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Fuzzy Decision Making System and the Dynamics of Business Games

by

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Declaration of Authorship

I, Festus Oluseyi Oderanti, declare that this thesis titled, 'Fuzzy Decision Making System and the Dynamics of Business Games' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.

- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.

- Where I have consulted the published work of others, this is always clearly attributed.

- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.

- I have acknowledged all main sources of help.

- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

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Abstract

Effective and efficient strategic decision making is the backbone for the success of a business organisation among its competitors in a particular industry. The results of these decision making processes determine whether the business will continue to survive or not. In this thesis, fuzzy logic (FL) concepts and game theory are being used to model strategic decision making processes in business organisations. We generally modelled competition by business organisations in industries as games where each business organization is a player. A player formulates his own decisions by making strategic moves based on uncertain information he has gained about the opponents. This information relates to prevailing market demand, cost of production, marketing, consolidation efforts and other business variables. This uncertain information is being modelled using the concept of fuzzy logic.

In this thesis, simulation experiments were run and results obtained in six different settings. The first experiment addresses the payoff of the fuzzy player in a typical duopoly system. The second analyses payoff in an n-player game which was used to model a perfect market competition with many players. It is an extension of the two-player game of a duopoly market which we considered in the first experiment.

The third experiment used and analysed real data of companies in a case study. Here, we chose the competition between Coca-cola and PepsiCo companies who are major players in the beverage industry. Data were extracted from their published financial statements to validate our experiment. In the fourth experiment, we modelled competition in business networks with uncertain information and varying level of connectivity. We varied the level of interconnections (connectivity) among business units in the business networks and investigated how missing links affect the payoffs of players on the networks.

We used the fifth experiment to model business competition as games on boards with possible constraints or restrictions and varying level of connectivity on the boards. We also investigated this for games with uncertain information. We varied the level of interconnections (connectivity) among the nodes on the boards and investigated how these affect the payoffs of players that played on the boards. We principally used these experiments to investigate how the level of availability of vital infrastructures (such as road networks) in a particular location or region affects profitability of businesses in that particular region.

The sixth experiment contains simulations in which we introduced the fuzzy game approach to wage negotiation in managing employers and employees (unions) relationships. The scheme proposes how employers and employees (unions) can successfully manage the deadlocks that usually accompany wage negotiations.

In all cases, fuzzy rules are constructed that symbolise various rules and strategic variables that firms take into consideration before taken decisions. The models also include learning procedures that enable the agents to optimize these fuzzy rules and their decision processes. This is the main contribution of the thesis: a set of fuzzy models that include learning, and can be used to improve decision making in business.

Acknowledgements

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0.1 Publications

The following articles have been been published (with some submitted) for publication:

1. **Festus Oluseyi Oderanti and Philippe De Wilde**, Dynamics of Business Games with Management of Fuzzy Rules for Decision Making, 2010, *International Journal of Production Economics*, **128(1):96-109, 2010.** ScienceDirect Link: http://dx.doi.org/10.1016/j.ijpe.2010.06.002

2. **Festus Oluseyi Oderanti and Philippe De Wilde**, Automated Fuzzy Decision Making System with Learning in Business Networks of Varying Connectivity, *the 2009 UK Workshop on Computational Intelligence, University of Nottingham England, (UKCI 2009), 7th-9th September 2009* pp. 31–36.

3. **Festus Oluseyi Oderanti and Philippe De Wilde**, Fuzzy Game Approach to Investigate Impact of Poor Infrastructures on Business Profitability in Developing Economies, *Proceedings of the IEEE International Conference on Emerging and Sustainable Technologies for Power and ICT in a Developing Society, Abuja, Nigeria (NIGERCON 2010), 17-19th June 2010*, pp. 131–145.

4. **Festus Oluseyi Oderanti and Philippe De Wilde**, Automatic Fuzzy Decision Making System with Learning for Competing and Connected Businesses, *Expert Systems With Applications*, **Submitted on Apr 10, 2010**.

5. **Festus Oluseyi Oderanti and Philippe De Wilde**, Automated Game Approach to Wage Negotiation and Decision Problems. To appear in *IEEE-NAFIPS 2011 - Annual Conference of the North American Fuzzy Information Processing Society (NAFIPS- 2011),El Paso, Texas, March 18-20, 2011*

6. **Festus Oluseyi Oderanti and Philippe De Wilde**, Wage Increment Problems and Fuzzy Inference Solutions. To appear in *IEEE-NAFIPS 2011 - Annual Conference of the North American Fuzzy Information Processing Society (NAFIPS- 2011),El Paso, Texas, March 18-20, 2011*

This work is dedicated to the Glory of God Almighty through his Son and Saviour, Jesus Christ....

Chapter 1

Introduction

Decision making processes are generally an integral part of our everyday lives. In every situation, we make one or more decisions regarding what to do, how to do it, what not to do. In order to make a decision, we choose from among available options to take actions. However, how to select a proper action when facing other agents is quite unclear [2].

A lot of researchers have used game theory to model various decision processes in firms [3, 4, 5, 6, 7, 8, 9, 10, 11, 12], in military [13, 14].

Many authors have used fuzzy logic concepts to analyze various decision making processes [13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. For instance, Ngai and Wat in [29] described a fuzzy decision support system in e-commerce (EC) development for the purpose of assessment of risk. In the paper, they designed and developed a model which is web based and that can assist an e-commerce project planner to identify some of the risk factors as well as project risks in their corresponding e-commerce projects.

In [27] Ding and Liang extended the concepts of fuzzy logic applications by proposing a model that involves the utilization of fuzzy set theory fundamental principles to analyse and consider a multiplicity of complex criteria and then make decisions on the most suitable partner in strategic shipping alliances in shipping industry. The project developed a practical model for business purposes that used the membership functions features in fuzzy set theory [30, 31] that suitably set the definition, conversion and treatment of vague and multi-level criteria in liner shipping. They argued that using the model would help a shipping business decision-maker

in identifying and recognising effective partner selection criteria prior to forming a joint service pact in liner shipping industry.

Jose Naranjo et al in [32] illustrated a model that implements fuzzy inference systems in designing a system that mimics human decisions while driving on roads and the model automatically controls vehicles. The prototype was able to automatically adjust speeds, select routes and is able to make decisions in performing some other more sophisticated tasks such as maneuvering and overtaking.

Philippe De Wilde in his work in [33] extended the applications of fuzzy logic in making micro-economic decisions by studying the rationality of fuzzy choice and introduced fuzzy constraints. This framework for fuzzy-decision making was different from previous attempts in that he showed how this could be easily combined together with maximizing a fuzzy utility. He then implemented fuzzy Cournot adjustment, defined equilibria and also studied their stabilities.

Also, some authors have applied these fuzzy logic concepts on different types of games [13, 15, 24, 25, 34, 35, 36]. For example, Borges et al, in [15] extended the two moves that are conventionally possible in traditional iterated prisoner's dilemma[37, 38] with the aid of fuzzy sets.

However, for the first time, we are combining the concepts of game theory [39, 40] and fuzzy logic theory to address the uncertainties of anticipated or prevailing market demand information [41] and production cost with respect to competition through commodity price. We are finding how a firm should appropriately respond to them so as to maximize its profit and position itself in a competitive advantage over its competitors. In other words, we are using the theories of games and fuzzy logic to model decision making processes of firms in industries.

Moreover, the famous laws of demand and supply, which can be found in many works such as [42, 43, 44], have explained the behaviours of consumers and firms with response to market prices of commodities. However, these laws treat market demands in the form of information that is readily available with sharp distinction and certainty. In the real world, the situation and conditions of the markets are not always known with certainty as according to [43], "the assumption of perfect knowledge is an unsatisfactory one in economics and to assume full knowledge of future profit streams seems particularly unsatisfactory". Therefore, a firm needs to base its strategic decisions on this uncertain (fuzzy) information available at its disposal. The law provides answers to what happens in the market when a

commodity demand takes on the two crisp variables *higher than a or lower than a* for a constant *a*. It does not tell us what would happen if the demand takes on uncertain or fuzzy variables, such as *very low, low, medium, high, very high*, and how a firm should strategically respond to them.

Taking decisions that solve the problems in this kind of situation is the essence of our research. We are designing an algorithm (simulation) which combines the concepts of game theory and fuzzy decision making systems [22, 45] to model the scenario described above, in which a firm makes strategic moves in marketing [46] based on uncertain market information [41] available to the firm and on strategy moves of those of its competitors.

Also, the laws of demand and supply explain how firms take decisions based on the market information only but in the real world, a firm rarely bases its decisions on market demand information only. Rather, it takes into consideration some factors internal to itself [47] such as cost of production which may include cost of raw materials, logistics, research and development. Therefore, our research also attempts to address this issue. That is, our algorithm analyses how a firm can successfully fix the price of a commodity by taking into consideration the uncertain market information as well as some factors that are internal to the firm such as cost of production. This is with the aim of making strategic moves that will enable the firm to maximize its payoffs through maximization of its market share as well as profits. These can be seen as an attempt to redefine the law of demand and supply in a more practical way as applicable to firms' competition in industries.

This research aims at developing an efficient decision support scheme simulated in the form of a non-cooperative zero-sum game with imperfect information, using fuzzy logic concepts that can assist a business organization in making an effective decision in a competitive market environment. We used a general illustration to describe the model and we verified the validity of our results with a case study using Coca-Cola and PepsiCo companies who are major players in the beverage industry. The thesis extends knowledge in the area of decision support functionalities through extension of methods for modeling underlying functionalities of fuzzy logic and game theory concepts. It also supports decision making processes in economics, measures impacts on individual users, multiparticipant users, organisations and in evaluating the fuzzy decision support system. In accordance with the aim of classical economists, our interests are concerned with answering questions of how agents in a market could interact so as to gather maximum monetary

wealth (profits) for themselves. This will mainly be based on a decision making scheme developed in [13]. The payoff of the game relies on the concept of theory of fuzzy moves (TFM) in which, according to Kandel and Zhang in [24], a player not only strives to take a strategy that is advantageous to himself but that is also at the same time, disadvantageous to his opponents.

1.1 Objectives

Our main objectives are:

- To advise the management of a business organization on certain marketing strategic decision policies that will keep the business in a strategic advantage over its competitors in the market.

- To investigate how a firm can successfully compete with its peers in the market by determining how much of its resources or efforts should be dissipated on our three adopted strategies of marketing: consolidation efforts (C), reserved resources/wealth/capital (W) and marketing aggressiveness (M) in such a way that its profit (accumulated wealth A_w) will be maximized.

- Given the uncertain (fuzzy) and prevailing market demand (D) information, the cost of producing a commodity (C_P), and considering the traditional laws of demand and supply, to find out what strategy $[C, W, M]$ a firm should adopt to maximize its payoffs and minimize those of its competitors.

- To provide trained and optimized fuzzy rules that establish the relationship between demand (D), production cost (C_P) as well as those marketing strategies above (i.e. $[C, W, M]$) that an entrepreneur can follow in forecasting the selling price of a commodity and thereby, the profit or wealth to be generated or accumulated (A_w).

- To investigate the validity of the research via a case study using real data of known companies.

- To analyse interaction and competition among networked business organisations.

- To examine various network characteristics [48] such as level of interaction or connectivity, number of nodes (players), location of a player with respect to those of his opponents, strength of individual player's strategy and those of his immediate opponents. We will investigate how these characteristics affect the payoff of players in a business network.

The board game results are further used to investigate the following:

- How the level of availability of vital infrastructure such as transportation in a geographical location can affect the profitability of business enterprises.

- To investigate situations where there are constraints imposed by regulatory authorities such as when two or more players are forbidden (possibly by law) from interacting to prevent collusion. This leads to constrained optimisation. Constraints can be between variables, or can be constraints imposed on communication between players.

- Why industries tend to concentrate more in highly developed locations than in less developed ones.

- Why developing nations are less attractive to industrialists [49] when compared to developed ones.

- How fuzzy reasoning or fuzzy inference systems (FIS) can help to improve the performance of businesses in an environment that is clouded with uncertainty and adverse conditions such as low level of infrastructural development.

- How performance of these business enterprises can be improved or enhanced through adaptation or learning of the fuzzy rules.

The fuzzy game approach to wage negotiation simulations are used to investigate the following:

- How deadlocks that usually associate with wage negotiation and employment contracts can be resolved by using the concepts of fuzzy logic and game theory.

- How to facilitate smooth relationships between employers and employees with respect to wage negotiation.

1.2 List of Contributions

- For the first time, we are employing the concepts of fuzzy logic and game theory to model decision processes of firms in industries with respect to strategic competition. We are providing a model that can serve as an effective tool in the hands of a business executive that will enable him to effectively utilize the uncertain (fuzzy) and anticipated market demand (D) information, cost of producing a commodity (C_P) and other fuzzy information at his disposal to maximize his market share as well as profit in the industry.

- For the first time, competition and uncertainties in business networks and on boards are being modelled through the combination of fuzzy logic concepts and game theory.

- We have investigated situations where there are constraints imposed by regulatory authorities such as when two or more players are forbidden (possibly by law) from interacting to prevent collusion.

- For the first time, we have employed concepts of fuzzy logic and game theory to investigate and explain why industries tend to concentrate more in highly developed locations than in less developed ones.

- Why developing nations are less attractive to industrialists when compared to developed ones.

- How fuzzy reasoning could help entrepreneurs, who are operating in locations with low level of infrastructures, make effective and competitive business decisions.

- We are giving a new perspective on the common laws of demand and supply with a more practical approach which takes cognisance of the uncertain (fuzzy) nature of most information at the disposal of business decision makers.

- We are introducing the fuzzy logic concepts and game theory in managing employers and employees relationships with respect to employment contract and wage negotiation.

- We illustrating how effective learning of the fuzzy membership functions can be achieved to enable the fuzzy player achieve the set goal.

1.3 Assumptions

1.3.1 Gender and Economic terms

Throughout this thesis, we shall be using he/his or him as appropriate to represent agents of any gender. Also, since this work represents a model of a real system, some of the economic terms and formulas used in this research such as demand (D), cost of production ratios, strategy variables [C W M], modelling equations, other variables as well as the fuzzy rule base may be modified by anybody adopting the model to suitably represent the situation in question. What we are trying to show is that the uncertainty in business environments can be suitably modelled or represented using fuzzy logic and game theory concepts.

We have further explained these strategic variables and what they represent in Section 4.1 on page 63. The models can work for systems that have more strategic variables than those that we have used in the models.

In Section 7.4.1 (page 110), we have demonstrated that the variables in our models can be tailored to the business situations in the real world and therefore are not limited to those variables that we have used in designing the systems.

These models can be used as effective and efficient decision tools by business organisations that are operating in different scenarios similar to those we have described in this thesis. However, in using the models as decision tools, the entrepreneur will need to adapt, adjust and modify the variables and the decision rules to suit the situations in question as well as his business environments.

For example, rather than competing with capital resources (say £5M), the organisation's competing resources may be in terms of roles assigned to personnels in the organisation. For instance, due to persistent reduction in sales over the last few weeks, an organisation may decide to assign more personnels to the marketing department (M) and less to the operation department (C) of the organisation. The organisation will then change these roles until desirable results are attained in the business.

Therefore, the models could be used as decision tools but the variables may need to be modified to adequately represent the situations in question as we have done in chapter 5 of Cola War simulations between Coca-Cola and PepsiCo companies.

1.3.2 Sources of Fuzzy Rules

As in many applications of fuzzy rule-based systems, the fuzzy if-then rules used in our models have been solicited from human experts [50, 51]. We sought knowledge from human experts in the fields that are related to each scenario described in this thesis. For example, in wage negotiation games in chapter 9, we sought knowledge from both the employers' sides and also from those of the unions.

In all the simulations, the accuracy of these solicited rules are judged and amended by searching related data from published economic and fuzzy inference literatures such as [42, 43, 52, 53, 54].

However, various other methods have been proposed in different publications for automatically generating fuzzy *if-then* rules from numerical data. According to Nozaki et al in [50], most of these methods have involved iterative learning procedures or complicated rule generation mechanisms such as gradient descent learning methods, genetic-algorithm-based methods and least-squares methods.

Therefore in this thesis, the fuzzy rule base we have adopted in formulating the fuzzy if-then rules used in our models have been solicited from human experts [50, 51] in the related fields.

1.4 Layout or Outline of the Thesis

This thesis is structured as follows:

Chapter 1 explains the research background, chapter 2 contains the summary of the literature review and chapter 3 contains the fuzzy set theory and the optimization technique used in the research. Chapter 4 gives the research general methodology with general illustrations. Chapter 5 verifies the validity of our model that we explained in chapter 4 by using companies' real data and we used a case study of competition between Coca-Cola and PepsiCo companies who are major players in the beverage industry.

Chapter 6 examines the payoff of the fuzzy player in n-player game which was used to model a perfect market competition with many players and as an extension of the two-player game of a duopoly market which we considered in chapter 4. It

investigates how the payoff of fuzzy player is affected with increasing number of competitors.

Chapter 7 explains how we modelled competitions on business networks with uncertain information and varying levels of connectivity. There, the level of interconnections (connectivity) among business units in the business networks were varied and how their payoffs are affected were investigated.

Chapter 8, models business competitions as games on boards and we investigated how various constraints on the boards affected players' payoffs.

Chapter 9 contains work on wage negotiation which proposes how fuzzy logic and game theory concepts could help to successfully reduce problems that usually accompany wage negotiation in employers and employees relationships.

Chapter 10 highlights summaries, conclusions and the future work will intend to do at later time after this PhD programme.

1.5 Other activities during PhD Programme

The bulk of the work done during the PhD Programme is as summarised in Section 1.4.

However, during this research, a lot of time was spent on reading literature related to the work from journals, papers and text books and also, enormous time were spent on running simulations in the laboratory using both Java programming language and Matlab. Some of the literature that I read have been summarised in chapter 2 of this thesis under literature review and a comprehensive list of them are as contained in the bibliography section of the thesis as well. I also took some time to study LATEX. I attended several research seminars, workshops and conferences organised by Educational Development Unit (EDU) of the university including: how to be an effective researcher. Also, I attended all the three stages of learning enhancement and development skills (LEADS 1, 2 and 3) which trained participants to become an approved tutor of the university, research development programs (RDP) and so on.

Outside Heriot-Watt University, I also attended some seminars, workshops and conferences at the University of Edinburgh and University of St Andrews, Fife

including SICSA PhD Conference 2009 which was organised for PhD students in Informatics and Computer Science in Scotland and held at St Andrews University on 3rd June 2009.

A full length paper was published from my work by a journal; *International Journal of Production Economics*. In this paper [52], we summarised all the topics and simulations covered from chapters 1 to 6 and it was titled *Dynamics of Business Games with Management of Fuzzy Rules for Decision Making*.

Also, I equally presented conference papers on my research and these are highlighted in publications' section at the beginning of this thesis. Another full length paper has been submitted to the journal of *Expert Systems with Applications*.

I also served, very often, as reviewer of academic papers for Journals such as *IEEE Transactions on Systems, Man, and Cybernetics–Part B: Cybernetics*.

Chapter 2

Literature Review

2.1 Decision Making Processes

A *decision* is a goal-directed behaviour made by the individual, in response to a certain need, with the intention of satisfying the motive that the need occasions [55]. The decision process begins with identification of a problem and ends with a *choice*. The problem arises when a sought-after goal can be obtained via alternative and sometimes competing avenues. In every behaviour or step we take, we are involved in at least simple decisions. For example, about four years ago, I was involved in a personal decision process on whether to go for a PhD degree or to continue with my work and enjoy my salary. Then, after I had made a decision to go for PhD, I was involved in another stage of decision processes which was based on which country (Nigeria, United States, United Kingdom and others) to do the PhD. This decision stage favoured United Kingdom. After I had overcome that decision stage, then another came in, on whether to go to University of Glasgow or Heriot-Watt University both in United Kingdom and Heriot-Watt University finally became the product or *choice* of my three-stage decision process.

The decision maker is an individual at the simplest level and a decision process must have a purpose in so far as it only exists to further a particular objective or goal of the decision maker. When faced with certain problems, an individual rational decision maker will make attempts to order or rank his goals or objectives in some certain relative order. The decision maker will then be in a position to examine various alternative means in order to achieve the desired goals. He will

then choose the best strategy which either minimizes the costs of any possible failure or maximizes the set objectives to achieve the desired goals[55].

There are many theories that provide advice to an economic man and among these are economic theory and decision theory. According to Martin Shubik in [10], microeconomic theory involves the study of the optimization process for a "rational" individual decision-maker an economic man-usually modeled as though he were confronted with a completely known set of certain or probabilistic outcomes. He asserted that the individual rational decision-maker of economic theory has been, on the one hand, a singularly simple individual and, on the other, an extremely complex one. His pristine simplicity comes about in his good fortune in knowing what he wants. For an entrepreneur-owner of a firm, his principal economic decision role is to maximize profits.

Decision processes in firms have been modelled in different research papers such as in [10, 33, 56, 57, 58]. Shubik in [59], summarizes the basis of the concepts of an economic man and his near relatives in decision making processes as follows:

1. A decision is a (conscious) choice of a move (or action) from among a well-defined set of alternatives.

2. The individual decision maker can attach a value to the outcomes arising from any set of moves.

3. The individual decision maker is motivated to act in such a manner that the expected value to him of the outcome is as high as possible.

2.1.1 View of a Rational Decision Maker

The rational man school of thought asserts that decisions are made by an individual and rational decision maker that is usually consistent and having considered economic factors, is cognizant of relevant of related cost and benefit ratios. It is in the assumption of this school of thought that a decision-maker has all the information and tools required for implementing and making a decision. This school also depends on assumption of an ideal situation that is not always available in the dynamic and business world. Hossein Bidgoli in [60] highlighted the progressive steps that the rational actor school follows in making its classic approach to decision making processes and these steps are listed as follows:

- Definition of the problem

- Generation of the alternatives

- Evaluation of the alternatives

- Implementation of the best out of the alternatives

- Evaluation of the solution to investigate how it is working by performing a systematic follow up.

However, a decision maker is frequently confronted with fuzzy constraints, fuzzy utility maximization, and fuzziness about the state of competitors[33]. There are many decision situations when we cannot process the information contained precisely in a quantitative form but which may need to be rather accessed or processed in qualitative form and therefore, the need for us to adopt a *linguistic approach* [61] . Decision-makers in a conflict must often make often make their decisions under risk and under unclear or fuzzy information[25]. In this thesis, Section 3.5.2 on page 41 has been dedicated to decision making under uncertainty or fuzziness.

2.2 Game Theory

2.2.1 Game Theory Framework

Game theory is a method for the study of decision-making in situations of conflict and it deals with problems in which the individual decision-maker is not in complete control of the factors influencing the outcome [9].It was developed to quantify, model and explain human behavior under conflicts between individuals and public interests[62]. A decision-maker in a game faces a cross-purposes maximization problem. He must plan for an optimal return, taking into account the possible actions of his opponents. A game is a model of a situation where two or more groups are in dispute over some issues or resources[25]. A player in a game is an autonomous decision-making unit. Tapan Biswas in [63] also stated that a large part of the decision making processes under uncertainties can be covered by game theory.

Game theory is the study of the ways in which strategic interactions among rational players produce outcomes with respect to the preference (or utilities) of those players, none of which might have been intended by any of them [64]. It is part of a large body of theories concerning decision making [11]. It deals with decision-making processes involving two or more parties, also known as players with partly or completely conflicting interest [9, 25] and it is one of the methodologies designed for application to the social sciences [10]. All situations in which at least one agent can only act to maximize his utility through anticipating (either consciously, or just implicitly in his behaviour) the responses to his actions by one or more other agents are called games and agents involved in games are referred to as players [44, 64] and could represent people, military, firms, countries or other organisations [13, 14, 25].

A game can be described in terms of the game's rules, individual decision-makers or the players, outcomes of the game or the payoffs, values of the players' payoffs, players' strategies, the type and the condition of information that is available during the game. All these components can be found in all situations of conflict and are therefore the major constituents of game theory. They are all elements and building blocks of game theory.

John von Neumann and Oskar Morgenstern in 1944 invented the mathematical theory of games [64] and despite the fact that game theory has been rendered mathematically and logically systematic only since 1944, the game-approach to solving various problems can be found among commentators from the ancient times. The participants in a game are called the players. In a non-cooperative game, the possible courses of action available to the players are referred to as options. Any set of options that can be taken by a particular player is called a strategy. When each player has selected a strategy, the result is referred to as an outcome. What is essential in a game is that two or more players are involved with partly or completely conflicting interests [25].

Participants (players) in games are assumed to be rational. Classical decision theory assumes that a man that is rational would choose the optimum out of the alternatives available to him from the universe of choices.

Each player in a game is concerned with maximizing his payoff and the players therefore need to also consider the possible reactions of the opponents to his every move in order to achieve his own optimal move [63]. Since the players do not

know the moves of the opponents with certainty, they therefore need to take decisions about their moves with some levels of rational justification. The essence of decision-making under uncertainty is indeed, the search for this rational decision [63].

Games are extensively used in modelling and understanding complex behaviours in a wide range of fields including theoretical biology, social interactions, economics, politics, defense and security. In spite of their simple structures, games are successful in capturing real life complex dynamics and have proved to be a powerful tool for analysing interesting phenomena [65, 66]. Classical game theory uses the extensive form and the strategic (or normal) form to describe a game [25, 64]. Each player in a game faces a choice among two or more strategies and a strategy is a predetermined *programme of play* that tells him what actions to take in response to every possible strategy other players might use [8, 24, 44, 64]. According to Fisher in [5], bright young theorists today tend to think of every problem in game theoretic terms, including problems that are easier to deal with in other forms and every department feels it needs at least one game theorist or at least one theorist who thinks in game theoretic terms. As a result, business strategy is not left out in this context as it is mostly dominated by the game theoretic approach.

Concepts of game theory investigate individuals that have different objectives or goals which are somehow interlocked. It must be noted that not all decision-making scenarios are games in nature. For example, an accountant who has been given certain sum of money to carry out a project or an engineer who has been mandated by his supervisor to reduce an industrial design complexity in order to minimize cost. These two scenarios do not portray game situations. Both the accountant and the engineer are faced with the problems of minimization and maximization which is a field in operation research. Both the accountant and the engineer can control fully, the relevant variables that are involved in these two situations and therefore do not have any human opponents to contend with that may want to oppose, jeopardize or act in contrary of their set objectives or goals.

2.2.2 Terms in Game Theory

For any game, there are three very important requirements and these are listed as follows [63]:

1. Players

2. Strategies which are permitted with respect to the rules of the game and

3. Payoffs (that is, utilities or outcomes)

A crucial aspect of the specification of a game involves the information that players have when they choose strategies. The simplest games (from the perspective of logical structure) are those in which agents have *perfect information* [67], meaning that at every point where each agent's strategy tells him to take an action, he knows everything that has happened in the game up to that point. A board game of moves in which both players watch all the actions (and know the rules in common), such as chess, is an instance of such game [8, 64]. In contrast, games in which players do not know everything that has happened in their games up to that point when they take actions, are referred to as with *imperfect information*. A games may also be distinguished based on the *order of play* that is, based on when or the *order* with which players choose their strategies with respect to those of the opponents. With respect to this, games are classified into *sequential-move* games and *simultaneous-move* games. We explained this as follows: consider two firms that are planning marketing campaigns, one of the firms might have allocated to its strategy some months or weeks earlier and if neither knows when the other (competitor or opponent) has allocated to its strategy or will allocate to its strategy (that is, when the campaign decision will be made), then such game is referred to as *simultaneous-move* games. In a sequential game however, such as in a chess game, players see what their opponents have done before taking their actions and these types of games are therefore referred to as *sequential-move* games [64].

With the two concepts described above, one may be thinking that the distinctions between games of *imperfect information* with that of *perfect information* and the games of *sequential-move* games with that of *simultaneous-move* games are the same. However, it is true that all games of *simultaneous-move* are also games of *imperfect information*. In some cases however, some games may be observed to contain mixed traits of *sequential-move* games and that of *simultaneous-move* games. For examples, if two competing firms allocates to their marketing campaign strategies without the knowledge of each other, then after that initial allocation, if they then engage in price competition in full view of each other, then we will need to analyze these two stages as a single game that has a stage of *simultaneous-move* followed by a stage of *sequential-move*. Therefore, if a game comprises

combined stages of this kind, then such game is referred to as a game of *imperfect information.*

On the other hand, for a game to be classified as that of *perfect information*, there must be no stages of *simultaneous-move*. That is, all players know and remember what have happened before and at each stage of the game.

Games of *perfect information* are simple to analyse by both the players and the analysts. This is because since they are finite and stop (terminate) after a certain and known number of steps, the games can be represented by using a straightforward procedure for predicting the outcomes by both the players and the analysts. A player in such game, before choosing an action, would have considered series of counter actions or reactions from his opponent, that may result from each action open to him. The player will then consider and choose, out the available action open to him, that action that is likely to earn him highest *payoff.*

The *rules of the game* specify the complete structure of the game. They indicate the span of the alternatives faced by a player at any point during the play, his information state and the payoffs resulting from any play [8]. A *play* of a game is a path followed down the game tree. The *payoff* is the resultant allocation from the play of a game. In chess, this is the value attached to a win, loss or draw and in poker, it is money. A *strategy* is a complete plan of actions for a player [24, 44, 64]. A *move* is the selection of one among a set of alternatives at a choice point in a game. In a game in which each player has a single move and these are made simultaneously, a strategy and a move are equivalent. The players have no contingencies to plan for [8].

2.2.3 Game Representation

Neumann and Morgenstern employed two major ways of representing game and these are known as the normalized form and the *extensive form*[68]. The normalized form can be displayed by means of a *payoff matrix* while the extensive form can be displayed by means of a *game tree.* Examples are used as follows to illustrate the two forms of game representation.

2.2.3.1 Payoff Matrix

To illustrate the normalized form of game representation, let us consider a simple game which consists of two players in which each player has only one move. The moves must be made simultaneously and a player must not have knowledge of actions of each other. Each of the players must choose and select a green or yellow card. If the two players select cards of the same colour, the first player wins £1. If however, cards with different colours are selected then, the second player will win £1. In any case, any player that loses will have to pay £1 to the opponent. Figures 2.1 and 2.2 illustrate the payoff matrix for player 1 and player 2 respectively in their conventional forms.

	Yellow	Green
Yellow	£1	- £1
Green	- £1	£1

FIGURE 2.1: Payoff of player 1.

	Yellow	Green
Yellow	-£1	£1
Green	£1	- £1

FIGURE 2.2: Payoff of player 2.

We can also equivalently combine the two matrices in Figures 2.1 and 2.2 into one matrix as shown in Figure 2.3. In each cell, the first figure represents the payoff of the first player while the second figure represents the payoff of the second player.

2.2.3.2 Game Tree

The extensive form of game representation with the use of game trees can take two forms. In the strict sense of game theory, these two forms are equivalent but

	Yellow		Green	
Yellow	£1	- £1	- £1,	£1
Green	- £1	£1	£1	- £1

FIGURE 2.3: Matrix showing payoffs of players 1 and 2.

may be viewed psychologically as being different. The two forms are as portrayed in Figures 2.4 and 2.5. As shown in the figures, the points during which players can make their choices are represented by the vertices on the game trees. Next to each vertex are shown the numbers that indicates which of the players must make the choice. The difference between Figures 2.4 and 2.5 is that in Figures 2.4, the first player has the topmost vertex as his choice point while the second player has the other two vertices to make his choice. The situation is opposite in Figure 2.5.
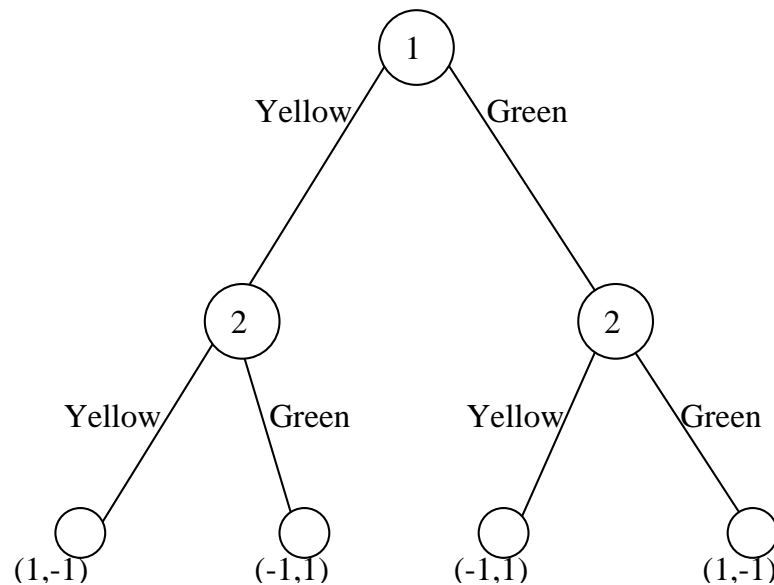
FIGURE 2.4: Game tree with first player at topmost vertex.

2.2.4   Branches and Methodologies of Game Theory

In order to understand fully, the many ways of applications of game theory in the field of microeconomics, it will be useful to highlight the major five out of many branches of game theory. According to [8, 10] these are identified as follows:

1. Theories of solution for two-person constant-sum games,
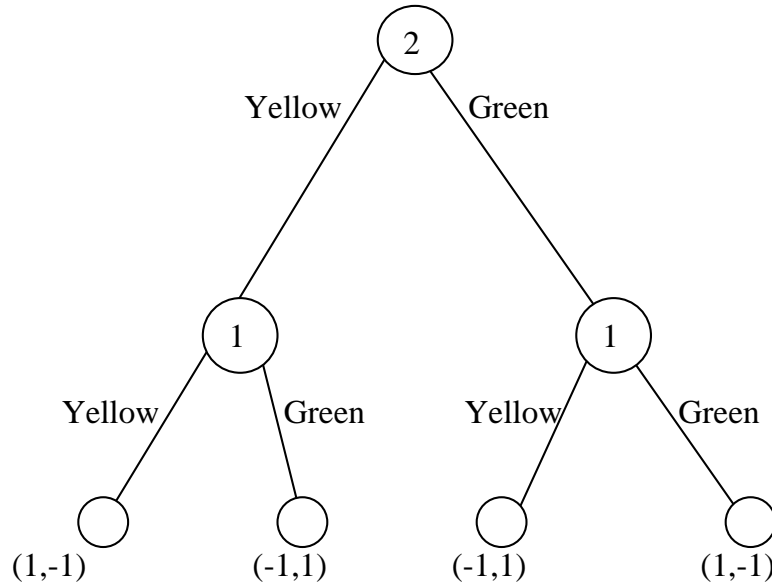
FIGURE 2.5: Game tree with second player at topmost vertex.

2. Description of the extensive form of a game,

3. Theories of solution for n-person games (where $n \geq 2$ for non-constant-sum games; $n \geq 3$ for constant-sum games),

4. Theories of solution for games against nature (games in which the rules are not completely specified),

5. Theories of solution for dynamic games.

Since there are very numerous publications on the above listed branches of game theory, we will not offer much explanation on them but we will however explain briefly on the first item on the list and highlight some of its aspects or divisions. More information on the other items on the list can be found in [8, 10].

2.2.4.1 Two-Person Constant-Sum and Two-Person Zero-Sum Games

With constant-sum games, the aggregate of players' payoff is the same with every combination of the players' strategies. In constant-sum, the payoffs of all players add up to a fixed constant *for all possible outcomes* [44]. The zero-sum game is therefore a peculiar case of constant-sum games. In two-person zero-sum game, the amount that one player wins is exactly the amount that the other player loses. Two-person zero-sum game are essentially betting games where the loss of one is

the gain of the other[63] .Examples of this include the matching pennies and two-person poker. Most of the other two-person games also belong to this category [9]. Competition between two firms may be modelled using this concepts and the payoffs of players can be represented by a *payoff matrix* described in Figures 2.1, 2.2 and 2.3 on page 18.

Games that have duel characteristics can be classified as typical direct applications of two-person zero-sum games. In a duel, one of the properties is that the goals of the players are diametrically opposed. In businesses or in any market where the size of the marked demand is somehow fixed by regulation agents such as the governments or fixed by habits, any additional customers that are gained by a particular firm will result in another firm losing an equivalent number of customers in that market.

2.2.4.2 Non-Zero-Sum Games

It must be noted that not all games are zero-sum in nature and in fact, many of the competitions that are more interesting in the market, business and economics are not zero-sum [63]. A very large market that contains many players (competitors) may not be zero-sum because instead of players fighting or opposing each other, there may be *room for all*. Example of these markets can be found in banking industries where there are so many players and the gain of one player may not necessarily affect another.

2.2.4.3 Goals of Gaming

There are different goals of gaming and in [65] few of them are identified and these are listed as follows:

- Training

- Experimentation

- Entertainment

- Therapy and Diagnosis

- Operations

- Training

Further and extensive readings on these goals of gaming can be found in [65]

2.2.4.4 Example of Popular Strategic Games

Scientists have used wide range of strategic games to analyze different phenomena or situations and common examples of these popular games are:

- Prisoner dilemma game

- Snowdrift game

- Game of chicken

- Battle of the Sexes game

- The stag-hunt game

- Free-rider game

Also, since there are many publications that have extensively discussed items on this list and other strategic games, no further explanation of them shall be offered in this thesis. Further readings can be found in different publications such as [15, 25, 26, 63, 66, 69, 70, 71, 72, 73]

2.2.4.5 Nash Equilibrium

Nash equilibrium [74] is one of the most important concepts of solution in classical game theory. It denotes an outcome at which none of the players would likely want to unilaterally depart because doing so may result in worst outcomes, or at least would not result in better outcomes that what has been earlier achieved. This may be viewed as the stable state of the game since none of the players would have any reason to defect to a different strategy if the opponent player does not defect. However, it has been argued that the rationality of moving or departing from outcomes-at least beyond an immediate departure- is not considered in this concept[25, 72].

Nash equilibrium ('NE') applies (or fails to apply, as the case may be) to whole sets of strategies, one for each player in a game. A set of strategies is a *Nash equilibrium* just in case no player could improve his payoff, given the strategies of all other players in the game, by changing his strategy [63, 64, 75].

2.3 Board Games

Since our experiments in Chapter 8 is based on board games in which we used various characteristics and constraints on boards to investigate how restrictions affect businesses, we shall give a brief introduction on board games in this chapter.

The definition of a particular game is generally considered or otherwise transparent by listing the rules of the game. Board games are games with a fixed set of rules that limit the number of pieces on a board, the number of positions for these pieces and the number of possible moves [76]. The limitations set by these rules contrast with games of skill where the number of positions may be endless. Also, in a board game, there must be indeed a board with pieces on it and moves or placement of pieces may influence the situation on a board and the pieces relate to one another on that board. This is however in contrast with most lottery games, such as roulette, where each bet or contract is commonly independent from the other contracts that have been made on the table, and by definition, are not moving around the board. A die in a board game such a ludo, shown in Figure 2.6 from [77], limits the movement of pieces on the board.

Board games have intrigued researchers in a number of sciences either as object of study or as models for developing analogies [76]. This is because unlike other games, board games present more opportunities for thinking, memory, and studying perceptions. Like other games, all board games require players who are mostly two. This characteristic sets board games apart from puzzles which usually involve one player.

2.3.1 Classification of Board Games

Board games have been classified by many authors based on the purpose of the game. For instance, war games require captures while players in race games race each other to reach the end of the board.

There are many variants of board games and these are not all as intellectually as demanding as chess or Go. Many board games involve two players, and are deterministic (no random element such as dice), and they also provide full information about the game's state to each player. Most board games do not also have hidden elements such as cards in the opponent's hand as we have in card games [78]. Research on board games can be found in numerous sources ranges from journals on psychology, cognition to historical works on board games.

An overview of board games as it is used and understood in a particular discipline exists for the field of artificial intelligence and computer science and such an overview was long ago provided for historical research [76]. The research on chess players has so far been generalised to several other domains of expertise and the domain of board games has received attention in its own right from other disciplines. There are board games such as Go, gomoku, bao and awele that have enabled comparative studies that put theories of cognition in different cultural contexts. Meanwhile, these areas of studies might not have been possible without the increasing interest in board games as another area of study [76]. Board games, most especially, Go, checker and chess have often been used to investigate and illustrate emergent theory which studies how complex behaviour emerges from simple components.

2.3.1.1 Description of the Chess Game

We shall give a brief description of the chess game as a sample of board games while full description on it and other games can be found in [76, 78, 79].

The conventional chess is made of $8X8$ board and the objective of the game is to checkmate or capture the opponent player's king. At the beginning of the game, the arrangements of the white pieces follow the following order on the first row: Rook, Knight, Bishop, Queen, King, Bishop, Knight, and Rook. On the second row, the eight white Pawns are then arranged. For the player black (second player), the arrangement of his pieces follow the same pattern on rows eight and seven respectively.

On the players' movements, the Bishops move in diagonal while the Rooks move horizontally and vertically in straight lines. The Queen combines the movements of both the Bishop and the Rook while the King can move one square in a direction.

Knight will first move horizontally by one square and vertically and it will then move one square ahead diagonally. Knight happens to be the only piece that can jump over pieces on the board. For all pieces, a piece is captured when a move ends on a square that is occupied by an opponent's piece. Pawns move one square forward but can also move two square forward from their starting point and can capture one square diagonally. There are special rules such as castling in which both the King and the Rook can move, taken enpassant (this happens when Pawn can be taken as if it had moved only one square when it has actually moved two squares from its starting location), and stalemate which is a game draw condition in which one side cannot move but is not however in a check.

Figure 2.7 on page 32 illustrates the picture of a conventional chess board while Figure 2.8 on page 32 illustrates fuzzy chess board as designed by Professor De Wilde. For instructions on how to play the fuzzy chess game, reader should please go to Professor Philippe De Wilde channel on Youtube and to play the game, please go to the following link on his home page:

(http://www.macs.hw.ac.uk/ pdw/fuzzychess/fuzzychess.html).

Another popular example of board games is *Ayo-Olopon* also know as *Oware*, *Awele*, *Mancala*, *Adji-Boto* and many more names. A typical *Ayo-Olopon* game is as shown in Figure 2.9 on page 33. Figure 8.1 on page 125 show the author of this thesis playing the board game with his wife *Adesola* in the computer laboratory to investigate his research results. Other popular board games are *Ludo* games shown in Figure 2.6 on page 32 and *Nine Men's Morris* shown in Figure 2.10 on page 33.

2.4 Business Games

2.4.1 Why Business Games?

Business gaming and case studies are commonly used in training and education in both business schools and companies. In a business school, the objective is to let the students know practical knowledge. On the other hand, the purpose of corporate training is to improve the behaviors and attitudes of employees in a company [80]. It is one of the educational techniques to train skills for managerial decisions within a limited time under the virtual business environment.

In business games, the firm identifies the moves that the rival could make in response to each of its strategies. The firm can then plan counter-strategies [42]. As Doug Ivester, Coca-Cola's president put it [43] "I look at the business like a chessboard. You always need to be seeing three, four, five moves ahead; otherwise, your first move can prove fatal". Game theory helps explore the impact of calculations about future market advantages on a firm's current market strategies.

In business games, the conflicting interest of a firm may be to minimize the cost function, maximize the market share, or maximize the profit [25]. In this game, profit maximization of the fuzzy player is to be achieved through learning by the fuzzy agent, and minimization of the payoffs of the opponents.

Game theory has had a deep impact on the theory of industrial organization. The reason it has been embraced by a majority of researchers in the field is that it imposes some discipline on theoretical thinking. It forces economists to clearly specify the strategic variables, their timing, and the information structure faced by firms. As is often the case in economics, the researcher learns as much from constructing the model (the "extensive form") as from solving it because in constructing the model one is led to examine its realism. Is the timing of entry plausible? Which variables are costly to change in the short run? Can firms observe their rivals' prices, capacities, or technologies in the industry under consideration? and so on [5].

Many authors have attempted to describe business games in different contexts. In [81], the author discussed the use of business game simulations as tools for teaching Information Systems. He argued that even though, the traditional teaching method may be useful for the foundational knowledge dissemination, but they do not provide the students, the platform that is optimal for implementing the IS concepts. The author acknowledged the effectiveness of business games simulations in designing Decision Support Systems (DSS) [82] and highlights his works which consists of a game he designed to engage students in decision-making systems that involve entrepreneurial decisions. However, this paper did not capture the uncertainties that surround the business environment and it mainly focused on teaching students rather than focusing on the business decision makers [83] which our work has thus addressed.

Martin Shubik in his paper [84] stated that the most common types of teaching games in existence are business games. Several definitions of business games were

offered in the paper [84] and few of them are as highlighted below:

- A business game is a contrived situation which imbeds players in a simulated business environment, where they must make management-type decisions from time to time, and their choices at one time generally affect the environmental conditions under which the subsequent decisions must be made. Further, the interactions between decision and environment are determined by a refereeing process which is not open to argument from the players.

  The statement concerning the refereeing process presents a factor which differentiates teaching from operational gaming.

- A business simulation or game may be defined as a sequential decision-making exercise structured around a model of a business operation in which participants assume the role of managing the simulated operation.

The relevant features of an organisation and its environment can be simulated using business games. In playing the game, the manager, the businessman, entrepreneur or others involved in the decision-maker making process may be required to make decisions in a very short period of time. The business gaming model will then portray the following characteristics:

1. A description of the internal features of the organisation or firm to be considered.

2. The firm's environment. This may comprise the customers, state of the economy, the market structure, and other business environmental variables [85].

3. Organisation's decision set. This may comprise marketing decision variables such as advertising, production policy, pricing, employment procedures and contracts and other variables on which the decision-makers could have some direct control over.

4. Set of possible outcomes. This will be determined by the choice of strategy selected by the decision-maker together with some other environmental factors. These sets of outcomes of a business game may include metrics such as level of market or industry shares, volume of sales, profits, and other metrics which are referred to in game theory as *payoffs* of the game.

2.4.2 Decision Making Processes in a Firm

Decision makers in an organisation are expected to be aware of and to be able to assess the information they generate and the potential use (or otherwise) of that information [86]. Nowadays, decision making [87] processes are becoming increasingly very complex for managers [86]. Therefore, the information needs of a manager are becoming more complex and demanding also as a result of this increasingly complexity of the business environment in which organisations have to function. Figure 2.11 on page 34 highlights some of the major pressures that are responsible for making decision making processes increasingly problematic in business environments [86].

For decisions to be adequately made in a firm [88], decision makers of the firm are assumed to have access to three different types of information; product-demand information, factor-supply information and production-technology information [44]. Under the assumptions of neoclassical marginal analysis, product-demand information usually takes one of two possible forms. Either the firm knows the prices of each of its products (and these prices are assumed to be constant) or it knows its total revenue function. According to [44], figure 2.12 below represents the common flow pattern of decision making process of a firm. In this research however, we are analysing product-demand information while we also combining production-technology information and factor supply information together as production function or cost.

2.4.3 Economic Theories of Market Structures, Demand and Supply

Demand and supply information are two of the most important market information to any firm and perhaps, the most fundamental concepts in Economics. The relationship between the two determines how resources are allocated. Demand refers to the number (quantity) of good or service is desired by buyers. Quantity demanded is the amount of a good or service that consumers are willing to buy at a certain price in a particular period of time. Supply of a commodity refers to the quantity that the market can offer. Quantity supplied denotes the amount of the good or service the manufacturers are willing to supply at a certain price.

From these, it can be inferred therefore, that price is a determinant of demand and supply [42, 44, 89, 90].

2.4.3.1 Law of Demand

The law of demand states that given that all factors remain constant, the higher the price, the lower the quantity demanded [42, 55]. This means that the consumer will demand less of a commodity at high price because as the price of the commodity goes up so does its opportunity cost. Therefore there exists at every time a particular relationship between the price of a good in the market and the quantity demanded of that good. The relationship between the quantity of a good bought and the price is what the economists refer to as *demand curve*, or *demand schedule*. For normal commodities, the demand curve will always have a negative slope. Figure 2.13 illustrates this relationship between demand and price of a commodity.

2.4.3.2 Law of Supply

The law of supply illustrates the quantities of a commodity that will be sold at a given price. However, unlike demand that slopes downwards, the slope for the law of supply goes positively upward. That is, the higher the price, the higher the quantity supplied [42, 44]. This means, the supplier will be willing to supply more at higher price so as to accumulate higher revenue and directly, a better profit. Figure 2.14 shows the relationship between price and the supply of a commodity.

2.4.3.3 Oligopoly

This is a market in which the number of firms is small enough for the behaviour of one firm to affect the behaviour of other firms in the market [44]. It is a market structure in which a few firms dominate the industry. Crucially, these firms recognise their rivalry and interdependence, fully aware that any action on their part is likely to induce counter-actions by their rivals [42]. This leads us into a consideration of strategies and counter-strategies between market participants, some of which can be modelled in terms of 'game playing' situations.

Oligopoly is a market structure that forms an intermediate between the two extremes of pure monopoly [42] and perfect market competition 2.4.3.5 [91]. There are key characteristics of oligopoly market that differentiates it from other market structure and these are listed as follows:

- Few sellers in the market with difficult entry for new entry.

- In oligopoly, products may be either homogeneous or non-homogeneous (product differentiation).

- Interdependence among firms is recognized in oligopoly competition.

- In oligopoly, prices tend to be *sticky* or *rigid*.

2.4.3.4 Duopoly

This is an extreme form of oligopoly with just two firms in the market. This is the type of market structure that the first game in this thesis (2-player game) illustrates and this leads us into a consideration of strategies and counter-strategies between market participants which are modelled in terms of 'game playing' situations.

2.4.3.5 Perfect Market

This is a market in which there are many sellers and buyers with homogeneous products and complete information about prices. In this thesis, this is illustrated in Chapter 6 as a game of multiple players (i.e. n-player game). Perfect competition is defined by the economist as a technical term and this only exist in a market where no businessman, farmer or labourer is big enough to have any personal influence on market price.

According to Griffiths and Wall in [42], there are key assumptions of a *perfect market competition* and these are listed as follows:

- Large number of buyers (purchasers): None of these purchasers must be significant enough to the extent of being able to influence the market price of the commodity by an individual purchasing decision.

- Large number of small firms: None of the firms must be significant enough by itself to influence the supply of the commodity in the market. Also, all the firms must produce identical (homogeneous) products.

- Each firm is a *price taker* on the demand curve for its product as being perfectly elastic at the going market price.

- Availability of perfect information: The price of the identical (homogeneous) product must effectively convey all the necessary information required by the buyers and the consumers.

- There must be freedom of entry into (as well as exit from) the market or industry

However, since there are many well known works on microeconomics that address those economic terms briefly explained above, no further discussion will be given of them in this thesis.
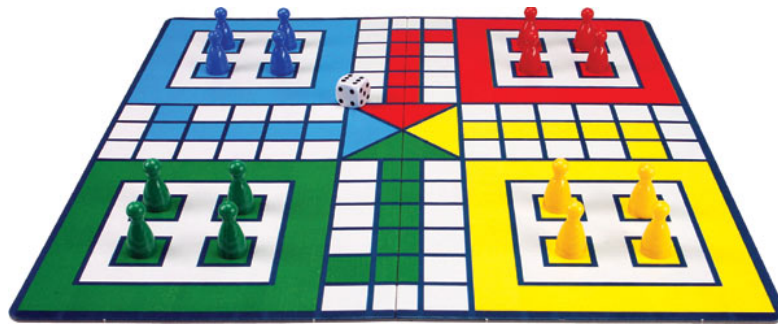
FIGURE 2.6: Ludo board game with pieces and one of the dice on the board.
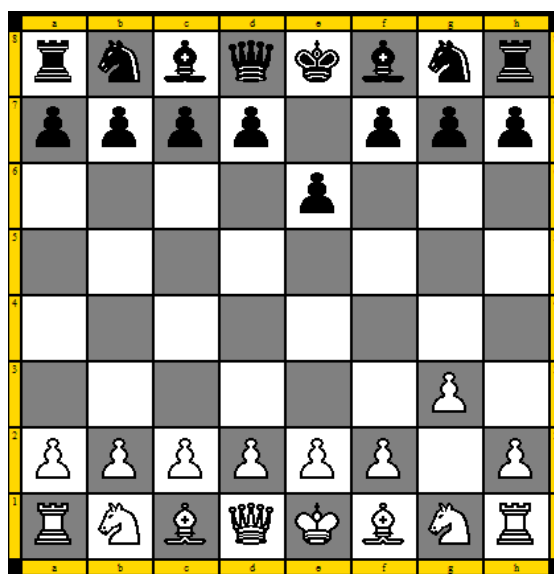
FIGURE 2.7: Chess board game with pieces shown on the board.

FIGURE 2.8: Fuzzy chess game developed by Prof. Philippe De Wilde. This uses the fuzzy inference system in making the moves on the board.
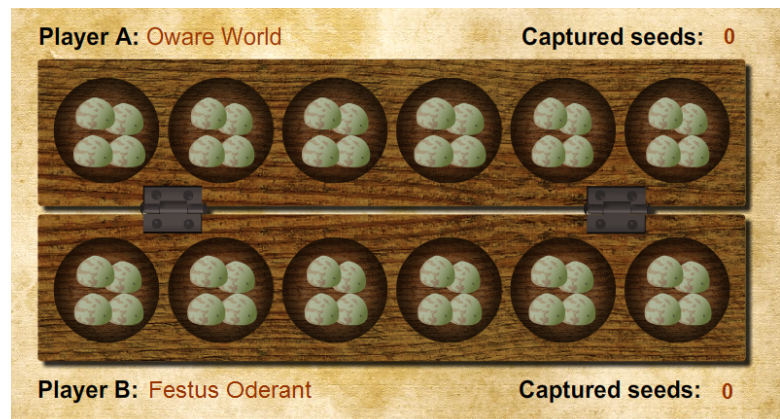
FIGURE 2.9: *Ayo-Olopon or Oware* game showing initial arrangement of seeds.
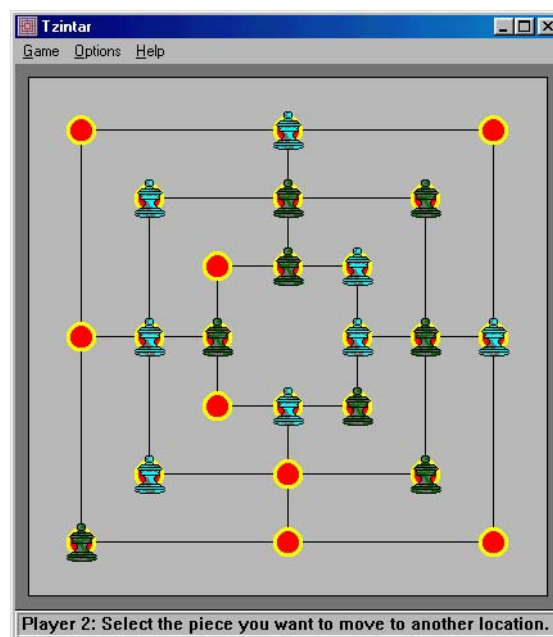
FIGURE 2.10: A board game of Nine Men's Morris .
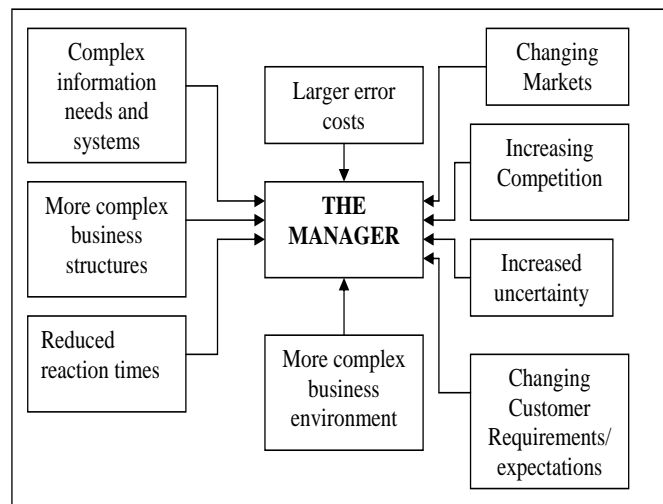
FIGURE 2.11: Decision-making environment and the manager.
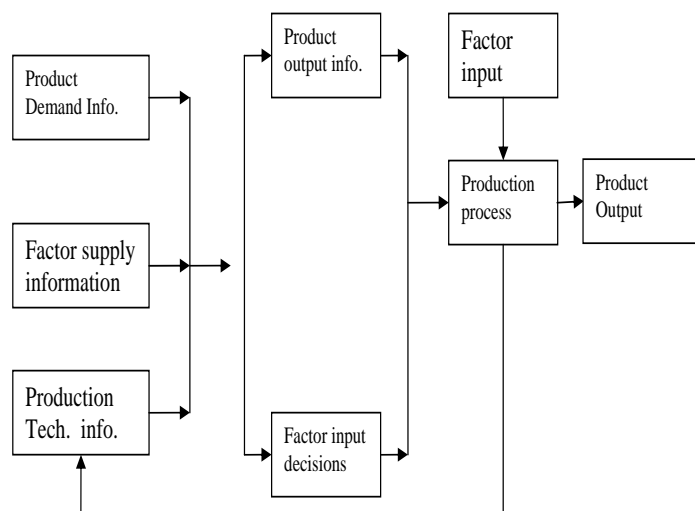
FIGURE 2.12: A flow diagram of decision process of a firm.
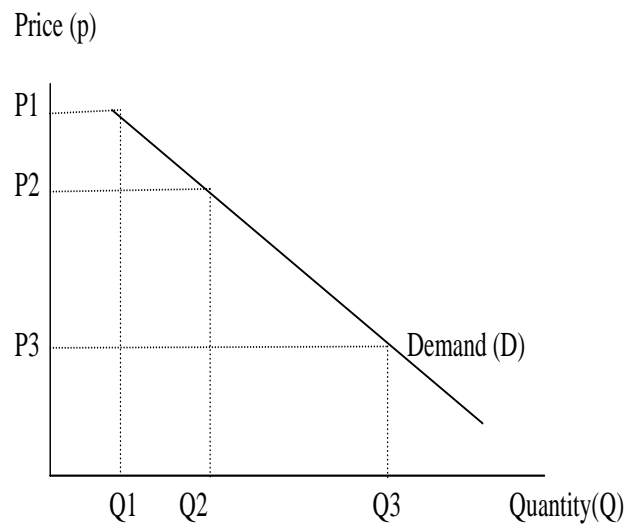
FIGURE 2.13: Law of Demand.
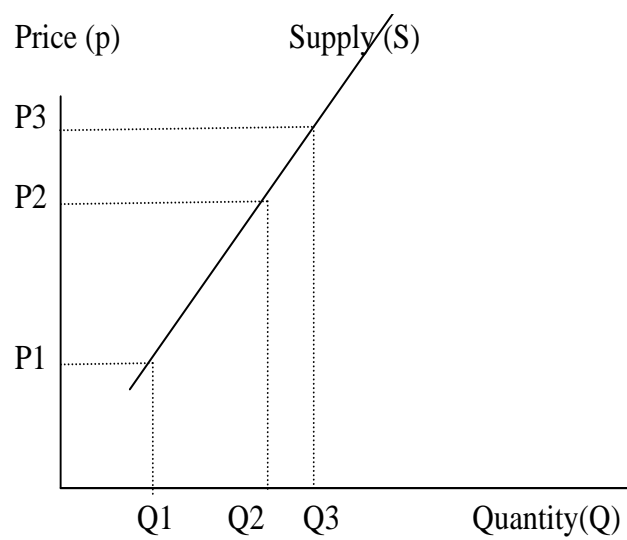
FIGURE 2.14: Law of Supply.

Chapter 3

Fuzzy Logic Concepts

3.1 Fuzzy Logic and Fuzzy Sets

As the complexity of a system increases, the utility of fuzzy logic as a modeling tool increases. For very complex systems, few numerical data may exist and only ambiguous and imprecise information and knowledge is available. Fuzzy logic allows approximate interpolation between input and output situations [92]

Fuzzy logic is a problem solving technique that was introduced by Lotfi Zadeh in [93] to deal with vague or imprecise problems [17, 18, 20, 22, 23, 29, 36, 94, 95, 96]. It provides a framework that attempts to define a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables [93]. It is used to model human reasoning and knowledge that do not have well defined boundary. Although fuzzy logic covers a wide range of theories and techniques, it is mainly based on four concepts: fuzzy sets, linguistic variables [97], possibility distributions (membership functions), and fuzzy if-then-rules [22]. The values of a linguistic variable are both quantitatively described by a fuzzy set. Possibility distributions or membership functions are constraints on the value of a linguistic variable imposed by assigning it a fuzzy set. Fuzzy if then rules are a knowledge representation scheme for describing a functional mapping between antecedents and consequents. A fuzzy inference system employs fuzzy if-then rules and can model the qualitative aspects of human knowledge and reasoning processes without employing precise quantitative analysis. Fuzzy inference systems are generally

understandable because the knowledge in these systems is contained in the form of fuzzy if-then rules containing membership functions [98].

The classical theory of crisp sets can describe only the membership or non-membership of an item to a set[99]. While, fuzzy logic is based on the theory of fuzzy sets which relates to classes of objects with unsharp boundaries in which membership is a matter of degree. In this approach, the classical notion of binary membership in a set has been modified to include partial membership ranging between 0 and 1. The membership function is described by an arbitrary curve suitable from the point of view of simplicity, convenience, speed, and efficiency. A sharp set is a sub set of a fuzzy set where the membership function can take only the values 0 and 1. The full range of the model input values, which are judged necessary for the description of the situation, can be used in fuzzy sets. The process of formulating the mapping from a given input to an output using fuzzy logic is called the fuzzy inference. The basic structure of any fuzzy inference system is a model that maps characteristics of input data to input membership functions, input membership function to rules, rules to a set of output characteristics, output characteristics to output membership functions, and the output membership function to a single-valued output or a decision associated with the output. In rule based fuzzy systems, the relationships between variables are represented by means of fuzzy if-then rules such as "**IF** antecedent proposition **THEN** consequent proposition" [100].

3.2 Fuzzy Thinking

Fuzzy logic is not logic that is fuzzy, but logic that is used to describe fuzziness [54]. Fuzzy logic theory is the theory of fuzzy sets, sets that calibrate vagueness. Fuzzy logic is based on the idea that all things admit of degrees. Temperature, height, speed, distance, beauty- all come on a sliding scale. For example, description such as the music is very loud, the car is speeding very fast, *Adesola* is very beautiful, *Joshua* is really tall, *Modakeke* is quite a long distance from *Kaduna*, *Abuja* is a very large and beautiful city, the weather is really very cold. All these examples fall on sliding scales which often makes them impossible to distinguish members of class from non-members. Take for another instance, how do we answer a question: when does weather becomes too cold? At what speed can a driver be accused of speeding too fast? When does water becomes too hot? All these forms of vagueness or uncertainty in a situation are very essential decisions that engineers should ponder

before determining appropriate procedure or method to express the vagueness. Boolean or conventional logic uses sharp distinctions, governed by a logic that uses one of two values: true or false; it forces us to a line between members of a class and non-members [54] whereas fuzzy logic provides a mathematical way to represent vagueness and fuzziness in humanistic systems [53]. As a specific example, in Boolean logic, we can easily determine when somebody is tall or short based on the calibration of our measuring device. If we draw a line of $1.70m$, then in Boolean or binary logic sense, anybody below $1.70m$ is short and his membership of the class of tall men in that regard is zero "0" and his membership of the class of short men is one "1". This will be the case for somebody such as *Seyi*, who is $1.69m$ tall or *Funmilayo* who is 0.8m tall. They both belong to same class. Similarly, *Adesola* and *Peter* who are $1.71m$ and $3.20m$ tall respectively would be classified, in binary or crisp sense, as each having a membership value of one "1" in the class of tall men and a membership value of zero "0" in the class of short men. Fuzzy logic however, attempts to take human reasoning beyond a crisp value of black and white or zero and one by introducing the degrees of membership. In the notion of fuzzy logic, those four people mentioned above would be recognised as being members of both short and tall men classes but to a certain degree or membership values denoted as μ in the interval between 0 and 1 (i.e. $[0, 1]$).

3.3 Fuzzy Sets

A fuzzy set is a set containing elements that have varying degree of memberships in the set [16, 22]. It can simply be defined as a set with fuzzy boundary [54]. Fuzzy set theory has been applied to many disciplines such as control theory, management sciences, mathematical modelling, operations research and many industrial applications [101]. A key difference between crisp and fuzzy sets is their membership function; a crisp set has unique memberships, whereas a fuzzy set may have an infinite number of memberships to represent it [53]. For fuzzy sets, the uniqueness is sacrificed, but flexibility is gained because the membership function can be adjusted to maximize the utility for a particular application. Elements of a fuzzy set are mapped to a universe of membership values using function theoretic form. Fuzzy sets are denoted by different symbols in different publications, however, in this thesis; a fuzzy set will be represented by a letter with a tilde on top of it. That is, fuzzy set A will be represented by $\tilde{A}$ and membership of a set

will be represented by μ. Therefore the functional mapping given by:

$$\mu_{\tilde{A}}(x) \in [0,1]$$

denotes the degree of membership of element x in fuzzy set $\tilde{A}$. Therefore, $\mu_{\tilde{A}}(x)$ is a value on the unit interval that measures the degree to which element x belongs to fuzzy set $\tilde{A}$.

A *fuzzy set* is defined by a membership function, it consists of some elements x of a universe of discourse X together with their membership values (or *degrees*) $\mu_a(x)$ [102] .

3.4 Membership Functions

In order to represent a fuzzy set in a computer, the membership function must be determined first. The membership function embodies the mathematical representation of membership of elements in a set [53]. All information contained in a fuzzy set is described by its membership function and it is useful to develop a lexicon of terms to analyse various special features of this function. There are a number of methods that can be used here such as seeking the knowledge of a single or multiple experts in the field. The use of artificial neural networks can also be implemented. This learns available system operation data and then derives the fuzzy sets automatically [54]. Membership functions can be represented graphically by different shapes such as triangle, trapezium and so on. In this research, we shall restrict ourselves to the use of triangular and trapezoidal membership functions [103] as shown in Figure 3.1 and 3.2.

We choose triangular membership functions because they can be specified by just three parameters, and this speeds up the learning procedure when the membership function shapes are adapted. Triangular membership functions are very general, and their versatility has been studied in [103, 104]. The range of our fuzzy variables is arbitrary; in a practical application such as in Section 5, the actual range of demand, production cost and so on would be re-scaled. What is important is that on the range, we define four or five membership functions. The number of membership functions is our choice of granularity in the examples.

The universe of discourse is a set X, discrete ($\{x_1, \ldots, x_n\}$), or continuous (union of intervals on the real line). Therefore, a *membership function* is a function $\mu_A : X \to [0, 1]$ [102].
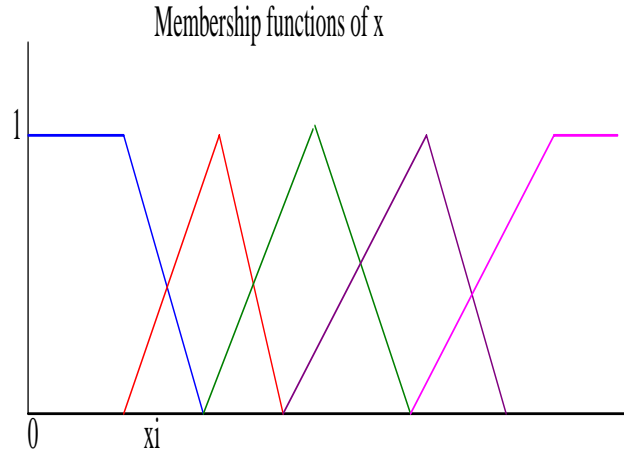
FIGURE 3.1: Membership functions of set x.
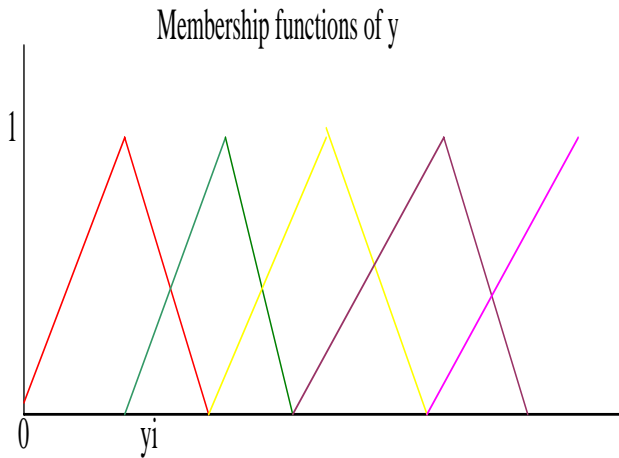
FIGURE 3.2: Membership functions of set y.

3.5 Linguistic Variables and Hedges

Linguistic variables are fuzzy variables while hedges are concentrations which tend to concentrate the elements of a fuzzy sets by reducing the degree of all elements that are only 'partly' in the set. The less an element is in a set (i.e. the lower its original membership value), the more it is reduced in membership through concentration [53]. For example, in a statement such as '*Adesola* is very beautiful' means that the linguistic variable *Adesola* takes *beautiful* as its linguistic value and has *very* as its hedge. Other examples of edges are *slightly, very very, plus,*

minus, moderately. They are generally useful operators which can also be used to break down continuums into fuzzy intervals.

3.5.1 Operations on Fuzzy Sets

Most of all the properties and operations on crisp sets are applicable to fuzzy sets. Examples of these operations are: intersection, union, complementary, containment, commutativity, associativity, indempotency, identity, transitivity, involution and De Morgan's laws.

3.5.1.1 Basic Logic Operations on Fuzzy Logic

The following are some of the basic operations on fuzzy logic [105]:

- *AND Operation*: $\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x)), \quad \forall x \in X.$

- *OR Operation*: $\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x)), \quad \forall x \in X.$

- *NOT Operation*: $\mu_{\neg A}(x) = 1 - \mu_A(x), \quad \forall x \in X.$

- *Extension Principle*: A function transforming a set into another set will transform a membership function into another membership function, using the extension principle [102].

- If $f : X \rightarrow Y$ is a function transforming universe of discourse X into Y, then fuzzy set $\mu_A(x)$ is transformed into $\mu_B(y)$:

$$\mu_B(y) = \begin{cases} \max_{y=f(x)} \mu_a(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise.} \end{cases}$$

3.5.2 Decision Making under Uncertainty or Fuzziness

According to Lofti Zadeh in [106], much of the decision-making in the real world takes place in an environment in which the goals, the constraints and the consequences of possible actions are not known precisely. Before fuzzy set theory was introduced, to deal quantitatively with imprecision, we usually employ the concepts and techniques of probability theory [107] and, more particularly, the tools

provided by decision theory, control theory and information theory. In so do-ing, we are tacitly accepting the premise that imprecision-whatever its nature-can be equated with randomness [106]. Although, probability theory is appropriate for measuring randomness of information, it is inappropriate for measuring the meaning of information[108].

Linguistic decision analysis is based on the use of the linguistic approach and it is applied for solving decision making problems under linguistic information by employing the theory of fuzzy sets. Its application in the development of the theory and methods in decision analysis is very beneficial because it introduces a more flexible framework which allows us to represent the information in a more direct and adequate way when we are unable to express it precisely. In this way, the burden of quantifying a qualitative concept is eliminated [61].

Let X be a set of options. A *fuzzy goal* is a fuzzy set $\mu_G(x), x \in X$. A *fuzzy constraint* is a fuzzy set $\mu_C(x), x \in X$.

A *fuzzy decision* is a fuzzy set $\mu_D(x), x \in X$, with

$$\mu_D(x) = \min(\mu_G(x), \mu_C(x)).$$

A crisp decision x^* can be derived from a fuzzy decision by *defuzzification*:

$$x^* = \arg\max_{x \in X} \mu_D(x).$$

There are several ways to defuzzify of fuzzy set, for example the *centre of gravity* of the area under the curve can be taken to get the defuzzified results. An example of a decision making graph according to Philippe De Wilde in [102] is as shown in Figure 3.3

3.5.2.1 Uncertainty of a Functional Dependency

- Uncertainty can be represented by additive noise, e.g.

$$y = x^2 + \xi,$$

with ξ a random variable.
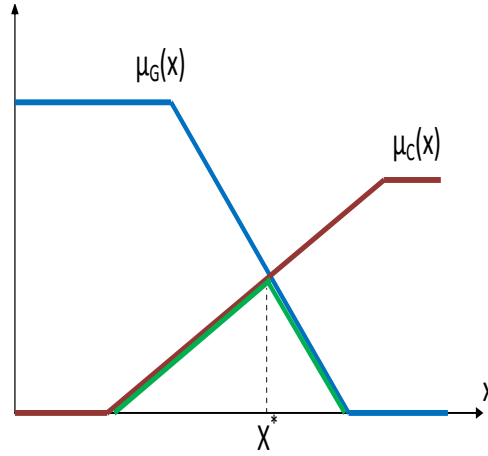
*x** is the decision, subject to constraints *C* and goal *G*.

FIGURE 3.3: x^* is the decision, subject to constraints C and goal G.

- The noise can also be on the *parameters* of the functional relationship, e.g.

$$y = x^{(2+\xi)},$$

or

$$y = \xi x^2.$$

- Zadeh proposed a radically different way of looking at this, where the curve of a function becomes a union of squares, and each point in the union belongs to the function to a certain degree. This is the *fuzzy graph* shown in Figure 3.4 [1, 102].

3.5.2.2 Union of Cartesian Products

The *fuzzy graph* is a union: $(A_1 \times B_1) \cup (A_2 \times B_2) \cup \ldots (A_n \times B_n)$.

Also the fuzzy graph could also be expressed as a union:

$(A_1 \times B_1) \cup (A_2 \times B_2) \cup \ldots (A_n \times B_m)$.

For example, if y-axis has four numbers and x-axis has five numbers, then the fuzzy graph could be expressed as:

$(A_1 \times B_1) \cup (A_2 \times B_2) \cup (A_3 \times B_3) \cup (A_4 \times B_4) \cup (A_5 \times B_4)$.

If X and Y are universes of discourse, $f^* : X \to Y$ is a fuzzy graph if:
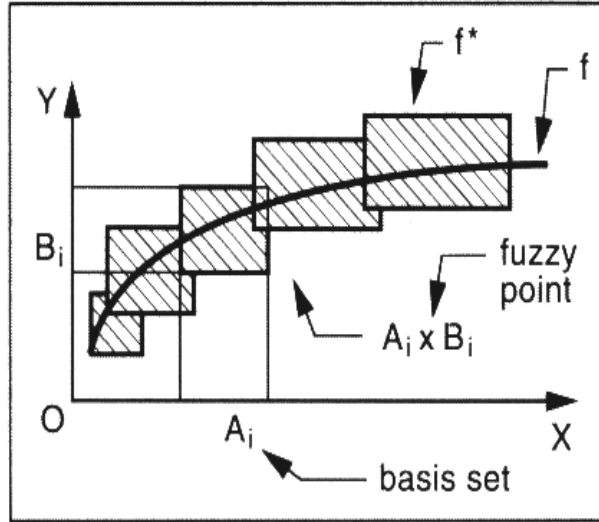
FIGURE 3.4: Fuzzy graph [1].

- $f^* = \bigcup_{\substack{i=1,n \\ j=1,m}} A_i \times B_j,$

- $\mu_{f^*}(u, v) = \max_i \min(\mu_{A_i}(u), \mu_{B_j}(v)), \quad u \in X, v \in Y.$

3.5.3 Fuzzy Associative Memory

To further illustrate the functions of fuzzy associative memory (FAM) [109], consider sampled fuzzy rules below which illustrate the application of fuzzy logic in the automobile:

3.5.3.1 Fuzzy Control: Rules for Stopping a Car

- 1 input, 1 rule, 1 output
  If you go too fast, brake hard.

- 1 input, 2 rules, 1 output
  If you go too fast, brake hard, or, if you go fast, brake.

- 2 inputs, 4 rules, 1 output
  If you go too fast and the wall is very close, brake hard, or
  If you go fast and the wall is very close, brake, or
  If you go too fast and the wall is close, brake, or
  If you go fast and the wall is close, slow down.

	too fast	fast
very close	brake hard	brake
close	brake	slow down

TABLE 3.1: Fuzzy associative memory (FAM) table for a two-input one-output rule system.

	μ_1^2	μ_2^2	μ_3^2	$\ldots$
μ_1^1	μ_{11}	μ_{12}	μ_{13}	$\ldots$
μ_2^1	μ_{21}	μ_{22}	μ_{23}	$\ldots$
μ_3^1	μ_{31}	μ_{32}	μ_{33}	$\ldots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

TABLE 3.2: Fuzzy associative memory (FAM) table for n-inputs and n-outputs rule system.

Based on the rules above, the fuzzy associative memory (FAM) table can be formed as shown in Table 3.1 for two inputs fuzzy inference system while Table 3.2 shows the FAM table for many inputs, many outputs fuzzy inference systems. The tables illustrate how an expert makes use of fuzzy rules in making decisions.

3.5.3.2 Two Inputs Fire Rules

From Tables 3.1 and 3.2 the two inputs firing rules of the fuzzy inference system can be illustrated as follows:

- 2 inputs x^1 and x^2

- x^1 belongs to the input membership functions:
  $\mu_1^1, \mu_2^1, \mu_3^1, \ldots$ to degrees $\mu_1^1(x^1), \mu_2^1(x^1), \mu_3^1(x^1), \ldots$.

- x^2 belongs to the input membership functions:
  $\mu_1^2, \mu_2^2, \mu_3^2, \ldots$ to degrees $\mu_1^2(x^2), \mu_2^2(x^2), \mu_3^2(x^2), \ldots$.

- output membership function μ_{ij} fires at degree
  $\min[\mu_i^1(x^1), \mu_j^2(x^2)]$, using min because of the 'and' in the rules.

- output membership function μ_{ij} is *truncated* at $\min[\mu_i^1(x^1), \mu_j^2(x^2)]$.

The steps involved in truncation of the membership functions which is illustrated in Figure 3.5 are as itemized below:
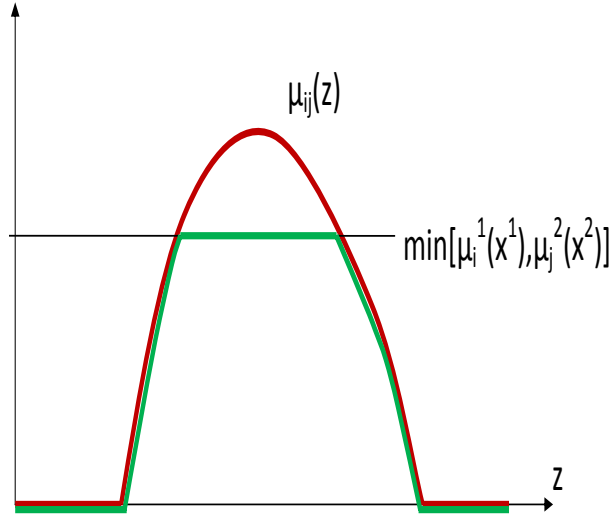
FIGURE 3.5: Truncated output membership function.

- $= \min \left\{ \mu_{ij}(z), \min[\mu_i^1(x^1), \mu_j^2(x^2)] \right\}$

- $= \min \left[\mu_{ij}(z), \mu_i^1(x^1), \mu_j^2(x^2) \right]$

3.5.3.3   Combination of all Output Membership Functions

All output membership functions are then combined as shown in Figure 3.6 and the processes involved are as summarized below:

- Max, because a collection of rules is combined with 'or'.

- $\max_{i,j} \min \left[\mu_{ij}(z), \mu_i^1(x^1), \mu_j^2(x^2) \right]$

- Sum can be used instead of max.

3.5.3.4   Defuzzification Using Centre of Gravity

The final stage of the fuzzy inference system is the defuzzification stage and the details involved are as summerized below:

- $f(z) = \max_{i,j} \min \left[\mu_{ij}(z), \mu_i^1(x^1), \mu_j^2(x^2) \right]$

- Centre of gravity $y = \frac{\int_{-\infty}^{\infty} z f(z) dz}{\int_{-\infty}^{\infty} f(z) dz}$

- y is the defuzzified output, the control
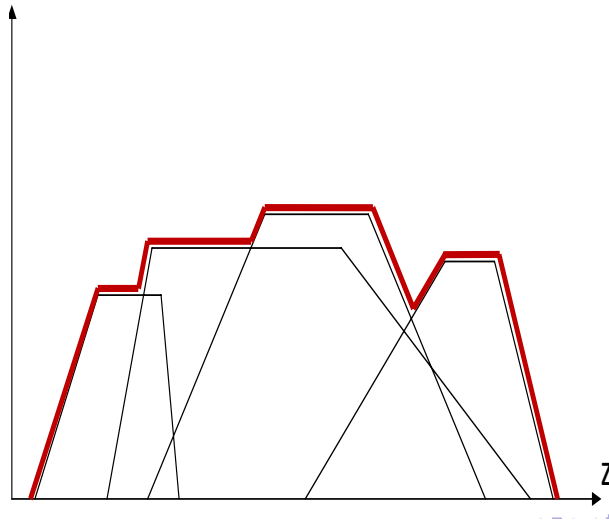
FIGURE 3.6: Combination of all Output Membership Functions.

3.5.4 Fuzzy Rules

Generally, fuzzy rules are conditional statements in the form of *IF-THEN* statements [110]. The simplest fuzzy rules are of the form If X is $\tilde{A}_i$ THEN Y is $\tilde{E}_j$ where $\tilde{A}_i$ and $\tilde{E}_j$ are fuzzy sets for the domains of X and Y [13]. More complex rules which will be used in this research will consist of several input and output variables. For example; If X is $\tilde{A}_i$ and Y is $\tilde{E}_j$ THEN Z_1 is $\tilde{C}_{1k}$ and Z_2 is $\tilde{C}_{2k}$. The statements before THEN is referred to as the antecedent while that after is referred to as consequent part. X, Y and Z are linguistic variables while $\tilde{A}_i$, $\tilde{E}_j$, $\tilde{C}_{1k}$ and $\tilde{C}_{2k}$ are linguistic values.

3.6 Fuzzy Decision Making System

In general, a fuzzy decision making system (FDMS) uses a collection of fuzzy membership functions (Figure 3.1 on page 40) and decision rules [111] that are solicited from experts in the field to reason about data [22]. Typical components of a fuzzy decision making system are as shown in Figure 4.1(a) on page 76. The components of an FDMS, as shown in the figure are; a fuzzification section, a fuzzy rule base, fuzzy decision logic and defuzzification section [19].

1. *Fuzzification section*: This is the section where the process of making a crisp quantity fuzzy [53] is carried out. This is done by simply recognising that

Input$B$/Input A	A_1	A_2	A_3	A_4	A_5
B_1	X_1	X_4		X_1	X_3
B_2	X_4	X_5	X_1		X_2
B_3	X_4	X_4	X_3	X_2	X_2
B_4	X_1	X_1	X_4	X_4	X_1

TABLE 3.3: Fuzzy associative memory (FAM) table for a two-input one-output rule system.

many of the quantities that we considered to be crisp and deterministic are actually not deterministic at all. They carry considerable uncertainty. If the form of uncertainty happens to arise because of imprecision, ambiguity, or vagueness, then the variable is probably fuzzy and can be represented by a membership function.

2. *Fuzzy rule base*: These rules are expressed in conventional antecedent-consequent form. The collection of such rules constitutes the fuzzy logic knowledge base that is used for inference of the decision agent. In a fuzzy system, if the antecedent is true to some degree, then the consequent is also true to that same degree. For a small number of inputs, there exists a compact form of representing a fuzzy rule-based system which consists of a tabular format with different partitions representing different inputs. This compact graphical form is called fuzzy associative memory table, or FAM table as shown in Table 3.3. Further explanations on FAM are offered in Section 3.5.3 on page 44 of this thesis.

In FAM, the linguistic values of one input variable form the horizontal axis and the linguistic values of the other input form the vertical axis. At the intersection of a row and a column lies the linguistic value of the output variable. A rule is said to 'fire', if the degree of truth of the premise part of the rule is not zero [22]. The implication is implemented for each rule and in Matlab [112], many built-in methods are supported such as the functions that are used by the AND method: min(minimum), which truncates the output fuzzy set, *prod* (product), which scales the output fuzzy set. Here, the AND method was used and the centroid was computed using the Mamdani-type inference system which requires the output membership functions to be fuzzy sets after the aggregation process. It (Mamdani FIS) integrates, using Equation 3.1 on page 50, across a two-dimensional function to find the centroid [113].

3. *The decision making logic (DML)*: The decision making logic is analogous to classical logic for reasoning [53] and it is similar to simulating human decision making in inferring fuzzy control actions based on the rules of inference in fuzzy logic [22].

4. *Defuzzification process*: This is the procedure that converts the fuzzy results into a crisp output. It converts a fuzzy control action (a fuzzy output) into a non-fuzzy control action (a crisp output) [22, 114]. Defuzzification has the result of reducing a fuzzy set to a crisp single-valued output, or to a crisp set; of converting a fuzzy matrix to a crisp matrix; or making a fuzzy number a crisp number. Fuzziness helps to evaluate the rules, but the final output of a fuzzy system has to be a crisp number and the input for the defuzzification process is the aggregate output fuzzy set and the output is a single number [54]. Mathematically, the defuzzification of a fuzzy set is the process of 'rounding off' from its location in the unit hypercube to the nearest (in a geometric sense) vertex. If one thinks of a fuzzy set as a collection of membership values, or a vector of values on the unit interval, defuzzification reduces this vector to a single scalar quantity - presumably to the most typical (prototype) or representative value [53]. The fuzzy output is obtained from aggregating the outputs from the firing of the rules [92]. Subsequent defuzzification methods on the fuzzy output produce a crisp value.

Several defuzzification methods have been discussed in the literature such as [22, 53, 54]. Among these methods are the following:

- Max membership principle.
- Centroid method.
- Weighted average method.
- Mean max membership.
- Center of sums.
- Center of largest area.
- First (or last) of maxima

For extensive explanation on each of the above methods, please see[22, 53, 54]. In this research, we used the centroid method and we shall give a brief explanation of it.

In fuzzy logic control systems, the defuzzification step involves the selection of one value as the output of the controller. More specifically, starting with a fuzzy subset (possibility distribution) F over the output space X of the controller, the defuzzification step uses this fuzzy subset to select a representative element x^*. The two most often used methods of defuzzification found in the literature are the *center of area (COA)* and *mean of maxima (MOM)* methods. The MOM method takes as its defuzzified value, the mean of the elements that attain the maximum membership grade in F [115]

Centroid defuzzification method: This method is also referred to as centre of area (COA) or centre of gravity (COG). It is the most commonly used [22], most popular [54], most physically intuitive [53] defuzzification technique and it finds the point where a vertical line would slice the aggregate set into two equal masses. In theory, the centroid method of defuzzification is calculated over a continuum of points in the aggregate output membership function but in practice, a reasonable estimate can be obtained by calculating it over a sample of points. Mathematically, the centroid method can be expressed as:

$$COG = \frac{\int \mu_{\tilde{A}}(x) x \, dx}{\int \mu_{\tilde{A}}(x) \, dx}. \tag{3.1}$$

$$COG = \frac{\sum \mu_{\tilde{A}}(x) x}{\sum \mu_{\tilde{A}}(x)} \tag{3.2}$$

Fuzzy inference techniques: In general, fuzzy decision making system can be implemented using any of the three common methods of deductive inference for fuzzy systems based on linguistic rules [53] and these methods are listed as follows:

- Mamdani system

- Sugeno systems

- Tsukamoto models

3.6.1 Fuzzy Inference Techniques

In fuzzy inference experiments there are two main types of Inference techniques (FIS) and these are: the Mamdani-type [113] and the Takagi-Sugeno (T-S)-type [116]. These are later referred to as Type I and Type III inference techniques

respectively . The differences between these two FISs lie in the consequents of their fuzzy rules, and thus their aggregation and defuzzification procedures differ accordingly [117].

3.6.1.1 Comparison of Mamdani and Takagi-Sugeno Model

In terms of use, the Mamdani FIS is more widely used, mostly because it provides reasonable results with a relatively simple structure, and also due to the intuitive and interpretable nature of the rule base. Since the consequents of the rules in a T-S FIS are not fuzzy this interpretability is lost; however, since the T-S FIS's rules' consequents can have as many parameters per rule as input values, this translates into more degrees of freedom in the design than a Mamdani FIS thus providing the system's designer with more flexibility in the design of the system. However, it should be noted that the Mamdani FIS can be used directly for either MISO systems (multiple input single output) as well as for MIMO systems (multiple input multiple output), while the T-S FIS can only be used in MISO systems [118].

In currently available adaptive fuzzy inference systems the fuzzy rules follow the Takagi, Sugeno, and Kang (T-S) style, sometimes called Type III. T-S rules have been shown to be more robust than Mamdani style rules, sometimes called Type I. However, Type I rules are easier to understand and can sometimes generalize better than Type III rules. A Type I rule is more intuitive than a Type III rule; the consequence is a fuzzy variable. The consequences of Type III rules are a linear combination of the antecedent labels therefore these rules may not be any easier to understand [98]. T-S adopts a linear equation in consequent part, which can not exhibit human's judgment reasonably.

Ebrahim Mamdani in 1975 [113] proposed the scheme which was the very first attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators [117] and Takagi-Sugeno (T-S) FIS was proposed to develop a systematic approach to generate fuzzy rules from a given input-output data [116]. The T-S fuzzy inference system works well with linear techniques and guarantees continuity of the output surface. But the T-S fuzzy inference system has difficulties in dealing with the multi-parameter synthetic evaluation; it has difficulties in assigning weight to each input and fuzzy rules.

In the T-S fuzzy model, the rule consequents are usually taken to be either crisp numbers or linear functions [100]. In general, T-S fuzzy modeling involves structure identification and parameter identification. The structure identification consists of initial rule generation after elimination of insignificant variables, in the form of IF-THEN rules and their fuzzy sets. Parameter identification includes consequent parameter identification based on certain objective criteria. In many situations, such rules are difficult to identify by manual inspection and therefore are usually derived from observed data using techniques known collectively as fuzzy clustering [100].

The Takagi-Sugeno scheme is a data driven approach where membership functions and rules are developed using a training data set. The parameters for the membership functions and rules are subsequently optimized to reduce training error. The relationship in each rule is represented by a localized linear function. The final output is a weighted average of a set of crisp values.

In this research, we used the Mamdani inference system. We adopted the Mamdani method which according to *Chai et al* in [119] has advantages in consequent part. According to [119], other advantages of the Mamdani fuzzy inference system are:

- It is intuitive.

- It has widespread acceptance.

- It's well suited to human cognition

3.6.1.2 Mamdani Fuzzy Inference Technique

The Mamdani scheme is a type of fuzzy relational model where each rule is represented by an IF-THEN relationship. It is also called a linguistic model because both the antecedent and the consequent are fuzzy propositions. The model structure is manually developed and the final model is neither trained nor optimized. The output from a Mamdani model is a fuzzy membership function based on the rules created. Since this approach is not exclusively reliant on a data set, with sufficient expertise on the system involved, a generalized model for effective future predictions can be obtained [92].

Since Mamdani fuzzy inference system shows its advantage in the output expression, we therefore used Mandani inference system (FIS) in this thesis. The process

truncates the output membership functions at their maximum value [53]. The model has the ability of learning because of differentiability during computation and has greater superiority in expression of consequent part and intuitive of fuzzy reasoning.

Moreover, the Mamdani model is a universal approximator because of its infinite approximating capability by training. All parameters in Mamdani FIS are non-linear parameters which can be adjusted by learning rules discussed above. The experimental results show that this model is superior to others in amount of adjusted parameters, scale of training data, consume time and testing error. It does well in non-linear modeling and forecasting [119].

3.7 Optimization of Fuzzy Membership Functions

Optimization deals with the ideas of tracking optimum operating conditions of systems [120]. The subject of function minimization is both important and ubiquitous in the physical sciences. This is easily demonstrated by noting that it is involved in a very wide variety of areas ranging from finding roots of polynomials and solving simultaneous equations to estimating the parameters of non-linear functions [121, 122]. In a fuzzy logic system, the membership functions can be parameterized by a few variables and the membership optimization problem can be reduced to a parameter optimization [123] problem if we constrain the membership functions to a specific shape such as triangles and trapezoids [124].

Fuzzy parameters refer to the parameters that define the membership functions of a fuzzy logic system. For instance, if we are using triangular membership functions, then the fuzzy parameters would be the centers and half-widths of the triangles [124].

Researchers have used many different methods over the past decade to optimize fuzzy membership functions. According to Dan Simon in [124], the methods can be broadly divided into two types: those that explicitly use the derivatives of the fuzzy system's performance with respect to the fuzzy parameters, and those that do not use these derivatives. Derivative-free methods include genetic algorithms [125, 126], neural networks, evolutionary programming, geometric methods, fuzzy equivalence relations, and heuristic methods. Derivative-based methods include

gradient descent [127], Kalman filtering [128], least squares, back propagation, and other numerical techniques [124].

Derivative-based methods are limited by dependency on analytical derivatives. They are also limited to specific objective functions, specific types of inference, and specific types of membership functions [124].

In this research, we have used the Nelder-Mead simplex method to optimize the fuzzy logic membership functions. The algorithm is as explained in the sections that follow.

3.7.1 Nelder-Mead Method of Optimization

Nelder-Mead algorithm [120] is a simplex method for finding a local minimum of a function of several variables [129]. The method [120] describes the minimization of a function of n variables, which depends on the comparison of function values at the $(n + 1)$ vertices of a general simplex, followed by the replacement of the vertex with the highest value by another point. The simplex adapts itself to the local landscape, and contracts on to the final minimum. The method is shown to be effective and computationally compact. A procedure is given for the estimation of the Hessian matrix in the neighbourhood of the minimum, needed in statistical estimation problems.

The Nelder-Mead simplex method also refer to as "amoeba algorithm" in [130] is a "direct" method requiring no derivatives [121]. The objective function is evaluated at the vertices of a simplex, and movement is away from the poorest value. The process is adaptive, causing the simplexes to be continually revised to best conform to the nature of the response surface.

It is an enormously popular direct search method for multidimensional unconstrained minimization [131]. In the method, the simplex adapts itself to the local landscape, elongating down long inclined planes, changing direction on encountering a valley at an angle, and contracting in the neighbourhood of a minimum. The criterion for stopping the process has been chosen with an eye to its use for statistical problems involving the maximization of a likelihood function, in which the unknown parameters enter non-linearly [120].

The Nelder-Mead method attempts to minimize a scalar-valued nonlinear function [132] of n real variables using only function values, without any derivative information (explicit or implicit). The Nelder-Mead method thus falls in the general class of direct search methods. A large subclass of direct search methods, including the Nelder-Mead method, maintain at each step a nondegenerate *simplex*, a geometric figure in n dimensions of nonzero volume that is the convex hull of $n + 1$ vertices [131].

Each iteration of a simplex-based direct search method begins with a simplex, specified by its $n+1$ vertices and the associated function values. One or more test points are computed, along with their function values, and the iteration terminates with a new (different) simplex such that the function values at its vertices satisfy some form of descent condition compared to the previous simplex. Among such algorithms, the Nelder-Mead algorithm is particularly parsimonious in function evaluations per iteration, since in practice, it typically requires only one or two function evaluations to construct a new simplex [131].

3.7.2 Why Nelder-Mead Algorithm?

Since Nelder Mead algorithm uses simplex methods and our fuzzy membership functions are of simplex shapes (triangles and trapeziums), therefore the algorithm proves more effective and is computationally more efficient than other optimization methods for our model. The method is selected because it is simple, can be programmed on a computer fairly easily and it is derivative-free [133]. Derivative-free method is desired since they do not use numerical or analytical gradients and can be applied to a wide range of objective functions and membership function forms.

Other main points of using the Nelder-Mead algorithm are the generality of the method, its accuracy, and the simplicity of the information required for the computer input statements. It can handle a wide variety of optimization problems, without requiring any modifications tailored to the problem at hand [121]. It can accommodate angle and angle rate as input variables [134].

Since its publication in 1965, the Nelder-Mead "simplex" algorithm has become one of the most widely used methods for nonlinear unconstrained optimization [131].

Further applications of this method which show other types of problems it can solve in comparison with alternative algorithms can be found in different papers such as [120], [131] and [121].

Lagarian et al in [131], offered three other reasons the Nelder-Mead Algorithm is widely acceptable and so extraordinarily popular and these are: First, in many applications, for example in industrial process control, one simply wants to find parameter values that improve some performance measures; the Nelder-Mead algorithm typically produces significant improvement for the first few iterations. Second, there are important applications where a function evaluation is enormously expensive or time-consuming, but derivatives cannot be calculated. In such problems, a method that requires at least n function evaluations at every iteration (which would be the case if using finite difference gradient approximations or one of the more popular pattern search methods) is too expensive or too slow. When it succeeds, the Nelder-Mead method tends to require substantially fewer function evaluations than these alternatives, and its relative "best-case efficiency" often outweighs the lack of convergence theory. Third, the Nelder-Mead method is appealing because its steps are easy to explain and simple to program.

3.7.3 Nelder-Mead Concepts

If we have two variables, then a simplex is a triangle, and the method represents a pattern search that compares the values of the function at the three vertices of the triangle. The model will reject the vertex, where value of the function $f(a, b)$ is highest and it will then replace this vertex with a new vertex. This translates into a new triangle and the search will continue from there. This algorithm will generate many triangles and these triangles may be of different shapes. In each subsequent triangle, the function values at the vertices become gradually smaller from each iteration. The triangles' sizes are reduced and we then find the coordinates of the minimum point. The algorithm is stated using the term simplex (a generalized triangle in N dimensions) and will find the minimum of a function of N variables. It is effective and computationally compact [129].

3.7.3.1 Formulating the Initial Triangle with Vertices P, T and Q

Consider $f(a, b)$ as a function that needs to be minimized. Let us start with a triangle with three vertices: $M_r = (a_r, b_r), r = 1, 2, 3$. At each of the three points: $e_r = f(a_r, b_r)$ for $r = 1, 2, 3.$, we will then evaluate $f(a, b)$. Then we will reorder the subscripts such that $e_1 \leq e_2 \leq e_3$. The following notations are used:

$$P = (a_1, b_1), \ T = (a_2, b_2) \ and \ \ Q = (a_3, b_3) \tag{3.3}$$

In equation 3.3, $\boldsymbol{P}$ is considered to be the best vertex, this follows by $\boldsymbol{T}$ as the as good vertex (meaning that vertex $\boldsymbol{T}$ is next to the best) and finally, the vertex $\boldsymbol{Q}$ is considered to be the worst of the three vertices.

3.7.3.2 Calculating the center of the good side

Considering the initial triangle shown in Figure 3.7 on page 57, in using Nelder-Mead algorithm, the iteration processes begin by calculating the midpoint of the segment line that joins points $\boldsymbol{P}$ and $\boldsymbol{T}$ of the triangle $\boldsymbol{PTQ}$. We will get this by finding the average of the two points.
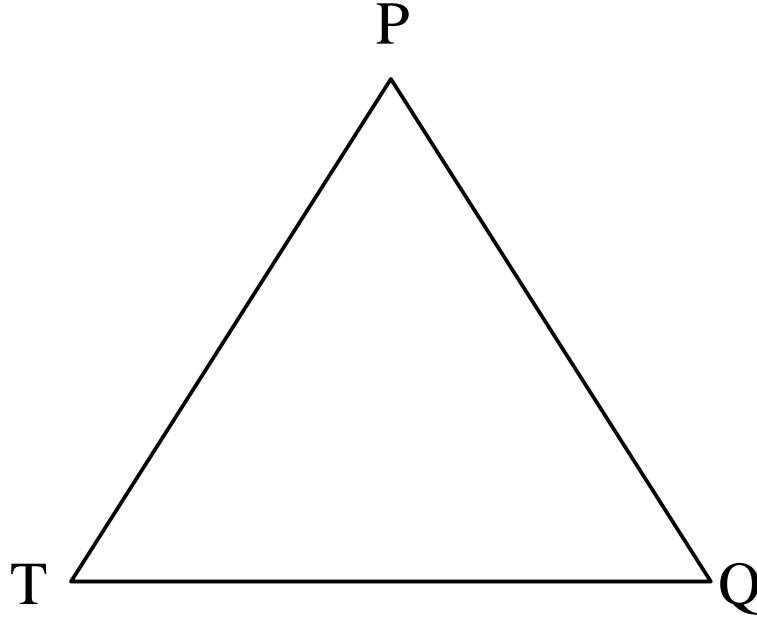
FIGURE 3.7: Initial triangle PTQ of Nelder-Mead algorithm.

$$N = \frac{P+T}{2} = \left(\frac{a_1 + a_2}{2}, \frac{b_1 + b_2}{2} \right) \tag{3.4}$$

3.7.3.3 Reflect to Point Z through PT

The value of the function $f(a,b)$ reduces as it proceeds on the side of the triangle along the line segment QP starting from point Q to point P. The function also reduces when proceeding from point Q to T. It can then be thought that $f(a,b)$ values may be smaller at a point that lies on the opposite side of line PT and away from point Q. This is as illustrated in Figure 3.8. By *reflecting* the triangle through the side of the line segment PT, we can choose a point Z as a test point. In order to calculate the point Z, we need to first determine the center of the line segment PT. We will denote this center point of PT as N. We can then draw a line segment of length u from point Q to point N. In order to locate our earlier chosen point Z, we need to extend the last segment QN by a distance u further, through N. These are as shown in Figure 3.8. We can then obtain the formula for the vector Z as follows:

$$Z = N + (N - Q) = 2N - Q. \tag{3.5}$$
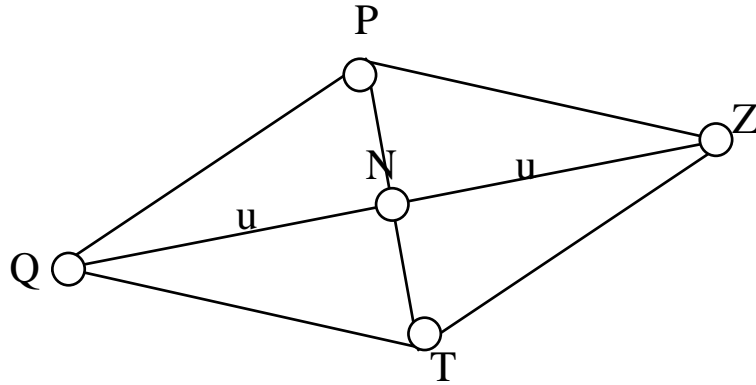
FIGURE 3.8: Modified triangle PTQ, the center point N and reflected point Z.

3.7.3.4 Expanding the line QZ to H

To find out if we have moved in the right direction of finding the minimum, we will need to compare the values of the function $f(a,b)$ at point Z to its value at point

Q. If it is smaller at point Z than at point Q, then we have moved in the right way. We will then need to expand the triangle PTZ to form another triangle PTH by extending the line joining point N and Z to another point H. The length of the newly formed line segment ZH is of distance u as shown in Figure 3.9. We then need to compare the values of the function $f(a, b)$ at the newly formed point H to that of the former point Z. We will know that we have found a better vertex at point H if the function value is smaller there than its value at point Z. We can then obtain the formula for the vector H as follows:

$$H = Z + (Z - N) = 2Z - N. \tag{3.6}$$
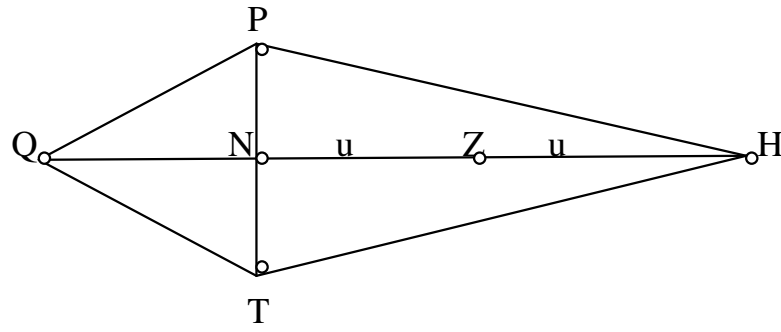
FIGURE 3.9: New triangle PTQ, points Z, and H.

3.7.3.5 Shrinking toward Point C

If however the function value at point Z is not smaller than its value at point Q but both rather have the same values, then we need to test another point. It may be that the function value is smaller at point N and since we must have a triangle, therefore, we cannot replace point Q with point N. We will need to consider two center points $K1$ and $K2$ for the line segments QN and NZ, respectively. These are as shown in Figure 3.10. We will consider the function $f(a, b)$ values at these two new points and we will refer to the point with smaller function value as point K. A new triangle PTK is then formed.

3.7.3.6 Contraction toward Point P

We will again check the function value at the new point K and should the value at K found to be not less than the function value at Q, we will then need to shrink
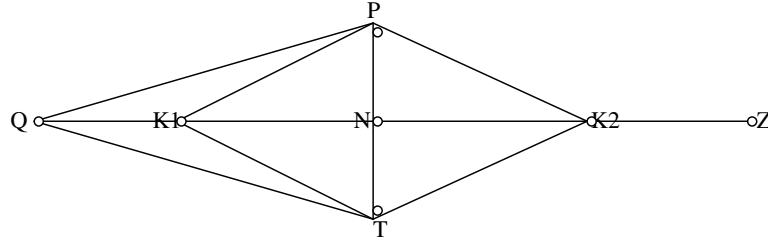
FIGURE 3.10: The triangle $\boldsymbol{PTQ}$, showing the shrunk points $\boldsymbol{K1}$, and $\boldsymbol{K2}$.

the points $\boldsymbol{T}$ and $\boldsymbol{Q}$ toward $\boldsymbol{P}$ as shown in Figure 3.11. Then we replace point $\boldsymbol{T}$ with $\boldsymbol{N}$ and $\boldsymbol{Q}$ with $\boldsymbol{F}$. This new point $\boldsymbol{F}$ is the center of the line segment that joins $\boldsymbol{P}$ with $\boldsymbol{Q}$.
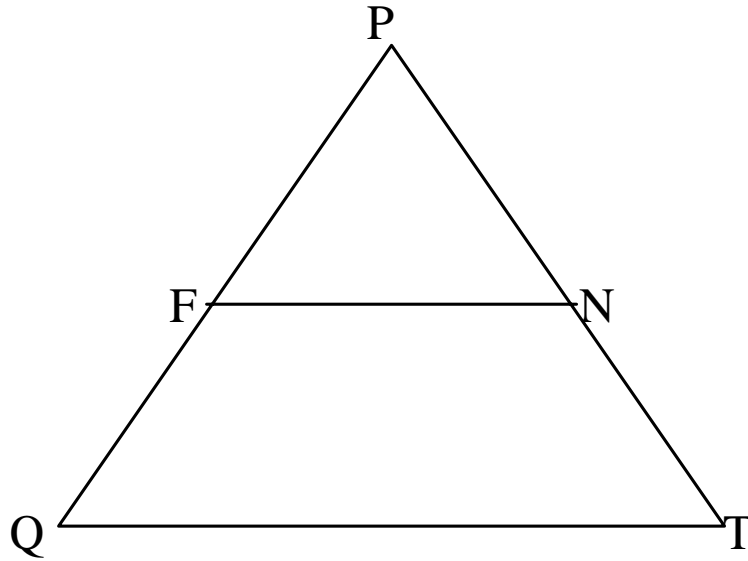
FIGURE 3.11: The triangle $\boldsymbol{PTQ}$, showing line $\boldsymbol{QT}$ shrunk toward point $\boldsymbol{P}$.

3.7.3.7 Computation and Decision at each Vertex

In this PhD research, we used computer programs in Matlab [135] to perform these computations and at each step, new vertices are produced, the program checks the function value at each new vertex and compares it to the previous vertex. These processes continue until a desirable solution has been achieved.

A simple flow chart that illustrates the steps involved in Nelder-Mead simplex algorithm which we used for our fuzzy logic membership functions optimization is as shown in Figure 3.12 on page 61.
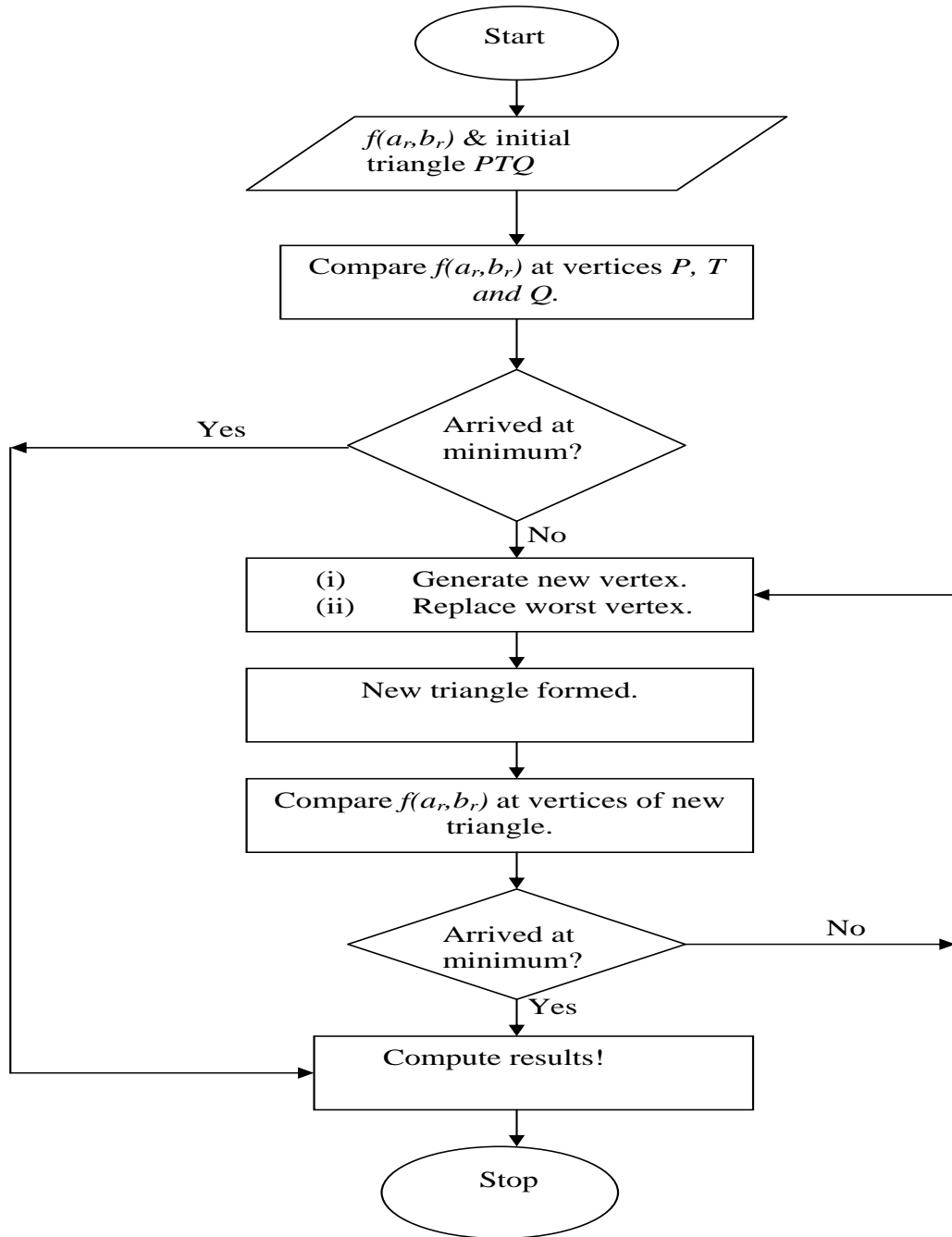
FIGURE 3.12: Summary of steps involved in Nelder-Mead simplex search method.

In this research, the design and implementation of this fuzzy decision making system was achieved with the aid of Matlab software. Matlab is a menu driven software [135] that allows the implementation of fuzzy constructs like membership functions and a database of decision rules [22].

3.8 Research Background

The background work for this research was based on extensive search of numerous publications as listed in the reference section of this thesis. These include those on game theory, fuzzy logic concepts, microeconomic theories, membership functions optimization techniques, decision making processes in different situations such as in firms, military, and other related situations.

However, specific attention was paid to the research that was carried out by Braathen and Sendstad in [13]. In that research, they used fuzzy logic theory and a constraint satisfaction problem approach to model automatic decision making processes in a military context. The decision agent was applied in two different types of simulation games to prove the general applicability of the design. The first game discussed a two sided zero sum application sequential resource allocation game with imperfect information which was interpreted as an air campaign game while the second was a network flow stochastic board game designed to capture important aspects of land maneuver operations. The *fuzzy logic/constraint satisfaction problem (FL/CSP) decision agent* was trained to optimize its performance against some measures.

In [13], two players; blue and red played against each other in which each player chose to allocate its resources on three roles: defense, profit and attack. The game comprised a two stage fuzzy inference system (FIS). The design of the automatic decision making in the simulation game considered both the generation of the set of moves and the evaluation of strategies based upon a ranking measure. It was labelled a modular mixed approach because the agent decomposed the implementation into several modules and rendered possible the combination of a constructive (human) and an evaluation (machine) type move generation approach. The training of the decision agent was done as an optimization of the FL parameters by playing a series of training games with performance measure based on game payoffs. The procedure that was adopted optimized the fuzzy logic (FL) parameters by minimizing the difference between the agent's performance measure value and game theoretic payoff value (or its estimates). It was concluded that the design could be generally useful for designing automatic decision agents in simulation game models.

Chapter 4

Research Methodology and General Illustration

In the first year of the PhD, we designed a model which was termed as fuzzy decision making system for business games (FDMSB) [52], and this is what we have summarised in this chapter.

4.1 Players' Strategies

A strategy is a decision rule that specifies how the player will act in every possible circumstance [136]. It is a specific course of action taken by the firm. This will involve the firm allocating values to its policy variables. These policy variables are generally those aspects of its activities that the firm can directly affect and may include price, spending on promotion, marketing, research and development and so on. For each strategy of this firm, its rival (or rivals) may adopt counter-strategies [42].

The outcome of a game will depend upon the strategies employed by every player. In games, any pure strategy, which can be rejected by comparing it with the other pure strategies and finding that there are others which are always better under every circumstance, is a dominated strategy and will not enter into a solution [9]

In our experiment, each player is given five units of initial resources which may represent capital, time, personnel or other business resources. In this case study, we

assume capital (say £5M). In each round, the players may choose to allocate their resources to one of three roles: *consolidation efforts* (C), *reserved or generated wealth* (W) and *aggressive marketing efforts* (M). These resource allocations will be done simultaneously by both players. Only the opponent's move history will be known, but without knowledge of the opponent's current choice of strategy. The allocations are denoted as a vector $[C, W, M]$ for each player and constitute the *strategy* of that player.

Consolidation efforts C refer to the proportion of resources that are spent to retain existing customers (if any) such as various customer service improvements, customer care, satisfaction, delight and customer retention initiatives. Marketing aggressiveness M denotes the part of these resources that are allocated to various advertising, marketing and promotional campaigns. These are principally targeted towards getting new customers. Reserved wealth W refers to part of the resources that are kept unused in the firm's coffer.

As examples of players' strategies, consider a firm **Y** that is a new entrant into a market. **Y** does not have existing customers to consolidate at the start of the game and therefore has $C = 0$. It may then decide to allocate all or most of its resources on advertising (marketing) campaigns M. If it chooses to allocate all to marketing $M = 5$, then its strategy $[C_y, W_y, M_y]$ becomes $[0, 0, 5]$. This is considered to be the strongest strategy. We refer to it in Section 4.2 Step 11, as globally optimizing player (Geq).

Assume **Y** enters the market with a much reduced price E_{sp} (probably as a result of new technology which leads to reduced production cost C_p). If it is economically impossible for the incumbent (existing) firm **G** to cut its price to the same level due to its high production cost C_p, **G** may decide to devote most of its resources (say £4M) to consolidate its existing customers in order to retain its market share. It may then decide to allocate the remaining resources M_g to market new customers. Therefore, **G**'s strategy $[C_g, W_g, M_g]$ becomes $[0, 4, 1]$.

The difference between strategies of different players is the proportion, number or amount that each player decides to allocate to each component of his own strategy $[C, W, M]$ out of his available total resources (say £5M). This is how firms allocate resources to their core strategies for competition (such as advertisement) and how these allocations could affect their payoffs in an uncertain or fuzzy market environment.

The variables in our models can be tailored to the business situations in the real world and therefore are not limited to those variables that we have used in designing the system as we further explained in Section 1.3.1 on page 7. Therefore, this model can be applied to any real business situation and the variables can be adapted to suit the situation in question.

The model can also work for systems that have more strategic variables than those that we have used in this model.

4.2 Fuzzy Decision Making System for Business Games (FDMSB)

The model for our proposed fuzzy decision making system for business games (FDMSB) is as shown in Figure 4.1(b) on page 76.

Our FDMSB involves two players (firms) in a typical duopoly market which we shall represent as green (g) and yellow (y) which represents the *fuzzy agent*. Each player is given five units of initial resources which may represent capital, time, personnel or other business resources. In our case we assume capital (say £5M). The number of rounds the game must be played is five which denotes a sequence of five possible moves for each player. In each round, the players may choose to allocate their units between three roles (strategies): *consolidation efforts* (C), *reserved or generated wealth* (W) and *aggressive marketing efforts* (M). These resources allocation will be done simultaneously with only the opponent move history that will be known but without knowledge of the opponent's current choice of strategy and are denoted as vector $[C, W, M]$ for each player.

The general procedures necessary for designing the proposed decision support system (FDMSB) are as listed in the steps below:

1. List all uncertain (fuzzy) factors that will be considered in taking the business decision: the uncertain or fuzzy information (factors) we are taking into consideration in this illustration are anticipated market demand information (D) and the production costs(C_P).

D/C_P	**Low**	**Medium**	**High**
Very Low	very low	very low	very low
low	low	low	low
Medium	medium	medium	low
High	medium	high	medium

TABLE 4.1: FAM table for expected market consolidation(E_c) efforts as output.

2. Determine the strategies of the players: Here, we are adopting three strategies for each player and these strategies are consolidation effort, wealth created or reserved and aggressive marketing efforts denoted as a vector with three elements $[C, W, M]$. As an illustration of a duopoly system, we have two players (firms) represented as green (g) with strategy represented as $[C_g, W_g, M_g]$ and yellow (y) with strategy represented as $[C_y, W_y, M_y]$.

3. Determine the input and output variables of FDMSB FIS: The inputs are market demand information (D) and production costs (C_P) and the outputs are expected consolidation efforts (E_c), expected wealth (E_w) and expected aggressive marketing efforts (E_m) where: $E_m = 5 - (E_w + Ec)$ (Because the total (expected) resources of each player at any point is five). Figure 4.2 on page 77 shows the Mamdani FIS interface of the simulation.

4. Develop fuzzy sets, subsets and membership functions for all the input and output variables: This can be accomplished by soliciting knowledge from the experts or searching through literature data. Our adopted fuzzy sets, subsets and membership functions are as shown in Figure 4.3 on page 78.

5. Formulate decision rules for the rule base: These also, ought to be solicited from experts [18, 94]. In this case study however, our adopted decision rules are as stated in Tables 4.1 and 4.2 while Figure 4.4 on page 79 shows the rules as coded using Mamdani fuzzy inference system.

6. Establish relationships between input values and their fuzzy sets and applying the decision rules: Tables 4.1 and 4.2 show the FAM tables for the rule base and the fuzzy rule base can be coded into fuzzy inference system (FIS) using Matlab toolbox.

7. Play the game: The procedure for playing the game is as follows: The game state is represented as vector $[g, y, A_w, r]$. g represents green player's amount of resources, y represents yellow player's amount of resources, A_w represents green's accumulated wealth (profit) and r is the number of rounds the game

D/C_P	**Low**	**Medium**	**High**
Very Low	medium	very large	very large
Low	medium	large	large
Medium	small	medium	large
High	small	medium	medium

TABLE 4.2: FAM table for expected wealth created(E_w) as output.

is played. Green player strategy is denoted as $[C_g, W_g, M_g]$ and yellow player strategy is denoted as $[C_y, W_y, M_y]$ where:

$$C + W + M = 5. \tag{4.1}$$

Because the total resources of each player at any point is five. As explained in Section 1.3.1 on page 7 and Section 4.1 (page 63), our choices of the number five in Equation 4.1 and for variable r are arbitrary. In a real system, any number that suitably represents the process can be chosen.

General rules of the game are as follows:

- Initial stage of the game is $[5, 5, 0, 5]$ (i.e according to vector $[g, y, A_w, r]$)

- At every state $[g, y, A_w, r]$, green chooses his moves by allocating to his strategy $[C_g, W_g, M_g]$ where $C_g + W_g + M_g = g = 5$ and yellow who is the fuzzy agent chooses his strategy $[C_y, W_y, M_y]$ where $C_y + W_y + M_y = y = 5$.

- The game changes states as follows:

$$r = r - 1, \tag{4.2}$$

$$A_w = A_w + W_g - W_y, \tag{4.3}$$

$$g = g + C_g + M_g r - (y + C_y + M_y r), \tag{4.4}$$

$$y = y + C_y + M_y r - (g + C_g + M_g r), \tag{4.5}$$

$$temp = A_w + g - y; \tag{4.6}$$

Where temp represents game payoff. Then,

$$E_m = 5 - (E_w + E_c) \tag{4.7}$$

Because the total resources or expected resources of each player at any point is five. Now,

$$D = M_y/M_g, \qquad (4.8)$$

$$C_P = (M_y + C_y + k)/(M_g + C_g + k), \qquad (4.9)$$

Where k represents other costs which are taken to be zero to avoid needless complication (i.e $k = 0$). We define E_w (expected profit/Wealth)= $E_{sp} - C_P$, where

$$E_{sp} = E_w + C_P \qquad (4.10)$$

and E_{sp} represents the expected selling price of the product.

- The game ends when $r = 0$ and if *temp* is greater than zero ($temp > 0$), the green player wins, if less than zero ($temp < 0$), then the fuzzy agent player (yellow) wins else, the game is draw (i.e. if $temp = 0$).

- This 2-player game is a zero sum game and therefore, yellow loses whenever green wins and vice versa and since our aim is to develop an agent that would win as much as possible, maximize his payoff and minimize that of the opponents, Nash equilibrium [137, 138] is not considered in this context.

8. Evaluate the fuzzy inference system (FIS): Using Matlab fuzzy toolbox, all the fuzzy inputs are passed into the Mamdani type FIS.

9. Get the defuzzified output from the FIS: The crisp output for the FDMSB is computed using centre of gravity method (COG) and sampled results are as shown in Figure 4.5 using rule view from Matlab FIS editor and the surface view is as shown in Figure 4.6 on page 80.

10. Determine whether the conditions for the end of the game have been met: In this case study, the condition for the end of the game is when the number of rounds r reaches 1 counting down from 5 (i.e. when $r = 1$). This is the first loop in the FDMSB game as shown in Figure 4.7 on page 81.

11. Training and performance evaluation: As explained in Section 3.7 on page 53, training and learning [139] of the FDMSB decision agent was accomplished through the optimization of the fuzzy logic parameters while using the game payoff as the basis for the performance measure after playing a series of the game as in [13]. This training or learning of the fuzzy agent to optimize

its performance was achieved through the use of the *fminsearch* function in Matlab having considered other optimization algorithms such as gradient descent and genetic algorithm.

Learning or training of the fuzzy player forms the second loop in the game. The flowchart in Figure 4.7 on page 81 shows the two loops of the FDMSB game. The first loop stops when $r = 1$ (this means the fifth round of the game) and the second loop represents learning of the fuzzy player and it stops when the set performance criteria have been met.

Fminsearch uses the Nelder-Mead Simplex Search Method for finding the local minimum x of an unconstrained multivariable function $f(x)$ using a derivative-free method and starting at an initial estimate. This is generally referred to as unconstrained non-linear optimization. If n is the length of x, a simplex in n-dimensional space is characterized by the $n+1$ distinct vectors that are its vertices. In two-space, a simplex is a triangle; in three-space, it is a pyramid. At each step of the search, a new point in or near the current simplex is generated. The function value at the new point is compared with the function's values at the vertices of the simplex and, usually, one of the vertices is replaced by the new point, giving a new simplex. This step is repeated until the diameter of the simplex is less than the specified tolerance [131]. We maximized the fuzzy agent's payoff based on the fuzzy membership functions (MFs) and therefore, algorithm stops when opponent's wealth is minimized. However, during the algorithm, the membership functions need to retain a valid shape as shown in Figure 4.8 on page 82 in comparison with those in Figure 4.5 on page 79.

Meanwhile, a better optimization result may be achieved through *simulated annealing* [140, 141, 142, 143] but this is outside the focus of this research and may be considered as an avenue for further research.

Furthermore, in this FDMSB game, we do not employ a *maxmin* strategy but rather, we attempted to maximize the number of times that the fuzzy agent wins, and his payoff, while at the same time minimize those of the opponents.

Consider two players G and Y playing the game, the expected outcome or payoff of a game can be denoted as $E_x(G, Y)$, using the notation of [13]. As a training performance measure, the minimum expected payoff of an entire game taken over the class of all opponents S was used as in [13]. If the fuzzy

agent encounters the *strongest* opponent choice of strategy (say an opponent with strategy [0 0 5] as in iteration 12 in Table 4.3 (page 75)), the outcome of this play will result in a minimum payoff and the opponent may win the game. In [13], this very strict global performance measure was regarded as equity against globally optimizing opponent (*Geq*).

$$Geq(G) = \inf_{X \in S} \{E_x(G, X)\} \tag{4.11}$$

Another extreme opponent which may be regarded as weakest opponent will be that which reserves all his resources i.e. an opponent with strategy [0 5 0] (as in iteration 13 in Table 4.3 (page 75)) with respect to the strategic vector $[C \ \ W \ \ M]$, this results in FDMSB fuzzy agent winning the game with highest payoff and we regard this as *equity against a locally optimizing opponent (Leq).*

$$Leq(G) = \sup_{X \in S} E_x(G, X) \tag{4.12}$$

These combined *global* and *local* performance measures are the basis for the rating of our FDMSB decision agent.

The results of training are shown in Figure 4.8 on page 82. When compared to the output triangles of Figure 4.5 on page 79, it can be observed that after training, the membership functions (triangles) of the fuzzy sets have shifted considerably towards left to minimize the opponent's payoff and thereby maximize the fuzzy agent's payoff. Also, the surface view of the trained system is as shown in Figure 4.9 on page 82.

4.3 Results Discussion for 2-Player Games

Sampled results of a typical FDMSB experiment in accordance with the procedure highlighted above are as shown in Table 4.3 on page 75 . The pie chart in Figure 4.10 (page 83) and data on Table 4.3 (page 75) show that the fuzzy player (**Yellow**) was able to win more than the competitor (**Green**) because he made use of the fuzzy inference system in making his business decisions.

From equations 4.4 and 4.5 on page 67 and from the results in Table 4.3, it will be seen that for any of the players to win the game, he must allocate a substantial

part of his resources to aggressive marketing and this allocation must outweigh that of the opponent's allocation.

According to this model and with respect to the two equations, since the number of rounds r decreases as the game is played, this reduces the strength of marketing aggressiveness. An entrepreneur who is a new entrant into an industry, is best advised to try as much as possible to devote much of his resources on aggressive marketing campaigns (M) than other strategies (i.e. efforts on consolidation (C) and reserved wealth (W)). This will enable him to have a strong footing in the industry and to be able to have a large market share as early as possible as the game is played and thus, will result in winning the game.

However, because the fuzzy player is able to capture the uncertainty in the business environment more effectively and efficiently as a result of the fuzzy rules in the fuzzy inference system, he is able to override the system and wins more often than the opponent. From the results in column four and five of Table 4.3, out of thirteen iterations shown on the table, the fuzzy player (yellow) wins in eight iterations (iterations 1, 2, 3, 6, 9, 10, 11 and 13) while the opponent (green) wins in only five iterations which are iterations 4, 5, 7, 8 and 12.

As shown in columns six and seven of Table 4.3, we verified these results by designing control experiments (simulations) in which the fuzzy player does not change his moves in accordance with the fuzzy rule base. The results obtained from the control experiments show that the game follows conventional trends, that is, the fuzzy player wins only where he allocates more units of resources to his marketing strategy at the start of the game than those of his competitors and his payoff also depends on this. The payoff of the fuzzy player in the control experiments (where he did not use fuzzy rule base) are far less than what he got when he used fuzzy rule base to make his business decisions.

Moreover, after learning, as stated in Section 7.4.1 step 11 and as shown in Table 4.3 and pictured in Figure 4.11 on page 83, the fuzzy agent performs much better as the agent was able to win more than he won before training.

Results in columns eight and nine show the the performance of the players after learning (training). The columns show that after learning, out of the same thirteen iterations that were used before learning, the fuzzy player wins a total of ten iterations (additional wins of two iterations and these therefore means losses to the opponent) while the opponent wins only three iterations. After learning, the two

additional iterations won by the fuzzy player, as shown in the table, are iterations 4 and 8. Therefore, these means that the opponent has lose two additional iterations as a result of zero sum concept.

A typical example is when the two players chose $[4, 0, 1]$ and when they both chose $[3, 1, 1]$. In both cases and some other cases, before training, it was green that won the game while after training, it was the fuzzy agent (yellow) that won. Moreover, in all cases, even when green wins, his payoff ($temp$) is always smaller (minimized) after learning of the fuzzy agent than what it was before learning.

For examples, in iterations 5, 7 and 12 (the only three iterations where green player wins after learning), before leaning of the fuzzy player, green's payoffs were 351.6, 136.8 and 1054.5 for those three iterations respectively. However, after learning, these were reduced (minimized) by the **learned fuzzy player** and therefore, green's payoffs for those iteration become 302.2, 94.9, and 1012.0 respectively.

From the results explained above, it can be observed that training (learning) of the fuzzy agent was really important and the training algorithm was very effective because it enables the agent to learn and reach the performance criteria.

At the end of the game, the estimated price for the commodity can be forecast with Equation 4.10: ($E_{sp} = E_w + C_P$).

4.4   Conclusion

We have modelled decision making processes under uncertainty in business games, using fuzzy logic concepts and game theory. Our model was termed fuzzy decision making system for business games (FDMSB). We illustrated this for 2-player games that represent duopoly market structure. A fuzzy decision making system for business games was designed and implemented using Matlab software. Fuzzy rules were constructed in developing the FDMSB model using the Matlab toolbox and the implementation of this model heavily depends on expert knowledge and experience to facilitate the development of a reasonable fuzzy rule base for the determination of the if-then rules that denote the relationship between inputs and the output variables.

Furthermore, we have applied a learning algorithm to the decision processes which enables the decision agent to optimize his performance in the decision processes

as the games were played so as to meet the set criteria. To do the learning, the Nelder-Mead simplex method for finding the minimum of an unconstrained multivariable function was used.

Results of the learning showed that the learning algorithm works very effectively and efficiently as the fuzzy player (yellow) was able to perform much better after learning with higher payoffs and this enables him to reach the set criteria

We verified these results by designing a control experiment (simulation) in which the fuzzy player does not change his moves in accordance with the fuzzy rule base. The results obtained from the control experiments show that the game follows conventional trends, that is, the fuzzy player wins only where he allocates more units of resources to his marketing strategy at the start of the game than those of his competitors and his payoff also depends on this. The payoff of the fuzzy player in the control experiment (where he did not use fuzzy rule base) are far less than what he got when he used fuzzy rule base to make his business decisions.

Our FDMSB procedure has practical uses in business contexts as it can serve as very useful tools in the hands of an entrepreneur to:

- Advise him on certain marketing strategic decision policies that can keep his business in strategic advantage over his competitors in the market.

- Give him insight on how his firm can successfully compete with its peers in the market by determining how much of its available resources or efforts could be dissipated on our three adopted strategies of marketing in such a way that his profit (accumulated wealth) will be maximized.

- Effectively utilize the uncertain (fuzzy) and prevailing or anticipated market demand (D) information, cost of producing a commodity (C_P) and other fuzzy information at his disposal to achieve the set goal of his business.

Also, we have been able to supplement the laws of demand and supply with a more practical approach which takes into consideration the uncertain (fuzzy) nature of most information available to business decision makers. While the traditional laws of demand and supply address the nature of decision processes by consumers and suppliers respectively, our own approach extends them further. This is to address the nature of decision processes by an intending entrepreneur or manufacturer to forecast the prospect of the proposed business through profit prediction from

estimated selling price given the fuzzy market or industry information available to him. This allows him to determine price and marketing strategies in function of a *very low, medium, high, very high*, etc. demand.

In arriving at our results, the simulations are based on assumptions and conditions that the players involved in the decision processes are rational players (Section 2.1.1 on page 12) and that only the fuzzy player (yellow), at the moment, uses fuzzy moves [144]. This is in accordance with our overall aim of designing models that illustrate how an entrepreneur could make effective and efficient business decisions by using fuzzy inference systems (FIS) in capturing uncertainties that may surround his business environments. This will therefore help the entrepreneur to have competitive advantages over his competitors who are unaware of the usefulness of these tools and therefore are not making use of the fuzzy inference models in their decision making processes.

Agent Moves			Untrained		Control Expt		Trained	
S/N	Green	Yellow	Winner	Payoff	Winner	Payoff	Winner	Payoff
1	1, 1, 3	1, 0, 4	Yellow	-22.0	Yellow	-14.7	Yellow	-63.9
2	2, 1, 2	1, 1, 3	Yellow	-52.7	Yellow	-35.2	Yellow	-94.8
3	3, 0, 2	2, 0, 3	Yellow	-26.7	Yellow	-17.9	Yellow	-68.8
4	4, 0, 1	4, 0, 1	Green	40.8	Green	61.3	Yellow	-8.2
5	1, 0, 4	2, 0, 3	Green	351.6	Green	527.5	Green	302.2
6	3, 1, 1	4, 0, 1	Yellow	-16.1	Yellow	-10.7	Yellow	-65.2
7	3, 0, 2	2, 1, 2	Green	136.8	Green	205.2	Green	94.9
8	3, 1, 1	3, 1, 1	Green	14.8	Green	22.34	Yellow	-34.2
9	3, 1, 1	2, 0, 3	Yellow	-289.3	Yellow	-192.9	Yellow	-305.0
10	0, 5, 0	1, 4, 0	Yellow	-99.8	Yellow	-66.6	Yellow	-142.2
11	0, 5, 0	0, 1, 4	Yellow	-704.8	Yellow	-469.9	Yellow	-747.2
12	0, 0, 5	0, 5, 0	Green	1054.5	Green	1581.8	Green	1012.0
13	0, 5, 0	0, 0, 5	Yellow	-863.8	Yellow	-575.9	Yellow	-906.2

TABLE 4.3: **Results of simulations of the untrained and trained agent in 2-player game**: From the table, the first column shows the serial numbers of the iterations, the second column contains player *green's* strategies while the third column contains that of *yellow*. For example, in the first iteration, *green's* strategy shows $[1, 1, 3]$, this indicates how resources are allocated to strategy $[C, W, M]$: $C = 1$, $W = 1$ and $M = 3$. The forth column gives the winners for the untrained simulations while the fifth column gives the payoffs of those simulations. Column six shows the winners for the *control experiment* simulations while the payoffs of players for those simulations are displayed in column seven. The control experiments show the results where both players did not use fuzzy inference systems in playing the games. Column eight shows the winners for the *trained* simulations while the payoffs of players for those simulations are displayed in column nine. These results show that that the fuzzy player (**Yellow**) was able to win more than the competitor (**Green**) because he made use of the fuzzy inference system in making his business decisions. Also, it can be observed that the trained agent is able to perform better after training as he wins more often than when he was not trained and where he does not win, opponent's payoff is minimized considerably and thereby maximized his own. The strongest opponent (*Geq*) and weakest opponent (*Leq*) are shown in iterations 12 and 13 respectively. The minus sign on yellow payoffs merely shows zero-sum. These results are as summarized in the pie charts of Figure 4.10 (page 83) and Figure 4.11 (page 83).
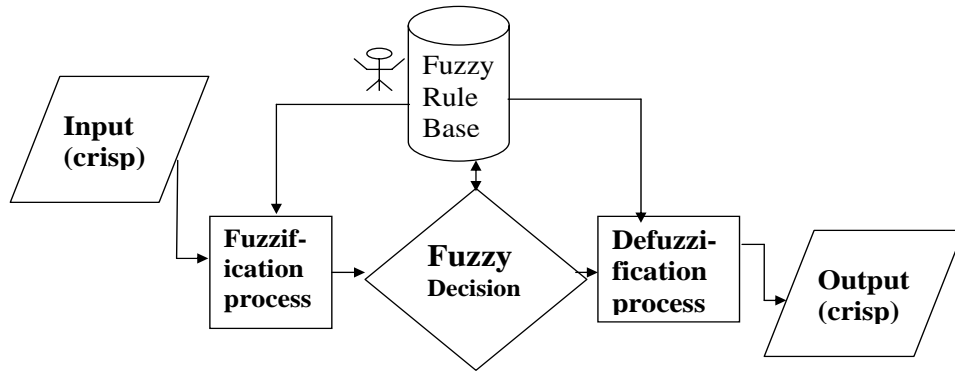
(a) Fuzzy decision making system (FDMS)
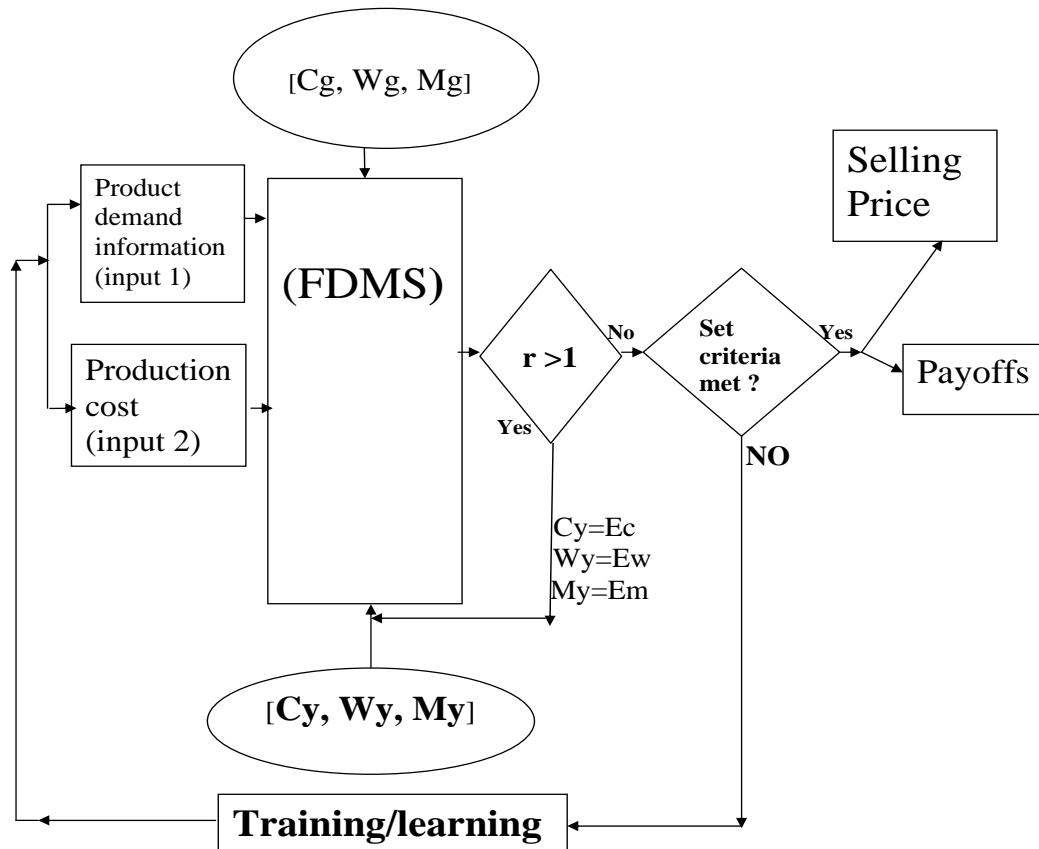
FIGURE 4.1: (a)Fuzzy decision making system (FDMS), (b)Fuzzy decision making system for business games (FDMSB) model.
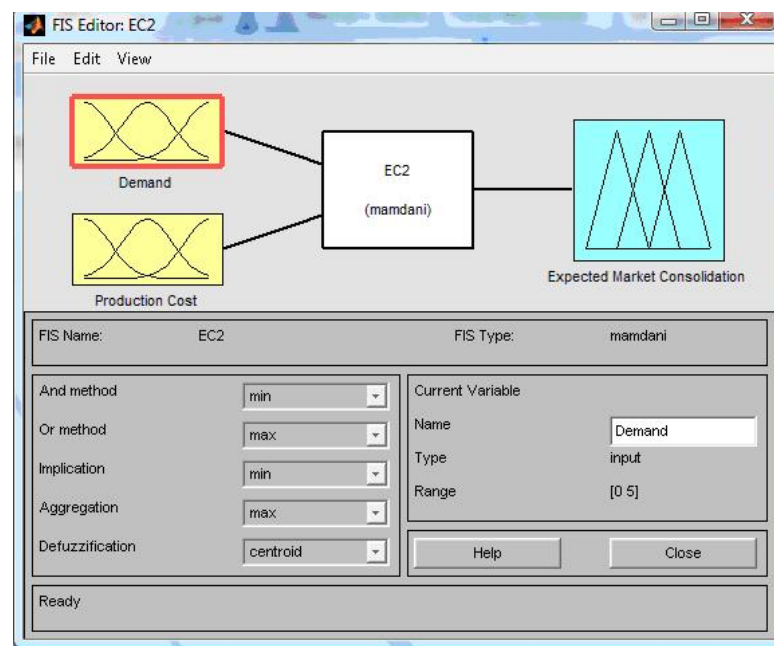
FIGURE 4.2: Mamdani fuzzy inference system (FIS) of the results of the simulation.
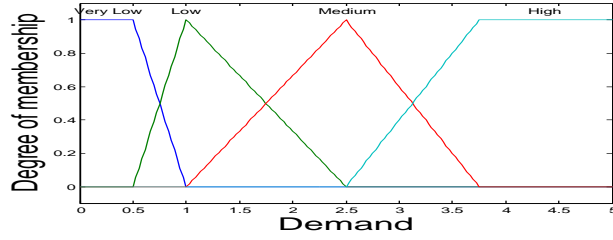
(a) Demand
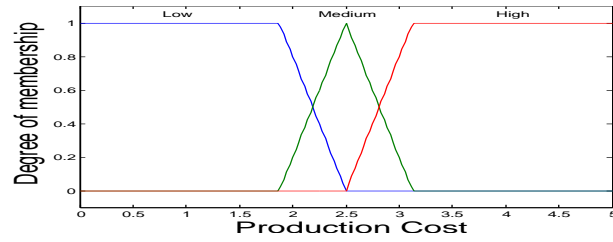
(b) Production cost
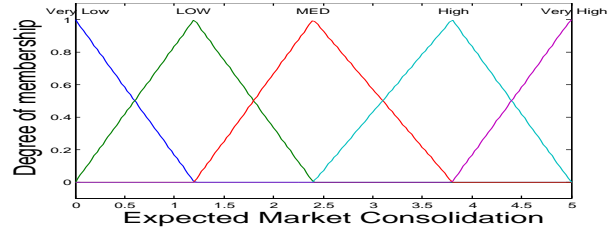
(c) Expected consolidation effort
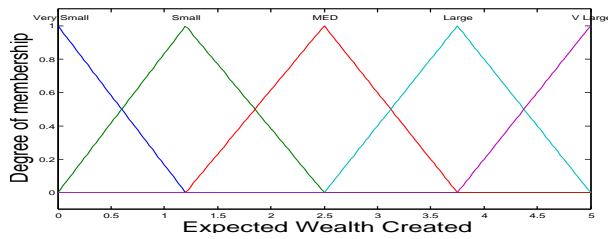
(d) Expected wealth

FIGURE 4.3: Membership functions for fuzzy variables of FDMSB rule base-inputs: Demand(D) and Production cost(C_P) and outputs:Expected consolidation efforts(E_c) and Expected wealth(E_w).
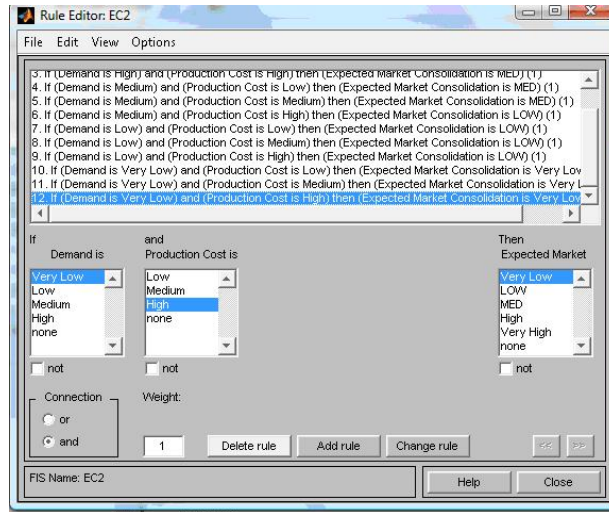
FIGURE 4.4: Rule base with Matlab rule editor for *expected consolidation efforts*(Ec) as output.
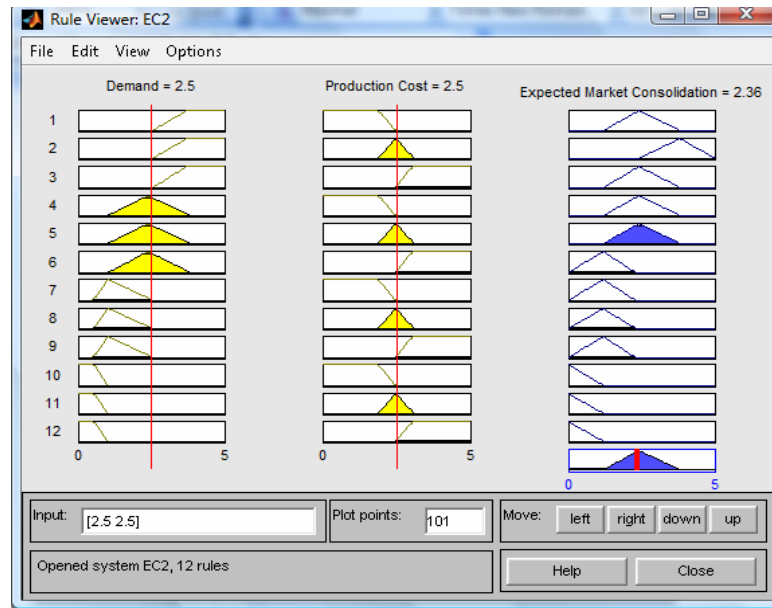
FIGURE 4.5: Defuzzified (crisp) values for *expected wealth generated Ew* at inputs $D = C_P = 2.5$.
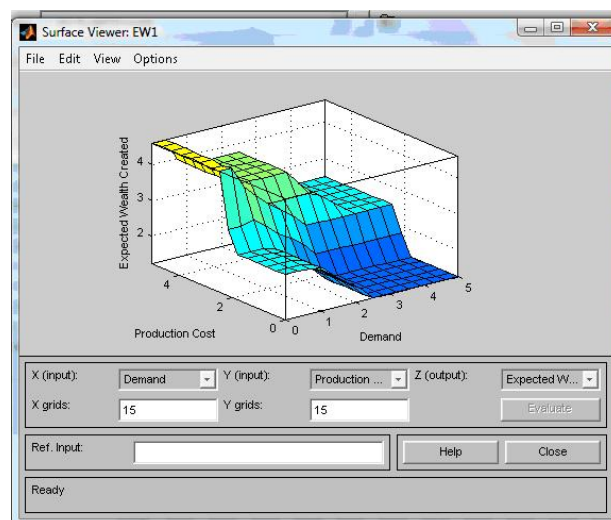
FIGURE 4.6: Surface views of *expected wealth generated Ew*.
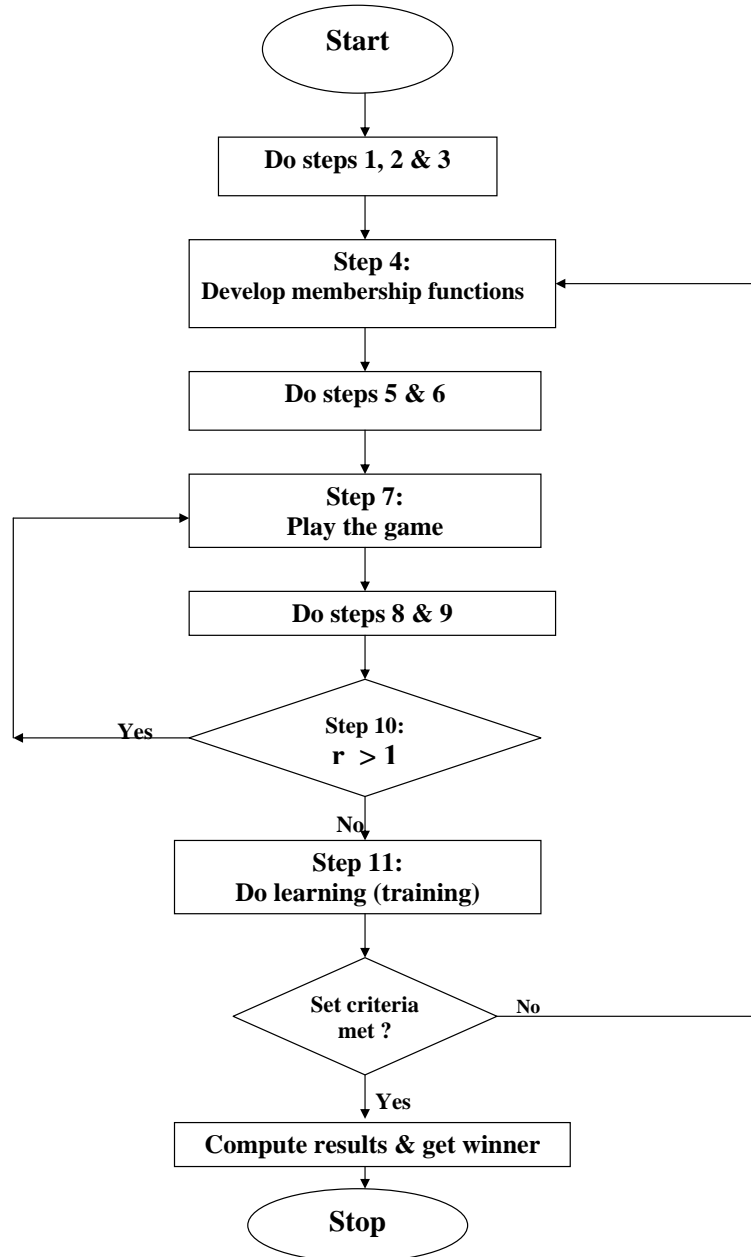
FIGURE 4.7: Chart showing the two loops of the FDMSB game. The first loop stops when $r = 1$ (this means the fifth round of the game) and the second loop represents learning of the fuzzy player and it stops when the set performance criteria have been met as explained in step 11 on page 68.
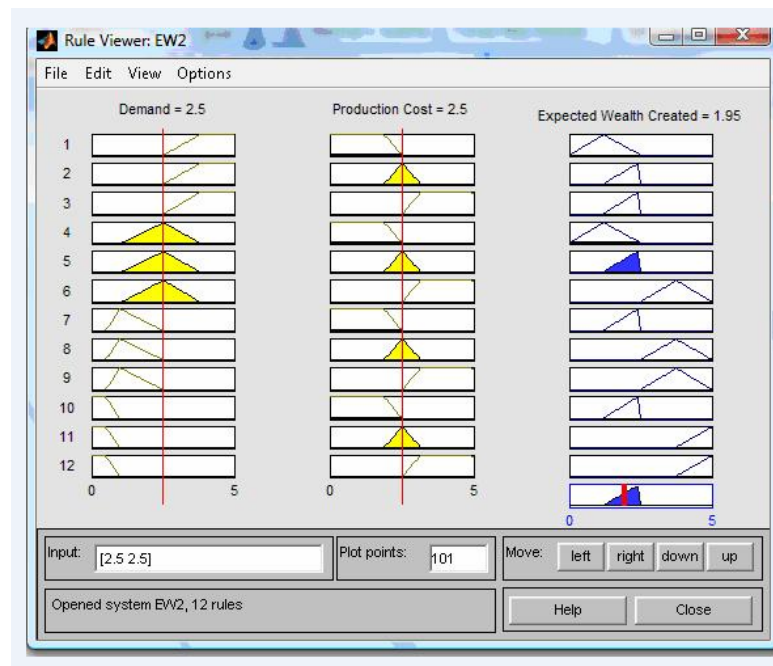
FIGURE 4.8: Output of the trained FDMSB fuzzy agent: It can be seen that the triangles of the membership functions have changed considerably and thereby minimized opponent's wealth accordingly.
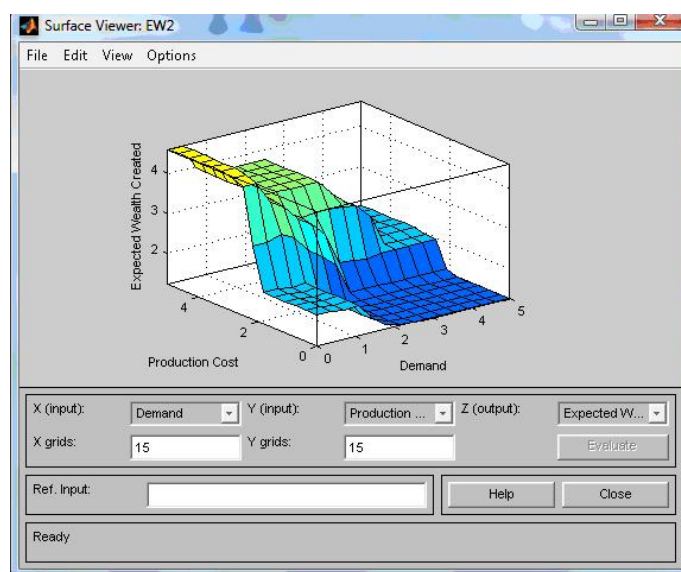
FIGURE 4.9: Surface view of *expected wealth generated Ew* after training.

Yellow (Fuzzy Player) Vs Green Wins
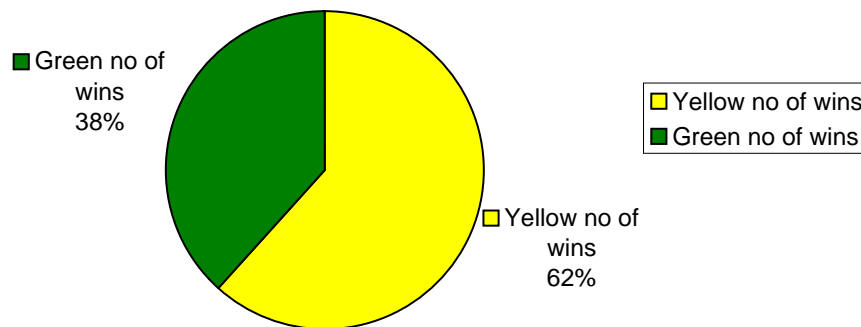
FIGURE 4.10: Results show that the fuzzy player (yellow) wins more often than the competitor (Green) because he made use of the fuzzy inference system (FIS) in making his business decisions from the results in Table 4.3.

Wins of Untrained and Trained Fuzzy Players
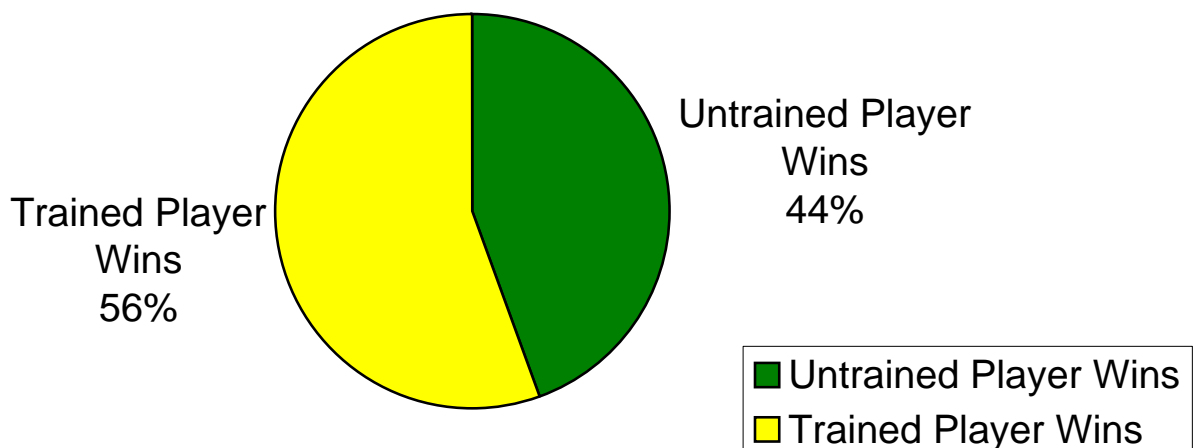
FIGURE 4.11: This chart shows how the performance of the fuzzy player increased after training as it won more often than it won before training from the results in Table 4.3 of page 75.

Chapter 5

FDMSB Case Study

5.1 PepsiCo Vs Coca-Cola Company

In this chapter, we illustrated the FDMSB model by taking competition in the beverage industry as a case study and this will be between Coca-Cola and PepsiCo who are the major players in the industry. Since these two companies are well known, we shall give no further introduction on them. We chose the two companies after we have considered companies in other industries but most of them do not have uniform means of reporting their financial data which made their data comparison very difficult

In running the FDMSB simulations, we used the companies' data available in their annual financial statements for the year 2003 as our initial values and input for the first round (year 2004). We then ran the FDMSB simulation for five rounds (representing five years) according to the procedures highlighted in Section 4.2 (page 65) and we compared the results obtained in the simulations to the two companies' data published for the year 2008. According to the published annual financial statements, PepsiCo had lower profits (payoffs) in both years. The chart in Figure 5.1 compares their profits for that year. Therefore, we took PepsiCo as our fuzzy player. After five rounds, we compared the results obtained to those which were published in the 2008 financial year and we discovered that had PepsiCo implemented our FDMSB approach, it would have outperformed its rival (Coca-Cola) with higher profit (payoff) in the year 2008 and eventually would have won the cola war.

As we explained in Section 1.3.1 (page 7) and Section 4.1 (page 63), the variables used in Section 4.2 (page 65) are modified to suit the situation in question and for this purpose, the following variables were used in the simulations: working capital (W), equity turnover or working capital turnover (W_t), current assets (c_a), current liabilities (c_l), market capitalization (m_c), profit margin (P_m), profit or net income (A_w), sales or revenue (L), cost or expenditure (C). The number of rounds ($r = 5$) the game is played represents years 2004 to 2008.

Also, we made use of the following standard accounting ratios and formulas for the variables:

Working Capital (W) *equals* Current Assets (c_a) *minus* Current liabilities (c_l),

$$W = c_a - c_l$$

Cost or expenditure (C) *equals* cost of sales C_{os} *plus* Selling, general and administrative expenses S_{ga}.

$$C = C_{os} + S_{ga}$$

Profit Margin (P_m) *equals* Net Income(profit A_w) *divided by* Sales or Revenue (L)

$$P_m = A_w/L. \tag{5.1}$$

Working Capital Turnover (W_t) *equals* Sales(L) *divided by* Working Capital (W)

$$W_t = L/W. \tag{5.2}$$

Our inputs, in this case study, for the fuzzy inference system (FIS) in the FDMSB model in Figure 4.1(b) (page 76) are profit margin (P_m) and working capital turnover (W_t). Input strategies for the two companies are cost (C), working capital (W), and sales (L) represented by vector $[C\ W\ L]$ for each player. Values for these variables are extracted from the two companies' financial statement for the year 2003 which were used as input data for the year 2004 (which represents the first round of the game) and are as shown in Table 5.1. Output strategies of the inference system of the FDMSB are expected cost (E_c), expected working capital (E_w) and expected sales (E_L) represented as vector $[E_c E_w E_L]$. Coca-Cola company is represented as the green player (g) with strategies $[C_g W_g L_g]$ while PepsiCo is represented as yellow player (y) with strategies $[C_y W_y L_y]$. After each round of

S/N	Variables	Coca-Cola	PepsiCo
1	Cost or expenditure (C)	15,250	21,839
2	Working capital (W)	24,825	23,625
3	Sales(L)	21,044	26,971
4	Working capital turnover (W_t)	0.8476	1.1416
5	Net income or profit (A_w)	4,347	3,568
6	Profit margin (P_m)	0.2065	0.1323

TABLE 5.1: Data and variables used in the simulation were obtained from year 2003 financial statement of PepsiCo and Coca-Cola companies. A pie chart that shows the difference in their profits is as shown in Figure 5.1.

the game, PepsiCo, who is the fuzzy agent, changes his strategies to the output of the FIS $[E_c E_w E_L]$ and this is used for the next round of the game. A simple model that shows the relationship between the input and the output variables of the FDMSB is as shown in Figure 5.2 on page 87.

Profits of Coca-cola and PepsiCo in 2003
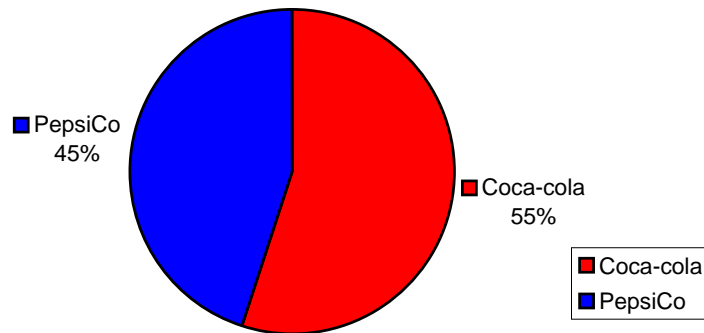
FIGURE 5.1: This chart compares Coca-Cola and PepsiCo profits published for the year 2003.

The procedures for the simulations follow those highlighted in Section 4.2 on page 65. Following these procedures, steps 4, 5 and 6 are now illustrated in the membership functions shown in Figure 5.3 (page 88). The decision rules for the rule base, as stated before, would normally be solicited from experts. In this case study however, our decision rules are as stated below:
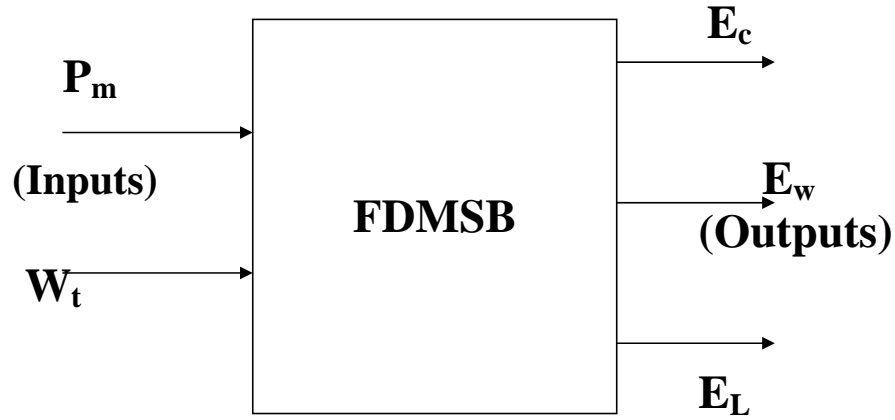
FIGURE 5.2: A basic model of the FDMSB engine showing the two inputs P_m, W_t and the three outputs E_c, E_w and E_L.

Rule Base 1:

1. If (Profit Margin is High) and (Working Capital Turnover is Low) then (Expected Cost or expenditure is Slightly Low)

2. If (Profit Margin is High) and (Working Capital Turnover is Medium) then (Expected Cost or Expenditure is Low)

3. If (Profit Margin is High) and (Working Capital Turnover is High) then (Expected Cost or Expenditure is Slightly Low)

4. If (Profit Margin is Medium) and (Working Capital Turnover is Low) then (Expected Cost or Expenditure is Slightly Low)

5. If (Profit Margin is Medium) and (Working Capital Turnover is Medium) then (Expected Cost or Expenditure is Slightly Low)

6. If (Profit Margin is Medium) and (Working Capital Turnover is High) then (Expected Cost or Expenditure is Very Low)

7. If (Profit Margin is Low) and (Working Capital Turnover is Low) then (Expected Cost or Expenditure is Very Low)

8. If (Profit Margin is Low) and (Working Capital Turnover is Medium) then (Expected Cost or Expenditure is Very Low)

9. If (Profit Margin is Low) and (Working Capital Turnover is High) then (Expected Cost or Expenditure is Very Low)
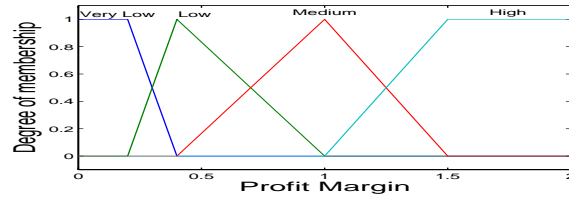
(a) Profit Margin
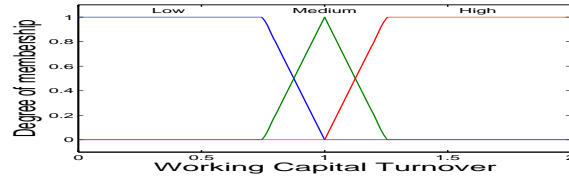
(b) Working Capital Turnover
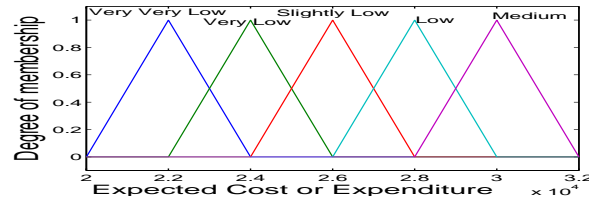
(c) Expected Cost
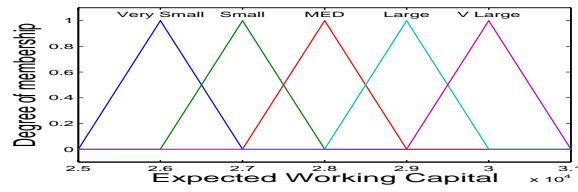
(d) Expected Working Capital
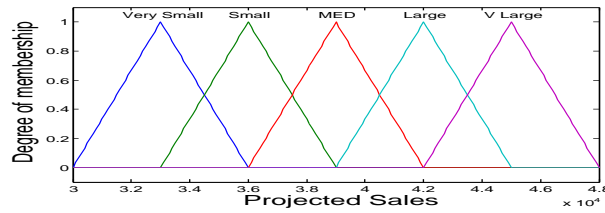
(e) Expected Sales

FIGURE 5.3: Membership functions for fuzzy variables of FDMSB rule base-inputs: Profit margin(P_m) and Working capital turnover (W_t) and outputs:Expected Cost or Expenditure(E_c), Expected working capital E_w and Expected sales(E_L).

10. If (Profit Margin is Very Low) and (Working Capital Turnover is Low) then (Expected Cost or Expenditure is Very Very Low)

11. If (Profit Margin is Very Low) and (Working Capital Turnover is Medium) then (Expected Cost or Expenditure is Very Very Low)

12. If (Profit Margin is Very Low) and (Working Capital Turnover is High) then (Expected Cost or Expenditure is Very Very Low)

Rule Base 2

1. If (Profit Margin is High) and (Working Capital Turnover is Low) then (Expected Working Capital is Small)

2. If (Profit Margin is High) and (Working Capital Turnover is Medium) then (Expected Working Capital is Medium)

3. If (Profit Margin is High) and (Working Capital Turnover is High) then (Expected Working Capital is Medium)

4. If (Profit Margin is medium) and (Working Capital Turnover is Low) then (Expected Working Capital is Small)

5. If (Profit Margin is medium) and (Working Capital Turnover is Medium) then (Expected Working Capital is Medium)

6. If (Profit Margin is medium) and (Working Capital Turnover is High) then (Expected Working Capital is Large)

7. If (Profit Margin is Low) and (Working Capital Turnover is Low) then (Expected Working Capital is Medium)

8. If (Profit Margin is Low) and (Working Capital Turnover is Medium) then (Expected Working Capital is Large)

9. If (Profit Margin is Low) and (Working Capital Turnover is High) then (Expected Working Capital is Large)

10. If (Profit Margin is Very Low) and (Working Capital Turnover is Low) then (Expected Working Capital is Medium)

11. If (Profit Margin is Very Low) and (Working Capital Turnover is Medium) then (Expected Working Capital is Very Large)

12. If (Profit Margin is Very Low) and (Working Capital Turnover is High) then (Expected Working Capital is Very Large)

Rule Base 3

1. If (Profit Margin is High) and (Working Capital Turnover is Low) then (Expected Sales is Small)

2. If (Profit Margin is High) and (Working Capital Turnover is Medium) then (Expected Sales is Medium)

3. If (Profit Margin is High) and (Working Capital Turnover is High) then (Expected Sales is Medium)

4. If (Profit Margin is medium) and (Working Capital Turnover is Low) then (Expected Sales is Small)

5. If (Profit Margin is medium) and (Working Capital Turnover is Medium) then (Expected Sales is Medium)

6. If (Profit Margin is medium) and (Working Capital Turnover is High) then (Expected Sales is Large)

7. If (Profit Margin is Low) and (Working Capital Turnover is Low) then (Expected Sales is Medium)

8. If (Profit Margin is Low) and (Working Capital Turnover is Medium) then (Expected Sales is Large)

9. If (Profit Margin is Low) and (Working Capital Turnover is High) then (Expected Sales is Large)

10. If (Profit Margin is Very Low) and (Working Capital Turnover is Low) then (Expected Sales is Medium)

11. If (Profit Margin is Very Low) and (Working Capital Turnover is Medium) then (Expected Sales is Very Large)

12. If (Profit Margin is Very Low) and (Working Capital Turnover is High) then (Expected Sales is Very Large)

The fuzzy rules are coded using Matlab software as shown in Figure 5.4 (page 91). The Mamdani type fuzzy inference system (FIS) shown in Figure 5.5 (page 91) shows the basic input/output system of the FDMSB model for the rule base 3 while samples of the defuzzified outputs are as shown in Figure 5.6 (page 93) with surface view in Figure 5.7 (page 93).
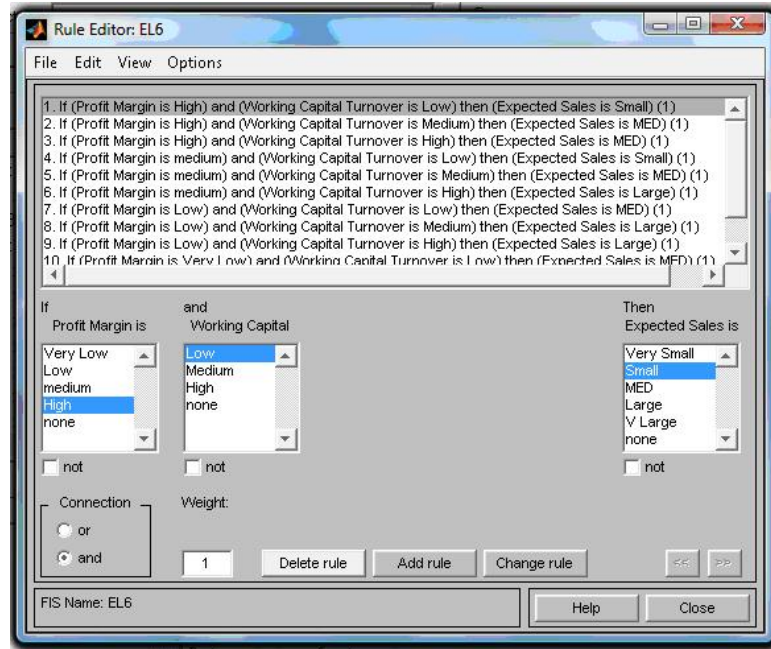
FIGURE 5.4: The rule base 3 coded using Matlab software.
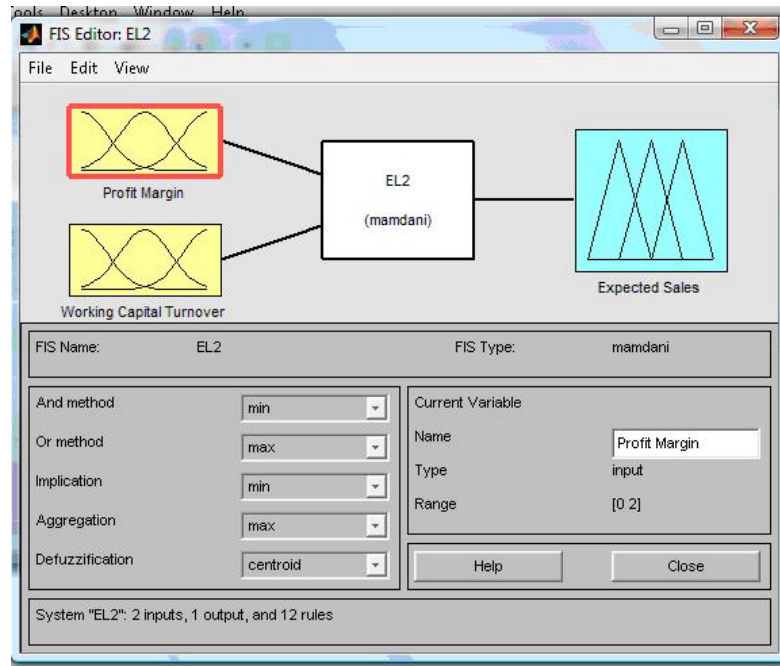
FIGURE 5.5: The fuzzy inference system for rule base 3 of FDMSB case study.

- Step 7, which explains how players change strategies, goes as follows:

  The game state (S) is represented as vector

  $$S = [g, y, A_{wg0}, A_{wy0}, r],$$

  where g and y represent initial resources of green and yellow respectively,

A_{wg0} is the green (Coca-Cola) initial (2003) net income or profit, A_{wy0} represents yellow (PepsiCo) initial (2003) net income or profit. Then

$$g = W_g + L_g - C_g, \tag{5.3}$$

$$y = W_y + L_y - C_y. \tag{5.4}$$

These resources change in each round of the games as follows:

$$g = W_g/W_y + L_g r/L_y - C_g/C_y, \tag{5.5}$$

$$y = W_y/W_g + L_y r/L_g - C_y/C_g. \tag{5.6}$$

At the end of each round, the new state (S) of the game becomes:

$$S = [g, y, A_{wg}, A_{wy}, r]$$

New profit (payoff or Net income) of yellow (A_{wy}) *equals* expected sales (E_L) *minus* expected cost E_c (outputs of FIS):

$$A_{wy} = E_L - E_c$$

and that of green implies: Net profit (A_{wg}) *equals* Sales of green (L_g) *minus* Cost of green (C_g):

$$A_{wg} = L_g - C_g$$

After each round, strategies of the fuzzy player change from $[C_y W_y L_y]$ to output of FIS: $[E_c E_w E_L]$.

The game ends when $r = 0$ and if payoff (profit or net income) of y (A_{wy}) is greater than that of g (A_{wg}), then PepsiCo (yellow) who is the fuzzy player wins. If A_{wy} is less than A_{wg}, then PepsiCo lost and if A_{wg} is equal to A_{wy} then the game is a draw.

- After this modified step 7, the rest of the game follows the rest of the procedural steps in Section 4.2 on page 65. Sampled defuzzified outputs are as shown in Figure 5.6 (page 93) and the surface view in Figure 5.7 (page 93).
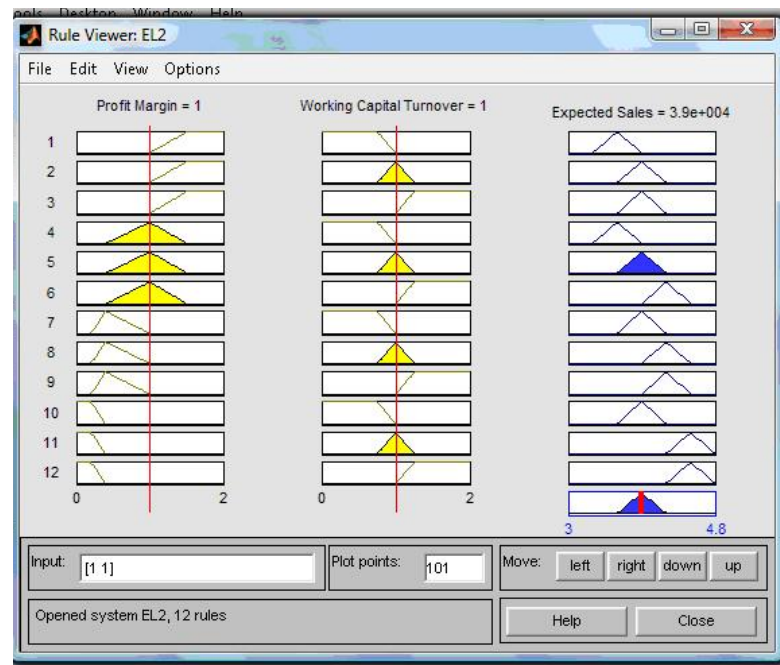
FIGURE 5.6: Defuzzified (crisp) output of the FDMSB.
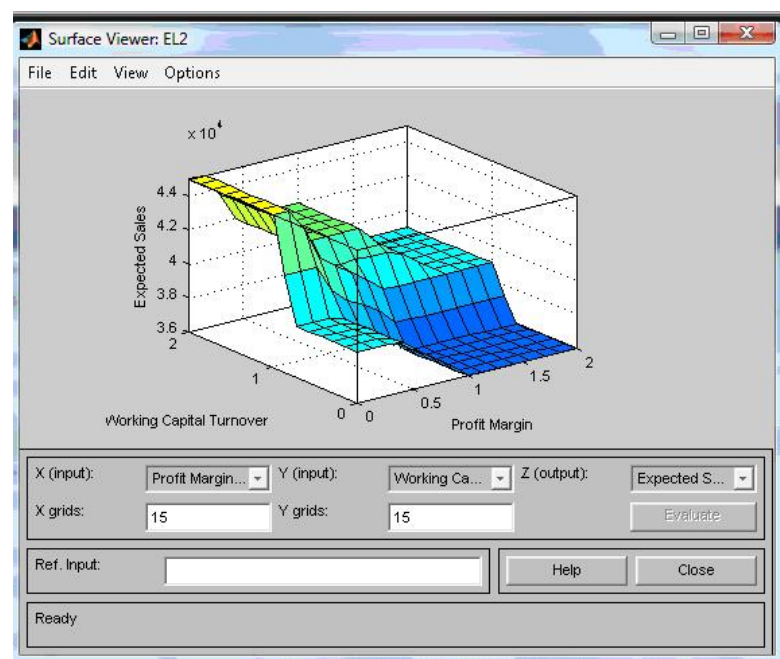
FIGURE 5.7: Surface view of the FDMSB case study.

5.2 Results of FDMSB Case Study of the Cola War

Recorded (published) data of the two companies in year 2008 (that is, after five years) are as shown in Table 5.2 on page 94.

S/N	Variables	Coca-Cola	PepsiCo
1	Cost or expenditure (C)	23,148	36,252
2	Working capital (W)	37,738	28,136
3	Sales(L)	31,944	43,251
4	Working capital turnover (W_t)	0.8465	1.5372
5	Net income or profit (A_w)	5,807	5,142
6	Profit margin (P_m)	0.1818	0.1189

TABLE 5.2: This table shows the data published by the two companies in their year 2008 financial statements. Pie chart in Figure 5.8 (page 94) also compares their published profits. When these are compared to the results of FDMSB simulations shown in Table 5.3 (page 95) as summarised in Table 5.4 (page 95), it would be observed that even though, both players performed better in the game, however, the fuzzy player PepsiCo won the game with much higher profits.

Profits of Coca-cola and PepsiCo in 2008
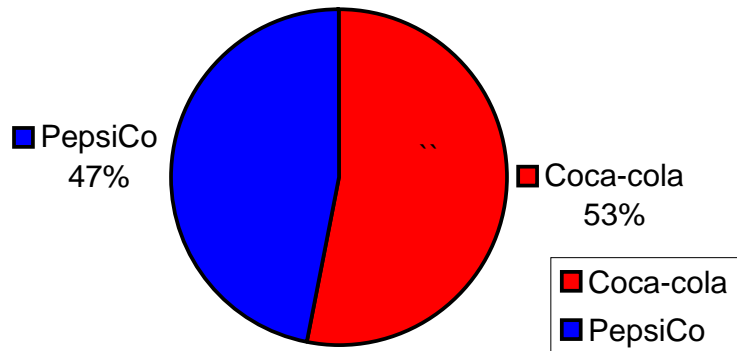
FIGURE 5.8: This chart compares Coca-Cola and PepsiCo profits published for year 2008. We then compared these with the results of our model as shown in Figure 5.10. We concluded that had PepsiCo implemented our model, it would have substantively won the Kola War.

Results of the FDMSB simulations after five rounds which represent five years (year 2008) are as shown in Table 5.3 on page 95. This has been compared to the results of the FDMSB simulations in Table 5.4 on page 95.

It was observed that both players performed better in the game while the fuzzy player who made use of the fuzzy reasoning performed much better despite his weaker initial 2003 financial data that were used as input and starting strategies

S/N	Variables	PepsiCo
1	Cost or expenditure (C)	23,999
2	Working capital (W)	29,000
3	Sales(L)	42,001
4	Working capital turnover (W_t)	1.4483
5	Net income or profit (A_w)	18,002
6	Profit margin (P_m)	0.4286

TABLE 5.3: This table shows the results of FDMSB simulation after five rounds and when compare the results to the data published by PepsiCo at the end of year 2008 (shown in Table 5.4), it will be observed that the fuzzy player (PepsiCo) performed better in terms of the profit gained (game payoffs), and eventually won the cola war due to his ability to grasp the uncertainties in the market through the implementation of the fuzzy rules and reasoning aided by FDMSB framework.

Variables	Coca-cola	PepsiCo	$PepsiCo^1$
Cost or expenditure (C)	23,148	36,252	23,999
Working capital (W)	37,738	28,136	29,000
Sales(L)	31,944	43,251	42,001
Working capital turnover (W_t)	0.8465	1.5372	1.4483
Net income/profit (A_w)	5,807	5,142	18,002
Profit margin (P_m)	0.1818	0.1189	0.4286

TABLE 5.4: Data published by the two companies in 2008 statements (in columns 2 and 3) and when compared to the results of FDMSB simulation in column 4, fuzzy player (PepsiCo) has better payoffs in terms of the profit gained, and eventually won the cola war due to his ability to grasp the uncertainties in the market through implementation of fuzzy rules and reasoning aided by FDMSB framework.

for the first (2004) round of the game. Therefore, we concluded that if PepsiCo or any company could make use of our model, they will be able to perform much better while competing with their peers in the market and possibly win the market.

5.3   Conclusion

We have modelled decision making processes under uncertainty in business games, using fuzzy logic concepts and game theory. We have shown that our model works very effectively and efficiently in a real world by using a case study of the competition between Coca-Cola and PepsiCo companies. These two companies are the major players in the beverage industry.

**Cola-Cola 2008 Published Profit Vs PepsiCo
Simulated Profit from FDMSB Model**
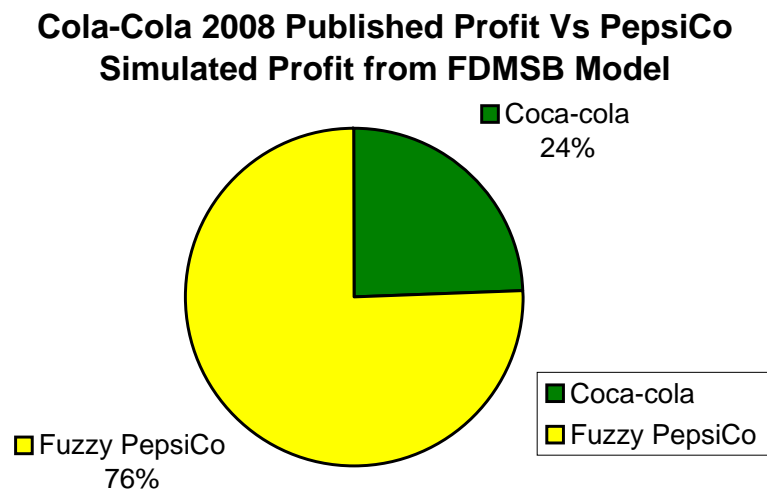
FIGURE 5.9: Results shown on this chart compares the profit from Coca-Cola 2008 published data and that of the simulated profit for PepsiCo from the FDMSB game after five rounds (years) starting from year 2003. It was observed that PepsiCo had higher profits in our model because it made use of fuzzy rules aided by the fuzzy inference system (FIS) in making its business strategic decisions.

Our model was termed fuzzy decision making system for business games (FDMSB). A fuzzy decision making system for business games was designed and implemented using Matlab software. Fuzzy rules were constructed in developing the FDMSB model using the Matlab toolbox and the implementation of this model heavily depends on expert knowledge and experience to facilitate the development of a reasonable fuzzy rule base for the determination of the **if-then** rules that denote the relationship between inputs and the output variables.

In running the FDMSB simulations, we used the companies' data available in their annual financial statements for the year 2003 as our initial values and input for the first round (year 2004). We then ran the FDMSB simulations for five rounds (representing five years) and we compared the results obtained in the simulations to the two companies' data published for the year 2008. According to the published annual financial statements, PepsiCo had lower profits (payoffs) in both years. Therefore, we took PepsiCo as our fuzzy player. After five rounds, we compared the results obtained to those which were published in the 2008 financial year and we discovered that had PepsiCo implemented our FDMSB approach, it would have outperformed its rival (Coca-Cola) with higher profit (payoff) in the year 2008 and eventually would have won the cola war.

**PepsiCo Published 2008 Profit Margin
and Simulated Fuzzy Profit Margin**
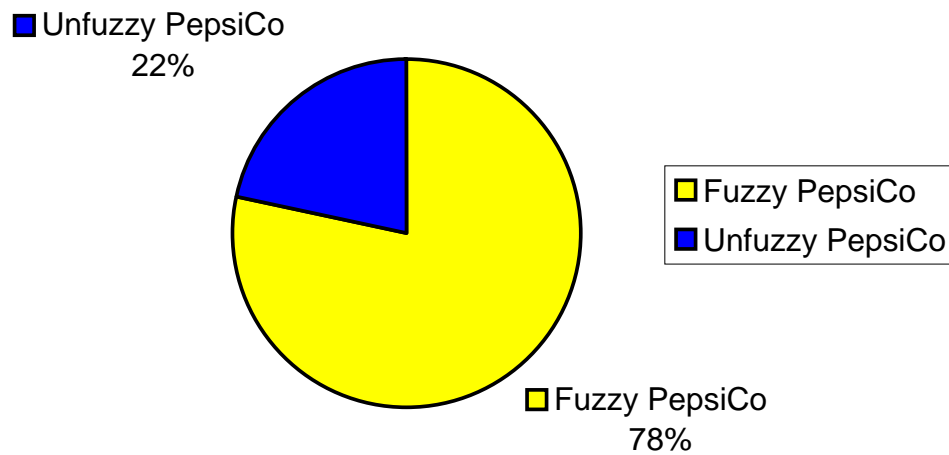
FIGURE 5.10: This chart compares PepsiCo profit margin that came out our simulation and compares it to that published by the company for year 2008. It was observed that PepsiCo had higher profits magin in our model because it made use of fuzzy rules aided by the fuzzy inference system (FIS) in making its business strategic decisions.

From the results of the FDMSB simulations after five rounds, which represents five years (from year 2003 to 2008), the fuzzy player (PepsiCo) who made use of the fuzzy reasoning performed much better than its major competitor despite his weaker initial 2003 financial data that were used as input and starting strategies for the first (2004) round of the game. Therefore, we concluded that if PepsiCo or any company could make use of our model, they will be able to perform much better while competing with their peers in the market and possibly win the market.

We have further shown that the variables in our FDMSB model can be tailored to the business situations in the real world and therefore are not limited to those variables that we have used in designing the system as we explained in Section 1.3.1 on page 7 and Section 4.1 (page 63). Therefore, this model can now be applied to any real business situation.

We showed that our model can also work for systems that have more (and varied) strategic variables than those that we have used in our general business model in chapter 4.

Furthermore, we have applied a learning algorithm to the decision processes which enables the decision agent to optimize his performance in the decision processes as the games were played so as to meet the set criteria. To do the learning,

the Nelder-Mead simplex method for finding the minimum of an unconstrained multivariable function was used.

Results of the learning showed that the learning algorithm works very effectively and efficiently as the fuzzy player (PepsiCo) was able to perform much better after learning with higher payoffs and this enables him to reach the set criteria.

In arriving at our results, the simulations are based on assumptions and conditions that the players involved in the decision processes are rational players (Section 2.1.1 on page 12) and that only the fuzzy player (PepsiCo), at the moment, uses fuzzy moves [144]. This is in accordance with our overall aim of designing models that illustrate how an entrepreneur could make effective and efficient business decisions by using fuzzy inference systems (FIS) in capturing uncertainties that may surround his business environments. This will therefore help the entrepreneur (PepsiCo) to have competitive advantages over his main competitor (Coca-Cola) who is unaware of the usefulness of these tools and therefore is not making use of the fuzzy inference models in his decision making processes.

Chapter 6

N-Player Game

6.1 Procedures for n-Player Game

In this chapter, we examined n-player games that represent perfect market competitions with many players (please see page 30). Our fuzzy player is still represented as *yellow*, the n-th player, who faces $n-1$ opponent players (competitors). The n-player games also follow the procedural steps of 2-player FDMSB general illustrations in Section 4.2 (page 65) with exceptions to steps 2 and 7 which are modified as follows:

- Step (2) Determining the strategy: as an example of a perfect market competition, we have n players. For $j = 1$ to $n-1$, the opponents $P(j)$ strategies are denoted as $[C(j), W(j), M(j)]$ and the fuzzy agent (yellow) strategy as $[C(n), W(n), M(n)]$.

- Step (7) Play the game: procedures for playing the game are as follows: The game state is represented as vector $S = [P_1, P_2, \cdots, P_{n-1}, P_n, A_w, r]$. Where P_1 to P_{n-1} represent opponent players' (competitors) amount of resources, P_n represents fuzzy agent player (yellow) amount of resources, A_w represents opponents' accumulated wealth (profit) and r is the number of rounds the game is played. Both the competitors and fuzzy player strategy are as stated in step 2 above (page 99) and:

$$C(j) + W(j) + M(j) = P(j) = 5. \tag{6.1}$$

As explained in Section 1.3.1 on page 7 and Section 4.1 (page 63), our choices of the number five in Equation 6.1 and for variable r are arbitrary. In a real system, any number that suitably represents the process can be chosen.

General rules of the game are as follows:

- Initial state of the game is $[5, 5, \cdots, 5, 5, 0, 5]$ according to the vector $[P_1, P_2, \cdots, P_{n-1}, P_n, A_w, r]$.

- At every state $[P_1, P_2, \cdots, P_{n-1}, P_n, A_w, r]$, for $j = 1$ to $n - 1$, the opponents $P(j)$ choose their moves (strategies) $[C(j), W(j), M(j)]$ where: $C(j) + W(j) + M(j) = P(j) = 5$ and yellow who is the fuzzy player chooses his strategy $[C(n), W(n), M(n)]$.

- The game changes states as follows:

$$While \ r > 0$$

$$for \ j = 1 \ to \ n$$

$$A_w = A_w + \sum_{j=1}^{n-1} W(j) - W(n), \qquad (6.2)$$

Where W(n) is the fuzzy agent's wealth

$$P(j) = P(j) + C(j) + M(j)r - (\sum_{i=1}^{n} P(i) -$$

$$P(j) + \sum_{i=1}^{n} C(i) - C(j) + (\sum_{i=1}^{n} M(i) - M(j))r, \qquad (6.3)$$

$$temp = A_w + \sum_{j=1}^{n-1} P(j) - P(n), \qquad (6.4)$$

$$E_m = 5 - (E_w + E_c),$$

$$D = M(n)/(\sum_{j=1}^{n-1} M(j)), \qquad (6.5)$$

$$C_P = (M(n) + C(n) + K)/((\sum_{j=1}^{n-1} M(j))$$

$$+ (\sum_{j=1}^{n-1} C(j)) + K, \qquad (6.6)$$

$$E_{sp} = E_w + C_P$$

.

- – The game ends when $r = 0$ and if $temp$ is greater than zero, $(temp > 0)$, then one of the opponent players wins, if less than zero $(temp < 0)$, then the fuzzy agent player (yellow) wins else, the game is a draw (i.e. if $temp = 0$).

- – To avoid repetition and to limit the size of this thesis, the rest of the procedures follow those steps highlighted in Section 7.4.1 (page 110) above for the 2-player FDMSB game case study.

6.2   Results Discussion for n-Player Game

From the n-player simulations, it was observed that because of the ability of the fuzzy player to grasp effectively the uncertainty in the business environment by changing his strategy based on the information provided by the fuzzy rule base, the fuzzy player wins more often as the number of competitors (opponent players) increases.

As shown in Table 6.1 with three players $(n = 3)$, very interesting cases are seen in those iterations where one expected the fuzzy player to lose because he started the game with weaker strategies than those of his competitors (as it happens in 2-player games), but because the player reasons in accordance with the fuzzy engine (rule base) and changes his strategies accordingly, the fuzzy player wins in those cases, and better than he wins in the 2-player game results shown on Table 4.3 of page 75.

From the results in column five and six of Table 6.1, out of thirteen iterations shown on the table, the fuzzy player (yellow) wins in ten iterations (iterations 1, 2, 3, 4, 5, 8, 9, 10, 11 and 13) while the opponent (green) wins in only three iterations which are iterations 6, 7, and 12.

This shows that the fuzzy player performs better in the 3-player game than in the 2-player games where he won only eight iterations out of thirteen iterations as shown in Table 4.3 of page 75. This shows that because of the fuzzy inference reasoning being used by the fuzzy player, the more the number of competitors, the better the payoffs of the fuzzy player. These trends continue with large number

of competitors as we investigated up to one hundred competitors as illustrated in the graph of Figure 6.2 on page 105.

Moreover, after learning, as stated in Section 7.4.1 step 11 and as shown in columns seven and eight of Table 6.1, the fuzzy agent performs much more better as the agent was able to win more than he won before training.

Results in columns nine and ten show the performance of the players after learning (training). The columns show that after learning, the payoffs of opponent player (green) were considerably reduced (minimized) by the **learned fuzzy player** while the payoffs of the fuzzy player were increased.

For examples, in iterations 1, 2, 3, 4, and 5, before leaning of the fuzzy player, fuzzy player payoffs were 59.1, 96.4, 328.1, 4.5 and 138.5 respectively. After learning, the fuzzy player payoffs increased to 128.2, 163.4, 385.8, 72.7 and 205.4 respectively.

Also, the green player payoffs were considerable reduced by the learned fuzzy player. As examples, in iterations 6, 7 and 12, before learning, green's payoffs were 82.9, 235.4 and 1397.4 respectively but these were reduced (minimized) to 22.3, 170.5 and 1330.7 respectively after the fuzzy player has learned.

As shown in columns seven and eight of Table 6.1, we verified these results by designing a control experiment (simulation) in which the fuzzy player does not change his moves in accordance with the fuzzy rule base. The results obtained from the control experiments show that the game follows conventional trends, that is, the fuzzy player wins only where he allocates more units of resources to his marketing strategy at the start of the game than those of his competitors and his payoff also depends on this. The payoff of the fuzzy player in the control experiment (where he did not use fuzzy rule base) are far less than what he got when he used fuzzy rule base to make his business decisions.

Therefore, after running several simulations with the number of players n ranging from 1 to 100, the results of the n-player FDMSB game show that the larger the number of opponent players (competitors), the better the fuzzy player performs, as illustrated in graph of Figure 6.2 on page 105 (with up to fifty competitors), due to the fact that he is able to adequately capture the uncertain information at his disposal which was modelled using the concepts of fuzzy reasoning.

.

	Agent Moves			Untrained Agent		Control Expt		Trained Agent	
S/N	Green	Brown	Yellow	Winner	Payoff	Winner	Payoff	Winner	Payoff
1	1, 1, 3	1, 1, 3	1, 0, 4	Yellow	-59.1	Yellow	-39.4	Yellow	-128.2
2	2, 1, 2	2, 1, 2	1, 1, 3	Yellow	-96.4	Yellow	-64.3	Yellow	-163.4
3	3, 0, 2	0, 4, 1	2, 0, 3	Yellow	-328.1	Yellow	-218.8	Yellow	-385.8
4	4, 0, 1	4, 0, 1	4, 0, 1	Yellow	-4.5	Yellow	-3.0	Yellow	-72.7
5	1, 0, 4	3, 2, 0	2, 0, 3	Yellow	-138.5	Yellow	-92.3	Yellow	-205.4
6	3, 1, 1	3, 0, 2	4, 0, 1	Green	82.9	Green	124.4	Green	22.3
7	3, 0, 2	2, 0, 3	2, 1, 2	Green	235.4	Green	353.1	Green	170.5
8	3, 1, 1	3, 1, 1	3, 1, 1	Yellow	-34.5	Yellow	-23.0	Yellow	-102.7
9	3, 1, 1	0, 1, 4	2, 0, 3	Yellow	-26.5	Yellow	-17.7	Yellow	-95.8
10	0, 5, 0	0, 5, 0	1, 4, 0	Yellow	-117.1	Yellow	-78.1	Yellow	-182.6
11	0, 5, 0	0, 0, 5	0, 1, 4	Yellow	-243.6	Yellow	-162.4	Yellow	-309.9
12	0,0,5(Geq)	0, 0, 5(Geq)	0, 5, 0 (Leq)	Green	1397.4	Green	2096.1	Green	1330.7
13	0,5,0 (Leq)	0, 5, 0 (Leq)	0, 0, 5(Geq)	Yellow	-1145.1	Yellow	-763.4	Yellow	-1210.6

TABLE 6.1: **Results of simulations of n-player game when** $n = 3$: From the table, the first column shows the serial numbers of the iterations, the second column contains player *green's* strategies, third column contains those of player *brown*, while the forth column contains that of *yellow*. For example, in the first iteration, *green's* strategy shows $[1, 1, 3]$, this indicates how resources are allocated to strategy $[C, W, M]$: $C = 1$, $W = 1$ and $M = 3$. The fifth column gives the winners for the untrained simulations while the sixth column gives the payoffs of those simulations. Column seven shows the winners for the *control experiment* simulations while the payoffs of players for those simulations are displayed in column eight. The control experiments show the results where both players did not use fuzzy inference systems in playing the games. Column nine shows the winners for the *trained* simulations while the payoffs of players for those simulations are displayed in column ten. It can be observed that the agent performs better than it does in 2-player game results shown on Table 4.3 of page 75. These two tables are better compared on the pie chart shown in Figure 6.1 (page 104). For example, in iterations 5, 9 and 11 where one of the opponents allocated higher strategy to marketing which is the strongest strategy, one expects the fuzzy player to lose but it won. It also happened in many other iterations which are not shown here for lack of enough space. Also, the fuzzy player has higher payoffs than in 2-player game. The strongest opponents (*Geq*) and weakest opponents (*Leq*) are shown in iterations 12 and 13 respectively. The minus sign on yellow payoffs merely shows the zero-sum concept.

3-Player Vs 2-Player Wins
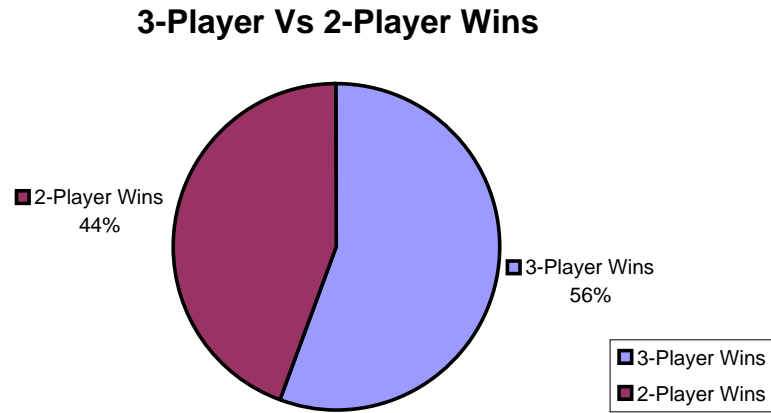
FIGURE 6.1: This chart compares the performance of the fuzzy (yellow) player in both 2-player (Table 4.3) and 3-player (Table 6.1)games. These trends extend to n players as discussed in chapter 6. However, more simulations on these are shown in Table 7.1 of page 116 for the business network games.

6.3   Conclusion

We have modelled decision making processes under uncertainty in business games, using fuzzy logic concepts and game theory. Our model was termed fuzzy decision making system for business games (FDMSB).

We have demonstrated that our model works well with large number of players. We illustrated this by using n-player games that represent perfect market structure. A fuzzy decision making system for business games was designed and implemented using Matlab software. Fuzzy rules were constructed in developing the FDMSB model using the Matlab toolbox and the implementation of this model heavily depends on expert knowledge and experience to facilitate the development of a reasonable fuzzy rule base for the determination of the if-then rules that denote the relationship between inputs and the output variables.

The results of our n-player simulations showed that an entrepreneur needs not to worry about the proliferation of competitors in the industry because by adopting this FDMSB model, the results of the n-player games show that the larger the number of opponent players (competitors), the better the fuzzy player performs in the games as shown in Table 6.1 and Figure 6.2 (page 105).

We verified these results by designing control experiments (simulations) in which the fuzzy player does not change his moves in accordance with the fuzzy rule base. The results obtained from the control experiments show that the game follows
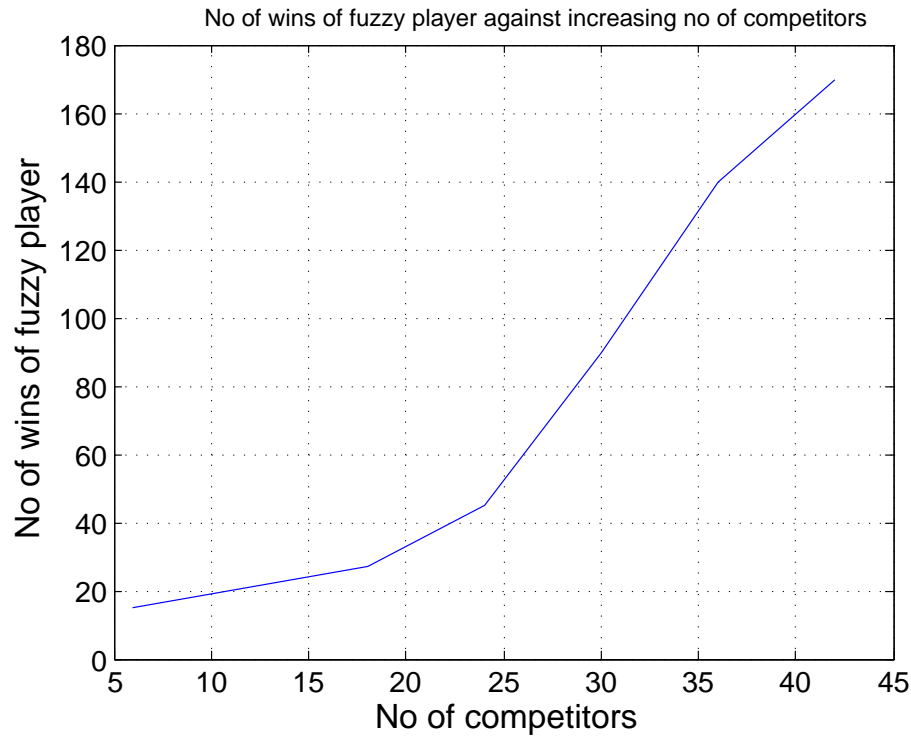
FIGURE 6.2: **A graph showing the strength of the fuzzy player with respect to increasing number of competitors**: It can be observed that the fuzzy player performance in the games improve as the number of competitors (opponent players) increases.

conventional trends, that is, the fuzzy player wins only where he allocates more units of resources to his marketing strategy at the start of the game than those of his competitors and his payoff also depends on this. The payoff of the fuzzy player in the control experiments (where he did not use fuzzy rule base) are far less than what he got when he used fuzzy rule base to make his business decisions.

Furthermore, we have applied a learning algorithm to the decision processes which enables the decision agent to optimize his performance in the decision processes as the games were played so as to meet the set criteria. To do the learning, the Nelder-Mead simplex method for finding the minimum of an unconstrained multivariable function was used.

Results of the learning showed that the learning algorithm works very well as the fuzzy player was able to perform much better after learning with higher payoffs and this enables him to reach the set criteria.

Our FDMSB procedure has practical uses in business contexts as it can serve as very useful tools in the hands of an entrepreneur to:

- Advise him on certain marketing strategic decision policies that can keep his business in strategic advantage over his competitors in the market.

- Give him insight on how his firm can successfully compete with its peers in the market by determining how much of its available resources or efforts could be dissipated on our three adopted strategies of marketing in such a way that his profit (accumulated wealth) will be maximized.

- Effectively utilize the uncertain (fuzzy) and prevailing or anticipated market demand (D) information, cost of producing a commodity (C_P) and other fuzzy information at his disposal to achieve the set goal of his business.

Also, we have been able to supplement the laws of demand and supply with a more practical approach which takes into consideration the uncertain (fuzzy) nature of most information available to business decision makers. While the traditional laws of demand and supply address the nature of decision processes by consumers and suppliers respectively, our own approach extends them further. This is to address the nature of decision processes by an intending entrepreneur or manufacturer to forecast the prospect of the proposed business through profit prediction from estimated selling price given the fuzzy market or industry information available to him. This allows him to determine price and marketing strategies in function of a *very low, medium, high, very high*, etc. demand.

In arriving at our results, the simulations are based on assumptions and conditions that the players involved in the decision processes are rational players (Section 2.1.1 on page 12) and that only the fuzzy player (yellow), at the moment, uses fuzzy moves [144]. This is in accordance with our overall aim of designing models that illustrate how an entrepreneur could make effective and efficient business decisions by using fuzzy inference systems (FIS) in capturing uncertainties that may surround his business environments. This will therefore help the entrepreneur to have competitive advantages over his competitors who are unaware of the usefulness of these tools and therefore are not making use of the fuzzy inference models in their decision making processes.

Chapter 7

Fuzzy Decision Making System on Business Networks

In this chapter, we are modelling uncertainty in business competitions in the form of games on networks [62, 145, 146, 147, 148], using fuzzy logic concepts and game theory. We investigate how the level of connectivity or number of links, number of opponent players (competitors), as well as choice of strategies adopted by opponent players affect the payoff of another player in the network. We shall call this player the fuzzy player. Learning is also introduced to investigate how the agent adapts over time during the game.

7.1   Introduction

Empirical work suggests that the pattern of social interaction has an important influence on economic outcomes [149, 150]. Analysis of interaction and cooperation among people in a group has been performed in the prisoners' dilemma game [15, 151, 152, 153, 154], snowdrift game [62, 71] and other games. For the first time, we have combined the concepts of fuzzy logic [155, 156, 157], and game theory to model interaction and decision making processes in business networks.

We would like to analyse interaction and competition among business organisations as games on networks and we would like to examine various network characteristics such as level of interaction or connectivity, number of nodes (players), location of a player with respect to those of his opponents, strength of individual player's

strategy and those of his immediate opponents. We will investigate how these characteristics affect the payoff [158, 159] of players in a business network [160, 161, 162].

7.2 Networks and Business

In a simple sense, a network is an interconnection of two or more nodes and a business network can be regarded as a network where these interconnected nodes are business units [163].

The search for models that account for the complex behaviour of biological, social, and economic systems has been the motivation of much interdisciplinary works in the last two decades [151, 164]. Strategists have been concerned with joint ventures, strategic alliances and strategic networks and the words "network" and "relationship" indicate that there is some kind of special organisational form at an aggregate level above that of individual components [158, 165, 166].

7.3 Fuzzy Decision Making System in the Business Network (FDMBN) Games Model

The design of an automatic FDMBN decision agent used a combination of game theory, fuzzy logic concepts, and training or learning of the fuzzy player to optimize his performance. This is achieved through the use of the *Nelder-Mead* Simplex Search Method for finding the local minimum x of an unconstrained multivariable function $f(x)$ using a derivative-free method and starting at an initial estimate.

In this section, we will analyse some fundamental concepts needed for designing the business network game.

We will consider two different sets of business networks as a case study [163]. One network has three players and the other has six players with different connectivities, as shown in Figures 7.1 A and B respectively on page 110.

7.3.1 Network Variables

The following variables will be used in the simulations: l is the number of connections or links to a particular node, d is the total number of nodes in the network, t is the total number of links in the fully connected network, k is the number of other nodes in the network that a particular node is directly connected to, while w is the number of nodes in the network that a particular node is not connected to. The *payoff* P of a player y will be represented as $P(y)$. We have

$$w = t - k. \tag{7.1}$$

For a fully connected network

$$t = (d - 1)/2. \tag{7.2}$$

7.3.2 Agents Payoffs

The fuzzy player will be denoted by y, we will call him the yellow player. In this simulation, with respect to figure 7.1 above, the payoff of the fuzzy player denoted as yellow y is calculated as follows:

The *payoff* of y in the partially connected network, $P(y_l)$, is the payoff it would get in the fully connected network $P(y_d)$ *MINUS* the payoff it would get if it was connected to only those nodes that it is not connected to, $P(y_w)$. That is:

$$P(y_l) = P(y_d) - P(y_w) \tag{7.3}$$

Using the diagrams in Figure 7.1, the payoff of network ii is the payoff of network i minus the payoff of network iii. This holds for networks A as well as B.
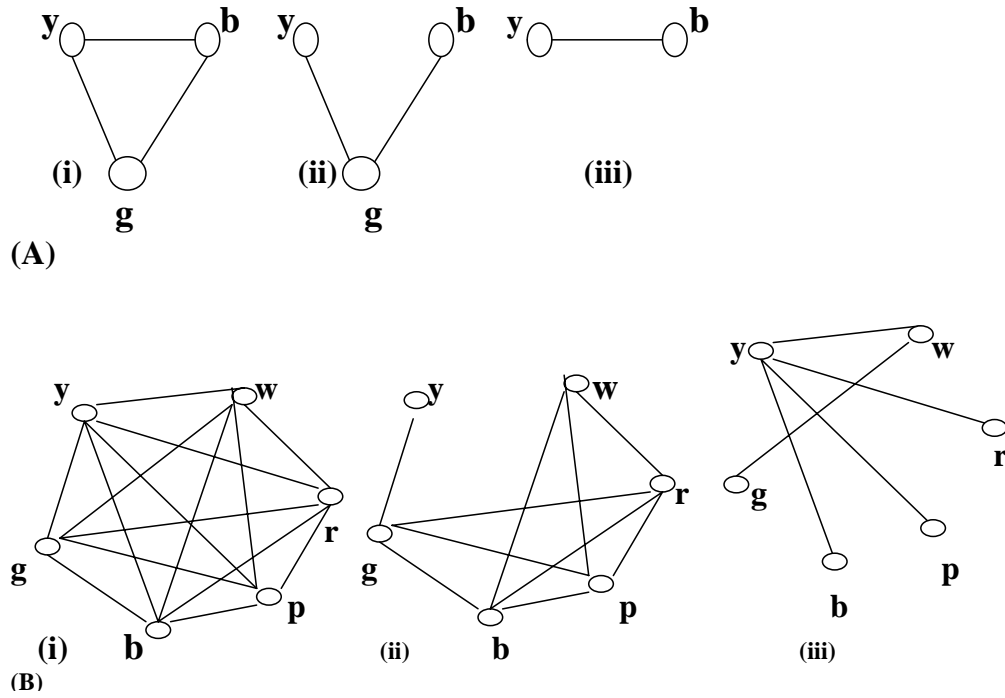
FIGURE 7.1: **The two figures A and B show two different sets of business networks with three and six players respectively**. Figures A(i) and B(i) are networks with full connectivity while figures A(ii), A(iii), B(ii) and B(iii) are networks with partial connectivity. Our results showed that the higher the connectivity, the higher the payoff of the fuzzy player. That is, the payoffs in A(i) and B(i) are greater than those in A(ii) and B(ii) respectively which in turn greater than the payoffs in A(iii) and B(iii) respectively. Also, we discovered that for the two sets of networks in A and B, the payoffs in (ii) are equal to the payoffs in (i) minus the payoffs in (iii). The payoff of y, the fuzzy player, also depends on the strategies of his neighbours as well as their positions in the network. For example in B(ii), when y faces a stronger (strategy that has highest resource allocation to marketing M) immediate opponent g, he has low payoffs. However, when g in turn faces a stronger immediate opponent b, this reduces the effects of g on fuzzy player y which results in y having higher payoffs. This is further explained in Section 7.5 on page 117.

7.4 Research Methodology

7.4.1 A Case Study of a Fuzzy Decision Making System for Business Network Games (FDMBN)

We shall illustrate our fuzzy decision making system in business network (FDMBN)[163] games using a 3-player network model shown in Figure 7.1 *A* above. The model involves three players (firms) in a typical market and we shall represent the players as green g, brown b and yellow y. Yellow represents the fuzzy player. Each player

is given five units of initial resources which may represent capital, time, personnel or other business resources. In our case we assume capital (say £5M). The number of rounds the game must be played is five, which denotes a sequence of five possible moves for each player. In each round, the players may choose to allocate their units between three roles (strategies): consolidation efforts C, reserved or generated wealth W and aggressive marketing efforts M. These resource allocations will be done with the knowledge of the opponents move history, but without knowledge of the opponent's current choice of strategy. They are denoted as a vector $[C, W, M]$ for each player. These strategies are fully explained in Section 4.1 on page 63.

7.4.1.1 Game Procedures

The procedures necessary for designing the proposed automatic decision system (FDMBN) are as listed in the steps below:

1. List all uncertain (fuzzy) factors that will be considered in taking the business decision: the uncertain or fuzzy information we are taking into consideration are anticipated market demand information (D) and the production costs (C_P).

2. Determine the strategies of the players: here, we are adopting three strategies for each player and these strategies are consolidation efforts, wealth created or reserved and aggressive marketing efforts, denoted as a vector with three elements $[C, W, M]$. As an example of a 3-player business network, we have three players (firms) represented as green g, whose strategy is represented as $[C_g, W_g, M_g]$, brown b with strategy represented as $[C_b, W_b, M_b]$ and yellow y with strategy represented as $[C_y, W_y, M_y]$.

3. Determine the input and output variables of FDMBN Fuzzy Inference System (FIS): The inputs are market demand information D and production costs C_P, and the outputs are expected consolidation efforts E_c, expected wealth E_w and expected aggressive marketing efforts E_m where: $E_m = 5 - (E_w + E_c)$ (Because the total (expected) resources of each player at any point is five) The variables E_c, E_w, and E_m relate to the fuzzy player y, and we will not index them by y.

4. Develop fuzzy sets, subsets and membership functions for all the input and output variables. This can be accomplished by soliciting knowledge from the experts or searching through literature data. Our adopted fuzzy sets, subsets and membership functions are as shown in Figure 7.2 on page 112.
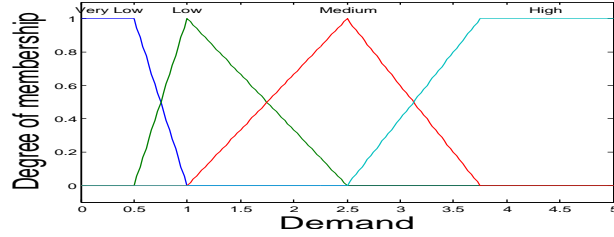
(a) Demand
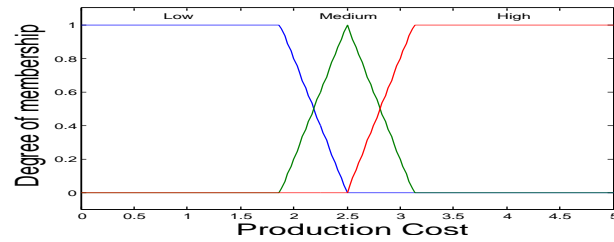
(b) Production cost
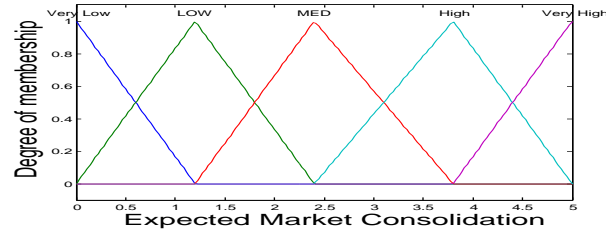
(c) Expected consolidation effort
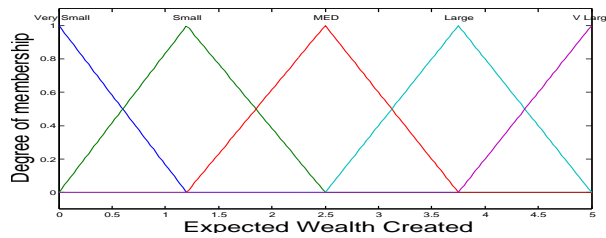
(d) Expected wealth

FIGURE 7.2: Membership functions for fuzzy variables of FDMBN rule base-inputs: Demand (D) and production cost (C_P) and outputs: expected consolidation efforts (E_c) and expected wealth (E_w).

5. Formulate decision rules for the rule base. These also ought to be solicited from experts [94]. The rules shown in Figure 7.2 depict our adopted decision rules.

6. Establish relationships between input values and their fuzzy sets and applying the decision rules. Using the relationships shown in Figure 7.2. The fuzzy rule base was coded into a Fuzzy Inference System (FIS) using the Matlab toolbox.

7. Play the game: The procedure for playing the game is as follows. The game state is represented as a vector $[g, b, y, A_w, r]$. Where g, b and y represent the amount of resources of the green, brown and yellow players respectively, A_w represents accumulated wealth (profit) and r is the number of rounds the game is played. Green, brown and yellow strategies are respectively denoted as $[C_g, W_g, M_g]$, $[C_b, W_b, M_b]$ and $[C_y, W_y, M_y]$ where:

$$C + W + M = 5. \tag{7.4}$$

As explained in Section 1.3.1 on page 7 and Section 4.1 (page 63), our choices of the number five in Equation 7.4 and for variable r are arbitrary. In a real system, any number that suitably represents the process can be chosen.

General rules of the game are as follows:

- Initial state of the game is $[5, 5, 5, 0, 5]$ (i.e according to vector $[g, b, y, A_w, r]$)

- At every state $[g, b, y, A_w, r]$, green and yellow choose their respective moves by allocating their strategies where: $C_g + W_g + M_g = 5$ and $C_b + W_b + M_b = 5$ and yellow who is the fuzzy player chooses his strategy $[C_y, W_y, M_y]$ where $C_y + W_y + M_y = 5$.

- The game changes states as follows: for the full network $A(i)$ shown in Figure 7.1,

$$r = r - 1 \tag{7.5}$$

$$A_w = A_w + W_g + W_b - W_y \tag{7.6}$$

$$g_d = g + C_g + M_g r - (b + C_b + M_b r + y + C_y + M_y r) \tag{7.7}$$

$$b_d = b + C_b + M_b r - (g + C_g + M_g r + y + C_y + M_y r) \tag{7.8}$$

$$y_d = y + C_y + M_y r - (b + C_b + M_b r + g + C_g + M_g r) \tag{7.9}$$

$$P(y_d) = A_w + g_d + b_d + y_d \tag{7.10}$$

Where g_d, b_d, and y_d are the resources of the players g, b, and y in a fully connected network. P_{y_d} represents the *payoff* of the fuzzy player

in the fully connected network. Next we have

$$E_m = 5 - (E_w + E_c), \tag{7.11}$$

because the total resources or expected resources of each player at any point is five. Also,

$$D = M_y/(M_g + M_b), \tag{7.12}$$

$$C_P = (M_y + C_y + k)/(M_g + M_b + C_g + C_b + k), \tag{7.13}$$

Where k represents other costs which are taken to be zero to avoid needless complication (i.e. $k = 0$).

We define E_w (expected profit/Wealth)= $E_{sp} - C_P$, where

$$E_{sp} = E_w + C_P \tag{7.14}$$

Where E_{sp} represents the expected selling price of the product of the fuzzy player. The output of the fuzzy inference system are expected consolidation efforts E_c, expected wealth E_w and expected aggressive marketing efforts E_m. In the subsequent rounds (r), the fuzzy player changes his strategy to $[E_c, E_w, E_m]$ based on the output of the FIS.

For our partially connected networks shown in Figure 7.1, $A(ii)$ and that shown in Figure 7.1 $A(iii)$ (page 110), the iteration follows similar steps but the effects of a player unconnected to a particular player are taken as zero. For example, in Figure 7.1 $A(ii)$, the iteration is as follows:

$$g_l = (g + C_g + M_g r) - (y + C_y + M_y r + b + C_b + M_b r), \tag{7.15}$$

$$b_l = (b + C_b + M_b r) - (g + C_g + M_g r) + 0, \tag{7.16}$$

$$y_l = (y + C_y + M_y r) - (g + C_g + M_g r) + 0, \tag{7.17}$$

$$P(y_l) = A_w + g_l + b_l + y_l, \tag{7.18}$$

Where $(P(y_l))$ represents the payoff of y in the partly connected network. Similarly, for Figure 7.1 $A(iii)$,

$$b_w = b + C_b + M_b r - (y + C_y + M_y r), \tag{7.19}$$

$$y_w = (y + C_y + M_y r) - (b + C_b + M_b r), \qquad (7.20)$$

$$(P(y_w)) = A_w + b_w + y_w, \qquad (7.21)$$

and $P(y_w)$ represents the *payoff* of y in the partly connected network. These iterations were combined into one simulation and we calculated the *payoff* based on Equation 7.3 above: $P(y_l) = P(y_d) - P(y_w)$.

- The game ends when $r = 0$. If $P(y_l)$ is greater than zero, the fuzzy player wins, if less than zero, then one of the opponents wins. Else, the game is a draw.

- This game is a zero sum game and therefore, yellow loses whenever opponents win and vice versa and since our aim is to develop an agent that would win as much as possible, maximize his payoff and minimize those of the opponents, Nash equilibrium [137, 138] is not considered in this context.

8. Evaluate the FIS: Using Matlab fuzzy toolbox, all the fuzzy inputs are passed into the Mamdani type FIS.

9. Get the defuzzified output from the FIS: the crisp output for the FDMBN is computed using centre of gravity method (COG).

10. Determine whether the conditions for the end of the game have been met: In this case study, the condition for the end of the game is when the number of rounds r reaches 1 counting down from 5 (i.e. when $r = 1$).

11. Training and performance evaluation: Training and learning of the FDMBN decision agent was accomplished through the optimization of the fuzzy logic parameters while using the game payoff as the basis for the performance measure after playing a series of the game as in [13]. Details of the training method are as explained in Section 3.7 (page 53) and Section 11 (page 68).

	Players					2-Player;(g,y)		3-Player;(g,b,y)		4-Player;(g,b,p,y)		5-Player;(g,b,p,r,y)	
S/N	Green(g)	Blue(b)	Purple(p)	Red (r)	Yellow(y)	Winner	Payoff	Winner	Payoff	Winner	Payoff	Winner	payoff
1	0, 0, 5	0, 0, 5	0, 0, 5	0, 0, 5	0, 0, 5	Green	249.00	Yellow	112.00	Yellow	3062.90	Yellow	29507.00
2	0, 5, 0	0, 5, 0	0, 5, 0	0, 5, 0	0, 5, 0	Yellow	117.00	Yellow	15.00	Yellow	517.93	Yellow	4564.00
3	1, 1, 3	1, 1, 3	1, 1, 3	1, 1, 3	1, 1, 3	Green	96.00	Yellow	73.00	Yellow	2134.90	Yellow	20367.00
4	2, 0, 3	2, 0, 3	2, 0, 3	2, 0, 3	2, 0, 3	Green	122.00	Yellow	96.00	Yellow	2254.90	Yellow	21370.00
5	0, 0, 5	0, 0, 5	0, 0, 5	0, 0, 5	0, 5, 0	Green	1006.00	Green	27.00	Yellow	2272.90	Yellow	23441.00
6	0, 5, 0	0, 5, 0	0, 5, 0	0, 5, 0	0, 0, 5	Yellow	912.00	Yellow	55.00	Yellow	1307.90	Yellow	10629.00
7	2, 2, 1	2, 2, 1	2, 2, 1	2, 2, 1	2, 2, 1	Yellow	63.00	Yellow	32.00	Yellow	1197.40	Yellow	11220.00
8	0, 0, 5	0, 5, 0	0, 5, 0	0, 5, 0	0, 0, 5	Green	249.00	Green	92.00	Yellow	1872.10	Yellow	15230.00
9	0, 5, 0	0, 0, 5	0, 0, 5	0, 0, 5	0, 0, 5	Yellow	912.00	Yellow	312.00	Yellow	2462.90	Yellow	24691.00
10	0, 0, 5	0, 5, 0	0, 5, 0	0, 5, 0	0, 5, 0	Green	1006.00	Green	212.00	Yellow	1042.90	Yellow	8966.00
11	0, 3, 2	1, 3, 1	1, 3, 1	1, 3, 1	1, 3, 1	Green	69.00	Yellow	8.92	Yellow	1176.40	Yellow	10999.00
12	4, 1, 0	1, 4, 0	1, 4, 0	1, 4, 0	1, 4, 0	Green	12.00	Yellow	8.47	Yellow	656.93	Yellow	5804.00
13	2, 3, 0	1, 0, 4	1, 0, 4	1, 0, 4	1, 1, 3	Yellow	566.00	Yellow	240.08	Yellow	2048.90	Yellow	20562.00
14	0, 5, 0	1, 4, 0	1, 4, 0	1, 4, 0	0, 0, 5	Yellow	912.00	Yellow	78.61	Yellow	1370.70	Yellow	10872.00
15	3, 1, 1	1, 3, 1	1, 3, 1	1, 3, 1	1, 2, 2	Yellow	136.00	Yellow	26.09	Yellow	1325.90	Yellow	11808.00
16	4, 0, 1	4, 0, 1	4, 0, 1	4, 0, 1	4, 0, 1	Yellow	11.00	Yellow	78.29	Yellow	1437.40	Yellow	13226.00
17	0, 4, 1	0, 4, 1	0, 4, 1	0, 4, 1	0, 4, 1	Yellow	73.00	Yellow	13.71	Yellow	957.45	Yellow	9213.00
18	5, 0, 0	5, 0, 0	5, 0, 0	5, 0, 0	5, 0, 0	Yellow	54.00	Yellow	62.87	Yellow	1052.90	Yellow	9249.00
19	1, 0, 4	1, 0, 4	1, 0, 4	1, 0, 4	1, 0, 4	Green	183.00	Yellow	104.03	Yellow	2658.90	Yellow	25438.00
20	0, 1, 4	0, 1, 4	0, 1, 4	0, 1, 4	0, 1, 4	Green	157.00	Yellow	81.03	Yellow	2538.90	Yellow	24435.00
21	1, 4, 0	1, 4, 0	1, 4, 0	1, 4, 0	1, 4, 0	Yellow	135.00	Yellow	29.17	Yellow	572.93	Yellow	5237.00
22	1, 3, 1	1, 3, 1	1, 3, 1	1, 3, 1	1, 3, 1	Yellow	89.05	Yellow	9.29	Yellow	1077.00	Yellow	10217.00
23	0, 0, 5	0, 0, 5	0, 5, 0	0, 5, 0	0, 0, 5	Green	249.96	Yellow	112.03	Yellow	2433.00	Yellow	19846.00
24	1, 2, 2	1, 3, 1	1, 2, 2	1, 3, 1	1, 2, 2	Green	10.13	Yellow	10.48	Yellow	1491.00	Yellow	13364.00
25	3, 0, 2	3, 0, 2	3, 0, 2	3, 0, 2	3, 0, 2	Green	62.13	Yellow	87.71	Yellow	1851.00	Yellow	17302.00
26	0, 1, 4	0, 1, 4	3, 0, 2	3, 0, 2	0, 1, 4	Green	157.56	Yellow	81.03	Yellow	2389.00	Yellow	21723.00
27	0, 0, 5	0, 1, 4	0, 1, 4	0, 1, 4	0, 0, 5	Green	249.96	Yellow	61.56	Yellow	2817.00	Yellow	26611.00
28	0, 0, 5	0, 0, 5	0, 1, 4	0, 1, 4	0, 0, 5	Green	249.96	Yellow	112.00	Yellow	2937.00	Yellow	27575.00
29	0, 0, 5	0, 0, 5	0, 0, 5	0, 1, 4	0, 0, 5	Green	249.96	Yellow	112.00	Yellow	3062.90	Yellow	28538.00
30	0, 5, 0	0, 0, 5	0, 1, 4	0, 1, 4	0, 5, 0	Yellow	117.00	Yellow	202.03	Yellow	1547.00	Yellow	16695.00
31	0, 5, 0	0, 0, 5	0, 0, 5	0, 1, 4	0, 5, 0	Yellow	117.00	Yellow	202.03	Yellow	1673.00	Yellow	17657.00

TABLE 7.1: **Results of simulations of FDMBN showing how payoffs of fuzzy player increase as competitors increase on a business network**. From the table, the first column shows the serial numbers of the iterations, the second column to the sixth contain players' strategies. For example, in the first iteration, *green's* strategy shows $[0, 0, 5]$, this indicates how resources are allocated to strategy $[C, W, M]$: $C = 0$, $W = 0$ and $M = 5$. Columns 7, 9, 11 and 13 show the winners in 2-player, 3-player, 4-player and 5-player games respectively while columns 8, 10, 12 and 14 give their respective payoffs. Each iteration involves the fuzzy player *yellow* and one, two, three or four other players depending on the values of n on the result columns. It will be observed from the results *that the more the number of competitors on the business network, the better (more) the payoffs of the fuzzy player.* Figure 7.4 on page 118 gives a graphical explanation on this table.
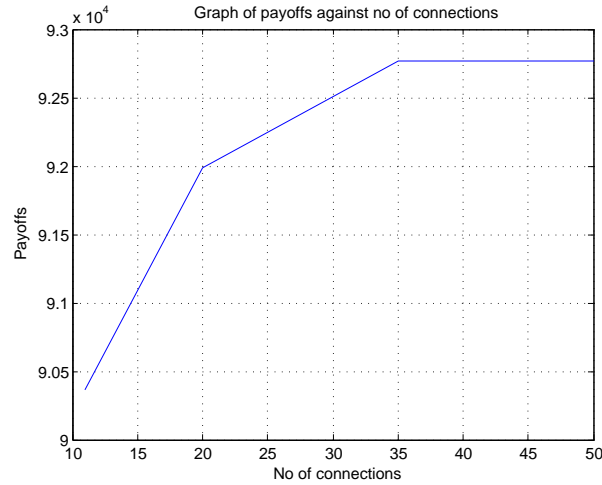
FIGURE 7.3: **A graph showing the strength of the fuzzy player with respect to different levels of connectivity on the network**. It can be observed that the fuzzy agent performance in the games improve as the level of connectivity increases. The higher the level of connectivity on the business network, the higher the payoffs of the fuzzy player. This trend continues for connections with up to 100 competitors and beyond. Table 7.2 on page 119 contains the data from which this graph was obtained.

7.5 Results Discussion for Business Games on Networks

Based on the procedure highlighted in Section 7.4.1.1, several simulations of FDMBN were run with different number of connections (levels of connectivity) and number of players.

As shown in Figure 7.3 (page 117) and Table 7.2 (page 119), it was observed that the higher connectivity among players, the higher the payoff of the fuzzy player.

From Table 7.2, l^1 represents the number of missing links. Therefore, $l^1 = 1$ represents the case when one link is missing from the business network. The winners on these iterations are shown in column nine and the corresponding payoffs in column ten.

Also, in the columns of the table, $l^1 = 2$ represents the case when two links are missing from the business network. The winners on these iterations are shown in column eleven and the corresponding payoffs in column twelve. The last two columns represent similar cases when $l^1 = 3$.
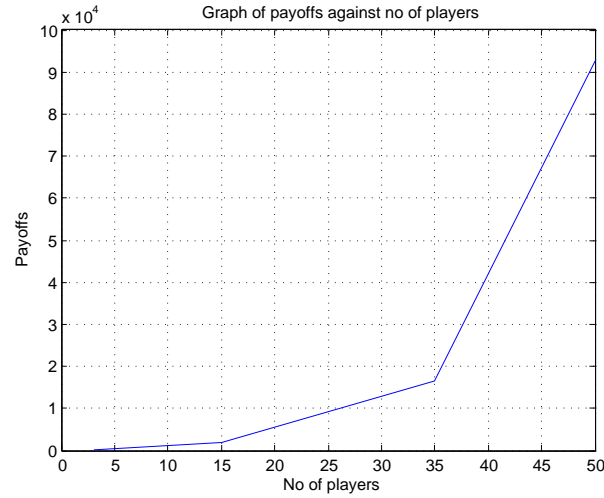
FIGURE 7.4: **A graph showing the strength of the fuzzy player with respect to an increasing number of competitors on the business networks**. It can be observed that the fuzzy player performance in the game improves as the number of competitors (opponent players) increases. This trend continues for games with up to 100 competitors and beyond. Table 7.1 on page 116 contains the data from which this graph was obtained.

From the result on these columns, it can be observed that the higher the number of missing links l^1 on the network, the lower the payoffs of the fuzzy player and vice-versa. That is, the the higher the level of connectivity, the higher the payoffs of fuzzy player on the business networks.

For examples, in the second iteration in which all the players have strategy $[0, 5, 0]$, the payoff of fuzzy player in the fully connected network was 4,586.00. This is as shown in column eight for the second iteration. However, when one link was missing from the network, the payoff of fuzzy player in this iteration reduced to 4,564.00. When two links were missing as shown in column twelve, the payoff reduced to 4,445.00 and finally for that iteration, when the number of missing links increased to three ($l^1 = 3$), the payoff reduced to 4,133.00. These trends continue for all the simulations and for a large networks.

We therefore concluded that the higher the level of connectivity among the players on the business networks, the higher the payoffs of players.

Chart in Figure 7.5 (page 120) shows the pictorial trends of these results. Also, results shown on the graph of Figure 7.4 (page 118) and Table 7.1 (page 116) confirm that because of the ability of the fuzzy player to grasp effectively the uncertainty in the business network environment by changing his strategy based on the information provided by the fuzzy rule base, the fuzzy player wins more

	Players					Full Network		$l^1 = 1$		$l^1 = 2$		$l^1 = 3$	
S/N	Green(g)	Blue(b)	Purple(p)	Red (r)	Yellow(y)	Winner	Payoff	Winner	Payoff	Winner	Payoff	Winner	Payoff
1	0, 0, 5	0, 0, 5	0, 0, 5	0, 0, 5	0, 0, 5	Yellow	29504.00	Yellow	29507.00	Yellow	29021.00	Yellow	27313.00
2	0, 5, 0	0, 5, 0	0, 5, 0	0, 5, 0	0, 5, 0	Yellow	4586.00	Yellow	4564.00	Yellow	4445.00	Yellow	4133.00
3	1, 1, 3	1, 1, 3	1, 1, 3	1, 1, 3	1, 1, 3	Yellow	20370.00	Yellow	20367.00	Yellow	20035.00	Yellow	18857.00
4	2, 0, 3	2, 0, 3	2, 0, 3	2, 0, 3	2, 0, 3	Yellow	21368.00	Yellow	21370.00	Yellow	21021.00	Yellow	19773.00
5	0, 0, 5	0, 0, 5	0, 0, 5	0, 0, 5	0, 5, 0	Yellow	23434.00	Yellow	23441.00	Yellow	22531.00	Yellow	19848.00
6	0, 5, 0	0, 5, 0	0, 5, 0	0, 5, 0	0, 0, 5	Yellow	10657.00	Yellow	10629.00	Yellow	10936.00	Yellow	11598.00
7	2, 2, 1	2, 2, 1	2, 2, 1	2, 2, 1	2, 2, 1	Yellow	11227.00	Yellow	11220.00	Yellow	11040.00	Yellow	10390.00
8	0, 0, 5	0, 5, 0	0, 5, 0	0, 5, 0	0, 0, 5	Yellow	15258.00	Yellow	15230.00	Yellow	15578.00	Yellow	16313.00
9	0, 5, 0	0, 0, 5	0, 0, 5	0, 0, 5	0, 0, 5	Yellow	24689.00	Yellow	24691.00	Yellow	24206.00	Yellow	22498.00
10	0, 0, 5	0, 5, 0	0, 5, 0	0, 5, 0	0, 5, 0	Yellow	8989.00	Yellow	8966.00	Yellow	8906.00	Yellow	8698.00
11	0, 3, 2	1, 3, 1	1, 3, 1	1, 3, 1	1, 3, 1	Yellow	11012.00	Yellow	10999.00	Yellow	10837.00	Yellow	10259.00
12	4, 1, 0	1, 4, 0	1, 4, 0	1, 4, 0	1, 4, 0	Yellow	5825.00	Yellow	5804.00	Yellow	5718.00	Yellow	5423.00
13	2, 3, 0	1, 0, 4	1, 0, 4	1, 0, 4	1, 1, 3	Yellow	20559.00	Yellow	20562.00	Yellow	20060.00	Yellow	18387.00
14	0, 5, 0	1, 4, 0	1, 4, 0	1, 4, 0	0, 0, 5	Yellow	10897.00	Yellow	10872.00	Yellow	11195.00	Yellow	11836.00
15	3, 1, 1	1, 3, 1	1, 3, 1	1, 3, 1	1, 2, 2	Yellow	11821.00	Yellow	11808.00	Yellow	11730.00	Yellow	11345.00
16	4, 0, 1	4, 0, 1	4, 0, 1	4, 0, 1	4, 0, 1	Yellow	13223.00	Yellow	13226.00	Yellow	13012.00	Yellow	12222.00
17	0, 4, 1	0, 4, 1	0, 4, 1	0, 4, 1	0, 4, 1	Yellow	9231.00	Yellow	9213.00	Yellow	9067.00	Yellow	8558.00
18	5, 0, 0	5, 0, 0	5, 0, 0	5, 0, 0	5, 0, 0	Yellow	9250.00	Yellow	9249.00	Yellow	9095.00	Yellow	8521.00
19	1, 0, 4	1, 0, 4	1, 0, 4	1, 0, 4	1, 0, 4	Yellow	25436.00	Yellow	25438.00	Yellow	25021.00	Yellow	23543.00
20	0, 1, 4	0, 1, 4	0, 1, 4	0, 1, 4	0, 1, 4	Yellow	24438.00	Yellow	24435.00	Yellow	24035.00	Yellow	22627.00
21	1, 4, 0	1, 4, 0	1, 4, 0	1, 4, 0	1, 4, 0	Yellow	5258.00	Yellow	5237.00	Yellow	5152.00	Yellow	4856.00

TABLE 7.2: **Results of simulations of FDMBN showing how payoffs of fuzzy player decrease as the number of missing links (l^1) increases on a 5-player business network**. In other words, the more the level of connectivity (links) on the business network, the better (more) the payoffs of the fuzzy player. From the table, the first column shows the serial numbers of the iterations, the second column to the sixth contain players' strategies (for full explanations on strategies, please, see Section 4.1 on page 63). For example, in the first iteration, *green's* strategy shows $[0, 0, 5]$, this indicates how resources are allocated to strategy $[C, W, M]$: $C = 0$, $W = 0$ and $M = 5$. Column 7 and 8 give the winner and the payoffs when the network is fully connected. Columns 9, 11 and 13 give the winners when 1, 2 or 3 links are missing from the network while columns 10, 12 and 14 indicate their payoffs respectively. Figure 7.3 on page 117 and Figure 7.5 on page 120 give graphical explanations on this table.

often or has a higher payoff as the number of players (competitors) in the game increases.

This is as demonstrated in Table 7.1. The payoffs in 5-player networks (column 14) is greater than that of 4-player networks (column 12) and this is also greater that the payoffs of players in 3-player networks (column 10) and the payoffs in 3-player network is greater that that of 2-player networks (column 8).

This confirms that the higher the number of competitors on the networks, the higher the payoffs of the fuzzy player.

Concerning the position or location of a player in a network with respect to the strategies of his neighbours, when y faces a stronger (strategy that has highest
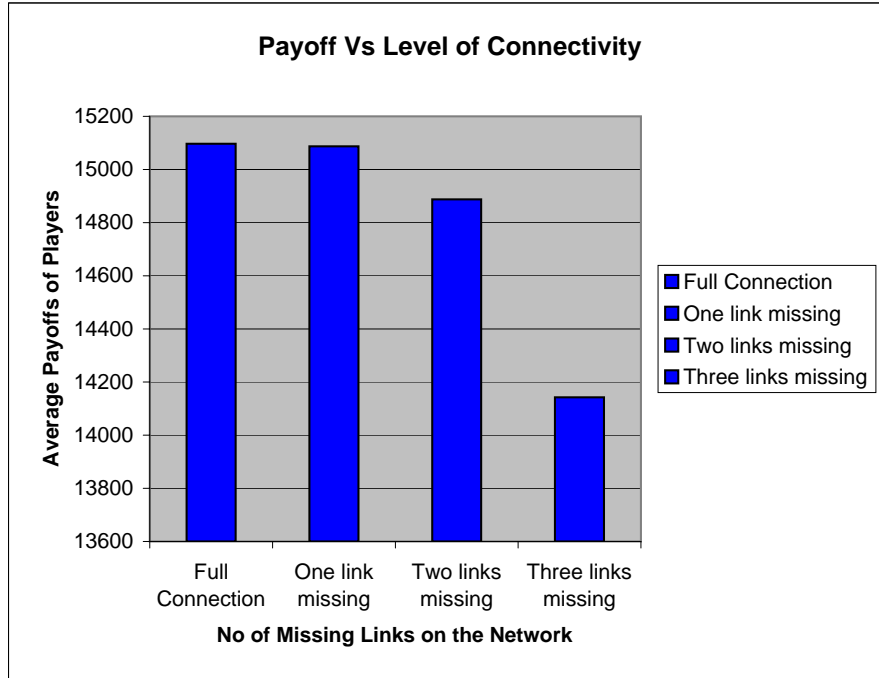
FIGURE 7.5: Pictorial view of the trends of the fuzzy player payoffs with respect to different levels of connectivity. This shows that the payoffs of the fuzzy player on the business network increase as the level of connectivity increases. Table 7.2 on page 119 contains the data from which this chart was obtained.

resource allocation to marketing M) immediate opponent g in the network as shown in Figure 7.1B(ii) (page 110), then y has low payoffs. However, when g in turn faces a stronger immediate opponent b, this reduces the effects of g on fuzzy player y which results in y having higher payoffs.

Also, it was observed that due to Equations (7.15)-(7.17),(7.19) and (7.20), for any of the players to win the game, he must allocate a substantial part of his resources to aggressive marketing (M) and this allocation must outweigh those of the opponents' allocations. According to this model and with respect to the equations, since the number of rounds r decreases as the game is played, this reduces the strength of marketing aggressiveness. An entrepreneur who is a new entrant into the networked industry, is best advised to try as much as possible to devote more of his resources on aggressive marketing campaigns (M) than other strategies (i.e. efforts on consolidation (C) and reserved wealth (W)). This will enable him to have a strong footing in the industry and to be able to have a large market share as early as possible as the game is played and thus, will result in winning the game. At the end of the game, the estimated price for the commodity can be forecast with Equation (7.14) ($E_{sp} = E_w + C_P$).

As in Tables 4.3 and 6.1 of chapters 4 and 6 respectively, we verified these results by designing control experiments (simulations) in which the fuzzy player does not change his moves in accordance with the fuzzy rule base. The results obtained from the control experiments show that the game follows conventional trends, that is, the fuzzy player wins only where he allocates more units of resources to his marketing strategy at the start of the game than those of his competitors and his payoff also depends on this. The payoff of the fuzzy player in the control experiments (where he did not use fuzzy rule base) are far less than what he got when he used fuzzy rule base to make his business decision.

After training, as stated in Section 7.4.1.1 step 11 (page 68), the fuzzy player performs better as the player was able to win more often than he won before training. From the results explained above, it can be observed that training (learning) of the fuzzy player was really important and the training algorithm was very effective because it enables the fuzzy player to learn and reach the performance criteria.

7.6 Conclusion

We have modelled decision making processes under uncertainty on business networks, using fuzzy logic and game theory. Our model was termed fuzzy decision making system for business networks (FDMBN). We illustrated this firstly with 3-player, 6-player and n-player network games to capture perfect market structures (please, see chapter 6). Also, we examined this model via a case study. The system was designed and implemented using Matlab software. Fuzzy rules were constructed in developing the FDMBN model using Matlab toolbox and the implementation of this model heavily depends on expert knowledge and experience to facilitate the development of a reasonable fuzzy rule base for the determination of the if-then rules that denote the relationship between inputs and the output variables. Furthermore, we have applied a learning algorithm to the decision process which enables the decision agent to optimize his performance in the decision process as the game is played so as to meet the set criteria. To do the learning, *Nelder-Mead* simplex method for finding the minimum of an unconstrained multivariable function was used.

Our FDMBN model has practical use in a business context as it can serve as a very useful decision tool in the hands of an entrepreneur. Given the fuzzy demand and

cost of production information, the estimated selling price (E_{sp}) can be predicted according to Equation 7.14.

In arriving at our results, the simulations are based on assumptions and conditions that the players involved in the decision processes are rational players (Section 2.1.1 on page 12) and that only the fuzzy player (yellow), at the moment, uses fuzzy moves [144]. This is in accordance with our overall aim of designing models that illustrate how an entrepreneur could make effective and efficient business decisions by using fuzzy inference systems (FIS) in capturing uncertainties that may surround his business environments. This will therefore help the entrepreneur to have competitive advantages over his competitors who are unaware of the usefulness of these tools and therefore are not making use of the fuzzy inference models in their decision making processes.

Chapter 8

Fuzzy Decision Making Systems using Board Games with Constraints as Models of Business Games

8.1 Introduction

We will now study uncertainties surrounding competition in businesses using board games[76, 79, 167] as illustrations. We investigated these uncertainties using concepts of fuzzy logic, percolation theory and game theory. These investigations focus on how the level of connectivity or number of links, number of opponent players (competitors), possible constraints or restrictions on the boards as well as choice of strategy[168] adopted by opponent players affect the payoff of another player on the boards. This other player is referred to, in this thesis, as the fuzzy player. We introduced learning to train and analyze how the fuzzy player adapts over time during the game.

This chapter contains experiments on the economic effects of the pattern of social interactions modelled as fuzzy board games and we will refer to the model as *fuzzy strategy* decision making system on *business board* (FSBB) games [169].

8.2 Chapter Objectives

Our main objectives are:

- To analyse competitions among business organisations as games on boards and we would like to examine various board characteristics such as level of connectivity, number of nodes (players), location of a player with respect to those of his opponents, patterns of board connections and moves, strength of individual player's strategy and those of his immediate opponents. We would investigate how these characteristics affect the payoffs of players in games played on boards.

- To investigate situations where there are constraints imposed by regulatory authorities such as when two or more players are forbidden (possibly by law) from interacting to prevent collusion. This leads to constrained optimisation. Constraints can be between variables, or can be constraints imposed on communication between players.

- To analyse how level of availability of vital infrastructures such as transportation (also communication) in a geographical location can affect the profitability (known here as payoffs) of business enterprises.

- To investigate why industries tend to concentrate in highly developed locations rather than less developed ones.

- To investigate why developing nations are less attractive to industrialists when compared to the developed ones.

- To study how fuzzy reasoning or fuzzy inference system (FIS) can help to improve the performance of businesses in an environment that is clouded with uncertainties and adverse conditions such as low level of infrastructural development.

- To investigate how performance of these business enterprises can be improved or enhanced through adaptation or learning (training of the fuzzy players) of the fuzzy inference system. We are providing trained and optimized fuzzy rules that simulate the relationship between demand (D), production cost (C_P) as well as those marketing strategies that an entrepreneur can follow in forecasting the selling price (E_{sp}) of a commodity and thereby, the profit or wealth to be generated or accumulated (Aw).

8.3 Board Games and Business

Board games have a universal appeal and there can be few people who have not at some time, been excited or stimulated by a board game [79]. There are different types of board games and in [79], they are grouped according to the following categories: Games of position, Mancala games, War games, Race games, Dice, Calculation and other games. Figure 8.1 (page 125) and Figure 8.2 (page 126) show the author, Festus, and his wife, Adesola, playing different types of board games in the computer laboratory in their attempts to investigate the outcome of the experiments of this research. The games played in the two Figures 8.1 and 8.2 are Ayo-Olopon (also called Mancala) and Ludo games respectively.

FIGURE 8.1: The author, Festus, playing Ayo-Olopon (also known as Mancala) game with his wife, Adesola, in the laboratory to investigate the outcomes of research experiments.

In this research, we have formulated a board game named; *fuzzy strategy* decision making system on *business board* (FSBB) games to simulate strategic competitions in business environments and we used this to investigate the impacts of basic infrastructures such as transportation networks [170, 171] on the profitability of businesses in particular geographical locations.

FIGURE 8.2: The author is playing Ludo game with his wife, Adesola, in the computer laboratory in order to investigates the outcomes of his research experiments.

8.4 Fuzzy Strategy Decision Making System on Boards (FSBB) Games Models

The general model for our proposed FSBB is as shown in Figure 8.3 on page 127. The design of the automatic FSBB decision agents used a combination of game theory, percolation theory[155, 156, 172], fuzzy logic concepts and training or learning of the fuzzy player to optimize his performance. This is achieved through the use of the *Nelder-Mead* Simplex Search Method [131, 173, 174] for finding the local minimum x of an unconstrained multivariable function $f(x)$ using a derivative-free method and starting at an initial estimate [163].

In the sections that follow, we will firstly analyse some fundamental concepts needed for designing the *fuzzy strategy* decision making system on *business board* (FSBB) games.

8.5 Difference between the Network Games (FDMBN) and the Board Games (FSBB)

In networks, the nodes represent the players and edges are links between players. Existence of links or non-existence determines whether the players can compete or not respectively. However, in board games, nodes are locations while edges are
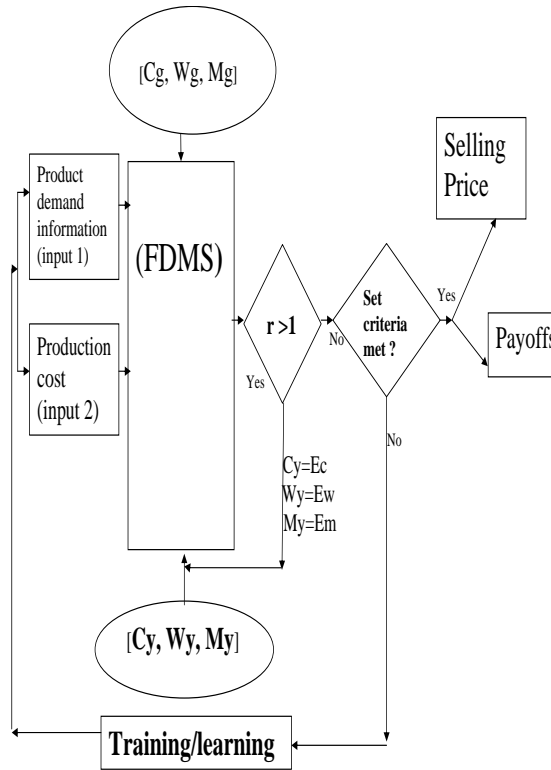
FIGURE 8.3: A model of FSBB game showing inputs, processes and outputs.

links between these locations. Existence of links or their non-existence between locations determines whether players can move between these locations or not respectively.

8.6 Fuzzy Strategic Decision Making System using Board Games with Constraints as Model of Businesses(FSBB)

Fuzzy strategic decision making system on business board (FSBB) game is an abstract experimental and strategic board game that is played on 5X5 board among two players whom we shall represent as yellow and green players. Each of the two players has ten pieces which represent trucks which are loaded with firms' products. The trucks are positioned initially, as shown in Figure 8.4 (page 129), at the start nodes which are at row 1 and row 5 for yellow and green players respectively. As shown in the figure, each node at the start row contains two pieces (trucks) each at the initial (start) stage of the game. These ten trucks owned by each player contain products which are to be distributed at their respective goal

nodes (destinations). The goal node for each player resides at the opponent's side of the board which necessitates for each player to travel across the board in order to take as many of his resources as possible to his destination (the goal node).

The board is used here to represent the road networks in a particular geographic location (such as between Edinburgh and London in the United Kingdom) and where players represent companies (involve in logistics with trucks) at each of these mentioned locations.

We varied level of connectivity (number of links) on the board by removing links arbitrarily and we investigated how these restrictions (missing links l^1) affect payoffs (profitability of businesses). This level of connectivity is used to investigate how the level of availability of vital infrastructures such as transportation networks in a geographical location can affect the profitability (known in the research as game payoffs) of business enterprises.

8.7 Board Variables

The following variables would be used in the simulation: n represents the number of players ($n = 2$), l^1 is the number of missing links on the board, d is the total number of nodes on the board, t is the total number of links in the fully connected network, Cost of production is represented as C_p, estimated demand of the product at destination node is represented as D . The *payoff* P of a player y will be represented as $P(y)$.

8.8 A Case Study of a Fuzzy Strategic Decision Making System on Business Board (FSBB) Games

We shall illustrate our fuzzy strategic decision making system on business board (FSBB) games with different board connections in Figure 8.4 (page 129). The model involves two players which represent firms that deal in logistics by road and are based at different geographical locations in a particular country or region. We shall represent the players as green (g) and yellow (y). Yellow represents the
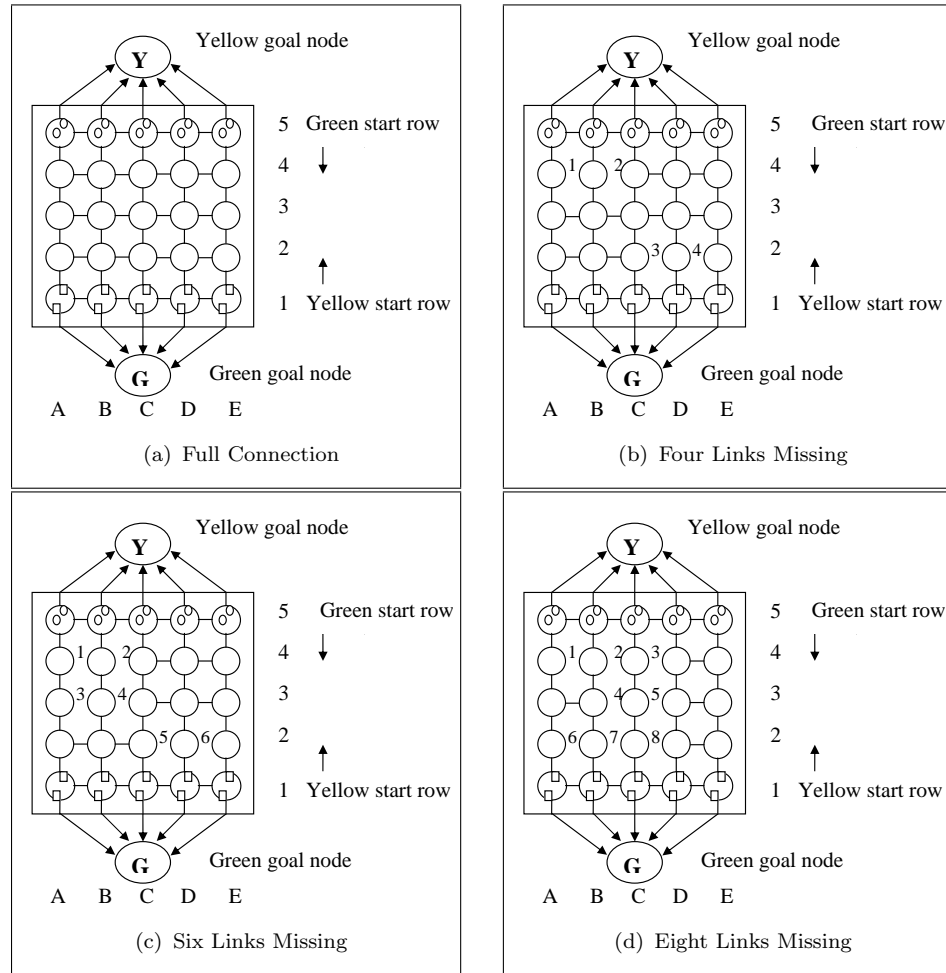
FIGURE 8.4: Different board connectivity figures: (a) shows a board with complete links/connectivity (i.e. fully connected), (b)board with four missing links, (c)board with six missing links and (d) board with eight missing links. Our results thus show that the less the connectivity on the board, the less the payoffs of the players. That is, the payoff of fuzzy player in (a) is greater than his payoff in (b) which in turn is greater than that of (c) and finally, the payoffs in (d) turn to be the lowest.

fuzzy player. Given the boards with different patterns of connections as shown in Figure 8.4, the game state is represented as vector $[g, y, A_w, r]$. Where g represents green player's amount of resources, y represents yellow player's amount of resources, A_w represents green's accumulated wealth (profit) and r is the number of rounds the game is played. Green player strategy is denoted as $[C_g, W_g, M_g]$ and yellow player strategy is denoted as $[C_y, W_y, M_y]$ where:

$$C + W + M = 10. \tag{8.1}$$

Because the total resources of each player at any point is ten. These resources are trucks of products to be taken to the players' respective goal nodes. For more information on players' strategies, please see Section 4.1 on page 63.

8.8.1 FSBB Game Rules

From Figure 8.4 (page 129), *row* 1 is the yellow player start row while *row* 5 is the green player start row. As shown on the diagram, the goal node of player y is represented as **Y** while the goal node of player g is represented as **G**.

Other rules are as follows:

- A node can only contain a maximum of three trucks at a time.

- A node can only contain a maximum of two trucks from same firm.

- A player that has two trucks in the same node (other than the start node) would get his profit reduced to half because his goods are in excess for that particular location. This means the less the connectivity among the nodes, the more the need for branching of a truck into neighbours' paths and therefore the more the risk of that player having two trucks in same node and thereby reducing his profits.

- A player must follow legal moves through connected nodes and cannot jump nodes.

- At any particular location, a player seeks the shortest path to move to the next location.

- For game to exist, there must be a minimum valid connection on the board. A minimum valid connection is the connection such that there exists at least a single path for a player to take his resources (trucks) from the start node to reach the destination node.

8.8.2 Game procedures

Following the FSBB general model in Figure 8.3 (page 127) and with respect to board diagrams in Figure 8.4 (page 129), the procedures that are necessary

for designing the proposed automatic business decision system (FSBB) are as explained below:

From the knowledge of percolation theory[155, 156, 172], the probability that there exist a connected link through which a truck will move from its start node to the destination node is denoted as p. Therefore, the probability that a piece will not arrive at the destination node due to lack of links between the nodes is given as $1 - p$.

On a fully connected board of twenty five nodes as shown in Figure 8.4a (page 129), the total connections the board would have if fully connected is 40; ($t = 40$). Since l^1 represents the number of missing links, therefore;

$$p = 1 - (l^1/t) \tag{8.2}$$

In the FSBB game our fuzzy player is still represented as *yellow*. The methodology of FSBB game simulation also follows the procedural steps of FDMBN general illustration in Section 7.4.1.1 (page 111) with exception to step 2, step 3 and step 7 which are modified as follows:

- Step(2) Determining the strategy: We still adopt our previous strategic vector $[CWM]$. This represents products being taken to the destination node to consolidate existing customers (*Consolidation C*), those that are not moved or reserved at the base as unused wealth (*Wealth W*) and those being taken to the goal nodes to market new customers (*Marketing M*).

  We have two players (firms) represented as green (g) whose strategy is represented as $[C_g, W_g, M_g]$, and yellow (y) with strategy represented as $[C_y, W_y, M_y]$.

- Step(3) Determine the input and output variables of FSBB FIS: As before, the inputs are market demand information (D), production costs (C_P) and the outputs are expected consolidation efforts (E_c), expected wealth (E_w) and expected aggressive marketing efforts (E_m) where: $E_m = 10 - (E_w + E_c)$ (Because the total (expected) resources of each player at any point is ten). These variables E_c, E_w and E_m relate to the fuzzy player y, and we will not index them by y.

Figure 8.5 on page 134 shows the FIS interface for the membership functions of the input variable *demand* (D), while Figure 8.6 on page 134 shows the FIS interface for input variable *Production Cost* (C_P).

- Step(7) Play the game: Procedures for playing the game are as follows: The game state is represented as vector $S = [g, y, A_w, r]$. Where g represents green player's amount of resources, y represents fuzzy player (yellow) amount of resources, A_w represents opponents' accumulated wealth (profit) and r is the number of rounds the game is played. Both the green and fuzzy player strategies are as stated in step 2 (page 130).

  Figure 8.7 on page 135 shows the Mamdani-type FIS interface for the board games. The interface shows the inputs variables *demand* (D) and *Production Cost* (C_P) as well as the expected wealth outputs (E_w).

  General rules of the game are as follows:

  - Initial state of the game is $[10, 10, 0, 10]$ (i.e. according to vector $[g, y, A_w, r]$).

  - At every state $[g, y, A_w, r]$, green chooses his move by allocating to his strategies $[C_g, W_g, M_g]$ where: $C_g + W_g + M_g = g = 10$ and yellow who is the fuzzy player chooses his strategy $[C_y, W_y, M_y]$ where:

  $$C_y + W_y + M_y = y = 10 \tag{8.3}$$

  As explained in Section 1.3.1 on page 7 and Section 4.1 (page 63), our choices of the number ten in Equation 8.3 and for variable r are arbitrary. In a real system, any number that suitably represents the process can be chosen.

  - The game changes states as follows: for different board connections shown in Figure 8.4a-d on page 129,

  $$r = r - 1 \tag{8.4}$$

  $$A_w = A_w + W_g - W_y \tag{8.5}$$

  $$g = ((g + C_g + M_g r) - (y + C_y + M_y r)) * d * p \tag{8.6}$$

  (where d is the total number of nodes and p as in equation 8.2)

  $$y = ((y + C_y + M_y r) - (g + C_g + M_g r)) * d * p \tag{8.7}$$

$$temp = A_w + g - y; \tag{8.8}$$

Where *temp* represents game payoff. Then,

$$E_m = 10 - (E_w + E_c) \tag{8.9}$$

Because the total resources or expected resources of each player at any point is ten. Now,

$$D = M_y/M_g, \tag{8.10}$$

$$C_P = (M_y + C_y + v)/(M_g + C_g + v), \tag{8.11}$$

Where v represents other costs which are taken to be zero to avoid needless complication (i.e. $v = 0$). We define E_w (expected profit/Wealth)= $E_{sp} - C_P$, where:

$$E_{sp} = E_w + C_P \tag{8.12}$$

and E_{sp} represents the expected selling price of the product.

– The game ends when $r = 0$ and if *temp* is greater than zero ($temp > 0$), the green player wins, if less than zero ($temp < 0$), then the fuzzy player (yellow) wins else, the game is draw (i.e. if $temp = 0$). Also, the game can end when one of the players has successfully taken all his resources to the goal node and in that case, such player wins.

Figure 8.8 on page 135 shows the FIS interface for the membership functions of output variable *expected consolidation efforts* (E_c).

– This 2-player game is a zero sum game and therefore, yellow loses whenever green wins and vice versa and since our aim is to develop an agent that would win as much as possible, maximize his payoff and minimize that of the opponents, Nash equilibrium [137, 138] is not considered in this context.

The remaining steps follow those stated in Section 7.4.1.1 on page 111.
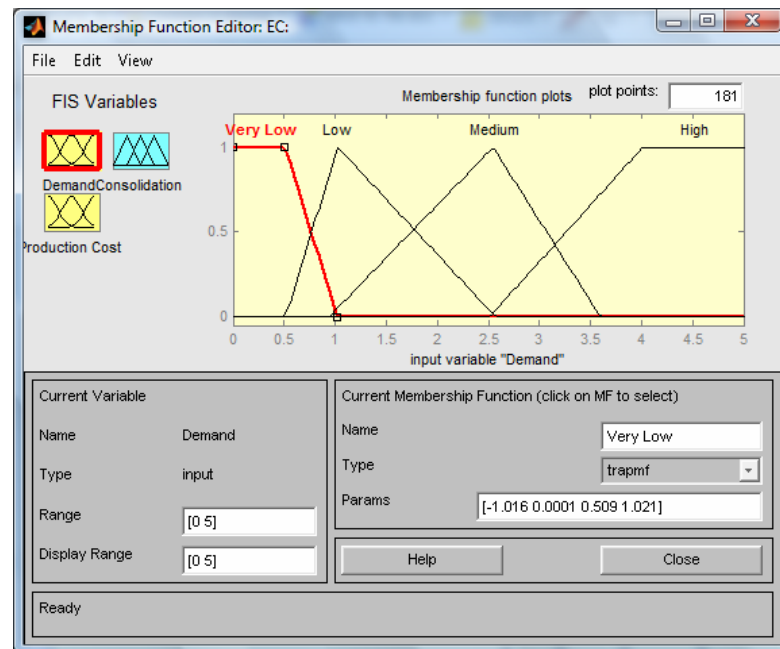
FIGURE 8.5: FIS interface for the membership functions of the input variable *demand* (*D*) for the board games.
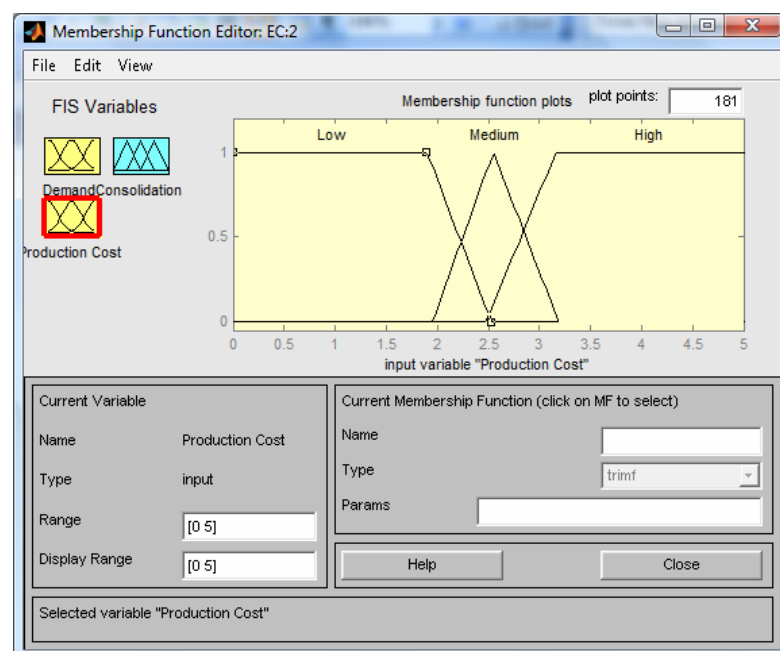
FIGURE 8.6: FIS interface for the membership functions of the input variable *Production Cost* (C_P) for the board games.
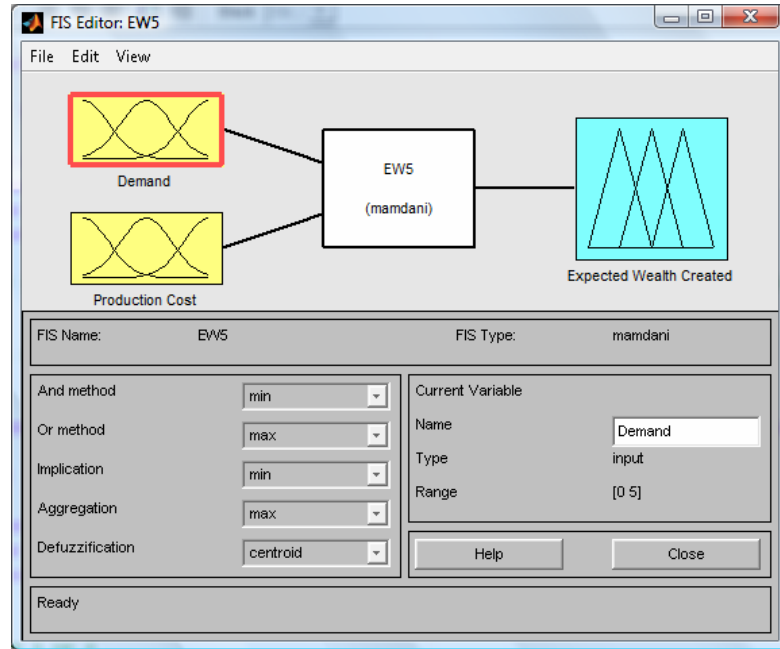
FIGURE 8.7: Mamdani-type FIS interface for the board games showing inputs *demand* (D) and *Production Cost* (C_P) as well as expected wealth outputs (E_w).

FIGURE 8.8: FIS interface for the membership functions of the output variable *expected consolidation efforts* (E_c) for the board games.

8.9 Results Discussion for Business Games on Boards

Based on the procedures highlighted in Section 8.8.2 above and from the board diagrams in Figure 8.4, we varied level of connectivity (number of links) on the
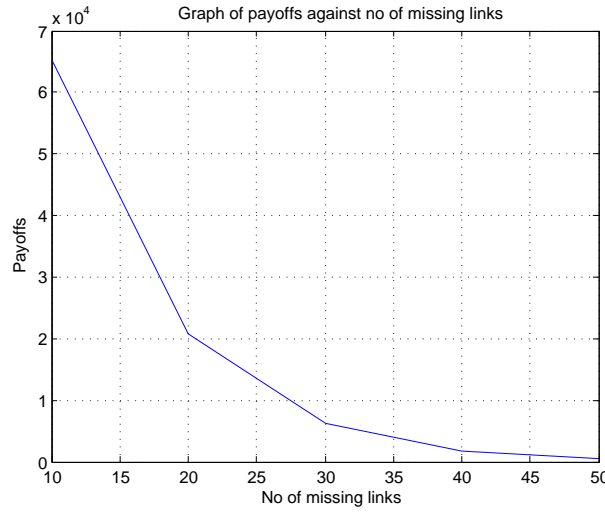
FIGURE 8.9: A graph showing how payoff of fuzzy player decreases as links on the board decrease. This figure illustrates data on Table 8.1 of page 139.

boards by removing links arbitrarily and we investigated how these restrictions (missing links) affect payoffs (profitability of businesses) [169]. This level of connectivity was used to simulate (investigate) how the level of availability of vital infrastructures such as transportation networks in a geographical location can affect the profitability of business enterprises and to achieve other objectives stated in Section 8.2 on page 124.

In the simulation, the number of links is the most important factor as this grossly affects the movements of the players' resources (the trucks). This is analogous to how poor road networks affect transportation of goods across a geographical location. In the simulation, other network characteristics such as the positions of the missing links are taken into consideration by the fuzzy variables in the fuzzy rule base. For example, if the position of the missing links affect the network such that a player needs to take longer route to reach destination, this increases the cost of production. The cost of production (CP) has been taken care of in the fuzzy rule base.

The results obtained as shown on Table 8.1 (page 139) show that the higher the level of connectivity on the boards, the higher the payoff of the players and vice-versa. That is, as the number of missing links on the board increases, the payoffs of players decrease.

From Table 8.1, l^1 on the columns represents the number of missing links on the board. Therefore, $l^1 = 4$ represents the case when four links are missing from

the board. The payoffs on these iterations are shown in column four. $l^1 = 6$ represents the case when six links are missing from the board. The payoffs on these iterations are shown in column five. $l^1 = 8$, $l^1 = 10$ and $l^1 = 12$ represent the cases when eight, ten and twelve links respectively are missing from the board and their corresponding payoffs are as shown in columns six, seven and eight respectively.

As shown in the table (Table 8.1), the payoffs in column four are greater than the payoffs in column five and these are also greater than the payoffs in columns six. The payoffs in column six are greater than the payoffs in column seven. The payoffs in column eight are the least because the boards have highest number of missing links in column eight. That is the lowest level of connectivity on the board.

From the result on these columns, it can be observed that the higher the number of missing links l^1 on the board, the lower the payoffs of the fuzzy player and vise-versa. That is, the the higher the level of connectivity, the higher the payoffs of fuzzy player on the business boards.

For example, in the sixth iteration (and same in all iterations) in which both players have strategy $[0, 10, 0]$, the payoff of fuzzy player in the board games with four missing links was 11,570.00. This is as shown in column four for the sixth iteration.

However, when six links were missing from the board, the payoff of fuzzy player in this iteration reduced to 3,708.80. This is as shown in column five.

When eight links were missing as shown in column six, the payoff reduced further to 1,110.50. Moreover, when ten and twelve links were missing from the boards, the payoffs further reduced to 307.90 and 78.20 respectively.

We therefore concluded that the higher the level of connectivity on the boards, the higher the payoffs of players.

This means that the higher the availability of transportation networks in a particular geographical location, the higher the profitability of business enterprises in such location.

This shows why developing nations that have low level of infrastructures such as transportation networks are less attractive to investors.

The graph in Figure 8.9 (page 136) illustrates these trends. This means that the lower the availability of road networks in a geographical location, the lower the prospects of businesses in such location.

Also, yellow wins more often than green because he takes his decisions based on the output of fuzzy reasoning from the fuzzy inference system (FIS). This shows the extent to which fuzzy reasoning can benefit a business operating in an adverse business environment that is clouded with diverse uncertainties as in developing nations.

We also observed that the stronger the strategy, the higher the payoff. That is, an agent that allocates more resources to marketing has stronger strategy and is more likely to have higher payoff.

Yellow, the fuzzy player begins to lose when the links on the board are extremely low. This shows the extent to which extremely poor road networks (and other infrastructures) can run a once prosperous business down.

As in Tables 4.3 and 6.1 of chapters 4 and 6 respectively, we verified these results by designing control experiments (simulations) in which the fuzzy player does not change his moves in accordance with the fuzzy rule base. The results obtained from the control experiments show that the game follows conventional trends, that is, the fuzzy player wins only where he allocates more units of resources to his marketing strategy at the start of the game than those of his competitors and his payoff also depends on this. The payoff of the fuzzy player in the control experiments (where he did not use fuzzy rule base) are far less than what he got when he used fuzzy rule base to make his business decisions.

After training, the fuzzy player performs better with higher payoffs as shown in Table 8.2 (page 140). This shows that the learning is important as the fuzzy player is able to adapt with fuzzy reasoning over time as also previously shown in FIS interface in Figure 4.8 on page 82.

The difference between the average payoffs before learning and the average payoffs after learning have been further summarised in Table 8.3 on page 141. The table shows that the fuzzy player payoffs increased in all the iterations after the fuzzy player has learned.

	Player Moves		Players Payoff Vs Missing Links l^1 (in "000)				
S/N	Green	Yellow	$l^1 = 4$	$l^1 = 6$	$l^1 = 8$	$l^1 = 10$	$l^1 = 12$
1	0, 0, 10	10, 0, 0	-12954.00	-3954.10	-1117.00	-288.60	-67.20
2	10, 0, 0	0, 0,10	-130410.00	-41844.00	-12541.00	-3480.30	-885.10
3	0, 10, 0	0, 0, 10	-131450.00	-42196.00	-12653.00	-3513.40	-894.10
4	0, 0, 10	0, 10, 0	-965.40	-105.40	37.30	32.00	14.40
5	0, 0, 10	0, 0, 10	-120830.00	-38588.00	-11503.00	-3173.00	-801.30
6	0, 10, 0	0, 10, 0	-11570.00	-3708.80	-1110.50	-307.90	-78.20
7	10,0,0	10, 0, 0	-22520.00	-7205.30	-2152.60	-590.00	-150.80
8	8, 0, 2	0, 8, 2	-32590.00	-10402.00	-3099.10	-854.30	-215.60
9	5, 0, 5	3, 0, 7	-93253.00	-29824.00	-8905.30	-2461.10	-622.90
10	0, 5, 5	4, 5, 1	-23071.00	-7302.40	-2154.90	−587.50	-146.40
11	7, 0, 3	10, 0, 0	-19659.00	-6233.20	-1843.00	-503.63	-125.81
12	9, 0, 1	3, 3, 4	-61123.00	-19581.00	-5857.90	-1622.40	-411.66
13	6, 0, 4	6, 0, 4	-61843.00	-19758.00	-5892.70	-1626.30	-410.99
14	4, 0, 6	4, 0, 6	-81506.00	-26034.00	-7762.80	-2141.90	-541.10
15	1,9 0	0, 1, 9	-119360.00	-38312.00	-11602.00	-3189.50	-811.59
16	1, 9, 0	1, 0, 9	-120560.00	-38696.00	-11602.00	-3221.50	-819.75

TABLE 8.1: **Results of simulation of the fuzzy business board games**: l^1 represents number of missing links on the board. From the table, the first column shows the serial numbers of the iterations, the second column contains player *green's* strategies while the third column contains that of *yellow*. For example, in the first iteration, *green's* strategy shows $[0, 0, 10]$, this indicates how resources are allocated to strategy $[C, W, M]$: $C = 0$, $W = 0$ and $M = 10$. Columns 4, 5, 6, 7 and 8 give the fuzzy player's payoffs when 4, 6, 8, 10 and 12 links (connections) are removed from the board respectively. It can be observed from the results that as l^1 increases, the payoff of fuzzy player decreases. This means that the less the level of connectivity on the business board, the less the payoff of fuzzy player and vice-versa. This implies that the less the availability of vital infrastructures such as road networks in a geographical location, the less the profitability of businesses in such location. The minus signs on payoffs merely show zero-sum. When it is minus, fuzzy player y wins but otherwise, green wins. Figure 8.9 on page 136 gives a graphical explanation on this table.

8.10 Conclusion

We have modelled decision making processes under uncertainty on boards using concepts of fuzzy logic, percolation theory and game theory. Our general model was termed fuzzy strategy decision making system for business boards (FSBB). The FSBB was used to investigate how various board characteristics such as level of connectivity or restrictions on the board affect the payoffs of players on the boards. Also, we examined these models with examples. The system was designed and implemented using MATLAB software. Fuzzy rules were constructed in developing

Player Moves			Players Payoff After Training (in "000)				
S/N	Green	Yellow	$l^1 = 4$	$l^1 = 6$	$l^1 = 8$	$l^1 = 10$	$l^1 = 12$
1	0, 0,10	10, 0, 0	-12965.00	-3958.20	-1118.40	-289.00	-67.31
2	10, 0, 0	0, 0, 10	-130420.00	-41848.00	-12542.00	-3480.70	-885.23
3	0, 10, 0	0, 0, 10	-131460.00	-42201.00	-12654.00	-3513.90	-894.26
4	0, 0, 10	0, 10, 0	-976.72	-109.44	35.89	31.53	14.28
5	0, 0, 10	0, 0, 10	-120850.00	-38593.00	-11505.00	-3173.50	-801.48
6	0, 10, 0	0, 10, 0	-11583.00	-3713.60	-1112.10	-308.38	-78.37
7	10, 0, 0	10, 0, 0	-22531.00	-7209.40	-2154.00	-595.74	-150.92
8	8, 0, 2	0, 8, 2	-32604.00	-10407.00	-3100.80	-854.86	-215.76
9	5, 0, 5	3, 0, 7	-93267.00	-29829.00	-8907.10	-2461.60	-623.06
10	0, 5, 5	4, 5, 1	-23082.00	-7306.50	-2156.30	-587.92	-146.48
11	7, 0, 3	10, 0, 0	-19670.00	-6237.20	-1844.40	-504.06	-125.98
12	9, 0, 1	3, 3, 4	-61134.00	-19585.00	-5859.30	-1622.90	-411.79
13	6, 0, 4	6, 0, 4	-61853.00	-19761.00	-5893.90	-1626.70	-411.10
14	4, 0, 6	4, 0, 6	-81517.00	-26038.00	-7764.20	-2142.30	-541.22
15	1, 9, 0	0, 1, 9	-119370.00	-38316.00	-11489.00	-3190.00	-811.74
16	1, 9, 0	1, 0, 9	-120570.00	-38701.00	-11604.00	-3222.00	-819.90

TABLE 8.2: **Results of simulation of the trained FSBB fuzzy player payoffs**: From the table, the first column shows the serial numbers of the iterations, the second column contains player *green's* strategies while the third column contains that of *yellow*. For example, in the first iteration, *green's* strategy shows [0, 0, 10], this indicates how resources are allocated to strategy $[C, W, M]$: $C = 0$, $W = 0$ and $M = 10$. Columns 4, 5, 6, 7 and 8 give the fuzzy player's payoffs after training when 4, 6, 8, 10 and 12 links (connections) are removed from the board respectively. It can be observed from the results that the trained agent is able to perform better after training as he wins more often than when he was not trained as compared to the results obtained on table 8.1. Where he does not win (such as in iteration 4), opponent's payoff is minimized considerably and thereby maximized his own. The strongest opponent (Geq) and weakest opponent (Leq) (explained in Section 7.4.1.1 step 11 page 68) are shown in iterations 4 and 3 respectively. The minus sign on payoffs merely shows zero-sum. When it is minus, fuzzy player y wins but otherwise, green wins. Table 8.3 on page 141 summarizes and compares the average results for the trained and untrained simulations.

the FSBB model using MATLAB toolbox and the implementation of this model heavily depends on expert knowledge and experience to facilitate the development of a reasonable fuzzy rule base for the determination of the if-then rules that represent the relationship between inputs and the output variables. Furthermore, we have applied a learning algorithm to the decision process which enables the decision agent to optimize his performance in the decision process as the game is played so as to meet the set criteria. To do the learning, *Nelder-Mead* simplex method for finding the minimum of an unconstrained multivariable function was

Average Payoffs After Training Vs Before Training (in "000)						
	Missing Links (l^1)	$l^1 = 4$	$l^1 = 6$	$l^1 = 8$	$l^1 = 10$	$l^1 = 12$	Total
1	Before Training	-65229	-20859	-6234	-1720	-435	-94479
2	After Training	-65240	-20863	-6229	-1721	-435	-94490

TABLE 8.3: This table summarizes and compares average results of fuzzy player's payoffs before training (Table 8.1 on page 139) and the payoffs after training (Table 8.2 on page 140). It can be observed that the fuzzy player performs better after training. The table therefore shows that training is very important.

used.

Our FSBB models have practical uses in business contexts as they can serve as very useful decision tools in the hands of entrepreneurs trading in environments similar to the scenarios. The experiments show that businesses are less profitable in situations where there are restrictions such as lack of availability of vital infrastructures or by constraints which may be imposed by regulatory authorities such as when two or more players are forbidden (possibly by law) from interacting to prevent collusion. Constraints can be between variables, or can be constraints imposed on communication between players. Also, given the fuzzy demand and cost of production information, the estimated selling price (E_{sp}) can be predicted according to Equation 8.12.

In arriving at our results, the simulations are based on assumptions and conditions that the players involved in the decision processes are rational players (Section 2.1.1 on page 12) and that only the fuzzy player (yellow), at the moment, uses fuzzy moves [144]. This is in accordance with our overall aim of designing models that illustrate how an entrepreneur could make effective and efficient business decisions by using fuzzy inference systems (FIS) in capturing uncertainties that may surround his business environments. This will therefore help the entrepreneur to have competitive advantages over his competitors who are unaware of the usefulness of these tools and therefore are not making use of the fuzzy inference models in their decision making processes.

Chapter 9

Fuzzy Game Approach to Wage Negotiation Decision Problems (FGAW)

In most cases, annual escalation clauses in employment contracts do specify future percentage increases in wages which are not tied to any index or rules. However, very often employers do find it difficult to meet these rigid [175, 176, 177] percentages and therefore, on various occasions, these have resulted into industrial disputes between employers and employees (or their unions) [177, 178, 179]. The percentages are mostly based on predictions of future inflation which are mostly misleading and based on historical data. In [180], Flood and Marion demonstrated that in an open economy under optimal wage indexation, in a world of one good, floating rates are preferred to fixed rates, regardless of the stochastic structure of the economy[175].

Many authors have agreed that wages ought to be positively linked to financial performance of the business and some also have detected some links between wages and profits[176].

In this thesis however, rather than pre-setting a rigid future and yearly percentage increase in wages, we propose a flexible scheme for employers and employees which they can use as decision support system for their future salary increase and this scheme uses a fuzzy inference system in arriving at more agreeable decisions on wage increase. For example, rather than specifying 5% *yearly increase* of wages, we propose that the wage increase formula needs to take into consideration other

factors which are mostly difficult to predict with certainty. These include inflation rate, business revenues or (profit), cost of production, number of competitors and other uncertain factors that may affect business operations. The accuracy of the fuzzy rule base would help to mitigate the adverse effects that a business may suffer from these uncertain factors. Based on our scheme, we propose that employers and employees should calculate their future wage increase by using a fuzzy rule base that takes into consideration these future variables which are mostly uncertain and that could affect their decisions.

9.1   Problem Definition

Wage negotiation has always been a persistent problem in business organisations [181, 182]. On many occasions, there have been cases in which the entire workforce of countries embarked on industrial strikes that resulted from wage negotiation problems. Gielen and VanOurs in [181] investigated what determines quits and layoffs that usually result as problems of poor wage negotiations by using a unique matched worker-firm dataset from the Netherlands. They concluded that in wage negotiation, the wage growth of a worker that stays in the firm is larger if that worker had a high quit probability and smaller when that worker had a high layoff probability.

In most cases, the root causes of wage negotiation [181, 183] disputes are not unconnected with inability of either of the two parties involved (employers and employees' unions) to sustain or maintain the *status quo* contained in their earlier agreement on wage increase[177]. This may be as a result of many reasons and some of these reasons are explained below on both sides.

9.1.1   Employers' Perspective

On the employers' parts, the once prosperous business might have run into an economic turbulence as a result of diverse and adverse uncertainties that surround the business environment. Several of these cases were witnessed during the recent global economic recession which affected several businesses globally and during which many businesses went underground (closed).

Therefore, when the revenue of a business goes down, then it may be economically impossible for managements to sustain the earlier agreements signed when the revenues of the companies were booming.

The same situation may occur if the rate of inflation adversely and grossly affect the cost of production (CP) in a firm without a corresponding increase in revenue.

9.1.1.1 Wage Negotiation in Developing Nations

In developing nations[184] such as Nigeria, wage negotiation has always been a chronic problem [185, 186] and this menace has been directly and indirectly running down the nation's economy for many decades. This is because many trade unions have always insisted on international wage scales for their respective professions but irrespective of whether the revenues of their countries attain those standards on which those scales were designed.

For example, the Medical Doctors, under the umbrella of the Nigeria Medical Association (NMA)[187, 188] will always insist on World Health Organisation (WHO) salary scales standard for their profession not minding whether the revenues and resources of the nation match those expected by these standards.

Also the academic staff in the universities under their union, Academic Staff Union of Universities (ASUU) [186, 187, 188] have always insisted on a *very high* salary scale called the University Academic (Staff) Salary Scale (UASS)[189, 190] as well as the international pension scheme called universities superannuation scheme (USS) which may certainly be obtainable in developed nations like the United State of America (USA) and the United Kingdom (UK). However, these unions would never consider the fact that their country's revenues are *very very far below* those of the developed nations mentioned.

9.1.2 Employees' Perspectives

Generally in any country and on the side of the employees however, the rates of inflation in the country mighty have shot up astronomically such that earlier wage increase agreement becomes no more realistic. This is because inflation affects the purchasing powers of the consumers. Example of this high inflation otherwise

known as hyperinflation is what is being currently experienced in Zimbabwe, a Southern African country.

9.2   Our Proposed Fuzzy Model for Wage Negotiation (FGAW)

Implementing and agreeing on our fuzzy reasoning (FGAW) model approach to wage negotiation would eliminate all the concerns mentioned in Section 9.1 above. The model takes effective cognisance of the factors that affect wage negotiation and effectively grasps and captures the uncertainty therein using fuzzy rules solicited from experts in the field. That is, the model considers varying ranges of inflation trends as they affect both parties and also considers the varying ranges of possible revenue increase of the organisation and arrives at an agreeable rate for wage increase which can be more sustainable for both present and in the future. This will also be more agreeable and acceptable to both parties.

For instance, rather than specifying 5% *yearly increase*, our work proposes a scheme such as:

IF Inflation is **very high** *AND Revenue is* **very low** *THEN Wage increase is* **medium**.

We verified this scheme and proved its validity with our algorithm and we discovered that it could be an invaluable tool in the hands of entrepreneurs. Details of the scheme are as explained in the sections that follow.

9.3   Justifications for the Scheme

- The scheme will reduce level of industrial disputes and revenue or profit losses. This is because both the employers and the employees already know the factors on which their wage increase are based and both parties can calculate the expected wage increase for a particular year right from their own desk based on the factors specified in the fuzzy rule base.

- Rather than management pushing or driving workers to work hard, for the betterment or success of the firm, this scheme would indirectly rest these

duties in the hands of the workers or their unions who will encourage employees to work hard so as to increase the revenues of the firm and hence, directly increase their wages.

- The scheme will reduce man hours lost on yearly wage negotiation.

- It puts the fate of the workers regarding salary increase in their own hands. The harder they work, the better the firm's revenue and the better the increase in their wages.

- It will reduce unemployment rate. This is because firms will no longer embark on sudden staff cut [181] as a result of unregulated agitation for wage increase which firms are occasionally forced to pay.

- There will be no need for staff to take abrupt pay cuts [177, 181, 182] in bids to keep the company afloat as was the case in Highland Airways [191], British Broadcasting Corporation (BBC) [182] and many other companies during the 2009 economic recession.

9.4   Factors in Wage Negotiation

In competitive labour markets, wage rates are determined by the forces of supply and demand for labour[42]. Even though, there may be many factors to be considered during wage negotiation, two major factors: inflation and revenue, and their concepts are as explained below.

In this simulation, while we are considering only inflationary trends and business revenue as the most important factors in determining wage increase, we are assuming that other factors remain constant and that decision makers are rational in their views (section 2.1.1). These other factors that are kept constant include the labour force and the market trends.

We are also assuming that the labour force of the organisations are represented jointly by their unions and that all necessary information about the company (such as the company account details) are available to both the union and the employer's representatives in the decision processes.

9.4.1   Inflation

Inflation in simple terms, can be defined as a decline in the purchasing power of money for goods and services. It is a rise in the aggregate level of prices of goods and services in a particular economy over a certain period of time[42].

Inflation is one of the major factors that are usually considered in wage bargaining [192, 193]. Den Butter and van de Wijngaert in [193] defined *wage space* as the sum of price inflation and labour productivity growth. In economics, inflation is calculated using consumer price index (CPI) [194]. Raffaela Giordano in [192] stated that the relationship between labour cost and inflation is statistically significant and quantitatively non-trivial. He further explained that high inflation countries are those where the cost of labour is lower.

9.4.1.1   Inflation Calculation

Inflation can be calculated by recording the prices of goods and services over certain years, we then take a particular year as a base year and then calculate the percentage rate changes of those prices over certain number of years. There exist many different price indices that can be used in calculating inflation, the most popular are:

- Consumer price index (CPI)

- Producer price index (PPI)

- GDP deflator

- Cost of living index (COLI)

- Commodity price index

Consumer Price Index (CPI) is the most commonly used in calculating inflation in an economy. CPI measures the prices of particular goods and services for a typical consumer.

To use CPI in calculating inflation, there must be a base year (say year 2005) and the commodity we want to use say a bottle of Coca-Cola of 25cl. For example, if

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
2010	112.4	112.9	113.5	114.2	114.4	114.6	114.3	114.9					
2009	108.7	109.6	109.8	110.1	110.7	111	110.9	111.4	111.5	111.7	112	112.6	110.833
2008	105.5	106.3	106.7	107.6	108.3	109	109	109.7	110.3	110	109.9	109.5	108.483
2007	103.2	103.7	104.2	104.5	104.8	105	104.4	104.7	104.8	105.3	105.6	106.2	104.7
2006	100.5	100.9	101.1	101.7	102.2	102.5	102.5	102.9	103	103.2	103.4	104	102.325

TABLE 9.1: United Kingdom Consumer Price Index (CPI) published by UK Office for National Statistics (ONS).

in year 2005, the price of a bottle of Coca-Cola was £1.00, year 2005 becomes our base year and the *CPI* then has index value of 100.

If by year 2006, the price of a bottle of Coca-Cola had become £1.25, then in year 2006, the value of our CPI is 125. If in 2007, the price had become £1.31, then the value of our CPI for that year is 131. This will be done for every year and a table of consumer price index (CPI) will be established.

In order to calculate the inflation for a particular year, we simply calculate the percentage change rate as follows:

$$Inflation(I) = \frac{CPI_1 - CPI_0}{CPI_0} * \frac{100}{1} \tag{9.1}$$

Where CPI_0 is the initial value and CPI_1 is the final value.

Table 9.1 (page 148) gives United Kingdom Consumer Price Index (CPI) published by UK Office for National Statistics (ONS).

9.4.2   Company's Profit

On wage bargaining and company's profit, many authors have worked on the idea of profit (Net income) sharing schemes to replace simple wage rates. One of such is [195] in which Norman Ireland explained the argument that profit sharing concerns microeconomic efficiency and relates to incentives in the place of work. He further explained that if workers see how their labour turns into profit from which they benefit, and particularly if they have some say in determining their work practices, then work will be better motivated, better performed and more highly valued.

Reinhilde Veugelers in [196] reports a model that applies a generalized Nash-Zeuthen-Harsanyi asymmetric bargaining theory [197]. He explained that the

bargaining outcome from this scenario is that workers receive the competitive wage plus a fraction of the firm's price-cost margins.

9.5   Methodology and Game Description

In accordance with the principal aim of our research which is based on ensuring the success of business organisations through effective decision processes. Our objective of fuzzy approach to wage negotiation is to develop an effective decision system that takes efficient cognizance of the uncertainty in the market and helps in achieving success in wage bargaining to ensure continued survival of the business.

9.5.1   Players' Strategies in Wage Negotiation

As defined and comprehensively explained in Section 4.1 (page 63), a strategy is a decision rule that specifies how the player will act in every possible circumstance [136]. It is a specific course of action taken by the firm. This will involve the firm allocating values to its policy variables. These policy variables are generally those aspects of its activities that the firm can directly affect and may include price, spending on promotion, marketing, research and development and so on. For each strategy of this firm, its rival (or rivals) may adopt counter-strategies [42].

In this experiment, the business has five units of initial resources (profit say £5M). Both the employer (represented as fuzzy player y) and the employees (represented as opponent player g) are deliberating on how this profit should be spent and also, how subsequent (future) profits generated by the company should hence be spent. Both players agreed on three variable-vector $[C, W, M]$. This forms the strategies for both players. That is, employer (y) strategy is $[C_y, W_y, M_y]$ while that of the employees' union is $[C_g, W_g, M_g]$. The bone of contention is "what proportion of this £5M should be allocated to each of these strategic variables C, W and M?".

In each round, the players may choose to allocate their resources to one of these three roles: *consolidation efforts* (C), *reserved wealth* (W) and *market expansion* (M). The allocations are denoted as a vector $[C, W, M]$ for each player and constitute the *strategy* of that player.

Consolidation efforts (C) refer to the proportion of the profit that adds to the wage increase of the employees. This is the most important variable to the employees as they would want to maximise this as much as they could. The reverse is the case with the employer. Employer would want to minimise allocation to this variable as much as they could. Market expansion (M) denotes the part of this profit designated for *market expansion* of the business including various advertising, marketing and promotional campaigns. These are principally targeted toward getting new customers and the most important variable to employers which they would like to maximise as much as they could. To the employees, it is less important. Reserved wealth W refers to part of the resources that are kept unused or those distributed to the firm's shareholders.

As examples of players' strategies, consider a case where the employer **Y** decides to allocate £4M out of the £5M on market expansion $M = 4$, and remaining £1M to be distributed to shareholders as shares $W = 1$. This means that for that financial year, there would be no wage increase for (or to *consolidate* the) workers $C = 0$. Then, employer strategy implies:

$$[C_y, W_y, M_y] = [0, 1, 4]$$

On the other hand, workers, represented by their union, may embark on negotiation with employer with a proposal that £3M be allocated to wage increase ($C = 3$), £1M to shareholders ($W = 1$) and remaining on market or business expansion $M = 1$. Therefore, workers strategy becomes:

$$[C_g, W_g, M_g] = [3, 1, 1]$$

The variables in our models can be tailored to the business situations in the real world and therefore are not limited to those variables that we have used in designing the system as we further explained in Section 1.3.1 on page 7 and Section 4.1 (page 63). Therefore, this model can be applied to any real business situation and the variables can be adapted to suit the situation in question.

The model can also work for systems that have more strategic variables than those that we have used in this model.

9.5.2   Sources of Fuzzy Rules

As stated in Section 1.3.2 (page 8), as in many applications of fuzzy rule-based systems, the fuzzy if-then rules used in our models have been solicited from human experts [50, 51]. We sought knowledge from human experts in the fields that are related to each scenario described in this thesis. For example, in these wage negotiation games, we sought knowledge from both the employers' sides and also from those of the unions.

In all the simulations, the accuracy of these solicited rules are judged and amended by searching related data from published economic and fuzzy inference literatures such as [42, 43, 52, 53, 54].

However, various other methods have been proposed in different publications for automatically generating fuzzy *if-then* rules from numerical data. According to Nozaki et al in [50], most of these methods have involved iterative learning procedures or complicated rule generation mechanisms such as gradient descent learning methods, genetic-algorithm-based methods and least-squares methods.

Therefore in this thesis, the fuzzy rule base we have adopted in formulating the fuzzy if-then rules used in our models have been solicited from human experts [50, 51] in the related fields.

9.5.3   The Model

From Equation 9.1, Inflation in an economy is calculated as follows:

$$Inflation(I) = \frac{CPI_1 - CPI_0}{CPI_0} * \frac{100}{1}$$

An entrepreneur may want to base his own inflation on the changes in the cost of production (CP) of his goods or services such that inflation is calculated as:

$$Inflation(I) = \frac{CP_1 - CP_0}{CP_0} * \frac{100}{1} \tag{9.2}$$

Where CP_0 is the initial value and CP_1 is the final value.

Therefore, change in inflation (ΔI) is calculated as:

$$(\Delta I) = \frac{I_1 - I_0}{I_0} * \frac{100}{1} \tag{9.3}$$

and change in profit (ΔR) of the business is calculated as:

$$\Delta R = \frac{R_1 - R_0}{R_0} * \frac{100}{1} \tag{9.4}$$

Where R_0 is the initial profit value and R_1 is the final value.

We assume that the company has initial resources (say £5M) profit. These resources are what are being deliberated upon by the two parties namely:

1. Employer (represented as fuzzy player y)

2. Employees (or their union as representative and therefore represented as player g)

These resources are to be allocated between three variables $[C, W, M]$ that form the strategy of each player. After the initial allocation which represents wage negotiation. The subsequent allocation will follow the outcome of the fuzzy rules from the fuzzy inference system and the expected outcome will determine the winner.

9.5.4   Play the Game

Following the general procedures highlighted in Section 7.4.1.1 on page 111, from Equations 9.1 to 9.4, and the *FGAW* general model in Figure 9.2 on page 154, the procedures necessary for designing the proposed automatic business decision system for wage negotiation (FGAW) are as explained below:

In the FGAW game our fuzzy player is still represented as *yellow*. The methodology of FGAW game simulation also follows the procedural steps of FDMBN general illustration in Section 7.4.1.1 (page 111) with exception to step 2, step 3 and step 7 which are modified as follows:

- Step(2) Determining the strategy: The game strategies are as explained in Section 9.5.1 (page 149). The business has five units of initial resources
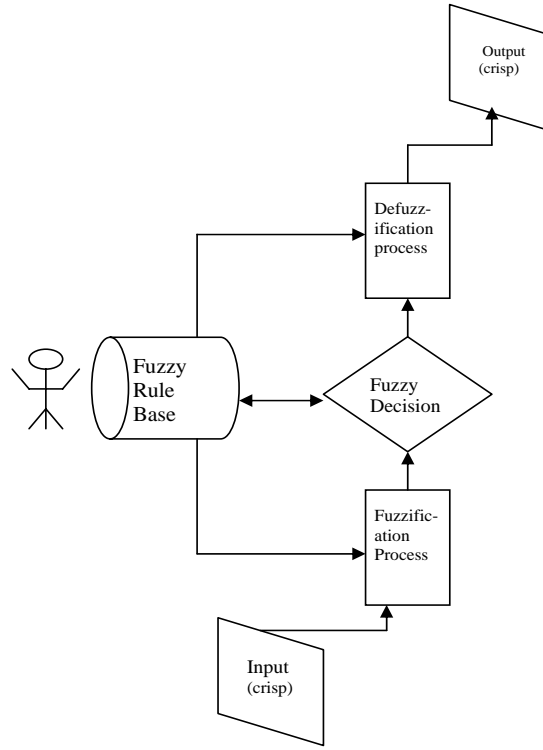
FIGURE 9.1: Fuzzy decision making system (FDMS) for fuzzy inference. This is used as part of the components of the FGAW model shown in Figure 9.2 (page 154).

(profit say £5M). Both the employer (represented as fuzzy player y) and the employees (represented as opponent player g) are deliberating on how this profit should be spent and also, how subsequent profits generated by the company should hence be spent. Both players agreed on three variable-vector $[C, W, M]$. This forms the strategies for both players. That is, employer (y) strategy is $[C_y, W_y, M_y]$ while that of the employees is $[C_g, W_g, M_g]$. The bone of contention is "what proportion of this £5M should be allocated to each of these strategic variables C, W and M?". Further details on player strategies are as explained in Section 9.5.1.

- Step(3) Determine the input and output variables of *FGAW* FIS: The inputs are the values of *change in inflation* (ΔI), and *change in business profit* (ΔR) and the outputs are *expected wage increase (consolidation efforts)* (E_c), expected wealth (E_w) and expected market expansion efforts (E_m) where: $E_m = 5 - (E_w + Ec)$ (Because the total (expected) resources of each player at any point is five).
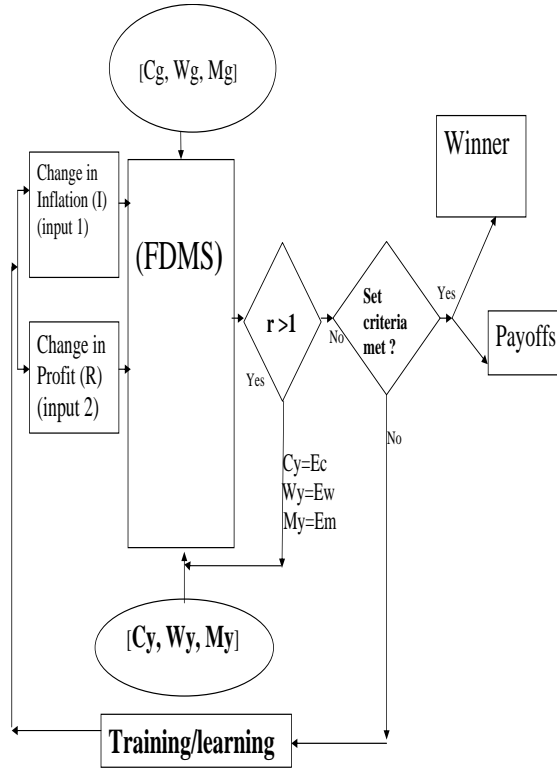
FIGURE 9.2: A model of fuzzy game approach to wage negotiation (FGAW) game showing inputs, processes and outputs. The FDMS components are as shown in Figure 9.1 (page 153).

- Step(7) Play the game: Procedures for playing the game are as follows: The game state is represented as vector $S = [g, y, A_w, r]$. Where g represents green player's amount of resources, y represents fuzzy player (yellow) amount of resources, A_w represents opponents' accumulated wealth (profit) and r is the number of rounds the game is played. Both the green and fuzzy player strategy are as stated in step 2 above.

  - Initial state of the game is $[5, 5, 0, 5]$ (i.e. according to vector $[g, y, A_w, r]$).
  - At every state $[g, y, A_w, r]$, green chooses his move by allocating to his strategies $[C_g, W_g, M_g]$ where: $C_g + W_g + M_g = g = 5$ and yellow who is the fuzzy player chooses his strategy $[C_y, W_y, M_y]$ where:

$$C_y + W_y + M_y = y = 5 \tag{9.5}$$

  As explained in Section 1.3.1 on page 7 and Section 4.1 (page 63), our choices of the number five in Equation 9.5 and for variable r are arbitrary. In a real system, any number that suitably represents the process can be chosen.

– The game changes states as follows:

$$r = r - 1, \tag{9.6}$$

$$A_w = A_w + W_g - W_y, \tag{9.7}$$

$$g = g + C_g + M_g r - (y + C_y + M_y r), \tag{9.8}$$

$$y = y + C_y + M_y r - (g + C_g + M_g r), \tag{9.9}$$

$$temp = A_w + g - y; \tag{9.10}$$

Where temp represents game payoff. Then,

$$E_m = 5 - (E_w + E_c) \tag{9.11}$$

This is because the total or expected resource of each player at any point is five. Now, the outputs of each round of the game are *expected wage increase* (*consolidation efforts*) (E_c), expected wealth (E_w) and expected market expansion efforts (E_m). This then forms the input strategies for the fuzzy player in the subsequent rounds of the game.

The game ends when $r = 0$ and if *temp* is greater than zero ($temp > 0$), the green player (employees) wins, if less than zero ($temp < 0$), then the fuzzy player (yellow or employer) wins else, the game is draw (i.e. if $temp = 0$).

– Evaluate the fuzzy inference system (FIS): Using Matlab fuzzy toolbox, all the fuzzy inputs are passed into the Mamdani-type FIS and a de-fuzzified (crisp) output interface is as shown in Figure 9.5 on page 158.

– The remaining steps follow those stated in Section 7.4.1.1 on page 111.

Sample rules of the fuzzy inference system are as copied below:

Rule Base 1:

1. If (Net Profit is High) and (Inflation is Low) then (Expected Wage Increase is Large)

2. If (Net Profit is High) and (Inflation is Medium) then (Expected Wage Increase is Medium)

3. If (Net Profit is Medium) and (Inflation is Low) then (Expected Wage Increase is Small)

4. If (Net Profit is Medium) and (Inflation is Medium) then (Expected Wage Increase is Medium)

5. If (Net Profit is Medium) and (Inflation is High) then (Expected Wage Increase is Large)

6. If (Net Profit is Low) and (Inflation is Low) then (Expected Wage Increase is Medium)

7. If (Net Profit is Low) and (Inflation is High) then (Expected Wage Increase is Small)

8. If (Net Profit is Very Low) and (Inflation is Medium) then (Expected Wage Increase is Small)

Rule Base 2

1. If (Net Profit is High) and (Inflation is Low) then (Expected Market Expansion is Medium)

2. If (Net Profit is High) and (Inflation is Medium) then (Expected Market Expansion is High)

3. If (Net Profit is Medium) and (Inflation is Medium) then (Expected Market Expansion is Medium)

4. If (Net Profit is Medium) and (Inflation is High) then (Expected Market Expansion is Low)

5. If (Net Profit is Low) and (Inflation is Medium) then (Expected Market Expansion is Very High)

6. If (Net Profit is Very Low) and (Inflation is High) then (Expected Market Expansion is Very Low)

During implementation, the fuzzy rules would be solicited from experts in the field.

The fuzzy rules for the change in inflation and business profits are coded using Matlab software as shown in Figure 9.3 (page 157). The Mamdani type fuzzy inference system (FIS) showed in Figure 9.4 (page 158) shows the basic input/output system of the FGAW model for the rule base.
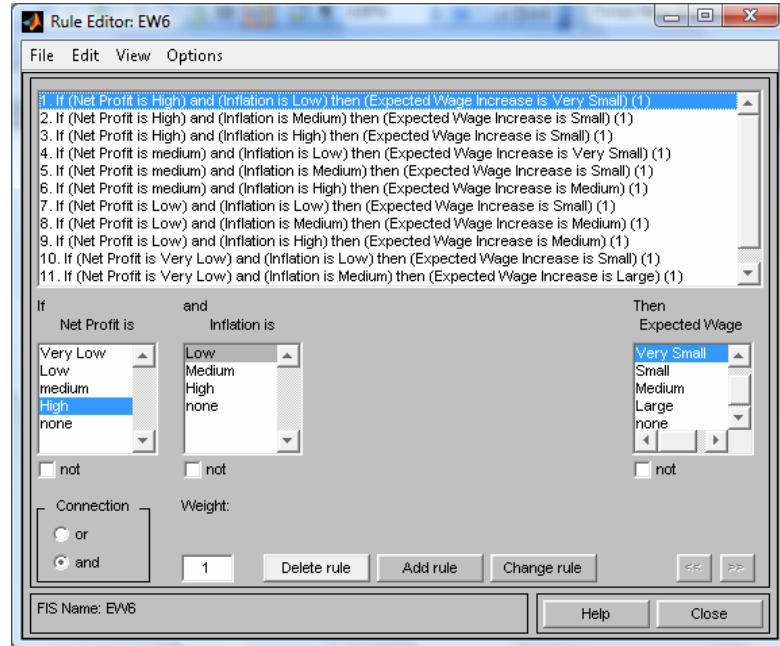
FIGURE 9.3: The rule base for the inflation rate and business profit coded using Matlab software.

Figure 9.6 on page 159 shows the FIS interface for the membership functions of the input variable *Change in inflation* (ΔI), Figure 9.7 on page 159 shows the FIS interface for input variable *Change in Profit* (ΔR) and Figure 9.8 on page 160 shows the FIS interface for the output variable *Expected Wage Increase or consolidation* (E_c).

This 2-player game is a zero sum game and therefore, yellow loses whenever green wins and vice versa.
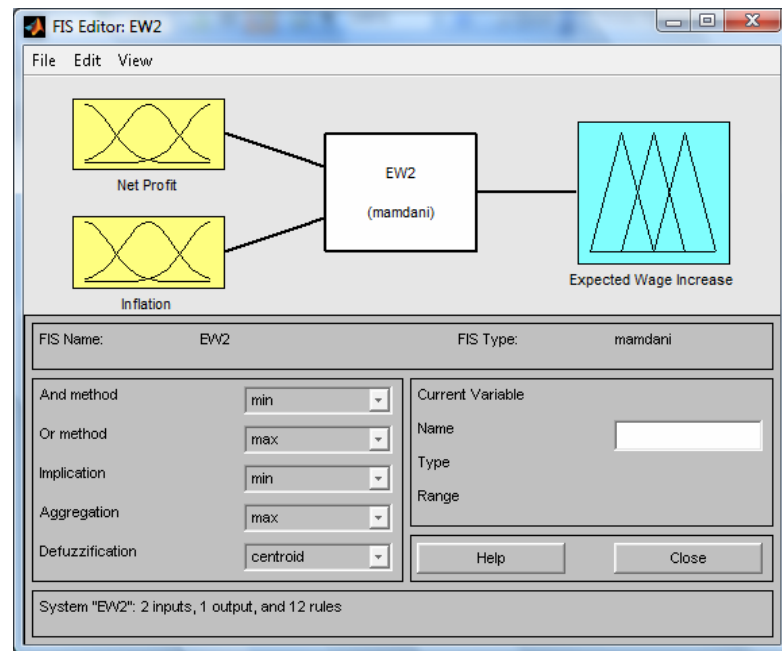
FIGURE 9.4: Mamdani type fuzzy inference system for the fuzzy decision system for wage negotiation.
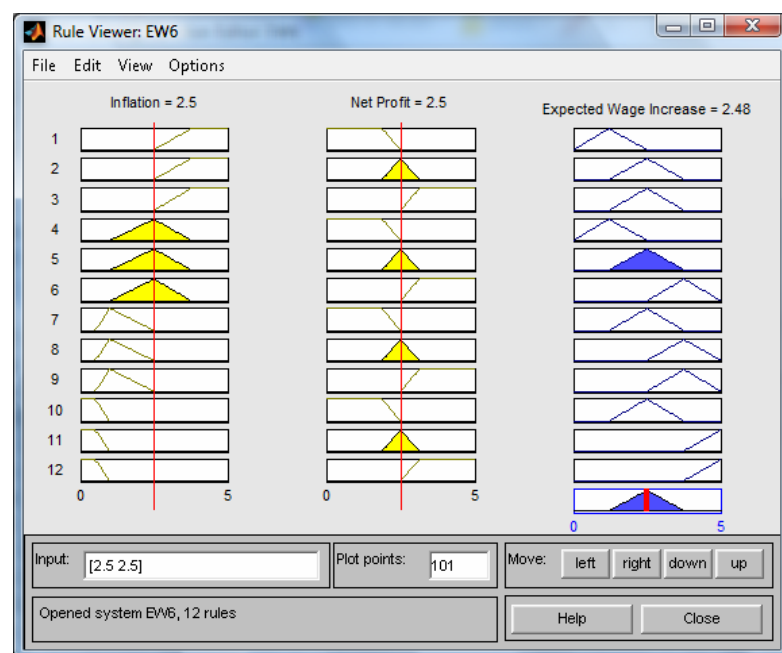
FIGURE 9.5: Defuzzified (crisp) values for *Expected Wage Increase or Consolidation* (E_c) at inputs $I = R = 2.5$.
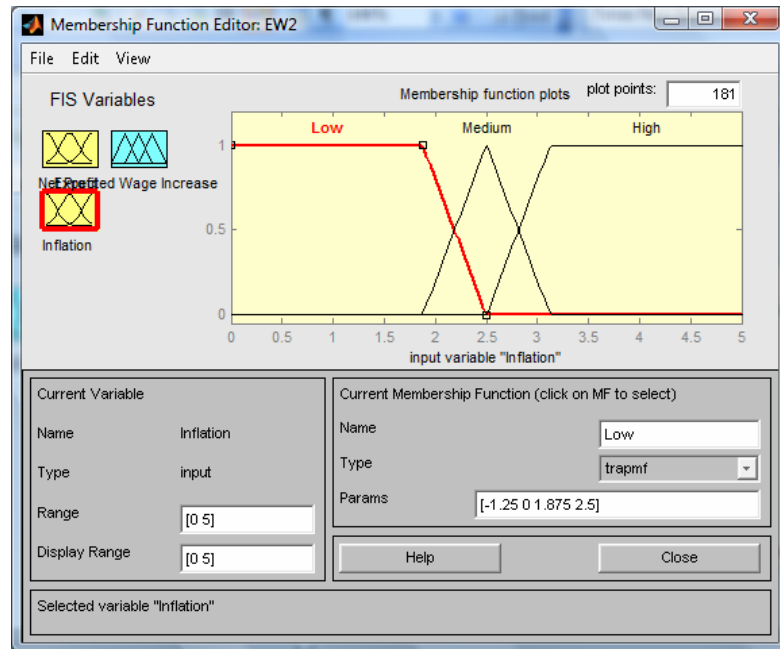
FIGURE 9.6: FIS interface for the membership functions of the input variable
Change in inflation (ΔI) for the FGAW games.
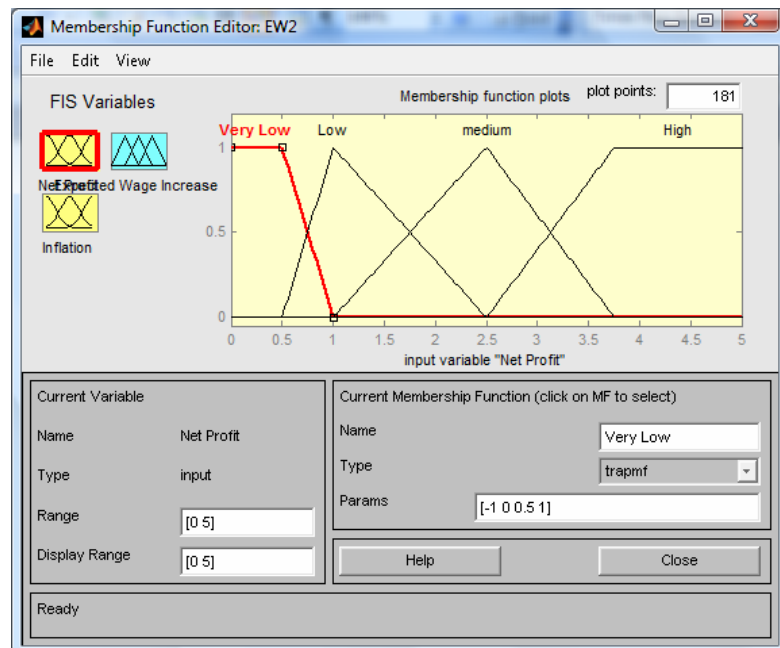
FIGURE 9.7: FIS interface for the membership functions of the input variable
Change in Profit (ΔR) for the FGAW games.

9.6 Results Discussion for Fuzzy Game Approach to Wage Negotiation Decision Problems

Based on the procedures highlighted in Section 9.5.4 above and from the results
on Table 9.2 on page 161, the results of the game shows that yellow (employer)
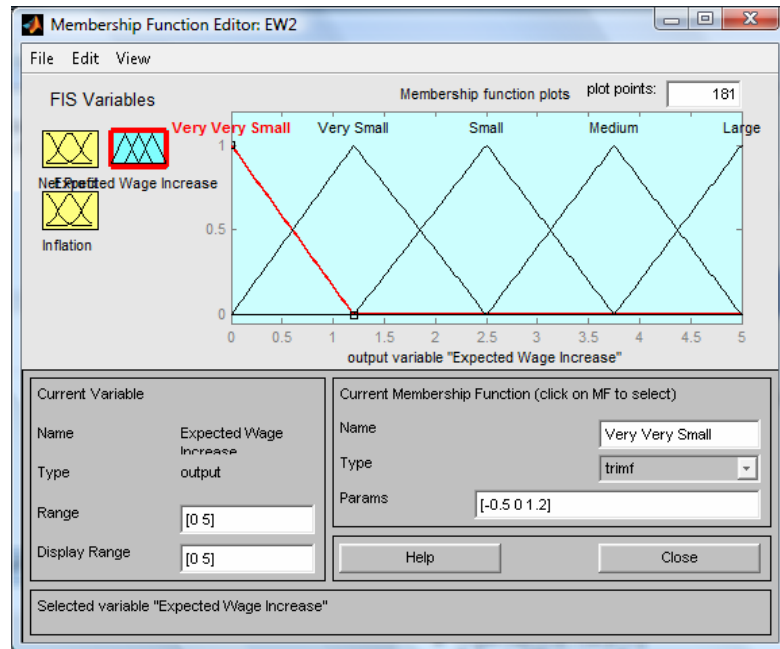
FIGURE 9.8: FIS interface for the membership functions of the output variable
Expected Wage Increase or consolidation (E_c).
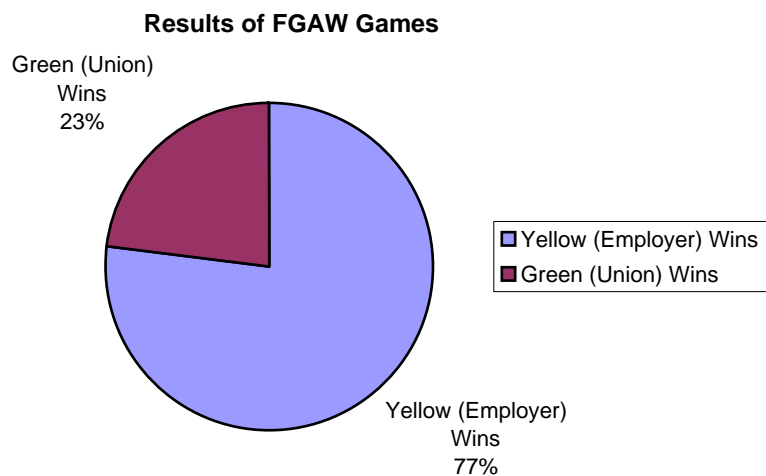
FIGURE 9.9: This chart summarises the performance of the fuzzy player (employer) and the union in the FGAW simulations shown on Table 9.2 of page 161.

wins more often than green (employees' union) because the business decision was based on the output of fuzzy reasoning from the fuzzy inference system (FIS). This shows the extent to which fuzzy reasoning can help a business if they make use of fuzzy rules as their decision support tools. Fuzzy rules make the wage negotiation more flexible and were able to capture both the present and the future uncertainty inherent in the business environment.

These results on Table 9.2 (page 161) show that the fuzzy player (**Yellow** which represents the employer) in FGAW iterations was able to win more often than the

	Player Moves		FGAW Players		Control Expmt	
S/N	Green	Yellow	Winner	Payoff	Winner	Payoff
1	1, 1, 3	1, 0, 4	Yellow	-22.0	Yellow	-63.9
2	2, 1, 2	1, 1, 3	Yellow	-94.8	Yellow	-52.7
3	3, 1, 1	4, 0, 1	Yellow	-65.2	Green	03.1
4	3, 0, 2	2, 1, 2	Green	94.9	Green	136.8
5	3, 0, 2	2, 0, 3	Yellow	-68.8	Yellow	-26.7
6	4, 0, 1	4, 0, 1	Yellow	-8.2	Green	40.8
7	0, 5, 0	0, 1, 4	Yellow	-747.2	Yellow	-704.8
8	0, 5, 0	0, 0, 5	Yellow	-906.2	Yellow	-863.8
9	1, 0, 4	2, 0, 3	Green	302.2	Green	351.6
10	3, 1, 1	3, 1, 1	Yellow	-34.2	Green	14.8
11	0, 0, 5	0, 5, 0	Green	1012.0	Green	1054.5
12	3, 1, 1	2, 0, 3	Yellow	-305.0	Yellow	-289.3
13	0, 5, 0	1, 4, 0	Yellow	-142.2	Yellow	-99.8

TABLE 9.2: **Results of simulations for the fuzzy game approach to wage negotiation ($FGAW$) decision system**: From the table, the first column shows the serial numbers of the iterations, the second column contains player *green's* strategies while the third column contains that of *yellow*. For example, in the first iteration, *green's* strategy shows $[1, 1, 3]$, this indicates how resources are allocated to strategy $[C, W, M]$: $C = 1$, $W = 1$ and $M = 3$. The fourth and fifth columns show FGAW players and the results, that is, the simulations in which the business uses fuzzy rules in taking its decisions. The sixth and seventh columns contain the control experiments in which the business management did not use fuzzy rules in the wage negotiation.
These results show that that the fuzzy player (**Yellow** which represents the employer) in FGAW iteration was able to win more often than the employees' union (**Green**) because the management of the business made use of the fuzzy inference system in making the business decisions. Out of the thirteen FGAW iterations on the table, yellow won a total of ten iterations. The control experiment in columns sixth and seven in which business did not use fuzzy rules show that the green wins as often as yellow does. These results are as summarized in the pie chart of Figure 9.9.

employees' union (**Green**) because the management of the business made use of the fuzzy inference system in making the business decisions. Out of the thirteen FGAW iterations on the table, yellow won a total of ten iterations.

The results of the control experiments in columns six and seven of the table (in which business did not use fuzzy rules to make decisions) show that green wins as often as yellow does. These two results (the FGAW games and the control experiment simulations) are as summarized in the pie chart of Figure 9.9 on page 160. The results of the control experiment (where fuzzy rules were not used) may be considered dangerous for the business. This is because if employees continue to

win and wages continue to grow without corresponding market expansion, the trend may lead to gradual demise of the business.

We also observed, as shown on the table, that the stronger the strategy, the higher the payoff. This means that the more the yellow player allocates to the market expansion variable, the better the payoffs. That is, for the business to continue to survive, decision makers must allocate more resources to marketing.

After training, the fuzzy player performs better with higher payoffs. This shows that the learning is important as the fuzzy player is able to adapt with fuzzy reasoning over time as also previously shown in FIS interface in Figure 4.8 on page 82.

9.7   Conclusion

We have used a fuzzy inference system in designing an effective and efficient decision system that models wage bargaining processes in organisations. The model took effective cognisance of the two parties involved and effectively grasped and captured the uncertainties in wage negotiation using fuzzy rules solicited from experts in the field. The model considers varying ranges of inflation trends as it affects both parties and also considers the varying ranges of possible revenue increase of the organisation and arrives at an agreeable rate for wage increase which will be sustainable for both present and future and also agreeable and acceptable to both parties.

The results of the model showed that the employer wins most often because the management implemented a fuzzy rule base in taking their wage decisions. This helped to formulate sustainable wage agreements between employers and employees.

The fact that the employer wins most often does not mean that the employees are cheated but rather guarantees the continued survival of their firm (or organisation) and therefore guarantees the continuity of their jobs.

If our scheme could be employed by entrepreneurs, it would help to greatly reduce deadlocks that usually plague wage negotiations between employers and employees (or their union) and will therefore increase productivities.

In arriving at our results, the simulations are based on assumptions and conditions that the players involved in the decision processes are rational players (Section 2.1.1 on page 12) and that only the fuzzy player (yellow), at the moment, uses fuzzy moves [144]. This is in accordance with our overall aim of designing models that illustrate how an entrepreneur could make effective and efficient business decisions by using fuzzy inference systems (FIS) in capturing uncertainties that may surround his business environments. This will therefore help the entrepreneur to have competitive advantages over his opponent players who are unaware of the usefulness of these tools and therefore are not making use of the fuzzy inference models in their decision making processes.

Chapter 10

Conclusions and Future Work

This chapter concludes the thesis. A summary of the research is presented and we highlight the main contributions of the thesis.

10.1   Summary

The main objectives of this thesis were to design and develop efficient and effective decision support schemes simulated in the form of non-cooperative zero-sum games with imperfect information. We used fuzzy logic theory, business concepts and concepts of game theory to develop decision processes (schemes) that can assist business organizations in making effective decisions in their competitive market environments.

Furthermore, we have applied a learning algorithm to the decision process which enables the decision agent to optimize his performance in the decision processes as the games were played so as to meet the set criteria. To do the learning, the Nelder-Mead simplex method for finding the minimum of an unconstrained multivariable function was used.

Results of the learning showed that the learning algorithm works very effectively and efficiently as the fuzzy player (yellow) was able to perform much better after learning with higher payoffs and this enables him to reach the set criteria

In chapter 4, we used a general illustration to describe the model and we gave the general methodology that we used throughout the research. Chapter 5 contained

experiments in which we used a case study to verified the validity of our model, that we explained in chapter 4, by using companies' real data and we used a case study of competition between Coca-Cola and PepsiCo companies who are major players in beverage industry.

In chapter 6, we carried out several experiments to investigate how the payoffs of the fuzzy player are affected as the number of competitors increased. There, we used an n-player game to model perfect market competition situations with many players and as an extension of the two-player game of a duopoly market which we considered in chapter 4.

We showed that our models can also work for systems that have more (and varied) strategic variables than those that we have used in our business models.

Chapter 7 explains how we modelled competitions on business networks with uncertain information and varying levels of connectivity. In that chapter, we varied level of interconnections (connectivity) among business units on the networks and we investigated how their payoffs were affected.

In chapter 8, we modelled business competitions as games on boards and we investigated how various constraints on the boards affected players' payoffs.

Our last experiments were in chapter 9 which contains work on wage negotiation. We proposed how fuzzy logic and game theory concepts could help to successfully reduce the problems that usually accompany wage negotiation in employers and employees relationships.

We also concluded that our FGAW model could also be applied for pension negotiations in determining what percentages the employers and employees should contribute toward their pension pots.

This research generally extended knowledge in the area of decision support functionalities through extension of methods for modeling underlying functionalities of fuzzy logic and game theory concepts. It also supports decision making processes in economics, measures impacts on individual users, multi-participant users and organisations in evaluating the fuzzy decision support systems. In accordance with the aim of classical economists, our interests were concerned with answering questions of how agents in a market could interact so as to gather maximum monetary wealth (profits) for themselves. We used the decision support scheme developed in [13] as our background research. The payoffs of the game relied on

the concepts of theory of fuzzy moves [144] (TFM) in which, according to Kandel and Zhang in [24], players were not only striving to take strategies that were advantageous to themselves but that were also at the same time, disadvantageous to their opponents.

In each simulation, we verified these results by designing a control experiment (simulation) in which the fuzzy player does not change his moves in accordance with the fuzzy rule base. The results obtained from the control experiments show that the game follows conventional trends, that is, the fuzzy player wins only where he allocates more units of resources to his marketing strategy at the start of the game than those of his competitors and his payoff also depends on this. The payoff of the fuzzy player in the control experiment (where he did not use fuzzy rule base) are far less than what he got when he used fuzzy rule base to make his business decisions.

10.2 Conclusions and Contributions

The following sections summarise the conclusions on different experiments in this thesis and also highlight the main contributions of the research.

10.2.1 The 2-Player Game

In chapter 4, sampled results of our FDMSB experiments in 2-player games which represent the *duopoly* market showed that the fuzzy player (**Yellow**) was able to win more often than the competitor (**Green**) because he made use of the fuzzy inference system in making his business decisions. This shows the effectiveness and efficiency of our model in capturing the uncertainty that surrounds business environments.

We observed from Equations 4.4 and 4.5 on page 67 and from the results on Table 4.3 on page 75 that for any of the players to win the game, he must allocate a substantial part of his resources to aggressive marketing and this allocation must outweigh that of the opponent's allocation.

According to this model and with respect to the two equations, since the number of rounds r decreases as the game is played, this reduces the strength of marketing

aggressiveness. An entrepreneur who is a new entrant into an industry, is best advised to try as much as possible to devote much of his resources on aggressive marketing campaigns. This will enable him to have a strong footing in the industry and to be able to have a large market share as early as possible as the game is played and thus, will result in winning the game.

At the end of the game, an entrepreneur can successfully forecast the estimated price for the commodity with the help of Equation 4.10: ($E_{sp} = E_w + C_P$).

Also, after training, the fuzzy player performs better as the agent was able to win more than he won before training.

From the results, it can be observed that training (learning) of the fuzzy player was really very important and the training algorithm was very effective because it enables the agent to learn and reach the performance criteria.

10.2.2   FDMSB Case Study of the Cola War

In this chapter (chapter 5), we showed that our model works very effectively and efficiently in a real business system. We illustrated the FDMSB model via a case study by taking competition in the beverage industry as our case study and this was between Coca-Cola and PepsiCo who are the major players in the industry. We chose the two companies after we have considered companies in other industries but most of them do not have uniform means of reporting their financial data which made their data comparison very difficult.

In running the FDMSB simulations, we used the companies' data available in their annual financial statements for the year 2003 as our initial values and input for the first round (year 2004). We then ran the FDMSB simulations for five rounds (representing five years) and we compared the results obtained in the simulations to the two companies' data published for the year 2008. According to the published annual financial statements, PepsiCo had lower profits (payoffs) in both years. Therefore, we took PepsiCo as our fuzzy player. After five rounds, we compared the results obtained to those which were published in the 2008 financial year and we discovered that had PepsiCo implemented our FDMSB approach, it would have outperformed its rival (Coca-Cola) with higher profit (payoff) in the year 2008 and eventually would have won the cola war.

From the results of the FDMSB simulations after five rounds, which represents five years (from year 2003 to 2008), the fuzzy player (PepsiCo) who made use of the fuzzy reasoning performed much better than its major competitor despite his weaker initial 2003 financial data that were used as input and starting strategies for the first (2004) round of the game. Therefore, we concluded that if PepsiCo or any company could make use of our model, they will be able to perform much better while competing with their peers in the market and possibly win the market.

We have further shown that the variables in our FDMSB model can be tailored to the business situations in the real world and therefore are not limited to those variables that we have used in designing the system as we explained in Section 1.3.1 on page 7 and Section 4.1 (page 63). Therefore, this model can now be applied to any real business situation.

We showed that our models can also work for systems that have more (and varied) strategic variables than those that we have used in our business models.

Furthermore, we have applied a learning algorithm to the decision processes which enables the decision agent to optimize his performance in the decision processes as the games were played so as to meet the set criteria. To do the learning, the Nelder-Mead simplex method for finding the minimum of an unconstrained multivariable function was used.

Results of the learning showed that the learning algorithm works very effectively and efficiently as the fuzzy player (PepsiCo) was able to perform much better after learning with higher payoffs and this enables him to reach the set criteria.

10.2.3 Conclusion on n-Player Game

Chapter 6 examined our model with many players using n-player game concepts which represents perfect market competition scenarios.

From the n-player simulation results, it was observed that because of the ability of the fuzzy player to grasp effectively the uncertainty in the business environment by changing his strategy based on the information provided by the fuzzy rule base, the fuzzy player wins more often than normally expected. Very interesting cases were seen in those iterations where one expected the agent to lose because it started the game with weaker strategies than those of his competitors (as it happened in

the 2-player games), but because the player reasoned in accordance with the fuzzy engine (rule base) and changes his strategies accordingly, the agent won in those cases, and more decisively than he won in the 2-player game.

We verified these results by designing a control experiment (simulations) in which the agent did not change his moves in accordance with the fuzzy rule base. The results obtained from the control experiments showed that the game follows conventional trends, that is, the agent wins only where he allocates more units of resources to his marketing strategy at the start of the game than those of his competitors and his payoff also depends on this.

Therefore, after running several simulations with the number of players n ranging from 1 to 100, the results of the n-player FDMSB game showed that the larger the number of opponent players (competitors), the better the fuzzy agent performs, as illustrated in graph of Figure 6.2 on page 105, due to the fact that he is able to adequately capture the uncertain information at his disposal which was modelled using the concepts of fuzzy reasoning and game theory.

10.2.4 Fuzzy Decision Making System on Business Networks

In chapter 7, we modelled uncertainties in business competitions in the form of games on networks. We investigate how the level of connectivity or number of links, number of opponent players (competitors), as well as choice of strategies adopted by opponent players affect the payoff of another player on the network. We called that player the fuzzy player. Learning was equally introduced to investigate how the agent adapts over time during the game. Several simulations of FDMBN were run with different number of connections (levels of connectivity) and number of players.

From the results, it was observed that the higher level of connectivity among the players, the higher the payoff of the fuzzy player. Also, results shown on the graph of Figure 7.4 on page 118 and Table 7.1 on page 116 confirm that because of the ability of the fuzzy player to grasp effectively the uncertainty in the business network environments by changing his strategy based on the information provided by the fuzzy rule base, the fuzzy agent wins more often or has a higher payoff as the number of players (competitors) in the game increases.

Concerning the position or location of a player in a network with respect to the strategies of his neighbours, when y faces a stronger (strategy that has highest resource allocation to marketing M) immediate opponent g in the network as shown in Figure 7.1B(ii) on page 110, then y has low payoffs. However, when g in turn faces a stronger immediate opponent b, this reduces the effects of g on fuzzy player y which results in y having higher payoffs.

10.2.5 Business Games on Boards

In this chapter (chapter 8), we studied uncertainties surrounding competitions in businesses using boards games as illustrations. Our investigations focused on how the level of connectivity or number of links, number of opponent players (competitors), possible constraints or restrictions on the boards as well as choice of strategies adopted by opponent players affect the payoff of another player on the boards. We also introduced learning to train and analyze how the fuzzy player adapts over time during the game.

The chapter contains experiments on the economic effects of patterns of social interactions modelled as fuzzy board games and we referred to the model as *fuzzy strategy* decision making system on *business board* (FSBB) games.

To re-emphasize how these experiments were different from those of the business networks studied in chapter 7, main objectives and our results from this chapter are as follows:

- We analysed competitions among business organisations as games on boards and we examined various board characteristics such as level of connectivity, number of nodes (players), location of a player with respect to those of his opponents, patterns of board connections and moves, strength of individual player's strategy and those of his immediate opponents. We investigated how these characteristics affected the payoffs of players in games played on boards.

- We investigated situations where there were constraints imposed by regulatory authorities such as when two or more players are forbidden (possibly by law) from interacting to prevent collusion. This led to constrained optimisation. Constraints could be between variables, or could be constraints imposed on communication between players.

- We principally used the experiments to investigate how the level of availability of vital infrastructures such as transportation (also communication) in a geographical location can affect the profitability (known here as payoffs) of business enterprises.

- We investigated why industries tend to concentrate in highly developed locations than less developed ones.

- We analysed why developing nations are less attractive to industrialists when compared to the developed ones.

- We studied how fuzzy reasoning or fuzzy inference system (FIS) can help to improve the performance of businesses in an environment that is clouded with uncertainties and adverse condition such as low level of infrastructural development.

- We also investigated how performance of these business enterprises could be improved or enhanced through adaptation or learning (training of the fuzzy players) of the fuzzy inference system.

10.2.5.1 Board Games Results

We developed an abstract, experimental and strategic board game that is played on 5X5 board among two players whom we represented as *yellow* and *green* players. Each of the two players had ten pieces which represented trucks which were loaded with firms' products. The trucks were positioned initially as shown in Figure 8.4 on page 129. As shown in the figure, each node at the start row contained two pieces (trucks) each at the initial (start) stage of the game. These ten trucks owned by each player contained products which were to be distributed at their respective goal nodes (destinations). The goal node for each player resided at the opponent's side of the board which necessitated for each player to travel across the board in order to take as many of his resources as possible to his destination (the goal node).

The board was used here to represent the road networks in a particular geographic location (such as between Edinburgh and London in the United Kingdom) and where each player represents companies (involve in logistics with trucks) at each of these mentioned locations.

We then varied level of connectivity (number of links) on the boards by removing links arbitrarily and we investigated how these restrictions (missing links) affected payoffs (profitability of businesses) on the boards. This level of connectivity was used to simulate (investigate) how level of availability of vital infrastructures such as transportation network in a geographical location can affect the profitability of business enterprises and to achieve other objectives stated above.

10.2.5.2 Conclusion on Board Games

- The results obtained showed that the higher the level of connectivity on the boards, the higher the payoffs of the players and vice-versa. This means that the lower the availability of vital infrastructures, such as road networks, in a geographical location, the lower the prospects of businesses in such location.

- Also, yellow wins more often than green because he takes his decisions based on the output of fuzzy reasoning from the fuzzy inference system (FIS). This shows the extent to which fuzzy reasoning can benefit a business operating in an adverse business environment that is clouded with diverse uncertainties as in developing nations.

- We also observed that the stronger the strategy, the higher the payoff. That is, agent that allocates more resources to marketing campaigns has stronger strategy and is more likely to have higher payoff.

- Yellow, the fuzzy player began to lose when the links on the boards were extremely low. This showed the extent to which extremely poor road networks (and other poor infrastructures) could run a once prosperous business down.

- After training, the fuzzy player performed better with higher payoffs. This showed that the learning was important as the fuzzy player was able to adapt with fuzzy reasoning over time.

10.2.6 Fuzzy Game Approach to Wage Negotiation Decision Problems

Our final experiments were carried out in chapter 9 in which we applied our model to wage negotiation and in managing employer and employees relationships.

The model described the interests of the two players (employers and employees) and effectively grasped and captured the uncertainties associated with wage decisions by using fuzzy rules solicited from experts in the field. The model considers varying ranges of inflation trends as it affects both parties and also considers the varying ranges of revenue increase of the organisation and arrives at a mutually agreeable rate for wage increase which will be sustainable for both present and future and also agreeable and acceptable to both parties.

If our scheme could be employed by the entrepreneurs, it would help to greatly reduce deadlocks that usually plague wage negotiations between employers and employees (or their unions) and will therefore increase productivity.

It must be noted that the fact that the employer wins most often does not mean that the employees are cheated but rather, it guarantees the continue survival of their firm (or organisation) and therefore guarantees the continuity of their jobs.

Our FGAW model can also be applied for pension negotiations in determining what percentages the employers and employees should contribute toward their pension pots.

In each simulation, we verified these results by designing control experiments (simulations) in which the fuzzy player does not change his moves in accordance with the fuzzy rule base. The results obtained from the control experiments show that the game follows conventional trends, that is, the fuzzy player wins only where he allocates more units of resources to his marketing strategy at the start of the game than those of his competitors and his payoff also depends on this. The payoffs of the fuzzy player in the control experiments (where he did not use fuzzy rule base) are far less than what he got when he used fuzzy rule base to make his business decision.

10.2.7 Assumptions and General Remarks

In arriving at our results, the simulations are based on assumptions and conditions that the players involved in the decision processes are rational players (Section 2.1.1 on page 12) and that only the fuzzy player (yellow), at the moment, uses fuzzy moves [144]. This is in accordance with our overall aim of designing models that illustrate how an entrepreneur could make effective and efficient business decisions by using fuzzy inference systems (FIS) in capturing uncertainties that may

surround his business environments. This will therefore help the entrepreneur to have competitive advantages over his competitors who are unaware of the usefulness of these tools and therefore are not making use of the fuzzy inference models in their decision making processes.

These models can be used as effective and efficient decision tools by business organisations that are operating in different scenarios similar to those we have described in this thesis. However, in using the models as decision tools, the entrepreneur will need to adapt, adjust and modify the variables and the decision rules to suit the situations in question as well as his business environments.

For example, rather than competing with capital resources (say £5M), the organisation's competing resources may be in terms of roles assigned to personnels in the organisation. For instance, due to persistent reduction in sales over the last few weeks, an organisation may decide to assign more personnels to the marketing department (M) and less to the operation department (C) of the organisation. The organisation will then change these roles until desirable results are attained in the business.

Therefore, the models could be used as decision tools but the variables may need to be modified to adequately represent the situations in question as we have done in chapter 5 of Cola War simulations between Coca-Cola and PepsiCo companies.

10.3 Future Work

The thesis has made its investigation and concluded, however there is much scope for future work for all the models presented in this work.

Future research may be developed along the following lines. Applying this model in a wider range of micro and macroeconomic models that are targeted to specific industries and international trade among countries. More specifically, the business network games could be used in modelling the adverse effects of international sanctions (disconnections) on the economies of nations. The model can also be applied to other different strategic games.

Experiments may be carried out to determine the actual duration and number of steps in the business games. In our model, we arbitrarily chose the steps based on

expert advice and from the game experiments in [13]. However, further work may be carried out to determine the actual duration for the business games.

To replace the adaptation of the membership functions by operations on type-2 fuzzy sets [198]. Type-2 fuzzy sets address the issues concerning uncertainty about the value of the membership functions and it allows incorporating uncertainty about the membership function into fuzzy set theory [199].

Also, the model can be applied for optimizing bidding in auctions and other areas of economics such as trading.

This FGAW model may also be applied for pension negotiations in determining what percentages the employers and employees should contribute toward their pension pots.

This automatic decision system can also be extended to capture human activities where available data are mostly uncertain or fuzzy such as in meteorology or weather forecasting and in designing embedded systems [200] for business enterprises.

Future work of this nature can also be channelled toward applications in robotics which is an area in artificial intelligence that is concerned with the practical uses of robots. A robot is a machine that is guided automatically and that is capable of doing tasks on its own. One of the other major characteristics of a robot is that by its movements or appearance, it often conveys a sense that it has intent or agency of its own. Therefore, fuzzy logic concepts and game theory may be introduced to integrate further intelligence into robots to enable them capture and grasp various uncertain events in their movements.

Learning (training) of the fuzzy system will also help robot to learn in making better decisions using fuzzy inference systems and also to deal with systems requiring advanced decision making in unpredictable environments [201].

Also, future work may be carried out to test the system behaviour toward other fuzzy inference techniques. In our model, we have used Mamdani-type fuzzy inference system. However, there are other inference techniques that can be tested on the system and evaluate its performance.

As explained in Section 3.6 (page 50), other popular common methods of deductive inference for fuzzy systems [53] that can be tested on this model are:

- Sugeno systems

- Tsukamoto models

Other areas of future work may also be channelled toward trying other optimization algorithms on the system and evaluate the performance of the models. Other optimization techniques that may be tried on the models are as suggested in Section 3.7 (page 53) and Step 11 (page 68).

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