Strathprints Institutional Repository

Clemente, Carmine and Pallotta, Luca and Gaglione, Domenico and De Maio, Antonio and Soraghan, John J. (2016) Automatic recognition of military vehicles with Krawtchouk moments. IEEE Transactions on Aerospace and Electronic Systems. ISSN 0018-9251 (In Press) ,

This version is available at http://strathprints.strath.ac.uk/57206/

Automatic Target Recognition of Military Vehicles with Krawtchouk Moments

Carmine Clemente*, Luca Pallotta†,
Domenico Gaglione,* Antonio De Maio‡ and John J. Soraghan*
*University of Strathclyde, CESIP, EEE, 204, George Street, G1 1XW, Glasgow, UK
E-mail: carmine.clemente@strath.ac.uk, domenico.gaglione@strath.ac.uk, j.soraghan@strath.ac.uk
†Centro Regionale Information Communication Technology (CeRICT) scrl, via Cinthia c/o Complesso Universitario di Monte Sant'Angelo, Fabbr. 8B - 80126 Napoli, Italy
E-mail: luca.pallotta@unina.it
‡Università di Napoli "Federico II", DIETI, via Claudio 21, I-80125 Napoli, Italy
E-mail: ademaio@unina.it

***Abstract*—The challenge of Automatic Target Recognition (ATR) of military targets within a Synthetic Aperture Radar (SAR) scene is addressed in this paper. The proposed approach exploits the discrete defined Krawtchouk moments, that are able to represent a detected extended target with few features, allowing its characterization. The proposed algorithm provides robust performance for target recognition, identification and characterization, with high reliability in presence of noise and reduced sensitivity to discretization errors. The effectiveness of the proposed approach is demonstrated using the MSTAR dataset.**

I. INTRODUCTION

Target recognition of military vehicles is a topic of increasing interest and demanding requirements [1], [2]. The knowledge of the vehicles deployed in a specific area of interest is fundamental to the understanding of the threat that exists (e.g: Small Intercontinental Ballistic Missile launcher rather than a theatre missile launcher). Furthermore it also allows a better understanding of the activities in a specific site. Currently there is a growing interest in the ability to increase the level of knowledge to the identification or characterization stage, where the actual capabilities of the vehicle/object can be better understood. Many current ATR algorithms for vehicles require the ability to identify small differences among targets like a specific configuration of a multirole vehicle. Furthermore, ATR represents one of the multiple tasks in which modern platforms are involved, for example an Unmanned Aerial Vehicle (UAV) will be acquiring the radar echoes, performing the imaging using High Performance Computing (HPC) capabilities [3], maintaining constant communication with a control centre or other platforms, while managing other systems like Electro-Optical (EO) sensors. For this reason the processing and the information extraction have to comply with the low Size Weight And Power (SWAP) paradigm.

Various approaches have been proposed to address the ATR challenge. A general approach has been investigated in [4], where L_2 normalization is applied to the image thereby preserving all the information of the image whilst assigning to the classifier the task of deriving the model and separation of targets. After L_2 normalization, the SAR chips containing the target are passed to the SVM that uses a Gaussian kernel, with the kernel size set to be the average Euclidean distance between training patterns. The SVM approach was tested on the MSTAR dataset and compared with other classifiers such as model matching and neural network. The work developed at MIT Lincoln Laboratory [1] provides a complete analysis by investigating both detection and classification of stationary ground targets using high resolution, fully polarimetric SAR images. The algorithm comprises three main stages: detection (or prescreening), discrimination, and classification. In particular a Mean-Square Error (MSE) classifier is exploited in this algorithm, whose minimum acceptable value is tresholded and targets that differ more than the threshold from the target model are labelled as clutter. The main drawback of the algorithm is the fact that it relies on a single metric (MSE), meaning that an accurate knowledge of the target models are required, otherwise the algorithm would incur in misclassification. A robust algorithm has been proposed in [5] in which an increased number of scattering centres are selected while retaining low computational complexity. The approach uses a relatively large number of scatterers with a variability reduction technique. To reduce the effect of the variabilities, a novel grid cell structure is developed by considering the information of potential targets, such as target sizes, structures, and relative positions of the strongest scatterers. Furthermore, features related to scatterer angular stability information are extracted. Discriminative graphical models have been used in [6] with the aim to fuse different features and allow good performance with small training datasets. A two-stage framework is proposed to model dependencies between different feature representations of a target image. The approach has been tested using the MSTAR dataset and the performance resulted to overcome EMACH, SVM, AdaBoost and Conditonal Gaussian Model classifiers.

In this paper an algorithm for ATR based on Krawtchouk moments is proposed. The characterization capability, and reliability of the new method are investigated in the paper.

Krawtchouk moments were introduced in [7] and [8] for image processing application purposes. Krawtchouk moments have been applied to 1, 2 and 3D signals [9]–[12]. In [9] a method using Krawtchouk moments was proposed to enhance noise corrupted speech signals. In particular Wiener filtering was applied after representing a noisy signal in the Krawtchouk and Tchebychev domains. Image super resolution was proposed in [10] for the specific case of low resolution video sequences. The authors of [10] used Krawchouk moments as they do not need for co-ordinate transformation, are orthogonal over a square region and are discrete moments in order to create a high resolution image sequence from a given low resolution image sequence. A Krawtchouk based noise resilient gait recognition from videos was proposed in [11]. In this approach the orthogonality of the moments was exploited in order to ensure minimal redundancy. Finally, the extension to 3D of Krawtchouk polynomials was used for shape search and retrieval in [12]. In particular, the property of low order Krawtchouk moments to capture edges was exploited in order to obtain enhanced discrimination of 3D objects with low complexity.

A common issue of most families of image moments [13], is the level of discretization error and poor robustness in low Signal to Noise Ratio (SNR) conditions. This error builds up as the order increases, limiting the accuracy of the computed moments. This drawback results in target recognition algorithms with less accuracy in discriminating between targets that differ in small components, that would be possible if only robust higher order moments are used.

Krawtchouk moments have some peculiar characteristics [8], in particular they are discretely defined, thus there is no requirement of spatial normalization and the discretization error is nonexistent. This translates in a relaxation on the amount of resources required to represent and store the polynomials. Moreover, the computational cost is reduced due to the orthogonality property of Krawtchouk polynomials that relaxes the requirements of feature selection to mitigate overfitting. These characteristics, together with the capability to pre-compute the polynomials, makes this family of image moments compatible with SWAP systems.

The remainder of the paper is organized as follows, Section II introduces Krawthcouk moments and describes the proposed ATR algorithm. Section III discusses the results obtained using the MSTAR dataset in different noise conditions and Section IV concludes the paper.

II. ATR Algorithm Based on Krawtchouk Moments

This section describes the ATR algorithm that is based on Krawtchouk moments. First the analytical formulation of the Krawtchouk moments is provided in order to support the understanding of the algorithm functional blocks described successively in detail.

A. Krawtchouk Moments

The classical formulation of Krawtchouk polynomials introduced in [7] suffers from numerical instability. For this reason the weighted Krawtchouk polynomials that were introduced in [8] have been selected for the purpose of representing the target in the proposed ATR approach.

The classical Krawtchouk polynomials of order n are defined as [8]

$$K_n(x;p,N) = \sum_{k=0}^{N} a_{k,n,p} x^k = \ {}_2F_1\left(-n,-x;-N;\frac{1}{p}\right) \quad (1)$$

where x and n belong to $(0,1,2,\cdots,N)$, $N \in \mathbb{N}$, where $\mathbb{N}$ is the set of natural numbers, p is a real number belonging to the set $(0,1)$, and ${}_2F_1$ is the Gauss hypergeometric function

$$ {}_2F_1\left(a,b;c;z\right) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{z^k}{k!} \quad (2)$$

where $(\cdot)_k$ is the Pochhammer symbol given by

$$(a)_k = a(a+1)\ldots(a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)} \quad (3)$$

and $\Gamma(\cdot)$ is the Eulerian Gamma function.

To overcome the numerical instability of these polynomials a weight [8] can be used leading to the weighted Krawtchouk Polynomials i.e.

$$\bar{K}_n(x;p,N) = K_n(x;p,N)\sqrt{\frac{w(x;p,N)}{\rho(n;p,N)}} \quad (4)$$

with $w(x;p,N) = \binom{N}{x} p^x (1-p)^{N-x}$ and $\rho(n;p,N) = (-1)^n \left(\frac{1-p}{p}\right)^n \frac{n!}{(-N)_n}$.
The polynomials defined in (4) are orthogonal i.e.

$$\sum_{x=0}^{N} \bar{K}_n(x;p,N)\bar{K}_m(x;p,N) = \delta_{nm}, \quad \forall p, N \quad (5)$$

with $(n,m) \in (0,1,\ldots,N)^2$. Furthermore, the parameter p represents a shift parameter, in particular, as p deviates from the value 0.5 by Δp the weighted Krawtchouk polynomials are approximately shifted by $N\Delta p$ [8]. This characteristic can be exploited to focus on a specific area of interest within the image, for example by increasing the number of features related to a specific section of a target (e.g. a tank turret) in order to improve the target characterization capabilities. Considering a 2D function of interest $\psi(x,y)$, e.g. a SAR image, with x and y natural numbers in the sets $(1,N)$ and $(1,M)$ respectively, and M and N representing the image width and height in samples, the Krawtchouk moments of order (n,m) are defined as

$$Q_{nm} = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} \bar{K}_n(y;p_1,N-1)\bar{K}_m(x;p_2,M-1)\psi(x,y) \quad (6)$$

The moments in (6) provide a powerful tool for representing 2D functions with a limited set of values and have been previously used for image compression and recognition [14].

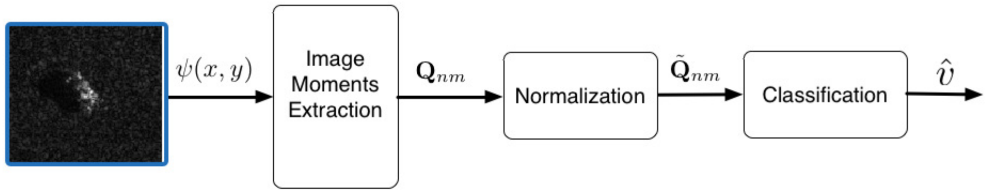


Figure 1. Block diagram of the proposed feature extraction and classification algorithm.

B. Algorithm Description

The functional blocks of the proposed ATR algorithm are depicted in Figure 1. The starting point is the intensity SAR image, $\psi(x, y)$, of a target from the set of J possible targets of interest. Equation (6) can be applied to $\psi(x, y)$ for each order up to (n, m),

to form the vector $\mathbf{Q}_{nm}$ containing the Krawtchouk moments

$$\mathbf{Q}_{nm} = [Q_{00}, \ldots, Q_{nm}] \,. \tag{7}$$

From equation (6) it is also possible to estimate the computational complexity of the proposed approach for feature extraction that results to be equal to $(N \times M)^2$. The feature vector, $\mathbf{Q}_{nm}$, has $(n+1) \times (m+1)$ elements and is normalized using the following standardization to ensure that any particular feature will not have higher impact on the classification stage [15]

$$\tilde{\mathbf{Q}}_{nm} = (\mathbf{Q}_{nm} - \mu_{\mathbf{Q}_{nm}})/\sigma_{\mathbf{Q}_{nm}}, \tag{8}$$

where $\mu_{\mathbf{Q}_{nm}}$ and $\sigma_{\mathbf{Q}_{nm}}$ are, respectively, mean and standard deviation of $\mathbf{Q}_{nm}$.

The feature vectors are then used as input to a classification algorithm, such as k-Nearest Neighbours, Support Vector Machine, or Bayesian Classifier. The output of the classifier is $\hat{v}$ with values in $(1, J) \in \mathbb{N}$ containing the output target class identifier of the image under test.

III. Performance Analysis on the MSTAR dataset

In this section the performance analysis of the proposed algorithm is assessed on real data. The MSTAR Dataset is a collection of SAR images of 14 different military targets [16], [17], that represents a useful test-bench for ATR algorithms. This dataset can be used for the different levels of target classification. According to the NATO AAP-6 Glossary Terms and Definitions, with “recognition” is meant the classification of the type/category of target; “identification” regards the capability to assign the target to a subclass; “characterization” takes into account the class variants. Following this definition, Table I reports the different targets and their grouping in the MSTAR dataset, together with the number of available images acquired with $15°$ and $17°$ of depression angle.

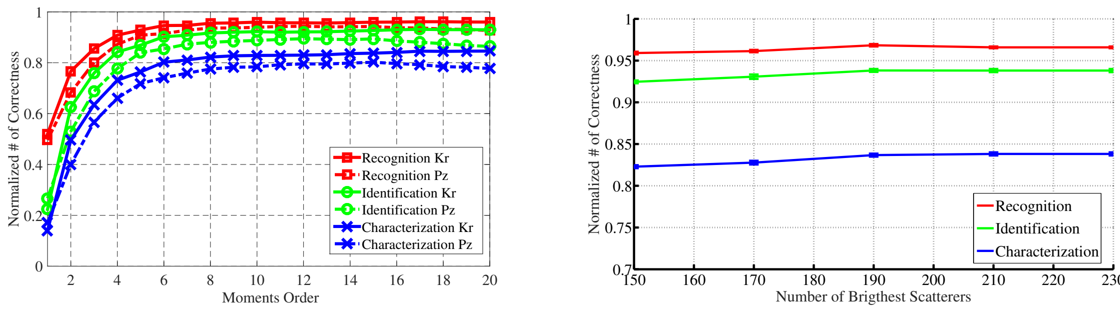
The images are supposed to cover the full $360°$ azimuth angle. However, due to missing images in the dataset, the total number of observations does not always cover each aspect angle. Moreover, different targets have different number of images. In the performance analysis 191 samples are used as the minimal number of images available for all the targets. The training images are selected randomly and the same number of images for each target from the set of images acquired at $15°$ of depression angle is considered. In order to investigate the robustness of the algorithm for different training sets available, the selection of the images used for testing and those used for training is randomized in each run. In this way a different subset of training images is drawn and, consequently, a different subset of test images is used in the testing stage. Specifically, a total of 100 Monte Carlo runs are performed for each analysis in order to be able to draw randomly a wider set of training and test images for the targets with more than 191 images available.

In order to investigate the capabilities and the robustness of the proposed approach, the results of the new algorithm are compared to those obtained using the pseudo-Zernike moments [18] and the approach proposed in [5]. In the experiments a k-NN classifier with $k = 3$ and $p_1 = p_2 = 0.5$ for the computation of the Krawtchouk polynomials have been used. Figure 2-a shows the normalized average number of correct Recognition, Identification and Characterization obtained for both Krawtchouk and pseudo-Zernike approaches. In the analysis all the moments available up to a selected order are considered.

It is seen that the Krawtchouk based algorithm is superior to the pseudo-Zernike based algorithm for all three levels of target discrimination. For example, considering moments of order up to 20 (441 features) the percentage of correct target recognition reaches 96.02% using the proposed algorithm while is 92.64% for the pseudo-Zernike algorithm. A similar trend is seen for the target identification case with performance going from 92.97% to 86.42% of correct identification when switching from Krawtchouk to pseudo-Zernike approach. This performance difference is confirmed in the target characterization case with correct target characterization of 84.58% using Krawtchouk versus the 77.74% obtained with the pseudo-Zernike. The identification and characterization results, with 6.55% and 6.89% of improvements in performance respectively, confirms the capability of Krawtchouk moments to represent with higher fidelity smaller details of the targets. Analysing the performance in the best case (190 brightest scatterers case) of the approach presented in [5] that are reported in Figure 2-b, it shows that the brightest scatterers based approach provides 96.83%, 93.81% and 83.67% of correct target recognition, identification and characterization.

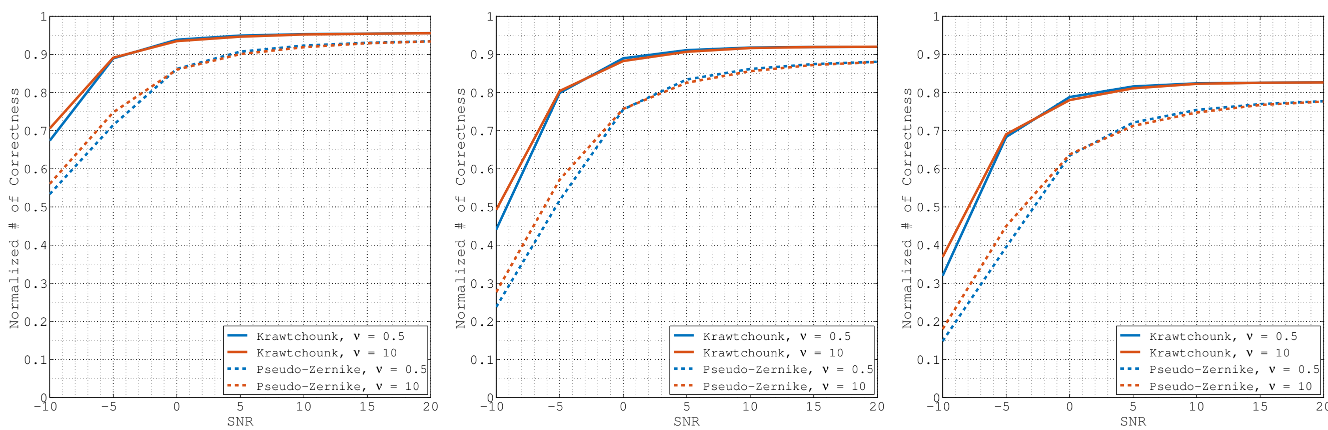
Table I. MSTAR DATASET.

Target	Type	# of Images $15^\circ - 17^\circ$	Recognition	Identification	Characterization
BMP2 9563	Tank	195-233	R1	I1	C1
BMP2 9566	Tank	196-232			C2
BMP2 C21	Tank	196-233			C3
T72 132	Tank	196-232		I2	C4
T72 812	Tank	195-231			C5
T72 S7	Tank	191-228			C6
2S1	Tank	276-299		I3	C7
T62	Tank	273-299		I4	C8
ZSU	Tank	274-299		I5	C9
BTR70 C71	Personnel Carrier	196-233	R2	I6	C10
BTR60	Personnel Carrier	195-256		I7	C11
ZIL131	Truck	274-299	R3	I8	C12
BRDM	Reconn. Vehicle	274-298	R4	I9	C13
D7	Bulldozer	274-299	R5	I10	C14



(a) (b)

Figure 2. Performance in terms of normalized correct number of Recognition, Identification and Characterization on the MSTAR dataset for (a) the proposed algorithm using Krawtchouk moments vs the algorithm introduced in [18] using pseudo-Zernike moments and (b) compared with the performance achievable using the approach in [5] for various number of brightest scatterers selected.



(a) Recognition (b) Identification (c) Characterization

Figure 3. Performance using Krawtchouck (continuous lines) and pseudo-Zernike (dashed lines) approaches, moments up to order 10, different SNR levels and values of the parameter ν in (10) equal to 0.5 and 10. (a) Recognition, (b) Identification, (c) Characterization.

This performance is comparable with those achievable using the proposed Krawtchouk based algorithm.

To demonstrate the higher robustness to noise of the Krawtchouk based approach, a stress analysis under different noise conditions has been performed. In the experiments additive and multiplicative noise are added to the dataset that are assumed to initially contain noise free images.

A. Additive Compound Gaussian Noise

For each pixel of the image, the additive noise, d, is modelled as a compound-Gaussian random variable [19], [20], which can be written in the form

$$d = \sqrt{\tau} g \tag{9}$$

where τ is a positive real random variable, and g is a complex circularly symmetric zero-mean Gaussian variable, whose variance is set in order to achieve a certain SNR.

As the variable τ follows a Gamma distribution

$$f(x) = \frac{1}{\Gamma(\nu)} \frac{1}{\mu^{\nu}} x^{\nu-1} e^{-x/\mu} u(x), \tag{10}$$

where $u(\cdot)$ is the unit-step function, μ and ν are the scale and shape parameters, respectively (we set $\mu = 1/\nu$ in order to have a gamma distribution with unit mean). Equations (9) and (10) ensure that the amplitude probability density function of d is K-distributed. SNR levels between -10 dB and 20 dB and values of ν of 0.5 and 10 are considered. Figure 3 shows that the results using the Krawtchouk moments approach is more reliable and robust to noise than the pseudo-Zernike one. For example it is noticed from Figure 3 that considering a SNR level of 0 dB and $\nu = 0.5$ (impulsive noise), Krawtchouk moments performance is 93.86%, 88.99%, 78.83% for recognition, identification and classification respectively, while using pseudo-Zernike moments the performance dropped to 86.20%, 75.58% and 63.48%. In this case the use of the proposed approach is able to provide more robust results in the presence of additional noise in the images, with performance improved of 7.66% in recognition, 13.41% in identification and 15.35% in characterization.

The confusion matrices showing the percentage of correct characterization obtained for $\nu = 0.5$ and moments up to order 10 (121 features) case are reported in Tables II and III, for Krawtchouk and pseudo-Zernike approaches respectively. A figure of merit for the overall performance of ATR algorithms considers the ratio of the sum of the values appearing in the diagonal of the confusion matrix to the sum of all the other values. This should have a value as high as possible, which is infinite for a perfect algorithm [21]. In this paper this figure of merit will be referred to β, which is computed as 3.65 and 1.69 from Tables II and III respectively.

Moreover, the tables show that in the presence of different configurations of the same vehicle (like BMP2 and T72), the capability of target characterization of the Krawtchouk based algorithm is superior compared to pseudo-Zernike. For example, considering the two 3×3 top left matrices of the confusion matrices relative to the BMP2 and T72 targets, and marked in red and blue for clarity, it is seen that both exhibit a more "diagonal" behaviour in the Krawtchouk case than in the pseudo-Zernike one. In particular the figure of merit β is 1.56 and 1.02 when the red matrices are considered and 4.62 and 2.83 for the blue matrices in the Krawtchouk and pseudo-Zernike cases respectively. These latest results demonstrate the capability of Krawtchouk moments to maintain a good representation of details in presence of noise.

B. Multiplicative Noise

In the multiplicative noise case the modulus of each pixel is multiplied with a square root of a Gamma random variable whose scale and shape parameters are chosen as $\mu = 1/\nu$. Moments of order between 1 and 20 and values of ν of 0.5 and 10 have been considered, and the results obtained in this analysis are shown in Figure 4. As seen in Figure 4 in this situation the performance obtained using the Krawtchouk approach results in higher reliability and robustness to noise compared to those obtained using pseudo-Zernike. For example, considering a SNR level of 0 dB and $\nu = 0.5$, the algorithm using Krawtchouk moments results to be correct in 91.43%, 84.57%, 73.65% of cases for recognition, identification and classification respectively, while using pseudo-Zernike moments the performance dropped to 88.46%, 79.50% and 67.25%.

In this case the use of the proposed approach is able to provide more robust results in presence of multiplicative noise in the images, with performance improved of 2.97% in recognition, 5.07% in identification and 6.40% in characterization capabilities. For completeness the confusion matrices showing the percentage of correct characterization obtained for the $\nu = 0.5$ and moments up to order 10 are reported in Tables IV and V, for Krawtchouk and pseudo-Zernike respectively. In these cases β results to have the values of 2.66 and 1.96 respectively. Again, in red and blue the variations of BMP2 and T72 are reported. Also for the multiplicative noise case, Krawtchouk moments show a better capability to maintain a good representation of details compared to those obtained using pseudo-Zernike moments, in particular, β results to be 1.35 and 1.05 for the BMP2 target and 3.61 and 2.71 for the T72 case for the Krawtchouk and pseudo-Zernike approaches respectively.

These results demonstrate the higher robustness to the presence of noise of Krawtchouk moments, making the proposed approach particularly suitable for more noisy SAR images like, those acquired with low cost sensors mounted on UAVs and low frequency SAR images (e.g.: Foliage Penetrating SAR).

IV. Conclusions

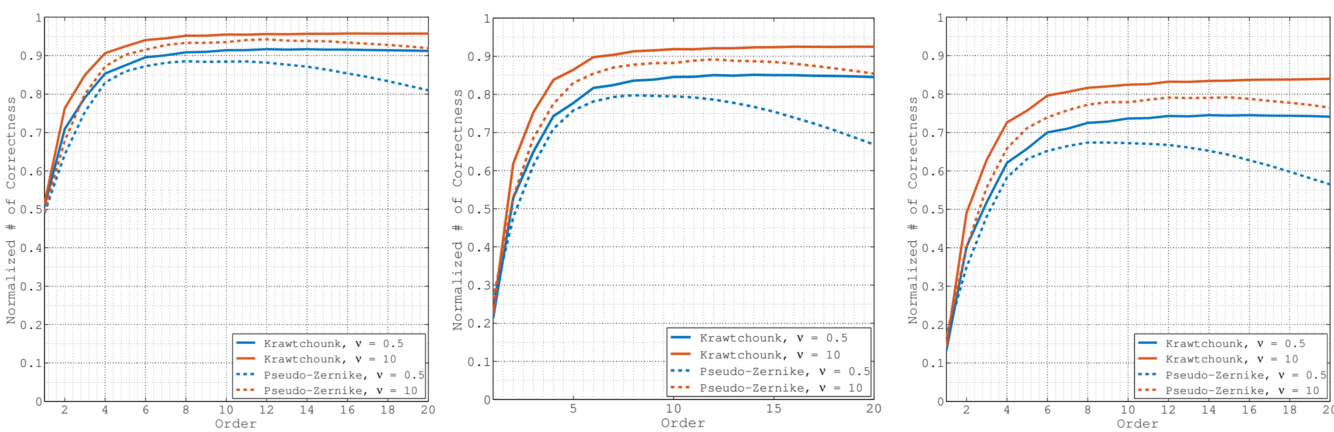
In this paper an algorithm for automatic target recognition based on Krawtchouk moments has been presented. The proposed approach was shown to provide a more reliable solution to the automatic target recognition challenge from SAR images with higher capabilities in discriminating between different sub-classes of targets and in noisy environments. The performance of the proposed algorithm were assessed using the real MSTAR dataset that contain different vehicles in various configurations. The superior performance and robustness of the Krawtchouk based algorithm have been confirmed by the experimental results, demonstrating improvements, in particular on the characterization of targets, over the approach using pseudo-Zernike moments that suffers from discretization errors and is less robust in presence of noise. Hence the proposed approach is particularly suitable for SWAP systems and with potential to be used on SAR images acquired with

Table II. CONFUSION MATRIX SHOWING THE PERCENTAGE OF CORRECT CHARACTERIZATION USING KRAWTCHOUK, SNR 0 DB, ORDER 10, ADDITIVE COMPOUND GAUSSIAN NOISE, $\nu = 0.5$

	BMP2 9563	BMP2 9566	BMP2 C21	T72 132	T72 812	T72 S7	2S1	T62	ZSU	BTR70 C71	BTR60	ZIL131	BRDM	D7
BMP2 9563	65.06%	11.38%	18.62%	0.20%	1.00%	0.45%	1.12%	0.27%	0.33%	0.16%	0.06%	0.06%	0.27%	1.00%
BMP2 9566	23.38%	59.69%	11.37%	0.25%	1.08%	1.12%	0.28%	0.42%	0.59%	1.31%	0.21%	0.22%	0.09%	0.00%
BMP2 C21	30.97%	14.91%	48.58%	0.91%	0.92%	0.55%	1.29%	0.29%	0.19%	0.84%	0.02%	0.20%	0.24%	0.12%
T72 132	0.58%	0.53%	1.38%	84.75%	4.20%	5.13%	0.08%	0.54%	2.13%	0.02%	0.04%	0.43%	0.03%	0.17%
T72 812	0.44%	0.69%	0.93%	3.89%	76.90%	13.79%	0.22%	0.97%	1.61%	0.22%	0.03%	0.29%	0.00%	0.02%
T72 S7	0.48%	1.40%	0.79%	9.04%	14.23%	70.76%	0.12%	0.52%	0.91%	0.32%	0.51%	0.80%	0.02%	0.08%
2S1	1.16%	2.08%	2.64%	0.41%	0.84%	0.15%	85.96%	0.28%	0.28%	3.75%	0.48%	0.81%	0.45%	0.71%
T62	2.14%	1.67%	3.00%	2.86%	1.98%	2.04%	2.93%	77.15%	2.90%	0.66%	0.48%	1.01%	0.82%	ashed0.35%
ZSU	1.13%	0.55%	1.37%	1.47%	0.04%	0.20%	0.47%	2.67%	89.03%	0.85%	0.14%	0.02%	0.45%	1.61%
BTR70 C71	0.25%	0.29%	0.74%	0.28%	0.48%	0.06%	0.70%	0.09%	0.01%	92.75%	2.30%	1.00%	1.04%	0.00%
BTR60	0.68%	0.37%	1.53%	0.21%	0.14%	1.14%	0.29%	0.83%	1.09%	3.42%	88.13%	0.49%	1.35%	0.34%
ZIL131	2.15%	0.87%	1.69%	0.95%	0.86%	1.40%	5.08%	0.90%	0.32%	2.59%	0.51%	79.22%	2.90%	0.57%
BRDM	1.31%	2.15%	1.65%	0.84%	0.47%	0.94%	3.17%	1.65%	3.27%	3.39%	1.11%	0.98%	78.13%	0.95%
D7	0.26%	0.54%	0.72%	0.08%	0.00%	0.05%	0.12%	0.08%	0.32%	0.00%	0.32%	0.01%	0.59%	96.91%

Table III. CONFUSION MATRIX SHOWING THE PERCENTAGE OF CORRECT CHARACTERIZATION USING PSEUDO-ZERNIKE, SNR 0 DB, ORDER 10, ADDITIVE COMPOUND GAUSSIAN NOISE, $\nu = 0.5$

	BMP2 9563	BMP2 9566	BMP2 C21	T72 132	T72 812	T72 S7	2S1	T62	ZSU	BTR70 C71	BTR60	ZIL131	BRDM	D7
BMP2 9563	49.06%	15.43%	22.36%	1.54%	0.80%	1.38%	1.91%	0.79%	0.29%	2.60%	0.61%	1.34%	1.53%	0.35%
BMP2 9566	24.53%	42.68%	18.41%	1.82%	1.29%	2.38%	1.78%	0.73%	0.33%	2.90%	0.71%	1.13%	1.20%	0.11%
BMP2 C21	28.45%	18.47%	39.30%	0.99%	1.07%	2.37%	2.24%	1.31%	0.41%	2.06%	0.61%	0.85%	1.20%	0.66%
T72 132	2.85%	3.05%	3.04%	65.09%	6.29%	10.04%	0.79%	2.79%	1.45%	0.51%	1.09%	1.73%	0.53%	0.74%
T72 812	2.57%	2.77%	2.06%	7.25%	55.82%	19.74%	1.36%	2.43%	1.62%	0.79%	1.13%	1.55%	0.20%	0.72%
T72 S7	2.08%	3.46%	2.67%	9.93%	10.66%	60.46%	0.74%	1.68%	1.64%	1.37%	0.94%	2.35%	0.48%	1.54%
2S1	1.90%	5.91%	3.64%	1.30%	1.44%	2.26%	60.06%	2.31%	2.93%	10.78%	0.70%	4.49%	0.83%	1.46%
T62	5.51%	4.55%	8.71%	4.34%	3.29%	4.75%	4.62%	49.85%	2.88%	2.98%	2.30%	1.83%	2.20%	2.20%
ZSU	0.84%	0.89%	1.17%	1.47%	0.39%	0.95%	0.77%	7.06%	77.18%	0.02%	0.61%	0.31%	0.98%	7.35%
BTR70 C71	2.47%	2.36%	2.32%	0.27%	0.11%	0.12%	1.68%	0.70%	0.02%	85.52%	1.16%	0.95%	2.27%	0.05%
BTR60	1.43%	1.96%	2.02%	1.27%	0.36%	1.76%	0.99%	1.59%	0.86%	8.46%	75.01%	1.05%	2.70%	0.54%
ZIL131	1.83%	2.34%	3.76%	2.60%	1.76%	1.36%	6.47%	2.85%	0.58%	6.73%	3.43%	63.84%	1.66%	0.79%
BRDM	1.70%	2.92%	1.23%	0.72%	0.44%	1.04%	3.30%	1.75%	1.94%	4.30%	3.31%	0.85%	74.72%	1.78%
D7	1.84%	0.71%	1.79%	1.16%	0.40%	2.17%	1.45%	2.22%	5.18%	0.15%	0.44%	0.40%	0.70%	81.38%



(a) Recognition (b) Identification (c) Characterization

Figure 4. Performance using Krawtchouck (continuous lines) and pseudo-Zernike (dashed lines) approaches, for different moments orders and multiplicative noise levels with $\nu = 0.5$ and $\nu = 10$. (a) Recognition, (b) Identification, (c) Characterization.

low cost sensors mounted on UAVs and Foliage Penetrating SAR images.

ACKNOWLEDGMENT

This work was supported by the Engineering and Physical Sciences Research Council (EPSRC) Grant number EP/K014307/1 and the MOD University Defence Research Collaboration in Signal Processing and the University of Naples “Federico II”.

REFERENCES

[1] Novak, L. M., Halversen, S. D., Owirka, G. J., Hiett, M., “Effects of Polarization and Resolution on SAR ATR,” *IEEE Trans. on Aerospace and Electronic Systems*, vol. 33, no. 1, pp. 102–116, January 1997.

[2] Novak, L., Owirka, G., Weaver, A., “Automatic target recognition using enhanced resolution SAR data,” *IEEE Trans. on Aerospace and Electronic Systems*, vol. 35, no. 1, pp. 157–175, Jan 1999.

[3] Clemente, C., Di Bisceglie, M., Di Santo, M., Ranaldo, N., Spinelli, M., “Processing of Synthetic Aperture Radar data with GPGPU,” in *IEEE Workshop on Signal Processing Systems, 2009. SiPS 2009.*, Oct 2009, pp. 309–314.

[4] Qun, Z. Principe, J., “Support vector machines for SAR automatic

Table IV. CONFUSION MATRIX SHOWING THE PERCENTAGE OF CORRECT CHARACTERIZATION USING KRAWTCHOUK, ORDER 10, MULTIPLICATIVE NOISE, $\nu = 0.5$

	BMP2 9563	BMP2 9566	BMP2 C21	T72 132	T72 812	T72 S7	2S1	T62	ZSU	BTR70 C71	BTR60	ZIL131	BRDM	D7
BMP2 9563	55.57%	14.27%	20.48%	1.07%	1.55%	0.88%	1.79%	0.91%	0.62%	0.33%	0.36%	0.36%	0.60%	1.21%
BMP2 9566	20.85%	55.14%	14.81%	1.09%	1.02%	1.81%	1.40%	0.69%	0.96%	0.79%	0.21%	0.78%	0.38%	0.07%
BMP2 C21	26.77%	17.64%	45.04%	1.15%	0.98%	0.76%	2.33%	1.56%	0.68%	0.57%	0.27%	0.59%	0.73%	0.96%
T72 132	1.15%	1.25%	1.59%	76.36%	5.05%	6.99%	0.33%	1.63%	3.40%	0.06%	0.34%	1.41%	0.06%	0.38%
T72 812	0.48%	0.98%	1.38%	5.35%	69.12%	16.32%	0.42%	2.06%	2.36%	0.17%	0.23%	1.02%	0.03%	0.10%
T72 S7	0.93%	1.51%	1.07%	9.96%	14.58%	64.92%	0.51%	1.02%	2.39%	0.27%	0.30%	1.89%	0.36%	0.30%
2S1	1.79%	2.03%	2.92%	0.57%	0.88%	0.68%	82.40%	1.13%	0.60%	3.34%	0.61%	1.52%	0.30%	1.25%
T62	1.92%	1.37%	2.68%	3.17%	2.98%	3.02%	3.53%	70.35%	5.66%	0.43%	0.89%	1.64%	0.91%	1.46%
ZSU	0.78%	0.56%	1.16%	0.94%	0.16%	0.47%	0.71%	3.06%	87.99%	0.43%	0.36%	0.13%	0.44%	2.80%
BTR70 C71	1.06%	0.74%	0.96%	0.47%	0.88%	0.32%	2.96%	0.52%	0.16%	86.43%	2.95%	1.14%	1.36%	0.04%
BTR60	1.22%	1.07%	1.39%	0.69%	0.34%	1.22%	1.19%	1.68%	1.74%	4.35%	81.33%	1.16%	1.31%	1.31%
ZIL131	1.85%	1.19%	1.75%	1.43%	1.27%	1.50%	6.79%	2.30%	0.74%	2.13%	0.82%	74.85%	1.94%	1.43%
BRDM	2.45%	1.90%	2.40%	0.88%	0.32%	1.20%	5.00%	2.56%	4.12%	2.65%	1.89%	1.05%	72.53%	1.04%
D7	0.40%	0.23%	0.65%	0.08%	0.01%	0.10%	0.30%	0.30%	1.19%	0.01%	0.12%	0.18%	0.43%	96.00%

Table V. CONFUSION MATRIX SHOWING THE PERCENTAGE OF CORRECT CHARACTERIZATION USING PSEUDO-ZERNIKE, ORDER 10, MULTIPLICATIVE NOISE, $\nu = 0.5$

	BMP2 9563	BMP2 9566	BMP2 C21	T72 132	T72 812	T72 S7	2S1	T62	ZSU	BTR70 C71	BTR60	ZIL131	BRDM	D7
BMP2 9563	46.98%	17.16%	23.11%	1.76%	0.70%	1.68%	1.88%	0.96%	0.37%	1.89%	0.52%	1.03%	1.42%	0.54%
BMP2 9566	21.53%	44.78%	19.70%	1.67%	1.24%	2.89%	2.00%	0.72%	0.65%	2.32%	0.79%	0.95%	0.63%	0.12%
BMP2 C21	25.81%	18.69%	40.79%	1.59%	1.05%	2.39%	2.58%	1.28%	0.41%	1.46%	0.86%	1.09%	0.99%	1.02%
T72 132	2.25%	2.11%	2.19%	66.52%	5.98%	11.28%	0.80%	2.65%	1.59%	0.22%	1.28%	1.81%	0.18%	1.14%
T72 812	1.06%	1.47%	1.23%	7.49%	56.06%	21.91%	2.28%	3.34%	1.37%	0.35%	0.65%	1.86%	0.12%	0.79%
T72 S7	1.29%	2.16%	1.71%	11.26%	10.10%	61.79%	1.07%	2.33%	1.67%	0.71%	0.89%	3.01%	0.15%	1.85%
2S1	1.67%	3.85%	3.20%	0.97%	1.70%	2.57%	67.12%	3.18%	1.79%	6.58%	0.95%	4.42%	0.48%	1.52%
T62	2.32%	1.51%	4.07%	3.51%	2.94%	5.30%	5.77%	63.20%	4.29%	0.84%	1.44%	1.73%	0.99%	2.09%
ZSU	0.35%	0.49%	0.42%	0.63%	0.18%	0.67%	0.31%	5.65%	83.33%	0.00%	0.26%	0.58%	0.27%	6.86%
BTR70 C71	2.49%	2.55%	2.45%	0.17%	0.24%	0.41%	2.96%	0.80%	0.05%	83.92%	1.86%	0.47%	1.55%	0.08%
BTR60	1.54%	1.46%	1.93%	1.30%	0.62%	2.15%	1.91%	1.81%	1.37%	7.00%	75.61%	0.72%	1.73%	0.86%
ZIL131	0.87%	1.09%	2.16%	1.37%	1.20%	1.46%	7.42%	3.84%	0.82%	2.39%	2.12%	72.70%	0.86%	1.69%
BRDM	1.66%	2.53%	1.59%	0.87%	0.45%	0.89%	2.59%	1.85%	1.56%	3.47%	2.13%	0.69%	77.83%	1.89%
D7	0.58%	0.22%	0.89%	0.40%	0.16%	1.11%	0.76%	2.10%	6.02%	0.01%	0.24%	0.81%	0.33%	86.38%

target recognition," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 37, no. 2, pp. 643–654, April 2001.

[5] Doo, S. H., Smith, G., Baker, C., "Reliable target feature extraction and classification using potential target information," in *2015 IEEE Radar Conference (RadarCon)*, May 2015, pp. 0628–0633.

[6] Srinivas, U., Monga, V., Raj, R., "SAR Automatic Target Recognition Using Discriminative Graphical Models," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 50, no. 1, pp. 591–606, January 2014.

[7] Krawtchouk, M., "On interpolation by means of orthogonal polynomials," *Memoirs Agricultural Inst. Kyiv*, pp. 21–28, 1929.

[8] Yap, P.-T., Paramesran, R., Ong, S.-H., "Image analysis by Krawtchouk moments," *IEEE Trans. on Image Processing*, vol. 12, no. 11, pp. 1367–1377, Nov 2003.

[9] Jassim, W. A., Paramesran, R., Zilany, M. S. A., "Enhancing noisy speech signals using orthogonal moments," *IET Signal Processing*, vol. 8, no. 8, pp. 891–905, 2014.

[10] Raj, P. A., "A Krawtchouk moments based super resolution technique for multiframe image sequence," in *IEEE 16th International Symposium on Consumer Electronics (ISCE), 2012* , June 2012, pp. 1–3.

[11] Ioannidis, D., Tzovaras, D., Damousis, I. G., Argyropoulos, S., Moustakas, K., "Gait Recognition Using Compact Feature Extraction Transforms and Depth Information," *IEEE Transactions on Information Forensics and Security*, vol. 2, no. 3, pp. 623–630, Sept 2007.

[12] Mademlis, A., Axenopoulos, A., Daras, P., Tzovaras, D., Strintzis, M. G., "3D Content-Based Search Based on 3D Krawtchouk Moments," in *Third International Symposium on 3D Data Processing, Visualization, and Transmission*, June 2006, pp. 743–749.

[13] Teague, M., "Image Analysis via the General Theory of Moments," *Optical Society Am.*, vol. 70, no. 8, 1980.

[14] Teh, C. H. Chin, R., "On Image Analysis by the Methods of Moments," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 10, no. 4, pp. 496–513, July 1988.

[15] Fukunaga, K., *Introduction to Statistical Pattern Recognition*, Computer Science and Scientific Computing. Elsevier Science, 1990.

[16] Ross, T., Worrell, S., Velten, V., Mossing, J., Bryant, M., "Standard SAR ATR evaluation experiments using the MSTAR public release data set," *Proc. SPIE,*, vol. 3370, pp. 566–573, 1998.

[17] "MSTAR image database, https://www.sdms.afrl.af.mil/index.php?collection=mstar," .

[18] Clemente, C., Pallotta, L., Proudler, I., De Maio, A., Soraghan, J., Farina, A., "Pseudo-Zernike based multi-pass automatic target recognition from multi-channel Synthetic Aperture Radar," *IET Radar, Sonar Navigation*, vol. 9, no. 4, pp. 457–466, 2015.

[19] Conte, E. Longo, M., "Characterisation of radar clutter as a spherically invariant random process," *IEE Proceedings Communications, Radar and Signal Processing*, vol. 134, no. 2, pp. 191–197, April 1987.

[20] Greco, M. Gini, F., "Statistical Analysis of High-Resolution SAR Ground Clutter Data," *IEEE Trans. on Geoscience and Remote Sensing*, vol. 45, no. 3, pp. 566–575, March 2007.

[21] Tait, P., *Introduction to Radar Target Recognition*, IEE Radar Series. Institution of Engineering and Technology, 2005.

Carmine Clemente (S'09-M'13) eceived the Laurea cum laude (BSc) and Laurea Specialistica cum laude (MSc) degrees in telecommunications engineering from Universita' degli Studi del Sannio, Benevento, Italy, in 2006 and 2009, respectively. In 2012, he received the Ph.D. degree in the Department of Electronic and Electrical Engineering, University of Strathclyde, Glasgow, UK. Currently he is a Lecturer in the Department of Electronic and Electrical Engineering, University of Strathclyde, Glasgow, U.K working on advanced Radar signal processing algorithm, MIMO radar systems and micro-Doppler analysis. His research interests include synthetic aperture radar (SAR) focusing and bistatic SAR focusing algorithms development, micro-Doppler signature analysis and extraction from multistatic radar platforms, micro-Doppler classification and statistical signal processing.

Luca Pallotta (S'12-M'15) received the Laurea Specialistica degree (cum laude) in telecommunication engineering in 2009 from the University of Sannio, Benevento, Italy, and the Ph.D. degree in electronic and telecommunication engineering in 2014 from the University of Naples "Federico II", Naples, Italy. His research interest lies in the field of statistical signal processing, with emphasis on radar signal processing and radar targets classification. Dr. Pallotta won the Student Paper Competition at the IEEE Radar Conference 2013.

Domenico Gaglione (S'13) received his BSc and MSc from UniversitÃ degli Studi di Napoli "Federico II", Italy, in 2011 and 2013, respectively, in Telecommunications Engineering. He is currently a PhD candidate at the Centre for Signal and Image Processing (CeSIP), University of Strathclyde, Glasgow, UK, under the supervision of Professor John Soraghan and Dr Carmine Clemente. He has been recently appointed as Research Assistant for the work package 4 within the LSSCN consortium of the UDRC phase II initiative, whose focus is the development of novel paradigms for Distributed MIMO Radar Systems (DMRSs). Domenico was the recipient of the First Prize at the Student Paper Competition of the 2015 IEEE International Radar Conference, Arlington, VA, USA. His research interests include micro-Doppler based and SAR based classification and identification, compressive sensing based radar techniques, MIMO radar signal processing and joint radar-communication system.

Antonio De Maio (S'01-A'02-M'03-SM'07-F'13) was born in Sorrento, Italy, on June 20, 1974. He received the Dr.Eng. degree (with honors) and the Ph.D. degree in information engineering, both from the University of Naples "Federico II", Naples, Italy, in 1998 and 2002, respectively. From October to December 2004, he was a Visiting Researcher with the U.S. Air Force Research Laboratory, Rome, NY. From November to December 2007, he was a Visiting Researcher with the Chinese University of Hong Kong, Hong Kong. Currently, he is a Professor with the University of Naples Federico II. His research interest lies in the field of statistical signal processing, with emphasis on radar detection, optimization theory applied to radar signal processing, and multiple-access communications. Dr. De Maio is the recipient of the 2010 IEEE Fred Nathanson Memorial Award as the young (less than 40 years of age) AESS Radar Engineer 2010 whose performance is particularly noteworthy as evidenced by contributions to the radar art over a period of several years, with the following citation for "robust CFAR detection, knowledge-based radar signal processing, and waveform design and diversity". Dr. De Maio also is a Fellow member of IEEE.

John J. Soraghan (S'83-M'84-SM'96) received the B.Eng. (Hons.) and M.Eng.Sc. degrees in electronic engineering from University College Dublin, Dublin, Ireland, in 1978 and 1983, respectively, and the Ph.D. degree in electronic engineering from the University of Southampton, U.K., in 1989. His doctoral research focused on synthetic aperture radar processing on the distributed array processor. After graduating, he worked with the Electricity Supply Board in Ireland and with Westinghouse Electric Corporation in the United States. In 1986, he joined the Department of Electronic and Electrical Engineering, University of Strathclyde, Glasgow, U.K., as a Lecturer and became a Senior Lecturer in 1990, a Reader in 2000, and a Professor in Signal Processing in September 2003, within the Institute for Communications and Signal Processing (ICSP). In December 2005, he became the Head of the ICSP. He currently holds the Texas Instruments Chair in Signal Processing with the University of Strathclyde. He was a Manager of the Scottish Transputer Centre from 1988 to 1991, and a Manager of the DTI Parallel Signal Processing Centre from 1991 to 1995. His main research interests are signal processing theories, algorithms and architectures with applications to remote sensing, telecommunications, biomedicine, and condition monitoring. Prof. Soraghan is a member of the IET and IEEE.