A Novel Kalman Filter Based Technique for Calculating the Time History of Vertical Displacement of a Boat from Measured Acceleration

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Abstract

Accelerometers are used to measure velocity and displacement in many applications such as ship motion, monitoring of civil and mechanical structure, seismology and machine condition monitoring. However, using direct numerical integration to calculate velocity and displacement from the acceleration signal is known to suffer from low frequency noise amplification and wind-up. In this paper, a Kalman filter based method is proposed for calculating displacement from measured acceleration. Integration wind-up is eliminated by incorporating an additional state variable, namely the integral of the displacement whose "measured" value is assumed to be equal to the known average value of the displacement. In many applications, such as those in marine environment, this average value can be assumed to be constant, usually conveniently assigned to be zero if non-linear behaviour and permanent deformations are deemed negligible. The paper describes the technique and investigates its performance under different conditions of amplitude and frequency of vibrations and sampling rate and validates it by conducting two laboratory experiments. In the first experiment the displacement of a small shaker is calculated from a relatively high frequency (tens of Hz) acceleration signal sampled at 1 kHz with a resolution of 1 g. The calculated displacement of the shaker is found to agree well with that measured using a high resolution laser. In the second experiment, the proposed method is applied to the calculation of the vertical displacement of a boat from a low frequency (less than 1 Hz) acceleration signal sampled at 5 Hz and a resolution of 0.01g. An experimental set up designed to mimic typical motion of a boat is used to validate the results. Although the method explained in this paper is used to calculate the vertical displacement of a boat, it can be applied for calculating the displacement in a wide range of applications with reciprocating movement.

Keywords

Boat Motion; Displacement Calculation; Integration Wind-up; Kalman Filter; Non-stationary Signals

Introduction

Measurement of the vertical movement of a boat, i.e. heave motion, is important both for practical needs and for basic study of the interaction between boat and sea waves (Miles 1986; Manganelli 2006). For instance when the design of an energy harvester for extracting electricity from the vertical movement of a boat is of interest, obtaining the amplitude and frequency of boat's heave motion is necessary (Sharkh et al. 2011). However, with displacement sensors, direct measurement of displacement and velocity of a boat is not feasible as they require to be fixed on an inertial frame of reference. For this reason, accelerometers are often used and their output signal is subsequently integrated to obtain velocity and displacement. Accelerometers also have the additional advantages of lower cost, smaller size and higher bandwidth than electromagnetic velocity and displacement sensors. However, direct integration of an acceleration signal poses two main difficulties. The first one stems from the presence of low frequency noise and dc drift which are amplified by the integration process leading to integration wind-up. The second arises from not knowing the initial values of velocity and displacement, which are often not-zero. This could also cause integral wind-up. Further errors are caused by digital sampling, particularly if the sampling rate and the ADC resolution are poor

(Faulkner et al. 1996; Gavin 1998; Han 2003; Hong et al. 2010).

To overcome these problems, various methods have been investigated in the literature in the context of different engineering applications. There are two main basic methods: using either numerical integration of the time domain signal, i.e. direct integration method (Park et al. 2005); or integrating its Fourier series equivalent, i.e. the frequency domain method (S. Han 2003; S. Han 2010). Taira *et al.* (1971) utilized the frequency method to estimate the vertical displacement of a ship. They applied a Fast Fourier Transform (FFT) algorithm to the measured acceleration signal. However, since the motion of a ship is inherently random and irregular, the FFT method caused errors in the estimation of displacement which was referred to as leakage error. The maximum leakage error caused by all frequencies components which composed the signal was investigated and the frequency corresponding to the maximum amount of leakage error was found. After double integration of the Fourier series of the acceleration signal, the displacement amplitude for all frequencies below the maximum leakage error frequency were assumed to be zero. Although, the frequency domain method suffers from the problem of spectral leakage, especially when the signal is random and irregular, this method was demonstrated to be effective in accurately calculating the vertical displacement of a ship from acceleration. The estimated ship displacement was used to correct wave gauge measurements and accordingly estimate the waves' heights. However, it is difficult to apply this approach in real time.

Several techniques have been proposed by different authors to overcome the integral wind-up problem. Gavin *et al.* (1998) proposed the employment of an integrator in a loop, to feed back the average of the integrated signal by using a low pass filter. They demonstrated the technique using both analogue and hybrid analogue-digital circuits. The analogue circuit performed well in terms of linearity and hysteresis when integrating random wide-band signals, but less so with long-period, low frequency signals. The hybrid circuit had excellent accuracy when integrating long-period signals but produced phase and bias errors when integrating wide-band signals.

The method proposed by Park *et al.* (2005) basically repeats the direct integration for a range of initial velocity conditions in order to find a suitable value for

which the integrator wind-up is eliminated. However, this method cannot be used in real time and the authors found that in practice it was necessary to segment the signal and apply the method to each segment individually.

Zhou *et al.* (1997) suggested a multi-step scheme to correct the drift produced when calculating the displacement of soil from measured acceleration during a shaking table laboratory test. These steps include applying baseline correction before each integration step and then applying a high pass filter to remove long-period oscillations from the displacement signal. Yang *et al.* (2006) also use a direct integration and base line correction method by assuming the acceleration base line to be parabolic which is then integrated to calculate the mathematical formulae for the velocity and displacement base line correction. The coefficients of the trend line polynomials are calculated using least square curve fitting methods. A high-pass filter is finally used to remove long-period oscillations from the displacement signal. Again, this method cannot be used in real time.

This paper presents a Kalman filter based real time method for calculating displacement and velocity from an acceleration signal. The method is based on the fact that in many vibrating structures, the average of displacement remains constant, which is used to overcome the integrator wind-up problem. This is utilised in the Kalman filter as an additional measurement to overcome the integration difficulties of low frequency noise amplification and integral wind-up. The validity of the proposed method is demonstrated through two laboratory investigations on systems with different specifications in terms of amplitude, frequency and sampling rate. This proposed method is used to calculate the displacement time history of a boat from acceleration signal.

In the following sections, first the Kalman filter equations used to calculate displacement from measured accerelation signal are derived. Then, the validity of suggested method are assessed by conducting two different experiments. The validated method is then utilised to calculate the displacement of a boat from measued acceleration signal in a real environment.

Kalman Filter

The Kalman filter, as a recursive least-square observer, has been applied in areas as diverse as aerospace, marine navigation, nuclear power plant

instrumentation, demographic modeling and manufacturing. It uses a state-space model of the system together with actual measurements to optimally estimate the state variables of the system (Grewal et al. 2008). The calculation of displacement from acceleration can be formulated in state-space form as follows. Assume that the acceleration signal a is sampled at constant time intervals of T_s. The velocity can then be calculated by using the following discrete equation:

$$v(k) = v(k-1) + aT_s \qquad (1)$$

where k is the sample number. The displacement y can be calculated by integrating (1), resulting in:

$$y(k) = y(k-1) + v(k-1)T_s + \frac{1}{2}aT_s^2 \qquad (2)$$

In addition to the measured acceleration, in many applications the average value of the displacement of the system is constant (normaly assumed to be zero) if non-linear behaviour and permanent deformation can be neglected. Calculating the average value, by integrating over one period, requires knowing the frequency of the signal, which is not always possible. Alternatively, a low pass filter with a transfer function of $\dfrac{1}{s + \omega_o}$ may be used to extract the average value. However, to integrate the low frequency components of the signal accurately, the cut-off frequency ω_o needs to be small and in the limit, i.e. when $\omega_o \to 0$, the transfer function of the filter becomes that of an integrator. As a first approximation it is therefore reasonable to assume that average displacement could be approximated to be the integral of the displacement $z = \int y \, dt$ whose measured value is zero. In discrete form,

$$z(k) = z(k-1) + y(k-1)T_s + \frac{1}{2}v(k-1)T_s^2 + \frac{1}{6}aT_s^3 \qquad (3)$$

Considering z to be the output of the system and a to be the input then equations (1), (2) and (3) can be expressed in the following state space form whilst allowing for the process noise $\mathbf{w}(k)$:

$$\mathbf{x}(k) = \mathbf{A}\mathbf{x}(k-1) + \mathbf{B}(\mathbf{u}(k) + \mathbf{w}(k)) \qquad (4)$$

where:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ T_s & 1 & 0 \\ \frac{1}{2}T_s^2 & T_s & 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} T_s \\ \frac{1}{2}T_s^2 \\ \frac{1}{6}T_s^3 \end{pmatrix} \qquad (5)$$

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} v \\ y \\ z \end{pmatrix}, \quad \mathbf{u} = (a) \qquad (6)$$

In matrix form, the output equation is written as:

$$\mathbf{Y} = \mathbf{C}\mathbf{x} + \mathbf{v} \qquad (7)$$

where $\mathbf{C} = (0 \quad 0 \quad 1)$ and $\mathbf{v}$ is the output $\mathbf{Y}$ measurement noise. The algorithm for Kalman filter (Brown et al. 1997) assumes that the noise terms, $\mathbf{w}$ and $\mathbf{v}$, have normal probability distributions with zero mean and covariances of Q and R, respectively:

$$\begin{aligned} p(\mathbf{w}) &\sim N(0,Q) \\ p(\mathbf{v}) &\sim N(0,R) \end{aligned} \qquad (8)$$

In some applications, the process noise covariance Q and the measurment noise covariance R matrices might change with each time step or measurement, however, in this paper they are assumed to be constant. The Kalman filter is a predictor-corrector algorithm. The prediction step contains the time update equations which are utilized to obtain the current state and error covariance estimations. The correction step equations, based on the measurement, provide a feedback to improve the estimated value:

Prediction Step:

$$\hat{\mathbf{x}}^-(k) = \mathbf{A}\hat{\mathbf{x}}(k-1) + \mathbf{B}u(k) \qquad (9)$$

$$\mathbf{P}^-(k) = \mathbf{A}\mathbf{P}(k-1)\mathbf{A}^\mathbf{T} + \mathbf{B}Q\mathbf{B}^\mathbf{T} \qquad (10)$$

Correction Step:

$$\mathbf{K}(k) = \mathbf{P}^-(k)\mathbf{C}^\mathbf{T}\left(\mathbf{C}\mathbf{P}^-(k)\mathbf{C}^\mathbf{T} + R\right)^{-1} \qquad (11)$$

$$\hat{\mathbf{x}}(k) = \hat{\mathbf{x}}^-(k) + \mathbf{K}(k)\left(\mathbf{Y}(k) - \mathbf{C}\hat{\mathbf{x}}^-(k)\right) \qquad (12)$$

$$\mathbf{P}(k) = \left(\mathbf{I} - \mathbf{K}(k)\mathbf{C}\right)\mathbf{P}^-(k) \qquad (13)$$

The matrix $\mathbf{P}$ is the covariance of the error, given by:

$$\mathbf{P} = E\left\{\left(\mathbf{x} - \hat{\mathbf{x}}\right)\left(\mathbf{x} - \hat{\mathbf{x}}\right)^T\right\} \qquad (14)$$

As will be shown later, the assumption of $z = 0$ in the Kalman filter is effective in eleminating dc drift as well as overcoming the unknown initial value problem.

Experimental Methods and Results

Displacement of a Shaker

To validate the technique, two laboratory experiments were conducted. The apparatus used in the first

experiment is shown in Fig. 1. A piezoelectric accelerometer (manufactured by the PCB Company Pty Ltd) was attached to the shaft of an electrodynamic shaker to measure its acceleration. The accelerometer has a maximum range of ± 500 g (g is the gravitational acceleration), a sensitivity of 9.54 mv/g over the frequency range of 1 Hz -10 kHz, and a 10 bit resolution (approximately 1 g). A Keyence laser sensor installed on top of the shaker was used to measure its displacement directly. The sensor has a range of ±40 mm with a resolution of 10 µm. In this experiment, the acceleration and displacement signals were recorded simultaneously.
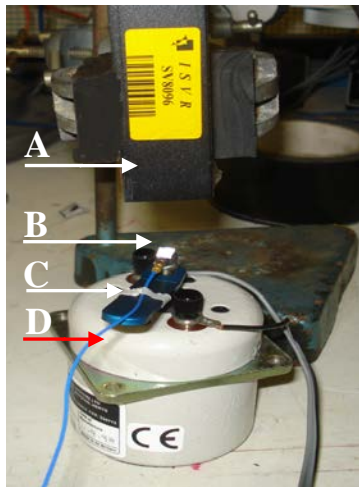
FIG. 1. LAB APPARATUS USED TO MEASURE ACCELERATION AND DISPLACEMENT OF A SHAKER. A) LASER SENSOR, B) ACCELEROMETER, C) SUPPORT, D) SHAKER
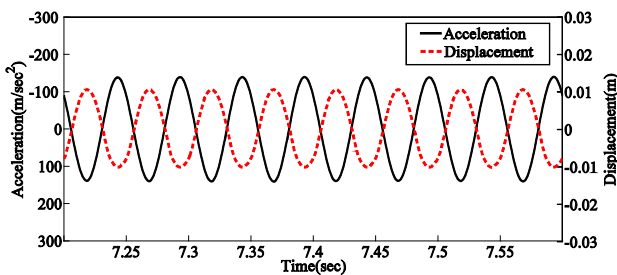
FIG. 2. MEASURED 20 HZ ACCELERATION AND DISPLACEMENT SIGNALS.
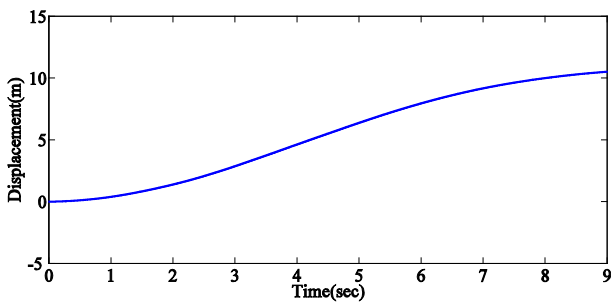
FIG. 3. CALCULATED DISPLACEMENT USING DOUBLE INTEGRATION OF THE ACCELERATION SIGNAL IN THE PREVIOUS FIGURE.
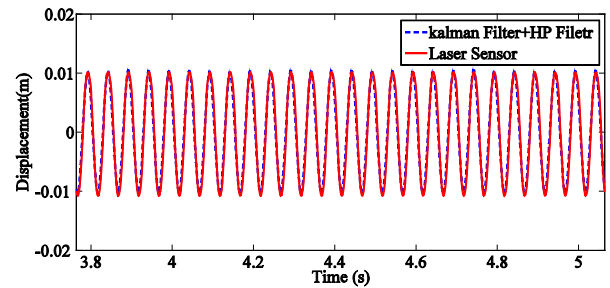

FIG. 4. CALCULATED AND MEASURED DISPLACEMENTS FOR THE ACCELERATION SIGNAL SHOWN IN FIG.1.

Fig. 2 shows a portion of the measured acceleration and displacement signals of the shaker when oscillating at 20 Hz and sampled at 1 kHz. The result of double integration of the acceleration signal, using the trapezoidal rule, is shown in Fig. 3. The figure clearly illustrates the integral wind-up problem. Fig. 4 shows the same displacement now calculated by our proposed Kalman filter method, which is in good agreement with the measured displacement.

The second scenario focuses on calculating the displacement of the shaker when performing random oscillations. The measured acceleration again sampled at a frequency of 1 kHz. Fig. 5 shows the recorded acceleration and displacement signals. The power spectral density of the acceleration by Welch's method, in Fig. 6, shows random excitation over the frequency range of 20-30Hz. A good agreement is again observed in Fig. 7 between the displacement measured by the laser sensor and that estimated using the proposed Kalman filter method. However, there are noticeable differences between the estimated and actual displacements in the vicinity of the peaks and troughs. The proposed method estimates the displacement with 4.8% Normalized RMS Error which is calculated by the following equation (M. D. Miles 1986):

$$NRE\% = 100 \frac{rms[\,y(t) - \tilde{y}(t)\,]}{4\,rms[\,\tilde{y}(t)\,]} \qquad (15)$$

where $y(t)$ is the displacement estimated from the Kalman filter method and $\tilde{y}(t)$ is the actual displacement measured by the laser sensor.
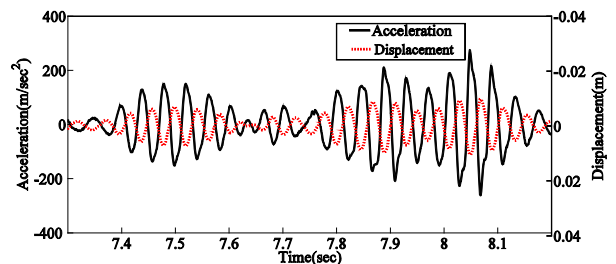

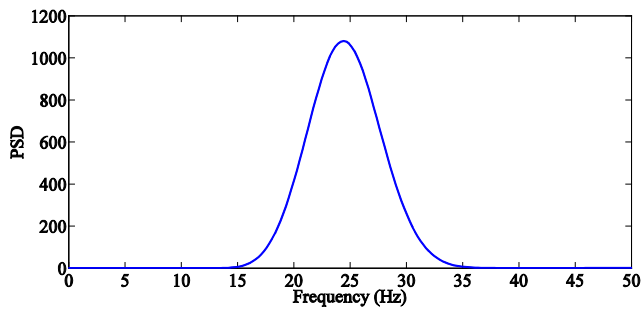FIG. 5. MEASURED ACCELERATION AND DISPLACEMENT WHEN THE SHAKER IS MOVING RANDOMLY

FIG. 6. POWER SPECTRAL DENSITY OF THE MEASURED
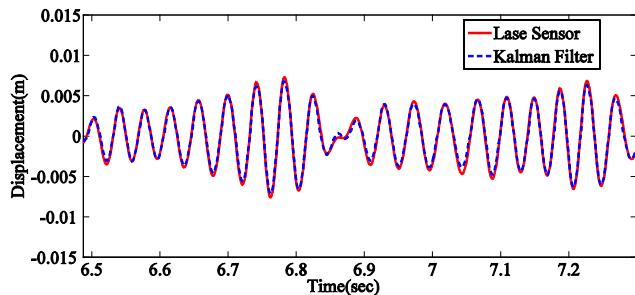RANDOM ACCELERATION OF THE SHAKER

FIG. 7. A MAGNIFIED VIEW OF THE CALCULATED AND
MEASURED RANDOM DISPLACEMENT OF THE SHAKER

a) Acceleration waveform
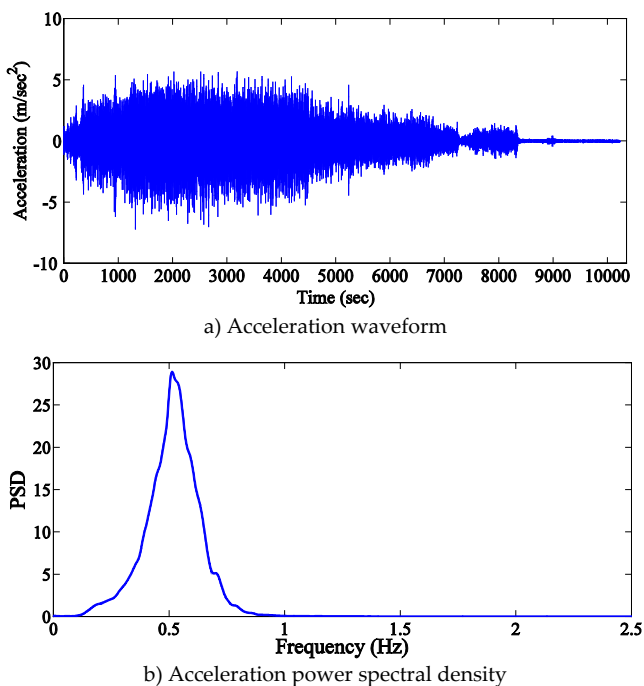

b) Acceleration power spectral density

FIG. 8. TYPICAL BOAT BOW VERTICAL ACCELERATION
MEASURED WHILE SAILING IN THE ENGLISH CHANNEL

Displacement of a Boat

A review of different studies has shown that the vertical movement of typical sailing boats is inherently random with the dominant frequency of vibration being less than 1 Hz (P. Manganelli 2006). This was confirmed by the authors' own boat acceleration measurement obtained while sailing in the English

Channel, as shown in Fig. 8. The boat was a double hull catamaran, 34 feet long, 14 feet wide with a total weight of approximately 3.5 tonnes. The micro-machined silicon static accelerometer was positioned about 1 m from the bow. An HC12 processor was used to record the acceleration of the boat at a rate of 5 Hz and a resolution of g/100 ms^{-2} or 0.0981 ms^{-2}, i.e., a relatively higher resolution but lower sampling rate than the accelerometer used in the shaker experiment described in the previous section.

To verify the accuracy of the proposed Kalman filter method when a low sampling rate accelerometer is used, a second experiment was devised to mimic the motion of the boat in the laboratory. In this experiment, the accelerometer and the associated HC12 processor and batteries were placed in a box attached to a seesaw board as shown in Fig. 9. The wooden board was swung manually and randomly at a frequency less than 1 Hz. The displacement of the accelerometer was measured from images recorded by a 25 frames per second video camera and associated image processing toolbox, which tracks the position of a black square attached to the accelerometer; the resolution is estimated to be less than 1.4 mm. Typical measured acceleration and position signals are shown in Fig. 10. The power spectral density of the acceleration waveform in Fig. 11 shows that it has a dominant frequency of 0.5 Hz, which is similar to that experienced by a boat.

Fig. 12 shows the performance of the proposed Kalman filter method when used to calculate the displacement of the seesaw board. Although the sampling frequency of the acceleration signal is only 5-10 times more than the excitation frequency range, the result shows a reasonable agreement between the measured and calculated displacements curves.

Here, the Kalman filter method estimates the displacement of the board relatively accurately with about 6.9% Normalized RMS Error. This percentage of error is greater than the error involved in the previous experiment. This is mainly due to the low sampling rate (5 Hz) of the data logger used here compared with the high sampling rate (1 kHz) utilised in the previous experiment.

Fig. 13 shows the estimated displacement of the real boat (whose acceleration is shown in Fig. 8), using our proposed Kalman filter method. The figure shows a maximum displacement of 1.25 m, which agrees with the visual estimates (actual boat displacement measurement was not possible). Using direct

integration would result in an parabolically growing displacement curve (i.e., approaching hundreds of meters) due to integral wind-up.
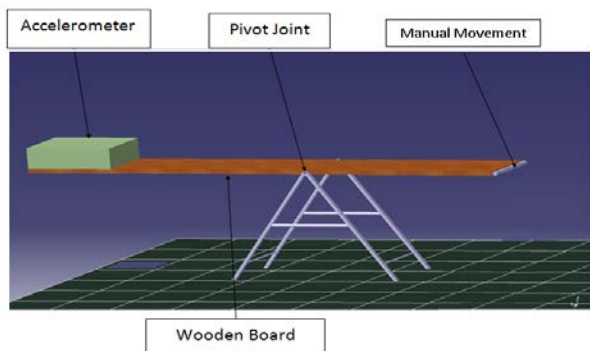

FIG. 9. SECOND EXPERIMENTAL SET-UP USED TO MIMIC THE MOTION OF A BOAT
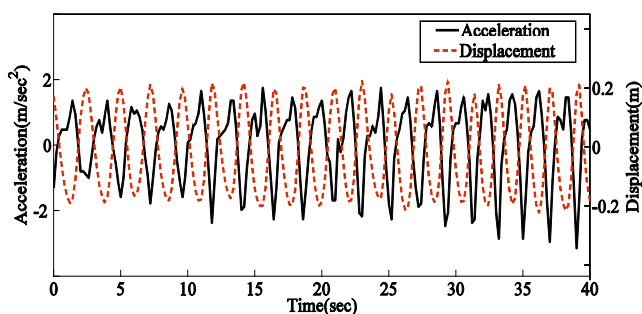

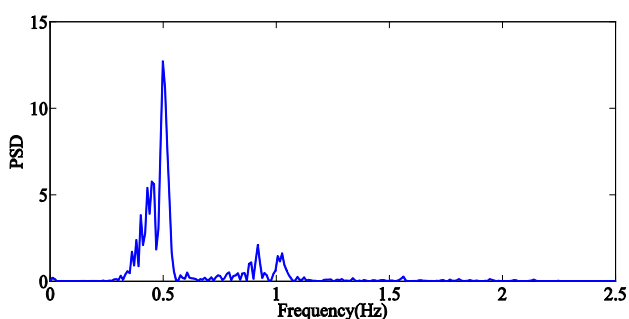FIG. 10. MEASURED ACCELERATION AND DISPLACEMENT OF THE RANDOM MOTION OF THE SEESAW BOARD

FIG. 11. POWER SPECTRAL DENSITY OF THE MEASURED RANDOM ACCELERATION OF THE SEESAW BOARD
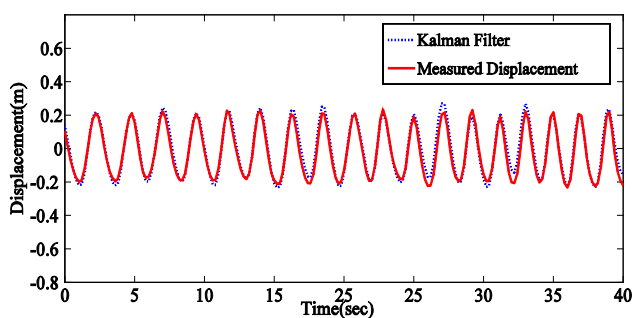

FIG. 12. COMPARISON BETWEEN MEASURED DISPLACEMENT WITH THAT ESTIMATED USING THE PROPOSED KALMAN FILTER FOLLOWED
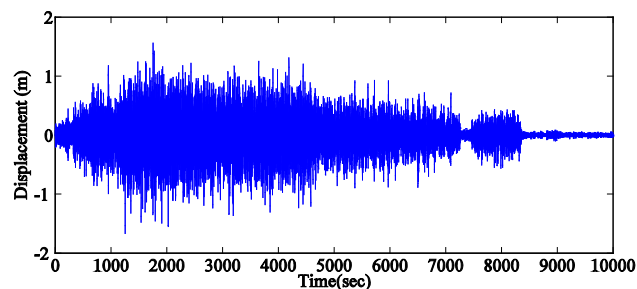

FIG. 13. CALCULATED DISPLACEMENT OF A BOW VERTICAL DISPLACEMENT OF A BOAT WHOSE ACCELERATION IS SHOWN IN FIG. 8.

Conclusions

A Kalman filter based method was introduced to overcome the problems associated with using double integration of an acceleration signal to calculate displacement, namely integration wind-up and amplification of low frequency noise. Integration wind-up is eliminated by incorporating an additional state variable, namely the integral of the displacement whose "measured" value is assumed to be equal to the known average value of the displacement. This, in many applications, can be assumed to be constant provided that permanent deformation and non-linear behaviour are negligible. The effectiveness and the accuracy of the technique were demonstrated experimentally. The accuracy of the method improves when the sampling rate of the acceleration is increased.

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