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Design and Tuning of Fractional-order PID Controllers for Time-delayed Processes

Emmanuel Edet

Technology and Innovation Centre
University of Strathclyde
99 George Street
Glasgow, United Kingdom
emmanuel.edet@strath.ac.uk

Reza Katebi

Industrial Control Centre
University of Strathclyde
Glasgow, United Kingdom
m.r.katebi@strath.ac.uk

Abstract— Frequency domain based design methods are investigated for the design and tuning of fractional-order PID for scalar applications. Since Ziegler-Nichol's tuning rule and other algorithms cannot be applied directly to tuning of fractional-order controllers, a new algorithm is developed to handle the tuning of these fractional-order PID controllers based on a single frequency point just like Ziegler-Nichol's rule for inter order PID. Critical parameters of the system are obtained at the ultimate point and the controller parameters are calculated from these critical measurements to meet design specifications. Thereafter, fractional order is obtained to meet a specified robustness criteria which is the phase-invariability against gain variations around the phase cross-over frequency. Results are simulated on second-order plus dead time plant to demonstrate both performance and robustness.

Keywords—robustness; performance; tuning; PID

I. INTRODUCTION

In recent years, interesting control engineering applications have been developed using differential equations of generalized order ' α ' where the derivative order α can be any non-integer. Control systems utilizing fractional dynamics termed fractional order control systems have been found to yield better performance than integer-order controllers under fair comparison [1].

A. Some Definitions of Fractional-order Derivative

Consider the generalised-order differential equation (1) shown below:

$$y(t) = \mathcal{D}^\alpha u(t) + u(t) \quad (1)$$

where $t > 0; n - 1 < \alpha < n; n \in \mathbb{R}$

Equation 1 is a fractional order differential equation as far as the order " α " is non-integer. There are two important definitions that allow for computation of fractional order derivative namely:

- Caputo's Fractional order derivative
- Riemann-Liouville Definition.

Fractional-order derivative with order (α) is defined by Riemann-Liouville as:

$$\mathcal{D}^\alpha f(t) \triangleq \mathcal{D}^n \ell^{n-\alpha} f(t)$$

$$= \frac{d^n}{dt^n} \left[\frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \right] \quad (2)$$

where $t > 0; n - 1 < \alpha < n; n \in \mathbb{R}$.

An alternative definition was also given by Caputo [2] as shown in equation 3:

$$\mathcal{D}^\alpha f(t) \triangleq \ell^{n-\alpha} \mathcal{D}^n f(t)$$

$$= \left[\frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^n(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \right] \quad (3)$$

t > 0; $n - 1 < \alpha < n; n \in \mathbb{R}$

However, the inclusion of the n th order derivative of $f(t)$ in Caputo's definition imposes restriction compared to Riemann-Liouville form [3].

B. Fractional-order PID

The theory of fractional dynamics has been utilized to design PIDs with non-integer order - $PI^\lambda D^\mu$. Fractional order PID Controllers ($PI^\lambda D^\mu$) have been extensively tested in demanding applications especially in mechatronic and automatic control applications and are found to yield very good results [2]. Basically, $PI^\lambda D^\mu$ is of the form:

$$C(s) = K_p + \frac{K_i}{s^\lambda} + K_d s^\mu \quad (4)$$

where:

λ and μ are the fractional orders of integral and derivative parts of the controller respectively. Therefore there are five parameters that are to be determined: K_p , K_i , K_d , λ , and μ . It can be observed that classical integer PID controller is a subset of this generalized-order $PI^\lambda D^\mu$ where λ and μ are set to one. In terms of frequency domain analysis, given the non-restriction of the orders of integral and derivative terms of the controller, more parameters are available for the manipulation of the slope of the magnitude curve and phase at both high and low frequencies. In terms of robustness to plant gain variations and uncertainties in the plant model, the fractional order PID controller is known to give better performance than the classical PID controller due to this inherent frequency domain property as well as availability of more degrees of freedom for the design engineer [2]. Also the high frequency noises, and load disturbances associated with plant models can be effectively handled using fractional controller [4].

II. SPECIFICATION OF DESIGN OBJECTIVES

The first step is to define design specifications expected to be satisfied by the fractional order controller. These design objectives are defined in frequency domain in order to take care of important control objectives: stability, performance and robustness. Each controller parameter can be tuned to satisfy each design specification.

A. Gain Margin and Phase Cross-over Frequency Specification.

Gain margin is a primary index of relative stability in classical control design. A pre-defined margin can be used to formulate a robustness constraint on the system gain. Equation I defines the relationship between gain margin and phase cross over frequency while equation 5 defines the gain margin constraint:

$$|C(j\omega_{cg})G(j\omega_{cg})|_{dB} = 0 \text{ dB} \quad (5)$$

$$|C(j\omega_{cp})G(j\omega_{cp})|_{dB} = \frac{1}{A_m} \quad (6)$$

where:

ω_{cg} - The gain crossover frequency

ω_{cp} - Phase cross over frequency

A_m - The Gain margin.

B. Phase Margin and Gain Crossover Frequency Specification

Phase margin and gain margin are extended to fractional order control as useful measures of robust stability. It is chosen to satisfy equation 7

$$\arg(C(j\omega_{cg})G(j\omega_{cg})) = -\pi + \phi_m \quad (7)$$

C. Robustness against Plant's Gain Variations

Bode's ideal loop defines the criteria for absolutely stable SISO loop. As long as the open loop gain is defined by a constant phase around the crossover frequency (useful band), robust stability against gain variations is guaranteed within that frequency range of constant phase.

$$\frac{d\{\arg(C(j\omega)G(j\omega))\}}{d\omega} = 0 \text{ around } \omega_{cg}. \quad (8)$$

The phase of the forward loop function will be flat around the cross over frequency. This improves robustness against gain-like variations in plant and the overshoot is nearly constant within that given frequency range.

D. High frequency noise rejection specification

In order to ensure satisfactory measurement noise rejection, appropriate bound for the complementary sensitivity function has to be defined:

$$\|T(j\omega)\|_{dB} \leq A \text{ dB}$$

$$\left\| \frac{C(j\omega)G(j\omega)}{1+C(j\omega)G(j\omega)} \right\|_{dB} \leq A \text{ dB for } \omega \geq \omega_t \text{ rad/s.} \quad (9)$$

where:

$T(j\omega)$ is the complementary sensitivity function.

A dB = the specified attenuation level in dB for the band $\omega \geq \omega_t \frac{\text{rad}}{s}$.

E. Output disturbance rejection specification.

Sensitivity bound can be defined to ensure satisfactory output disturbance rejection by the controller:

$$\left\| \frac{1}{1+C(j\omega)G(j\omega)} \right\|_{dB} \leq B \text{ dB} \quad (10)$$

where:

B dB = the specified magnitude of the sensitivity function in dB for the band $\omega \leq \omega_s \frac{\text{rad}}{s}$.

In addition, the nth-fractional integral term ensures steady state error rejection as effectively as an equivalent integer-order integral controller due to the implementation as $s^{-1}s^{1-n}$. Five parameters are available for tuning in order to meet five specifications. The proportional gain, integral gain, derivative gain, derivative (fractional) order and the fractional order of the integral term are available parameters that can be optimised to meet the defined specifications [3]. This is the primary reason behind the superior control action of fractional order controller type over the integer order types – more parameters are available for optimization to meet more control objectives. Monje [3, 5] used MATLAB's optimization tool (fmincon) to formulate an optimization problem that solves for these five parameters and it was a constraint non-linear optimization problem with five unknown variables. Another way of obtaining robust values for PID controller gains is to use sustained oscillation method as described next while the fractional order for the integral controller term can be chosen to satisfy robust performance criteria. This method reduces computational cost.

III. THE ULTIMATE POINT PARAMETER METHOD

In industrial circles, Ziegler-Nichols PID tuning rule and similar PID design methods are still dominant for simple process models. In this section, a similar technique is developed for SISO processes. The basic principle involves using a proportional controller in cascade with the plant under a closed loop configuration. Thereafter, the proportional gain is systematically increased from very small values until sustained oscillation or continuous cycling is observed at the process output. The value of the proportional gain that yields this sustained oscillation is recorded as the critical gain (K_u) while the period of oscillation is noted as the ultimate period T_u .

Several other important information about the process are obtainable from these critical point measurements. For instance, the phase cross over frequency can be obtained as:

$$\omega_{pco} = \frac{2\pi}{T_u} \quad (11)$$

The fractional PI controller is then obtained by relocating this ultimate frequency point on the complex plane to a more desirable point. This is illustrated on the Nyquist diagram of figure 1. The critical point or ultimate point is the stability limit signified by the point where the Nyquist curve intersects the unit circle (-1,0).

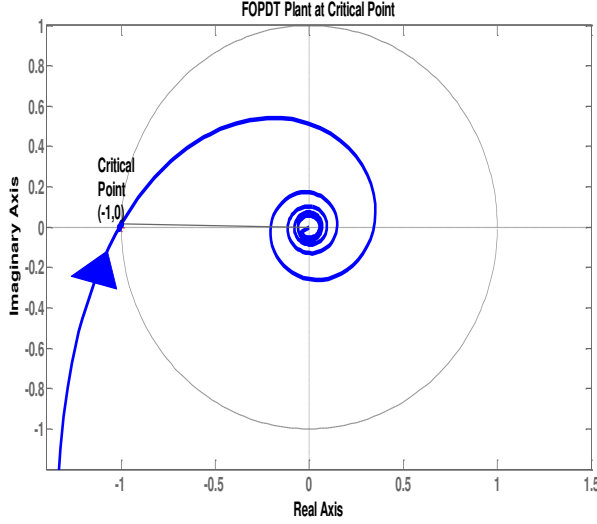


Fig. 1. Nyquist Diagram indicating Plant's Critical Point

A. Design of the Fractional-order PI controller

Consider a fractional PI controller of the form:

$$C(s) = K_p + \frac{K_i}{s^\alpha} \quad (12)$$

$$C(j\omega) = K_p + \frac{K_i}{(j\omega)^\alpha}$$

$$K_i = \frac{K_p}{T_i}$$

A desirable point on the complex plane is chosen as design point - B given as $(r_B e^{j(\pi+\phi_B)})$.

The controller is expected to move the ultimate point $(\frac{1}{K_u}, 0)$ to this desirable point. On the Nyquist plot, the ultimate point is - point A $(r_A e^{j\pi})$.

$$r_A = \frac{1}{K_u} \quad (13)$$

Finally, Let the frequency characteristic of the controller $C(s)$ be $r_C e^{j(\pi+\phi_C)}$. For this controller to move the ultimate point to the desirable point on the complex plane, equation 14 must be satisfied:

$$\Rightarrow r_A r_C e^{j\pi} e^{j(\pi+\phi_C)} = r_B e^{j(\pi+\phi_B)} \quad (14).$$

$$r_C e^{j(\pi+\phi_C)} = \frac{r_B}{r_A} (\cos \phi_B + j \sin \phi_B) \quad (15)$$

$$r_C e^{j(\pi+\phi_C)} = r_B K_u (\cos \phi_B + j \sin \phi_B)$$

Therefore, the fractional-order PI controller is chosen to satisfy the magnitude and phase conditions below in 16 and 17:

$$r_C = \frac{r_B}{r_A} \quad (16)$$

$$\phi_C = \phi_B - 0 \quad (17)$$

Recall the controller structure in frequency domain as given in equation 12:

$$C(j\omega) = K_p + \frac{K_i}{(j\omega)^\alpha}$$

$$C(j\omega) = K \left(1 + \frac{1}{(\tau_i)(j\omega)^\alpha} \right)$$

$$C(j\omega) = K \left(1 + \frac{1}{\omega^\alpha \tau_i} \left(\cos \frac{\alpha\pi}{2} - j \sin \frac{\alpha\pi}{2} \right) \right) \quad (18)$$

$$\Rightarrow K \left(1 + \frac{1}{\omega^\alpha \tau_i} \left(\cos \frac{\alpha\pi}{2} - j \sin \frac{\alpha\pi}{2} \right) \right) = r_B K_u (\cos \phi_B + j \sin \phi_B).$$

Comparing magnitudes:

$$r_B K_u \cos \phi_B = K \left(1 + \frac{1}{\omega^\alpha \tau_i} \left(\cos \frac{\alpha\pi}{2} \right) \right) \quad (19)$$

In the same vein, arguments are compared and the phase is expected to fulfil equation 20 below:

$$\tan \phi_B = \frac{-\omega^{-\alpha} \tau_i^{-1} \sin \alpha\pi/2}{(1 + \omega^{-\alpha} \tau_i^{-1} \cos \alpha\pi/2)} \quad (20)$$

If we set $y = \frac{1}{\tau_i}$,

$$y = \frac{-\tan \phi_B}{\omega^{-\alpha} (\tan \phi_B \cos \frac{\alpha\pi}{2} + \sin \alpha\pi/2)} \quad (21)$$

B. Justification of Design Point

The desired point B is chosen such that $r_B = 0.29$ and $\phi_B = 46 \text{ deg}$.

This is a very desirable point because it translates to $-0.2-j0.21$ point. The distance from this point to the stability limit $(-1,j0)$ is $d = \sqrt{(-0.2 + 1)^2 + (-0.21)^2}$ which is approximately 0.9. Closed loop sensitivity can be deduced from this information on the Nyquist curve of the forward loop because the maximum sensitivity M_s is the reciprocal of the shortest distance from the Nyquist curve to the critical point.

By definition: $M_s = \max_{0 \leq \omega \leq \infty} \left| \frac{1}{1+C(s)G(s)} \right|$.

However, on the Nyquist curve, $M_s = \frac{1}{d}$.

$$\Rightarrow M_s = \frac{1}{0.9} = 1.1$$

This is very desirable for robustness. For the closed loop system to be robust against variations in process dynamics, maximum sensitivity of the closed loop system is specified: $M_s < 2$. Reasonable values of M_s ranges from 1.05 to 1.95. This indicates good robustness for many practical control applications and it is therefore recommended as the design point as far as single frequency point method is concerned throughout this paper. This design point is also justified as far as closed loop stability is concerned. It meets Nyquist stability criterion as the critical stability $(-1,0)$ point will not be encircled. Therefore, the fractional order PI controller can be obtained from 22 and 23.

$$K = 0.202 K_u - \left(\frac{\tau_u}{2\pi} \right)^\alpha \cos \frac{\alpha\pi}{2} \quad (22)$$

$$|\tau_i| = 0.966 \left(\frac{\tau_u}{2\pi} \right)^\alpha \left(\sin \frac{\alpha\pi}{2} + 1.036 \cos \frac{\alpha\pi}{2} \right) \quad (23)$$

Fractional order is selected to meet the required resonant peak M_r condition as given by Monje for SISO-FOPDT systems. This is sufficient for SISO control problems with FOPDT process model.

$$G(s) = \frac{Ye^{-Ls}}{s\tau+1}. \text{ Relative dead-time is: } T = \frac{L}{L+\tau} \quad (24).$$

TABLE I. RELATIVE DEAD TIME AND FRACTIONAL-ORDER

Cases	α - order	Relative Dead-time T
1.	0.7	$T < 0.1$
2.	0.9	$0.1 \leq T < 0.4$
3.	1.0	$0.4 \leq T < 0.6$
4.	1.1	$T \geq 0.6$

For Second order processes such as SOPDT (Second Order Plus Dead-Time model), derivative component can be included for a more damped response. One possible solution is to design the fractional controller in the form of $PI^\mu D$ -structure. It is however tuned in a similar fashion as the fractional PI case using ultimate parameters.

C. The $PI^\mu D$ - Structure

The structure of the controller is given as:

$$C(j\omega) = K_p + \frac{K_i}{(j\omega)^\alpha} + K_D(j\omega) \quad (25)$$

$$C(j\omega) = K_p \left(1 + \frac{1}{\tau_i \left(\cos \frac{\alpha\pi}{2} + j \sin \frac{\alpha\pi}{2} \right) \omega^\alpha} + \tau_D(j\omega) \right) \quad (26)$$

Let the controller $C(j\omega)$ be characterised by $r_c e^{j(\pi+\theta_c)}$ in frequency domain and the desirable point on the Nyquist plane chosen as point B that is: $(r_B e^{j(\pi+\theta_B)})$ as earlier given.

The controller is expected to move the ultimate point $(\frac{1}{K_u}, 0)$ to this desirable point as earlier explained in previous section. On the complex plane, the ultimate point is - point A $(r_A e^{j\pi})$. Therefore, equations 14, 15 and 16 must be fulfilled as earlier shown. It follows that:

$$\begin{aligned} & \left(K_p \left(1 + \frac{1}{\tau_i \left(\cos \frac{\alpha\pi}{2} + j \sin \frac{\alpha\pi}{2} \right) \omega^\alpha} + \tau_D(j\omega) \right) \right) \\ &= \frac{r_B}{r_A} \cos(\theta_B - \theta_A) \quad (27) \end{aligned}$$

Consider the argument:

$$\tan(\theta_B - \theta_A) = \frac{\omega\tau_D - \frac{1}{\omega^\alpha\tau_i} \sin(\alpha\pi/2)}{1 + \frac{1}{\omega^\alpha\tau_i} \cos(\alpha\pi/2)}$$

Given that the ultimate point is to be moved; $\theta_A = 0$.

$$\tan \theta_B \left(1 + \frac{1}{\omega^\alpha\tau_i} \cos \left(\frac{\alpha\pi}{2} \right) \right) = \omega\tau_D - \frac{1}{\omega^\alpha\tau_i} \sin(\alpha\pi/2)$$

Making τ_i subject of formula and substituting $\omega = \frac{2\pi}{\tau_u}$ yields:

$$\tau_i = \frac{\tau_u^\alpha (\tan \theta_B \cos \frac{\alpha\pi}{2} + \sin \frac{\alpha\pi}{2})}{(2\pi)^\alpha \left(\frac{2\pi\tau_D}{\tau_u} - \tan \theta_B \right)} \quad (28)$$

$$K_p = r_B K_u \cos \theta_B - \frac{\tau_u^\alpha \cos \frac{\alpha\pi}{2}}{((2\pi)^\alpha \tau_i)} \quad (29)$$

Given that the order α is obtainable as before from the M_r table, there are now three parameters left to be obtained from two equations. This is clearly insufficient. One way to solve it is to treat the ratio of τ_D to τ_i as a constant [6]. For instance:

$$\tau_D = 0.25\tau_i.$$

All four parameters can therefore be calculated.

The second structure of fractional controller with derivative component is of PID^α form.

D. The PID^α Controller form.

$$C(j\omega) = K_p + \frac{K_i}{j\omega} + K_D(j\omega)^\alpha \quad (30)$$

$$C(j\omega) = K_p \left(1 - j \frac{1}{\tau_i \omega} + \tau_D(j\omega)^\alpha \right)$$

The controller structure is given in equation 30. It is expected to move the ultimate point to our new design point B implying that equations 14, 15 and 16 applies again as stated in previous sections. Therefore:

$$\begin{aligned} & (K_p (1 + \omega^\alpha \tau_D \cos(\alpha\pi/2))) = \frac{r_B}{r_A} \cos(\theta_B - \theta_A) \\ & K_p = \frac{\frac{r_B}{r_A} \cos(\theta_B - \theta_A)}{1 + \omega^\alpha \tau_D \cos \frac{\alpha\pi}{2}} \\ & K_p = \frac{r_B K_u \cos(\theta_B)}{1 + \omega^\alpha \tau_D \cos \frac{\alpha\pi}{2}} \quad (31) \end{aligned}$$

Considering the argument as before:

$$\tan(\theta_B) = \frac{\omega^\alpha \tau_D \sin(\frac{\alpha\pi}{2}) - \frac{1}{\omega\tau_i}}{1 + \omega^\alpha \tau_D \cos \frac{\alpha\pi}{2}} \quad (32)$$

The ratio of integral time to derivative time is set to constant as before: $\tau_D = 0.25\tau_i$ and equations 32 and 31 gives the two controller parameters. Fractional order are obtained in similar fashion to previous cases using the peak resonant condition.

IV. RESULTS OF SISO PLANT SIMULATIONS

Consider a FOPDT plant given as

$$G(s) = \frac{12.8e^{-s}}{16.7s + 1}$$

Three controllers C_1 , C_2 and C_3 are designed in the following forms: PI^μ , $PI^\mu D$ and PID^α respectively. This is done using the sustained oscillation method as described in this work:

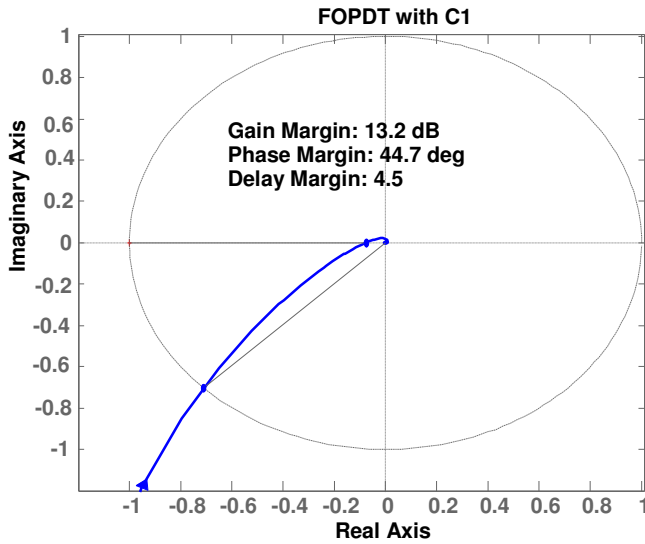


Fig. 2. Nyquist Diagram of FOPDT Plant with controller C1

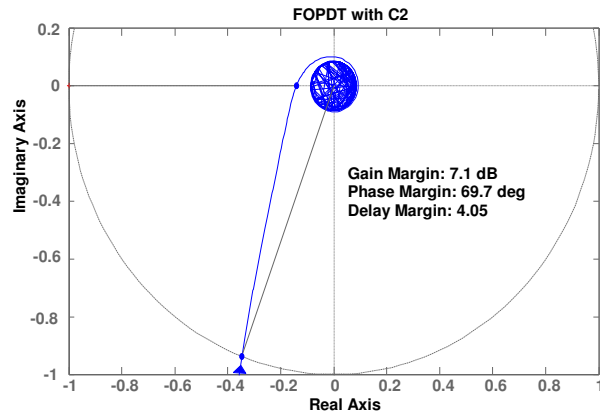


Fig. 3. Nyquist Diagram of FOPDT Plant with controller C2

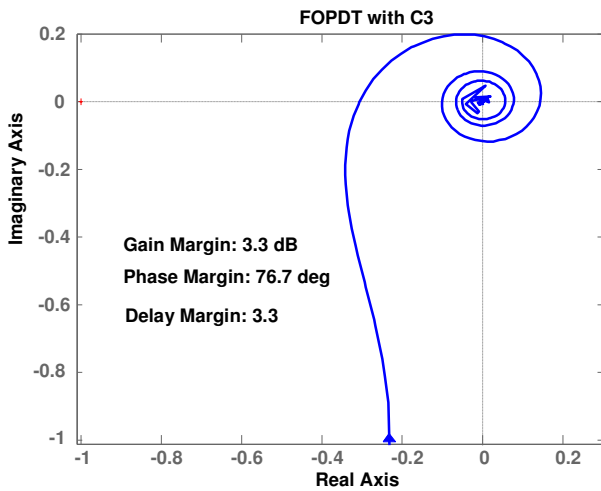


Fig. 4. Nyquist Diagram of FOPDT Plant with controller C3

$$C_1 = 0.04\left(1 + \frac{1}{1.12s^{0.7}}\right)$$

$s^{-0.7}$ is implemented as $s^{0.3}s^{-1}$ using the well-known ORA approximation (Oustaloup Recursive Approximation) in order to obtain integer order form of the controller with the same frequency domain properties [7, 8]. ORA has been reported by many authors as a good approximation to the fractional order. Therefore, it is used throughout this work to realise a rational and implementable function.

$$C_2 = 0.32\left(1 + \frac{0.1}{s^{0.7}} + 0.35s\right)$$

$$C_3 = 0.218\left(1 + \frac{1}{3.6s} + 1.08s^{0.7}\right)$$

It is observed that the controller moves the design point of the plant in figure 1 to a desired (stable) region as shown in Nyquist diagrams shown next:

Consider a second example - a SOPDT plant below:

$$G_2(s) = \frac{2e^{-0.2s}}{s(0.5s + 1)}$$

The controller realized using $PI^{\mu}D$ structure as earlier discussed is: $C_{22}(s) = 0.459 + \frac{0.2}{s^{0.9}} + 0.28s$.

The fractional controller of the form PID^{α} is also realized as C_{11} given below: $C_{11} = 0.444 + \frac{0.175}{s} + 0.3s^{0.9}$.

Performance is compared using Bode diagram of figure 5 and step response shown in figure 6.

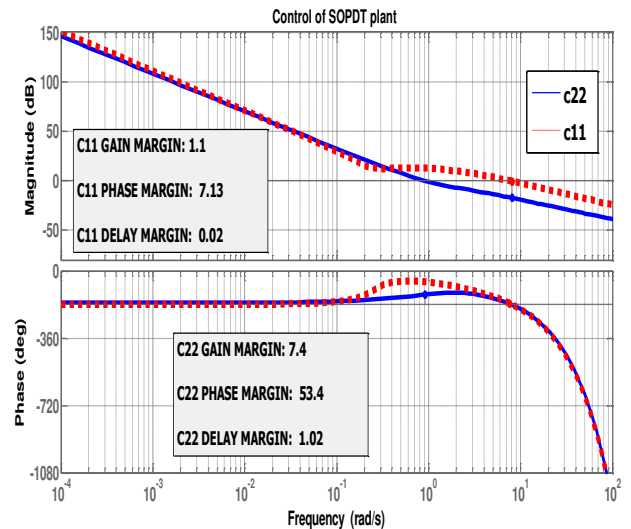


Fig. 5. Bode Diagram of SOPDT Plant with controller C11 and C22

V. DISCUSSION OF RESULTS AND CONCLUSION

These SISO based methods yield excellent response for the plants examined as reflected in the gain margin, phase margin and delay margin. The delay margin is an important robustness measure which gives information about how much time delay can be introduced to the system before it goes unstable. In addition, very little computation is required to yield the desired fractional controller. In summary, the main contribution of this paper is development of a new design method for fractional-order PID (FPID) controller. Extended simulations has been carried out on delayed processes with FOPDT and SOPDT dynamics. It has been found to yield control actions with good compromise between robustness and performance.

VI. REFERENCES

- [1] Z. Li and Y. Chen, "Ideal, Simplified and Inverted Decoupling of Fractional order TITO Processes," in IFAC, Cape Town, 2014.
- [2] I. Podlubny, "Fractional-Order Systems and PID Controllers," IEEE Transactions on Automatic Control, vol. 44, no. 1, pp. 208-214, 1999.
- [3] C. Monje, Y. Chen, B. D. Vinagre and V. Feliu, Fractional-order Systems and Controls: Fundamentals and Applications, London: Springer-Verlag, 2010.
- [4] G. Vinagre, D. Valerio and J. Costa, "Rule-Tuned PIDs and Fractional PIDs for a Three-Tank Liquid System," in Symposium on Fractional Signals and Systems, Lisbon, Portugal, 2009.
- [5] C. Monje, Y. Chen, D. Vinagre and V. Feliu, "Tuning and Auto-tuning of Fractional order Controllers for Industry Applications," Control Engineering Practice, vol. 16, pp. 798-812, 2008.
- [6] K. Astrom and T. Hagglund, Advanced PID Control, Durham, USA.: ISA, 2006.
- [7] D. Valerio and S. Costa, "Time domain implementation of Fractional order controllers," IEEE Transaction on Control Theory Application, vol. 152, no. 5, pp. 539-552, 2005.
- [8] F. Merrikh-Bayat, "Rules for Selecting the Parameters of Oustaloup Recurssive Approximation for the Simulation of Linear Feedback Systems containing Fractional PID controller," Nonlinear Science and Numerical Simulation, vol. 17, pp. 1852-1861, 2012.

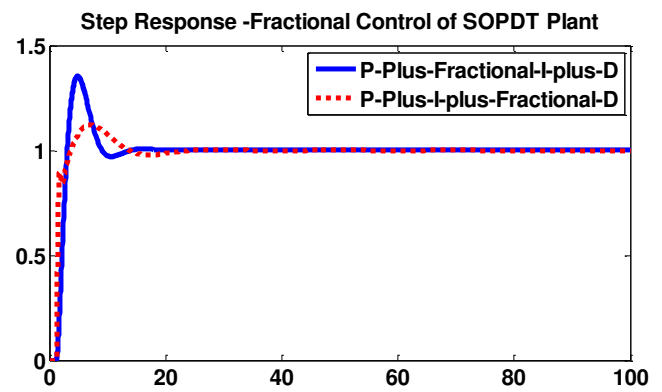


Fig. 6. Diagram of step response SOPDT Plant with controller C11 and C22