Analytical Modelling of Wells with Inflow Control Devices

Vasily Mihailovich Birchenko

Submitted for the degree of

Doctor of Philosophy

Institute of Petroleum Engineering

Heriot-Watt University

July 2010

Abstract

Inflow Control Devices (ICD) have been successfully used in hundreds of wells around the world during the last decade and are now considered to be a mature well completion technology. This work is dedicated to the methodology of making following three decisions with respect to ICD application:

1. Selection between ICD and Interval Control Valves (ICV), the other advanced completion technology.

2. Identification of whether particular well is likely to benefit from ICD.

3. Quantification of the anticipated positive effect.

Design of an advanced completion for a particular field application often includes feasibility studies on both ICV and ICD. The choice between these two technologies is not always obvious and the need for general methodology on making this choice is recognised by the petroleum industry. In this dissertation ICD has been compared against the competing ICV technology with particular emphasis on issues such as uncertainty in the reservoir description, inflow performance and formation permeability. The methodology of selection between ICD and ICV is proposed.

The benefits of ICD application can, by and large, be attributed to reduction of the following two effects detrimental to horizontal well performance:

- Inflow profile skewing by *frictional pressure loss* along the completion (heel-toe effect).

- Inflow variation caused by *reservoir heterogeneity*.

Frictional pressure drop along the completion is an important design factor for horizontal wells. It has to be taken into account in order to secure optimum reservoir drainage and avoid overestimation of well productivity. Many authors have previously addressed various aspects of this problem, but an explicit analytical solution for turbulent flow in wellbore has not so far been published. This dissertation presents such a solution based on the same assumptions as those of previous researchers.

New method to quantify the reduction of inflow imbalance caused by the *frictional pressure loss* along a horizontal completion is proposed. The equation describing this phenomenon in homogeneous reservoir is derived and two solutions presented: an analytical approximation and a more precise numerical solution. Mathematical model for effective reduction of the inflow imbalance caused by the *reservoir heterogeneity* is also presented.

The trade-off between well productivity and inflow equalisation is a key engineering issue when applying ICD technology. Presented solutions quantitatively addresses this issue. Their practical utility is illustrated through case studies.

To those who fought against the Nazis in 1939-1945.

Acknowledgements

I would like to express deep gratitude to my supervisor, Professor David R. Davies, for sharing with me his wide spectrum of petroleum engineering experience. His guidance, understanding, patience and meticulous remarks helped me very much throughout the course of this research.

I am thankful to the sponsors of the "Added Value from Intelligent Field & Well system Technology" JIP at Heriot-Watt University for the financial support and feedback on my work. I am especially grateful to Mike R. Konopczynski (Halliburton) who initiated the "ICV versus ICD" comparison project and advised me during its course.

I would like to thank my colleague Faisal T. Al-Khelaiwi for our fruitful discussions on advanced well completions and his help with the literature review on this subject.

With regards to the uncertainty chapter of this thesis I thank Dr. Vasily V. Demyanov, Dr. Jerome Vidal and Ivan Grebenkin. Vasily has kindly advised me on Geostatistics and Uncertainty Quantification methods. Jerome particularly appreciated this chapter from the reservoir engineering viewpoint and gave a number of valuable recommendations on its possible extension. My colleague Ivan continues research in this direction. He has verified my results at the introductory stage of his own work.

I would like to extend my gratitude to Dr. Alexandr V. Usnich (University of Zurich) and Dr. Andrei Iu. Bejan (University of Cambridge) who consulted me on certain mathematical aspects of this dissertation. Alexandr advised me on properties of the Weierstrass elliptic function. Discussion of Chapter 6 with Andrei have helped me to formulate my ideas more rigorously and succinctly. He also kindly checked the derivation of formulae presented in that chapter.

I am indebted to my colleague Khafiz Muradov who has helped me to find approximate analytical solution presented in Chapter 5.

I thank AGR Group, Schlumberger and Weatherford for providing access to their software.

Last, but not the least, I would like to acknowledge the organisations primarily responsible for the education I received prior to beginning of this research: Lyceum of Belarusian State University, Moscow Institute of Physics and Technology, Yukos oil company.

Table of Contents

List of Tables

List of Figures

Nomenclature

Fluid volumes are in standard conditions, fluid density and viscosity are in completion conditions.

A_p Function of h_p, see equation (4.62), page 70

A_q Function of h_q, see equation (4.54), page 68

B Formation volume factor

C_d Discharge coefficient for nozzle or orifice

C_f Unit conversion factor: $2.956 \cdot 10^{-12}$ in field units and $4.343 \cdot 10^{-15}$ in metric

C_p Function of h_p, see equation (4.61), page 70

C_q Function of h_q, see equation (4.52), page 68

C_r Unit conversion factor: $4/\pi$ in SI, 0.1231 in field, 0.01474 in metric

C_u Unit conversion factor: $8/\pi^2$ in SI units, $1.0858 \cdot 10^{-15}$ in metric units, $7.3668 \cdot 10^{-13}$ in field units

D Internal diameter of completion

E_p Productivity error

G_p Function of i_p, see equation (5.30), page 88

G_q Function of i_q, see equation (5.24), page 87

$I_U(j)$ An auxiliary function, see equation (6.12), page 102

$I_{Uj}(j)$ An auxiliary function, see equation (6.18), page 103

J	Well productivity index
L	Length of completion
P	Tubing (base pipe) pressure
P_a	Annulus pressure
P_e	Reservoir pressure at the external boundary
P_t	Pressure at the toe of the tubing i.e. $P(0)$
P_w	Flowing bottom hole pressure (at the heel of the tubing) i.e. $P(L)$
R_a	Average correlation radius along the anisotropy direction
R_d	The ratio of drawdown at the toe and the heel of the well
R_p	Average correlation radius perpendicular to the anisotropy direction
Re	Reynolds number
Re_c	The Reynolds number at which the transition between laminar and turbulent flow starts to take place
Re_h	Reynolds number at the heel of the well
$S_U(j)$	An auxiliary function, see equation (6.13), page 102
$S_{Uj}(j)$	An auxiliary function, see equation (6.19), page 103
U	Inflow per unit length of completion
U_e	Estimate of specific inflow to the well, see equation (5.18), page 86
U_h	Fluid inflow at the heel
U_t	Fluid inflow at the toe
ΔP	$\equiv P_e - P$
ΔP_r	Reservoir drawdown i.e. $P_e - P_a$

ΔP_w Total pressure drop at the heel i.e. $P_e - P_w$

ΔP_{ICD} Pressure drop across the ICD i.e. $P_a - P$

ΔP_{rh} Drawdown at the heel i.e. $P_e - P_a(0)$

ΔR_a Half width of uniform distributions of R_a

ΔR_p Half width of uniform distributions of R_p

arcsinh Inverse hyperbolic sine

$\eta(j)$ Probability density function of the specific productivity index

$\langle \ \rangle$ Angled brackets used to denote the average values of variables

μ Viscosity of produced or injected fluid

μ_{cal} Viscosity of calibration fluid (water)

ω_2 The omega2 constant (1.529954), real half-period of $\wp(z;0,1)$, see equation (4.29), page 62

ρ Density of produced or injected fluid

ρ_{cal} Density of calibration fluid (water)

$\wp$ The Weierstrass elliptic function $\wp(z;g_2,g_3)$ with invariants g_2 and g_3

a_{ICD} Channel ICD strength (Table 5.1)

d Effective diameter of nozzles or orifices in the ICD joint of length l_{ICD}

e Absolute roughness of pipe wall

f Fanning friction factor

f_a Average Fanning friction factor

f_h Fanning friction factor at the heel of the well

f_l	Average Fanning friction factor for the part of the wellbore occupied with laminar flow
h_p	Horizontal Well number for a pressure constrained well, see equation (4.60), page 69
h_q	Horizontal Well number for a rate constrained well, see equation (4.51), page 68
i	$\sqrt{-1}$ (the imaginary unit)
i_p	ICD well number for a pressure constrained well, see equation (5.29), page 88
i_q	ICD well number for a rate constrained well, see equation (5.23), page 87
j	Specific productivity index
j_1	Minimum value of specific productivity index
j_2	Maximum value of specific productivity index
j_m	The mode (peak) of the triangular p.d.f.
l	Distance between particular wellbore point and the toe
l^*	The characteristic length of horizontal well, see equation (4.66), page 71
l_{ICD}	Length of the ICD joint (typically 12 m or 40 ft)
q	Flow rate (in the tubing) at distance l from the toe of the well
q_w	Well flow rate i.e. $q(L)$
q_w^*	Flow rate of a horizontal well of l^* length
q_{inf}	Flow rate of a hypothetical infinitely long completion, see equation (D.1), page 133
q_{nof}	Well flow rate estimate neglecting friction
u	Normalised well flow rate (Seines et al., 1993)

v Fluid volumetric velocity

x Dimensionless distance from the toe

y_p Dimensionless flow rate for a pressure constrained well

y_q Dimensionless flow rate for a rate constrained well

z_0 The zero of $\wp(z; 0, 1)$, see equation (4.30), page 62

CoV Coefficient of variation

FBHP Flowing bottom hole pressure

GLR Gas-Liquid Ratio

HO Heavy Oil

HP High Permeability

ICD Inflow Control Device

ICV Interval Control Valve

ID Inside Diameter

IPR Inflow Peformance Relationship

IPR Inflow performance relationship

MP Medium Permeability

MRM Multiple Reservoir Management

OD Outside Diameter

ODE Ordinary differential equation

p.d.f. Probability density function

PI Productivity index

TVD True Vertical Depth

USD United States dollar

WWS Wire-Wrapped Screen

List of Publications

The research work towards this thesis resulted in the following publications and preprints:

Birchenko, V.M., Al-Khelaiwi, F.T., Konopczynski, M.R., and Davies, D.R., 2008. Advanced wells: How to make a choice between passive and active inflow-control completions. In *SPE Annual Technical Conference and Exhibition*. URL `http://dx.doi.org/10.2118/115742-MS`.

Birchenko, V.M., Demyanov, V.V., Konopczynski, M.R., Davies, D.R., 2008. Impact of reservoir uncertainty on selection of advanced completion type. In *SPE Annual Technical Conference and Exhibition*. URL `http://dx.doi.org/10.2118/115744-MS`.

Al-Khelaiwi, F.T., Birchenko, V.M., Konopczynski, M.R., Davies, D.R., 2010. Advanced Wells: A Comprehensive Approach to the Selection Between Passive and Active Inflow-Control Completions. *SPE Prod & Oper.* URL `http://dx.doi.org/10.2118/132976-PA`.

Birchenko, V.M., Usnich, A.V., Davies, D.R., 2010. Impact of frictional pressure losses along the completion on well performance. *J. Pet. Sci. Eng.* URL `http://dx.doi.org/10.1016/j.petrol.2010.05.019`

Birchenko, V.M., Muradov, K.M., Davies, D.R., 2009. Reduction of the horizontal well's heel-toe effect with Inflow Control Devices. Preprint PETROL2793 submitted to Journal of Petroleum Science and Engineering.

Birchenko, V.M., Bejan, A.Iu., Usnich, A.V., Davies, D.R., 2009. Application of Inflow Control Devices to heterogeneous reservoirs. Preprint PETROL2802 submitted to Journal of Petroleum Science and Engineering.

Chapter 1

Introduction

1.1 Well-Reservoir Contact

Increasing well-reservoir contact has a number of potential advantages in terms of well productivity, drainage area, sweep efficiency and delayed water or gas breakthrough. However, such long, possibly multilateral, Maximum Reservoir Contact (MRC) wells bring not only advantages by replacing several conventional wells, but also present new challenges in terms of drilling and completion due to the increasing length and complexity of the well's exposure to the reservoir (Salamy, 2005). The situation with respect to reservoir management is less black and white. An MRC well improves the sweep efficiency and delays water or gas breakthrough by reducing the localized drawdown and distributing fluid flux over a greater wellbore area, but it will also present difficulties when reservoir drainage control is required.

Production from a conventional well is normally controlled at the surface by the wellhead choke; increasing the total oil production by reducing the production rate of a high water cut, conventional well afflicted by water coning. Such simple measures do not work with an MRC well, since maximization of well-reservoir contact does not by itself guarantee uniform reservoir drainage. Premature breakthrough of water or gas occurs due to:

1. Frictional pressure losses along the completion (the "heel-toe effect").

2. Reservoir permeability heterogeneity.

3. Variations in the distance between the wellbore and fluid contacts e.g. due to multiple fluid contacts, an inclined wellbore, a tilted oil-water contact, etc.

4. Variations in reservoir pressure in different regions of the reservoir penetrated by the wellbore.

The "heel-toe effect" is the difference in the specific inflow rate between the heel and the toe of the well due to frictional pressure drop along the completion. The effect becomes significant when this frictional pressure drop is comparable with well drawdown. The "heel-toe" effect problem is greatest in reservoirs with Darcy permeability or when a small diameter flow conduit is employed while producing at high flow rates, resulting in significant frictional pressure drop along the length of the conduit. It can be mitigated via an increase in either the wellbore conduit diameter or by the use of shorter laterals, though such solutions are not always affordable or practical.

The remaining three above listed challenges can, in principle, be partially mitigated through proper design of the wellbore's trajectory. Such a design requires a good understanding of the reservoir's geology, its drive mechanism, etc. All these are parameters are often poorly known at the time of designing and even during the drilling of the well.

1.2 Advanced Well Completions

Downhole inflow control provided by advanced well completions have proven to be a practical solution to the above highlighted problems. The inflow is controlled by restricting the fluid's flow from annulus into tubing. The distribution and settings of these restrictions are designed to enhance sweep efficiency and restrict unwanted water or gas production from the "guilty" completion intervals where these parameters are non-optimum. The two major types of advanced completions are Interval Control Valves (Gao et al., 2007) and Inflow Control Devices (Al-Khelaiwi and Davies, 2007).

Interval Control Valve (ICV) is a key part of intelligent (or smart) well technology. The completion interval of intelligent well is divided into zones by packers and the inflow into each zone is controlled by an Interval Control Valve (Figure 1.1). Hundreds of wells around the world are now equipped with remotely operated ICVs of varying complexity and capabilities that are used to actively control inflow from multiple completion intervals (zones) producing a common reservoir or from different reservoirs.

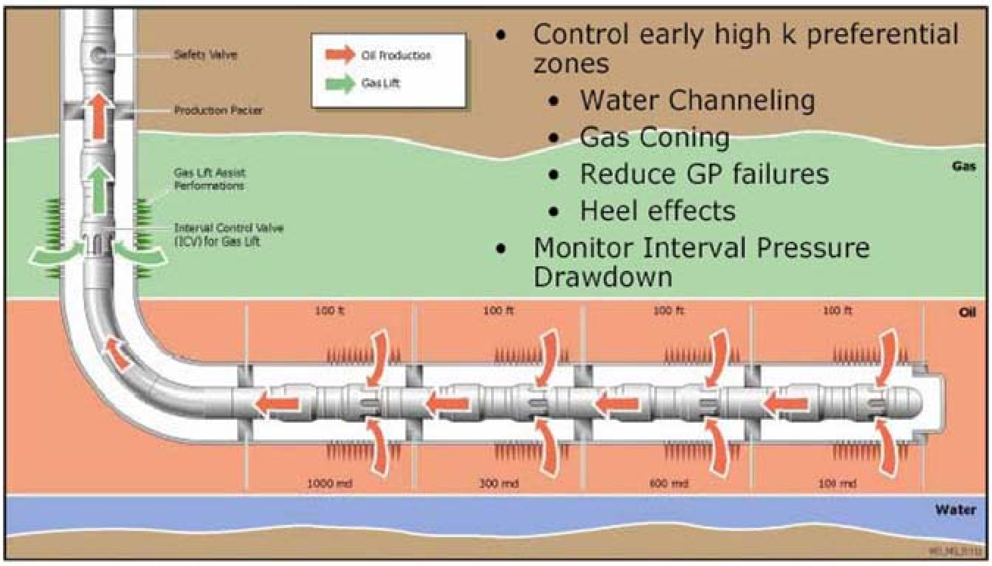

Figure 1.1: Schematics of intelligent well (courtesy WellDynamics)

An Inflow Control Device (ICD) is a well completion screen that restricts the fluid flow from the annulus into the base pipe. The restriction can be in form of channels (Figure 1.2) or nozzles/orifices (Figure 1.3), but in any case the ability of an ICD to equalise the inflow along the well length is due to the difference in the physical laws governing fluid flow in the reservoir and through the ICD. Liquid flow in porous media is normally laminar, hence the relationship between the flow velocity and the pressure drop is linear. By contrast, the flow regime through an ICD is turbulent, resulting in the quadratic velocity/pressure drop relationship.

The ICD's resistance to flow depends on the dimensions of the installed nozzles or channels. This resistance is often referred to as the ICD's "strength". It is set at the time of installation and can not be adjusted without recompleting the well.

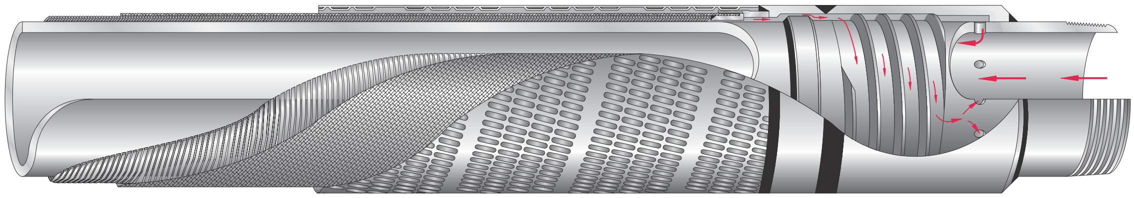

Figure 1.2: Channel ICD schematics (courtesy Baker Oil Tools)

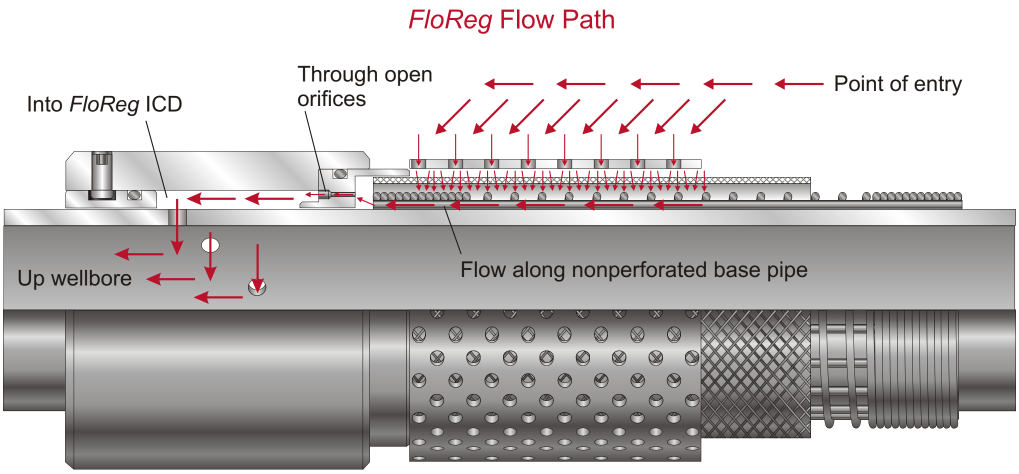

Figure 1.3: Orifice ICD schematics (courtesy Weatherford)

ICDs are especially effective in reducing the free gas production. This is because in-situ gas viscosity is usually at least an order of magnitude lower than that of oil or water while in-situ gas density is only several times smaller than that of oil or water. Hence gas inflow will dominate after the breakthrough if not restricted by gravity (Mjaavatten et al., 2008) or an advanced completion. ICDs introduce an extra pressure drop that is proportional to the square of the volumetric flow rate and can thus effectively reduce high velocity gas inflow.

ICDs have been installed in hundreds of wells during the last decade and are now considered to be a mature well completion technology.

The drivers that gave rise to the development of the ICV & ICD technologies were quite different. The first ICV applications were to allow the controlled, commingled production of multiple reservoirs via a single flow conduit (see, e.g., Nielsen et al., 2002)); while ICDs were developed to counteract the "heel-toe effect" (see, e.g., Haaland et al., 2005). The application area of both technologies has increased dramatically since these early applications. Reservoir simulation and subsequent field experience have confirmed that:

- ICV applications to a single reservoir add value (see, e.g., Brouwer and Jansen, 2004).
- ICDs can mitigate inflow or injection imbalance caused by permeability variations (see, e.g., Raffn et al., 2007).

1.3 The Scope of This Dissertation

This dissertation is focused on methodology of ICD completion design and justification. Chapter 2 compares the functionality and applicability of ICD against the competing ICV technology. Completion selection guidelines are developed based on multiple criteria drawn from reservoir, production, operation and economic factors. Reservoir engineering aspects, such as uncertainty management, formation heterogeneity, and the level of flexibility required by the development are analysed. Production and completion characteristics, such as tubing size, the number of separately controllable completion zones, the installation of multiple laterals and the value of real time information were also investigated. This systematic analysis forms the basis of a screening tool to identify the optimum technology for each particular situation.

This chapter provides a robust, comparative framework for both production technologists and reservoir engineers to select between passive (ICD) and active (ICV) inflow control for optimised, advanced well completions.

Chapter 3 extends the ICD vs ICD comparison into the field of uncertainty analysis. It illustrates the quantification of the long-term benefits of advanced completions using the probabilistic approach and shows how advanced completions reduce the impact of geostatistical uncertainty on the production forecast. Geostatistical realisations of a benchmark reservoir model were generated with a suitable level of data uncertainty. The reservoir was developed by a single horizontal well in a fixed location. The probabilistic (P10, P50, P90) oil recovery distributions were then obtained and compared for three completion options: an Open Hole with a sand control screen or a perforated pipe, Inflow Control Devices (ICDs) and Interval Control Valves (ICVs).

Steady-state performance of ICDs can be analysed in detail with well modelling software (Ouyang and Huang, 2005; Johansen and Khoriakov, 2007). Most reservoir simulators include basic functionality for ICD modelling; while some of them (Wan et al., 2008; Neylon et al., 2009) also offer practical means to capture the effect of the annulus flow. Thus, current numerical simulation software enables engineers to perform the design and economic justification of an ICD completion. However we hold a view that relatively simple analytical models still have a role to play in:

- Quick feasibility studies (screening ICD installation candidates).
- Verification of numerical simulation results.
- Communicating best practices in a non-product specific way.

Reduction of the “heel-toe effect” is one of the two main reasons for ICD application. In order to find out whether particular well may benefit from ICD installation one has to estimate frictional pressure losses along the completion. The flow regime in most of horizontal wells is turbulent (Dikken, 1990). There are many publications on frictional pressure losses along the completion available in the literature, but an explicit analytical solution for turbulent flow in wellbore has not so far been published. Chapter 4 presents such a solution and thus helps to define the area of ICD technology applicability.

Chapters 5 and 6 propose novel analytical models for reduction of inflow imbalance caused by the “heel-toe effect” and reservoir heterogeneity respectively. These models allow one to estimate the:

- ICD design parameters suitable for particular field application.
- Impact of ICD on the well’s Inflow Peformance Relationship (IPR).

The trade-off between well productivity and inflow equalisation is the key issue of the ICD technology application. The proposed models quantitatively address this issue. The practical utility of developed models is illustrated through case studies.

Chapter 7 presents the conclusions and possible extensions for this dissertation.

Chapter 2

How to Make the Choice between Passive and Active Inflow-Control Completions

2.1 Introduction

The application areas of the ICV and ICD technologies have developed so that they overlap (Gao et al., 2007). We therefore initially studied the main functional differences between ICVs and ICDs:

1. **Remote control** - ICVs deliver reservoir and production management advantages giving more flexible field development, increased value of information, improved clean-up etc.

2. **Flow conduit diameter** - The ICV's reduced inner flow conduit diameter increases the "heel-toe" effect compared to an ICD for comparable borehole sizes.

3. **Multilateral well applications** - ICVs, unlike ICDs, can currently only be installed in the well's mother bore due to limitations of available control

umbilical technology to connect to both the mother bore and laterals at the junction.

4. **Design, Installation procedure complexity, Cost and Reliability** - ICV technology is more complex; hence ICDs have the advantage in terms of simpler design and installation, and lower costs. Although the simplicity of the ICD would imply greater reliability, there is little or no available operational data to support this, particularly when considering the greater likelihood of ICD plugging, due to scale, asphaltenes, waxes, etc., compared to ICVs.

This initial study framework was extended to develop a comprehensive comparison of ICV and ICD application to an oil field (Figure 2.1 and Table 2.1) in terms of reservoir, production and cost engineering. The reasons behind the choices made are summarised in the following sections of this chapter.

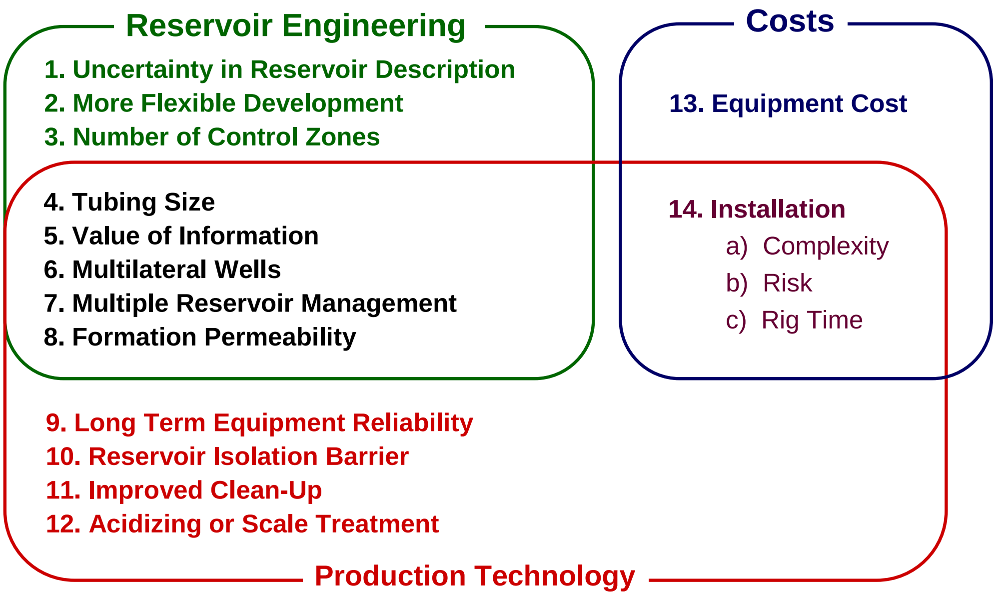

Figure 2.1: ICV vs ICD comparison framework for oil field applications

2.2 Uncertainty in the Reservoir Description

We have used a reservoir engineering uncertainty quantification methodology to demonstrate how advanced well completions can reduce the impact of the geosta-

Aspect		**ICD vs Cased Hole**	**ICD vs ICV**
1. Uncertainty in Reservoir Description		D	V
2. More Flexible Development		D	V
3. Number of Controllable Zones		D	D
4. Inner Flow Conduit Diameter		=	D
5. Value of Information		=	V
6. Multilateral Wells	Control **of** Lateral	=	V
	Control **within** Lateral	D	D
7. Multiple Reservoir Management		D	V
8. Formation Permeability	High	D	D
	Medium-to-Low	D	V
9. Long Term Equipment Reliability		C	D
10. Reservoir Isolation Barrier		=	V
11. Improved Well Clean-Up		D	V
12. Acidizing/Scale Treatment		D	V
13. Equipment Cost		D	D
14. Installation		D	D
15. Gas Production		C	V

Table 2.1: Conventional cased hole, ICD and ICV completions compared

tistical uncertainty on the production forecast. The study results are summarised below and described in detail in Chapter 3:

- ICD technology increased the mean recovery from 28.6% to 30.1% with a small decrease in risk (P10 - P90) from 6.3% to 5.3%.
- ICV technology further increased the mean recovery to 30.6% and reduced the risk compared to the base case by 50% (from 6.3% to 3.1%).

The impact of advanced completions on the probabilistic forecast of field oil recovery was studied using 8 reservoir realisations of the PUNQ-S3 reservoir (Floris et al., 2001). During this study it was found that the results were very dependent on the choice of the Base Case. Advanced completions often add little or no value if the degree of reservoir uncertainty is low and an optimum well trajectory is employed. Our Base Case well design and completion was chosen using a relatively complete knowledge of the reservoir, its geology, drive mechanisms and fluid contacts. Our results represent a conservative estimate of the advanced completion's value. This is especially true for the ICV case.

2.3 More Flexible Development

An ICV's downhole flow path's diameter can be changed without intervention while that for an ICD is fixed once it has been installed. The ICV thus has more degrees of freedom than an ICD, allowing more flexible field development strategies to be employed.

2.3.1 Reactive Control Based on "Unwanted" Fluid Flows

ICD completions restrict gas influx at the onset of gas breakthrough due to the (relatively) high volumetric flow rate of gas. Nozzle (orifice) type ICDs can also limit water influx due to the density difference between oil and water. However, an ICD's ability to react to unwanted fluids (i.e. gas and water) is limited compared to that of an ICV, especially a multi-set point ICV. ICVs allow the well to be produced at an optimum water or gas cut by applying the most appropriate (zonal) restrictions which maximises the total oil production with a minimum gas or water cut.

2.3.2 Proactive Control

ICD completions impose a proactive control of the fluid displacing oil. However, it is not possible to modify the applied restriction at a later date to achieve an optimum oil recovery, even if measurements were available that indicated an uneven advance of the flood front was occurring. ICVs, with their continuous flexibility to modify the inflow restriction, have the advantage here (see, e.g., de Montleau et al., 2006).

2.3.3 Real Time Optimization

Effective management of the reservoir sweep requires continuous adjustment of the injection and production profiles throughout the well's life. The continuous measurement of downhole and surface data (e.g. pressure, temperature and flow rate) in both injection and production wells, followed by the translation of this data into information and, finally, the carrying out of actions based on this information that require the ability to continuously adjust the fluid flow rate into or out of specific wellbore sections (see, e.g., van den Berg, 2007). For example, maintaining the required production rate from a thin oil column or from a reservoir with a declining pressure may require frequent flow rate adjustment (Meum et al., 2008). Similarly, adjusting injection distribution may be required over time to account for changing voidage replacement requirements. ICVs thus have the advantage here.

2.4 Number of Controllable Zones

The zonal flow length controlled by each ICV zone in horizontal and highly deviated wells is normally large due to the technical and economic limitations of the number of ICV that can be installed in a well. This limitation makes it difficult for ICVs to control the movement of an advancing flood front towards a well completion containing multiple sub-zones characterised by highly variable permeability values (e.g. fractures, heterogeneous reservoir with a short, permeability correlation length). A maximum of six ICVs have been installed in a well to date (Konopczynski, 2008). Various electrical and hybrid electro-hydraulic systems have been developed with the capability of managing many more valves per well. However, their high cost and operating temperature limitations have precluded their widespread acceptance by the market. The successful development of a low cost, reliable, single line, electrically-activated valve will increase this maximum number of ICV-controlled zones that can be installed in each well (Saggaf, 2008), though such a result will require radical changes in current technology.

The number of zones controlled with ICDs is limited by the number of annular flow isolation packers employed and the incremental cost of the additional ICD's and packers. For example, Saudi Aramco suggests installing them every 50-100 ft (Hembling et al., 2007). An ICD completion can thus potentially have many more control zones than an ICV completion. This makes the ICD the potentially preferred option for horizontal wells requiring many control intervals (e.g. wells completed in a fractured or a heterogeneous reservoir with a short correlation length).

Dividing the wellbore into ten or more separate zones has become a practical proposition since the development of swell packers (rings of rubber attached to the screen joint that significantly increase their volume on exposure to water or oil (see, e.g., Freyer and Huse, 2002; Ogoke et al., 2006)). Annular flow elimination is a necessary condition for achieving the regulatory effect of ICDs installed across heterogeneous formations. It can most easily be achieved by installing swell packers; though borehole collapse around the screen due to low formation strength or installation of a gravel pack (Augustine et al., 2008) can also reduce or eliminate annular flow. A practical consideration for the selection of swell packers is the inability to retrieve the ICD/swell packer completion once the rubber has reacted. Thus, after an ICD well has been completed, remedial mechanical actions to respond to problems with this type of well is usually limited to borehole abandonment and sidetracking.

2.5 Inner Flow Conduit Diameter

2.5.1 Completion Sizes

The "heel-toe" effect is one of the two primary reasons for ICD installation. The frictional pressure drop across a length of pipe is inversely proportional to the fifth power of its internal diameter when the flow is turbulent (and to the fourth when it is laminar). This strong dependence on the flow conduit diameter makes this parameter an important factor when comparing the production performance of various completion designs, particularly for high flow rate wells. An ICD completion is typically run in open hole. Its dimensions are often the same as that of the standard sand screen for that hole size; the Outside Diameter (OD) of the flow conduit being

Hole (bit) size, in.		$5\,{}^{7}/_{8}$	$7\,{}^{7}/_{8}$	$8\,{}^{1}/_{2}$ or $9\,{}^{1}/_{2}$
Maximal ICD OD, in.		$4\,{}^{1}/_{2}$	$6\,{}^{1}/_{2}$	$7\,{}^{1}/_{2}$
Flow Conduit	OD, in.	$3\,{}^{1}/_{2}$	$5\,{}^{1}/_{2}$	$6\,{}^{5}/_{8}$
	ID, in.	3.0	4.9	5.9

Table 2.2: ICD completion sizes

typically 2-3 inches smaller than the drill bit diameter (Table 2.2). By contrast, open hole ICV completions can only be applied in consolidated formation since an open annulus is required for fluid flow from the reservoir face to the valve. This inflow into the ICV will be severely hampered if the annulus collapses. The vast majority of ICV completions have been installed in cased holes, reducing the flow conduit diameter. The necessity to install (multiple) control lines imposes further restrictions on the tubing size. The production conduit installed in an ICV completion will typically be 2-3 inches smaller than that of an ICD completion in the same diameter hole. Table 2.3 illustrates this for an $8\,{}^{1}/_{2}$ in. hole size. Table 2.4 and Figure 2.2 present the flow conduit/drill bit diameter relationship for some other sizes.

	Flow Conduit		Casing	
	OD, in.	ID, in.	OD, in.	ID, in.
ICD	$6\,{}^{5}/_{8}$	5.9	No	No
ICV	$3\,{}^{1}/_{2}$	3	7	$3\,{}^{1}/_{2}$

Table 2.3: ICD and ICV completion sizes for $8\,{}^{1}/_{2}$ in. hole

It should be noted that the above limitations on flow conduit size apply to the completion interval only; larger tubing sizes can be used above the completion zone. Thus a two-zone, shrouded ICV system can be installed in the larger diameter casing above a production liner.

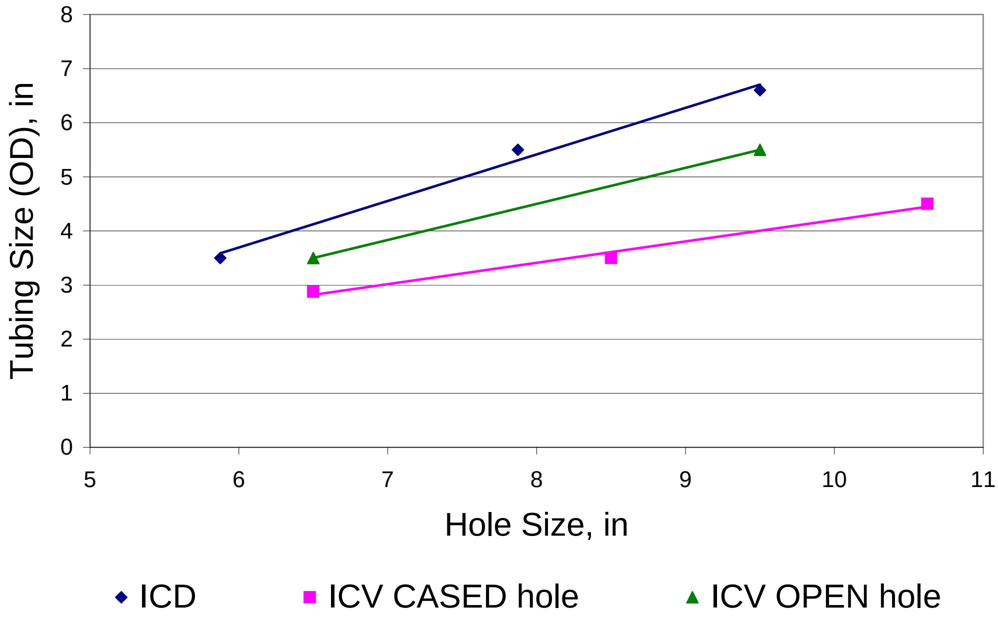

Figure 2.2: ICD and ICV tubing sizes vs wellbore hole size

2.5.2 Impact of the Inner Flow Conduit Diameter on Inflow Performance

Fluid flow is governed by pressure differences. An optimal design of an ICD completion requires a comparison of the pressure drop in the reservoir with that across the ICD. These two values should be of the same order of magnitude (Table 2.5). A reasonable level of inflow equalization can be achieved when the two pressure drops are equal, thus ICD installation may be worthwhile if the "heel-toe" effect is significant. A high level of inflow equalization requires the pressure drop across the ICD to be several times greater than the pressure drop across the reservoir. These simple considerations are in agreement with recommendations made by the major ICD suppliers.

The most influential parameters for the ICD completion design are the well's PI – both the absolute value and its variation as a function of the location along the wellbore, the length of the completion, the target drawdown or production rate and the in-situ, reservoir fluid's properties (density and viscosity). The optimum ICD strength (i.e. nozzle diameter or pressure drop ("bar") rating) for each particular well can be estimated using analytical formulae (Chapters 5 and 6); though

Hole size, in.	OD ICD flow conduit, in.	OD cased hole ICV flow conduit, in.	OD open hole ICV flow conduit, in.
$5\,^{7}/_{8}$	$3\,^{1}/_{2}$	–	–
$6\,^{1}/_{2}$	–	$2\,^{7}/_{8}$	$3\,^{1}/_{2}$
$7\,^{7}/_{8}$	$5\,^{1}/_{2}$	–	–
$8\,^{1}/_{2}$	–	$3\,^{1}/_{2}$	$5\,^{1}/_{2}$
$9\,^{1}/_{2}$	$6\,^{5}/_{8}$	–	–
$10\,^{5}/_{8}$	–	$4\,^{1}/_{2}$	–

Table 2.4: ICD and ICV tubing sizes vs hole size

ΔP_{ICD}	$\ll$	ΔP_r	ICD does not influence inflow profile
	$\gg$		Unjustified reduction of well deliverability
	$\approx$		Optimal ICD completion design

Table 2.5: Pressure drop in ICD completion design

a detailed analysis of the completion performance requires the use of numerical methods available via commercial, well modelling software. An example analysis was performed with appropriate well modelling software (Halliburton, 2009). Three homogeneous reservoir scenarios were used to illustrate the impact of flow conduit diameter on the inflow distribution along the wellbore and the IPR of the well is given below. In all the three cases

- Wellbore diameter was $8\,^{1}/_{2}$ in.
- Completion length was 3200 ft.
- k_v/k_h ratio was 0.1
- A two-zone ICV (Figure 2.3) and a channel ICD completion were compared using a Wire-Wrapped Screen (WWS) completion as a reference.

Table 2.6 outlines the key characteristics of three models: High Permeability (HP), Heavy Oil (HO) and Medium Permeability (MP).

Case name	Horizontal permeability, mD	Oil viscosity at reservoir conditions, cP
HP	1000	1.1
HO	1000	11
MP	100	1.1

Table 2.6: The three reservoir scenarios

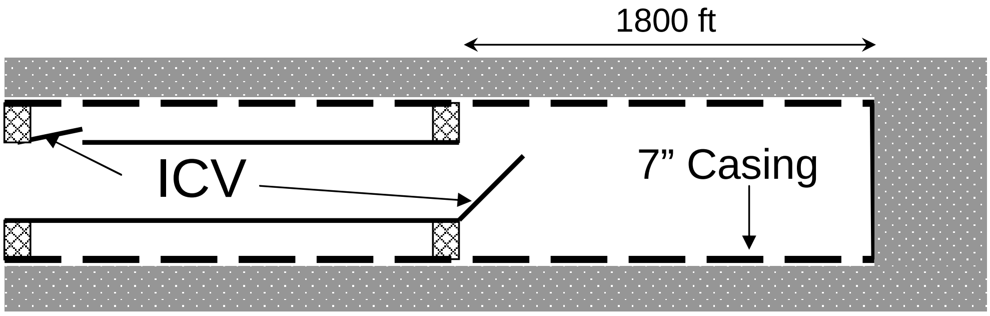

Figure 2.3: A two-zone ICV completion

2.5.3 Inflow Distribution along the Wellbore

High Permeability Case

The effects of the production conduit size and the “heel-toe” effect are usually dominant in high permeability, high productivity wells. The WWS completion on Figure 2.4 shows a moderate difference between the inflow rate from the heel and that from the toe for a high production rate (10,000 BOPD). The production from the heel zone of the ICV completion is three times higher than from the toe zone. An “0.2 bar” ICD decreases the small “heel-toe” effect observed in the WWS completion. It demonstrates the best performance out of the three completions. A stronger ICD (e.g. “3.2 bar”) would have completely equalized the inflow profile. The WWS and ICD completions gave practically the same, equalized inflow profile while the “heel-toe” effect for the ICV completion decreases to 20% at a lower production rate (3,000 BOPD).

The well’s specific PI (0.26 BOPD/psi/ft or 2 Sm^3/d/bar/m) for the High Permeability case is based on a Troll oil well (Haug, 1992). The “heel-toe” effect has been long recognised as a major challenge for the Troll-West oil development; the field seeing the first, large scale, deployment of ICD technology. However, the overwhelming majority of world's oil wells have a productivity index at least one order of magnitude lower than that encountered in Troll-West. The resulting reduction in the "heel-toe" effect will be demonstrated with the next two cases.

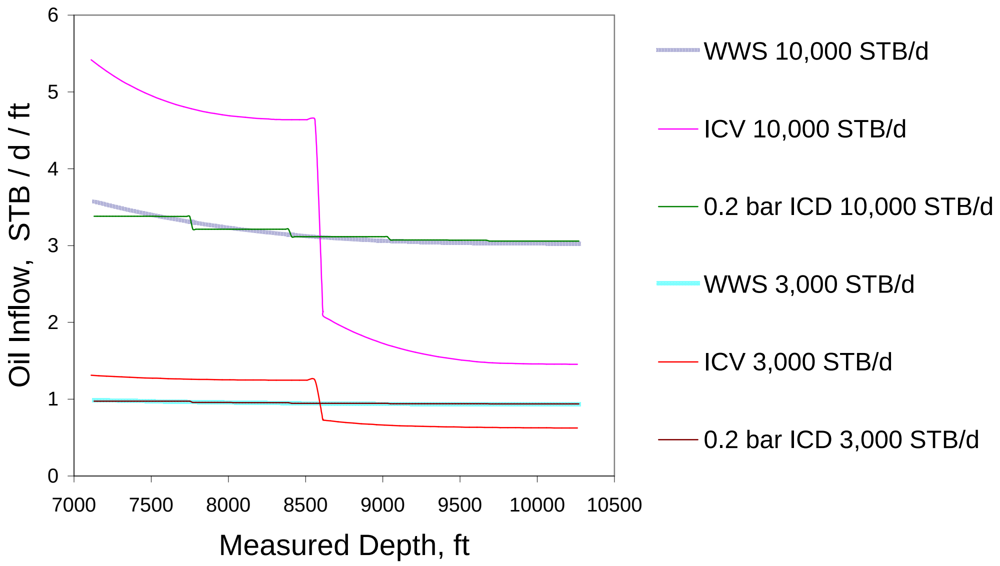

Figure 2.4: High Permeability case, inflow from reservoir to well

Heavy Oil Case

The WWS and the ICD completion demonstrate a high level of inflow equalisation at both the 3,000 and 10,000 BOPD production rates (Figure 2.5). The magnitude of the "heel-toe" inflow ratio in the ICV completion is reduced to 1.5 times (compared to 3 times in the High Permeability scenario). An increased oil viscosity decreases the "heel-toe" effect. This occurs because the drawdown is proportional to viscosity (Darcy's law) while frictional pressure loss depends weakly on viscosity if the flow is turbulent (as illustrated by the Moody diagram (Moody, 1944)). This combination of parameters allows the drawdown to increase while the frictional pressure drop remains almost the same. Hence the impact of frictional pressure drop on the inflow profile will become smaller.

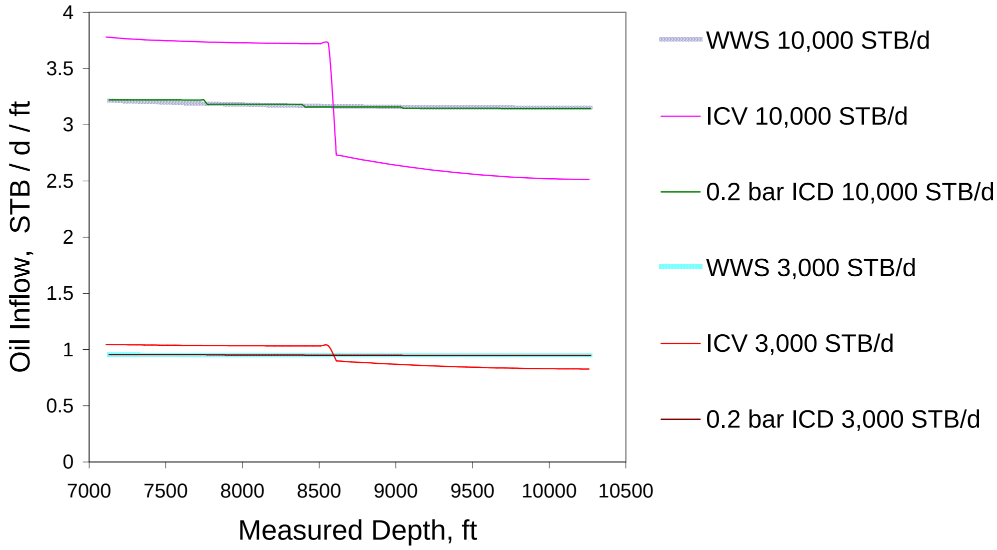

Figure 2.5: Heavy Oil case, inflow from reservoir to well

Medium Permeability Case

A reduction in reservoir permeability increases the drawdown (at the same production rate) while not influencing the pressure drop along the wellbore. Hence the Medium Permeability (Figure 2.6) and Heavy Oil (Figure 2.5) cases show similar results.

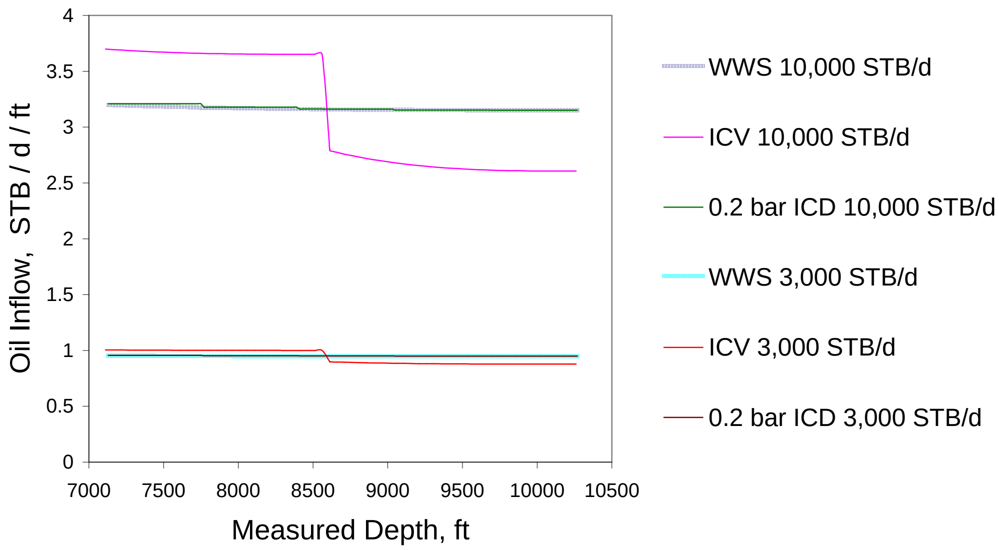

Figure 2.6: Medium Permeability case, inflow from reservoir to well

2.5.4 Inflow Performance Relationship

Our well performance calculations employed the nodal analysis technique with the node placed downstream of the completion. The WWS will thus have a better IPR than the advanced completions since they introduce an additional pressure drop into the fluid's flow path from the reservoir to the tubing. In high permeability formations the smaller diameter of the ICV completion's flow conduit will frequently limit the well's production rate due to its poorer outflow performance.

Figure 2.7 shows the High Permeability case's production performance based on the IPR curves for the three completions types. The WWS demonstrates the best inflow performance, as expected. The additional pressure drop imposed by the "0.2 bar" channel ICD is relatively small, its IPR is thus only slightly lower. The two-zone ICV completion, with both valves fully open, takes "third place" in this IPR comparison due to the smaller flow conduit diameter in the completion zone.

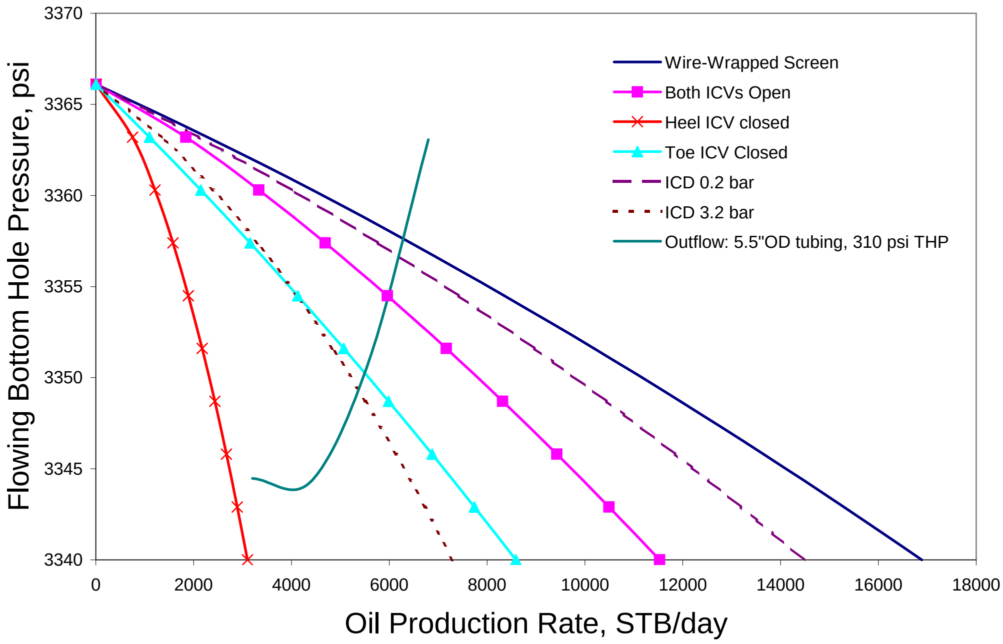

Figure 2.7: High Permeability case, impact of advanced completions on inflow performance

Shutting the heel ICV will (a) shorten the wellbore/reservoir exposure length by forcing the fluid to flow the "long way" via the toe ICV and (b) reduce the inner conduit diameter causing the total flow rate from the toe section to flow through the smaller diameter tubing. The IPR for the ICV completion is thus the lowest of all. The average flow path length becomes shorter when the toe valve closes, a scenario that improves the inflow performance. The performance of the "3.2 bar" channel ICD is similar to that of the ICV completion with the toe ICV closed. The overall conclusion is that the larger flow conduit diameter gives the ICD an advantage over ICV. This advantage plays an important role in high permeability, high production rate applications.

2.6 Formation Permeability

Table 2.7, a compilation of published ICD field applications, shows that they have been mainly applied to reservoirs with an average permeability of one Darcy or greater, the only exception being the Shaybah field (Salamy et al., 2006) where ICDs were applied to reduce the production of free gas from the gas cap. The requirement to create a completion pressure drop similar to the reservoir drawdown has two important consequences with respect to ICD applications in medium and low permeability reservoirs:

1. **Low permeability reservoirs** are normally produced at a higher drawdown than more permeable reservoirs; hence an ICD employed in such a field must also generate high pressure drop for effective equalisation while being sufficiently robust to withstand both the high pressure drop and, possibly, a high flow velocity throughout the well's active life. Any erosion (enlargement) of the ICD restriction will reduce inflow equalisation. Erosion is expected to preferentially occur at higher permeability zones in heterogeneous formations due to their higher production potential and weaker formation strengths. Selective erosion could thus reduce the pressure drop across the high permeability zone while it maintains that across the low permeability zones restriction. The level of flow equalisation will then be reduced. It is expected that suitable equipment design and proper choice of construction materials will mitigate this concern, as has been achieved for ICVs (Gao et al., 2007).

Well Type	Field	Permeability	Challenge
P	Troll	6 D	Gas
P	Grane	7 D	Gas
P	Zuluf	3.5 D	Water
P	Ringhorne	> 1 D	Water & Gas
P	Chayvo	> 1 D	Water & Gas
P	Etame	1.8 D	Water
P	Emlichheim	1-10 D	Water
P	West Brae	6 D	Water & Gas
I	Urd J-1H	0.1-2.5 D	Water
P	Shaybah-257	10-200 mD	Gas

Table 2.7: Published ICD field applications

Erosion is an important aspect of advanced completion design which should not be overlooked. Further, the extra pressure drop across the completion with an optimal ICD completion will significantly reduce the well's productivity / injectivity index throughout the complete life of the well. This reduction will become less acceptable as the permeability of the reservoir decreases.

2. ICV application in medium and low permeability reservoirs does not require such a reduction of the well's productivity / injectivity. ICVs have been qualified to operate with static pressure differentials of 690 bar and unloading pressures of 240 bar, though practical long term (years) production at high pressure differentials greater than 100 bar can result in significant erosion (Schlumberger, 2002). Use of a two position ICVs (fully open or fully closed) will significantly reduce the risk of erosion; while still giving near optimum hydrocarbon recovery in some circumstances (Zandvliet et al., 2007).

Reservoir permeability is an important parameter, both when making the choice between an ICV or ICD completion and when selecting the optimum type of ICV or ICD to be installed. A simultaneous analysis of the following reservoir-related aspects is required for this. Table 2.8 summarises the following discussion on the impact of reservoir permeability on the choice between an ICV and an ICD:

1. Inflow control objectives. An optimal development strategy does not always require complete uniformity of inflow that an ICD can provide. Thus flow equalization might not be required if the distance between the wellbore and aquifer (or injector) varies significantly for different parts of a long horizontal well. The required degree of inflow equalization must be determined if inflow equalization is not the sole control objective.

2. Well performance. A high ICD strength may be needed to achieve a high level of inflow uniformity, reducing the overall well productivity / injectivity. A compromise between these two criteria must be sought.

3. Fluid phases involved (oil, water, gas).

   (a) Both ICVs and ICDs can be used to manage gas flow distribution in a gas injection completion. ICD application to gas injection in oil fields is unlikely to pose:

      - erosion concerns since the injected gas is normally dry, sand-free and non-corrosive;
      - injectivity loss concerns since the viscosity of the gas at reservoir conditions is at least an order of magnitude lower than that of oil or water hence gas injectivity (in reservoir volumes) is considerably higher than that of water.

   (b) Limiting water production in a low permeability reservoir with an ICD presents practical difficulties due to the resulting high pressure drop across the completion.

   (c) ICDs can be useful in reducing volume of gas cap gas produced in low permeability fields. The ICD's pressure drop is proportional to the square of the volumetric flow rate; while the in-situ gas density is several times smaller than that of oil or water. Downhole, gas flow velocities are greater than those experienced during liquid production, hence an ICD than will restrict gas production more efficiently by water production.

(d) High viscosity emulsions can form within advanced completions incorporating a small diameter flow restriction. Emulsion formation depends on which surface active components present within the crude oil and the shear experienced by the fluid mixture during flow through the restriction. The emulsion can increase the fluid's viscosity several times, reducing the well's outflow performance.

4. Production / injection rate. The relationship between the pressure drop and the flow rate is linear for liquid flow in the reservoir and quadratic within the ICD. The ratio of these pressure drops, and ultimately the completion design, thus depends on the well's production or injection rate. The efficiency of the ICD will decrease if the well operates at a different flow velocity from the value for which the ICD completion was designed. Appropriate sensitivity analyses should study the implication of expected / possible flow velocity changes on the ICD completion's performance.

5. Productivity variations along the wellbore. An ICD allows many intervals to be controlled along the inflow zone - though high permeability contrasts can be difficult to smooth out with a constant strength, ICD completion.

2.7 Value of Information

Downhole pressure, temperature and flow rate measurements can now be made available on a real time basis due to advances in fiber optic sensing technology. This technology can be applied to conventional as well as advanced (ICD and ICV) completions. Measurements can be made both outside the completion (at the sandface) and within the flow conduit. Analysis of this data improves the surveillance engineer's understanding of the subsurface processes. Any required remedial actions can thus be more quickly identified and implemented based on up-to-date well data.

The ICV's advantages with respect to "Value of Information" stems from its remote control capability. Changing the well's total production rate via the surface choke is the only action that can be taken with conventional and ICD completions;

		Inflow Control Devices	Interval Control Valves
Prolific reservoirs	Oil producer	Prevent early water & gas breakthrough (+)	Similar to ICD (+) but small tubing size restricts production or injection rate. Can be mitigated by drilling larger hole (–).
	Gas/water injector	Equalise injection profile (+)	
Medium and low permeability reservoirs	Oil producer	Reduce gas-liquid ratio (+). Water cut not reduced (–).	Reduce both GLR and water cut (+)
	Gas/water injector	Suitable for gas injection (+). Application for water injection requires larger injection pressure to overcome ICD pressure loss (–) and erosion resistant ICD design (–).	Suitable for both water and gas injection (+). Small tubing size important if injection rate is high (–).

Table 2.8: The role of formation permeability in choice between ICV and ICD for oil production and water/gas injection wells

while ICV completions allow remote control of the individual zone what increases the value of information coming from downhole sensors.

The ICV itself is a source of information. Disturbing the well inflow e.g. by sequential ICV closure, allows identification of the zonal productivity. A well test can be performed during a planned or unplanned well or zone shut-in. The above advantages become increasingly important as a larger number of fields employ real-time production optimisation.

2.8 Multilateral Wells

An ICV installed in the mother-bore of a multilateral well can control the inflow from a lateral e.g. balance the flows from multiple laterals or react to changes in a particular laterals' performance (see, e.g., Haugen et al., 2006). Today's technology does not allow installation of an ICV within the lateral itself.

ICDs cannot control laterals in the same way as ICVs, however they can offer inflow control along the length of the lateral (Qudaihy et al., 2006). The different flow control capabilities offered by ICVs and ICDs result in both technologies being employed in multilateral wells (see, e.g., Sunbul et al., 2007).

2.9 Multiple Reservoir Management (MRM)

Simultaneously accessing multiple reservoirs from the same wellbore yields reduced capital and operational expenditure for field development. Both national petroleum legislation and good reservoir engineering practice require allocation of the field's or well's total daily production to a particular zone as well as prevention of reservoir cross-flow. The advantages of MRM are:

- Optimal sequential production (Akram et al., 2001)
- Commingled production via a single wellbore (see, e.g., Nielsen et al., 2002)
- Controlled fluid transfer between layers (for sweep improvement or pressure support) (Lau et al., 2001)
- In-situ (auto) gas lift (see, e.g., Al-Kasim et al., 2002)
- Prevention of cross flow between reservoirs during periods of well shut-in or low rate production. Such cross flow can damage reservoirs due to incompatibility of fluids or changing the fluid saturation levels of the rock as well as can resulting in loss of reserves to low pressure reservoirs.

These advantages have been achieved in the field with ICV completions. ICD MRM applications have not yet been published, although ICDs are capable, in principle, of limiting cross-flow due to the imposed additional, flow resistance. They are not capable of preventing cross flow between reservoirs. A dual completion (Figure 2.8) is the conventional solution for simultaneous management of two reservoir layers; while a Single-string, ICV completion (Figure 2.9) is the "advanced" option. A 121/4 in. hole, 95/8 cased well can be completed with two 31/2 tubing strings (a dual completion) or a $5\,{}^{1}/_{2}$ in. flow conduit with ICVs (the advanced completion).

ICVs have the advantage over ICDs for MRM (Table 2.9). This is well known as commingling was one of the original drivers behind many ICV installations.

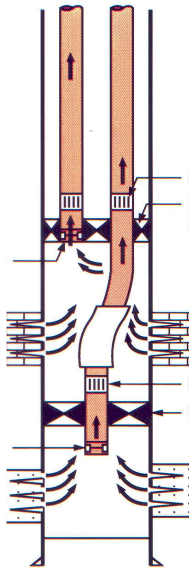

Figure 2.8: Multiple layer reservoir management with dual completion

Completion	Optimal sequential production	Commingling	Controlled fluid flow between layers	In-situ gas lift
Dual completion	+	+	–	–
ICV	+	+	+	+

Table 2.9: Interventionless production of a two layer reservoir of a conventional dual and a single string ICV completions compared

2.10 Long Term Equipment Reliability

Erosion or plugging of nozzle or channel in principle can cause ICD failure; however data on ICD reliability is not publicly available. There has been some field evidence of screen plugging during ICD installation.

An ICV is a more complex piece of equipment than an ICD, making it potentially less reliable. ICV failure is usually defined as the "inability to cycle the valve to

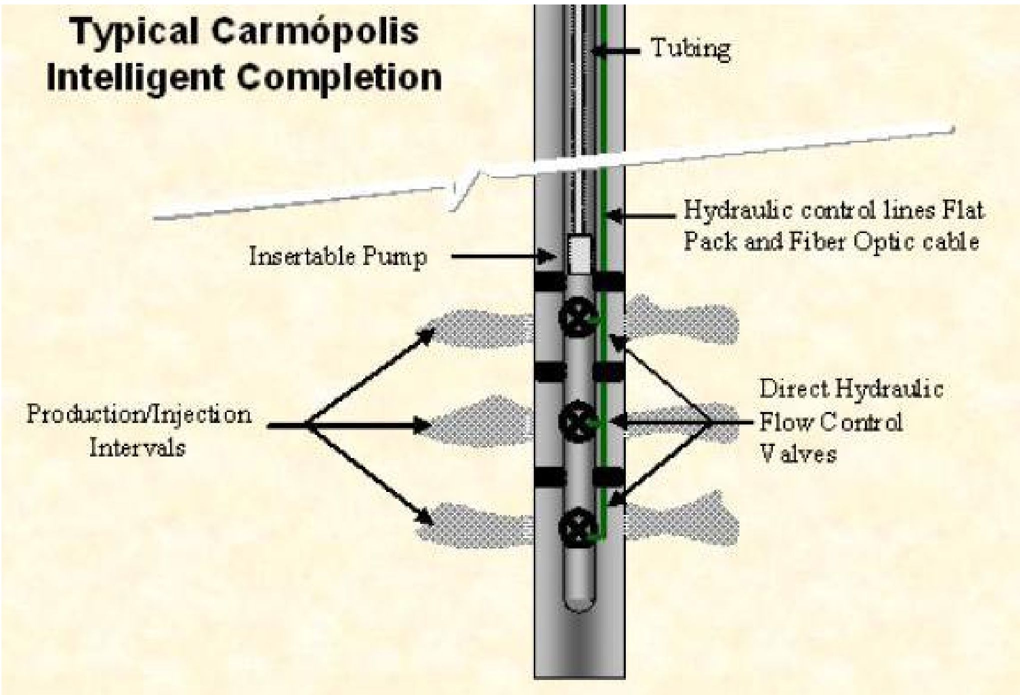

Figure 2.9: Multiple layer reservoir management with single string ICV completion (Silva et al., 2005)

the desired position". Such failure can be identified easily. This is not the case for an ICD completion since only ICD blockage is immediately apparent from the well performance. ICV reliability has been systematically studied (see, e.g., Drakeley et al., 2001; Mathieson et al., 2003).

ICV reliability depends on the type of actuation mechanism used. All-hydraulic valves are more reliable than electro-hydraulic ones. Figure 2.10 shows the improvement in reliability with the increasing number of valve systems installed in Shell. After an initial learning period, the technology has matured and no failures have occurred since 2002. Analysis of all hydraulic WellDynamics installations in Shell shows a 5 year probability of zonal control system survival of 96% (de Best and van den Berg, 2006). For comparison, 14 out of 36 electro-hydraulic sleeves failed in Snorre B in approximately five years (Kulkarni et al., 2007) making the survival of 61%.

No cases of ICD failure have been identified in the field; the ICD thus has an advantage over ICV with respect to reliability.

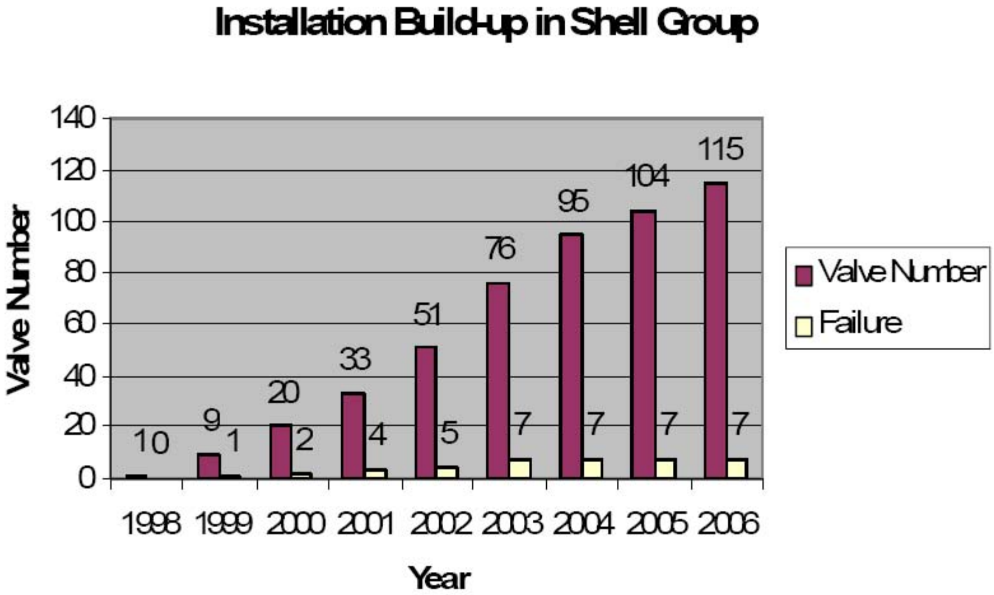

Figure 2.10: ICV reliability statistics for all-hydraulic systems (de Best and van den Berg, 2006)

2.11 Reservoir Isolation Barrier

An ICV has been accepted as reservoir isolation barrier during the initial stages of well intervention operations e.g. removal of wellhead (Stair et al., 2004). This reduces the required rig time and lowers the cost of the intervention. Another advantage is that the reservoir is not exposed to the workover fluid, reducing the risk of formation damage.

An ICD can not act as an isolation barrier.

2.12 Improved Well Clean-Up

Formation damage frequently significantly reduces well performance to a value below the well's "real" potential. Well clean-up is the process by which this damage may be removed. This is a particularly important issue in a low permeability environment. There is no available quantitative model of clean-up process, but an increased drawdown has proven to be successful (and has been observed by DTS). Sequential opening and closing of valves allows imposing higher drawdown on one zone after another. Thus ICV technology provides better clean-up than conventional completion.

The effect of ICD on clean-up is ambiguous. On one hand, ICD the maximum reservoir drawdown that can be applied will be reduced due to the choking effect of the completion (and the quadratic relationship between flow rate and pressure drop across ICD). On the other hand, the ICD equalises inflow. Hence, clean-up of the high productive and the heel intervals will be restricted, allowing a (relatively) greater drawdown – and better clean-up – of the remainder of the well. Which of these two factors will prevail depends on particular circumstances such as availability (and power) of artificial lift.

Due to above mentioned reasons I believe that ICVs will have advantage over ICDs in terms of clean-up in most of practical situations.

It should be noted, however, that it is an emerging area of research: large number of publications have addressed this question during last several years (see, e.g., Moen and Asheim, 2008; Sunbul et al., 2008; Al-Khelaiwi et al., 2009; Olowoleru et al., 2009; Shahri et al., 2009). The debate is still open.

2.13 Bullhead Selective Acidizing or Scale Treatment

Correct placement of acid or inhibitor is crucial for success of acidizing or scale treatment jobs in long horizontal wells. Difficulties in placement increase with well length and permeability variations. It is impossible to treat long completion intervals when the injection rate is too low. ICDs have advantage over conventional well in terms of acidizing and scale treatment due to their equalizing effect. ICVs can focus the treatment precisely on the target zone (Bellarby et al., 2003; Kavle et al., 2006), providing better placement and more economical use of the chemical agent compared to an ICD.

2.14 Equipment Cost

Completion costs vary greatly depending on the required functionality. The more complex nature of an ICV completion (compared to an ICD) results in the ICV being more expensive than the ICD in almost all cases. Similarly the ICD will be more expensive than a wire-wrapped screen.

A $5\,^1/_2$ inch ICD completion for 4800 ft horizontal well in 2002 was reported to cost $1.8 million (Augustine, 2002), a sum what was approximately 30% greater than a generic sand screen completion. Typical ICV completions would cost several million US dollars. Low (many tens or hundreds of thousands of USD) cost ICV applications have been reported (Silva et al., 2005). ICV equipment is more expensive than alternative ICD hardware. Quantitative comparison of ICV vs ICD costs is difficult to perform as these costs are very case specific and not publicly available.

2.15 Installation

ICV completion is more expensive in terms of installation complexity, rig time and installation risks. The last can be mitigated by proper planning and training as well as through system integration tests.

2.16 Gas Fields

The context of the above comparison is oil production, the current arena of ICV-ICD competition. However the same guidelines can be used for gas field applications, complemented by the following “gas specific” reasoning.

ICVs have been applied in gas fields (e.g. Canyon Express, Sapphire & Rosetta etc.) ICVs can control the gas production from multiple zones and shut-off water or high sand producing zones regardless of the reservoir rock or fluid properties (see, e.g., Katamish et al., 2005).

PVT properties of the gas and presence/absence of water in the reservoir are the two important factors in application of ICDs to gas fields.

2.16.1 Retrograde Condensate Gas

ICDs proved to be an efficient solution for recovery of a thin oil rim present in some gas condensate fields (Henriksen et al., 2006). Condensed liquid is often a major revenue earner in gas condensate fields and therefore ICD, favouring liquid to gas (section 1.2), has considerable potential in the development of gas condensate reservoirs.

However the phase behaviour of such reservoirs is extremely complex and the experience stemming from black oil reservoirs may be misleading in this context. The developement of gas condensate reservoirs should be based on understanding of underlying physics and meticulous economic planning, not "rule-of-thumb", cliché solutions.

2.16.2 Dry Gas

ICD applications for gas production per se have not yet been reported. We believe this is due to the ambiguity of the ICD effect in gas wells:

- On one hand ICDs completions can delay water production by equalising the inflow.
- On the other hand they will encourage water production, as discussed in section 1.2.

Also the drawdown in a gas well usually varies more with time than that of an oil well. And non-adjustable ICD completion will be less efficient in such scenarios. In principle, perforating (after water breakthrough) can convert ICD completion into a conventional slotted liner but this raises new concerns:

- Delayed production.
- Optimal moment for perforation may be not obvious since different completion intervals may exhibit different behaviour.
- Health, safety and environmental risks.

Thus, in case of dry gas production, water breakthrough deprives an ICD of one of its main advantages over an ICV – its simplicity.

2.16.3 Wet Gas

Phase behaviour of wet gas is somewere between that of a condensate and a dry gas. Liquids may condense in the completion zone and/or the wellbore depending on the produced fluid's PVT envelope, reservoir pressure and temperature and the conditions of flow through the ICD. A completion enhancing production of the liquid fraction may be either desirable or undesirable depending on particular circumstances (economics, lifting capability of the well, etc.). The value of ICDs in wet gas field applications requires additional investigation and is beyond the scope of this dissertation.

2.17 Conclusions

Major aspects dictating the choice between ICVs and ICDs have been reviewed. Figure 2.1 and Table 2.1 provide basis for the selection criteria. The value difference between ICV and ICD has been quantified where appropriate. As usual, full economic quantification of the value associated with each completion remains a field specific task.

Chapter 3

Impact of Reservoir Uncertainty on Selection of Advanced Completion Type

3.1 Introduction

Well performance prediction is one of the major tasks when preparing an oil or gas field development plan. The complexity and predictive quality of models used to support this activity have increased significantly during the last two decades, partly driven by the ever decreasing cost coupled with the increasing power of computers However, large discrepancies between the model and reality still frequently occur. They stem from:

- The lack of data (e.g. the unknown distribution of petrophysical properties in reservoir).
- Deliberate simplifications to make the problem more tractable (e.g. upscaling, black oil PVT models, neglect of thermal effects, etc.).
- Computational (sub-grid) errors and

- An incomplete understanding of the physics and chemistry of the subsurface. Petroleum researchers still work on developing more precise description of the laws governing hydrocarbon production (e.g. multiphase flow, relative permeability effects associated with gas condensate flow in porous media, effect of water salinity on oil recovery, etc.).

Many E&P development decisions are made under a high level of uncertainty. The degree of uncertainty and its impact on decision making is naturally greatest at the exploration stage of the field development process. This is one reason why a probabilistic analysis is part of reserves estimation and other standard workflows used in making early development decisions. The predictive accuracy of reservoir models should increase as the field development proceeds, since the quality and the quantity of reservoir data will continually increase. Reservoir models should be continually updated by field production data, history matching, and the ever increasing number of (logged) reservoir penetrations. However, uncertainty quantification always remains an important task; even during the later, more mature phase of reservoir development.

Reservoir engineers typically deal with multiple possible reservoir models of the real field. A single, or several, possible development strategies are evaluated and compared using all these models. The results of multiple simulations are processed to quantify the uncertainty in the production forecast. Special software tools (Schulze-Riegert et al., 2001; Manceau et al., 2001; Williams et al., 2004) have been developed to assist reservoir engineers in managing the required large number of reservoir simulations, the processing of their results and, ultimately, the quantifying of the uncertainty in the production forecast.

The parameters used in well completion design (e.g. well productivity index, deviation and skin, fluid composition, reservoir pressure etc.) can normally be measured or estimated more precisely than the macro-scale, reservoir characteristics which describe the complete reservoir. Completions are traditionally designed using well performance software which employs a reservoir inflow description that is often only valid over a short period at a particular pointing time during the reservoir's exploitation. Systematic implementation of the probabilistic approach to decision making, as is widely practised in Reservoir Engineering and Geoscience, has rarely been used in completion modeling where wellbore and near-wellbore parameters are typically used in a deterministic manner to choose the optimum tubing and casing dimensions, the well construction materials, the perforation density etc.

Vertical and slightly deviated (or conventional) wells commonly have a relatively small reservoir contact length. This limited length allows reservoir drainage control to be implemented by adjusting the production or injection rate of individual wells with a surface choke. Such an optimum reservoir drainage strategy will enhance the reservoir sweep efficiency and limit production of unwanted water or gas. The project's profitability will thus be enhanced along with other, long term, economic benefits.

Horizontal wells are characterised by a greatly increased reservoir contact compared to conventional wells. Surface control of the well's production is no longer sufficient, on its own, to implement an optimum reservoir drainage strategy. Advanced completions, with their control of the reservoir fluid inflow along the length of (long) completion interval, were developed to overcome this challenge.

Conventional well performance simulators and mechanical engineering packages have proven to be sufficient to address the major design issues associated with conventional completions, such as the:

- Well's ability to deliver a target production rate.
- Balance between reservoir inflow and tubing outflow within the well and
- Mechanical issues (e.g. sand-control, equipment reliability, etc.)

An evaluation of the (economic) benefit of the chosen, conventional well, completion design is traditionally based on a series of *short term* predictions (or "snapshots") of well performance at various periods in the well's life. (The input reservoir performance parameters are normally supplied by simulation or other reservoir engineering techniques e.g. material balance.) In addition, a simple sensitivity analysis is often carried out to evaluate the "robustness" of the chosen design if "possible" values for the above parameters are found to occur. However, a systematic analysis of realistic combinations of these parameters is not normally performed.

Justification of an advanced completion is a more complex process since the completion can react to the reservoir's inflow performance by adjusting its outflow. The significantly greater cost of the advanced completion along with its dynamic adjustment of the well's performance requires that the *long term* economic benefits should be analysed when considering installing an advanced completion. The completion's dynamic interaction with the reservoir displacement processes implies that a software tool that closely couples the simulation of the reservoir with an accurate description of the well performance is required to quantify these economic benefits. These benefits can then be expressed in terms of the cumulative oil production, risk reduction, etc. or in the standard economic terms such as net present value, internal rate of return, etc.

This combination of reservoir and wellbore modeling, economic evaluation and, sometimes, a sophisticated optimization routine are required to:

- Choose the number and location of inflow control zones.
- Make optimum use of the advanced completion's ability to adjust itself throughout the well's life to maximize the economic performance.
- Assess the economic implications of any reduction in the reliability of an advanced wells, more complex, completion components compared to that of a conventional well.
- Justify the extra cost of the advanced completion (compared to conventional one) to (a possibly sceptical) management.

The design of advanced completions thus requires an integrated knowledge of both well and reservoir technology.

Uncertainty quantification is an important and well established part of the reservoir engineering skill set. The remainder of Chapter 3 will show how the same approach can be used to design an advanced completion that can perform efficiently despite the uncertainty reflected by a range of possible reservoir models.

3.2 Literature Review

The impact of reservoir uncertainty on the selection of completion system has historically been addressed from several perspectives:

1. Wehunt (2006) presented a probabilistic distribution of skin values for three common completion types. These distributions were based on several hundred observations of vertical or slanted wells. The impact of other uncertain parameters was analyzed and the Monte Carlo technique used to assess their impact on the estimated well productivity.

2. Ouyang (2007) considered five common completion scenarios for a horizontal well in homogeneous formation. He performed a sensitivity study of the oil production rate with respect to both the completion type and the reservoir characteristics. The study illustrated that "uncertainty in input parameters may play an important role in well completion modeling and well performance prediction".

Well Inflow Performance analysis of conventional completions has normally employed a simple "snap-shot-in-time" approach in which the reservoir was represented by a single value of the productivity index, reservoir pressure and phase fractions. The traditional reluctance to use reservoir simulation in the evaluation of conventional completion performance is exemplified by the above two papers – they both included it as an optional rather than a compulsory step.

Advanced completions can be seen as an additional investment which "pays back" by improving the long term project economics through an increased operational flexibility. Sharma et al. (2002) and Han (2003) suggested applying real options theory to quantify the value of the flexibility offered by advanced completions. The key parameters for Real Options evaluation process are the present value of the project and its accompanying Uncertainty. These parameters can only be estimated via multiple reservoir simulations. Both these publications focus on the application of the real options concept rather than on the reservoir engineering aspects of the problem that are addressed in this chapter.

Real options analysis is a mathematical theory that allows the financial comparison of the risk and reward associated with a number of possible strategies, each of which is made-up of a portfolio of several possible investment opportunities or projects. Traditional Net Present Value evaluation methods use a predefined, fixed project scenario that does not quantify the value of flexibility that can allow a reduction in the level of uncertainty while maintaining the expected reward. The source of real options theory was the evaluation and optimisation of a range of possible portfolios of shares against a specified level of risk or uncertainty. Such an application naturally translates to the building of an optimum portfolio of multiple exploration licenses, each having a separate value and associated risk along with the right, but not the obligation, to develop a possible discovery.

A number of attempts have been made to popularize the real options theory within the petroleum industry. However, even financial experts (Leslie and Michaels, 1997) admit that "options theory is notoriously arcane" and "many discussions that go beyond the conceptual level get bogged down in the mathematics". Real options theory is still far from being accepted by petroleum industry as a standard evaluation method (Gai, 2002). Further, the necessary software is not widely available within the industry.

This chapter addresses the reservoir engineering aspects of obtaining the key input parameters for a real options evaluation. Completion technologies will be compared in terms of their impact on probabilistic distribution of oil recovery (P50, P90 and P10). It will be shown that the main objective of this comparison - to demonstrate how advanced completions can reduce the impact of geostatistical uncertainty on production forecasting - can be fulfilled without going into the depths of real options theory.

3.3 Advanced Well Completions

Both ICVs and ICD can react to the actual flow parameters as fluid flows from the reservoir into the tubing. It will be shown in this chapter how this "ability to react" can be used to reduce the uncertainty within a production forecast. "React" in this context implies restricting the inflow from the given completion interval. Table 3.1 outlines fluid flow parameters that the ICV and the ICD will react to. The ICV has technical advantage over the ICD for many applications (Chapter 2). However ICVs are more expensive, more complex and take a longer time to install. Hence the selection between an ICV and an ICD represents an operational, as well as a technical and an economic question.

Completion type	Total volumetric flow rate	Water Cut, GOR
ICD	+	–
ICV	+	+

Table 3.1: Fluid flow parameters ICV and ICD can react to

I have used the following workflow to assess the value of the two types of advanced completion in reducing the impact of reservoir uncertainty on production forecast (Figure 3.1:

1. Choose an appropriate reservoir and generate a number of its stochastic realisations (models).
2. Choose the base case by placing a horizontal well with a simple (perforated or open hole) completion in an optimal location with appropriate production rate, constraints, period etc.
3. Quantify the uncertainty (confidence interval) in oil recovery for the Base case with multiple realisations of the reservoir.
4. Upgrade the base case with the two advanced completion designs (ICV and ICD).
5. Compare the mean recovery predictions and the confidence intervals for the base, ICV and ICD cases.

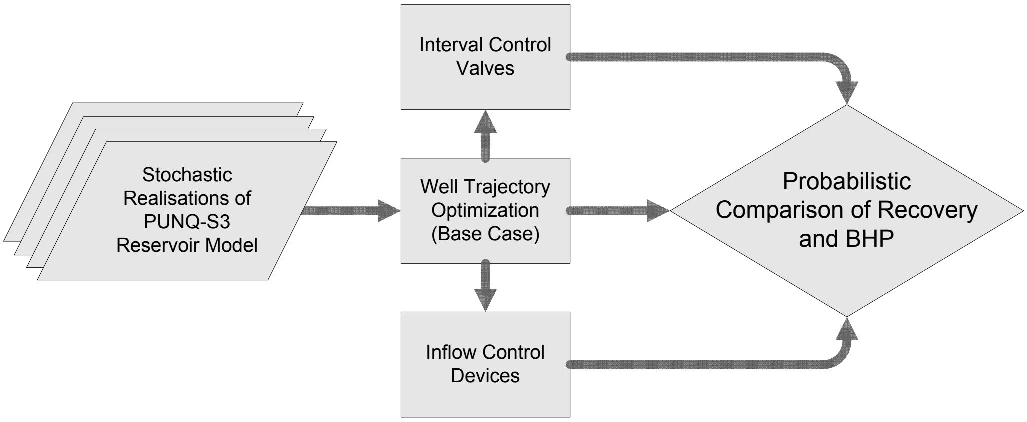

Figure 3.1: Uncertainty study workflow

3.4 Choice of Reservoir Model

The PUNQ-S3 reservoir model is based on a real field in North Sea operated by Elf Exploration & Production. It has been widely used as a benchmark model for uncertainty quantification studies (PUNQ stands for "Production forecasting with UNcertainty Quantification") (Floris et al., 2001). The model, previously used in more than 50 published papers, was chosen for this study since the:

- Reservoir is nearly 30 m thick (and hence a good candidate for exploitation by a horizontal well).
- Model has only 1761 active cells. The multiple runs required for the quantification of the geostatistical uncertainty and the optimization of the development strategy can be performed without access to high-end computing power.

Seven realisations were generated using a sequential Gaussian simulations/cosimulation algorithm in addition to the "truth" case (Carter, 2006). Sufficient variability of these realisations was ensured through use of a stochastic geostatistical algorithm (sequential Gaussian simulation) and random selection of the correlation lengths. The correlation length values were randomly generated based on parameters outlined in Table 3.2 and then used for generation of the seven models. All realisations were conditioned to match the data from the six original PUNQ-S3 well locations. My aim was to create a Base Case with a reasonably high level of geostatistical uncertainty.

Layer	R_a, m	ΔR_a, m	R_p, m	ΔR_p, m
1	800	200	3500	1500
2	1000	600	1000	600
3	1000	250	3500	1500
4	2750	1300	2750	1300
5	1000	250	3500	900

Table 3.2: Correlation length values used to generate the seven geostatistical realisations of PUNQ-S3 reservoir

3.5 The Base Case

The original six vertical production wells in the PUNQ-S3 model were substituted by a single horizontal well producing at the same total liquid rate (600 Sm^3/d). A range of production scenarios were studied. It was concluded that the outcome of this type of comparative study will be to a large extent determined by the decisions made during the selection of the base case such as:

- Well Location
- Liquid Production Rate
- Production Period

The field is bounded to the east and south by a fault with a fairly strong aquifer to the north and west. This aquifer is the main drive mechanism since the gas cap is weak and there is no injection. The primary direction of water front movement is from west to east. A well placed in the middle of the reservoir (Figure 3.2) will show early water breakthrough and poor drainage of the eastern part of the reservoir. An advanced completion equipped with ICVs can add significant value to a well located in such a non-optimal position (Figure 3.3). This illustrates how ICV equipped wells are particularly suited for delivering value in non-optimally located production wells. The increased value (maximum oil recovery) for the well producing under liquid limit control was achieved by limiting water production from watered-out zones.

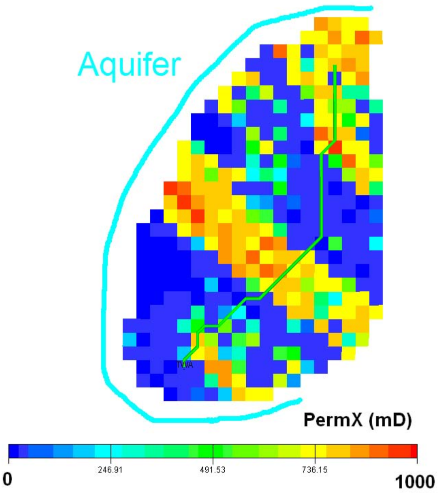

Figure 3.2: Non-optimal well location

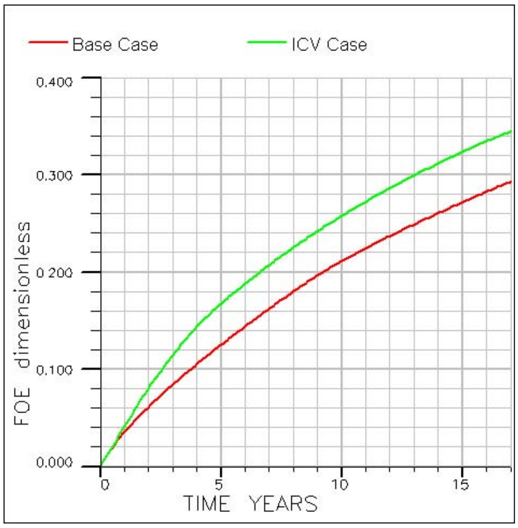

Figure 3.3: Increase in recovery for non-optimal well location

The optimal well location (Figure 3.4) positions the well close to the fault forming the eastern edge of the reservoir. The optimal well is located so that all zones are penetrated in the topmost layer, apart from the 2nd zone in layer 3. This exception was made so that the completion interval draining this part of the reservoir could be placed below both the gas cap and the low permeability, 2nd layer. The new, optimum well location allows a much improved production profile (Figure 3.5) with significant water production only being observed after 17 years. Table 3.3 outlines the characteristics of the Base Case horizontal well, while Figure 3.6 and Figure 3.7 illustrate the formation permeability in the grid blocks surrounding this optimal well's location. The significant improvement derived from the optimized trajectory results in limited additional value being achieved by the advanced completion if the evaluation period is restricted to the above 17 year production period. This period was therefore extended to a 30 year time span since any additional value due to advanced completions can only be derived after the 16 year, pure oil production phase has ended (Figure 3.5).

Table 3.4 summarises the probabilistic forecast for recovery from the Base Case well. Probability density of recovery (Figure 3.8) was estimated by interpolating the results of the simulations and assuming that the 8 reservoir realisations are equiprobable. The average recovery factor is 28.6%; with a recovery of more than

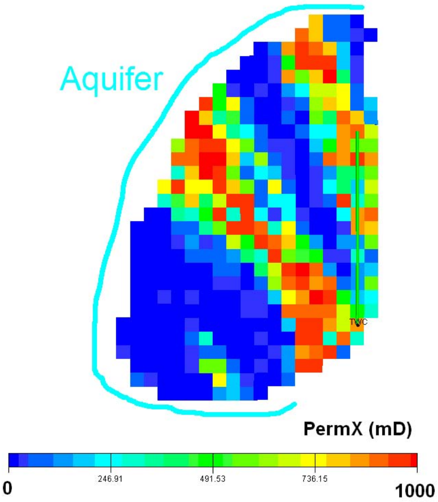

Figure 3.4: Optimal well location

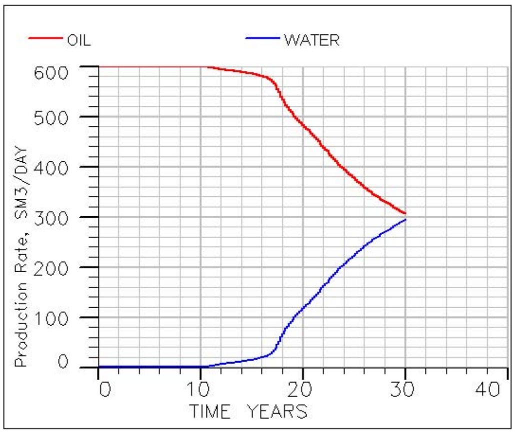

Figure 3.5: Oil and water production for optimal well location

Well length	2527 m
Wellbore diameter	0.216 m (8.5 in.)
Liner OD	0.178 m (7 in.)
Well productivity index	1000 $Sm^3/d/bar$
Drawdown at 600 Sm^3/d	0.6 bar
Oil density at surface conditions	912 kg/m^3
Oil viscosity at reservoir conditions	1.5 cP

Table 3.3: Base case well data

25.5% expected at the 90% confidence level while there is only a 10% chance that recovery will exceed 31.7% of original oil in place. The width of this confidence interval (P10 - P90) is then a measure of the production forecast's uncertainty.

3.6 Advanced Completion Cases

3.6.1 ICD Case

Design of an optimal ICD completion requires a comparison of the values of the reservoir drawdown with that of the pressure drop across the ICD. The conclusion that these two values should be of the same order of magnitude is based on both

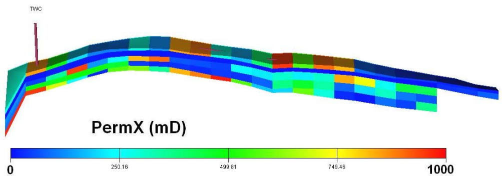

Figure 3.6: Vertical slice of reservoir at optimal well location

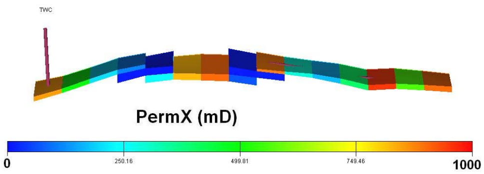

Figure 3.7: Permeabilities of the grid blocks connected to the well

Field Oil Recovery, %											
Simulation results for the eight geostatistical cases								Statistical Analysis			
1	2	3	4	5	6	7	8	Mean	P90	P10	P10 - P90
25.1	25.9	27.0	27.9	29.0	30.1	31.1	32.5	28.6	25.5	31.7	6.2

Table 3.4: Base case recovery distribution

analytical considerations (sections 2.5.2 and 5.5), as well as reservoir simulations. A "64 bar" channel- type ICD was chosen for the well completion design to achieve a reasonable level of inflow equalization. (The "64 bar" refers to the pressure drop created when a standard, 12 meter ICD screen joint is exposed to water inflow rate of 26 Sm^3/d). This ICD "strength" rating corresponds approximately to the inflow channel being replaced by a single, 2 mm diameter nozzle. The completion interval was divided into 210 zones. Each ICD equipped zone was 12 m long and each zone was segregated from adjacent zones by an external packer.

In 2007, when this work was performed, commercial reservoir simulators were unable to model annular and tubing flow with multiple connections between the two flow paths (in particular, flow loops were not allowed in the well completion nodes). It was technically possible to link a specially programmed, well simulator package with a reservoir simulator (see, e.g., Wan et al., 2008) to accurately model looping flow and allow its impact on the long term well/reservoir performance to be fully evaluated. Establishing such links require a substantial amount of time and may lead to convergence problems (Al-Khelaiwi, 2007). Since this work focused on the impact of uncertainty in the reservoir description, it was decided to simplify the study by assuming that annular flow was not present. This is equivalent to installing an external packer between each ICD joint; an assumption that will have a positive

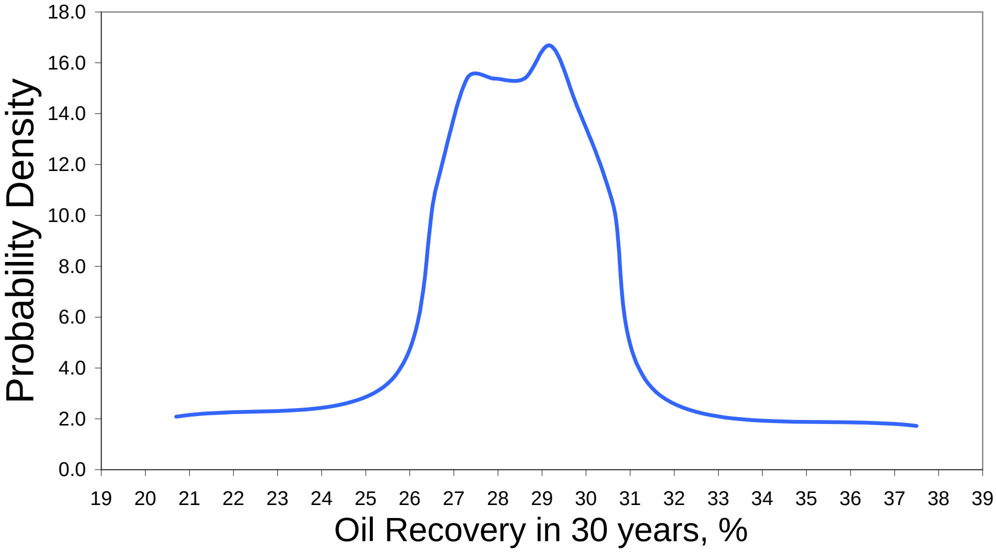

Figure 3.8: Probabilistic production forecast for the Base Case

impact on performance of the ICD completion.

3.6.2 ICV Case

The completion interval was divided into 5 zones, each of which was separately controlled with an infinitely variable, $3\,^1/_2$ in. diameter ICV installed in $8\,^1/_2$ in. hole. The well target production rate of 600 Sm^3/d was achieved at all times by employing a similar drawdown for all three completion cases (Figure 3.9 and Table 3.6). The maximum difference in the flowing bottom hole pressure (FBHP) created at equivalent stages in the reservoir life by the different completion designs had a value of 10 bar. The Base Case had the highest FBHP. The FBHP with ICDs installed was lower than the Base Case due to the imposed, additional pressure drop across the ICD. The ICV case had the lowest FBHP because, in addition to pressure loss across the valves, it also has a smaller, flow conduit diameter. The improved ICV well performance was achieved by limiting the production from zones with a higher water cut. The weak gas cap had a limited effect. The tested ICV operation policies that successfully limited gas production also reduced the oil recovery. More sensitive ICV gas cap management strategies might have proved more successful than author's attempts.

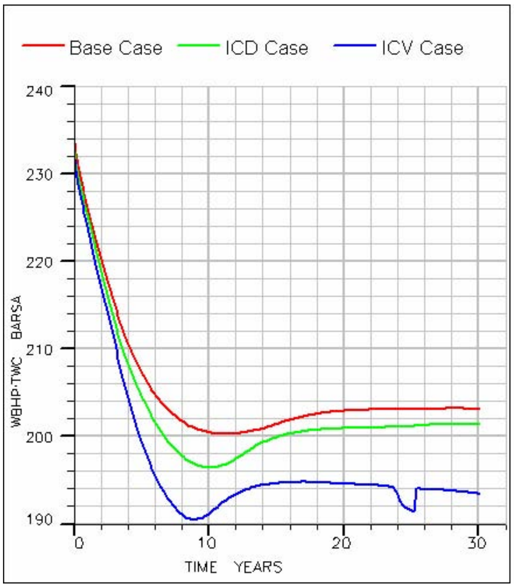

Figure 3.9: Flowing bottom hole pressure comparison for one of the 8 realisations

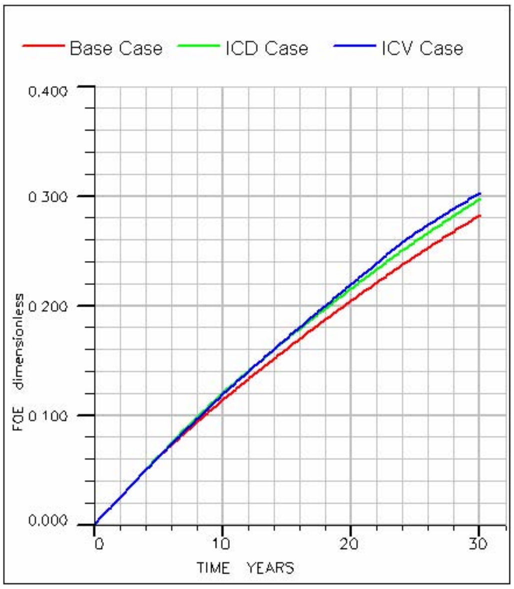

Figure 3.10: Recovery comparison for one of the 8 realisations

3.7 Results

Figure 3.10 compares the recovery achieved by the three completion options for the fourth of the eight geological realisations. It demonstrates that it may be difficult to tell which completion gives the greatest oil recovery if only a single reservoir model is employed. Out of the eight reservoir models used here ICDs gave the highest recovery in 6th, 7th and 8th cases; while the ICVs were most successful in the remaining 5 cases (Table 3.5).

A probabilistic comparison (Figure 3.11, Table 3.5 and Table 3.6) allows a clearer comparison of the 30 year oil recovery efficiency of the three alternative completions. The average predicted recovery for the ICD completion (the green line) was greater than the corresponding value for the Base Case. However, these two cases show only minor differences in the uncertainty (the difference between the P10 and the P90 cases) of the recovery forecast at all certainty levels. The ICV completion (red line) showed both an increased oil recovery as well as delivering a reduction in the uncertainty. Any increase in the oil production was reflected by a corresponding decrease in the water production since the well production was liquid rate constrained.

Field Oil Recovery, %												
	Simulation results for the eight geostatistical cases								Statistical Analysis			
Case	1	2	3	4	5	6	7	8	Mean	P90	P10	P10 - P90
Base	25.1	25.9	27.0	27.9	29.0	30.1	31.1	32.5	28.6	25.5	31.7	6.2
“64 Bar” ICD	27.0	27.9	28.7	29.6	30.4	31.4	32.2	33.4	30.1	27.4	32.7	5.3
5 ICVs	29.0	29.5	29.8	30.1	30.7	31.6	31.7	32.2	30.6	29.1	32.2	3.1

Table 3.5: Recovery distribution for Base, ICD and ICV cases

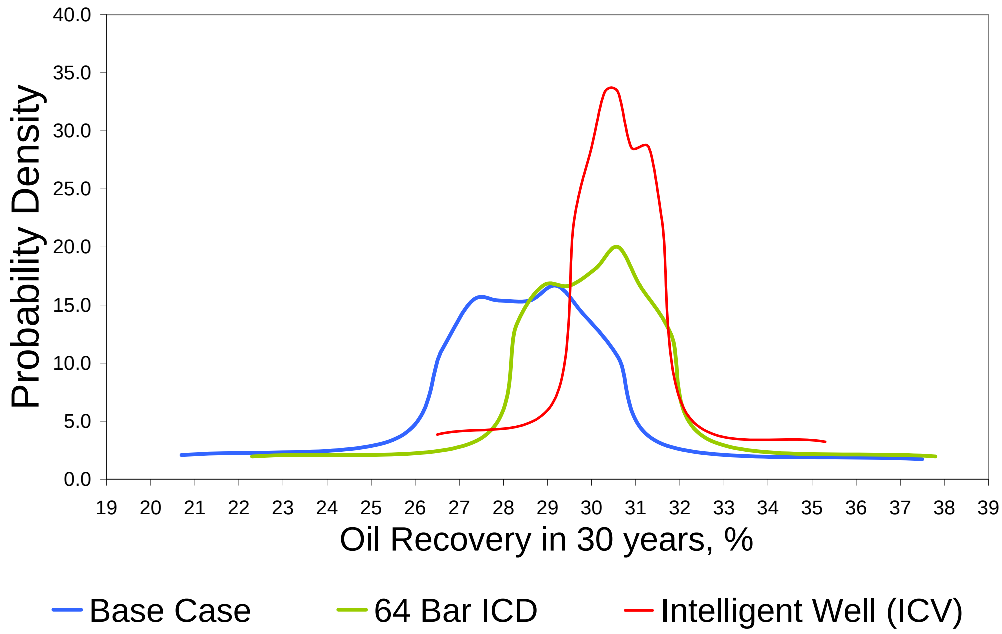

Figure 3.11: Impact of advanced completion on production forecast

Final flowing bottom hole pressure, bar				
Case	Mean	P90	P10	P10 - P90
Base	200	198	203	5
"64 Bar" ICD	198.5	196	201	5
5 ICVs	192	191	192.5	1.5

Table 3.6: The final flowing bottom hole pressure compared

3.8 Discussion

The author consider these results to be a conservative estimate of the value derived from an advanced completion since the Base Case selection process employed a good understanding of the reservoir geology, drive mechanisms and fluid contacts. However it should be borne in mind that advanced completions may not add value if the degree of reservoir uncertainty is sufficiently low and the well trajectory have been thoroughly optimized.

Careful selection of the Base Case is an important step in the process of determining how realistic the subsequently calculated potential for added value is.

3.9 Conclusions

Well completion design has been shown to reduce the impact of geostatistical uncertainty on the production forecast using the uncertainty quantification methodology as applied in reservoir engineering. The study of the PUNQ-S3 reservoir found that a well completion designed based on:

- ICDs increased the mean recovery with a limited decrease in of risk.
- ICVs further increased mean recovery and reduced the risk compared to the base case by 50%.

Chapter 4

Impact of Frictional Pressure Losses Along the Completion on Well Performance

4.1 Introduction

The impact of frictional pressure losses across the length of completion on well performance is usually negligible for a vertical or deviated well since the well-reservoir contact length is short, being normally of several tens of meters. Today's directional drilling technology allows increased well-reservoir contact, bringing a number of advantages in terms of the well's productivity, drainage area and sweep efficiency together with delay in the breakthrough of water or gas. However, frictional pressure losses along the completion are an important factor in long, possibly multilateral, wells. These losses have to be taken into account in order to secure optimum reservoir drainage and avoid overestimation of well productivity.

Commercial well and reservoir simulators enable engineers to model this (and many other complex phenomena) while designing long horizontal and multilateral wells. In terms of underlying assumptions these products are more general than analytical models. This allows engineers to quickly consider large number of options and thus optimise completion design in a timely manner. However, analytical solutions of fairly complicated situations are still needed to validate the results of numerical simulators and they often implemented in numerical simulators to provide the user with an analytical option.

4.2 Literature Review

The first published analysis of the pressure drop along a completion was performed by Dikken (1990). He used Blasius's correlation for the turbulent friction factor and presented solutions for both laminar and turbulent flow cases. The laminar flow problem was reduced to a linear ordinary differential equation (ODE) of the second order that has a rigorous closed form solution. Dikken developed analytical formulae for wells of infinite length for the turbulent flow case and also proposed a numerical solution for finite wells.

Joshi (1991) suggested a way of assessing the importance of pressure drop along the completion by assuming that all fluid enters the wellbore at the toe and applying a standard pipe flow correlation. This simple method largely overestimates the wellbore pressure drop: dividing Joshi's result by 3 still gives an upper estimate (Appendix C). The frictional pressure losses can be neglected if the upper estimate is small compared to the drawdown. Such calculations have great practical use as they provide a "quick-look" evaluation to determine whether a more sophisticated analysis is necessary in any particular case.

Joshi (1991) also considered the ratio of drawdown at the toe and the heel of the well as one of the key parameters describing the well performance. This parameter (the drawdown ratio) is designated R_d in this dissertation.

Seines et al. (1993) used Haaland's flow correlation (Haaland, 1983) and they noted that, for highly turbulent flow, the friction factor is mainly a function of the dimensionless wellbore roughness i.e. the dependence of the friction factor on Reynolds number can be neglected. This allowed reduction of the turbulent flow problem to the following second order non-linear ODE:

$$u'' = u^2 \tag{4.1}$$

where u is normalised well flow rate (Seines et al., 1993).

Eq. (4.1) was solved analytically for the case of infinite wells and numerically for wells of finite length. The author also introduced characteristic length for horizontal wells, l^*. The meaning of this parameter will be discussed in subsection 4.6.1 of this chapter.

Halvorsen (1994) observed that the solution of Eq. (4.1) for finite wells can be expressed through a Weierstrass elliptic function and expanded into a theta series. However Halvorsen's solution still contains an implicit relationship between the expansion's parameters which has to be resolved iteratively. The author calculated the value of q_w^*/q_{inf} to eight significant digits (0.96019421). This result was confirmed by Seines et al. (1993), allowing it to be used as a benchmark for the explicit solution developed in this thesis.

Landman (1994) extended Dikken's work by deriving an analytical solution for turbulent flow in wells of finite length. This solution establishes an implicit relationship between the well rate and the drawdown via use of the Gauss hypergeometric function. It has to be used iteratively to calculate the well rate for the given drawdown or vice-versa.

Novy (1995) used Jain's friction factor correlation (Jain, 1976) and solved the problem numerically for both liquid and compressible gas flow. He provided guidelines as to when friction can be ignored in a particular well.

Ozkan et al. (1995) proposed a quite general dynamic semi-analytical model that can account for fluid compressibility, various boundary conditions and reservoir anisotropy. The model takes the form of a fairly complex non-linear integral equation. The authors suggest solving it by discretizing in space and time with the recommendation to use a minimum of 40 well segments. This solution appears to be similar to the conventional numerical approaches in terms of implementation effort and computational efficiency.

The semi-analytical model developed by Penmatcha et al. (1999) is mathematically very similar to that suggested by Seines et al. (1993). Penmatcha's model is more complex and potentially more general because it does not employ the:

- Darcy-Weisbach equation.
- Assumption of a constant friction factor.

Penmatcha et al. (1999) introduced a dimensionless variable called productivity error, E_p , which is the error in the well productivity calculations due to neglecting frictional pressure drop in the wellbore:

$$E_p = (q_{nof} - q_w)/q_{nof} \tag{4.2}$$

where q_w well flow rate,
q_{nof} well flow rate estimate neglecting friction.

An extensive sensitivity analysis that illustrated the effect of well length, flow rate, wellbore roughness, reservoir drawdown, fluid viscosity and reservoir permeability on productivity error was also reported.

Hill and Zhu (2008) suggested an approach similar to that of Joshi (1991). The main difference is that Hill and Zhu (2008) used half the total flow rate (this is equivalent to dividing the Joshi's pressure estimate by 4). It is shown in Appendix C that this approach gives neither an upper nor a lower estimate of the frictional pressure losses. They did not formally address the question of their method's precision. It can not therefore be considered as a rigorous evaluation of well performance.

Many other authors addressed various aspects of frictional pressure losses along the completion. This review has been restricted to the most relevant works. An explicit analytical solution for turbulent flow in wellbore has not so far been published. This chapter presents such a solution based on the same assumptions as those of previous researchers.

4.3 Problem Formulation

Estimation of the two dimensionless variables R_d and E_p (see section 4.2 for definitions) can give answers to many practical questions related to frictional pressure losses in a long completion. For instance, the drawdown ratio, R_d, indicates whether friction skews the inflow profile. A non-uniform inflow profile may decrease oil sweep efficiency and recovery through a premature water or gas breakthrough. The productivity error, E_p, helps to define the well's inflow performance curve which is one of the key parameters used in well design.

The motivation for the present work was the development of an explicit analytical model that would describe turbulent flow in a highly deviated well with a long completion interval. In particular the model should provide formulae for the:

- Pressure and flow rate profiles along the completion.
- Drawdown ratio R_d.
- Productivity error E_p.

4.3.1 Assumptions

The model invokes the following assumptions with respect to the inflow from the reservoir. The:

- Well fully penetrates reservoir (or the edge effects due to partial penetration can be neglected).
- Flow is steady or pseudo-steady state, subject to Darcy's law.
- Distance between the well and the areal reservoir boundary is much longer than the well length (or the boundary is parallel to the well).
- Reservoir is homogeneous.
- Perpendicular-to-the-well components of the reservoir pressure gradients are much greater than the along-hole ones.

The above simplifications are required in order to introduce the term of Specific Productivity Index (PI per unit length) and assume that it is constant throughout the completion interval. The assumptions about the wellbore flow are that it is:

- Isothermal.
- Incompressible.
- Steady state.
- Homogeneous (no slip between the phases) and the:
- Friction factor is constant along the completion interval.
- Pressure drop due to acceleration is small compared to that of friction.
- Dependence of fluid's viscosity upon pressure can be neglected.

Note that the completion interval is not assumed to be perfectly horizontal. Under assumptions stated above, hydrostatic gradient in the reservoir is the same as in the wellbore what makes the model applicable for deviated wells. There are two assumptions however that I would like to discuss in more detail.

Constant Friction Factor

The friction factor for highly turbulent flow is determined primarily by the roughness of the pipe (Figure 4.1). The dependence of its value on Reynolds number is weak. I chose to neglect this dependency since this allows simplification of the problem by assuming that the friction factor is constant along the completion interval. In practice this implies that turbulent flow occupies the majority of the wellbore and the completion roughness is constant along its length. Appendix A shows how the average friction factor and the error associated with this averaging can be estimated.

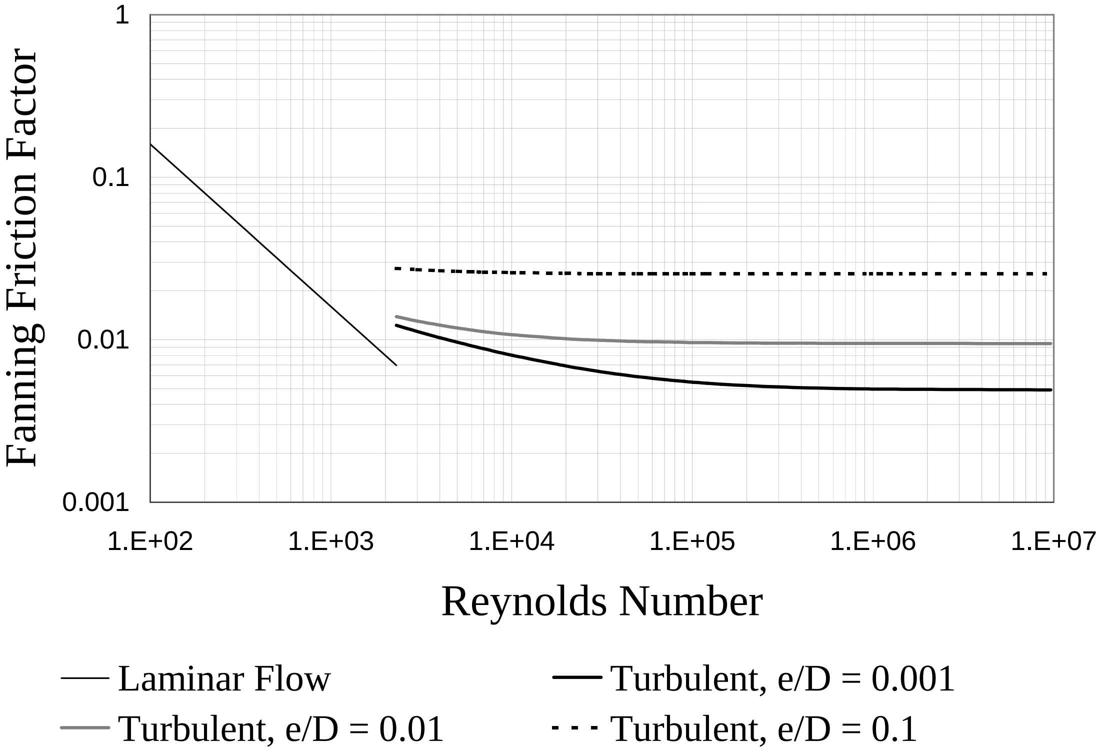

Figure 4.1: Fanning friction factor for rough pipes (Haaland's correlation)

Acceleration

Most authors agree that pressure drop due to acceleration in horizontal wells is usually small compared to that due to friction. However a quantitative analysis of conditions, under which the assumption is valid, is not readily available in the literature. Appendix B shows that acceleration can be ignored if:

$$f_a L \gg D \tag{4.3}$$

where L length of completion,
D internal diameter of completion,
f_a average Fanning friction factor.

This result can be further simplified for wells with predominantly turbulent flow: long completion intervals with a relatively high roughness (e.g. an open hole or sand screen completion) have a Fanning friction factor with an order of magnitude of 10^{-2}. The inequality (4.3) can then be transformed into:

$$L/D > 1000 \tag{4.4}$$

4.3.2 Mathematical Formulation

Fluid inflow from the reservoir and the pressure drop along the completion interval are the two interrelated phenomena that this work focuses on. The assumptions (subsection 4.3.1) allow the inflow to the well to be described as:

$$\frac{dq}{dl} = j\left(P_e - P(l)\right) \tag{4.5}$$

where q flow rate (in the tubing) at distance l from the toe of the well,
P_e reservoir pressure at the external boundary,
l distance between particular wellbore point and the toe,
j specific productivity index.

The specific productivity index, j, is an empirical parameter indicating that fluid inflow from reservoir to wellbore is proportional to pressure difference between the external reservoir boundary and wellbore. It can be estimated with a number of techniques. For example:

- Analytical PI models (see, e.g., Babu and Odeh, 1989; Goode and Kuchuk, 1991; Goode and Wilkinson, 1991);
- Well log data;
- Well test data.

The Darcy-Weisbach equation that defines the friction factor has the following form in SI units:

$$\frac{dP_f}{dl} = -\frac{2f}{D}\rho v^2 \tag{4.6}$$

where v is fluid volumetric velocity. Eq. (4.6) can be used for both laminar and turbulent frictional pressure losses. Note that the Fanning friction factor defined in (4.6) is four times smaller than the Darcy-Weisbach (Moody) friction factor. The Fanning friction factor will be referred to in the remainder of the dissertation as the "friction factor".

The following form of the Darcy-Weisbach equation (4.6) is more convenient in the context of the problem under consideration:

$$\frac{dP}{dl} = -\frac{C_f \rho f B^2}{D^5} q^2(l) \tag{4.7}$$

where B formation volume factor,

C_f unit conversion factor: $2.956 \cdot 10^{-12}$ in field units and $4.343 \cdot 10^{-15}$ in metric.

The system of equations (4.5) and (4.7) can be reduced to a single non-linear ODE of the second order:

$$\frac{d^2q}{dl^2} = \frac{jC_f \rho f_a B^2}{D^5} q^2 \tag{4.8}$$

which is mathematically equivalent to Eq. (4.1). Note that f_a was introduced in Eq. (4.8), an average value of friction factor (Appendix A).

In practice fluid production or injection is controlled either by the target flow rate or the pressure. Hence the problem may be formulated with one of two sets of boundary conditions. Flow rate control:

$$\begin{cases} q(0) = 0 \\ q(L) = q_w \end{cases} \tag{4.9}$$

or pressure control:

$$\begin{cases} q(0) = 0 \\ \frac{dq(L)}{dx} = j\Delta P_w \end{cases} \tag{4.10}$$

where P_w flowing bottom hole pressure (at the heel of the tubing) i.e. $P(L)$,

ΔP_w the total pressure drop at the heel i.e. $P_e - P_w$.

Eq. (4.8), with boundary conditions (4.9) or (4.10), defines the problem to be solved.

4.4 Derivation of the Solution

4.4.1 General Solution

Let us nondimensionalise Eqs. (4.8)-(4.10). The dimensionless distance from the toe is:

$$x = l/L \tag{4.11}$$

The dimensionless flow rate and the Horizontal Well number for a rate constrained well (Eq. (4.9)) are:

$$y_q(x) = q(Lx)/q_w \tag{4.12}$$

$$h_q = C_f \rho f B^2 J_s L^2 q_w / D^5 \tag{4.13}$$

The analogous variables for a pressure constrained well (Eq. (4.10)) are:

$$y_p(x) = \frac{q(Lx)}{J_s \Delta P_w L} \tag{4.14}$$

$$h_p = C_f \rho f B^2 J_s^2 L^3 \Delta P_w / D^5 \tag{4.15}$$

Eq. (4.8) may be transformed into

$$y'' = hy^2 \tag{4.16}$$

where

$$h = \begin{cases} h_q \text{ for a rate constrained well (Eq. (4.9))} \\ h_p \text{ for a pressure constrained well (Eq. (4.10))} \end{cases} \tag{4.17}$$

and the boundary conditions correspondingly take the form of:

$$\begin{cases} y_q(0) = 0 \\ y_q(1) = 1 \end{cases} \tag{4.18}$$

or

$$\begin{cases} y_p(0) = 0 \\ \frac{dy_p(1)}{dx} = 1 \end{cases} \tag{4.19}$$

The drawdown ratio R_d and the productivity error E_p can be expressed directly through the dimensionless flow rate:

$$R_d \equiv \Delta P(0)/\Delta P(L) = y'(0)/y'(1) \tag{4.20}$$

$$E_p = 1 - \frac{q(L)}{J_s L \Delta P(L)} = 1 - \left(\frac{dy_q(1)}{dx}\right)^{-1} = 1 - y_p(1) \tag{4.21}$$

Now let us find the general solution of Eq. (4.16). This ODE does not contain the independent variable x hence its order can be reduced by substitution $p = y'$ and taking y as a new independent variable:

$$y'' \equiv \frac{dy'}{dx} = \frac{dy'}{dy}\frac{dy}{dx} = p'p \tag{4.22}$$

Substituting (4.22) into (4.16):

$$d\left(p^2\right)/2 = hy^2 dy \tag{4.23}$$

$$p^2 = 2hy^3/3 + C \tag{4.24}$$

$$\left(y'\right)^2 = 2hy^3/3 + C \tag{4.25}$$

The general solution of (4.25) is expressed through a Weierstrass elliptic function. Indeed, this function satisfies the differential equation similar to Eq. (4.25):

$$\left(\wp'\right)^2 = 4\wp^3 - g_2\wp - g_3 \tag{4.26}$$

where $\wp\left(z; g_2, g_3\right)$ is a Weierstrass elliptic function with invariants g_2 and g_3.

Comparison of equations (4.25) and (4.26) suggests that in our case $g_2 = 0$.

Let us consider function $U(z) = t^2\wp(tz; 0, g_3)$ where t is an arbitrary number. One can show that $U(z)$ satisfies the following equation:

$$(U')^2 = 4U^3 - t^6 g_3 \tag{4.27}$$

This observation will be used below.

The Weierstrass elliptic function with the parameters $g_2 = 0$, $g_3 = 1$ has the following properties:

1. It is periodic with half periods of ω_1, ω_2 and that are related as follows

   $$\omega_1 = \omega_2 \left(1 + i\sqrt{3}\right)/2 \tag{4.28}$$

   $$\omega_2 = 1.5299540370571934\ldots \tag{4.29}$$

   In particular the points 0, $2\omega_1$, $2\omega_2$ are the vertices of an equilateral triangle.

2. It has a pole of second order at $z = 0$, and (because it is periodic) at the points $2m\omega_1 + 2n\omega_2$ for all integer numbers m, n.

3. $\wp(z; 0, 1)$ equals zero at the centre of that equilateral triangle, namely at the point

   $$z_0 = \omega_2 \left(1 + i/\sqrt{3}\right) \tag{4.30}$$

   It also equals zero at the point $2a$, and at all the points with coordinates $\pm a + 2m\omega_1 + 2n\omega_2$ (due to periodicity). Also, it should be borne in mind that $3a = 2\omega_1 + 2\omega_2$.

We are seeking solution in real values. Hence let us transform $\wp(z; 0, 1)$ into a new function so that it takes real values on the real line. Using the (4.27) we may take:

$$R(z) = z_0^2\wp(z_0\, z; 0, 1) \tag{4.31}$$

This new function (Figure 4.2) has the following properties:

1. It has real values on the real line.

2. It satisfies the following differential equation: $(R')^2 = 4R^3 - z_0^6$ and hence $R'' = 6R^2$.

3. It has a second order pole at points $3n$ for integer n.

4. It equals zero at points $3n \pm 1$

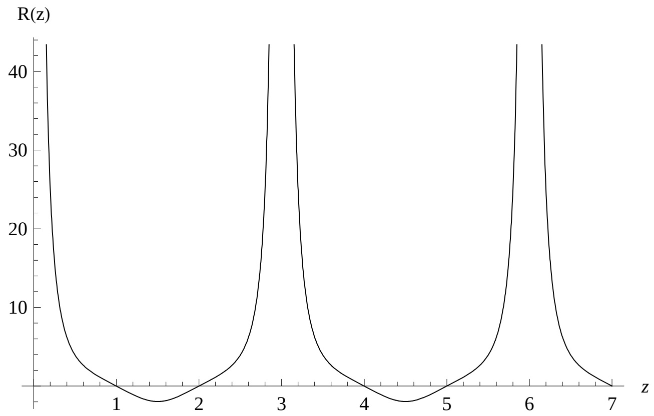

Figure 4.2: Plot $R(z) = z_0^2 \wp(z_0\, z; 0, 1)$

Thus $R(z)$ is a particular solution of $r'' = 6r^2$. Using the observation (4.27) one can readily write down the corresponding general solution:

$$r(x) = A^2 R(Ax + D) \tag{4.32}$$

Finally, the general solution of (4.16) can be obtained from (4.32) using substitution $y(x) = 6r(x)/h$:

$$y(x) = 6A^2 R(Ax + D)/h \tag{4.33}$$

which is equivalent to:

$$y(x) = 6z_0^2 A^2 \wp\left(z_0(Ax + D); 0, 1\right)/h \tag{4.34}$$

Constant C appearing in (4.25) is linked with A by the following relationship (Usnich, 2008):

$$C = \frac{2^8 \omega_2^6 A^6}{3h^2} \tag{4.35}$$

4.4.2 Boundary Value Problem of Rate Constrained Well

Using (4.33) one can readily transform the boundary conditions (4.18) into:

$$\begin{cases} R(D_q) = 0 \\ 6A_q^2 R(A_q + D_q) = h_q \end{cases} \tag{4.36}$$

The first equation in (4.36) implies:

$$D_q = -1 \tag{4.37}$$

(or any other number of the form $3n \pm 1$). Therefore system (4.36) can be reduced to:

$$6z_0^2 A_q^2 \wp\left(z_0 A_q - z_0; 0, 1\right) = h_q \tag{4.38}$$

This equation with respect to A_q has no apparent analytical solution. Following Wolfram Mathematica™code has been written to solve it numerically for $h_q \in [0.1, 100]$.

$e = I/2 + \sqrt{3}/2$; wt = Gamma[1/3]^3/(4 ∗ π)//N;

w1 = wt ∗ $\left(1 + I * \sqrt{3}\right)/2$; $a = 2e\mathrm{wt}/\sqrt{3}$;

U[z_, b_]:=$b^{2}a$^2WeierstrassP[$a(bz-1)$, {0, 1}]

Sol[k_, b0_]:=Re [ReplaceAll [b, FindRoot [$U[1, b] == \frac{k}{6}$, {b, b0}]]] ;

t = Sol[0.09, 0.1]; tbl = Reap[Do[Sow[t = Sol[Sow[i], t]], {i, 0.1, 100, 0.1}]];

tbl = Partition[Flatten[Delete[tbl, 1]], 2]

InputForm[%]

Export["Aq.xls", %, "XLS"]

Figure 4.3 presents the solution as well as $0.1/C_q$ value calculated using (4.35). One can see that dependence of $0.1/C_q$ vs. h_q is more linear than that of A_q; hence it is reasonable to initially find analytical interpolation for C_q:

$$C_q \approx 1/\left(1 + 0.1647\, h_q + 0.001793\, h_q^2\right) \tag{4.39}$$

and then use it and Eq. (4.35) to express A_q via h_q:

$$A_q \approx \frac{1}{\omega_2} \sqrt[6]{\frac{3h_q^2}{2^8\left(1 + 0.1647\, h_q + 0.001793\, h_q^2\right)}} \tag{4.40}$$

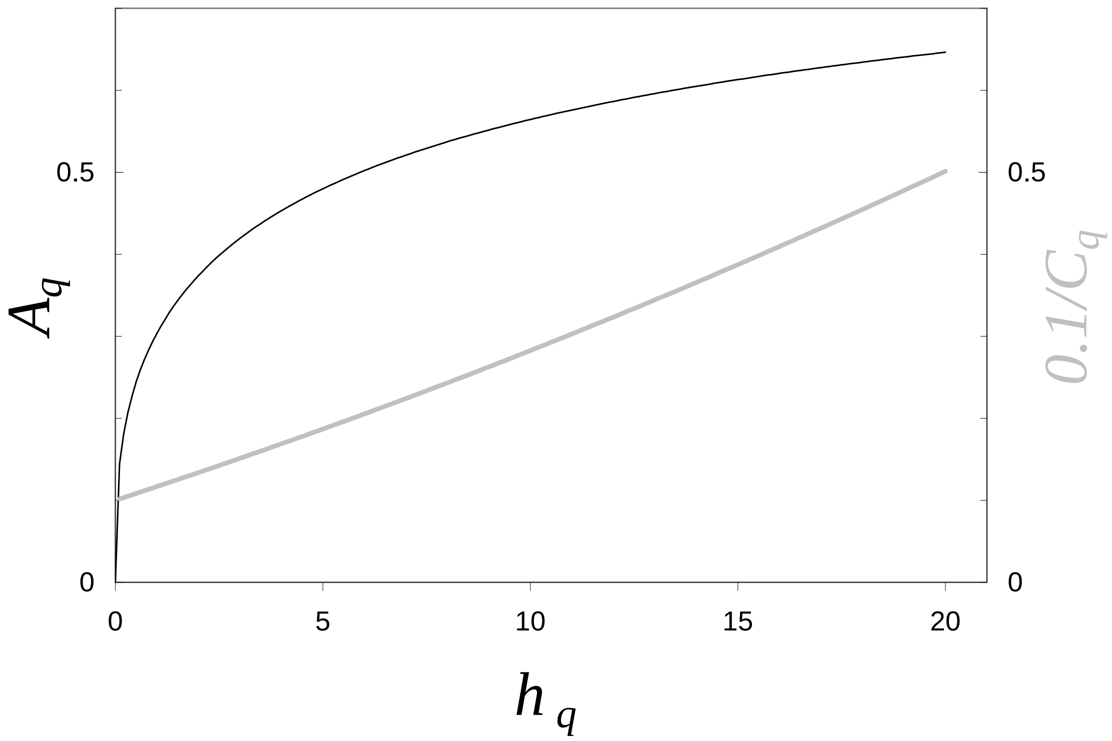

Figure 4.3: Numerical solution for A_q

The coefficient of determination (R^2) associated with interpolation Eq. (4.39) is 1.0000. Thus Formula (4.39) can be regarded as precise for most petroleum engineering applications. Figure 4.4 demonstrates the match between the numerical solution and its analytical interpolation. Formulae (4.34), (4.37) and (4.40) thus provide solution to Eq. (4.16) with boundary conditions (4.18).

4.4.3 Boundary Value Problem of Pressure Constrained Well

Eq. (4.25) allows expressing the function through its derivative. Hence the boundary value problem (4.19) can be reduced in a similar manner to that used previously for

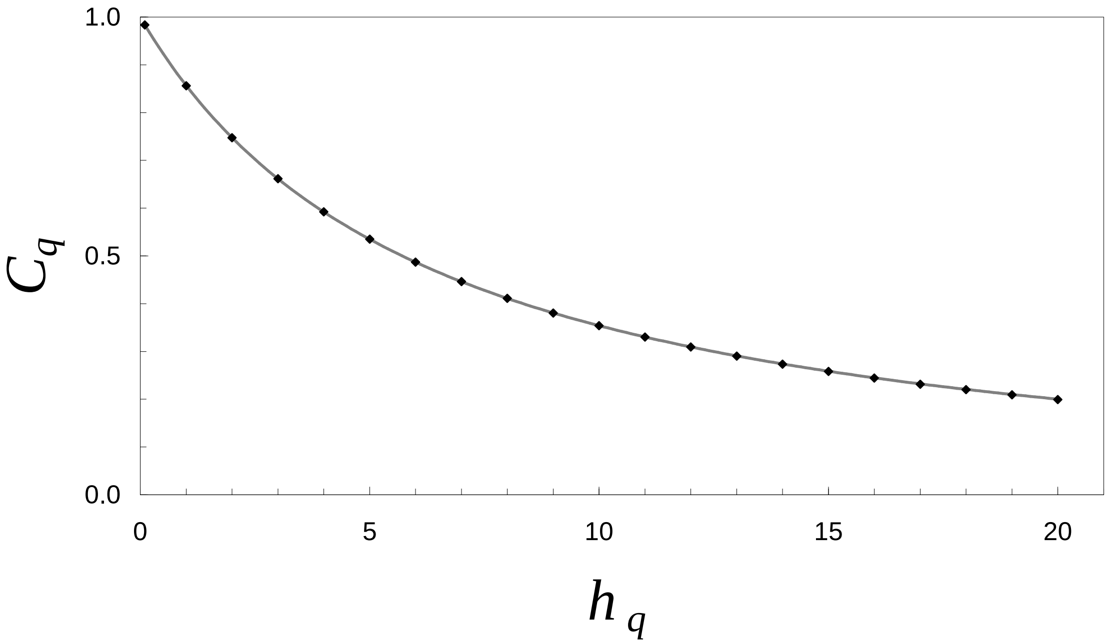

Figure 4.4: Comparison of the numerical solution and interpolation for C_q

(4.18):

$$\begin{cases} y_p(0) = 0 \\ y_p(1) = \sqrt[3]{\frac{3(1-C_p)}{2h_p}} \end{cases} \tag{4.41}$$

Applying (4.33) to (4.41) gives:

$$6A_p^2 R(A_q - 1) = \sqrt[3]{3(1 - C_p)h_p^2/2} \tag{4.42}$$

Expressing C_p via A_q (Eq. (4.35)) and R via $\wp$ (Eq. (4.31)) allows one to transform (Eq. (4.42)) to:

$$\wp\left(z_0 A_p - z_0; 0, 1\right) = \frac{1}{6z_0^2 A_p^2}\sqrt[3]{(3h_p^2/2 - 2^7\omega_2^6 A_p^6)} \tag{4.43}$$

This equation was solved with respect to A_p using the same numerical technique as Eq. (4.38). The only change to the code presented in subsection 4.4.2 is highlighted with bold font below:

$\text{Sol[k\_, b0\_]:=Re}\left[\text{ReplaceAll}\left[b, \text{FindRoot}\left[U[1,b] == \mathbf{\frac{1}{6}\sqrt[3]{\frac{3}{2}\left(\boldsymbol{k}^2 - \frac{2^8 wt^6 \boldsymbol{b}^6}{3}\right)}}, \{b, \text{b0}\}\right]\right]\right];$

The solution was interpolated as follows:

$$C_p \approx 1 - h_p/(1.5 + h_p) \tag{4.44}$$

$$A_p \approx \frac{1}{\omega_2} \sqrt[6]{3h_p^2(1 - h_p/(1.5 + h_p))/2^8} \tag{4.45}$$

The coefficient of determination (R^2) associated with interpolation Eq. (4.44) is 0.9999.

4.5 The Solution for Frictional Pressure Losses Along the Completion

In section 4.4 the Eqs. (4.8)-(4.10) were nondimensionalised and solved. Now one can write down the solution in a more practical original dimensional form.

4.5.1 Rate Constrained Well

Solution for a rate constrained well (boundary conditions (4.9)):

$$\Delta P_w = \frac{q_w}{jL} \sqrt{2h_q/3 + C_q} \tag{4.46}$$

$$R_d = \sqrt{C_q} / \sqrt{2h_q/3 + C_q} \tag{4.47}$$

$$E_p = 1 - 1/\sqrt{2h_q/3 + C_q} \tag{4.48}$$

$$q(l) = q_w 6 z_0^2 A_q^2 \wp \left(z_0(A_q l/L - 1); 0, 1\right)/h_q \tag{4.49}$$

$$\Delta P(l) = \frac{q_w}{jL} \sqrt{2h_q \left(q(l)/q_w\right)^3/3 + C_q} \tag{4.50}$$

where

$$h_q = C_f \rho f_a B^2 j L^2 q_w / D^5 \tag{4.51}$$

$$C_q \approx \left(1 + 0.1647\, h_q + 0.001793\, h_q^2\right)^{-1} \tag{4.52}$$

$$z_0 = \omega_2 \left(1 + i/\sqrt{3}\right) \tag{4.53}$$

$$A_q \approx \frac{1}{\omega_2} \sqrt[6]{\frac{3h_q^2}{2^8 \left(1 + 0.1647\, h_q + 0.001793\, h_q^2\right)}} \tag{4.54}$$

Horizontal Well number h_q can be qualitative interpreted as the ratio of reservoir and wellbore conductivity. This number approaches zero when wellbore is much more conductive then reservoir. Formulae (4.46)-(4.48) give an explicit analytical solution for the drawdown at the heel ΔP_w, the drawdown ratio R_d and the productivity error E_p. Figure 4.5 illustrates Formulae (4.47)-(4.48). One can see, for example, that:

- Friction reduces well's productivity by 19% when $h_q \approx 1$.
- The drawdown at the toe becomes half of that at the heel when $h_q \approx 3$.

Formula (4.49), describing the flow rate distribution in wellbore, employs a special function $\wp$ called the Weierstrass elliptic function (Abramowitz and Stegun, 1965, Ch. 18). Algorithms for calculating this function are available in literature (see, e.g., Eckhardt, 1980; Coquereaux et al., 1990; Baker, 1992). It can also be computed using specialised mathematical software (Maplesoft, 2009; Wolfram, 2009).

Formula (4.52) is, strictly speaking, an interpolation of a numerical solution (subsection 4.4.2). However, the high precision of this interpolation allows one to regard formula (4.52) as being precise since the maximum discrepancy associated with the interpolation is less than 0.2% for $h_q \leq 10$. The interpolation remains robust even for much higher values of h_q with the maximum discrepancy being about 2% for $h_q \leq 100$. I believe that scenario of $h_q > 10$ is of little practical interest since it corresponds to a productivity error $E_p > 60\%$ and a drawdown ratio $R_d < 23\%$.

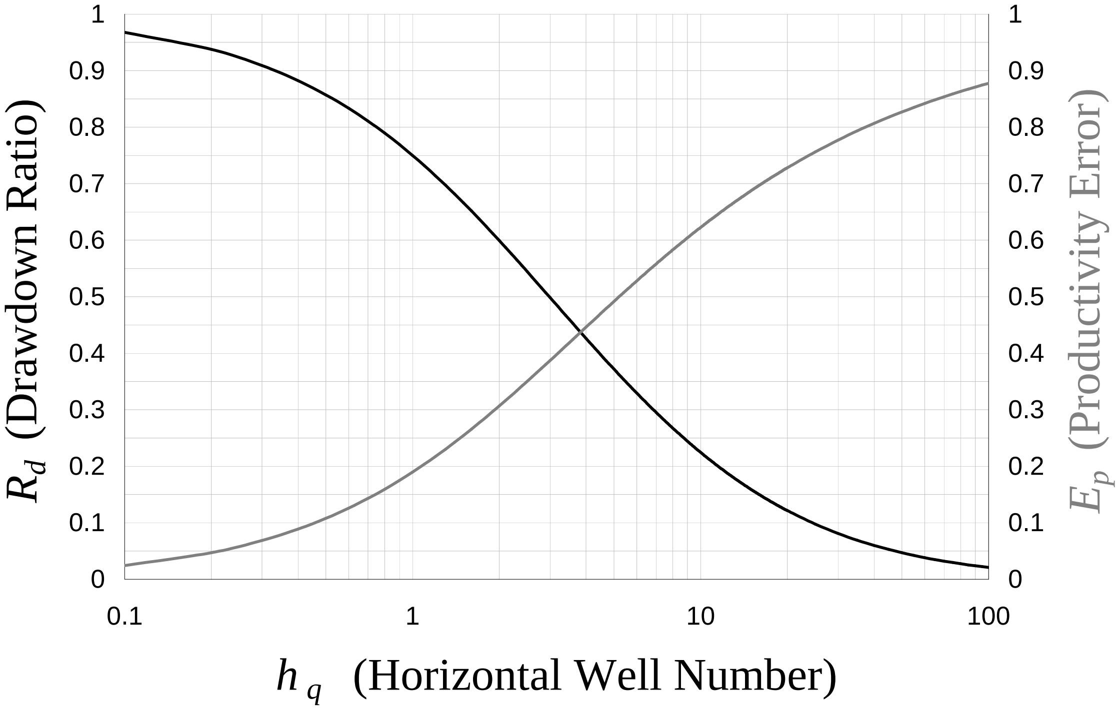

Figure 4.5: The drawdown ratio and the productivity error for a *rate* constrained well

4.5.2 Pressure Constrained Well

Solution to the problem of a pressure constrained well (boundary conditions (4.10)):

$$q_w = \Delta P_w j L \sqrt[3]{1.5(1 - C_p)/h_p} \tag{4.55}$$

$$R_d = \sqrt{C_p} \tag{4.56}$$

$$E_p = 1 - \sqrt[3]{1.5(1 - C_p)/h_p} \tag{4.57}$$

$$\Delta P(l) = \Delta P_w \sqrt{\frac{2h_p}{3}\left(\frac{q(l)}{\Delta P_w j L}\right)^3 + C_p} \tag{4.58}$$

$$q(l) = \Delta P_w j L \, 6 z_0^2 A_p^2 \wp\left(z_0(A_p l/L - 1); 0, 1\right)/h_p \tag{4.59}$$

where

$$h_p = C_f \rho f_a B^2 j^2 L^3 \Delta P_w / D^5 \tag{4.60}$$

$$C_p \approx 1 - h_p/(1.5 + h_p) \tag{4.61}$$

$$A_p \approx \frac{1}{\omega_2}\sqrt[6]{3h_p^2(1-h_p/(1.5+h_p))/2^8} \tag{4.62}$$

Formula (4.61) is less accurate than its analogue (4.52) for the rate constrained well since the author was unable to find a better interpolation for C_p. The average mismatch between precise numerical solution and its interpolation is about 3% for $h_p \leq 20$ (Table 4.1). The horizontal well number h_p does not normally exceed 20 in practical well designs. However the approximation still gives sensible results for values of h_p well above 20. E.g. the maximum mismatch is about 20% for $h_p \leq 50$. Figure 4.6 presents the *precise* solution for the drawdown ratio R_d and the productivity error E_p. The table of precise values of C_p is available in Excel format as supplementary data of this dissertation.

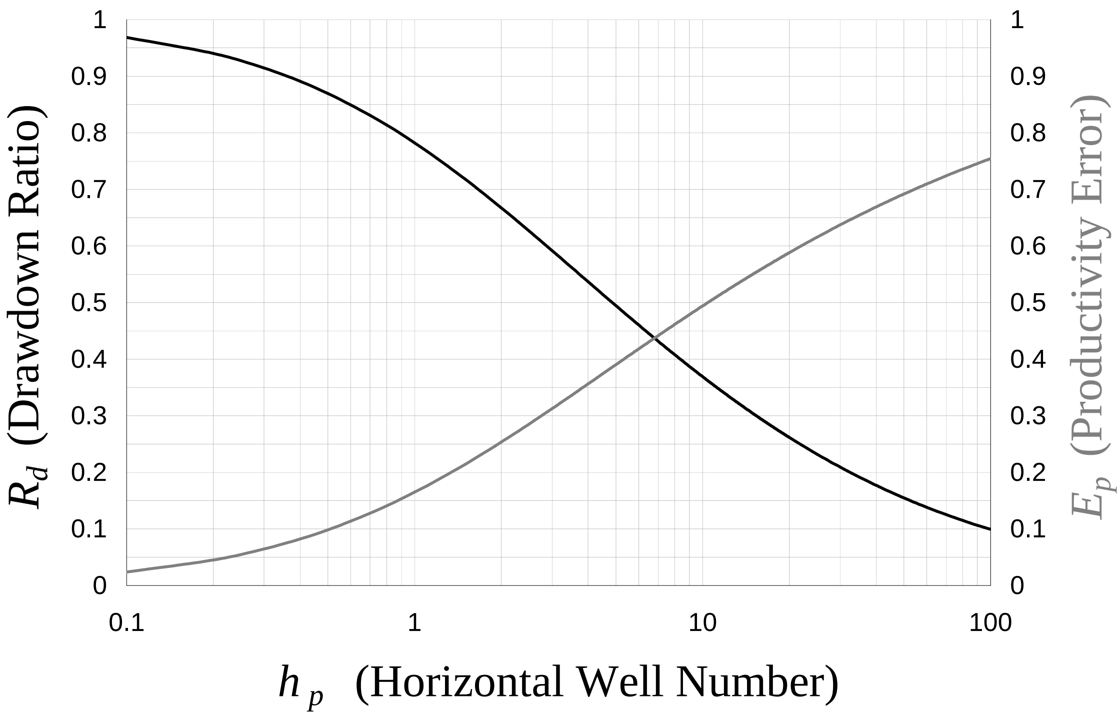

Figure 4.6: The drawdown ratio and the productivity error for a *pressure* constrained well

The relative simplicity of formula (4.61) allows one to conveniently express the

Interpolation	Average	Maximum
C_q for $h_q \leq 10$	0.03%	0.13%
C_p for $h_p \leq 20$	3.1%	5.6%

Table 4.1: Interpolation discrepancies for the two solutions

key parameters as a function of h_p:

$$q_w \approx \Delta P_w j L \sqrt[3]{1.5/(1.5+h_p)} \tag{4.63}$$

$$R_d \approx \sqrt{1 - h_p/(1.5+h_p)} \tag{4.64}$$

$$E_p \approx 1 - \sqrt[3]{1.5/(1.5+h_p)} \tag{4.65}$$

4.6 Model Verification

The model has been verified by comparing its example calculation results to those obtained with other published solutions and with a numerical simulation.

4.6.1 Seines et al. (1993)

The dependence of well rate on completion length $q_w(L)$ was presented in Figure A-6 of Seines et al. (1993) where L was normalised by the characteristic length l^*:

$$l^* = \sqrt[3]{12D^5/(j^2 C_f \rho f_a B^2 \Delta P_w)} \tag{4.66}$$

and q_w by the production rate of infinite well q_{inf}.

The solution presented in subsection 4.5.2 was used to reproduce the Figure A-6 of Seines et al. (1993). The authors did not publish tabular data corresponding to their Figure A-6 which makes precise comparison with this reproduction (Figure 4.7) problematic. However the work of Seines et al. (1993) agrees with Halvorsen's benchmark. Subsection 4.6.2 shows that the proposed solution also agrees with this benchmark. In addition to that I visually checked several data points on Figure A-

6 of Seines et al. (1993) and confirmed that they match the results given by the proposed solution. For example, L/l^* of 0.322 corresponds to q_w/q_{inf} of 0.591. On this basis, I believe that presented solution is consistent with the results of Seines et al. (1993).

The authors classified their characteristic length as being "optimal" because if:

- $L = l^*$, the well's production rate is almost identical to that of infinite length well ($q_w^* \approx q_{inf}$);
- $L < l^*$, then the well's production rate decreases rapidly with decreasing L;
- $L > l^*$, then the production rate increases only marginally with increasing L (Seines et al., 1993).

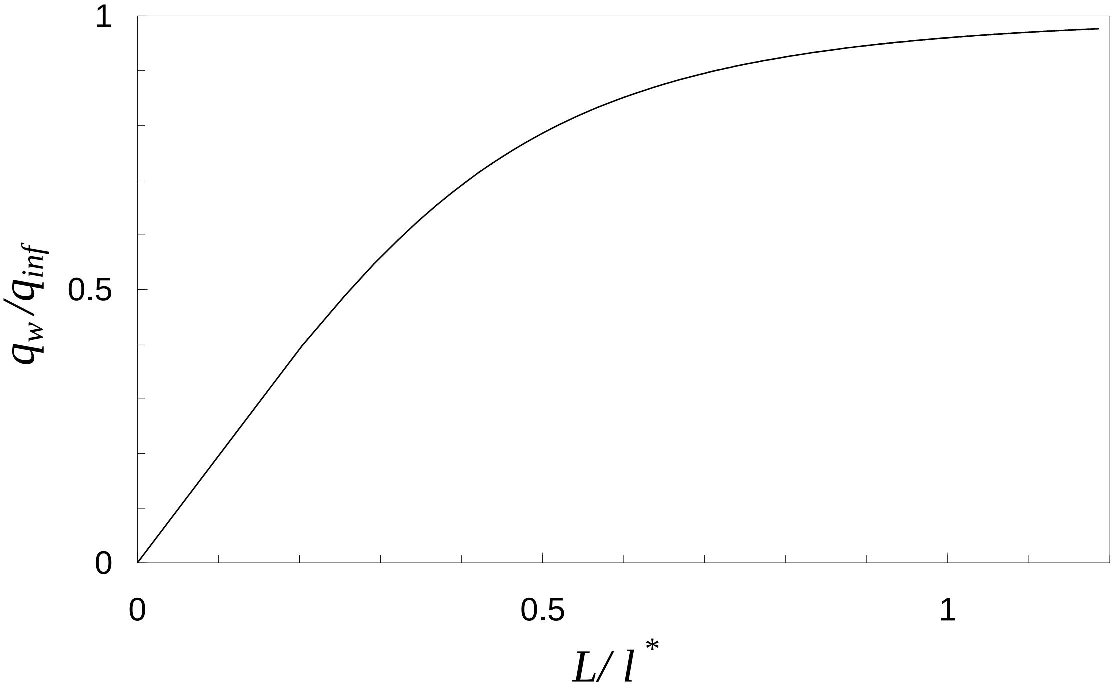

Figure 4.7: The dependence of well rate on completion length

Seines et al. (1993) used logarithmic scale for their Figure A-6. This may create the erroneous impression that the function $q_w(L)$ changes its curvature from concave up at small L to concave down at large L. In fact, it is concave down for all positive L (see Figure 4.7 with its linear scale). This is as expected from the basic physics:

the production gain from every extra metre extension to the completion interval is smaller than that from the previous one (due to frictional pressure losses).

I could not ascribe any specific mathematical significance to the point $L = l^*$ of function $q_w(L)$. I believe that in order to define a practical optimal completion length one has to broaden the definition of the problem itself, e.g. involve economical, reservoir and (or) risk analysis.

4.6.2 Halvorsen (1994)

According to Halvorsen (1994), a well of l^* length produces 96% of the production rate of an infinite well (more precisely $q_w^*/q_{inf} = 0.96019421$). The proposed explicit analytical solution gives 0.961 (Appendix D); a difference of 0.08% from Halvorsen's result. This minor discrepancy is due to the interpolation used to represent the solution explicitly. The precision can be further improved by use of look-up tables or more sophisticated interpolation techniques. However these approaches are considered unnecessary since the errors associated with model's assumptions and the uncertainty associated with the input parameters are, in practice, much higher than this mathematical error.

The drawdown ratio for a such well is 0.34, i.e. the inflow rate at the toe is only one third of that at the heel. This difference may affect reservoir sweep efficiency.

4.6.3 Penmatcha et al. (1999)

The example well case considered by Penmatcha et al. (1999) was used to compare his model to that presented in section 4.5. Table 4.2 outlines Penmatcha's parameters. According to my calculations, the average Fanning friction factor in this case is 0.0048 and the horizontal well number h_q equals 16.5. The mismatch in the drawdown estimates given by the two models (Table 4.3) is 10%. The mismatch in the productivity error is less than 5%. Unfortunately the level of detail provided by Penmatcha et al. (1999) was insufficient to allow me to identify the exact cause of these mismatches. The results of numerical simulation performed using commercial well modelling software (Weatherford, 2008) lie between Penmatcha's results and mine.

Production rate	q_w	10,000 stb/day
Well length	L	6,000 ft
Completion internal diameter	D	1/3 ft (4 in)
Specific PI (infinite conductivity)	j	0.25 stb/day/psi/ft
Fluid density	ρ	49 lb/ft^3
Fluid viscosity	μ	1 cP
Formation volume factor	B	1.05 rb/stb
Relative wellbore roughness	e/D	0.0005

Table 4.2: Well data used by (Penmatcha et al., 1999, App. A)

		Penmatcha et al.	This model	Numerical solution
Drawdown at the heel, psi	ΔP_w	25	22.5	23
Productivity error	E_p	0.73	0.70	0.71

Table 4.3: Results obtained using Table 4.2 well data.

4.6.4 Numerical Simulation

More than 30 well cases were studied using commercial well modelling software (Johansen and Khoriakov, 2007; Weatherford, 2008) in order to verify the model for a wide range of input parameters (Table 4.4). The results are presented in Figures 4.8 and 4.9. The average mismatch between the numerical simulation and proposed model is 4% with a maximum of 9%. This mismatch is due to the simplifications made in proposed model (use of a constant friction factor and the neglect of acceleration).

The practical value of the proposed model is illustrated in Table 4.5 by comparing the average and maximal mismatch to those of the upper estimate approach (Appendix C). One can see that proposed model predicts frictional pressure losses in completion much more accurately than simpler upper estimate approach (Appendix C).

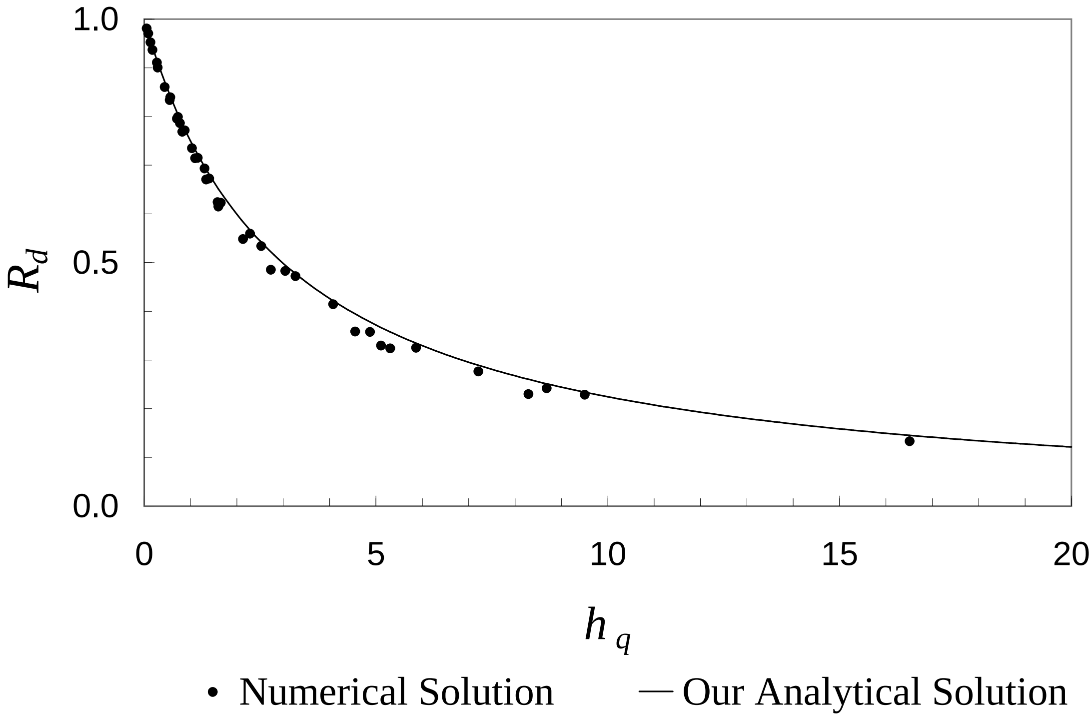

Figure 4.8: Numerical verification of Eq. (4.47) for the drawdown ratio, R_d, in a rate constrained well.

4.7 Discussion

The proposed closed form solution for turbulent frictional pressure drop along completions fills in the gap between the established analytical models that neglect this phenomenon and the numerical simulators that allow its detailed analysis. The dimensionless numbers used in the model introduce a succinct and comprehensive terminology for description of frictional pressure drop effects in horizontal wells.

Implementation of the solution is straightforward. It can be incorporated into engineering spreadsheets and used for applications such as:

- Identifying well design where the frictional pressure drop along the completion's length warps the inflow profile. Such wells are likely to benefit from ICD installation.

- Scoping economic analysis of the optimal well diameter and length.

Presented model is computationally efficient due to its explicit form. This makes it the preferred choice for use in computationally expensive applications such as

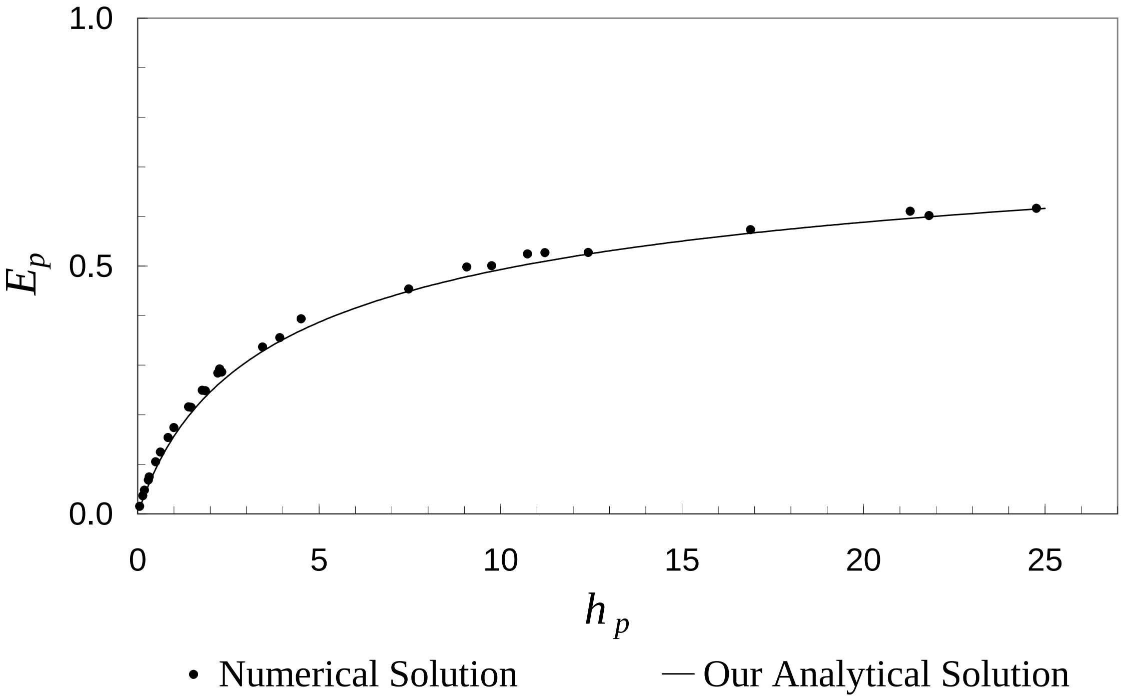

Figure 4.9: Numerical verification of Formula (4.57) for the productivity error, E_p, of a pressure constrained well

uncertainty analysis, history matching and optimisation.

The next chapter provides more details on how this model can be used in the context of ICD application.

4.8 Conclusions

An explicit analytical model for turbulent flow in a highly deviated wellbore has been developed and verified. The model is consistent with the semi-analytical models of Seines et al. (1993), Halvorsen (1994) and Penmatcha et al. (1999) as well as with the results of numerical simulations performed in commercial well modelling software.

	Parameter	Minimum	Maximum
L	Well length, ft	2 000	17 000
D	Completion internal diameter, in	4	8.5
j	Specific PI, STB/day/psi/ft	0.003	0.78
ρ	Fluid density, lb/ft^3	43	53
μ	Fluid viscosity, cP	0.7	12
ΔP_w	Drawdown at the heel, psi	1.1	450
q_w	Production rate, STB/day	150	13 000
Re_h	Reynolds number at the heel	$4 \cdot 10^3$	$3 \cdot 10^5$

Table 4.4: Range of parameters studied during the numerical verification process (subsection 4.6.4)

	Average	Maximum
Main model (section 4.5)	4%	9%
Upper estimate (Appendix C)	20%	60%

Table 4.5: Pressure mismatch with numerical simulation

Chapter 5

Reduction of the Horizontal Well's Heel-Toe Effect with Inflow Control Devices

5.1 Introduction

Increasing well-reservoir contact has a number of potential advantages in terms of well productivity, drainage area, sweep efficiency and delayed water or gas breakthrough. However, long, possibly multilateral, wells not only bring advantages, but also present new challenges in terms of drilling, completion and production. One of these challenges is the frictional pressure losses increasing with well length. The inflow profile becomes distorted so that the heel part of the well produces more fluid than the toe when these losses become comparable to drawdown. This inflow imbalance, in turn, often causes premature water or gas breakthrough. It should thus be avoided.

Installation of Inflow Control Devices (ICDs) is an advanced well completion option that provides a practical solution to this challenge. An ICD is a well completion device that directs the fluid flow from the annulus into the base pipe via a flow restriction. This restriction can be in form of channels (Figure 1.2), nozzles or orifices (Figure 1.3). In all cases the ability of an ICD to equalise the inflow along the well length is due to the difference in the physical laws governing fluid flow in the reservoir and through the ICD. Liquid flow in porous media is normally laminar, hence there is a linear relationship between the flow velocity and the pressure drop. By contrast, the flow regime through an ICD is turbulent, resulting in a quadratic velocity/pressure drop relationship.

The physical laws of flow through an ICD make it especially effective in reducing the free gas production. In-situ gas viscosity under typical reservoir conditions is normally at least an order of magnitude lower than that of oil or water; while in-situ gas density is only several times smaller than that of oil or water. Gas inflow into a well will thus dominate after the initial gas breakthrough if it is not restricted by gravity (Mjaavatten et al., 2008) or an advanced completion. ICDs introduce an extra pressure drop that is proportional to the square of the volumetric flow rate. The dependence of this pressure drop on fluid viscosity is weak for channel devices and totally absent if nozzle or orifice ICDs are used. These characteristics enable ICDs to effectively reduce high velocity gas inflow.

The magnitude of a particular ICD's resistance to flow depends on the dimensions of the installed nozzles or channels. This resistance is often referred to as the ICD's "strength". It is set at the time of installation and can not be changed without a major intervention to recomplete the well.

ICDs have been installed in hundreds of wells during the last decade, being now considered to be a mature, well completion technology. Steady-state performance of ICDs can be analysed in detail with well modelling software (Ouyang and Huang, 2005; Johansen and Khoriakov, 2007). Most reservoir simulators include basic functionality for ICD modelling. Some of them (Wan et al., 2008; Neylon et al., 2009) also offer practical means of capturing the effects of annular flow. Thus, current numerical simulation software enables engineers to properly perform the design and economic justification of an ICD completion. However relatively simple analytical models still have a role in:

- Quick feasibility studies (screening ICD installation candidates).

- Verification of numerical simulation results.
- Communicating best practices in a non-product specific manner.

This chapter proposes analytical and numerical solutions to the problem of reducing the heel-toe effect with ICDs. These solutions allows one to estimate the:

- ICD design parameters that reduce the heel-toe effect to the required level.
- Impact of ICD on the well's inflow peformance relationship (IPR).

5.2 Assumptions

The reservoir inflow model employed assumptions that:

- Flow is steady or pseudo-steady state and is described by Darcy's law.
- The distance between the well and the areal reservoir boundary is much longer than the well length (or the boundary is parallel to the well).
- The reservoir is homogeneous.
- Edge effects (due to semi radial inflow at the heel and the toe) are negligible.
- The perpendicular-to-the-well components of the reservoir pressure gradients are much greater than the along-hole ones.

The above simplifications are fairly typical for other analytical solutions that couple reservoir and wellbore flow using a specific productivity index (section 4.2). In the model presented in this chapter a specific productivity index, j, is assumed to be constant throughout the completion interval and equal to the ratio of well's Productivity Index (J) and the completion length (L):

$$j = J/L \tag{5.1}$$

Generally speaking, j is an empirical parameter that implies fluid inflow from the reservoir to wellbore is proportional to the pressure difference between the external reservoir boundary and the annulus:

$$\frac{dq}{dl} = j(l)(P_e - P_a(l)) \tag{5.2}$$

Eq. (5.2) is a repeat of Eq. (4.5) after rewriting in a notation more suited for this chapter.

The flow along the wellbore is assumed to be:

- Isothermal.
- Incompressible.
- Steady state.
- Homogeneous (no slip between the phases)

and the:

- Friction factor is constant along the completion interval.
- Pressure drop due to acceleration is small compared to that of friction.
- Dependence of fluid's viscosity upon pressure can be neglected.

Note that the completion interval is not assumed to be perfectly horizontal: True Vertical Depth (TVD) can vary along the completion since the reservoir pressure at the external boundary P_e is measured at the same TVD as the corresponding point l of the tubing.

Figure 1.2 and Figure 1.3 illustrate a typical ICD. The assumptions made about flow within ICD will now be described in more detail:

1. There is no flow in the annulus parallel to the base pipe i.e. fluid flows from reservoir directly into the ICD's screen. This assumption is reasonable when:
    - ICDs are combined with a number of intermediate packers or a gravel pack (Augustine et al., 2008) or

- The wellbore has collapsed around the screen so that annular flow is no longer possible.

2. ICDs of the same design (or "strength" of pressure drop) are installed throughout the completion length. This is the most common type of ICD application due to the relative simplicity of its design and installation (Henriksen et al., 2006). Variable "strength" ICD completions have been reported (Helmy et al., 2006; McIntyre et al., 2006), but they require a more complex design based on a detailed and reliable description of j along the wellbore at the time of completion installation in addition to increased operational risk of not installing them at the correct depth.

3. The flow distribution along the wellbore's internal flow conduit $q(l)$ is "smooth" (i.e. has a continuous derivative). Strictly speaking, this distribution is step-like because fluid enters wellbore through a number of point-like sources (ICD's nozzles or channels). However these steps are relatively small if the number of sources (or ICD completion joints of approximately 12 m length) is sufficiently large (e.g. > 50). This assumption is usually valid as a typical ICD well completion has more than 100 ICD joints. This assumption was introduced since dealing with continuous variables is mathematically more convenient.

5.3 Problem Formulation

5.3.1 General Formulation

Let us consider fluid flow through an ICD joint in order to obtain a mathematical formulation of the problem. The total pressure difference between the reservoir and the base pipe (tubing), ΔP, can be divided into the pressure drop in the reservoir, ΔP_r, and the pressure drop in ICD, ΔP_{ICD}:

$$\Delta P = \Delta P_r + \Delta P_{ICD} \tag{5.3}$$

In terms of mathematics, the inflow is the derivative of flow rate with respect to the measured depth. A separate notation U is designate to it here since it will be used extensively in this and next chapters:

$$U(l) \equiv \frac{dq(l)}{dl} \tag{5.4}$$

Then one can rewrite (5.2) as:

$$\Delta P_r(l) = U(l)/j \tag{5.5}$$

The pressure drop generated by an ICD is proportional to the second power of the flow rate through the ICD (Schlumberger, 2009). In terms of specific inflow it is:

$$\Delta P_{ICD} = aU^2(l) \tag{5.6}$$

where

$$a = \begin{cases} \left(\frac{\rho_{cal}\,\mu}{\rho\,\mu_{cal}}\right)^{1/4} \frac{\rho}{\rho_{cal}}\, l_{ICD}^2 B^2 a_{ICD} & \text{for channel ICDs} \\ \frac{C_u \rho l_{ICD}^2 B^2}{C_d^2 d^4} & \text{for nozzle or orifice ICDs} \end{cases} \tag{5.7}$$

The effective nozzle or orifice diameter, d, and the channel ICD rating, a_{ICD}, determine the pressure drop across the appropriate type of ICD. The industrial "bar" rating (Table 5.1) refers to the pressure drop created when a standard, 12 meter long ICD joint is exposed to a water flow rate of 26 Sm^3/d (Henriksen et al., 2006). It is used as a convenient method of comparing the pressure drop achieved by different types of ICDs.

Industrial "bar" rating	0.2	0.4	0.8	1.6	3.2
a_{ICD}, bar/(Rm^3/day)^2	0.00028	0.00055	0.00095	0.0016	0.0032
a_{ICD}, psi/(Rbbl/day)^2	0.00076	0.0015	0.0026	0.0044	0.0087

Table 5.1: Channel ICD strength

Substitution of equation (5.5) and (5.6) into (5.3) gives a quadratic equation with respect to the specific flow rate through the ICD completion:

$$\Delta P(l) = aU^2(l) + U(l)/j(l) \tag{5.8}$$

Eq. (5.8) has two real roots. The negative root has no physical meaning since the inflow rate must be positive as long as reservoir pressure, P_e, is greater than pressure in the base pipe P. Thus, the solution of Eq. (5.8) is:

$$U(l) = \frac{-1 + \sqrt{1 + 4a\Delta P(l) j^2(l)}}{2aj(l)} \tag{5.9}$$

The well production rate, q_w, is the integral of (5.9) over the well length:

$$q_w = \int_0^L \frac{-1 + \sqrt{1 + 4a\Delta P(l) j^2(l)}}{2aj(l)}\, dl \tag{5.10}$$

Formulae (5.9) and (5.10) are quite general as they account for both reservoir heterogeneity and frictional pressure losses in the base pipe. However, these formulae alone are of little practical utility without information or assumptions on pressure, $\Delta P(l)$, and specific productivity index, $j(l)$. The next section presents a practical formulation of the problem for the case when j can be treated as a constant (Eq. 5.1).

5.3.2 Formulation for a Homogeneous Reservoir

An explicit analytical model for the turbulent frictional pressure losses along a conventional well completion was presented in Chapter 4. The governing equations for this case are (4.5) and (4.7). They can be reduced to a single non-linear ODE of the second order which was solved in section 4.4.

Eq. (4.5) should be substituted with Eq. (5.8) when Inflow Control Devices are applied. This can then be rewritten as:

$$a\left(\frac{dq}{dl}\right)^2 + \frac{1}{j}\frac{dq}{dl} = (P_e - P(l)) \tag{5.11}$$

Let us now consider a system of Eqs. (4.7) and (5.11). It can be reduced to a single non-linear ODE of the second order:

$$2a\frac{dq}{dl}\frac{d^2q}{dl^2} + \frac{1}{j}\frac{d^2q}{dl^2} = \frac{C_f \rho f B^2}{D^5} q^2(l) \tag{5.12}$$

In practice, fluid production or injection is controlled either by a target flow rate or by a pressure condition. Hence Eq. (5.12) should be complemented with one of two sets of boundary conditions, i.e. for flow rate control:

$$\begin{cases} q(0) = 0 \\ q(L) = q_w \end{cases} \tag{5.13}$$

or for pressure control:

$$\begin{cases} q(0) = 0 \\ \frac{dq(L)}{dl} = j\Delta P_{rh} \end{cases} \tag{5.14}$$

Eq. (5.12), with boundary conditions (5.13) or (5.14) represents a mathematical formulation of the problem.

5.4 Solution

5.4.1 Qualitative Analysis

The general solution of Eq. (5.12) cannot be expressed via analytical functions. However certain propositions can be made about its properties:

1. $q(l)$ is a monotonically increasing, concave-up function:

$$\frac{dq}{dl} > 0, \quad \frac{d^2q}{dl^2} > 0$$

2. $q(a)$ is a monotonically decreasing, concave up function:

$$\frac{dq}{da} < 0, \quad \frac{d^2q}{da^2} > 0$$

3. When a is small ($a \to 0$), the solution of Eq. (5.12) approaches the solution for a conventional horizontal well proposed in Chapter 4.

4. With increase in a:

   - The productivity/injectivity of an ICD well decreases:

$$\frac{d}{da}\left|\frac{dq}{dP}\right| < 0$$

   - The magnitude of heel-toe effect decreases i.e. $q(l)$ becomes more linear:

$$\lim_{a\to\infty}\frac{d^2q}{dl^2} = 0 \tag{5.15}$$

Eq. (5.15) forms the basis of the simplifying assumption (subsection 5.4.2, Eq. 5.16) that allows an approximate analytical solution of the Eq. (5.12).

5.4.2 Approximate Analytical Solution

The purpose of ICD application to homogeneous reservoirs is to reduce heel-toe effect. Hence one can assume that inflow rates at the heel and toe will not greatly differ when the chosen ICD completion is installed:

$$U_h/U_t \approx 1 \tag{5.16}$$

Specific inflow term $\frac{dq}{dl}$ in Eq. (5.12) can now be substituted by a constant U_e:

$$(2ajU_e + 1)\frac{d^2q}{dl^2} = \frac{C_f\rho f B^2 j}{D^5}q^2(l) \tag{5.17}$$

U_e can be estimated as:

$$U_e = \begin{cases} q_w/L & \text{for flow rate control (Eq. (5.13))} \\ j\Delta P_{rh} & \text{for pressure control (Eq. (5.14))} \end{cases} \tag{5.18}$$

Eq. (5.17) is mathematically equivalent to Eq. (4.8) which was solved in Chapter 4; allowing solutions for a rate and pressure constrained wells as follows.

Rate Constrained Well

The solution for a rate constrained well (boundary conditions (5.13)) is:

$$U(l) \approx \frac{q_w}{L}\sqrt{2i_q\left(l/L\right)^3/3+G_q} \tag{5.19}$$

$$U_t \equiv U(0) \approx q_w\sqrt{G_q}/L \tag{5.20}$$

$$U_h \equiv U(L) \approx \frac{q_w}{L}\sqrt{2i_q/3+G_q} \tag{5.21}$$

$$\Delta P_w = aU_h^2 + U_h/j \tag{5.22}$$

where

$$i_q = \frac{C_f\rho f_a B^2 jL^2 q_w}{\left(2ajq_w/L+1\right)D^5} \tag{5.23}$$

$$G_q \approx \left(1+0.1647\, i_q+0.001793\, i_q^2\right)^{-1} \tag{5.24}$$

Pressure Constrained Well

The solution to the problem of a pressure constrained well (boundary conditions (5.14)) is:

$$q_w \approx J\Delta P_{rh}\sqrt[3]{1.5/(1.5+i_p)} \tag{5.25}$$

$$U(l) \approx j\Delta P_{rh}\sqrt{\left(1-G_p\right)\left(l/L\right)^3+G_p} \tag{5.26}$$

$$U_t \equiv U(0) \approx j\Delta P_{rh}\sqrt{G_p} \tag{5.27}$$

$$U_h \equiv U(L) = j\Delta P_{rh} \tag{5.28}$$

where

$$i_p = \frac{C_f\rho f_a B^2 j^2 L^3\Delta P_{rh}}{\left(2aj^2\Delta P_{rh}+1\right)D^5} \tag{5.29}$$

$$G_p \approx 1 - i_p/(1.5 + i_p) \tag{5.30}$$

5.4.3 Numerical Solution

I implemented a numerical solution of Eq. (5.12) using the shooting method with the starting boundary condition:

$$\frac{dq(0)}{dl} = U_e \tag{5.31}$$

where U_e is given by formula (5.18).

The corresponding Wolfram Mathematica^{TM}code is:

---

sol = NDSolve[{$2ny$"$[x]y'[x] + y$"$[x] == hy[x]$^$2, y[0] == 0, y'[1] == 1\}, y, x,$

Method $\rightarrow$ {"Shooting", "StartingInitialConditions" $\rightarrow$ $\{y[0] == 0, y'[0] ==$ $1\}\}$];

Section 5.6 compares the results of my numerical solution to those obtained with the most appropriate of the commercial well modelling software (Johansen and Khoriakov, 2007; Halliburton, 2009).

5.5 Choosing an Appropriate ICD Strength

An increase in the strength of an ICD will improve the inflow equalisation while at the same time leading to a reduction in the well's Inflow Performance Relationship (IPR). The trade-off between reducing well productivity and the increasing degree of inflow equalisation is the key issue in ICD technology application. One can choose an ICD strength value that gives a reasonable compromise between the inflow performance and the inflow equalisation by using either the analytical or the numerical solution of Eq. (5.12) or commercial well modelling software. In this section I present another analytical way of choosing ICD strength which is simpler but less rigorous than above mentioned methods.

Production from horizontal well is often constrained by the requirement that the drawdown at the heel should be small enough to avoid premature breakthrough of water or gas. Proposed analytical approach to the choice of the appropriate ICD strength is based on the supposition that the ICD should introduce an additional pressure drop of the same order of magnitude as the reservoir drawdown (Table 2.5). This translates, in context of the heel-toe effect, to the requirement that the pressure drop across the ICD should be of the same order of magnitude as the drawdown at the heel ΔP_{rh}:

$$\Delta P_{ICD} \approx n \Delta P_{rh} \tag{5.32}$$

where n is a positive dimensionless number (see Eq. 5.36).

The ICD strength required to produce such a pressure drop is obtained from Eqs. (5.5) and (5.6):

$$a \approx \frac{nL^2}{\Delta P_{rh} J^2} \tag{5.33}$$

Eq. (5.33) can take two forms depending on the type of flow restriction installed in the ICD (Eq. (5.7)). For a

1. Channel type of ICD:

$$a_{ICD} \approx \left(\frac{\rho\, \mu_{cal}}{\rho_{cal}\, \mu} \right)^{1/4} \frac{\rho_{cal}\, nL^2}{\rho\, l_{ICD}^2 B^2 \Delta P_{rh} J^2} \tag{5.34}$$

2. Nozzle/orifice ICDs:

$$d \approx \left(\frac{C_u \rho l_{ICD}^2 B^2 \Delta P_{rh} J^2}{C_d^2 n L^2} \right)^{1/4} \tag{5.35}$$

The higher values of n correspond to a greater ICD "strength" and hence a smaller difference between the heel and toe specific inflows. I suggest the following "rule of thumb" for choosing n (and hence the ICD "strength"):

$$n \approx (U_h/U_t)_{noICD} - 1 \tag{5.36}$$

where $(U_h/U_t)_{noICD}$ is the ratio of inflow at the heel and at the toe for an equivalent conventional (no ICD) well produced with the same drawdown at the heel. This ratio can be estimated with well modelling software or analytical model presented in section 4.5.

In author's opinion the pressure interaction between the reservoir and the ICD is the key factor for solving the problem of selecting the appropriate ICD strength. However, it should be borne in mind that this problem has no general or "correct" solution. The preferred solution will also be influenced by factors beyond the scope of this simple analysis (e.g. economics, field development strategy, annulus flow, gas production, clean-up, erosion etc.) which are only relevant to a specific field application.

5.6 Case Study

ICDs were invented by Norsk Hydro for use in the Troll field, one of the Norwegian continental shelf's largest oil and gas producing fields (Henriksen et al., 2006). The thin oil column (4 - 27 m) present across much of the field represented a tremendous challenge, both in terms of drilling and completion operations, due to the requirement to produce oil prior to large scale gas production from the gas cap. Only the thickest part of oil column was initially recognised as proven reserves, despite the large volume of oil in place in the part of the field with a thinner column. However, the construction of wells with increasingly longer horizontal sections, the implementation of multilateral well technology and the application of ICDs during the last decade have resulted in the successful development of an increasing fraction of the oil volume originally-in-place.

The Troll reservoir, a permeable homogeneous sandstone, is thus an ideal candidate for illustrating the practical utility of the proposed analytical solution for homogeneous reservoirs (see Table 5.2 for Troll field data). Being an oil rim reservoir, the Troll field has the initial reservoir pressure equal to the bubble point pressure. Strictly speaking, one should use Vogel's (non-linear) formula to calculate the inflow performance for this reservoir. However, the difference between a linear and a Vogel

IPR is less than 0.5% for the range of flow rates of interest here. The assumption of linear inflow from the reservoir thus remains valid.

Drawdown at the heel	ΔP_{rh}	1.2 bar
Well length	L	2 500 m
Completion internal diameter	D	0.15 m (5.9 in)
Well's PI estimate (neglecting friction)	J	5 000 Sm^3/day/bar
In-situ fluid density	ρ	800 kg/m^3
In-situ fluid viscosity	μ	1.7 cP
Formation volume factor	B	1.17 Rm^3/Sm^3
Absolute roughness of base pipe	e	0.05 mm

Table 5.2: Typical Troll oil well data

Both well modelling software and analytical methods (Chapter 4) indicate that frictional pressure losses have a great effect on the IPR in this case with the specific inflow at the heel being 4 times higher than that at the toe ($U_h/U_t \approx 4.4$). Eqs. (5.33) and (5.36) recommended a value of $a \approx 0.63$ bar$\cdot$day^2/Sm^4 to reduce this ratio. This is equivalent to $a_{ICD} \approx 0.003$ bar/(Rm^3/day)^2 in terms of channel ICD strength (Eq. (5.34)) and $d \approx 4.1$ mm in terms of nozzle/orifice ICD effective diameter (Eq. (5.35), assuming $C_d = 1$). These estimations agree well with actual value (0.0032 bar/(Rm^3/day)^2) of the channel ICD strength installed in the Troll field (Henriksen et al., 2006). This estimate will be further refer to as the "recommended ICD".

Figure 5.1 illustrates the effect of the recommended ICD on the performance of the Troll well described in Table 5.2: U_h/U_t ratio was reduced from 4.4 ("no ICD" case) to 1.9 ("recommended ICD" case). As can be seen from the figure, numerical solution of Eq. (5.12) matches very well with the solution given by well modelling software (Johansen and Khoriakov, 2007) while proposed approximate analytical solution (Eq. (5.26)) also gives a reasonable estimate of the inflow profile.

A further increase in the ICD strength gives an even more uniform inflow at the cost of a reduced well inflow performance. For instance, doubling the "recommended

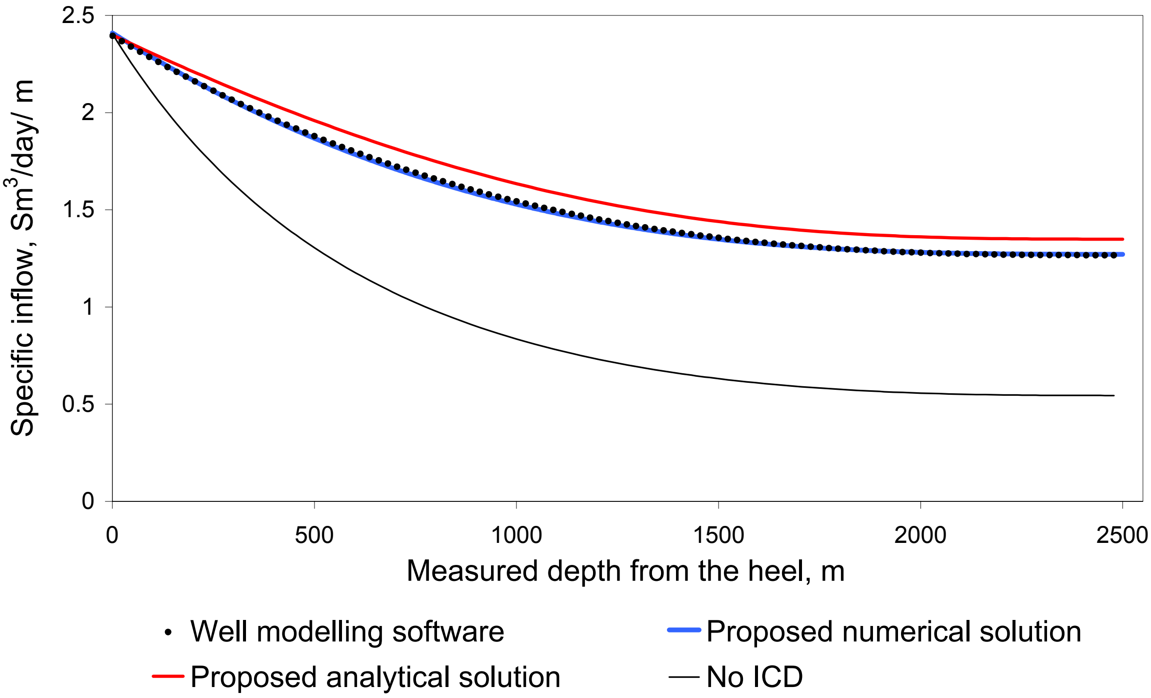

Figure 5.1: Impact of the "*recommended* ICD" on the specific inflow distribution for the Troll case

ICD strength" ($a_{ICD} \approx 0.006$ bar/(Rm^3/day)^2 or $d \approx 3.5$ mm) corresponds to a U_h/U_t ratio of 1.5 (Figure 5.2).

Figure 5.3 illustrates the general dependence of inflow equalisation on the ICD strength for the Troll case. This dependence agrees with preliminary judgements 3 and 4 made in subsection 5.4.1. Indeed:

- The heel-toe effect vanishes at small nozzle diameters (i.e. with an increase in ICD "strength").
- Large nozzle diameters (low ICD "strength") result in a well performance similar to that of conventional horizontal well.

Figure 5.3 is also a good illustration of the precision of the proposed analytical solution: it approaches the numerical solution for extremely high and low values of ICD "strength" while having an error of less than 10% for intermediate values.

Let us now discuss the impact of the recommended ICD completion on the well's IPR (Figure 5.4). The IPR curve for a well without ICDs (thin black line) can be calculated using either well modelling software or analytical methods (e.g.

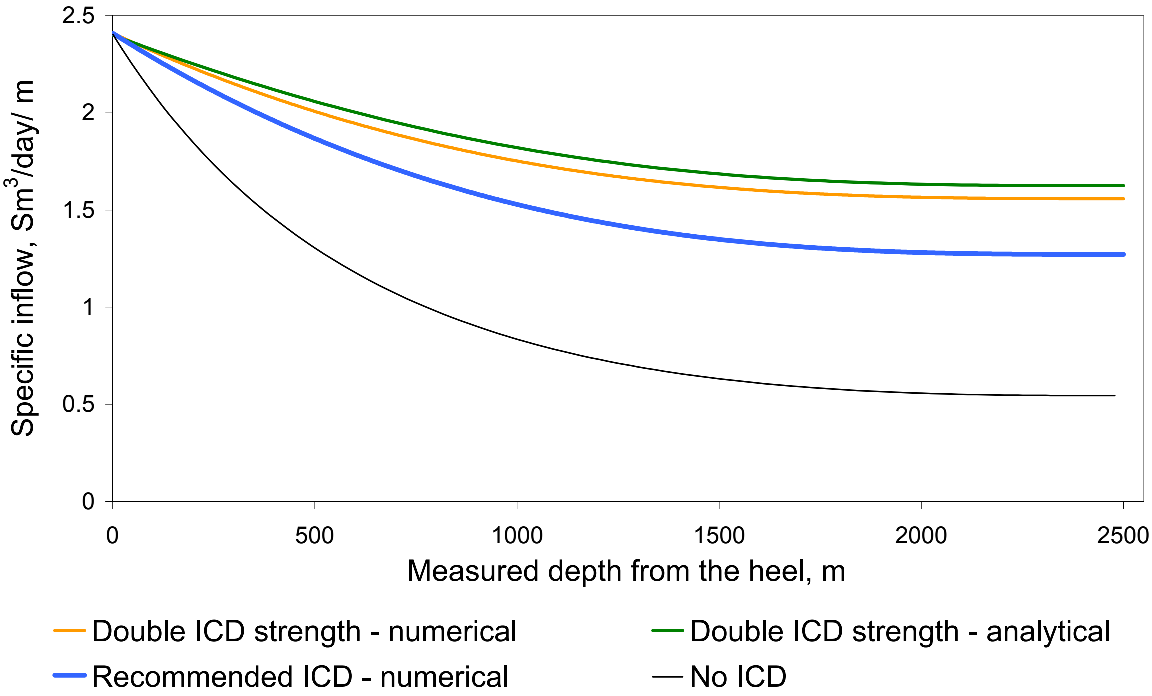

Figure 5.2: Impact of the "*double* ICD" strength on the specific inflow distribution for the Troll case

Chapter 4). It is non-linear due to the frictional pressure losses along the completion. The IPR of ICD completions has been calculated in two ways using:

1. Analytical formulae (5.21)-(5.24).

2. The numerical solution of Eq. (5.12).

The deviation of the proposed analytical solution from the more precise numerical one is quite small for the "recommended ICD". An increase in ICD "strength" result in even better agreement. One can also deduce from Figure 5.4 that a doubling of the ICD "strength" does not reduce the IPR by a factor of two. This results from $q(a)$ being a monotonically decreasing, *concave-up* function.

This case study has demonstrated how the analytical and numerical solutions presented in this chapter can be used to estimate the:

- Equipment design parameters of the "recommended ICD" completion that substantially reduces the heel-toe effect.

- Impact of ICD on well's inflow performance.

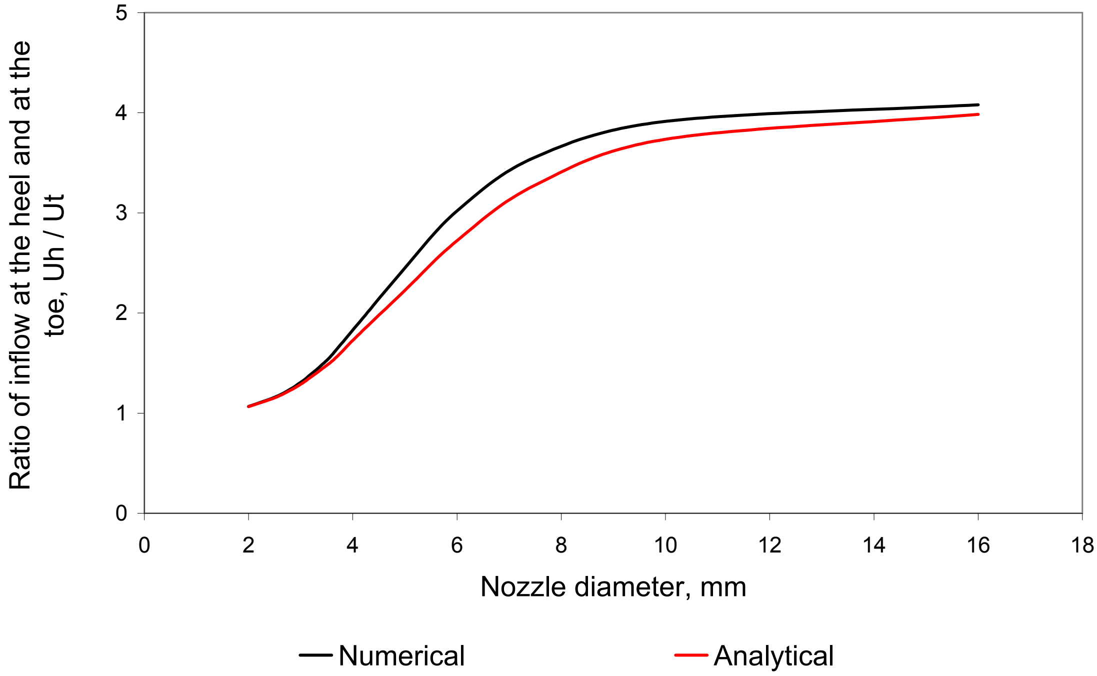

Figure 5.3: Dependence of inflow equalisation on ICD nozzle diameter

5.7 Discussion

A substantial number of papers have been published over the last decade addressing various aspects of the application of ICD technology. Most of them use a case study format with emphasis on the practical challenges and the positive effects of installing an ICD. However, as shown in Chapter 2, ICD technology is not universally applicable. The approach described here is one step in the development of a clear methodology for ICD completion design by proposing transparent analytical and numerical ICD inflow performance models.

The approximate analytical solution (subsection 5.4.2) highlights the dependencies between the key parameters influencing the resulting ICD completion design (e.g. nozzle diameter dependence on the drawdown at the heel). Its deviation from the more precise numerical solution is less than 10%, making it suitable for quantitative analysis of ICD completions. However, in author's opinion the main value of this analytical model is in providing a *qualitative* insight into the ICD-reservoir pressure interaction.

Accurate numerical solution of Eq. (5.12) is a more practical engineering tool.

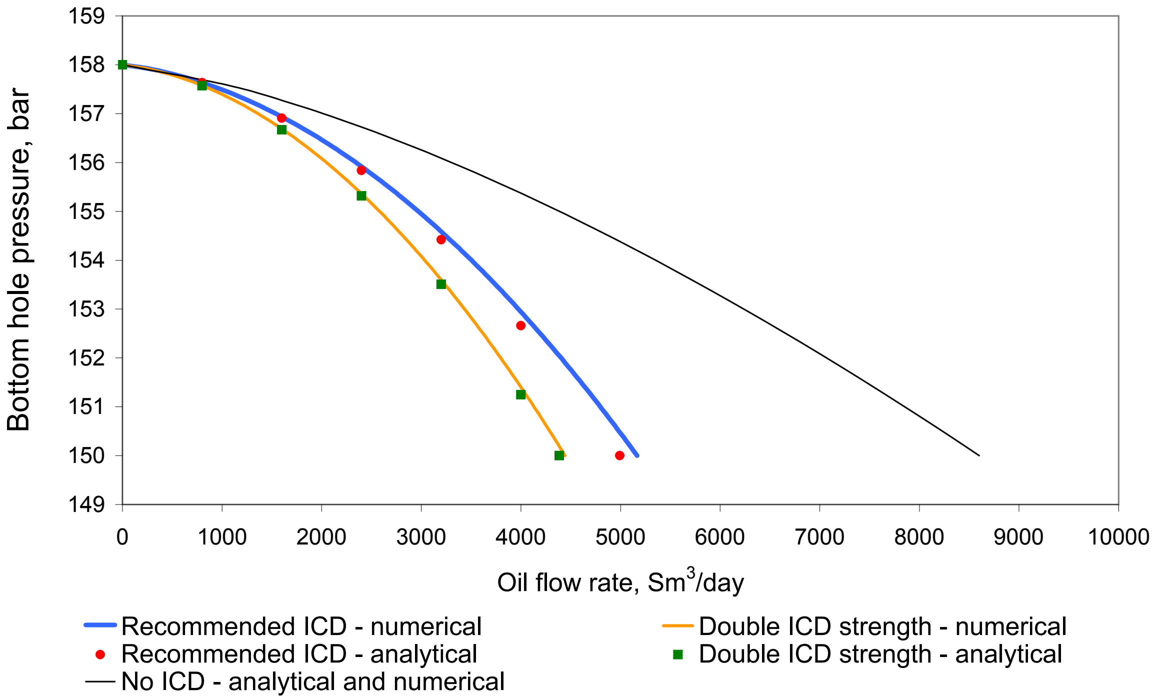

Figure 5.4: Impact of the “recommended ICD” on well’s IPR

The shooting method used is a common technique for solving multi-point boundary value problems. It is described in most textbooks on numerical methods and ready-to-use implementations are available in most programming languages.

Section 5.5 presented a simplified, but effective, alternative to analytical or numerical solution of Eq. (5.12). It suggests a simple rule for choosing ICD strength that will substantially reduce the heel-toe effect. This alternative approach may be used, if necessary, to check the validity of results obtained with commercial well modelling software or with the methods presented in subsections 5.4.2 and 5.4.3.

The next chapter extends this analytical approach to situations where reservoir heterogeneity (not heel-toe effect) is the primary reason for inflow variation.

5.8 Conclusions

The equation that quantifies the heel-toe effect reduction achieved by Inflow Control Devices installed in horizontal wells has the form of a second order, non-linear ODE. Two solutions have been presented: an approximate analytical and a more precise numerical one. These solutions allow one to estimate the:

- ICD design parameters that reduce the heel-toe effect to the required level.
- Impact of the ICD on the well's inflow peformance relationship.

The practical utility of this approach been illustrated using published Troll oil field data.

Chapter 6

Application of Inflow Control Devices to Heterogeneous Reservoirs

6.1 Introduction

This chapter proposes an analytical model for heterogeneous reservoirs that quantifies the reduction of inflow variation along a horizontal well with ICDs installed. This model allows one to estimate:

- The ICD design parameters that substantially reduce the inflow variation caused by reservoir heterogeneity.
- The impact of a specific ICD completion on Inflow Performance Relationship (IPR) of a long well completed in a heterogeneous reservoir.

6.2 Assumptions

The model presented in this chapter invokes the following assumptions with respect to the inflow from the reservoir:

- Flow through the reservoir can be described by Darcy's law.
- Steady or pseudo-steady state flow into the well.
- The distance between the well and the reservoir boundary is much longer than the well length (or the boundary is parallel to the well).
- The perpendicular-to-the-well components of the reservoir pressure gradients are much greater than the along-hole ones.

The chosen assumptions for the description of the wellbore flow are that:

- Friction and acceleration pressure losses between the toe and the heel are small compared to the drawdown.
- The fluid is incompressible.

The above assumptions imply that the difference between the reservoir external boundary pressure P_e and the tubing pressure P is constant throughout the completion length:

$$P_e(l) - P(l) = \Delta P_w = const \tag{6.1}$$

The assumptions about the ICDs are as follows:

1. There is no flow in the annulus parallel to the base pipe, i.e. the fluid flows from reservoir directly through ICD screens into the base pipe.
2. ICDs of the same "strength" are installed throughout the completion length.
3. The flow distribution along the wellbore's internal flow conduit $q(l)$ is "smooth" (i.e. it has a continuous derivative).

These assumptions were previously discussed in section 5.2.

6.3 Problem Formulation

Let us analyse the impact of an ICD completion on the well inflow profile when the well is completed in a heterogeneous reservoir.

The Specific Productivity Index, j, and hence the inflow, U, change stochastically along the completion interval. A coefficient of variation will be used to quantify the degree of these changes. Recall that the coefficient of variation of a random variable is defined as the ratio of its standard deviation and its mean.

The annulus pressure P_a is equal to the base pipe pressure P for a conventional completion (no ICD). Hence:

$$U(l) = j(l)\Delta P_w \tag{6.2}$$

where ΔP_w is a constant independent of l. Eq. (6.2) shows that, in case of conventional completion (no ICD), the coefficient of variation of specific inflow is equal to that of the specific PI:

$$\mathrm{CoV}\, U = \mathrm{CoV}\, j \tag{6.3}$$

ICD application reduces the variation of inflow so that:

$$\mathrm{CoV}\, U < \mathrm{CoV}\, j \tag{6.4}$$

Inequality (6.4) may seem intuitively obvious to engineers familiar with the ICD technology, however its rigorous mathematical proof requires considerable ingenuity. The proof was suggested by Alexandr V. Usnich (Birchenko et al., 2009).

Let us consider the ratio of the two coefficients of variation, $\mathrm{CoV}\, U/\,\mathrm{CoV}\, j$. This ratio equals unity for a conventional completion and decreases monotonically with increasing ICD strength. The magnitude of this decrease is a quantitative measure of the equalisation of the inflow along the completion length due to the ICD. The objective of this work is to develop a mathematical model linking the ratio of the two coefficients of variation with the well parameters (such as ICD "strength", drawdown, etc.)

6.4 Solution

The solution for inflow to an ICD well was presented in section 5.3. Eq. (5.10) is of limited use for a "quick-look" analysis if the local specific productivity index varies substantially along the completion interval since:

1. The exact shape of the productivity profile $j(l)$ is often unknown:
   - Detailed measurements (logging) are not always feasible.
   - The productivity index changes with time (e.g. due to fluid saturation changes).
2. In general one needs to evaluate the integral (5.10) numerically even if $j(l)$ is known (or can be estimated).

The engineering team that develops each well drilling proposal will normally define an expected range of values for the specific productivity index, j, as part of the proposal. These could be based on:

- Well log data.
- Reservoir models.
- Production performance of similar wells in the same field.

This range of values may take a number of forms. For instance, in its simplest form it could comprise of only three values: pessimistic (P90), most probable (P50) and optimistic (P10). Ideally, a complete specification of j would be available in the form of a probability density function (p.d.f.). In the case when j depends on some other parameters and information about their distribution is available this density can be estimated, for example, via Monte-Carlo simulation.

It is often easier to make a judgement about the statistical distribution of the specific productivity index, $\eta(j)$, rather than its spatial distribution $j(l)$. This allows one to transform formula (5.10) into

$$q_w = L \int_{j_1}^{j_2} \frac{-1 + \sqrt{1 + 4a\Delta P_w j^2}}{2aj} \eta(j)\, dj \tag{6.5}$$

Calculation of the coefficient of variation requires mean and mean square values of the specific inflow rate. The mean specific inflow rate is the ratio of the well flow rate to its length:

$$\langle U\rangle = q_w/L = \int_{j_1}^{j_2} \frac{-1+\sqrt{1+4a\Delta P_w j^2}}{2aj}\eta(j)\,dj \tag{6.6}$$

Similarly, its mean square value is calculated as follows:

$$\langle U^2\rangle = \int_{j_1}^{j_2} \left(\frac{-1+\sqrt{1+4a\Delta P_w j^2}}{2aj}\right)^2 \eta(j)\,dj \tag{6.7}$$

The choice of the method for solving the integrals (6.6) and (6.7) depends on the functional form of $\eta(j)$, the p.d.f. of the specific PI. Notably, these integrals can be solved analytically for a piecewise linear p.d.f. (e.g. a uniform or triangular distribution). The corresponding solutions are presented below in subsections 6.4.1 and 6.4.2. When the density function has a more complex form (e.g. a normal or log-normal distribution) the integrals have to be evaluated numerically.

6.4.1 Uniform Distribution of Specific Productivity Index

Generally speaking, a uniform distribution of the specific productivity index is unlikely to be encountered in practice. In fact, petro-physical quantities are usually modelled by a normal or log-normal distribution. However, the data required to determine the distribution parameters with sufficient precision is often unavailable. A uniform distribution may be a sensible starting assumption when data is scarce.

Assuming that j is uniformly distributed between two values j_1 and j_2, $j_1 \le j_2$, its density function is as follows:

$$\eta(j) = \begin{cases} 1/(j_2-j_1) & \text{for } j_1 \le j \le j_2 \\ 0 & \text{otherwise} \end{cases} \tag{6.8}$$

In this case:

$$q_w = \langle U \rangle L = \frac{I_U(j_2) - I_U(j_1)}{j_2 - j_1} L \tag{6.9}$$

and

$$\frac{\mathrm{CoV}\, U}{\mathrm{CoV}\, j} = \frac{(j_2 + j_1)\sqrt{3\left(\langle U^2 \rangle - \langle U \rangle^2\right)}}{\langle U \rangle (j_2 - j_1)} \tag{6.10}$$

where

$$\langle U^2 \rangle = \frac{S_U(j_2) - S_U(j_1)}{j_2 - j_1} \tag{6.11}$$

with

$$I_U(j) = \frac{\sqrt{1 + 4a\Delta P_w j^2} - \ln\left(1 + \sqrt{1 + 4a\Delta P_w j^2}\right)}{2a} \tag{6.12}$$

$$\begin{aligned} S_U(j) \quad = \quad & \frac{1}{2a^2 j}\Big(-1 + 2a\Delta P_w j^2 + \sqrt{1 + 4a\Delta P_w j^2} - \\ & -2j\sqrt{a\Delta P_w}\,\mathrm{arcsinh}\left(2j\sqrt{a\Delta P_w}\right)\Big) \end{aligned} \tag{6.13}$$

6.4.2 Triangular Distribution of Specific Productivity Index

The most probable, or modal value is often known within reasonable error margins in addition to knowledge about the minimum and maximum values of the specific PI. The specific productivity index j may then be modelled by the (more complex) triangular distribution. This is a legitimate approach if a triangular distribution can be fitted to the field data with accuracy similar to that of the more common normal or log-normal distributions.

The p.d.f. of a triangular distribution is as follows:

$$\eta(j) = \begin{cases} 2(j - j_1)/(j_2 - j_1)/(j_m - j_1) & \text{for } j_1 \le j \le j_m \\ 2(j_2 - j)/(j_2 - j_1)/(j_2 - j_m) & \text{for } j_m \le j \le j_2 \\ 0 & \text{otherwise} \end{cases} \tag{6.14}$$

Then

$$q_w = \langle U \rangle L = \frac{2L}{j_2 - j_1} \left(\frac{I_{Uj}(j_m) - I_{Uj}(j_1) - j_1 \left(I_U(j_m) - I_U(j_1) \right)}{j_m - j_1} + \right.$$
$$\left. + \frac{j_2 \left(I_U(j_2) - I_U(j_m) \right) - I_{Uj}(j_2) + I_{Uj}(j_m)}{j_2 - j_m} \right) \tag{6.15}$$

and

$$\frac{\mathrm{CoV}\, U}{\mathrm{CoV}\, j} = \frac{j_2 + j_m + j_1}{\langle U \rangle} \sqrt{\frac{2\left(\langle U^2 \rangle - \langle U \rangle^2 \right)}{j_1^2 + j_2^2 + j_m^2 - j_1 j_2 - j_1 j_m - j_2 j_m}} \tag{6.16}$$

where

$$\langle U^2 \rangle = \frac{2}{j_2 - j_1} \left(\frac{S_{Uj}(j_m) - S_{Uj}(j_1) - j_1 \left(S_U(j_m) - S_U(j_1) \right)}{j_m - j_1} + \right.$$
$$\left. + \frac{j_2 \left(S_U(j_2) - S_U(j_m) \right) - S_{Uj}(j_2) + S_{Uj}(j_m)}{j_2 - j_m} \right) \tag{6.17}$$

with

$$I_{Uj} = \frac{1}{2a} \left(-j + \frac{j\sqrt{1 + 4aj^2 \Delta P_w}}{2} + \frac{\operatorname{arcsinh}\left(2j\sqrt{a \Delta P_w} \right)}{4\sqrt{a \Delta P_w}} \right) \tag{6.18}$$

$$S_{Uj}(j) = \frac{\Delta P j^2/2 - I_U(j)}{a} \tag{6.19}$$

and functions I_U and S_U defined by (6.12) and (6.13) respectively.

6.5 Case Study

This case study shows how the proposed model for a uniform distribution can be used in practice for the following two cases:

1. Highly Productive Reservoir
2. Medium Productivity Reservoir.

This is done to quantitatively illustrate the dependence between the specific PI and ICD “strength” required to reduce inflow variations.

6.5.1 Highly Productive Reservoir

Let us consider a 1 km long well completed in a prolific heterogeneous reservoir (Table 6.1). The anticipated PI of the well is 2 000 Sm^3/day/bar. A drawdown, ΔP_r, of 0.5 bar is required for a conventional completion to achieve the target well rate of 1 000 Sm^3/day. Pressure drop introduced by conventional completion is usually negligible compared to the drawdown:

$$\Delta P_w \approx \Delta P_r = 0.5 \text{ bar} \tag{6.20}$$

Well length	L	1 000 m
Well PI	J	2 000 Sm^3/day/bar
Minimum value of specific PI	j_1	0.5 Sm^3/day/bar/m
Maximum value of specific PI	j_2	3.5 Sm^3/day/bar/m
Target well flow rate	q_w	1 000 Sm^3/day
In-situ fluid density	ρ	800 kg/m^3
In-situ fluid viscosity	μ	1.7 cp
Formation volume factor	B	1.2 Rm^3/Sm^3
Length of the ICD joint	l_{ICD}	12.2 m

Table 6.1: *Highly productive* reservoir case study data

The inflow distribution along the completion is expected to be highly uneven and uncertain due to complex reservoir geology. The local specific productivity index is anticipated to be within the range of 0.5 - 3.5 Sm^3/day/bar/m. Subject to the assumptions stated in section 6.2, the inflow to the conventional completion will be proportional to the local specific productivity index. This implies a 7-fold variation of specific inflow rate for the above case.

A completion combining ICDs and annular flow isolation will improve oil recovery by smoothing out the specific inflow rate variations and increasing oil sweep efficiency along the above horizontal well.

The uniform distribution model (formula (6.9), subsection 6.4.1) predicts that "1.6 bar" ICD completion with ΔP_w of 1 bar will produce 1 070 Sm^3/day. That is, the "1.6 bar" ICD completion reduced well productivity by approximately 50% (for the target rate), but also delivered an improved degree of inflow equalisation (Figure 6.1). The grey line in Figure 6.1 was obtained using formula (5.9). The specific inflow rate variation is considerably smaller than for a conventional completion. Namely, the CoV U/ CoV j ratio of 0.52 for the "1.6 bar" ICD case can be interpreted as almost a 50% reduction of the difference between regions of high and low specific inflow rate.

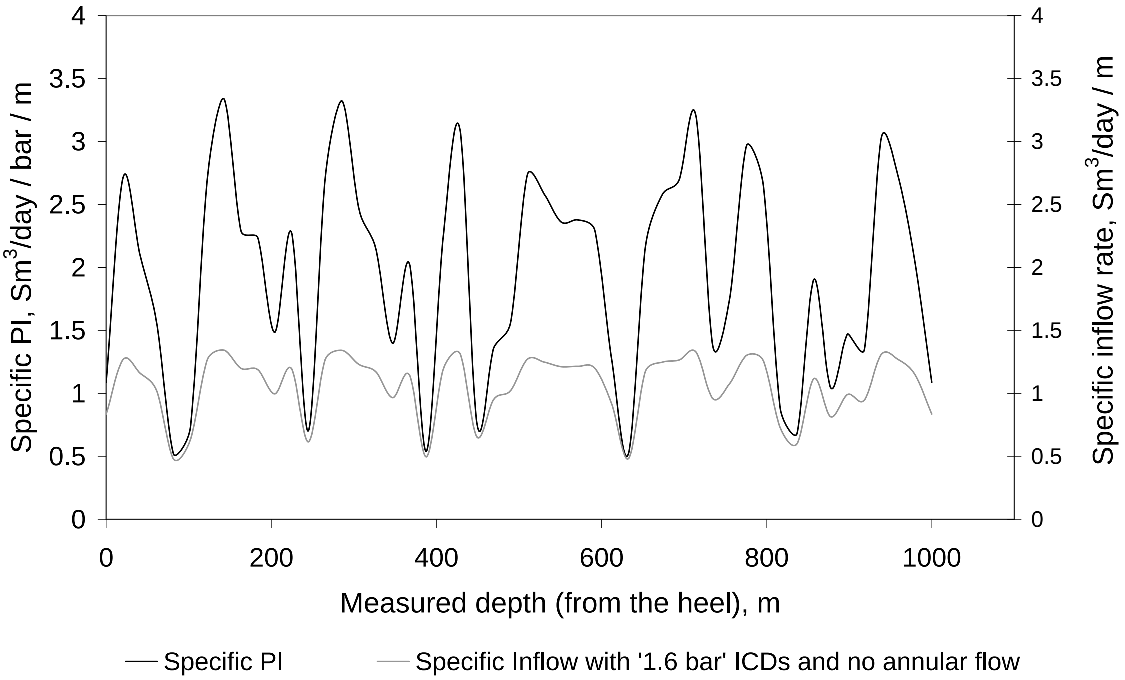

Figure 6.1: An example of inflow equalisation with ICDs

An increase in the ICD strength gives an even more uniform inflow at the cost of further reduction of well inflow performance. This is illustrated in figures 6.2 and 6.3 which were derived using formulae (6.9) and (6.10) for $\Delta P_w = 1$ bar.

6.5.2 Medium Productivity Reservoir

The specific productivity index is the key parameter in ICD completion design. The majority of ICD installations to date are in reservoirs with an average permeability

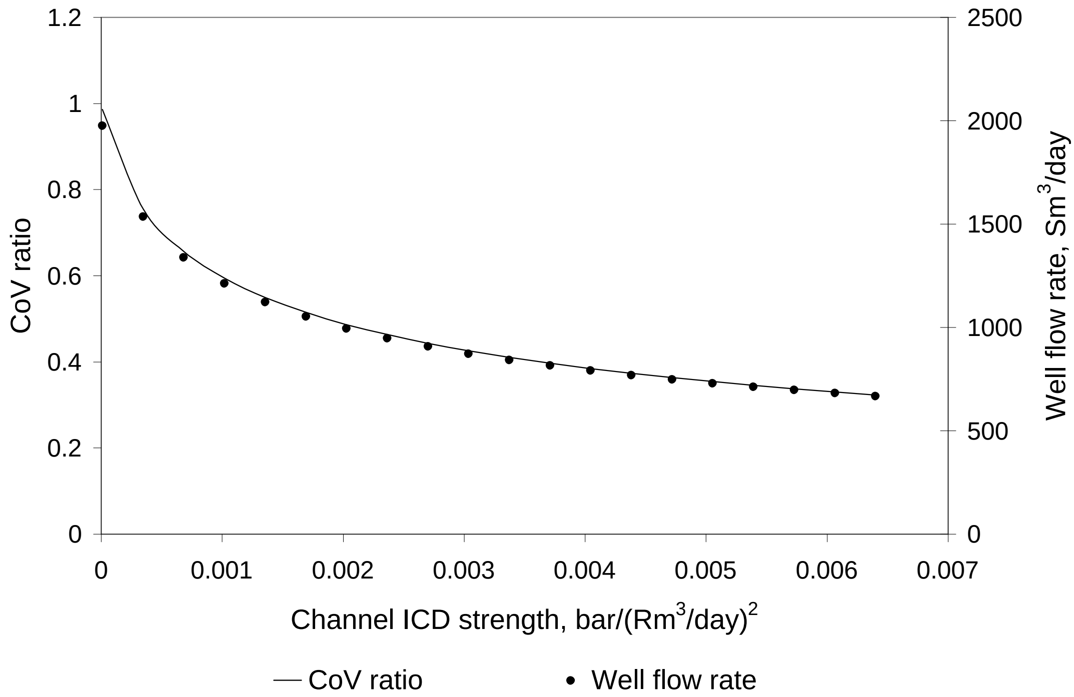

Figure 6.2: Dependence of inflow equalisation and well productivity on ICD strength for *channel* ICDs in a *highly productive* reservoir

of one Darcy or greater (Table 2.7). In order to illustrate the importance of this parameter let us now consider the case with 10 times lower PI (200 Sm^3/day/bar) and 10 times higher total pressure drop (10 bar). Such modifications (Table 6.2) would not change the inflow performance of conventional completion as it is the product of the PI and the pressure drop that determines the inflow rate. However, the performance of an ICD completion will be different since the inflow is no longer proportional to the above mentioned product in this case.

Well's PI	J	200 Sm^3/day/bar
Minimum value of specific PI	j_1	0.05 Sm^3/day/bar/m
Maximum value of specific PI	j_2	0.35 Sm^3/day/bar/m
Total pressure drop at the heel	ΔP_w	10 bar

Table 6.2: *Medium* productivity reservoir case study

According to formulae (6.9) and (6.10), the flow rate of a "1.6 bar" ICD well is 1 720 Sm^3/day and the (CoV U/ CoV j) ratio is 0.85 for the medium productivity reservoir. This implies that the well flow rate and specific inflow rate variation were

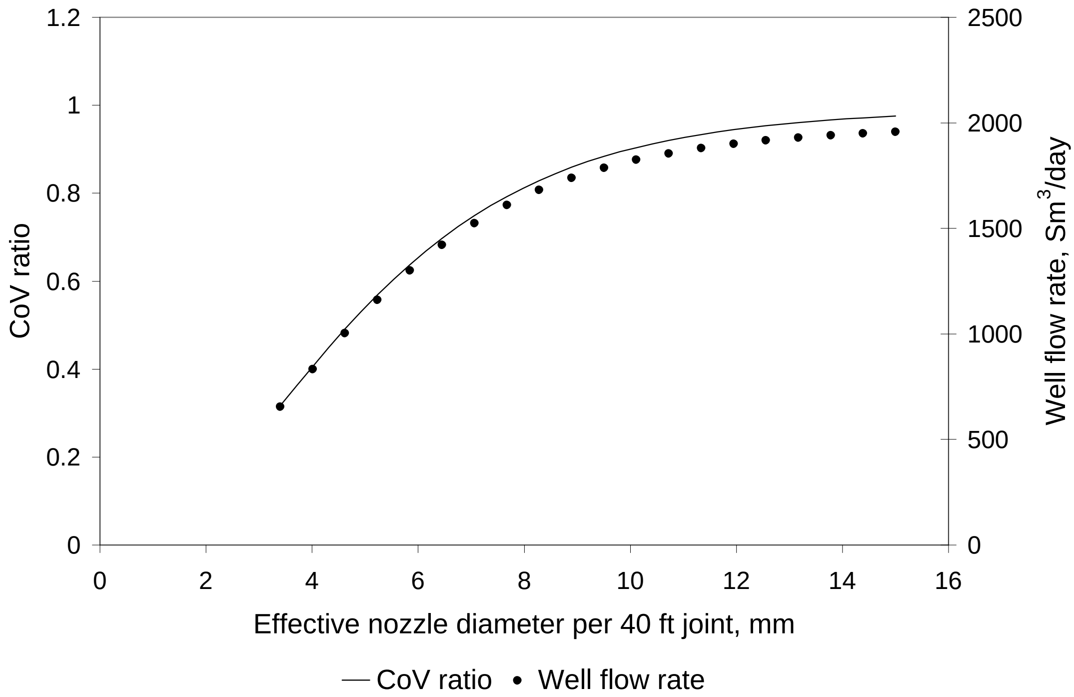

Figure 6.3: Dependence of inflow equalisation and well productivity on ICD strength for *nozzle/orifice* ICDs in a *highly productive* reservoir

reduced by only 15% (in contrast to 50% in the Highly Productive Reservoir case). Figures 6.4 and 6.5 illustrate that the ICD's efficiency of inflow equalisation generally decreases as reservoir permeability decreases. Medium productivity reservoirs require the installation of higher "strength" ICDs than those required for inflow equalisation in prolific reservoirs.

6.6 Discussion

The question of precision of this model ultimately depends on the validity of assumptions made in section 6.2. For instance, the formulae for the $(\mathrm{CoV}\, U / \mathrm{CoV}\, j)$ ratio should be regarded as a lower (optimistic) estimate since they were obtained by neglecting annular flow. Annular flow can technically be completely eliminated by using a large number of isolation packers or a gravel-pack. However, in practice annular flow occurs to a greater or lesser extent in almost all of wells. The implications of annular flow are very case specific; requiring help from a numerical simulator if they need to be studied (e.g. Neylon et al., 2009). This remark especially applies

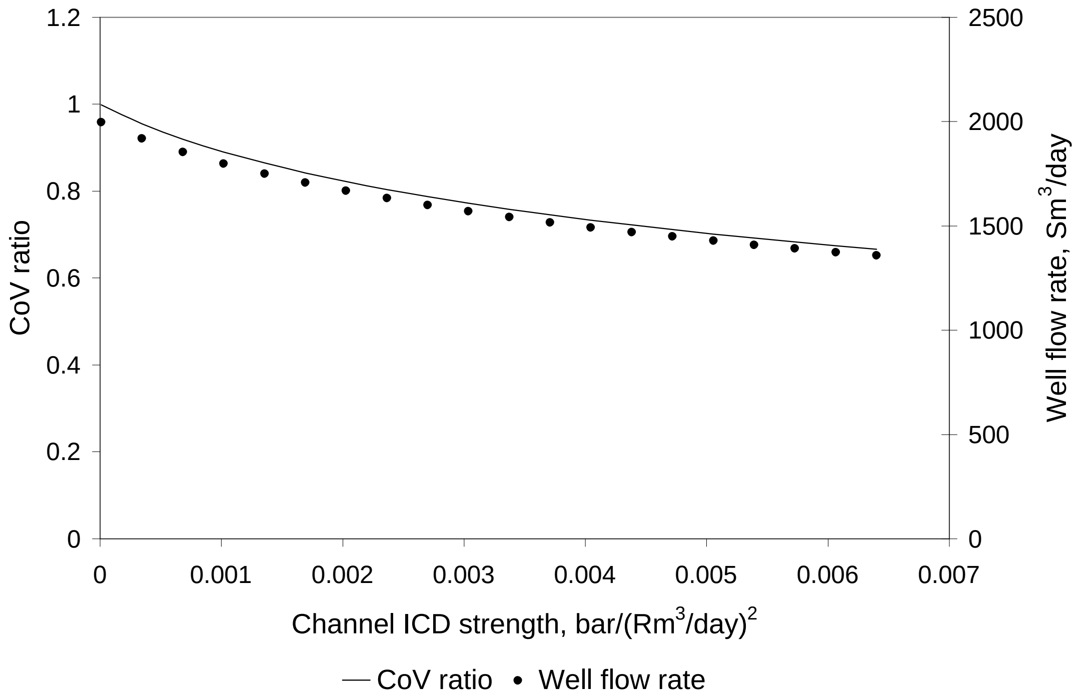

Figure 6.4: Dependence of inflow equalisation and well productivity on ICD strength for *channel* ICDs in a *medium* productivity reservoir

to fractured reservoirs where the characteristic length of reservoir heterogeneity (or width of the fracture) is considerably smaller than the length of an ICD joint. Adequate modelling of such cases was an extremely challenging task for both analytical and numerical methods at the time during which the majority of the work for this thesis was performed. Commercial petroleum engineering software that models annular flow has only recently became more widely available (Wan et al., 2008; Neylon et al., 2009).

The neglect of frictional pressure losses is a valid assumption in most practical cases. The model presented in this chapter is *not* applicable when both reservoir heterogeneity and friction have substantial impact on the inflow distribution. In such cases numerical simulation should be used for proper completion design.

With numerical simulators at hand, some engineers may question the practical utility of the present work. However, it is recognised to be a good practice to employ a number of models of different complexity rather than one complex model when solving a difficult engineering problem (Williams et al., 2004). It should also be borne in mind that the experience with, and more importantly, the availability of

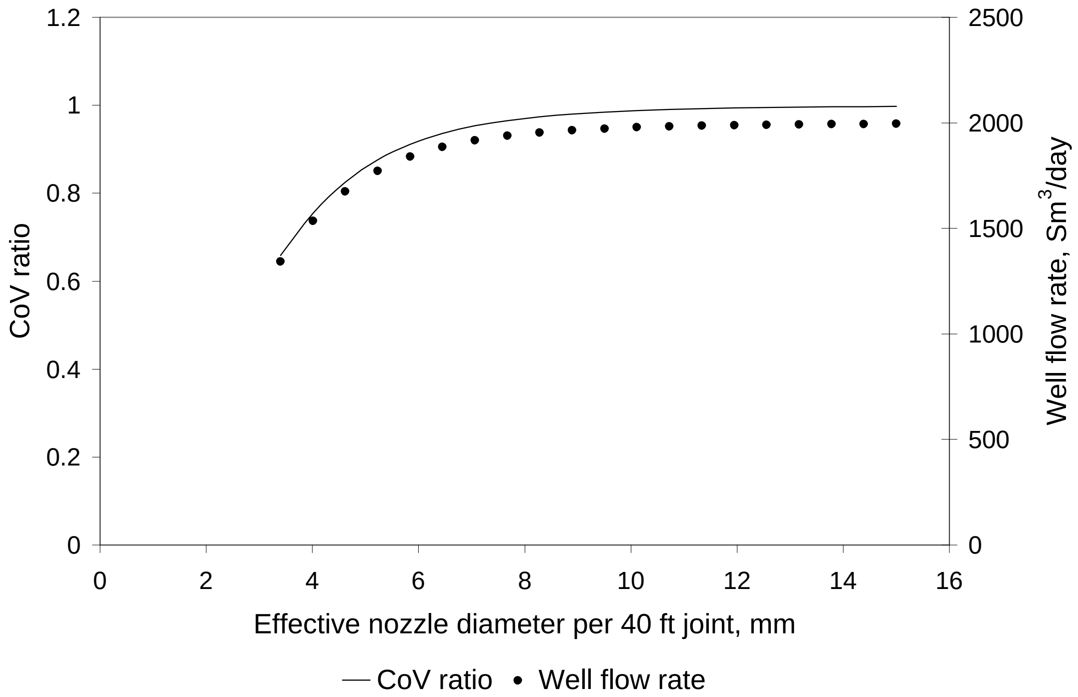

Figure 6.5: Dependence of inflow equalisation and well productivity on ICD strength for *nozzle/orifice* ICDs in a *medium* productivity reservoir

numerical simulation varies from company to company. Relying solely on numerical simulation can be an obstacle in transferring best practices. The proposed analytical model may require more simplifications than today's numerical models, but it is simpler, more transparent and provides insight into underlying physics in a more easily understood form. Analytical and numerical approaches thus complement one another.

6.7 Conclusions

An explicit analytical model for ICD application to heterogeneous reservoirs has been proposed. Eqs. (6.9) and (6.15) allow one to estimate the IPR of an ICD completion in a heterogeneous reservoir while Eqs. (6.10) and (6.16) quantify the ICD's equalisation effect. The model was used in a case study which quantitatively illustrated why medium permeability reservoirs require higher "strength" ICDs to achieve a given degree of inflow equalisation than prolific reservoirs.

Chapter 7

Conclusions and Future Work

7.1 Conclusions

The work presented in this thesis can be summarized as follows:

1. Major aspects dictating the choice between ICVs and ICDs have been reviewed. Figure 2.1 and Table 2.1 provide basis for the selection criteria. The value difference between ICV and ICD has been quantified where appropriate. As usual, full economic quantification of the value associated with each completion remains a field specific task.

2. Well completion design has been shown to reduce the impact of geostatistical uncertainty on the production forecast using the uncertainty quantification methodology as applied in reservoir engineering. The study of the PUNQ-S3 reservoir found that a well completion designed based on:

    - ICDs increased the mean recovery with a limited decrease in of risk.
    - ICVs further increased mean recovery and reduced the risk compared to the base case by 50%.

3. An explicit analytical model for turbulent flow in a highly deviated wellbore has been developed and verified in Chapter 4. The model is consistent with the

semi-analytical models of Seines et al. (1993), Halvorsen (1994) and Penmatcha et al. (1999) as well as with the results of numerical simulations performed in commercial well modelling software.

4. The equation that quantifies the heel-toe effect reduction achieved by Inflow Control Devices installed in horizontal wells has the form of a second order, non-linear ODE. Two solutions have been presented: an approximate analytical and a more precise numerical one. These solutions allow one to estimate the:

   - ICD design parameters that reduce the heel-toe effect to the required level.
   - Impact of the ICD on the well's inflow peformance relationship.

   The practical utility of this approach been illustrated using published Troll oil field data.

5. An explicit analytical model for ICD application to heterogeneous reservoirs has been proposed in Chapter 6. Eqs. (6.9) and (6.15) allow one to estimate the IPR of an ICD completion in a heterogeneous reservoir while Eqs. (6.10) and (6.16) quantify the ICD's equalisation effect. The model was used in a case study which quantitatively illustrated why medium permeability reservoirs require higher "strength" ICDs to achieve a given degree of inflow equalisation than prolific reservoirs.

6. The trade-off between well productivity and inflow equalisation is a key engineering issue when applying ICD technology. Solutions presented in Chapters 5 and 6 give general guidance for choosing an ICD design as well as provide specific quantitative recommendations.

7.2 Future Work

Some possible extensions for this work are:

Chapter 3.

1. Further investigate the dependence of results on the choice of the Base Case (more aggressive production strategy, gas reinjection etc.).
2. Investigate the impact of advanced well completion on other types of reservoir uncertainty (e.g. relative permeability curves, fluid contacts, aquifer etc.)
3. Employ a larger number of representative reservoir realisations, which could involve adaptive selection of the geomodel parameters to match the history data and assess their uncertainty (see, e.g., Demyanov et al., 2004).
4. Model the annular flow in ICD completion.

Chapter 5. Develop guidelines on acceptable magnitudes of the heel-toe effect.

Chapter 6.

1. Implement numerical solution of integrals (6.6) and (6.7) for arbitrary form of p.d.f. of specific productivity index.
2. Develop guidelines on acceptable values of inflow's coefficient of variation.

Bibliography

Abramowitz, M., Stegun, I., 1965. Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Table. Courier Dover Publications. 68

Akram, N., Hicking, S., Blythe, P., Kavanagh, P., Reijnen, P., Mathieson, D., 2001. Intelligent well technology in mature assets. In: Offshore Europe.
URL `http://dx.doi.org/10.2118/71822-MS` 26

Al-Kasim, F. T., Tevik, S., Jakobsen, K. A., Tang, Y., Jalali, Y., 2002. Remotely controlled in-situ gas lift on the norne subsea field. In: SPE Annual Technical Conference and Exhibition.
URL `http://dx.doi.org/10.2118/77660-MS` 26

Al-Khelaiwi, F., 2007. Private communication. 45

Al-Khelaiwi, F., Davies, D., 2007. Inflow Control Devices: Application and value quantification of a developing technology. In: SPE International Oil Conference and Exhibition in Mexico.
URL `http://dx.doi.org/10.2118/108700-MS` 2

Al-Khelaiwi, F. T., Muradov, K. M., Davies, D. R., Olowoleru, D. K., 2009. Advanced well flow control technologies can improve well clean-up. In: 8th European Formation Damage Conference.
URL `http://dx.doi.org/10.2118/122267-MS` 30

Augustine, J., Mathis, S., Nguyen, H., Gann, C., Gill, J., 2008. World's first gravel-packed inflow-control completion. SPE Drilling & Completion 23 (1), 61–67.
URL `http://dx.doi.org/10.2118/103195-PA` 13, 81

Augustine, J. R., 2002. An investigation of the economic benefit of Inflow Control Devices on horizontal well completions using a reservoir-wellbore coupled model. In: European Petroleum Conference.
URL http://dx.doi.org/10.2118/78293-MS 31

Babu, D. K., Odeh, A. S., November 1989. Productivity of a horizontal well. SPE Reservoir Engineering 4 (4), 417–421.
URL http://dx.doi.org/10.2118/18298-PA 58

Baker, L., 1992. C Mathematical Function Handbook. McGraw-Hill, Inc. New York, NY, USA. 68

Bellarby, J. E., Denholm, A., Grose, T., Norris, M., Stewart, A., 2003. Design and implementation of a high rate acid stimulation through a subsea intelligent completion. In: Offshore Europe.
URL http://dx.doi.org/10.2118/83950-MS 31

Birchenko, V., Bejan, A., Usnich, A., Davies, D., November 2009. Application of Inflow Control Devices to heterogeneous reservoirs, preprint PETROL2802 submitted to Journal of Petroleum Science and Engineering. 99

Brouwer, D., Jansen, J.-D., 2004. Dynamic optimization of waterflooding with smart wells using optimal control theory. SPE Journal 9 (4), 391–402.
URL http://dx.doi.org/10.2118/78278-PA 5

Carter, J., 2006. PUNQ-S3 Truth Case. Imperial College London.
URL http://www3.imperial.ac.uk/earthscienceandengineering/research/perm/punq-s3model/truthcase/ 41

Clemo, T., 2006. Flow in perforated pipes: A comparison of models and experiments. SPE Production & Operations 21 (2), 302–311.
URL http://dx.doi.org/10.2118/89036-PA 125

Coquereaux, R., Grossmann, A., Lautrup, B. E., 1990. Iterative method for calculation of the weierstrass elliptic function. IMA J Numer Anal 10 (1), 119–128. 68

de Best, L., van den Berg, F., 2006. Smart fieldsmaking the most of our assets. In: SPE Russian Oil and Gas Technical Conference and Exhibition.
URL http://dx.doi.org/10.2118/103575-MS xii, 28, 29

de Montleau, P., Cominelli, A., Neylon, K., Rowan, D., Pallister, I., Tesaker, O., Nygard, I., September 2006. Production optimization under constraints using adjoint gradients. In: 10th European Conference on the Mathematics of Oil Recovery. 11

Demyanov, V., Sabbey, S., Christie, M., 2004. Neighbourhood algorithm with geostatistical simulations for uncertainty quantification reservoir modelling: PUNQ-S3 case study. In: 9th European Conference on Mathematics in Oil Recovery. 112

Dikken, B. J., November 1990. Pressure drop in horizontal wells and its effect on production performance. Journal of Petroleum Technology 42 (11), 1426–1433.
URL http://dx.doi.org/10.2118/19824-PA 6, 52

Drakeley, B. K., Douglas, N. I., Haugen, K. E., Willmann, E., 2001. Application of reliability analysis techniques to intelligent wells. In: Offshore Technology Conference.
URL http://dx.doi.org/10.4043/13028-MS 28

Eckhardt, U., 1980. Algorithm 549: Weierstrass' elliptic functions. ACM Trans. Math. Softw. 6 (1), 112–120. 68

Floris, F., Bush, M., Cuypers, M., Roggero, F., Syversveen, A.-R., May 2001. Methods for quantifying the uncertainty of production forecasts: a comparative study. Petroleum Geoscience 7, 87–96. 10, 41

Freyer, R., Huse, A., 2002. Swelling packer for zonal isolation in open hole screen completions. In: European Petroleum Conference.
URL http://dx.doi.org/10.2118/78312-MS 13

Gai, H., 2002. A method to assess the value of intelligent wells. In: SPE Asia Pacific Oil and Gas Conference and Exhibition.
URL http://dx.doi.org/10.2118/77941-MS 39

Gao, C., Rajeswaran, T., Nakagawa, E., 2007. A literature review on smart well technology. In: SPE Production and Operations Symposium.
URL http://dx.doi.org/10.2118/106011-MS 2, 8, 21

Goode, P., Kuchuk, F., August 1991. Inflow performance of horizontal wells. SPE Reservoir Engineering 6 (3), 319–323.
URL http://dx.doi.org/10.2118/21460-PA 58

Goode, P., Wilkinson, D., August 1991. Inflow performance of partially open horizontal wells. Journal of Petroleum Technology 43 (8), 983–987.
URL http://dx.doi.org/10.2118/19341-PA 58

Haaland, A., Rundgren, G., Johannessen, ., 2005. Completion technology on troll-innovation and simplicity. In: Offshore Technology Conference.
URL http://dx.doi.org/10.4043/17113-MS 4

Haaland, S., 1983. Simple and explicit formulas for the friction factor in turbulent pipe flow. Journal of Fluids Engineering 105 (1), 89–90. 52, 127

Halliburton, 2009. Netool™ near wellbore and completion hydraulics simulation software. 16, 88

Halvorsen, G., July 1994. Discussion of considering wellbore friction effects in planning horizontal wells. Journal of Petroleum Technology 46 (6), 620. 53, 73, 76, 111, 133

Han, J. T., 2003. There is value in operational flexibility: An intelligent well application. In: SPE Hydrocarbon Economics and Evaluation Symposium.
URL http://dx.doi.org/10.2118/82018-MS 38

Haug, B., 1992. The second long-term horizontal well test in troll: Successful production from a 13-in. oil column with the well partly completed in the water zone. In: SPE Annual Technical Conference and Exhibition.
URL http://dx.doi.org/10.2118/24943-MS 17

Haugen, V., Fagerbakke, A.-K., Samsonsen, B., Krogh, P., 2006. Subsea smart multilateral wells increase reserves at Gullfaks South Statfjord. In: SPE/DOE Symposium on Improved Oil Recovery.
URL `http://dx.doi.org/10.2118/95721-MS` 25

Helmy, M., Veselka, A., Benish, T., Yeh, C., Asmann, M., Yeager, D., Martin, B., Barry, M., 2006. Application of new technology in the completion of ERD wells, Sakhalin-1 development. In: SPE Russian Oil and Gas Technical Conference and Exhibition.
URL `http://dx.doi.org/10.2118/103587-MS` 82

Hembling, D., Sunbul, A., Salerno, G., 2007. Advanced well completions result in enhanced well productivity and recovery in Saudi Aramco's offshore fields. In: Asia Pacific Oil and Gas Conference and Exhibition.
URL `http://dx.doi.org/10.2118/108877-MS` 13

Henriksen, K., Gule, E., Augustine, J., 2006. Case Study: The Application of Inflow Control Devices in the Troll Oil Field. In: SPE Europec/EAGE Annual Conference and Exhibition.
URL `http://dx.doi.org/10.2118/100308-MS` 32, 82, 83, 90, 91

Hill, A., Zhu, D., May 2008. The Relative Importance of Wellbore Pressure Drop and Formation Damage in Horizontal Wells. SPE Production & Operations 23 (2), 232–240.
URL `http://dx.doi.org/10.2118/100207-PA` 54, 132

Jain, A., 1976. Accurate explicit equation for friction factor. Journal of the Hydraulics Division 102 (5), 674–677. 53

Johansen, T. E., Khoriakov, V., 2007. Iterative techniques in modeling of multiphase flow in advanced wells and the near well region. Journal of Petroleum Science and Engineering 58 (1-2), 49 – 67.
URL `http://dx.doi.org/10.1016/j.petrol.2006.11.013` 6, 74, 79, 88, 91

Joshi, S. D., 1991. Horizontal Well Technology. PennWell Books. 52, 54

Katamish, H., Steel, N., Smith, A., Thompson, P., 2005. WDDM: The jewel of the Nile. In: SPE Europec/EAGE Annual Conference.
URL http://dx.doi.org/10.2118/94123-MS 32

Kavle, V., Elmsallati, S., Mackay, E., Davies, D., 2006. Impact of intelligent wells on oilfield scale management. In: SPE Europec/EAGE Annual Conference and Exhibition.
URL http://dx.doi.org/10.2118/100112-MS 31

Konopczynski, M., 2008. Private communication. 12

Kulkarni, R., Belsvik, Y., Reme, A., 2007. Smart-well monitoring and control: Snorre B experience. In: SPE Annual Technical Conference and Exhibition.
URL http://dx.doi.org/10.2118/109629-MS 28

Landman, M. J., 1994. Analytic modelling of selectively perforated horizontal wells. Journal of Petroleum Science and Engineering 10 (3), 179 – 188.
URL http://dx.doi.org/10.1016/0920-4105(94)90079-5 53

Lau, H. C., Deutman, R., Al-Sikaiti, S., Adimora, V., 2001. Intelligent internal gas injection wells revitalise mature S.W. Ampa field. In: SPE Asia Pacific Improved Oil Recovery Conference.
URL http://dx.doi.org/10.2118/72108-MS 26

Leslie, K. J., Michaels, M. P., No. 3, 1997. The real power of Real Options. The McKinsey Quarterly. 39

Manceau, E., Mezghani, M., Zabalza-Mezghani, I., Roggero, F., 2001. Combination of experimental design and joint modeling methods for quantifying the risk associated with deterministic and stochastic uncertainties - an integrated test study. In: SPE Annual Technical Conference and Exhibition.
URL http://dx.doi.org/10.2118/71620-MS 35

Maplesoft, 2009. Maple™ mathematical software. 68

Mathieson, D., Rogers, J., Rajagopalan, S., McManus, R., 2003. Reliability assurance, managing the growth of intelligent completion technology. In: SPE Annual Technical Conference and Exhibition.
URL http://dx.doi.org/10.2118/84327-MS 28

McIntyre, A., Adam, R., Augustine, J., Laidlaw, D., 2006. Increasing oil recovery by preventing early water and gas breakthrough in a West Brae horizontal well: A case history. In: SPE/DOE Symposium on Improved Oil Recovery.
URL http://dx.doi.org/10.2118/99718-MS 82

Meum, P., Tndel, P., Godhavn, J.-M., Aamo, O. M., 2008. Optimization of smart well production through nonlinear model predictive control. In: Intelligent Energy Conference and Exhibition.
URL http://dx.doi.org/10.2118/112100-MS 12

Mjaavatten, A., Aasheim, R., Saelid, S., Gronning, O., 2008. A model for gas coning and rate-dependent gas/oil ratio in an oil-rim reservoir. SPE Reservoir Evaluation & Engineering 11 (5), 842–847.
URL http://dx.doi.org/10.2118/102390-PA 4, 79

Moen, T., Asheim, H., 2008. Inflow Control Device and near-wellbore interaction. In: SPE International Symposium and Exhibition on Formation Damage Control.
URL http://dx.doi.org/10.2118/112471-MS 30

Moody, L., 1944. Friction factors for pipe flow. Trans. ASME 66 (8), 671–677. 18

Neylon, K., Reiso, E., Holmes, J., Nesse, O., 2009. Modeling well inflow control with flow in both annulus and tubing. In: SPE Reservoir Simulation Symposium.
URL http://dx.doi.org/10.2118/118909-MS 6, 79, 107, 108

Nielsen, V. B. J., Piedras, J., Stimatz, G. P., Webb, T. R., 2002. Aconcagua, Camden Hills, and King's Peak fields, Gulf of Mexico, employ intelligent completion technology in unique field-development scenario. SPE Production & Facilities 17 (4), 236–240.
URL http://dx.doi.org/10.2118/80292-PA 4, 26

Novy, R., 1995. Pressure drops in horizontal wells: When can they be ignored. SPE Reservoir Engineering Journal 10 (1), pp. 29–35.
URL http://dx.doi.org/10.2118/24941-PA 53

Ogoke, V., Aihevba, C., Marketz, F., 2006. Cost-effective life-cycle profile control completion system for horizontal and multilateral wells. In: SPE Annual Technical Conference and Exhibition.
URL http://dx.doi.org/10.2118/102077-MS 13

Olowoleru, D., Muradov, K., Al-Khelaiwi, F., Davies, D., 2009. Efficient intelligent well cleanup using downhole monitoring. In: 8th European Formation Damage Conference.
URL http://dx.doi.org/10.2118/122231-MS 30

Ouyang, L.-B., 2007. Uncertainty assessment on well performance prediction for an oil well equipped with selected completions. In: Production and Operations Symposium.
URL http://dx.doi.org/10.2118/106966-MS 38

Ouyang, L.-B., Aziz, K., 2000. A homogeneous model for gas-liquid flow in horizontal wells. Journal of Petroleum Science and Engineering 27 (3-4), 119 – 128.
URL http://dx.doi.org/10.1016/S0920-4105(00)00053-X 125

Ouyang, L.-B., Huang, B., 2005. An evaluation of well completion impacts on the performance of horizontal and multilateral wells. In: SPE Annual Technical Conference and Exhibition.
URL http://dx.doi.org/10.2118/96530-MS 6, 79

Ozkan, E., Haciislamoglu, C. S. M., Raghavan, R., 1995. Effect of conductivity on horizontal well pressure behavior. SPE Advanced Technology Series 3 (1), 85–94.
URL http://dx.doi.org/10.2118/24683-PA 53

Penmatcha, V., Arbabi, S., Aziz, K., 1999. Effects of pressure drop in horizontal wells and optimum well length. SPE Journal 4 (3), 215–223.
URL http://dx.doi.org/10.2118/57193-PA x, 54, 73, 74, 76, 111

Qudaihy, D. A., Qahtani, H. A., Sunbul, A., Hembling, D., Salerno, G., 2006. The evolution of advanced well completions results in enhanced well productivity and recovery in Saudi Aramco's offshore fields. In: IADC/SPE Asia Pacific Drilling Technology Conference and Exhibition.
URL `http://dx.doi.org/10.2118/103621-MS` 26

Raffn, A., Hundsnes, S., Kvernstuen, S., Moen, T., 2007. ICD screen technology used to optimize waterflooding in injector well. In: SPE Production and Operations Symposium.
URL `http://dx.doi.org/10.2118/106018-MS` 5

Saggaf, M., March 2008. A vision for future upstream technologies. Journal of Petroleum Technology 60 (3), 54–55, 94–98. 12

Salamy, S., Al-Mubarak, H., Hembling, D., Al-Ghamdi, M., 2006. Deployed smart technologies enablers for improving well performance in tight reservoirscase: Shaybah field, Saudi Arabia. In: Intelligent Energy Conference and Exhibition.
URL `http://dx.doi.org/10.2118/99281-MS` 21

Salamy, S. P., 2005. Maximum reservoir contact (MRC) wells: A new generation of wells for developing tight reservoir facies. In: SPE Distinguish Lecturer Series.
URL `http://dx.doi.org/10.2118/108806-DL` 1

Schlumberger, October 2002. TRFC-HN AP and TRFC-HN LP hydraulic flow control valves datasheet.
URL `http://www.slb.com/media/services/completion/intelligent/trfc_hn_ap.pdf` 22

Schlumberger, 2009. Eclipse user manual. 83

Schulze-Riegert, R., Axmann, J., Haase, O., Rian, D., You, Y.-L., 2001. Optimization methods for history matching of complex reservoirs. In: SPE Reservoir Simulation Symposium.
URL `http://dx.doi.org/10.2118/66393-MS` 35

Seines, K., Aavatsmark, I., Lien, S., Rushworth, P., October 1993. Considering wellbore friction in planning horizontal wells. Journal of Petroleum Technology 45 (10), 994–1000.
URL `http://dx.doi.org/10.2118/21124-PA` xviii, 52, 53, 54, 71, 72, 76, 111, 133

Shahri, A. M., Kilany, K., Hembling, D., Lauritzen, J. E., Gottumukkala, V., Ogunyemi, O., Moreno, O. B., 2009. Best cleanup practices for an offshore sandstone reservoir with ICD completions in horizontal wells. In: SPE Middle East Oil and Gas Show and Conference.
URL `http://dx.doi.org/10.2118/120651-MS` 30

Sharma, A., Chorn, L., Han, J., Rajagopalan, S., 2002. Quantifying value creation from intelligent completion technology implementation. In: European Petroleum Conference.
URL `http://dx.doi.org/10.2118/78277-MS` 38

Silva, M., Portella, R., Izetti, R., Campos, S., 2005. Technologies trials of intelligent-field implementation in Carmopolis field. In: SPE Annual Technical Conference and Exhibition.
URL `http://dx.doi.org/10.2118/95517-MS` xii, 28, 31

Stair, C., Bruesewitz, E., Shivers, J., Rajasingam, D., Dawson, M., 2004. Na kika completions overview: Challenges and accomplishments. In: Offshore Technology Conference.
URL `http://dx.doi.org/10.4043/16228-MS` 29

Su, Z., Gudmundsson, J. S., 1998. Perforation inflow reduces frictional pressure loss in horizontal wellbores. Journal of Petroleum Science and Engineering 19 (3-4), 223 – 232.
URL `http://dx.doi.org/10.1016/S0920-4105(97)00047-8` 125

Sunbul, A., Hembling, D., Qudaihy, D. A., Harbi, N. A., Salerno, G., 2007. The evolution of advanced well completions to enhance well productivity and recovery

in Saudi Aramco's offshore fields. In: SPE Middle East Oil and Gas Show and Conference.
URL http://dx.doi.org/10.2118/105036-MS 26

Sunbul, A., Lauritzen, J., Hembling, D., Majdpour, A., Raffn, A., Zeybek, M., Moen, T., 2008. Case histories of improved horizontal well cleanup and sweep efficiency with nozzle based Inflow Control Devices (ICD) in sandstone and carbonate reservoirs. In: SPE Saudi Arabia Section Technical Symposium. Paper SPE 120795. 30

Usnich, A., 2008. Private communication. 63

van den Berg, F. G., 2007. Smart fieldsoptimizing existing fields. In: Digital Energy Conference and Exhibition.
URL http://dx.doi.org/10.2118/108206-MS 12

Wan, J., Dale, B. A., Ellison, T. K., Benish, T. G., Grubert, M. A., 2008. Coupled well and reservoir simulation models to optimize completion design and operations for subsurface control. In: Europec/EAGE Conference and Exhibition.
URL http://dx.doi.org/10.2118/113635-MS 6, 45, 79, 108

Weatherford, 2008. Wellflo^{TM} petroleum engineering software.
URL http://www.ep-solutions.com/PDF/Literature/5943_WellFlo_Software.pdf 73, 74

Wehunt, C., September 2006. Well performance with operating limits under reservoir and completion uncertainties. SPE Drilling & Completion 21 (3), 200–211.
URL http://dx.doi.org/10.2118/84501-PA 38

Williams, G., Mansfield, M., MacDonald, D., Bush, M., 2004. Top-down reservoir modelling. In: SPE Annual Technical Conference and Exhibition.
URL http://dx.doi.org/10.2118/89974-MS 35, 108

Wolfram, 2009. Mathematica^{TM} mathematical software. 68

Zandvliet, M., Bosgra, O., Jansen, J., den Hof, P. V., Kraaijevanger, J., 2007. Bang-bang control and singular arcs in reservoir flooding. Journal of Petroleum Science and Engineering 58 (1-2), 186 – 200.
URL http://dx.doi.org/10.1016/j.petrol.2006.12.008 22

Appendix A

Friction Factor Calculation

Laboratory experiments suggest that inflow or outflow through perforations does not change the friction factor radically (see, e.g., Su and Gudmundsson, 1998; Ouyang and Aziz, 2000). However the debate over the pressure drop correlation for perforated pipes remains open (Clemo, 2006). Current versions of commercial petroleum software use the same correlations for perforated and blank (non-perforated) pipes. We used a non-perforated pipe correlation since the broad question of pressure drop correlation choice is beyond the scope of this work.

The model presented in the section 4.5 can be used with any correlation as long as the assumption of constant friction factor remains valid. The average friction factor and the error associated with the averaging can be also estimated with various techniques. The approach outlined below is an example of how such estimations can be done in principle. It is not an intrinsic part of the model and can be easily changed if appropriate for a specific case.

The flow regime is determined by the Reynolds number. The Reynolds number at the heel of a rate constrained well is:

$$Re_{hq} = \frac{C_r \rho B q_w}{\mu D} \tag{A.1}$$

Precise calculation of Reynolds number for a pressure constrained well is, generally speaking, an iterative process because the flow rate at the heel is unknown. A first approximation of the Reynolds number can be readily obtained by neglecting the frictional pressure losses:

$$Re_{hp_0} = \frac{C_r \rho B J_s L \Delta P_w}{\mu D} \tag{A.2}$$

Formula (4.55) can then be used to calculate the first approximation of flow rate. Repetition of this process gives further, more precise, estimates. Black arrows in Figure A.1 show the data that should be used repeatedly in such iterative process. This procedure was found to converge extremely rapidly. The precision obtained by the first approximation is usually sufficient for a "quick-look" deterministic analysis since the error it introduces is typically much smaller than that due to the uncertainties associated with the:

- Friction factor correlation.
- Input parameters.
- Validity of the assumptions.

Two iterations may be required if this model is to be used in a more sophisticated type of analysis (e.g. for uncertainty quantification or optimisation).

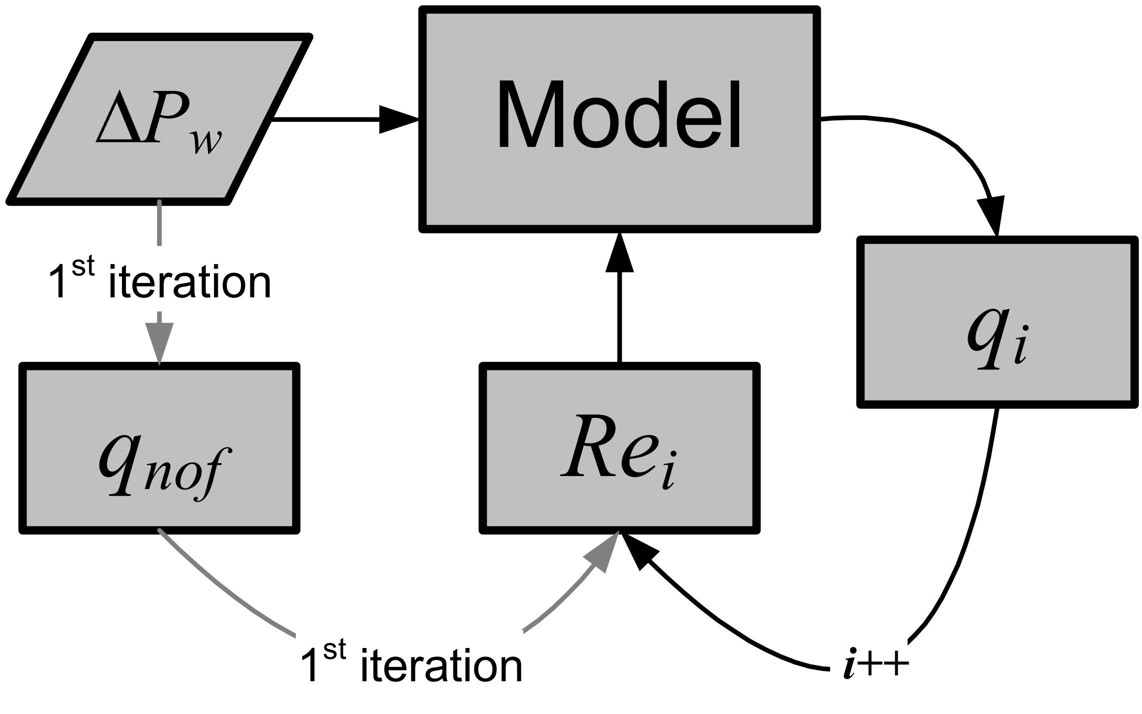

Figure A.1: Reynolds number calculation for a pressure constrained well

The above enables calculation of the Reynolds number at the heel of the well, Re_h. We will now discuss how this number is used to calculate the average friction factor.

Flow at the toe of the well is laminar and the required friction factor for circular pipes is:

$$f = 16/Re, \quad Re < Re_c \tag{A.3}$$

Using Eq. (4.6) we define the average friction factor for the toe part of the completion where laminar flow is present:

$$f_l \equiv \frac{\left\langle \frac{dP}{dx} \right\rangle}{2\rho \left\langle v^2 \right\rangle} D \tag{A.4}$$

One can show that it is equal to:

$$f_l = 24/Re_c \approx 0.01 \tag{A.5}$$

and the normalised mean-square error of the pressure gradient associated with this averaging process is $2/\sqrt{5}$.

The critical Reynolds number Re_c (the number at which the transition between laminar and turbulent flow starts to take place) depends on flow configuration and, strictly speaking, must be determined experimentally. The transition normally occurs over a range of Reynolds numbers between 2300 and 3600. In this work we will assume that Re_c has a value of 2300 and that the transition to fully developed turbulent flow happens rapidly at Re_c (Figure 4.1).

The Fanning friction factor for fully developed turbulent flow in rough pipes can be calculated using following formula (Haaland, 1983):

$$f = \left[3.6 \log_{10} \left(6.9/Re + (e/D)^{10/9} \right) \right]^{-2} \tag{A.6}$$

Figure A.2 illustrates the averaging procedure we used to estimate the overall friction factor:

1. The average friction factor for the section of the wellbore with laminar flow (black dotted line) is estimated using formula (A.5).

2. The friction factor for the remaining completion length where turbulent flow is present is assumed to be constant (grey dotted line) and equal to that at the heel, f_h, which calculated using Re_h.

3. The overall average is calculated as weighted sum of f_l and f_h:

$$f_a = f_h + Re_c(f_l - f_h)/Re_h \tag{A.7}$$

The normalised error associated with such averaging can be estimated as

$$\langle|f - f_a|\rangle/f_a = 2\,(1 - Re_c/Re_h)\,|f_l - f_h|Re_c/Re_h/f_a \tag{A.8}$$

For example, Eq. (A.7) gives $f_a = 0.0145$ for $f_h = 0.015$, $Re_h = 20\,000$ and, according to Eq. (A.8), the associated averaging error is 6%.

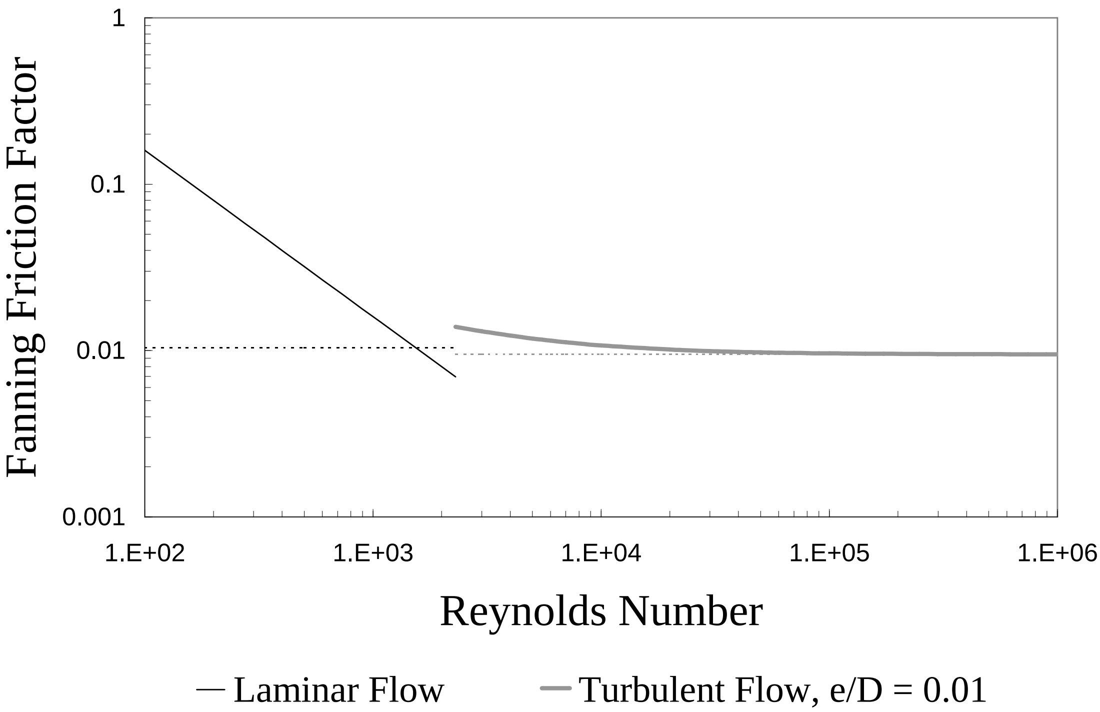

Figure A.2: Averaging the friction factor for rough pipes

Appendix B

Pressure Drop due to Acceleration

Fluid that enters wellbore with a velocity directed nearly perpendicular to wellbore has to be accelerated to acquire a velocity directed towards the heel. This causes some pressure loss. Let us estimate the resulting pressure loss and compare it to the frictional in order to determine whether the acceleration component of the pressure drop can be neglected. Other assumptions (subsection 4.3.1) remain valid. Also bear in mind that all formulae in this appendix are in SI units, however the final inequality (4.3) is also valid in metric and field units.

The pressure gradient due to acceleration according to energy conservation law is:

$$\frac{dP_a}{dl} = -\frac{d}{dl}\left(\frac{\rho v^2}{2}\right) \tag{B.1}$$

The distribution of flow along the completion $v(l)$ has to be known in order to evaluate acceleration pressure drop. The fluid velocity at the toe of the well can be assumed to be zero and flow rate at the heel is v_h. Neither acceleration nor friction normally have a significant effect on the flow rate along the completion's length. In fact, these effects are often neglected in conventional wells producing with flow regimes other than mist flow. Hence, for an order-of-magnitude estimate, one can reasonably assume that flow rate increases linearly with l from zero at the toe to v_h at the heel:

$$v(l) \approx v_h l/L \tag{B.2}$$

Substitution of formula (B.2) into (B.1) gives

$$\frac{dP_a}{dl} \approx -\frac{\rho v v_h}{L} \tag{B.3}$$

Dividing the Darcy-Weisbach equation (4.6) by (B.3) gives the ratio of the frictional to the acceleration pressure losses:

$$\frac{dP_f}{dP_a} \approx \frac{2fLv}{v_h D} \tag{B.4}$$

The average value of fluid velocity, $\langle v \rangle$, is $v_h/2$ leading to the average ratio of pressure drops due to friction and acceleration becoming:

$$\left\langle \frac{dP_f}{dP_a} \right\rangle \approx f_a L/D \tag{B.5}$$

The acceleration pressure loss can thus be neglected when $f_a L/D \gg 1$.

Appendix C

The Upper Estimate of Frictional Pressure Drop

Integration of Eq. (4.7) gives frictional pressure loss between the toe and the heel:

$$P_t - P_w = \frac{C_f \rho f B^2}{D^5} \int_0^L q^2(l)\, dl \tag{C.1}$$

The upper estimate of the frictional pressure loss can thus be obtained from the upper estimate of integral $\int_0^L q^2(l)\, dl$. Let us examine the function $q(l)$ describing the flow rate distribution in the completion. It has the following properties:

1. $q(0) = 0$ (no flow at the toe);
2. $q(L) = q_w$ (the flow rate at the heel is known);
3. for any $l_1, l_2 \in [0, L]$ $q'(l_2) > q'(l_1)$ if $l_2 > l_1$ (the function is concave up).

Hence (Figure C.1):

$$q^2(l) \leq (q_w l/L)^2 \quad \text{for any } l \in [0, L] \tag{C.2}$$

Integration of the inequality (C.2) gives:

$$\int_0^L q^2(l)\, dl \leq q_w^2 L/3 \tag{C.3}$$

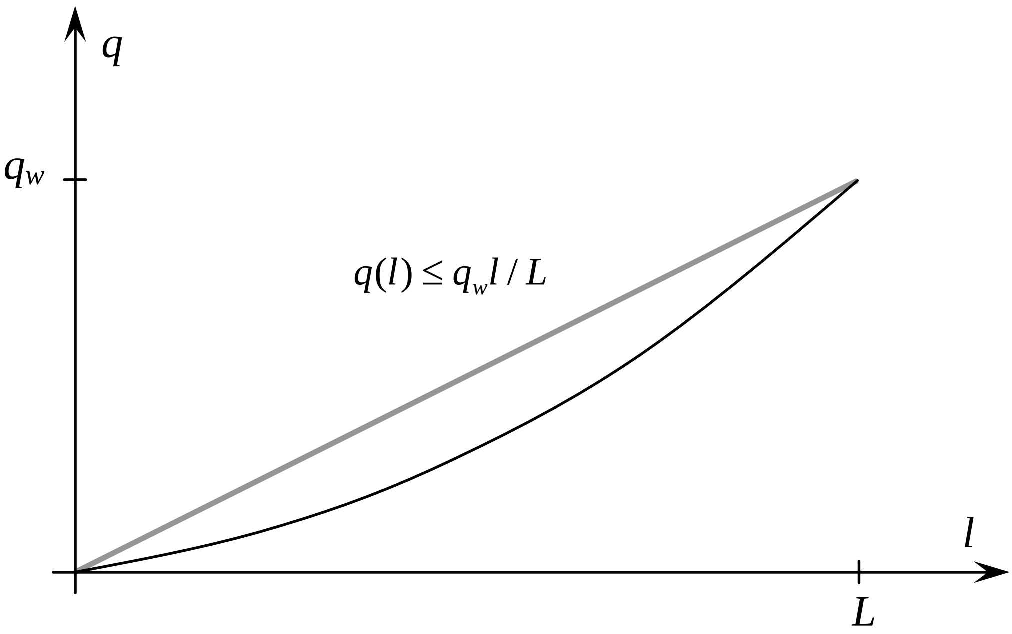

Figure C.1: Upper estimate of flow distribution along the completion interval

Finally, substituting inequality (C.3) into (C.1) indicates the upper estimate of the frictional pressure loss along the completion to be:

$$P_t - P_w \leq \frac{C_f \rho f B^2 L q_w^2}{3D^5} \tag{C.4}$$

The estimate of Hill and Zhu (2008) is equivalent to:

$$P_t - P_w \approx \frac{C_f \rho f B^2 L q_w^2}{4D^5} \tag{C.5}$$

Formula (C.5) is not an upper estimate because, if frictional pressure drop along the completion is much smaller than the drawdown, then $q(l) \rightarrow q_w l/L$ and the left hand side of the inequality (C.4) approaches the right hand side. Neither is it a lower estimate because this can not be formulated without reference to the reservoir properties.

Appendix D

Comparison to Halvorsen's Solution

According to Eqs. (A-23) and (A-24) of Seines et al. (1993):

$$q_{inf} = J_s l^* \Delta P_w / 2 \tag{D.1}$$

Substitution of formula (4.66) into (4.60) gives

$$h_p^* = 12 \tag{D.2}$$

while substitution of formula (D.2) into (4.63) gives:

$$q_w^* = 0.4805\, J_s l^* \Delta P_w \tag{D.3}$$

$$q_w^* / q_{inf} = 0.961 \tag{D.4}$$

The precision can be further improved by using look-up tables or other more sophisticated interpolation techniques instead of formula (4.61). For instance, using the look-up table provided as supplementary data for this work, one can obtain even more significant digits than provided by Halvorsen (1994): $C_p^* = 0.11472694553885$ and then

$q_w^*/q_{inf} = 0.96019420783774$.

Formula (4.56) gives the drawdown ratio of 0.34 for such a well (subsection 4.6.2).