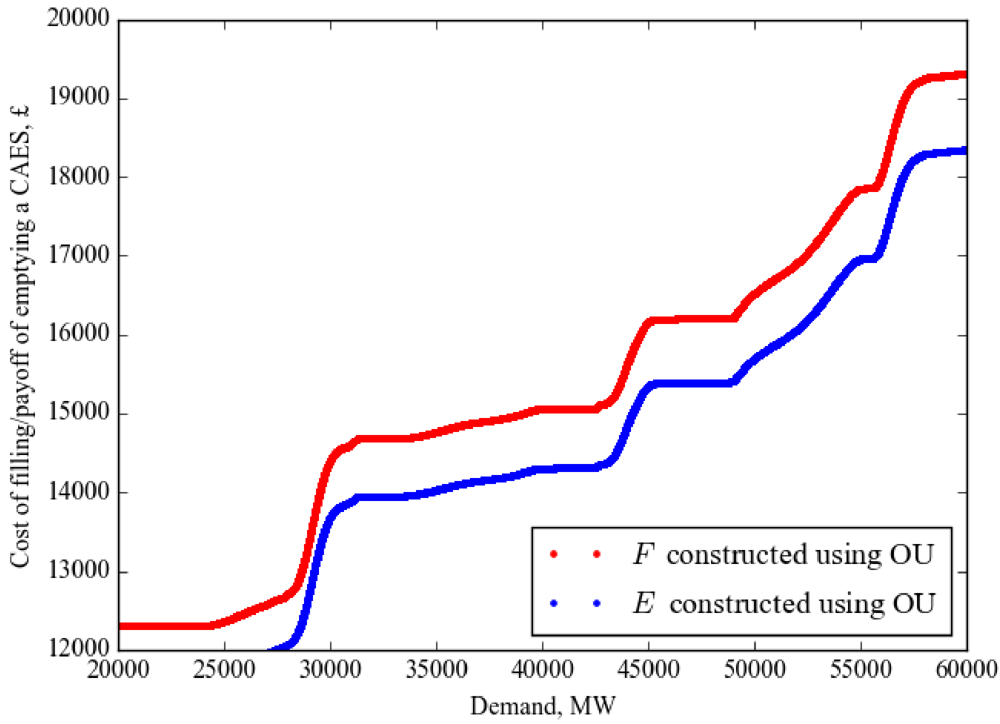


Figure 6.1: The cost/payoff of filling /emptying the CAES technology based on winter days data with $\eta = 0.95$.

where the facility is filled and emptied sequentially (sequential entry and exit).

6.2.1 Pure exit

The solution in this case is described in section 4.2. Considering having a full technology, the decision maker would like to know the optimal time to empty the technology. As inputs to the optimal stopping problem we consider, Ornstein-Unlebeck process for winter day with parameters $\theta = 42748$, $\kappa = 0.04$ and $\sigma = 276$. The simulated results are shown in Fig 6.2, the plots shows the simulated emptying value $E(x)$ and the function $E(x)/\psi(x)$. As discussed in section 4.2, the emptying boundary $b^{\star}$ is actually the turning point of the function $E(x)/\psi(x)$. Being endowed with a full CAES technology during a winter day, an optimal exit time strategy will be when net demand hits the boundary $b^{\star} = 45145$ MW. Looking at the merit order curve, this suggest that the CAES technology would start selling electricity to the grid at an approximated price of $p = £53.61/MWh$, that is the price of CCGT plant.

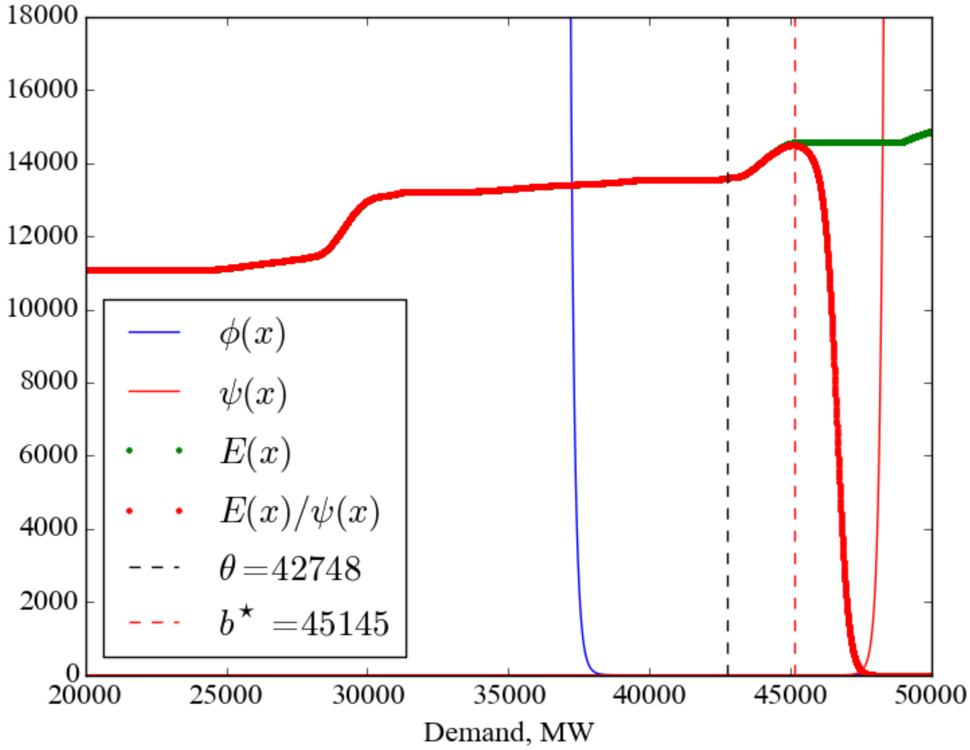


Figure 6.2: The solution of a pure exit problem for CAES technology during winter intraday.

6.2.2 Pure day entry and exit

Here we consider a semi-reversible problem, where we have an empty CAES technology that need to be filled with cost $F(x)$ before it can be emptied. This case is described in section 4.3. We consider a winter day parameters, and possess the sensitivity analysis with respect of the leakage level in the store captured by the parameter η. The higher the round trip efficiency, the less energy lost due to storage, the more efficient the technology. Fig 6.3 shows the optimal solution of the pure entry then exit case for winter intraday for different efficiencies. For a winter day with an exit boundary of $b^\star = 45145$ MW, 69% round trip efficiency gives an entry boundary of $a^\star = 11730$, 79% gives an entry boundary of $a^\star = 28310$ MW, 89% gives an entry boundary of $a^\star = 29980$ MW and 99% gives an entry boundary of $a^\star = 44620$ MW.

The results suggest that a technology with a round trip efficiency less than 69% may not have the chance to be filled during the day for a later delivery at $b^\star = 45145$

MW. For a technology with efficiency between 70% to 80% may have the chance to be filled when net demand is very low, that may happen if there is a lot of wind energy supply in the system. While a technology with more than 90% efficiency will have the chance to be filled up to an upper bound $a^\star = 44580$ MW. Intuitively based on the idea that the storage facility will be charged when demand is low and discharged later that day when demand is high, we can give the an upper bound for daily income P by $P \approx E(b^\star) - F(a^\star)$, this will be earned at most once in a daily basis. Therefore, the present value of the storage facility can be approximated as follow

$$\overline{v} \approx \sum_{\delta t=0}^{\infty} e^{-r_d\,\delta t} P \approx P \int_0^{\infty} e^{-r_d s} ds = \frac{P}{r_d}, \tag{6.4}$$

with δt correspond to one day and $r_d = 0.05/365$ is the one day discount rate, results are summarised in Table 6.1. The price column in the table present the upper bound price at which the facility have to buy electricity and store it, then sell it later when demand hits $b^\star = 45145\ MW$ at an approximated price of £53.61 $/MWh$. The results shown in the Fig 6.4, are the approximated present value of the storage facility as in the Equations 6.4. The present value of the CAES is not monotonic in efficiency, more precisely, we can observe from the Fig 6.4 a very low present value of a 79% and 94% efficient storage facility. This non-monotonicity is related to the filling boundary $a^\star$. As efficiency increases, the filling boundary also increases. Now looking at the cost F in Fig 6.1, we observe an increase in the cost around 28000 MW, and another increase in the cost around 43000 MW. We get a lower present value when the filling boundary $a^\star$ hits the levels where the cost is start increasing after being almost constant. And that happens when the facility is 79% efficient, and when it is 94% efficient (see Table 6.1). The benefit to be gained by the storage when it is more efficient is to have a smaller gap between filling and emptying boundaries. If the operator wishes to use the facility multiple times, then this smaller gap between the filling and emptying boundaries allow him/her to maximize the aggregate payoff from the multiple filling and emptying. This case will be discussed in details in Section 6.2.4.

η	$B^\star(\pounds)$	$a^\star$ (MW)	$E(b^\star)(\pounds)$	$F(a^\star)(\pounds)$	Price $(\pounds/MWh)$
0.69	13330.07	11730	13382.40	13375.22	37.12
0.74	14296.01	15735	14352.13	14342.25	39.62
0.79	15261.96	28310	15321.87	15321.70	45.85
0.84	16227.91	29175	16291.60	16287.35	47.75
0.89	17193.85	29980	17261.34	17211.71	48.56
0.94	18159.80	43235	18231.08	18230.88	52.62
0.99	19125.75	44620	19200.82	19156.20	53.61

Table 6.1: The upper bound of filling a CAES technology for different efficiencies, $b^\star = 45145$ MW for all efficiencies.

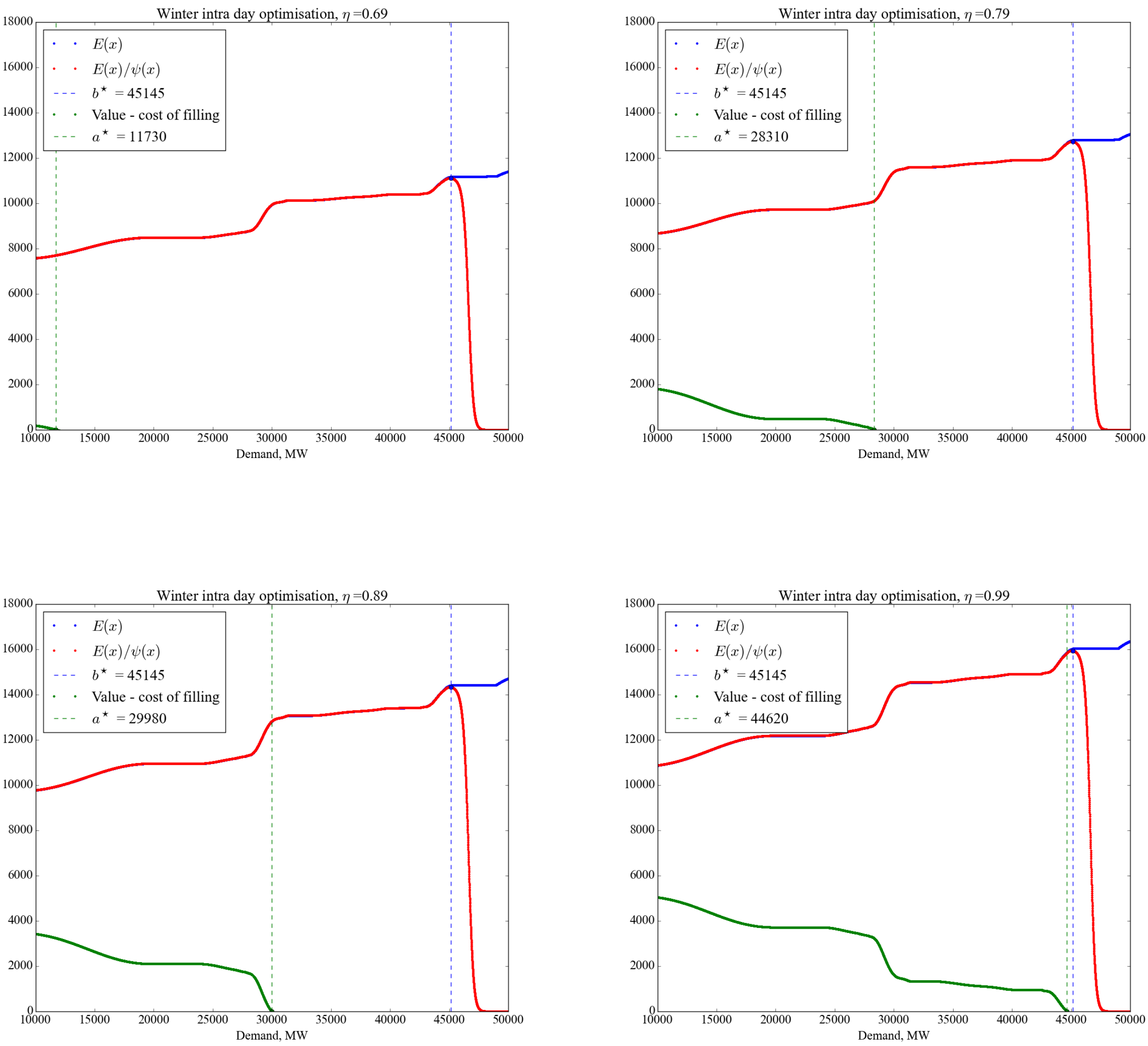


Figure 6.3: The filling boundary for CAES during winter day, with different efficiencies and an emptying point setting at $b^\star = 45145$ MW.

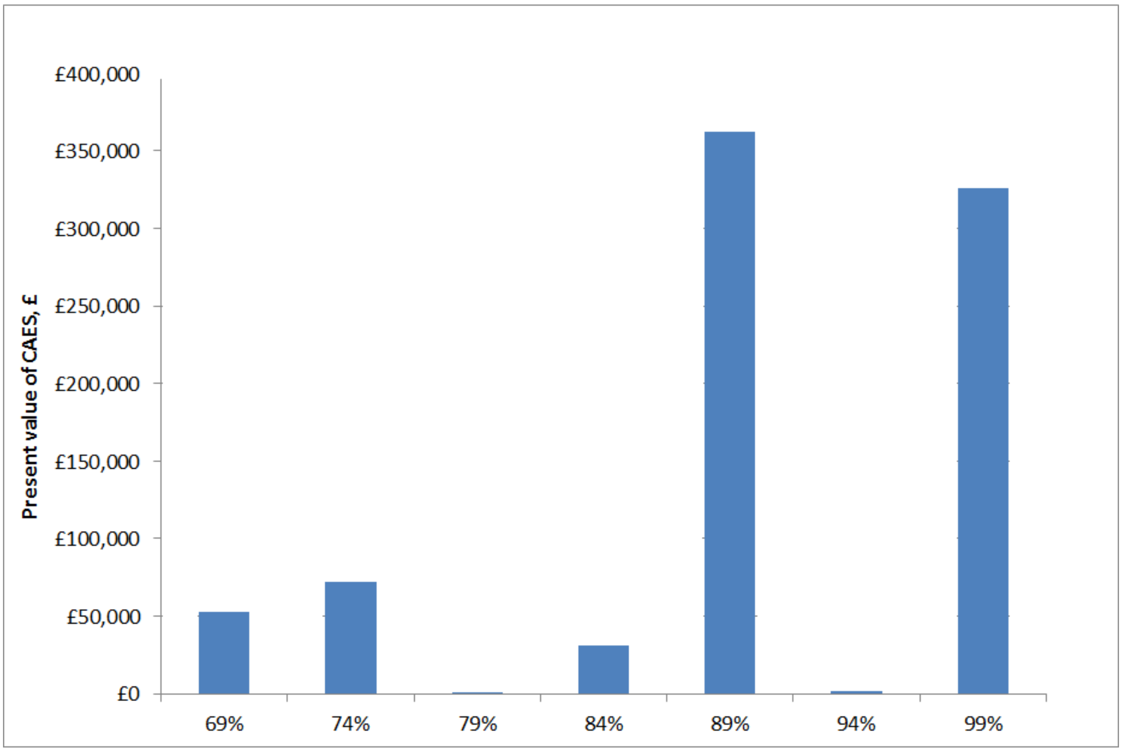


Figure 6.4: The present value for CAES technology during winter intraday with different efficiencies.

6.2.3 Pure night entry and exit

Considering the 'night' model where

$$\theta = 25067,\ \kappa = 0.253,\ \sigma = 223.$$

A new function E, F based on these parameters is constructed; the merit order curve for the night hours (Fig 5.12) is considered where the hydro generation is removed. This is because in the UK it is rare to utilise hydro power during night hours when prices are low. Utilizing the CAES technology once during night gives the results in Table 6.2. Fig 6.5 shows the present value for CAES with different efficiencies, operating during night time. The decrease in the present value for the facility with 99% efficiency is because the filling boundary $a^{\star}$ hits the level where the cost F have been increased (see Table 6.2 and Fig 6.1).

η	$B^\star(£)$	$a^\star$ (MW)	$E(b^\star)(£)$	$F(a^\star)(£)$	Price $(£/MWh)$
0.89	13321.39	10950.0	13337.59	13337.37	35.94
0.94	14069.78	14740.0	14086.89	14086.63	39.09
0.98	14668.49	18275.0	14686.34	14685.99	40.73
0.99	14818.17	25010.0	14836.20	14836.01	41.30

Table 6.2: The upper bound of filling a CAES technology during winter night for different efficiencies, $b^\star = 25945$ MW for all efficiencies.

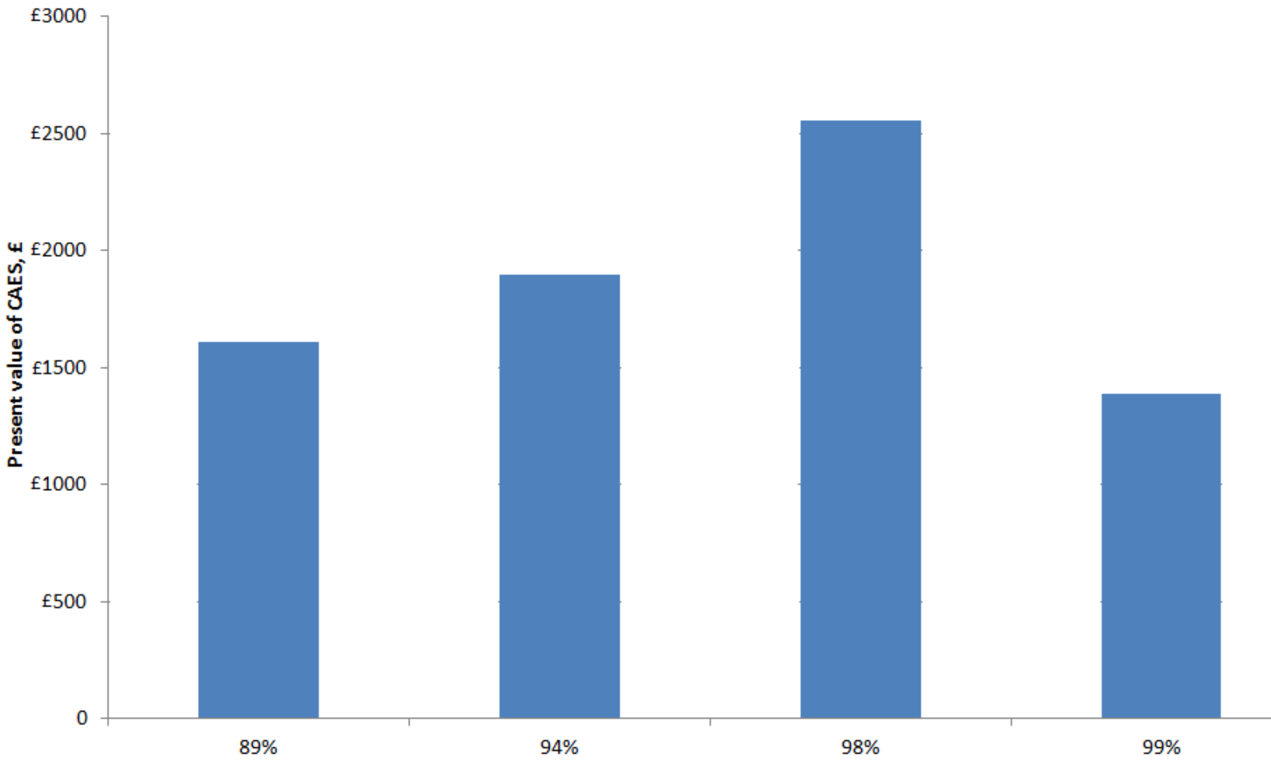


Figure 6.5: The present value for CAES technology during winter intra-night with different efficiencies.

6.2.4 Sequential entry and exit

Now we consider the case where the storage facility could have multiple use during the day, in this case the emptying would increase by amount $A^\star\psi(x)$, which has the potential of shifting $b^\star$ because ϕ is not linear, generating a positive feedback loop that simultaneously increase $A^\star$ to A and $B^\star$ to B. The interesting observation is that in the case of one single use, there is one single chance to maximize pay-off. However in the multi-use we looking for a and b that maximize the product of individuals pay-offs and the frequency of payoffs. This interesting case can be studied using a combination of the cost of filling the facility F, pay off of emptying E and ϕ and ψ representing the first hitting times of the stochastic net demand.

In order to find the sequential boundaries a and b for a CAES technology the methodology describes in section 4.4.1 is applied, setting $\eta = 0.99$ as efficiency of the storage facility. Results are summarized in the Table 6.3.

u	42495	42495	43250
$l^A(u)$	44646.058	42719.533	44495.217
$l^B(u)$	44646.593	42719.434	44495.223
% D	1.2×10^{-3}	2.3×10^{-4}	1.3×10^{-5}
a	42495	42495	43250
b	44645	42720	44495
A	196300.	196300	456604
B	211386	211386	471684
$B\psi(a) - F(a)$	196328.457	196328.457	456620.3
$A\phi(a)$	196328.217	196328.217	456620.2
% D	1.2×10^{-3}	1.2×10^{-4}	1.9×10^{-5}
$B\psi'(a) - F'(a)$	-0.0449	-0.0449	-0.00643
$A\phi'(a)$	-0.0449	-0.0449	-0.0643
% D	2.5×10^{-5}	5.2×10^{-6}	6×10^{-6}
$B\psi(b)$	211690.2	211404.52	472219.21
$A\phi(b) + E(b)$	212149.6	211276.72	472337.11
% D	0.217	0.0605	0.025
$B\psi'(b)$	0.4952	0.03798	0.8215
$A\phi'(b) + E'(b)$	0.5038	0.03935	0.7092
% D	1.72	3.615	15.8

Table 6.3: Candidate solution to the storage problem.

These results were obtained by employing a grid based on a separation in $\Delta x = 5\ MW$, this was five times the resolution of the constructed function F and resulted in some additional smoothing of F and E. The functions ϕ and ψ were constructed based on the estimated parameters for the Itô process X_t. To overcome the problem when ϕ and ψ are not defined we use an approximation by fitting ϕ' and ψ'. Then we apply **Algorithm 2** to find l^A, l^B, the fill boundary a and the empty boundary b.

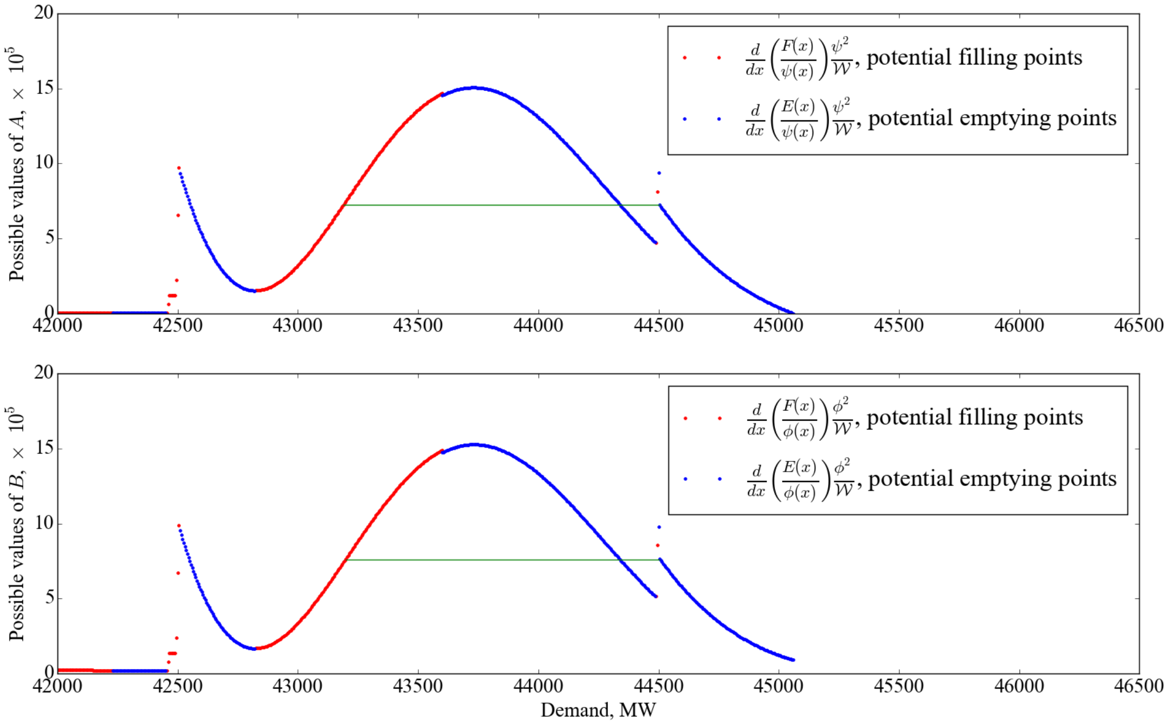


Figure 6.6: Finding a, b. The solid green line represents the proposed solution.

The potential emptying point b is then the intersection of l^A and l^B. The potential filling points and potential emptying points as well as the proposed solution a, b to the problem are presented in Figure 6.6. We start by using 4.51-4.56 to identify candidate values for a (the set $\mathcal{A}$) and b (the set $\mathcal{B}$). This is the coding of the curves in the two diagrams. For each candidate a, starting at the largest element of $\mathcal{B}$, we search for the solution 4.57 to give l^A, this is shown in the top graph. While for each candidate b, starting from the largest element of $\mathcal{B}$, we search for the solution 4.58 to give l^B, this is shown in the bottom graph. Given l^A and l^B we look for crossing point of the two lines. We choose a, $l^A(a)$ and a, $l^B(a)$ as the point where they cross (or come very close), this is the solution in green line in both graphs in Fig 6.6.

Nevertheless, we still have to check if points above b and all points below a belong to the optimal sets. In other words, we should check if 3.29 hold. For all points above b we have $(E/\psi)'' < 0$, so 3.29 holds. However, on the point of the intervals, $[39795, 40000]$, $[42820, 43100]$ we have $(F/\phi)'' < 0$, therefore these interval cannot be on the "charging" region. To overcome this problem and find a valid solution

u	$l^A(u)$	$l^B(u)$	% difference	A	B
39785	43150	43150	1.5×10^{-4}	786	457425
x	a_w	b_w			
$A\phi(x) + B\phi(x)$	458146.4	458275.5			
$g(x)$	456554	456623.2			
% difference	0.35	0.36			
$A\phi'(x) + B\phi'(x)$	-0.00786	0.36			
$g'(x)$	-0.0274	0.00315			
% difference	0.35	0.36			

Table 6.4: Candidate solutions to cover prohibited intervals.

we need to cover these intervals with a "wait to region" by repeating the algorithm, but replacing E and F by the charge payoff

$$g(x) = B\psi(x) - F(x), \tag{6.5}$$

then the value function is given by

$$v_j(x,0) = \begin{cases} A_j\phi(x) + B_j\psi(x) & , \text{ if } x \in \mathcal{C}_j^0 \subseteq \mathcal{C}^0 \\ g(x) & , \text{ if } x \in \mathcal{D}^0 \end{cases} \tag{6.6}$$

Executing the algorithm gives the results in Table 6.4.

The results shows that there is an intersection between l^A, and l^B, however the actual fit of $A\phi(x) + B\psi(x)$ and $g(x)$ are less satisfactory and the fit of $A\phi'(x) + B\psi'(x)$ and $g'(x)$ is very poor, given that based on the behaviour of g at a and b we would require smooth fit here. Given that there is no fit for the covering intervals means that there is no viable solution to the stopping problem.

6.3 Night filling/ day emptying for CAES technology

Intuition would suggest the storage facility would be charged at night during low demand at lower prices, then discharged on the day during high demand for a higher prices. We suggest treating this case as a model where the "filling" decision is made on the night time parameters and the "empty" decision is based on the day-time parameters. Technically, we consider two diffusions, day and night. We look for the entry during night and exit during day.

We start with an empty CAES technology that needs to be filled with cost $F(x)$ at night before it can be emptied and deliver the value $B\psi_d$ during the day. This case is described in section 4.4.2.

The one cycle use simulation results are shown in Fig 6.7 for winter night/-day model, the graph shows the function $\psi_d(x)$, $\phi_n(x)$ from Eq 5.9, the simulated emptying value $E(x)$ and filling value $B\psi_d - F(x)$, the function $E(x)/\psi_d(x)$ and $B\psi_d(x) - F(x)/\phi_n$. It also shows the entry boundary to fill the facility and the exit boundary to empty the facility. Table 6.5 summarize the upper bound boundaries at which the facility with different efficiencies could be charge at night for a delivery at $b^\star = 45035$ MW during the day, it shows also the cost of filling, the income of emptying and the price at which the facility buys electricity during night hours. The present value of the CAES in the case of the night/day model is presented in Fig 6.8. We observe monotonicity in efficiency as opposite to the results in Fig 6.4 (intra day) and Fig 6.5 (intra night). This explained by the fact that the cost F does not change for all the filling boundaries $a^\star$ related to each efficiency, see Table 6.5 (F column). We fill during the night where the cost is constant for all efficiencies, while we empty during the day where prices are increasing as the efficiency increase. That explained the Millions of pounds in the present value, as we get a higher difference between emptying (day) and filling (night).

To find the sequential boundaries a and b for day and night diffusion for the case of multiple use of the facility, the algorithm describes in section 4.4.2 is applied.

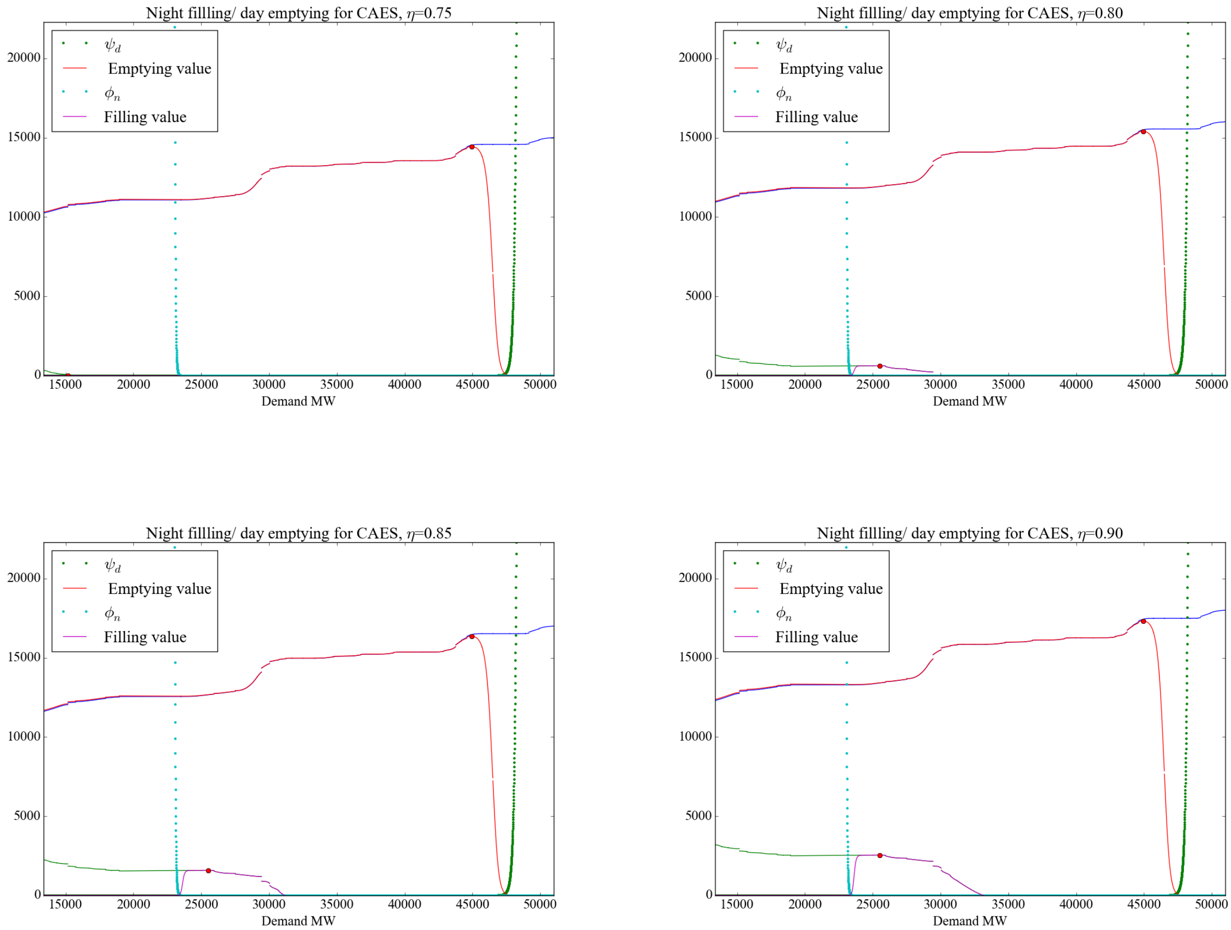


Figure 6.7: The filling boundary for CAES during winter day technology with different efficiencies and an emptying point setting at $b^{\star} = 45035$ MW

η	$B^{\star}(£)$	$a^{\star}$ (MW)	$A^{\star}$	$E(b^{\star})(£)$	$F(a^{\star})(£)$	Price $(£/MWh)$
0.75	14440.073	15165	$1.7394e-210$	14527.52	14299.89	39.62
0.80	15402.74	25520	6622.09	15496.02	14766.53	41.30
0.85	16365.41	25525	1583.90	16464.52	14766.57	41.30
0.90	17328.08	25530	2545.71	17433.02	14766.61	41.30
0.95	18290.75	25535	3507.53	18401.52	14766.66	41.30

Table 6.5: The upper bound of filling a CAES technology for night/day model for different efficiencies, $b^{\star} = 45035$ MW for all efficiencies.

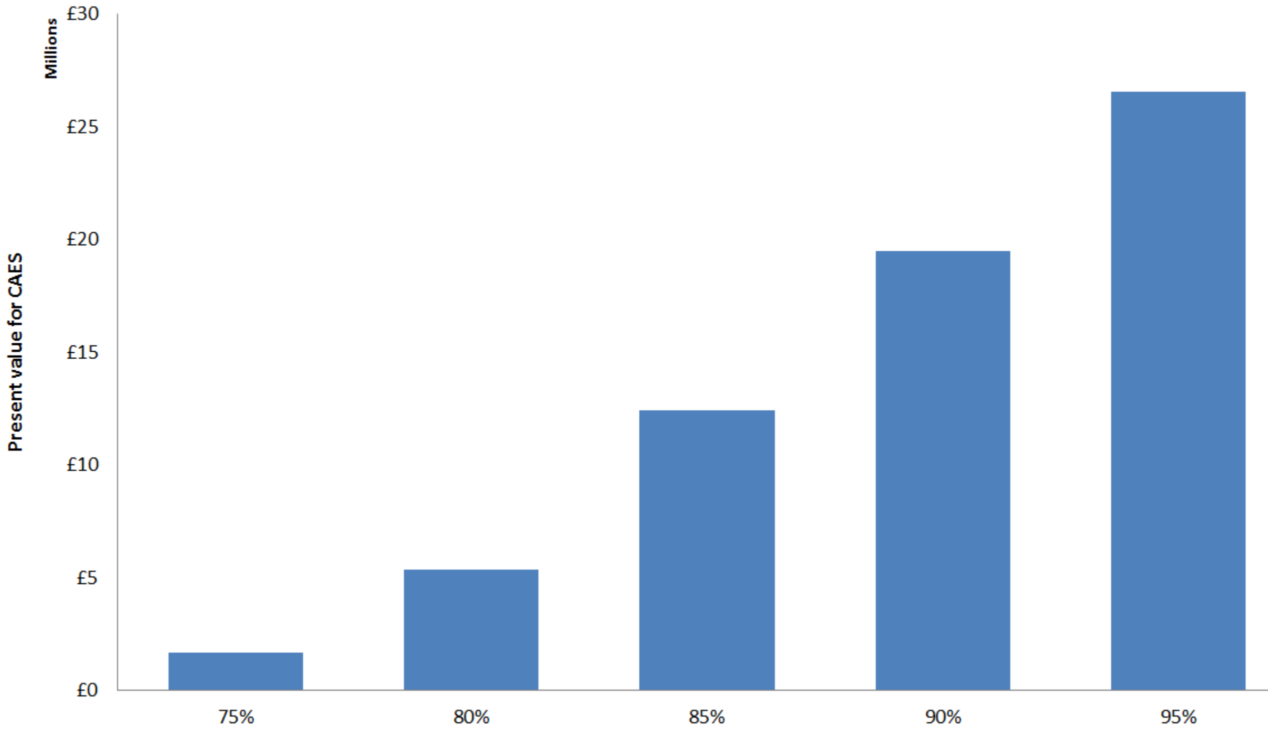


Figure 6.8: The present value for CAES technology for night/day model for different efficiencies.

The resulting filling point u such that (3.29) hold, is given by

$$u = 31380\ MW,$$

and $l^A(u)$, $l^B(u)$ that satisfies the Equations 4.65 and 4.66 are given by

$$l^A(u) = 43906.52\ MW \quad , l^B(u) = 43906.3635l\ MW, \tag{6.7}$$

with a difference of $3.7 \times 10^{-4}\%$, we also have

$$\begin{cases} B\psi_d(a) - F(a) = 1,099,056.4 \\ A\phi_n(a) = 1,099,056.4 \\ \text{D} = 4.2 \times 10^{-14}\% \end{cases}, \begin{cases} B\psi_d'(a) - F'(a) = -0.0039 \\ A\phi_n'(a) = -0.0039 \\ \text{D} = 4.4 \times 10^{-14}\% \end{cases} \tag{6.8}$$

and

$$\begin{cases} B\psi_d\left(b\right) = 1,114,684.5 \\ A\phi_n\left(b\right) + E\left(b\right) = 1,114,433.1 \\ \mathrm{D} = 0.022\% \end{cases}, \begin{cases} B\psi_d^{'}\left(b\right) = 0.7262 \\ A\phi_n^{'}\left(b\right) + E^{'}\left(b\right) = 0.7325 \\ \mathrm{D} = 0.868\%. \end{cases} \tag{6.9}$$

While the algorithm has delivered results, they are somewhat unsatisfactory. Intuition points to these results, but these have been delivered by splitting the data into 'day' and 'night' and then using a heuristic to propose the solution.

The root of the problem is the fact that the demand does not follow an Ornstein-Uhlenbeck process. This can easily be established by noting that the residuals in 5.20 should be Normally distributed. Both Shapiro-Wilk and Anderson-Darling tests reveal that the residuals are, in fact, not Normally distributed; the model is misspecified. The consequence of this misspecification is that the frequency of cycles between $a^\star$ and $b^\star$ estimated in the different models are too low.

Chapter 7

An empirical study

The results described in Chapter 6 was obtained by considering theoretical process for the demand, calibrating the observed data to this theoretical process, identify E, F, ψ, ϕ and building the solution on this basis. However, given the steps of the algorithm, there is no need to know the details of the diffusion, only that Assumptions 3.1.1-4.1.2 hold, and the functions E, F, ψ and ϕ exist. The cost of filling F and the income of emptying E can be constructed by combining the discount rate, the actual demand data with the merit order curve, while the functions related to the first hitting times of the diffusion ϕ and ψ can be identified by applying the discount rate and demand data to the equations in 3.8. The analysis we undertake is restricted to winter months in the period 1 August 2013 to 31 July 2015. All data was used, including that for weekends and the Christmas period and no day/night split was made.

7.1 Implementation

To generate the cost F, for each data point, F is constructed based on observed demand data and the merit order curve. These data are then fitted to a polynomial using least squares regression forming the charging cost function, F (Fig 7.1), with

$$F(x) = 5.1926 \times 10^{-35}x^9 - 1.7997 \times 10^{-29}x^8 + 2.7287 \times 10^{-24}x^7$$
$$- 2.3728 \times 10^{-19}x^6 + 1.3025 \times 10^{-14}x^5 - 4.6751 \times 10^{-10}x^4$$
$$+ 1.0960 \times 10^{-5}x^3 - 1.16169 \times x^2 - 1.3613 \times 10^3 x$$
$$- 4.9723 \times 10^6.$$

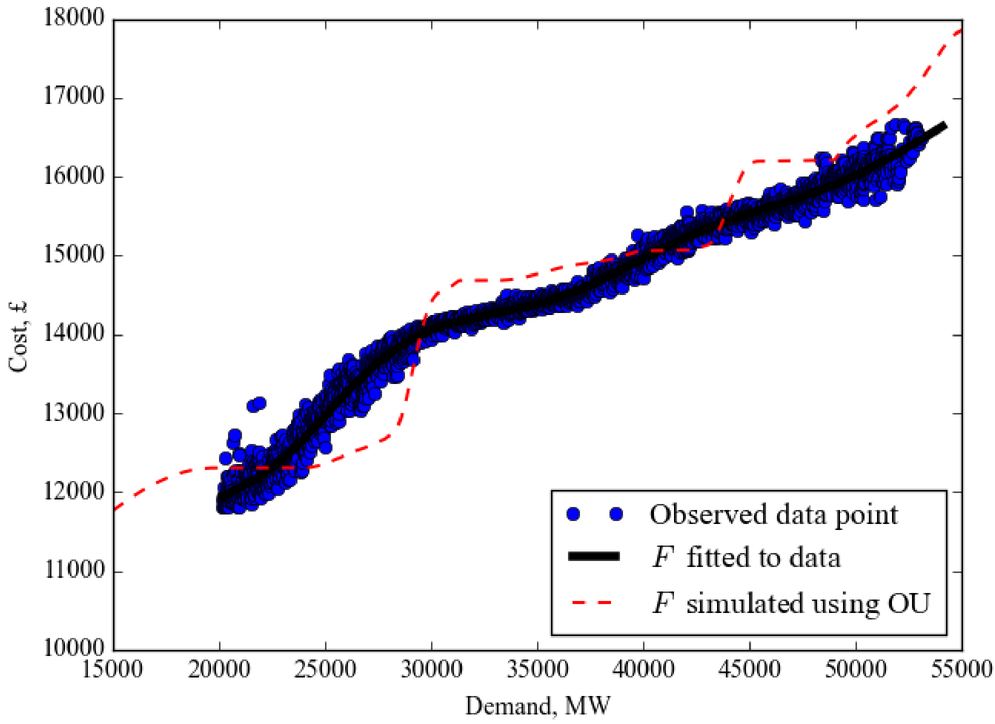


Figure 7.1: Construction of F based on observed demand data and the merit order curve.

To identify the estimates $\widehat{\phi}$, $\widehat{\psi}$ for ϕ, ψ, the following process was undertaken. Starting at each data point, running maxima and minima were noted and when these changed, the first hitting time of all points between the old and new maximum (or minimum) was set as the change time. If there was a time jump of more than 30 minutes between data points or data was repeated more than once, the process was stopped and the algorithm moved on to the next starting point. This generated a large number of samples, but because of the "re-sets" some points were 'isolated' and as a consequence the data needed to be manipulated (locally, by eye) to ensure that

	H	J	K	L	M	P	Q
$\widehat{\phi}$	1.00021	256.23253	1.65366	2.87953	1.0000	20156	54096.579
$\widehat{\psi}$	0.99986	-35.79520	1.38294	9.44348	1.15731	20156	54093

Table 7.1: Parameters for $\widehat{\phi}$ and $\widehat{\psi}$, with $\widehat{\alpha} = P_\phi = 20156$, $\widehat{\beta} = Q_\psi = 54093$.

ϕ was strictly decreasing and ψ was strictly increasing. Based on the observations the data was fitted to the functions

$$\widehat{\phi}(x) = H_\phi + \frac{J_\phi}{(x - P_\phi)^{K_\phi}} + \frac{L_\phi}{(x - Q_\phi)^{M_\phi}}$$
$$\widehat{\psi}(x) = H_\psi + \frac{J_\psi}{(x - P_\psi)^{K_\psi}} + \frac{L_\phi}{(Q_\psi - x)^{M_\psi}}$$

The boundary α is an inaccessible boundary point if $\psi(\alpha) = 0$, and similarly β is inaccessible if $\phi(\alpha) = 0$, the implication is that Assumption 3.1.2 hold. This constraint the fitting; to overcome this, $\widehat{\psi}$ was fitted for lower values of demand, giving a candidate solution for $\widehat{\alpha} = P_\phi$, this candidate is then used to fit the lower values of $\widehat{\phi}$. Similarly, $\widehat{\phi}$ was fitted for upper values of demand, giving a candidate solution for $\widehat{\beta}$, this candidate is then used to fit the upper values of $\widehat{\psi} = Q_\psi$. The results based on this process are summarized in Table 7.1 and Fig 7.2. Note that these results agreed with the observation in the previous section and suggest that the demand is not mean reverting, rather it is extreme avoiding.

It is important for the Equations 4.51, 4.54 to be hold whether we are using $\widehat{\phi}$ or $\widehat{\psi}$. To test the coherence of the data, the size of the union and the intersection of the points satisfying the two inequalities 4.51 and 4.54 should be compared. In theory the number of points in the union should be equal the number of points in the intersection. We find that for 4.51 the intersection is 98.45% of the union, while for 4.54 the intersection is 99.95% of the union.

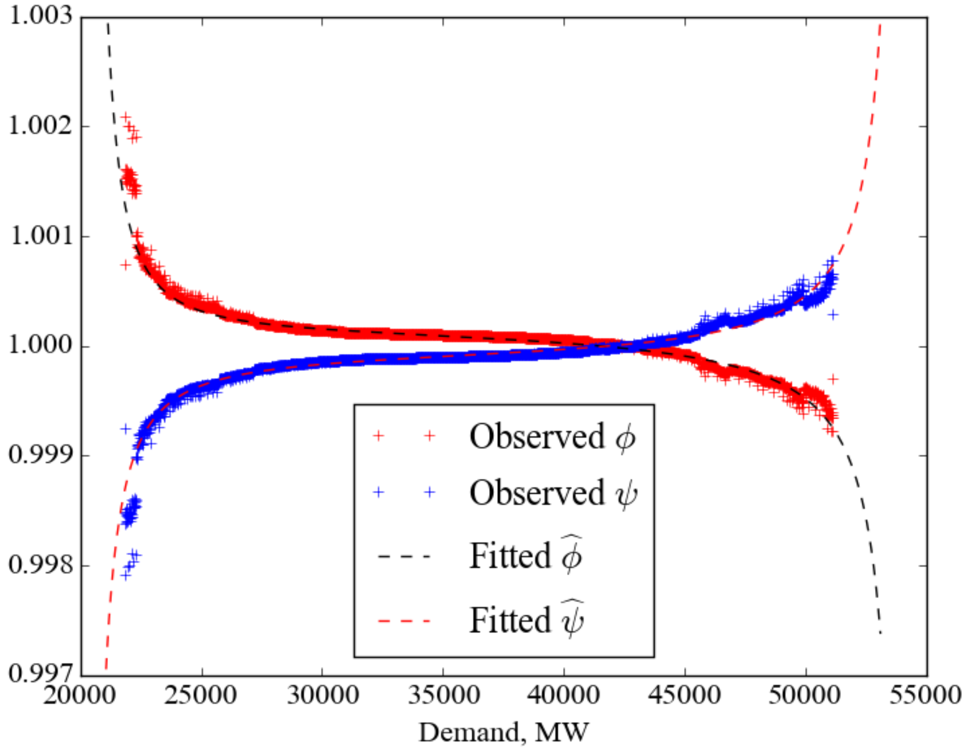


Figure 7.2: Construction of $\widehat{\phi}$ and $\widehat{\psi}$.

7.2 Finding the boundaries to fill/empty a storage technology

Now that we have the empirical data, we can find the filling boundary of a CAES technology in the case of one single cycle and the results are summarized in Table 7.2. In this case the empting boundary for all efficiencies is $b^{\star} = 53466$ MW, which correspond to an approximate selling price of $p = £58.00\ /MWh$. Assuming the facility will sell electricity at p, the results shows an upper bound constant buying price for all efficiencies to be $£43.73\ /MWh$, that is the marginal cost of coal plants (Aberthaw7, Aberthaw8 and Aberthaw9). The presented value is presented in Fig 7.3.

To investigate the multiple case use of the technology, **Algorithm 2** is used with the round trip efficiency set as $\eta = 0.90$, which is less that than the one used in the case of the theoretical OU process. A grid precision of $\Delta x = 1MW$ is used and the results are displayed Table 7.3. The first run of the algorithm gives the candidate

η	$B^{\star}(£)$	$a^{\star}$ (MW)	$A^{\star}$	$E(b^{\star})(£)$	$F(a^{\star})(£)$	Price $(£/MWh)$
0.74	12169	20776	79	12234	12028	40.73
0.79	12991	20816	893	13060	12033	40.73
0.84	13814	20856	1707	13887	12039	40.73
0.89	14636	20886	2522	14714	12043	40.73
0.94	15458	20916	3337	15540	12047	40.73
0.99	16280	20946	4152	16367	12051	40.73

Table 7.2: The upper bound of CAES technology for different efficiencies, $b^{\star} = 53466$ MW for all efficiencies.

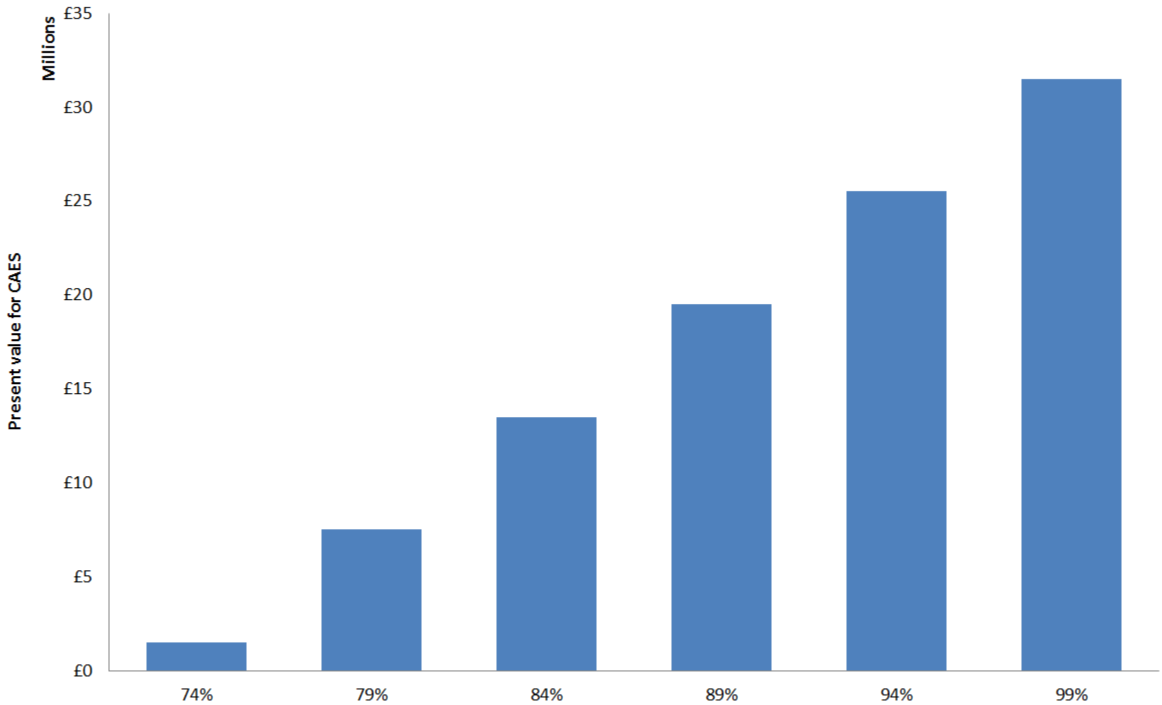


Figure 7.3: The present value for CAES technology for different efficiencies.

boundary value of $a = 33451MW$ and $b = 42781MW$, however this solution leave the interval [28121.32571] where filling is sub-optimal. Repeating the algorithm to cover delivers a poor fit since there is not a meeting point of $l^{A}(u)$ and $l^{B}(u)$. Given there is no solution here we adopt the candidate solution $a = 24293MW$, $b = 44677MW$.

To check the optimality of this solution, we consider the following strategy to trigger the filling and emptying of the CAES facility. The filling will take place when demand net wind is in $[20156, 24293]$, while emptying will take a place when

u	24293	33419
$l^A(u)$	44677.2069	42807.25
$l^B(u)$	44677.2048	42805.5
% D	4.5×10^{-6}	-0.004
a	24293	33419
b	44677	42807
A	1230671.8	2184098.3
B	1244447.7	2198884.5
$B\psi(a) - F(a)$	1231146.71	2184348.2
$A\phi(a)$	1231146.71	2184348.2
% D	1.9×10^{-14}	2.1×10^{-14}
$B\psi'(a) - F'(a)$	-0.13574	-0.0253
$A\phi'(a)$	-0.13574	-0.0253
% D	0.0	1.4×10^{-14}
$B\psi(b)$	1244533.08	2198928.7
$A\phi(b) + E(b)$	1244532.32	2197833.8
% D	6.1×10^{-5}	0.0498
$B\psi'(b)$	0.0384	0.04803
$A\phi'(b) + E'(b)$	0.0385	0.04807
% D	0.138	0.085

Table 7.3: Candidate solution to the storage problem.

demand net wind is in $[44677, 54093]$. The data for winter of 2013-2014 and of 2014-2015 was used and the facility started both seasons in the 'Empty' state and if it was full at the end of the season, its inventory value was based on emptying the technology at the emptying boundary b. The cost of filling was determined using the function F identified in Fig 6.2, while the value of emptying was determined by using $E(x) = F(x) \times \eta$. For example, the cost of filling the facility at the candidate value of $a = 24293\ MW$ is $12,762.62$ whereas the value of emptying at $b = 44677\ MW$ is $13,955.14$. On this basis the candidate strategy gives a net present value for the facility over two winter seasons as $47,113.59$, with the optimal strategy being $a^\star = 24000\ MW$ and $b^\star = 40800\ MW$ with a net present value of $50,295.41$. A contour plot showing these results presented in Fig 7.4. From the contour plot we can note that strategies with an discharging point around $40800\ MW$ performed significantly better than those discharging above 41000 MW. This can be explained by the fact that the test data included weekends and the Christmas period, and strategies discharging at around 40800 MW were able to cycle over these holidays, generating additional value. Repeating the experiment excluding holidays resulted in a net present value of the candidate strategy of $40,148.84$ while the empirically optimal strategy was identified as $a^\star = 24600MW$ and $b^\star = 42,900$ MW with a net present value of $44,514.16$. The contour plot without holidays and weekends is shown in Fig 7.5.

The empirical optimal strategies and optimal candidate strategies are compared in Figures 7.6, 7.7, 7.8 and 7.9. The cumulative NPV of winter season of 2013-2014 and 2014-2015 are shown respectively in Fig 7.10. The interesting observation is that it has been a significant increase in the number of strategies or the filling/emptying times of the storage facility in November 2014 comparing with November 2013, and that also could be observed comparing December 2013 with December 2014.

In winter of 2013-2015 a total of 20 filling and emptying empirical optimal strategy gives a cumulative NPV of £18698, and a total of 16 optimal candidate strategies gives a cumulative NPV of £18850. While during the winter of 2014-2015 a total of 29 filling and emptying empirical optimal strategy gives a cumulative NPV of £44514, and a total of 17 optimal candidate strategies give a cumulative NPV of £40140.

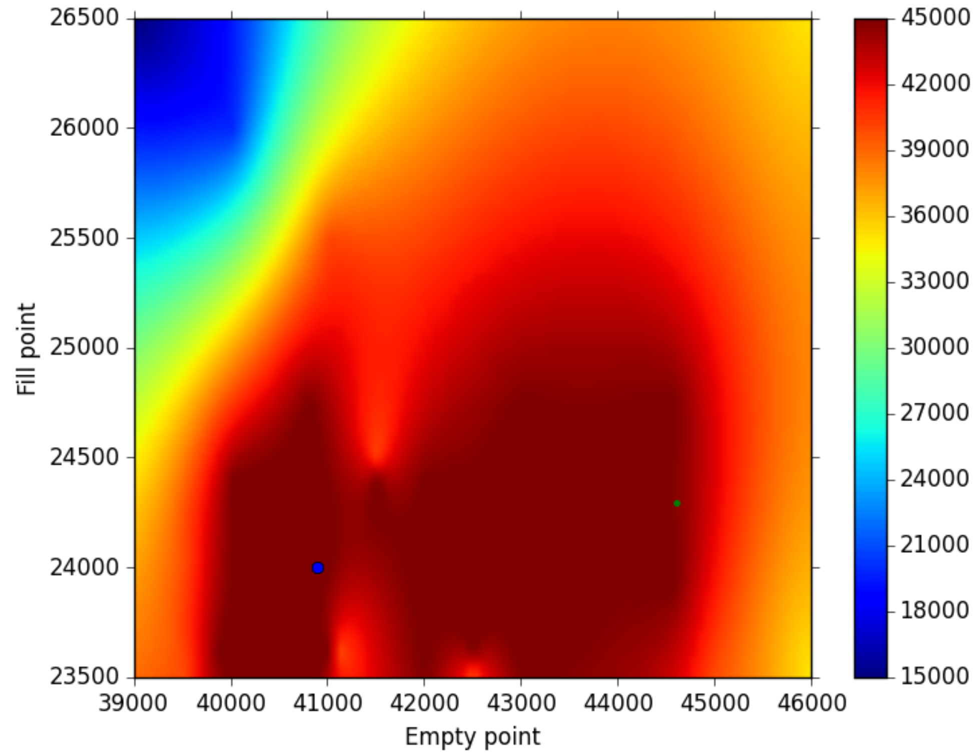


Figure 7.4: Contour plot showing values for different strategies. The empirically optimal strategy identifies by the blue dot, with value £50295, the green dot is the candidate strategy, with value £47114.

7.3 Conclusion

The candidate strategy does not appear to be optimal. However there are a number of points to consider in assessing the effectiveness of the methodology.

The algorithm did not identify an optimal solution. There is no meeting point of l^A and l^B, they only come very close. The results, both empirically optimal and the candidate strategy sit on a“plateau” suggesting that there is no optimal fill/empty pair. This reflect the fact that all the optimal emptying points lay in the demand range of 40-45 GW, which, with reference to the merit order curve, corresponds to a fairly constant marginal price of power. This interval is supplied by Combined Cycle Gas Turbines. More careful construction of the functions F and E might resolve this problem if necessary.

The data is not Markovian. The methodology rests on the assumption that the data is Markovian and the underlying diffusion is time-homogeneous; this is not

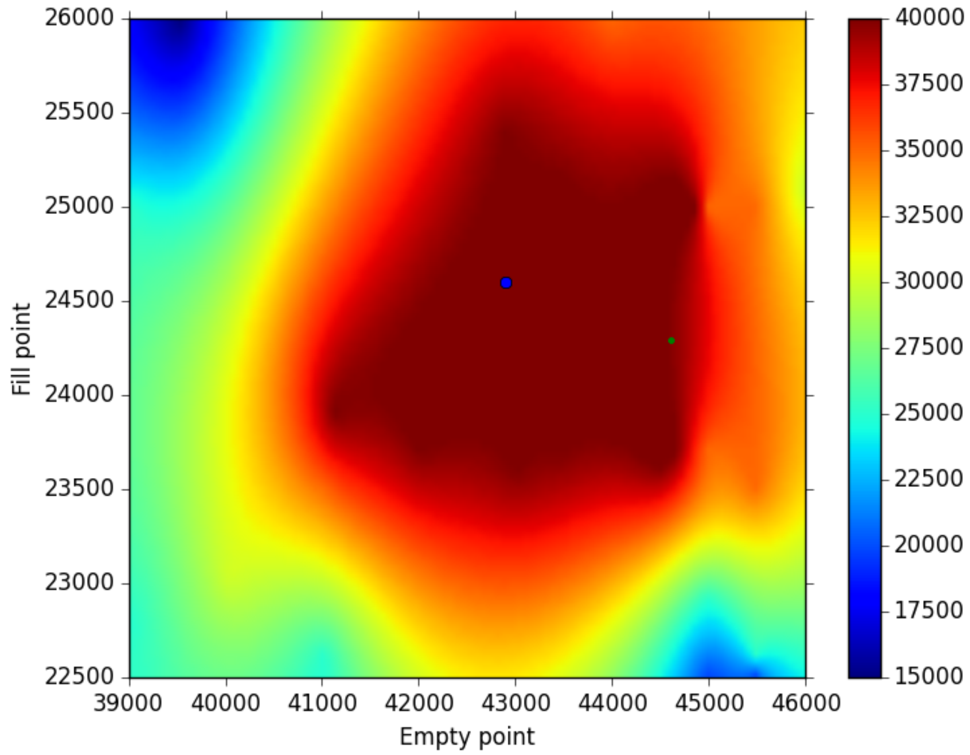
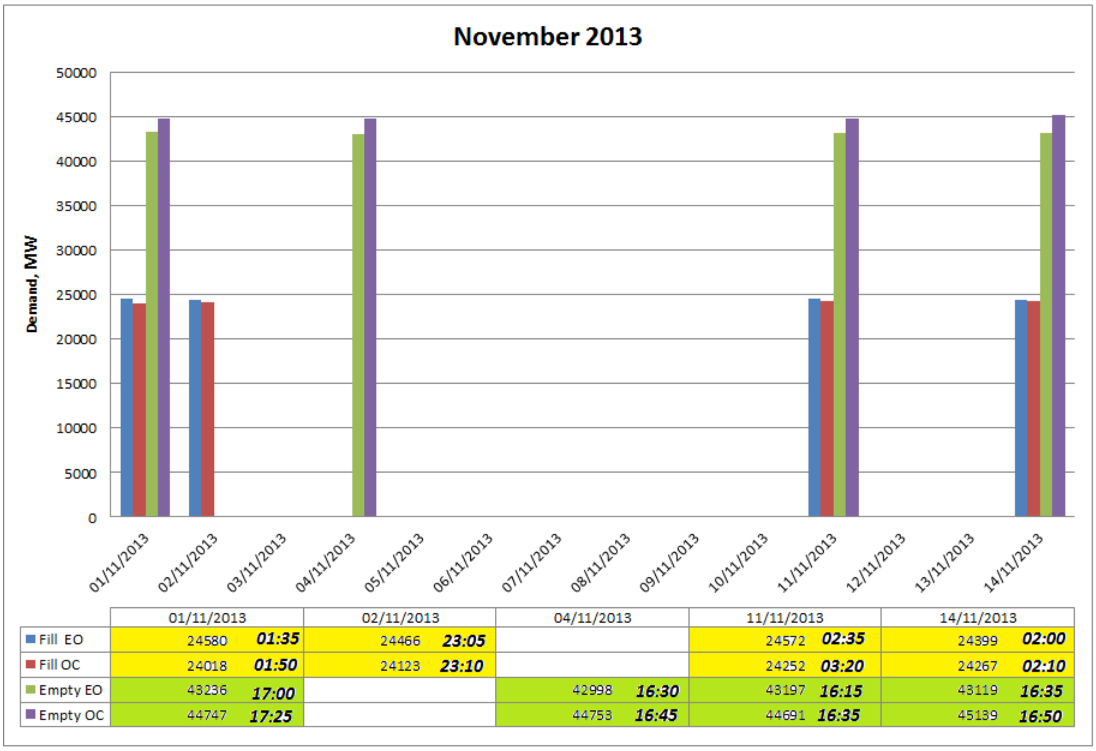


Figure 7.5: Contour plot showing values for different strategies, without holidays. The empirically optimal strategy identifies by the blue dot, with value £44514, the green dot is the candidate strategy, with value £40149.

the case. This assumption is the cost of a relatively straightforward methodology. The impact is that the periodicity of demand is not encoded in ϕ and ψ, and this results in the algorithm under-estimating the number of cycles the demand will pass through.

The observer results are subject to experimental error. We are only testing against two seasons. Over 2013-2014 the empirically optimal strategy under-performs the candidate strategy £18698 to £18850, while performing significantly better in 2014-2015, £44515 to £40149.



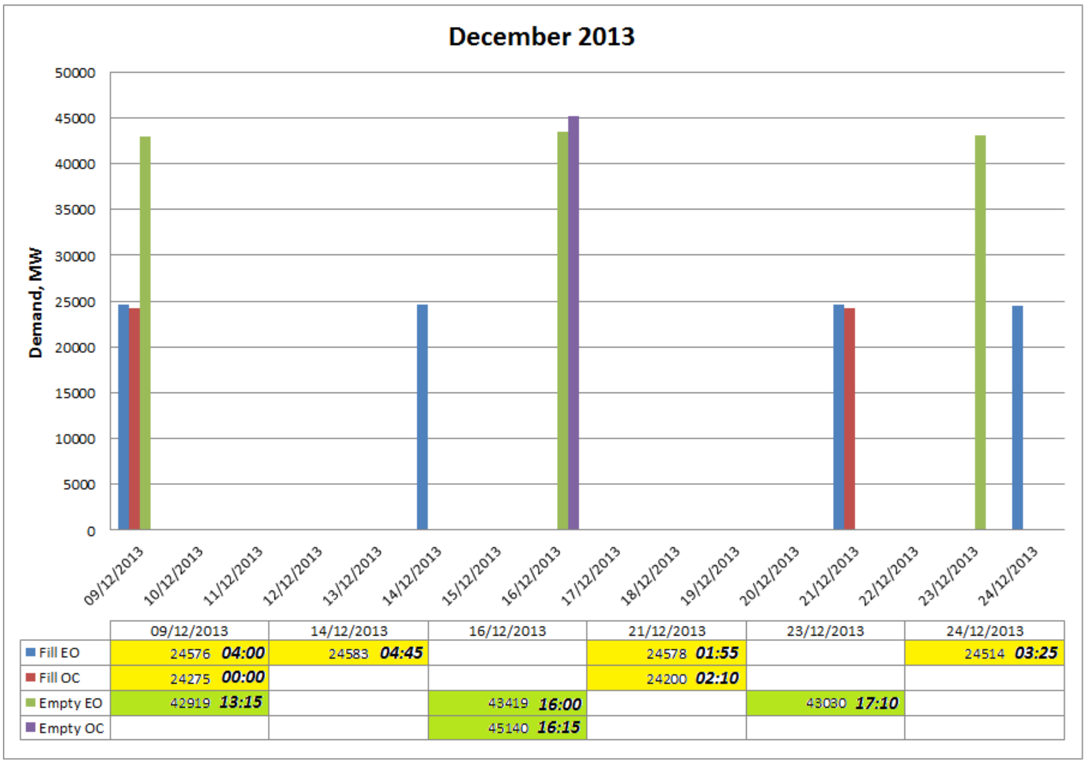
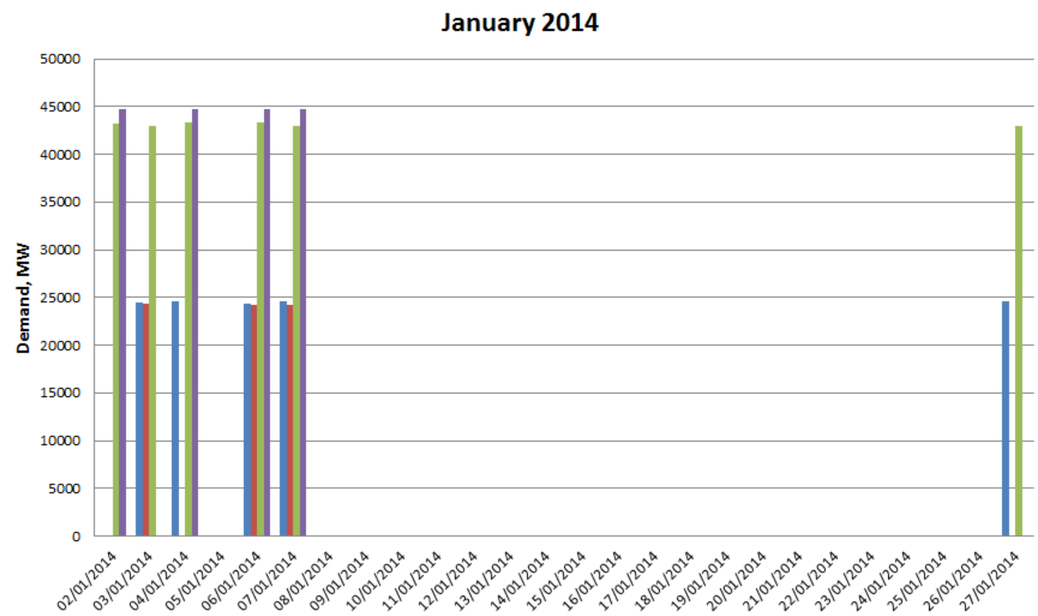
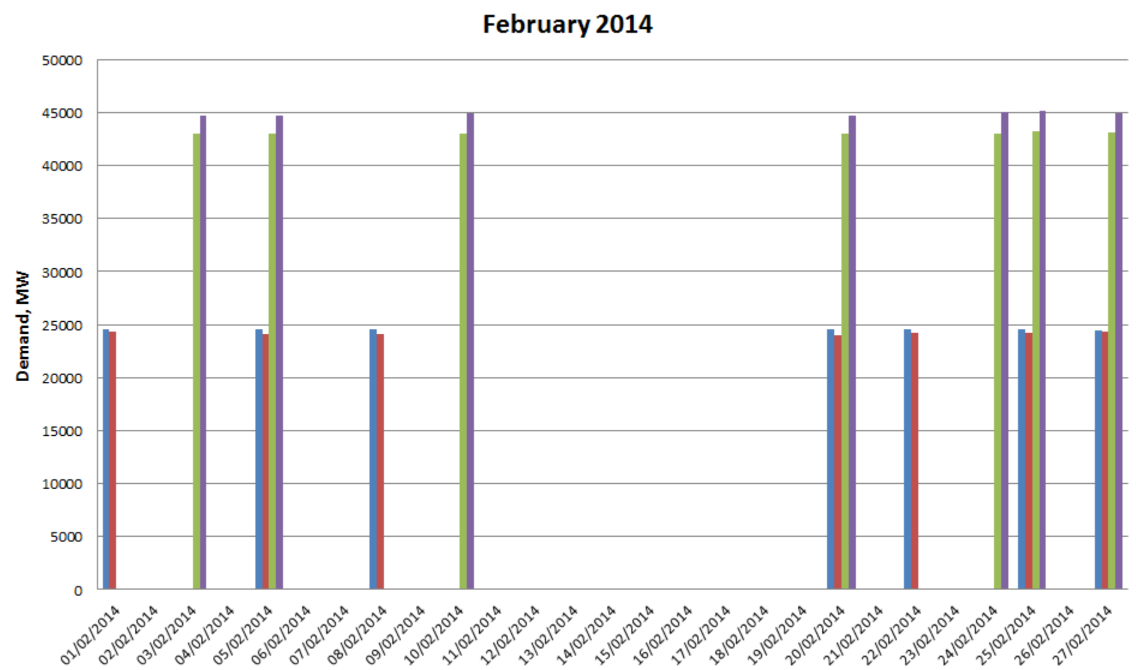


Figure 7.6: Empirical optimal (EO) and optimal candidate (OC) strategies, 11/2013-12/2013.

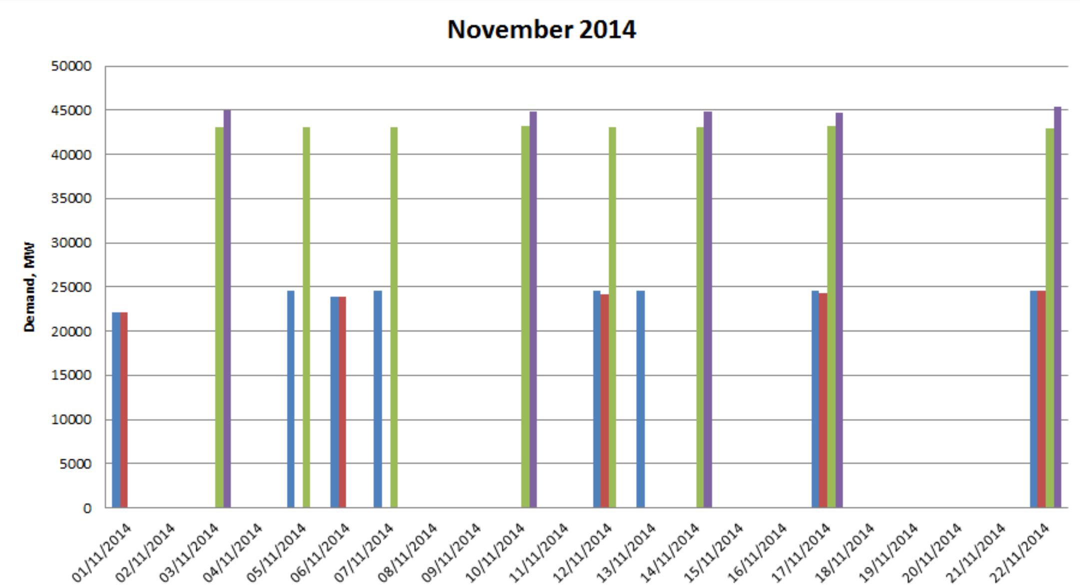


	02/01/2014	03/01/2014	04/01/2014	06/01/2014	07/01/2014	27/01/2014
Fill EO		24484 02:15	24553 03:40	24333 00:00	24552 03:05	24588 04:10
Fill OC		24288 02:25		24266 00:05	24170 03:30	
Empty EO	43202 16:40	42947 17:20	43287 16:45	43304 16:45	42947 16:30	42906 10:45
Empty OC	44738 17:15		44692 17:15	44704 17:30	44765 16:50	

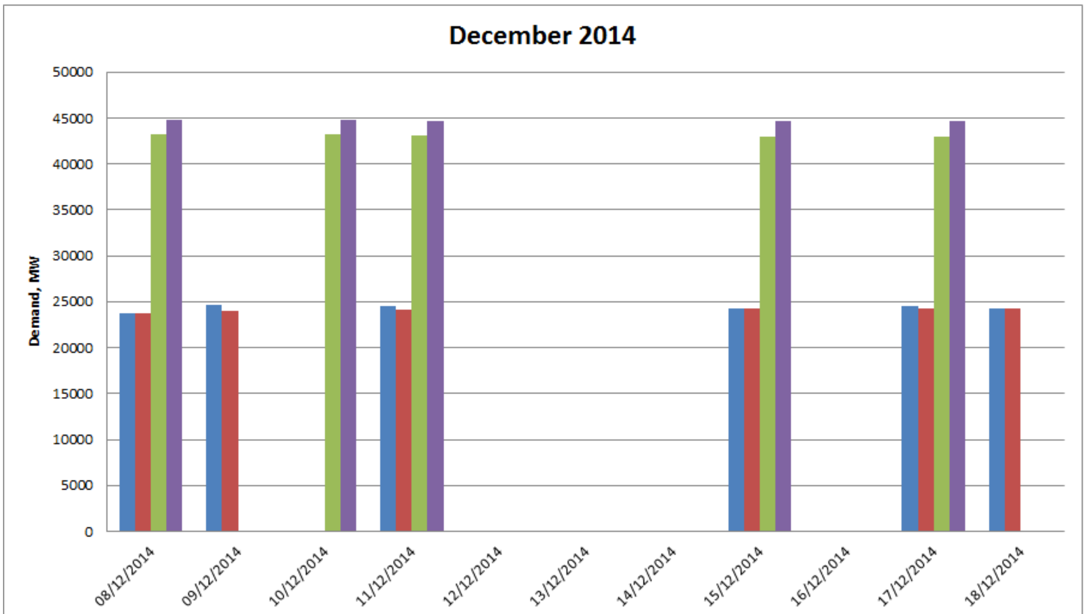


	01/02/2014	03/02/2014	05/02/2014	08/02/2014	10/02/2014	20/02/2014	22/02/2014	24/02/2014	25/02/2014	27/02/2014
Fill EO	24529 04:25		24508 03:45	24516 03:00		24554 03:20	24596 02:35		24507 02:05	24494 03:25
Fill OC	24288 05:00		24127 03:55	24079 03:20		24030 03:30	24261 03:10		24182 02:30	24287 03:30
Empty EO		43037 16:55	43046 16:25		43007 08:05	42955 17:45		42990 17:10	43180 18:00	43086 16:55
Empty OC		44737 17:20	44686 16:55		44864 09:15	44683 18:35		44991 17:55	45128 18:05	44893 17:40

Figure 7.7: Empirical optimal and optimal candidate strategies, 01/2014-02/2014.

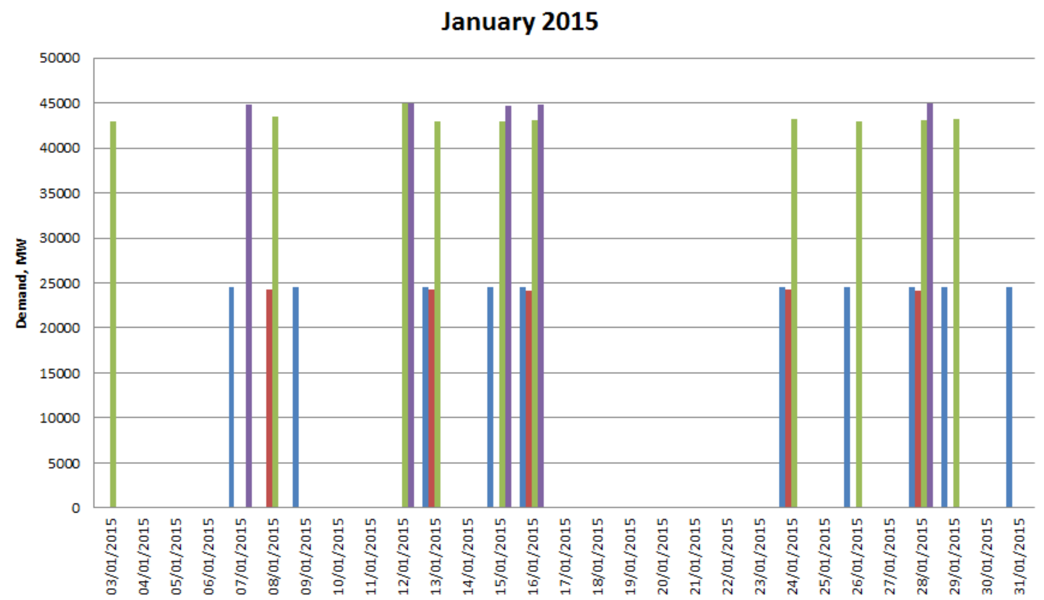


	01/11/2014	03/11/2014	05/11/2014	06/11/2014	07/11/2014	07/11/2014	10/11/2014	12/11/2014	13/11/2014	14/11/2014	17/11/2014	22/11/2014
Fill EO	22117 00:00		24598 03:55	23924 23:25		24581 23:50		24530 00:35	24538 00:00		24556 02:55	24537 00:25
Fill OC	22117 00:00			23924 23:25				24221 01:30			24281 03:15	24125 00:35
Empty EO		43073 16:25	43039 16:25		43018 17:20		43157 16:25	43126 17:05		43096 16:35	43227 16:05	42931 17:20
Empty OC		44938 16:55					44857 16:40			44843 16:50	44759 16:25	45356 18:04

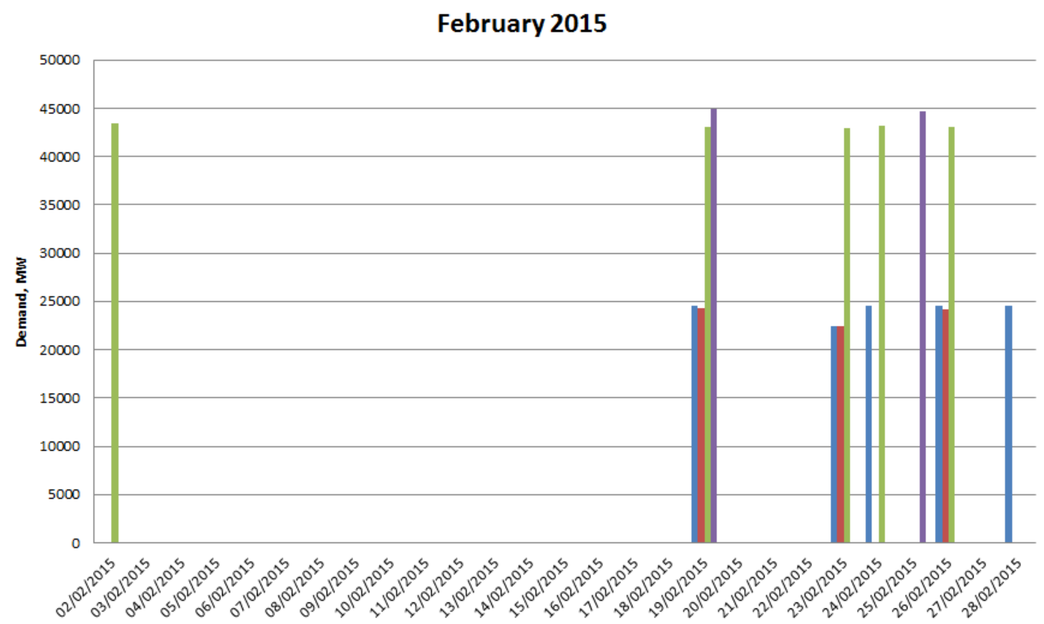


	08/12/2014	09/12/2014	10/12/2014	11/12/2014	15/12/2014	17/12/2014	18/12/2014
Fill EO	23778 00:00	24585 23:55		24569 03:40	24252 00:00	24525 02:05	24292 00:00
Fill OC	23778 00:00	24026 03:00		24183 03:50	24252 00:00	24210 02:35	24292 00:00
Empty EO	43214 15:55		43244 16:30	43052 16:20	42917 12:05	42926 16:05	
Empty OC	44795 16:10		44729 17:05	44707 16:35	44713 15:50	44714 16:35	

Figure 7.8: Empirical optimal and optimal candidate strategies, 11/2014-12/2014.



	03/01/2015	07/01/2015	08/01/2015	09/01/2015	12/01/2015	13/01/2015	15/01/2015	16/01/2015	24/01/2015	26/01/2015	28/01/2015	29/01/2015	31/01/2015
Fill EO		24578 23:40		24550 03:50		24590 03:50	24571 03:20	24554 00:00	24588 04:25	24518 04:35	24586 03:15	24475 04:20	24525 23:55
Fill OC			24260 00:00			24293 04:25		24180 00:30	24873 04:40		24170 03:50		
Empty EO	42975 17:15		43447 16:45		44883 09:40	42992 16:40	42949 16:45	43053 16:25	43272 17:25	42928 16:30	43041 17:05	43233 15:50	
Empty OC		44866 09:40			44883 09:40		44724 17:30	44786 16:45			44886 17:30		



	02/02/2015	19/02/2015	23/02/2015	24/02/2015	25/02/2015	26/02/2015	28/02/2015
Fill EO		24555 02:35	22460 00:00	24568 03:20		24564 03:15	24558 02:15
Fill OC		24282 03:25	22460 00:00			24198 03:30	
Empty EO	43378 07:35	43047 17:20	42918 18:05	43221 18:00		43092 18:05	
Empty OC		44916 17:45			44702 17:25		

Figure 7.9: Empirical optimal and optimal candidate strategies, 01/2015-02/2015.

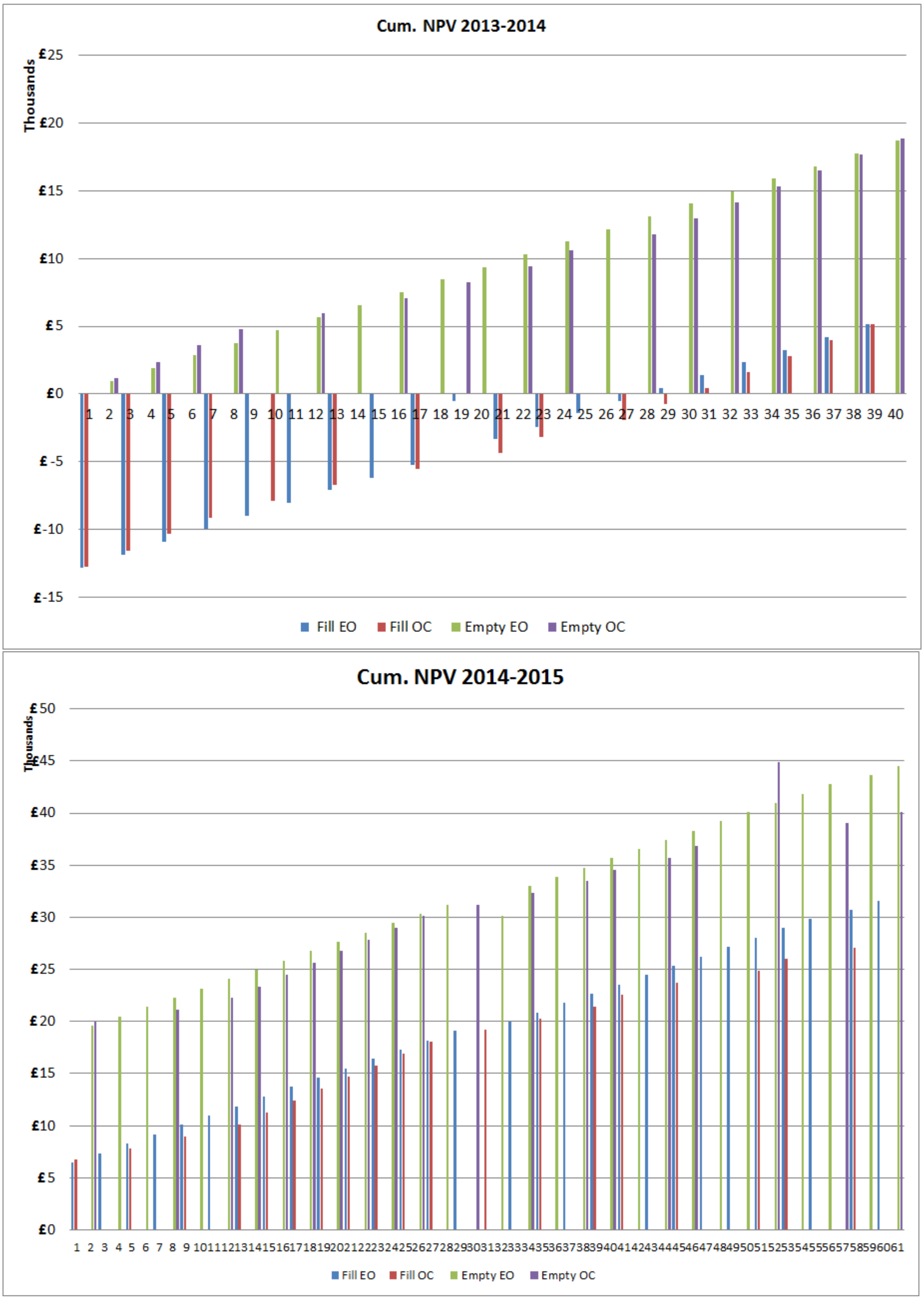


Figure 7.10: Cumulative NPV of CAES, EO vs OC for Winter 2013-2014 and 2014-2015.

Chapter 8

Conclusions and future work

8.1 Conclusions

The thesis presents a methodology for assessing the economic impact of power storage facilities that can be used by policy makers, in the spirit of the recommendations in [[119], 5.2.2]. The methodology identifies a viable optimal stopping strategy and by reflecting on the data it is suggested, that in the case considered, no actual optimal strategy exists.

The attributes of a good model in policy making are identified as ease of maintenance, adaptability and ease of interpretation at the expense of a loss of precision. This rule out many novel techniques that identify optimal strategies, those are complex and only really suitable for plant managers. We suggest that the main benefit of the methodology we present is that, while built of sophisticated mathematics, it only requires an understanding of calculus and the ability to implement the central algorithm.

The strategy identified is naive, and could be inferred by searching through simple entry and exit strategies, as we did in testing the optimality of our strategy. However, such an approach would not identify the existence of more complex, and better, strategies that involve discontinuous charging and discharging regions. This situation should not be disregarded given the staircase nature of the pay-offs encountered in the problem.

We note that the optimal strategy for the CAES plant is based on nightday arbitrage and does not appear to address the issue of variable wind generation. This is not an unusual result, the explanation being that wind is not that volatile: there are periods when the weather is windy and periods when it is not, and within these periods, lasting a few days, wind power generation is largely predictable. This suggests that the relevant power storage technology to address wind variability needs to be very large capacity that can store excess power generated over days and is able to compensate for similar periods of low wind power generation. These suggest focusing on hydro/pumped storage technologies, rather than chemical storage technologies, with significant capital costs. This presents the policy makers with the challenge of establishing a regulatory framework that incentives investment in such plant. The simplicity and flexibility of the methodology we present would play a part in taking on this challenge.

On this basis, we believe the utility of this methodology is in, for example: assessing the potential impact of feed in tariff levels in improving the economic viability of storage in supporting the integration of renewable electricity generation onto the grid; as part of studying capacity payments; or, to investigate what technological developments need supporting.

An important feature of the methodology used in Chapter 7 is that it is not based on identifying a particular Itô process driving the model; rather the analysis is based solely on empirical data. This approach, which mitigates for model risk, is shown to be better, in this case, than the standard approach of identifying an Itô process to which the observed data is calibrated.

While the methodology has been developed with a particular application in mind, it has broader utility in employing the standard theory of optimal stopping in other situations where the driving process cannot be easily identified.

8.2 Future extension

The methodology we studied in this thesis is based on a number of assumptions. We considered an Ornstein Uhlenbeck process for the dynamics of net demand, a continuous time homogeneous Markov process. In contrast, its discrete counterpart of autoregressive processes (AR(1)) may well present the dynamics of net demand. Though the theory of optimal stopping is not mature for AR(1), however, some existing results ([33]) might be a starting point.

In terms of the electricity market assumptions, we considered an approximation method (merit order curve approximation) for electricity prices based on the market under BETTA in 2012; however from autumn 2013 the UK market had some changes under the EMR like assuming the capacity market and the contract for difference (CfDs). These mechanisms will play an important role on the electricity prices, therefore for future studies, these should be taken into account. A perfect competitive market under which this study based on, may not be realistic, rather a model that takes into account the different agents participating in the market is a good extension, and here we recommend the use of agent modelling approach.

The main assumption on modelling the storage was to consider a "bang bang control". However a study that includes inventory is needed. A future extension to this work is to assume the costs of charging/discharging are related to the level of charge and $Z \in [0, 1]$ instead of assuming $Z \in \{0, 1\}$.

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