# Direct velocity feedback versus a geometric controller design of remotely located vibration control systems

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ABSTRACT: The mitigation of human induced vibrations in floors continues to be a key area of research particularly as a result of advancement in material and design technologies enabling the design of light, slender and more open plan structures. These floors are typically characterised by low and close natural frequencies as well as low modal damping ratios, and these combinations of factors contribute to their increased susceptibility to human induced vibrations. Amongst the remedial measures pursued to enhance their vibration serviceability performance, active vibration control (AVC) technologies are emerging as a viable technology and predominantly direct output feedback approaches have been pursued in past analytical studies and field trials.

It has often been assumed that actuators and sensors can be located where vibration attenuation is desired and this may not always be feasible. The research work presented in this paper compares the vibration mitigation performances of the direct velocity feedback scheme that has been extensively used in past floor vibration control researches against a geometric controller design approach that has been developed to provide a design freedom for reducing vibration in both local and remote locations. The geometric controller design approach assumes the inability to locate the actuators and sensors at the remote location but acknowledges that this measurement can be obtained during the commissioning stage and used during the design phase to enhance both local and remote locations. All the analytical and experimental studies are based on a laboratory structure. The work demonstrates comparable vibration mitigation performances of the dominant mode of vibration of the laboratory structure for both approaches but also demonstrates potential for additional enhancement to the second vibration mode of the laboratory structure with the geometric controller design approach. Approximately 20 – 25dB attenuation in the first and second vibration modes of the laboratory structure were achieved.

Keywords: active control, human-induced vibrations, geometric controller design

## 1 INTRODUCTION

Vibration serviceability is now the dominant design criterion for many floor structures, mainly in the commercial sector due to two key factors. Firstly, modern floors are more slender and lightweight than in the past as a result of improved design methods, structural materials and construction technologies. The drivers for improved construction technology include reduced economic cost, increased flexibility of usage, and lower carbon footprints due to reduced material usage. Secondly, there is a trend towards more open plan floor layouts that have fewer full height partition walls than in the past. This reduces the amount of inherent damping present in finished floor structures and hence increased levels of response to human induced excitation. These floors are also characterised by low and

close natural frequencies which fall within the range of frequencies produced by human activities as well as their harmonics.

Amongst the remedial measures pursued to enhance the vibration serviceability performance of office floors, AVC technologies are emerging as a promising and viable technology. This has been seen in some past field trials. In those investigations, combinations of direct output feedback (DOFB) and model-based AVC control schemes were successfully used to augment damping for selected vibration modes in each of the floors studied and in the process suppress human-induced vibrations  $[1,2,3,4,5,6]$ . Predominantly single-input-single-output (SISO) and multi-SISO collocated sensor and actuator pairs were utilized in both the DOFB, for example, direct velocity feedback (DVF) and model-based controller schemes. In most of these past implementations of

AVC in office floors, it was possible to site actuator and sensor pairs at required locations. There was one case in which it was not possible to site the actuators and sensor pairs at a desired location.

Recently, a geometric controller design approach for harmonic or broadband control of remotely located vibration has been developed for trials in the marine and aerospace sectors for large scale structures or those exposed to harsh environments  $[7,8,9]$ . The concept behind this controller design approach is that for large scale structures or where the system environment is harsh, it may not be feasible to locate sensors or even actuators where vibration attenuation is desired or this may be prohibitively expensive. The optimal control of local vibration may in turn result in an enhancement at remote locations. These researches therefore define a design freedom for reducing vibration both at local and remote points, and can be applied for either discrete frequency and/or broadband control and often involves an inversion of the local path plant dynamics. The geometric controller design approach assumes the inability to locate the actuators and sensors at the remote location but acknowledges that this measurement can be obtained during the commissioning stage and used during the design phase to enhance both local and remote locations. This approach may in turn be beneficial for floor vibration control in which, as the authors have found out, the presence of services or other facilities may often hinder the siting of actuators and sensors at desired locations.

The research work presented here compares vibration mitigation performances between the DVF control scheme and the geometric controller design approach. An overview of the design methodologies for each of the controller schemes and appropriate compensators designed to achieve desired closedloop specifications with the plant model known are presented. Some stability studies are shown as well as a brief insight into the use of the geometric controller approach for controlling both local and remote vibration. The work in this paper is organized as follows: section 2 introduces the plant and actuator dynamics as well as an overview of the design methodologies with DVF and the geometric controller design approach. Also shown are results of stability studies for both controller design approaches. In section 3, the results of uncontrolled and controller frequency response functions (FRFs) in both the analytical studies and experimental implementation are presented and some conclusions are included in section 4.

## 2 PLANT MODEL AND CONTROLLER SCHEMES

The laboratory structure used for the AVC studies is a simply-supported in-situ cast post-tensioned slab strip with a span of 10.8 m, width of 2.0 m and depth of 275 mm. The dynamic properties for the AVC studies are evaluated from a point accelerance FRF test in figure 1c using a collocated sensor and actuator pair (S1, A1) as shown in figures 1a and 1b. Figure 1c also shows the transfer function between actuator A1 and remote sensor S2 which is an additional measurement used for the geometric controller design. An analytical model of the point accelerance FRF is obtained in the Laplace domain using the modal expansion approach as shown in Eq. 1. *s* is a complex variable,  $\mu_i \geq 0$ ,  $\zeta_i$  and  $\omega_i$  are the inverse of the modal mass, damping ratio and natural frequency associated with the  $i^{\text{th}}$  mode of vibration. A summary of the modal properties of the first three bending modes of vibration estimated from the EMA test are shown in Table 1.

$$
G_s(s) = \sum_{i=1}^{3} \frac{\mu_i s^2}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}
$$
 (1)

Table 1. Estimated modal properties of laboratory structure.





Figure 1. Plan view of laboratory structure showing test location for AVC studies and measured point mobility FRF for AVC design  $(A1 - \text{actuator } 1, S1 - \text{local sensor } 1, S2 - \text{remote sensor } 2)$ 

The actuators are APS Dynamics model 400 electrodynamic shakers. Their dynamic characteristics, expressed by the transfer function between the force applied to the structure,  $f(t)$ , and the input voltage command,  $v(t)$ , can be described by the linear second order system in Eq. 2 when they are driven in

tion used here.  $K_c = 300$ ,  $\zeta_{act} = 0.07$  and  $\omega_{act} = 8.168$ rads/s are the force-voltage constant, damping ratio and natural frequency, respectively.

$$
G_{act}(s) = \frac{K_c s^2}{s^2 + 2\zeta_{act}\omega_{act}s + \omega_{act}^2}
$$
 (2)

DVF with an inner loop compensation for the actuator is shown in figure 2.  $C_0(s)$  is a lossy integrator that includes a feedback gain component,  $\gamma_{dc}$ , as shown in Eq. 3. This is designed to impart significant damping in the structure. The designed inner loop actuator compensator,  $C_I(s)$ , is shown in Eq. 4. This achieves a closed-loop transfer function of the actuator inner loop with a frequency of 1.30 Hz and a damping ratio of 0.60. As noted before, DVF is implemented in a SISO set-up with the collocated actuator and sensor pairs A1, S1 shown in figure 1b.  $\gamma_{dc}$  = 300 and the DVF scheme is designed to achieve damping ratios of 0.16 and 0.075 in modes 1 and 2, and ensures a gain margin (GM) of 9.5 dB and phase margin (PM) greater than  $30^0$ . A second-order Butterworth filter with cut-off frequency 1.0 - 100 Hz is also implemented in both analytical and experimental studies. Stability studies are evaluated using both the root locus plot  $G_s(s)G_{act}(s)G_{acomp}(s)C_o(s)$  as shown in figure 3.

$$
C_0(s) = \frac{\gamma_{dc}}{s + \beta}, \ C_I(s) = \frac{s^2 + 1.14s + 66.72}{s^2 + 9.80s + 66.72}
$$
 (3,4)



Figure 2. Direct velocity feedback with inner loop actuator compensation

Where:

 $G<sub>s</sub>(s)$  Floor model  $G_{act}(s)$  Actuator model  $G_{bp}(s)$  Band pass filter *f* (*t*) Actuator force  $d_i(t)$  Input disturbance *e*(*t*) Error signal

 $r(t)$  Reference signal  $g(v_e)$  $g(v_e)$ Saturation nonlinearity

 $\ddot{y}(t)$  Structural acceleration response

*y* (*t*) Structural velocity response

 $C_0(s)$  Transfer function of outer loop

 $v(t)$  Final control voltage signal

 $C_I(s)$  Transfer function of inner loop

 $v_e(t)$  Initial control voltage signal

 $x_a(t)$  Displacement of actuator moving mass

 $\ddot{x}_a(t)$  Acceleration of actuator moving mass



Figure 3. Root locus plot for DVF control scheme

The formulation of the geometric controller design approach is derived from the works of  $[8,9,10]$  assuming a two input two output system with the transfer function matrix relating the disturbance and control inputs to the remote (S2 in figure 1c) and local (S1 in figure 1c) vibration outputs as shown in Eq. 5.  $y(s)$  and  $z(s)$  are the locally measured and remote vibration whilst  $u(s)$  and  $d(s)$  are the local control force and remote disturbance force, respectively. Eq. 6 shows the control input as a function of the locally measured vibration. These past works have also been extended to deal with stable controller designs for non-minimum phase dynamics. Through the definition of a design variable,  $\alpha(s)$  in Eq. 7, the location vibration output,  $y(s)$  and remote vibration output,  $z(s)$ , in the presence of the feedback control signal in Eq. 6 can be expressed as shown in Eqs. 8 and 9. Various design objectives can be outlined through the selection of the design variable,  $\alpha(s)$ , for example, attenuation of remote vibration without increment in local vibration, or attenuation in vibration of both the remote and local vibration. Further details of specifying the design objectives for either harmonic or global controller can be seen in $^{[8,9,10]}$ .

$$
\begin{bmatrix} y(s) \\ z(s) \end{bmatrix} = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} u(s) \tag{5}
$$

$$
u(s) = -k(s)y(s) \tag{6}
$$

$$
\alpha(s) = -\frac{g_{11}(s)k(s)}{1 + g_{11}(s)k(s)}\tag{7}
$$

$$
y(s) = [1 + \alpha(s)]g_{12}(s)d(s)
$$
 (8)

$$
z(s) = \left[1 + \alpha(s) \frac{g_{12}(s)g_{21}(s)}{g_{11}(s)g_{22}(s)}\right]g_{22}(s)d(s)
$$
(9)

In the work presented here,  $k(s)$  is selected using a parametric design of  $\alpha(s)$  to attenuate vibration at both the local point (S1) and remote point (S2)

shown in figure 1b. The derived compensators for two different designs: geometric controllers 1 and 2 with this procedure are shown in figures 4a and 4b and figures 5a and 5b show the Nyquist contours for the stability studies. Controller 1 is tuned to offer similar vibration mitigation performance as DVF in both the first and second vibration modes of the laboratory structure whilst with controller 2, it is demonstrated that there is potential to achieve further attenuation in the second mode of vibration i.e. flexibility to isolate and enhance target modes of vibration. From figures 6a and 6b, since the centres of both design 'mobius' circles derived for both local and remote locations from Eqs. 5 to 9 almost over- $\text{lap}^{(7,8,9)}$ , this gives the design freedom to enhance damping at both locations simultaneously. The geometric controller design here includes a 3rd order band pass filter with cut-off frequency  $1.75 - 26.5$ Hz.



**b**)  $k_2(s)$ 

Figure 4. Bode plots of compensators  $k_1(s)$  and  $k_2(s)$  for geometric controller schemes 1 and 2



schemes 1 and 2







b) Circles at 16.8 Hz

Figure 6. Design 'mobius' transformation circles showing the lowest bending modes of vibration of the laboratory structure can be enhanced at both at the local (S1) and remote (S2) points with actuator at local location

## 3 UNCONTROLLED AND CONTROLLED FREQUENCY RESPONSE FUNCTIONS

The results of uncontrolled and controlled frequency response functions (FRFs) predicted analytically and measured experimentally for both the local and remote locations are shown in figures 7 and 8 for DVF, and geometric controller schemes 1 and 2. They reflect approximately  $20 - 25$  dB attenuations of the first and second modes of vibration of the laboratory structure. With the geometric controller (GC) design scheme, since the compensator design incorporates approximate inversion in the plant dynamics, feedback gains have to be curtailed so that the influence of the anti-resonances in the open loop FRF do not degrade the vibration mitigation performances as seen in figures 7 and 8 or in the worst case become sources of instability.



e) Analytical (GC2) f) Experimental (GC2) Figure 7. Uncontrolled and controlled FRFs with DVF and Geometric controllers 1 and 2 – FRFs at local location S1 (Local)





(Remote)

#### 4 CONCLUSIONS

The geometric controller scheme offers a new design freedom for controlling vibration at both a local and remote location by using design mobius circles. This can be tuned to guarantee that a controller designed does not degrade the vibration serviceability performance at both the local and remote location as outlined in the works of  $[7,8,9]$ . This approach to design can be beneficial to AVC of human induced office floor vibrations where the presence of services and other facilities would mean that there are restrictions on the potential locations of sensors and actuators. As a result of the approach making use of an inversion in plant dynamics, additional compensation or gain regulation can be used to ensure the antiresonances of the open loop system do not degrade the vibration mitigation performance objective. In this work, vibration mitigation performances of DVF and the geometric controller schemes have been found to be quite comparable and there is potential for further attenuation of the second mode of vibration with the geometric controller formulation.

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