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A novel model for Uncertainty Propagation analysis applied for human thermal comfort evaluation

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Abstract: The comfort sensation is mainly affected by six variables: air temperature, mean radiant temperature, air velocity, relative humidity, personal metabolism and clothing insulation. These are characterized by different mean values and distributions.

To analyze the uncertainty propagation three numerical models are used: the Fully Monte Carlo Simulation MCSs, the Monte Carlo Simulation Trials MCSt, and a novel model named "Adaptive Derivative based High Dimensional Model Representation" (AD-HDMR).

In the paper these three different methods are applied to the thermal comfort evaluation, through the PMV Index, they are analyzed and their efficiency was verified in terms of computational time. To allow a revision of this index, the effect of the different variables was then analyzed.

Key-Words: Mathematical models, Statistical analysis, Thermal comfort, Engineering physics.

1 Introduction

The steady growth of the performance of modern computers, accompanied by a gradual decrease of the associated costs, has been encouraging the numerical simulation of more complex physical problems.

The most general way to perform uncertainty propagation (UP), as well as global sensitivity analysis, is to use the Monte-Carlo (MC) approach, which basically follows three main steps:

1) sample the input random variable(s) from their known or assumed (joint) Probability Density Function (PDF),

2) compute deterministic output for each sampled input value(s), and

3) determine the statistical characteristics of the output distribution (e.g. mean, variance, skewness). The MC method has the property that it converges to the exact stochastic solution when the number of samples $n \rightarrow \infty$. In practice the value of n can be a finite number, but to have a highly converged process it should be very high, causing an excessive computational costs (even for modern computers). A way to reduce the computational time of the UP process could be to build a less expensive surrogate of the model and then use it to propagate the uncertainty. Among the surrogate based approaches, there are the Stochastic Collocation (SC) method, the Polynomial Chaos (PC) [1], and the Kriging surrogate model [2]. These methods are nonintrusive by nature, because they consider the model as a black box and try to approximate the implemented function. One of the major issues of the surrogate based approaches is the so called Curse of Dimensionality (CoD) [3], which limits their use to problems with low dimensionality. The cut-High Dimensional Model Representation (cut-HDMR) [4] was developed to decouple the interaction effects of chemical systems, and was successfully used in other fields such as uncertainty quantification [5], sensitivity analysis [6] and interpolation problems [7].

In this work a new method for UP is used and compared to the results given by MC. The used approach decomposes the stochastic space into subdomains, which are then interpolated separately by a selected interpolation technique Each [8]. interpolating model is built accordingly to the outcomes of a new derivation of the cut-HDMR. The contribution of each independent sub-domain to the final response is evaluated, and only important sub-domains are sampled and interpolated, causing a dramatically reduction of the necessary number of samples for high dimensional spaces.

The MC simulation is then applied on each important interpolated sub-domain to approximate the propagation of uncertainty, as well as the sensitivity of the global function w.r.t. the single variables or combinations of variables. The specific application in this work is implemented in human comfort topic.

The thermal comfort in indoor environment play an important role, but often neglected, in the design of new buildings, or in the renovation of old ones.

In particular in residential buildings, where designing only considering energy saving aspects could cause overheated or overcooled indoor climate condition [9], or in educational buildings, where it could cause loss of attention and learning ability problems, and if protract even asthmatic symptoms [10, 11].

In 1970 Fanger [12] introduced a method to predict the level of thermos-hygrometric comfort within a confined space as happens in residential buildings.

The UNI EN ISO 7730 [13] takes up this theory. Through the measurement of four physical quantities (air velocity, air temperature, radiant temperature and relative humidity) and two subjective (clothing and personal metabolism) by applying the mathematical relationship contained in this standard, it is possible to reach statistical index called Predicted Mean Vote (PMV index). Through a 7-point scale (Table 1) in which a score is associated with the thermal sensation of the human body [12, 13,14].

Table 1: Thermal sensation scale in ISO 7730:05

_		
	+3	Hot
	+2	Warm
	+1	Slightly warm
	0	Neutral
	-1	Slightly cool
	-2	Cool
	-3	Cold

The same ISO 7730 provides a further subdivision of the value of the PMV shown in Table 2.

Table 2: ISO 7730:05 in PMV Classification

Category UNI 7730	Thermal state of the body as a whole		
А	-0,20 < PMV < 0,20		
В	-0,50 < PMV < 0,50		
С	-0.70 < PMV < 0.70		

The characteristics of the measures in situ by the four mechanical parameters are imposed by the standard UNI EN ISO 7726 [15] such as field and uncertainty about the nature of physical measurements follow a Gaussian distribution.

The characteristics of the measures for the two sizes are imposed by subjective standards: the UNI EN ISO 8996 [16] takes into account the evaluation and measurement of human metabolism.

The UNI EN ISO 9920 [17] shows how to evaluate the thermal resistance of clothing through direct and indirect measures.

Olesen and others [18] associated with this evaluation an uncertainty distribution.

Some authors [19, 20, 21] have made the problem of verifying if starting from the uncertainties of the input variables to the system, which was the uncertainty of the PMV output.

Focus of this work is to apply the three numerical methods (fully Monte Carlo, Monte Carlo trials and the Derivative-HDRR) from common data, and to compare the results obtained and the computational costs.

2 The PMV Index

To evaluate the PMV, the model is established by UNI 13005 [12] and it is expressed by relation:

PMV=

$$= (0.303 \cdot e^{-0.036M} + 0.028) \cdot \begin{cases} (M - W) - 3.05 \cdot 10^{-3} \cdot [5733 - 6.99 \cdot (M - W) - p_a] - \\ -0.42 \cdot [(M - W) - 58.15] - 1.7 \cdot 10^{-5} \cdot M \cdot (5867 - p_a) - \\ -0.001 \cdot 4M(34 - t_a) - \\ -3.96 \cdot 10^{-8} \cdot f_{cl} \cdot [(t_{cl} + 273)^4 - (\overline{t_r} + 273)^4] - \\ - f_{cl} \cdot h_c(t_{cl} - t_a) \end{cases}$$
(1)

Where:

 $t_{cl} = 35.7 - 0.028 \cdot (M - W) - I_{cl} \left| 3.96 \cdot 10^{-8} \cdot f_{cl} \cdot \left[(t_{cl} + 273)^4 - (\overline{t_r} + 273)^4 \right] + f_{cl} \cdot h_c (t_{cl} - t_a) \right\}$

$h_{c} = \begin{cases} 2.38 \cdot (t_{cl} - t_{a})^{0.25} \\ 12.1 \sqrt{v_{ar}} \end{cases}$	$\begin{array}{l} for \ 2.38 \cdot \left(t_{cl} - t_{a}\right)^{0.25} > 12.1 \sqrt{v_{ar}} \\ for \ 2.38 \cdot \left(t_{cl} - t_{a}\right)^{0.25} < 12.1 \sqrt{v_{ar}} \end{array}$
$f = \int 1.00 + 1.290 \cdot I_{cl}$	$_{for} I_{cl} \leq 0.078 m^2 \cdot {}^{\circ}C/W$
$f_{cl} = \begin{cases} 1.00 + 1.290 \cdot I_{cl} \\ 1.05 + 0.645 \cdot I_{cl} \end{cases}$	for $I_{cl} > 0.078m^2 \cdot °C/W$

where the symbols are the same as in the table 3.

3 The simulation methods

A classical method to evaluate the uncertainties in the indirect measure is the Monte Carlo (MC), and in fact the UNI 13005 [22] prescribes to use this method to assess the uncertainties of indirect measures.

As stated in the introduction, since the MC approach can be very computationally expensive, other approximated methods can be preferred. In this paper we use one of the alternative approaches, which is based on the cut-High Dimensional Model Representation (cut-HDMR) decomposition [23].

This technique decomposes the stochastic space into sub-domains of lower dimensionality, and interpolates each sub-domain with the most appropriate technique. It determines the range of coverage with the approximation degree of 95%, which is a measure of the model fidelity and of the information quality on the independent variables, contained in the probability density function.

The instrumental accuracy required by the UNI EN ISO 7726 [15] was taken as the uncertainty of the physical variables. For the uncertainties associated with the variables M and Icl, reference was made to Olesen and others [18] and therefore, an associated uncertainty with rectangular distribution, and having the mean value equal to 15%, is assumed.

As regards the numerical methodology the Monte Carlo Methods "full" and "adaptive" according to the law are both applicable.

The main difference between them is that in the adaptive method the loop described in Figure 1 can be stopped when the variation of the statistical values with the iterations:

$$abs(f(x)_{nc}-f(x)_{nc-100})$$

 n_c is the current number of samples, and is < 1e-5 , bringing to an n << 10^6 . The full Monte Carlo is interrupted when N = 10^6 , with N, n_c are natural numbers.

In Figure 1 the block diagram of the Monte Carlo Simulation is represented. In order to let the reader understand the difference between the MC approaches and the approximate one, the AD-HDMR (Adaptive Derivative High Dimensional Model Representation) method is briefly described in what follows.

The AD-HDMR [8] approach proposed for this study is based on the cut-High Dimensional Model Representation (cut-HDMR) decomposition, and it allows a direct cheap reconstruction of the quantity of interest and analyses similar to an ANOVA (Analysis Of Variance) decomposition [23]. Basically, the core part of the Monte Carlo algorithm shown in Figure 1, in the AD-HDMR approach should be substituted by the loop in upper part of the figure 2: the function response, f(x), is decomposed in a sum of contributions given by each

stochastic variable and each one of their interactions through the model, considered as increments with the respect the nominal response, *f*c:

$$f(x) = f_c + \sum_{i=1}^{n} dF_i + \sum_{1 \le i < j \le n} dF_{ij} + \dots + dF_{1...n}(2)$$

where n is the number of variables.

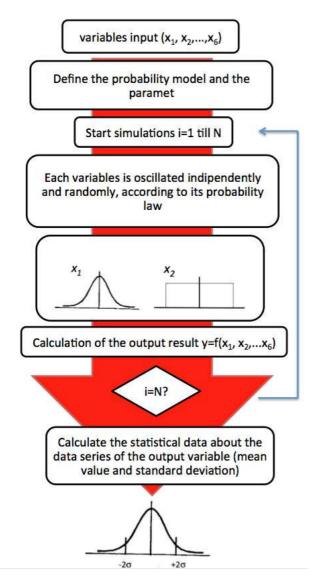


Figure 1: Block diagram of Monte Carlo Method.

Α surrogate model representation can be independently generated for each element of the sum (called "Increment Functions") and only for the non-zero elements, thus greatly reducing the complexity of sampling and building the model. Moreover, the contribution of each term of the sum the global response can be quantified to independently so that higher order interactions with low or zero contribution can be neglected already by analyzing the lower order terms.

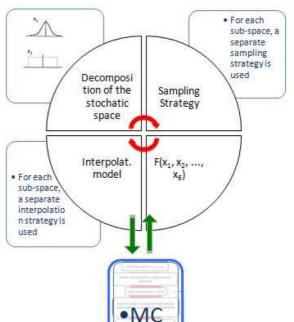


Figure 2: Scheme of the AD-HDMR method.

This particular approach can be used to propagate any known standard distribution, from the classic Gaussian and uniform, to Gumbell and Landau ones.

Not only is the output of this method the (multidimensional) distribution of the quantity of interest, but also the quantification of the global contribution of each term of the sum to the global response. This feature, allows for a complete analysis of the sensitivity of the response with respect to each of the stochastic variables, as well as their interactions. Moreover in the case that the objective function should be considered as a black box, the analysis of the single contributions can give an insight into the structure of the response function.

The selection of the important stochastic domains is done in two phases, the first phase predicts the importance of each sub-design domain and the second phase neglects the sub-design domains with little importance on the interpolation process. The prediction approach compares the importance of each stochastic variable and predicts the importance of its combination, i.e. interaction. The nonimportant interactions of variables are then neglected and the adaptive sampling interpolation process starts.

The adaptive approach is based on an adaptive sampling technique, which compares the interpolation process in each iterative step. The position of a new sample is then given by the largest difference between these two interpolations, where the difference is computed as the change of a shape of the selected interpolation technique. The selected interpolation technique is the so called Multisurrogate adaptive technique, which is able to interpolation combine and exploit various techniques. The convergence process of the adaptive part is based on the observation of the statistical properties of the weight function propagated through the interpolation technique. Being the weight function the input distribution in the case of the UQ propagation, the change of the expected value and the standard deviation of the weight function assures that the model is accurate around the area of interest.

The Multi-surrogate Adaptive interpolation technique is particularly suited when the behavior of the underlying function is unknown a priori. In this case, it is very difficult, or impossible, to say which of several one the available interpolation/approximation technique should or could be used, and the Multi-surrogate Adaptive method is able to select the best interpolation techniques for the given problem, based on the features of the function. The interpolation/approximation techniques already implemented include polynomials, RBF, kriging and splines.

The three methods are implemented for 20 cases in Table 4, according to the values of the uncertainties and the distribution function of the individual input variables (Table 3) determined following the ENV 13005:1999.

Table 3: Metrological Characteristics of The input Variables and Corresponding Distribution Type ISO 7730:05 in PMV Classification

Microclimate and subjective variables with uncertainty and corresponding distribution						
SymbolsStandardParameter(units)Deviation						
Metabolic rate	M (W/m ²)	±15%	Rectangular			
Static clothing insulation	Icl (m ² K/W)	±15%	Rectangular			
Vapour partial pressure	Pa (Pa)	±300 Pa	Gaussian			
Average radiant temperature	tr (°C)	±4°C	Gaussian			
Air temperature	ta (°C)	±1°C	Gaussian			
Absolute air velocity	Va (m/s)	2*(0,05+ 0,05Va)	Gaussian			

Table 4: 20 cases of simulation. The expected PMV
value, calculated through the equation (1), in red
color. "w" and "s" indicate the two weather
combination (winter and summer respectively), and
RH the relative humidity.

Case	RH	Season	Icl	Va	М	ta=tr	Pa	Exp. PMV
	[%]		[m ² K/W]	[m/s]	[W/m2]	[°C]	[Pa]	[•]
1	30	W	0.132	0.1	59.4	19.21	668.5	-0.6
2	30	W	0.132	0.1	59.4	20.43	721.1	-0.35
3	30	W	0.132	0.1	59.4	22.1	798.9	0
4	30	W	0.132	0.1	59.4	23.74	882.3	0.35
5	30	W	0.132	0.1	59.4	24.91	946.4	0.6
6	30	S	0.066	0.12	59.4	23.37	862.9	-0.6
7	30	S	0.066	0.12	59.4	24.22	908.1	-0.35
8	30	S	0.066	0.12	59.4	25.4	974.4	0
9	30	S	0.066	0.12	59.4	26.57	1044	0.35
10	30	S	0.066	0.12	59.4	27.4	1096	0.6
11	70	W	0.132	0.1	59.4	18.32	1475	-0.6
12	70	W	0.132	0.1	59.4	19.46	1584	-0.35
13	70	W	0.132	0.1	59.4	21.04	1747	0
14	70	W	0.132	0.1	59.4	22.59	1920	0.35
15	70	W	0.132	0.1	59.4	23.69	2053	0.6
16	70	S	0.066	0.12	59.4	22.54	1915	-0.6
17	70	S	0.066	0.12	59.4	23.35	2011	-0.35
18	70	S	0.066	0.12	559.4	24.48	2152	0
19	70	S	0.066	0.12	59.36	25.59	2299	0.35
20	70	S	0.066	0.12	59.36	26.38	2409	0.6

4 Results

Performing the 3 different simulation, it is possible to notice that the obtained Standard deviation is quite similar between the 3 models (table 5), but the best achievement is the reduction of the number of calls of the Adaptive MonteCarlo model, and even more those of the Adaptive Derivative High Dimensional Model Representation respect to the fully MonteCarlo simulation (Table 6). The number of calls of this last method is 4 orders of magnitude lower, allowing a relevant reduction of the computational costs. This approach can be very useful for the real time comfort prediction applied to control system [24] or also for more complex problems.

Table 5: Standard deviation of the 20 cases of simulation.MC-1e⁶ is the fully MonteCarlo method, MC-Adapt is the adaptive MonteCarlo and AD-HDMR the Adaptive Derivative High Dimensional Model Representation.

	MC-1e ⁶	MC-Adapt	AD-HDMR
Case	Standard deviation	Standard deviation	Standard deviation
1	0.5183	0.5176	0.5188
2	0.5033	0.5032	0.5080
3	0.4852	0.4843	0.4911
4	0.4707	0.4687	0.4769
5	0.4625	0.4604	0.4685
6	0.6747	0.6746	0.6721
7	0.6636	0.6630	0.6612
8	0.6500	0.6495	0.6496
9	0.6390	0.6381	0.6377
10	0.6329	0.6311	0.6316
11	0.5223	0.5217	0.5185
12	0.5072	0.5065	0.5084
13	0.4886	0.4876	0.4947
14	0.4731	0.4713	0.4794
15	0.4641	0.4622	0.4706
16	0.6783	0.6781	0.6750
17	0.6668	0.6662	0.6641
18	0.6525	0.6493	0.6515
19	0.6408	0.6402	0.6391
20	0.6339	0.6332	0.6326

Table 6: Number of calls of the three methods for the					
20 cases.					

Case	MC-1e6	MC-Adapt	AD-HDMR
1	1E+06	33800	75
2	1E+06	43900	74
3	1E+06	34200	72
4	1E+06	17000	74
5	1E+06	17000	74
6	1E+06	48800	73
7	1E+06	12000	74
8	1E+06	44300	80
9	1E+06	39700	74
10	1E+06	32600	74
11	1E+06	33800	76
12	1E+06	33800	75
13	1E+06	33800	81
14	1E+06	17000	74

15	1E+06	17000	74
16	1E+06	48800	73
17	1E+06	12000	74
18	1E+06	15200	80
19	1E+06	44300	74
20	1E+06	44300	71

In the figure 3 the distribution of the increment function for the six different variables is shown. This quantity is relevant because is related to the importance of the variables in the variation of the PMV, and in particular wider is the curve, more important is the variable. In particular the random variable t_r has the biggest influence on the final input in terms of uncertainty. Its high variance means that this variable creates the largest uncertainty on the final output. The partial mean of this variable is negligible, meaning that this variable brings only uncertainty into the final output and an increase in accuracy of the input (decrease of the input uncertainty) does not influence the final expected value. The curve confirms that the input distribution is a Gaussian one and the underlying function is mainly linear. Instead the random variable V_a has a small influence on the final uncertainty and its influence on the final uncertainty is negligible. Decrease in the input uncertainty would not affect the final uncertainty. On the other hand, the partial mean has a large influence on the global mean. The shape of the curve show that the underlying function has high steep gradient around the central point and flat region close to it. This suggests that near the central point there is a discontinuity (due to the non significant negative velocity values). Moreover, due to a small number of samples on the right side of the figure, the curve confirms that the input distribution is Gaussian. This random variable can be used to increase or decrease the final expected value.

Finally in the figure 4 the boxplot of the statistical distribution obtained with the Fully MonteCarlo simulation is shown. It can be observed that the PMV value and consequently the comfort prediction is very affected by the accuracy of the input variables. Starting from input quantities that generate a PMV index completely inside a comfort zone, applying the UP it is not difficult to reach the adjacent region or even more with the first or third quartile of the distribution.

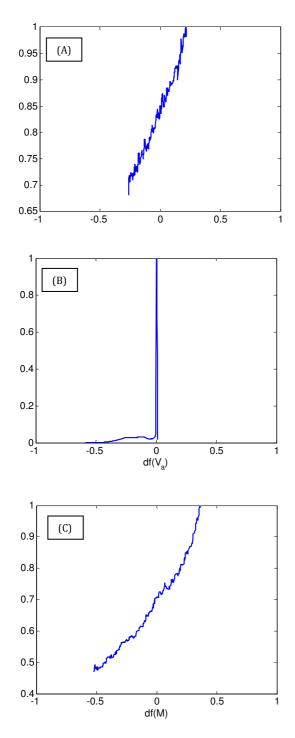


Figure 3: Distribution of the increment function for the variables: (A) Static clothing insulation, (B) Absolute Air Velocity, (C) Metabolic Rate. The ordinate value is the number of sample normalized for the maximum value.

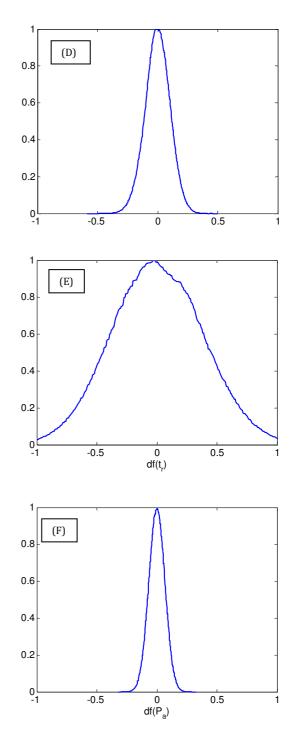
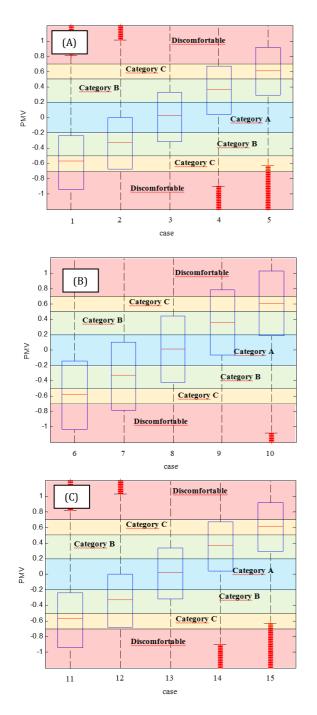


Figure 4: Distribution of the increment function for the variables: (D) Air temperature, (E) Average radiant temperature, (F) Vapor partial pressure. The ordinate value is the number of sample normalized for the maximum value.

As shown in the figure 4, this behavior is more relevant in summer than in winter, and it is less affected by the relative humidity. The PMV model is consequently to restrictive for professional use and need a revision.



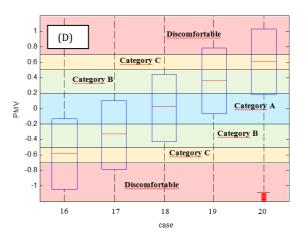


Figure 4: Boxplot of 20 the cases. (A) Cases from 1 to 5 with RH = 30% and winter season (B) Cases from 6 to 10 with RH = 30% and summer season (C)

Cases from 11 to 15 with RH = 60% and winter season (D) Cases from 16 to 20 with RH = 60% and summer season.

4 Conclusion

In the final evaluation of the uncertainty of a function dependent on several variables, the Monte Carlo method is one of the two methods proposed in the UNI ENV 13005. In particular, this method, unlike other methods, allows to simulate the variation of all the function independent variables through their mean value, its uncertainty and its distribution whatever the mathematical model used. This leads to the knowledge of the same parameters of the unknown variable and can do best considerations on the results. Also a new method named Adaptive Derivative High Dimensional Model Representation was compared with the other two to perform uncertainty propagation,

These models are applied to the computation of the PMV index under 20 different cases, the results are quite similar in all the simulation condition, but comparing the computational costs if the adaptive MonteCarlo simulation id 10^2 times faster than the fully MonteCarlo model, and the HDMR even more (10^4 times).

Another important consideration can be done analyzing the PMV statistical distribution, the classification according to the ISO 7730:05 is too restrictive for professional use, and the comfort prediction is strongly affected by the accuracy of the input variables so a revision of this classification is desirable, in particular the estimation of the PMV is strongly affected by the radiant temperature, an higher precision in the estimation of this parameter is recommended.

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