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Reasoning-and-proving in geometry in school mathematics textbooks in Japan

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*Detailed Response to Reviewers

Dear Gabriel and editor,

We have corrected minor points you made. Thank you for your support. It was great pleasure to work with you.

Reviewer's points	Our amendment
Page 3, line 1: Change "Secondly" to "Second". Page 4, lines 49-50: Some words are missing. Page 14, line 4: You probably meant "Table 5" and not "Table 4". Page 19, line 26: Change "student" to "students". Page 20, line 43: Change "focuses" to "focus".	We corrected these points in our revised manuscript.

Best wishes

Taro

Taro Fujita

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**Highlights (for review)

- We analyse the geometry chapters of a Grade 8 mathematics textbook from Japan
- the emphasis in the textbook is on 'direct' proofs of geometrical statements
- the textbook helps students construct suitable proofs
- students may not fully appreciate the necessity or generality of mathematical proof

Reasoning-and-proving in geometry in school mathematics textbooks in Japan

Abstract: In Japan it is in Grades 7-9, and primarily in geometry, that school students are introduced to the significance and methodology of proof in mathematics. As textbooks play a central role in everyday lessons in Japan, this paper presents an analysis of the geometry chapters of a selected mathematics textbook currently in common use with students aged 13-14 in Grade 8. We show that the emphasis in the textbook is on 'direct' proofs of geometrical statements, accompanied by activities which encourage students to form conjectures. Based on our analysis, we raise critical issues related both to the strengths and weaknesses of such a textbook design on students' understanding of reasoning-and-proving. The strengths, as evidenced by Japanese national data, are that most Grade 8 students in Japan are able to construct suitable proofs – usually based on congruent triangles. The weaknesses, as verified by other research, are that the same students may not fully appreciate the necessity or generality of mathematical proof.

Keywords: school textbooks, geometry education, *reasoning-and-proving*, Japan, congruent triangles

1. Introduction

The findings of international research confirm that teaching the key ideas of proof and proving to all students in lower secondary school (Grades 7-9) is not an easy task (see, for example, Harel & Sowder, 2007; Knuth, Choppin & Bieda, 2009; Mariotti, 2006). The aim of this paper is to contribute to international efforts to improve this situation by analysing the design of relevant components of mathematics textbooks that are in common use by teachers at this level in Japan. There are important reasons for choosing this focus. First, it is the case in Japan that textbooks play a central role in everyday mathematics lessons (for example, see Sekiguchi, 2006). Second, all students in Japan are introduced to the significance and methodology of mathematical proof during their Grade 8 geometry lessons (see Jones and Fujita, 2013). For these reasons, an analysis of the geometry component of a major-selling Grade 8 Japanese textbook should reveal much about the approach to the teaching of proof used in Japan that can help to inform international efforts to improve mathematics teaching. Our premise, as Yackel and Hanna (2003, p. 234) emphasise, is that one of the most challenging undertakings for mathematics educators in their efforts to help students acquire competency in proof is to "design means to support teachers in developing forms of classroom mathematics practice that foster mathematics as reasoning". Textbooks are one important source of such support.

In our research we focus on proof and proving, and, in particular in this paper, on what opportunities textbooks provide for what Stylianides (2009, p. 259) calls "*reasoning-and-proving*" (RP): that is, the classroom activities of "identifying patterns, making conjectures, providing non-proof arguments, and providing proofs". While some features of Japanese textbooks and geometry curriculum have been reported (for example, Howson, 1995; Hoyles et al., 2002), these studies examined features across the whole content of geometry across various countries. As such, the more in-depth study of proof in geometry that we present in this paper complements these earlier analyses. Within the selected textbook, *Mathematics International Grade 8* (Fujii and Matano, 2012) published by Tokyo Shoseki, a textbook that is in common use in Japan, we analyse the geometry content in addressing the following research question:

What characterises the way in which ideas of reasoning-and-proving *are introduced and developed in the geometry chapters of a selected Grade 8 textbook from Japan?*

To address this question, we developed a method of analysing RP in mathematics that enabled us to examine not only sets of exercises for students but also other parts of the textbook such as blocks of narrative. In revealing, through our analysis, the emphasis in the textbook on ‘direct’ proofs and, in these proofs, the central role given to congruency, we illustrate the instructional approaches that the textbook writers intend as ways to enrich students’ learning opportunities of RP. With Japanese mathematics textbooks increasingly available in English, our study should be valuable in an international arena because of the way we provide insight into the Japanese approach to teaching RP. We acknowledge that we report only on one textbook but our sample is from a major publisher and is the most popular textbook in use in Japan at this time. Hence we argue that the choice is appropriate.

2. Relevant research on textbooks in mathematics education

In recognition of the influence of textbooks on classroom practice, a number of studies have examined the content, structure and use of mathematics textbooks; examples include Foxman (1999), Gueudet et al. (2011), Herbel-Eisenmann (2007), Pepin and Haggarty (2001), Remillard et al. (2009), Reys et al. (2004), Rezat (2006), and Valverde et al. (2002). For the purposes of this paper, we restrict ourselves to studies that have focused on proof and proving in the content and structure of textbooks and/or on geometry, plus studies reporting on these aspects of Japanese textbooks.

In research on proof and proving in school mathematics textbooks, Vincent and Stacey (2008) examined a selection of three mathematical topics in a sample of nine Australian textbooks designed for Grade 8. While their focus was a range of issues including the procedural complexity of problems, they found an overall absence of problems requiring deductive reasoning. In a similar vein, Nordstroem and Loefwall (2006), in an analysis of two Swedish mathematics textbooks for students aged 16-18, found that the notion of proof was often left implicit or not defined in a meaningful and mathematically-appropriate way. Along the same lines, Hanna and de Bruyn (1999), in an analysis of a sample of Canadian textbooks for students age 17-18, found that only in the topic of geometry did the textbooks do a 'reasonable job' of providing opportunities to learn proof.

In research that has examined *reasoning-and-proving* (RP) in school mathematics textbooks in more depth, Stylianides (2009) reported on an analysis of tasks from a sample of U.S. textbooks designed for 6th to 8th Grade. This analysis found that less than half (about 40%) of the tasks offered "at least one opportunity for RP" (ibid p. 273). Stylianides also found that tasks in the geometry units in the textbooks "were more likely to design opportunities for proofs than were tasks in the algebra units and less likely to design opportunities for proofs than were tasks in the number theory unit" while the same geometry sections "were less likely to design opportunities for patterns than were tasks in the units in the other two content areas" of algebra and number theory (ibid p. 274). Thompson et al. (2012) also conducted an in-depth study, this time focussing on RP in three broad topics in algebra (specifically 'exponents', 'logarithms', and 'polynomials') across twenty U.S. textbooks aimed at high school students in Grades 9-12. They reported that while "approximately 50% of all the properties of exponents, logarithms and polynomials were justified by some argument in the textbooks", opportunities for students "to use proof-related reasoning in the exercise sets were ... rare" (ibid pp. 282-283).

In turning to research on textbooks from Japan, one thing to note is that the process of developing mathematics textbooks in Japan, as described by Shimizu and Watanabe (2010), is that such textbooks may be published by private publishers but the textbooks need to reflect the official Course of Study and accompanying Teaching Guide that is published by the Ministry of Education (see MEXT, 2008). What is more, all textbooks must pass through a textbook authorization process overseen by a Textbook Authorization Council. The development and review process is such that, according to Shimizu and Watanabe, it takes about three years from the time a publisher begins drafting their textbooks to the time teachers can begin using the books in the classroom. Through this process there are, in practice, usually around seven different textbook series on offer from different publishers.

Investigations by both Howson (1995) and Hoyles et al. (2002) included Japan in cross-national comparisons. Both reports point to the geometry components of textbooks for Grades 7-9 in Japan concentrating on congruence and thence similarity. Peterson (2008), in an analysis of several textbooks in use in Japan, reported that in each of the 7th, 8th and 9th Grade textbooks he examined there were two chapters devoted to geometry. Peterson noted that almost every section of each textbook, whether geometry or another topic in mathematics, began "with a deep thought-provoking question" (ibid, p. 216) aimed at provoking student thinking. Most recently, Miyakawa (2012) provided a brief report on a comparison of the way that proof in geometry is introduced in French and Japanese Grade 8 mathematics textbooks. Amongst other things, Miyakawa reported that congruency of triangles is "quite often proven as a step to prove other properties and plays an important role in the textbook" (ibid, p. 230), though he did not have space in his brief report to provide the relevant data.

Overall, this range of research illustrates how textbooks constitute an important component of what can be called the 'potentially implemented' curriculum; something which mediates between the intended and implemented curriculum. Such existing research suggests that opportunities for RP in textbooks can vary from some to very little, and that geometry and number theory can be topics where such opportunities occur most often. In particular, in textbooks in Japan, attention to *reasoning-and-proving* is likely to be concentrated in Grade 8 geometry, with the topic of triangle congruency predominating. As the detail of how triangle congruency is set out in school textbooks has yet to be the subject of research, this is a gap that this paper also addresses.

3. Reasoning-and-proving in Japanese mathematics education

The specification of the mathematics curriculum for Japan is given in the 'Course of Study' (MEXT, 2008). As no differentiation is stipulated in the 'Course of Study', mixed-attainment classes are common with all students in a Grade using the same textbook. In the 'Course of Study', the term 'proof' first appears in Grade 8 in the topic 'Geometrical Figures' (the other topics at this grade level being 'Numbers and Algebraic Expressions', 'Functions' and 'Making Use of Data). Table 1 gives the detail of the topic 'Geometrical Figures'.

Table 1

'Geometry' in Course of Study for Grade 8 in Japan [source: MEXT, 2008]

As evidenced by the 'Course of Study', established practice in Japan is that mathematical content related to RP is taught within the context of geometry, with the term 'proof' appearing explicitly only in the specification of the content for geometry. This does not mean that RP does not occur in other areas of the mathematics curriculum; rather, it means that RP is not stipulated explicitly.

In Japan, a Grade 8 textbook covers the entire curriculum for that grade level. The list of chapters in the selected textbook (*Mathematics International Grade 8* published by Tokyo Shoseki), and how many lessons each chapter entails, is given in Table 2. As can be seen from the table, around one third of the lessons are devoted to geometry; this is at least 34 lessons out of 105 (each lesson being

50 minutes). The teacher has some additional flexibility to use a few additional lessons for geometry (in total some 12 lessons a year can be used flexibly).

Table 2

Grade 8 mathematics textbook content

More detail of the content related to geometry (as laid out in chapters 4 and 5 of the textbook) is given in Table 3. This table shows that the Grade 8 students in Japan who use the selected textbook study ideas and methods of proof in mathematics through exploring the properties of basic 2D geometrical shapes (that is, triangles and parallelograms).

Table 3

Grade 8 mathematics textbook content in geometry

4. Theoretical framework and method of analysis

4.1. Theoretical framework

The theoretical framework for this study is derived primarily from the work associated with the Third International Mathematics and Science Study (TIMSS), especially the work of Robitaille et al. (1993) and Valverde et al. (2002). In order to provide an in-depth focus on RP, we augment an element of the TIMMS framework through using a conceptualisation of RP informed by work of Stylianides (2009) and of Thompson et al. (2012).

The first consideration of the TIMSS framework is that while textbooks are usually divided into sections that generally correspond to lessons to be taught in the classroom, it is necessary to break such sections down into smaller structures, called ‘blocks’, if a more fine-grained analysis is to take place. From Valverde et al. (2002), a ‘block’ is taken to be one or more paragraphs united by a theme, or one figure or group of figures. Such ‘blocks’ can be characterised as one of the following: 1) narrative (for example, providing the objectives of the lesson, or relevant mathematical terminology, etc), 2) narrative related to another block (for example, separate framed texts that supplements or explains a topic), 3) unrelated narrative (for example, a reminder to the student to do something), 4) a figure related to other block (for example, a figure that supplements a text block), 5) an unrelated figure (for example, a cartoon figure that provides a reminder to the student to do something), 6) a question or exercise set related to another block (a set of questions for students to answer; in the Japanese textbook we analysed exercises are clearly indicated as 'check' or 'Prob.'), 7) an unrelated question or exercise set (a set of questions not related to the main narrative block), 8) an activity (a suggested activity for students to work on or ask questions about; in the Japanese textbook we analysed for this paper, activities are indicated as 'Q'), 9) a worked example (something that, for example, illustrates a way of solving a problem), 10) a block not classified as any of the above.

The second aspect of the TIMSS framework is that it posits that any piece of curriculum material, such as a 'block' (as above) from a textbook, can be characterised in terms of three parameters; these are "subject matter content", "performance expectation" and "perspectives" (Robitaille et al., 1993, p. 43). The content in our case is, as might be expected, the mathematical content. Performance expectations comprise the "kinds of performances or behaviours that a … block of content might be expected to elicit from students" (ibid p. 44). The 'perspectives' aspect relates to "the nature of the discipline exemplified in the material" (ibid) and is not considered in this paper.

To guide our analysis, we utilised the idea of 'blocks' from the TIMSS framework because, as Thompson et al. (2012, p. 255) argued, "looking only at problems in exercise sets might ignore important opportunities for learning that exist in the narrative and worked-out examples".

In terms of RP the TIMSS codes for performance expectations are rather broad and, as Thompson et al. (2012) have shown, do not capture all the nuances of RP. Thus, in conducting our analysis, we modified the relevant performance expectation codes which relate to RP in the TIMSS framework by replacing these with expectations informed by the RP framework proposed by Stylianides (2009).

As noted above, RP appears within the TIMSS textbook analysis framework as one of the 'performance expectations' of a 'block' of text in a textbook chapter; the other performance expectations being 'knowing' and 'using routine procedures'. The approach we developed was to analyse the way in which RP was presented in the textbook by using a set of performance expectation categories informed by the work of Stylianides (2009), specifically: *identifying a pattern*, *making a conjecture*, *providing a proof*, and *providing a non-proof argument*. In terms of identifying a pattern, Stylianides further classified this into either *definite* (it is possible to a draw conclusive statement from some data) or *plausible* (a relation seems evident but it is not possible to draw a conclusive statement). In our analysis we took *identifying a definite pattern* as something which would lead students to conjecture or discover a geometrical statement that might be provable. Following Stylianides, we took non-proof arguments as encompassing *empirical arguments* and *rationales*; the former entailing "validating the claim in a proper subset of all the possible cases covered by the claim" (ibid) and latter being "valid arguments for or against mathematical claims that do not qualify as proofs" (ibid). We took a *direct proof* as, in the usual meaning, a straightforward combination of already-established facts. Reasoning such as refutation was also taken as a proof argument.

Thus, to illustrate our approach to utilising this RP framework, consider the following narrative block (on page 98 of the textbook): "In elementary school, we learned that the sum of the interior angles in a triangle is 180 by actually measuring the angles or by rearranging the angles of a triangle …". We classified this as a non-proof *rationale* as the argument contains no theoretical justification for why the three angles of a triangle sum to 180 degrees.

By taking these considerations, and following a pilot analysis, we utilised the following in terms of possible forms of RP in the textbook:

- Identifying definite patterns;
- Conjecturing and discovering;
- Non-proof argument: empirical;
- Non-proof argument: rationale
- Proof argument: direct proof;
- Proof argument: other reasoning such as refutation.

In summary, Table 4 shows our framework for analysis. To illustrate our use of this framework, we provide an example of our analysis in the next section.

Table 4

Analysis codes – Performance expectations for RP

4.3 Analysis examples

Using the TIMSS framework, we analysed the textbook by following these steps:

- Identifying individual lessons in textbook chapters 4 and 5 (informed by using the teacher guide);
- Dividing each lesson into 'blocks';
- Coding each 'block' in terms of 'content' and those aspects of 'performance expectation' not related to RP (specifically 'knowing' and 'using routine procedures').
- Coding proof related blocks by using RP performance expectations.

To illustrate the steps in identifying ‘block types’ and ‘content’, we take a lesson on isosceles triangles (lesson 224, pp. 122-3) as an example. In this lesson, students learn the conditions for a triangle to be isosceles and how to prove that ‘if the base angles of a triangle are equal, then it is isosceles’. In the lesson we identified 11 blocks in total, including graphics. Three blocks were coded as ‘narrative’, three as ‘graphics’, one as ‘activity’, one as ‘exercise sets’, and one as ‘worked example’. The analysis of the lesson in terms of ‘block’ type and ‘content’ is shown in Table 5.

Table 5
Example of lesson analysis

As mentioned above, in terms of ‘performance expectations’ of a ‘block’ of text, we retained the TIMSS coding for ‘knowing’ and ‘using routine procedures’ as these are important when characterising RP in textbooks. For example, without students knowing definitions they are unlikely to be able to investigate geometrical properties further. To exemplify our analysis, Table 6 shows the coding using our overall RP framework of various ‘blocks’ taken from several sections of the selected textbook. Where appropriate, the justification of each coding is given *in italics* in the table.

Table 6
Example of lesson analysis using the RP framework

To complement the analysis of the textbooks into ‘blocks, we also noted the overall structure of the individual ‘lessons’ in the textbook. We did this by looking at general tendencies of ‘block’ type patterns. We also recorded the way in which RP was built up in the textbook across the relevant geometry chapters through the proving of various geometrical statements.

4.4. Overall method of analysis

Our complete set of analysis codes is listed in Table 4. In coding the 34 lessons in the two selected geometry chapters, we identified 151 ‘blocks’ in Chapter 4 and 148 ‘blocks’ in Chapter 5. In analysing all these 299 ‘blocks’ we used a four-stage process. The first stage was for the two authors to undertake a joint initial coding of all the ‘blocks’ of text in a sample of lessons from each of the two selected chapters on geometry. The second stage was for the first author, as a native speaker of Japanese, to do an initial coding of all the other blocks in the selected chapters. For the third stage, the second author worked through a sample of coded lessons and the two authors then met to discuss and agree the final coding. In the fourth stage, the first author revisited all the coding to make any adjustments in the light of the discussion during the third stage, with a sample of any adjusted codes being checked with the first author. Through this procedure we are confident of the accuracy, validity and reliability of the coding.

5. Findings

5.1. Overall textbook design for geometry

We first report on the overall textbook design in terms of arrangements of block types, content and performance expectations. Our analysis is summarised in Table 7 (to be read in conjunction with the list of ‘block’ type shown in Table 4). Table 7 shows that a variety of block types appear in the textbook geometry chapters, including narratives, activities, worked examples, and exercise sets that include diagrams or make explicit references to related diagrams. Despite this variety, the chapters are well-focused on the selected geometrical content as there are no ‘unrelated instructional narratives’ (coded 3), no ‘unrelated exercise sets’ (coded 7a and 7b) and no ‘other’ blocks (coded 10).

While 'activity' blocks (coded 8a or 8b) are relatively small in terms of the total number of blocks, it is noticeable that about 38% of lessons set out in the geometry chapters of the textbook have a problem-solving activity close to the start of the lesson. A typical textbook lesson, then, has some form of narrative at the start and then proceeds Activity → Exercise or Worked example → Narrative. The lesson shown in Table 5 illustrates this format. In general, by starting from a problem-solving situation, students are expected to form some initial conjectures about the geometrical statements explored in each lesson. In the layout of a lesson, a narrative block which recalls or summarises some facts or theorems accompanied by some exercise sets then follows.

Table 7

Block Type frequency

As to the geometry content, our analysis shows that the geometry chapters concentrated on 1 or 2 topics in each lesson (for example, 2-D basic geometry, 2-D polygons and circles, congruency, construction, measurement of areas and angles, etc) rather than covering a greater variety of geometrical topics (as happens in the textbooks in some countries; see Jones and Fujita, 2013). The major content consisted of '2-D geometry: Polygons and circles' (coded 1.1.3), 76% of 299 blocks, '2-D geometry: Basics (point, line, and angles)' (coded 1.1.2), 34% of 299 blocks), and 'Congruence' (coded 1.2.3), 28% of 299 blocks. Only two of the geometry lessons contained topics from 'Number and Algebra' (the 12th lesson in chapter 4, and the 5th lesson in chapter 5). It was particularly noticeable that content codes for 'transformations' (code 1.2.1) and for 'symmetry' (code 1.2.2) were zero.

Table 8 summarises the performance expectations, and the following were found to be the major ones (noting that almost all blocks could have more than one performance expectation); 'Recalling properties and theorems' (coded

2.1.3), 69.6% of 299 blocks, and ‘Consolidating notation and vocabulary’ (coded 2.1.4), 29.8% of 299 blocks. In addition, 7% of the blocks were coded as ‘Consolidating notation and vocabulary (proof)’.

In terms of RP, 35.5% of the 299 blocks was coded as being related to RP (coded 2.3.1 to 2.3.6). In general, direct proof arguments were dominant: over half of all arguments coded as RP were coded as ‘Proof argument - Direct proof’ (coded 2.3.5). The other significant form of RP was ‘Conjecturing and discovering’ (coded 2.3.2); this comprised about a quarter of the arguments coded as RP. We give a more detailed analysis further below.

When ‘lessons’ are considered, the RP performance expectation is prominent in that 32 out of the 34 geometry lessons (94%) provided activities and exercises that contained at least some RP. These results indicate that Japanese G8 geometry teaching expects students to engage in mathematical *reasoning-and-proving* rather than solely ‘performing routine procedures’ (coded 2.2.2), the latter being 7.7% of 299 blocks (though note that we are not saying that performing routine procedures is not important).

Table 8

Performance expectations frequency

Over the next three sections, we show how we completed our analysis of the forms of RP in the textbook. We do this by first examining RP in non-exercise blocks and then reporting on RP in exercise blocks. We conclude our findings by reporting on the overall approach to RP through triangle congruency in the textbook from Japan.

5.2. RP in non-exercise blocks

The non-exercise blocks are those coded as ‘Central instructional narrative’ (coded 1), ‘Activity’ (coded 8), and ‘Worked example’ (coded 9). Table 9 summarises the results, with the non-exercise blocks (coded 1, 8 and 9) as the rows, and the performance expectations (detailed in the Table 4) as the columns (noting that codes beginning 2.1 entail 'knowing', those beginning 2.2 entail 'routine procedure', and those beginning 2.3 are '*reasoning-and-proving*').

Table 9

Performance expectations in non-exercise blocks

From Table 9 it is evident that many blocks included 'recalling properties and theorems' (the column coded 2.1.3); for example, 95% of activity blocks (the row coded 8) included this, as did 92% of exercise sets (the row coded 9). In contrast, few blocks entailed 'routine procedures' (the column codes beginning with 2.2).

In terms of RP (columns coded 2.3.1 to 2.3.6) there were a number of relevant findings. For example, 'central instructional narrative' (coded as row 1) provided a small number of opportunities of non-proof arguments (shown in columns 2.3.2 and 2.3.3). An example of this is, as noted above, that the lesson on proving the base angles of an isosceles triangle are equal began with a narrative stating that students would have verified this by measurement and folding paper in primary schools (textbook p. 118). Blocks of 'activity' (coded in row 8) and 'worked example' (coded in row 9) were found to have different roles. In terms of the 'activity' blocks, 62% of these blocks (out of 21) were designed to encourage students to form conjectures to be proved. In contrast, 65% of 'worked example' blocks (of 26 blocks) offered concrete examples of how to proceed with a direct proof (coded as column 2.3.5). None of these 'worked example' blocks offered opportunities for other types of proof argument such as finding counter-examples (coded as column 2.3.6).

5.3. RP in exercise blocks

We now turn to consider the 50 exercise blocks that we identified across the two chapters (each block containing an average of 1.8 questions). Table 10 summarises our analysis.

Table 10

Performance expectations in exercise blocks

Here the performance expectation of recalling theorems and properties (coded as 2.1.3 in Table 10) appeared in 100% of this block type. This is because in any exercise block at least one question required the recalling of theorems and/or known properties. A total of 36% of the blocks (adding the blocks coded 2.2.2 and 2.2.3) required the use of routine procedures such as calculating angles in given triangles and/or sets of parallel lines.

In terms of RP (blocks coded 2.3.1 to 2.3.6 in Table 10), it is evident that students are expected to be able to undertake direct proofs of geometrical statements; 70% of the 50 blocks were coded 2.3.5. Some 24% of exercise sets (of 50 blocks) provided opportunities to make conjectures (coded as 2.3.2 in table 10) and then prove them. An example of this was 'in a triangle ABC, let O be a point on side AC such that OA=OB=OC. What is the measure of angle ABC? Explain why' (textbook p. 120); here students would be first expected to make a conjecture that angle ABC was a right angle and would then be expected to know how to prove it. Opportunities for other types of arguments (coded as 2.3.6), though small in number, appeared in exercises such as examining the converse of 'In ΔABC and ΔDEF, if ΔABC is congruent to ΔDEF, then AB=DE'. Even then, some of these opportunities for other types of arguments occurred in content that was coded as 'number and algebra' rather than geometry. It is also noticeable that opportunities for non-proof argument were very small (coded 2.3.3 and 2.3.4 in Table 9).

6. Discussion

From the results and analysis in the previous section, the characteristics of RP in geometry teaching in Japanese textbook can be summarised as follows:

- In Japanese textbooks, direct proof arguments are provided mainly in exercise sets and worked examples, but students experience various aspects of RP in narrative blocks (developing their knowledge about definitions, facts, theorems, and so on, and the differences between proof and non-proof arguments), and activity blocks (used for conjecturing). Congruency plays a key role in providing RP opportunities.

In terms of the general instructional approaches in RP teaching in geometry in Japan, we can characterise this as follows:

- Lessons start from a problem solving situation, with the geometrical facts to be proved and learnt often coming later. A sequence conjecturing → proving is prominent in the process of RP in the textbook.

This latter feature matches what has been reported about Japanese mathematics education (e.g. Shimizu, 1999; Stigler and Hiebert, 1999). Shimizu, for example, reported that, for Japanese teachers, the 'summing up' stage, which summarises facts learnt in a lesson, is very important, and by the time that students reach this stage of a lesson they have spent considerable time investigating or thinking through the facts for themselves and that this is often through a problem-solving situation rather than through performing routine procedures (Shimizu, 1999, p. 192). Thus, as our analysis shows, geometrical facts and theorems studied in lessons do not often come first in a lesson. Rather, such facts and theorems are shown after students have worked on them. This approach to lessons, incorporated into the design of the Japanese textbook, might build up students' view of mathematics that an important thing in learning mathematics is to make a conjecture and then try to prove it.

Our analysis shows that the manner of mathematical proof is built up in the textbook is through proving various geometrical statements. Given the evidence about how curricula approaches influence students' views of geometry of students, this fits with what we report elsewhere (Kunimune, Fujita and Jones, 2010) that Japanese students tend to see geometry as a very formal subject for study and it is this issue that needs some attention. The reason why further attention is needed to this issue is that, notwithstanding the design of Japanese textbooks, research indicates that Japanese students can have difficulties in fully understanding proof in geometry (see, for example, Kunimune, 1987; 2000). For example, data collected between 1987 and 2005 show that, while most 14-15 year-old students in Japan (in the third year of secondary school) can write down a proof, around 70% cannot understand why proofs are needed. This is because the students tend to accept both proof arguments and non-proof arguments as valid 'proofs' which would cover all possible cases to verify or explain a mathematical statement. Even so, given that in Japan up to the age of 15 there is no differentiation by attainment, the fact that most Grade 9 students can write a proof in geometry can be regarded as an achievement thanks to the efforts of Japanese teachers and educators. The issue that we hope our research highlights is the need to help more students understand why proofs are needed.

Considering that the textbook continues to be one of the most influential artifacts for student learning, there are opportunities to improve the design of textbooks for the teaching of proof in geometry. In terms of Japanese textbooks, an improvement is likely to involve providing students with more effective instructional activities so that they can more fully appreciate proof arguments in geometry, not only just emphasising that it is impossible to cover 'all cases' by using only, for example, a few empirical data. From this point of view, our analysis in this paper using the RP framework can provide some insights. As we have shown in this paper, the Grade 8 textbook from Japan that we analysed mainly provides direct proof arguments. While it is understandable that geometry teaching in Japan tries to concentrate on more formal ways of mathematical reasoning, this aspect of proof might be somewhat over-emphasised. This might result in rather too many students not, in fact, fully grasping why arguments based on empirical evidence or other rationales are inappropriate when they learn to write proofs in geometry. As such, textbook writers and teachers could consider providing students with opportunities to consider why, for example, cutting the corners of a triangle and fitting these together is not a mathematical explanation of the sum of the interior angles of a triangle.

Another suggestion is that some exercise sets could offer some learning opportunities to examine the difference between non-proof and proof arguments. By taking such instructional approaches and textbook redesign, students' understanding towards more formal proofs might be enriched. Also there could be more opportunities for types of proof arguments other than direct proofs. For example, while it is reasonable to start from simple and direct proofs for students who have just started learning proving in geometry, opportunities for refutations might be increased as learning progresses. This is because utilising the idea of counterexamples is very powerful in mathematics (e.g. Stylianides and Al-Murani, 2010; Thompson et al., 2012).

Finally, it is clear from our analysis that the geometry chapters in the textbook from Japan focus almost entirely on triangles and parallelograms. Yet as Usiskin (2012, p. 2501) explains "In school geometry in the past half-century, greater attention has been given to coordinate geometry, transformations, applications of geometry, and dynamic geometry technology". It is noticeable that none of this appears in the Japanese textbook. Usiskin concludes his article by arguing that "the more shapes that the geometry covers and the more ideas that relate to the shapes of figures, the better the shape of the geometry" (ibid, p. 2509). It could be that while the Grade 8 Japanese textbook analysed in this paper provides a carefully-crafted exposition of proof in geometry, what is sacrificed is any wider notion of the geometrical by there being no space to include coordinate geometry, transformations, applications of geometry, and dynamic geometry technology.

Author note

The contributions of the authors to this paper were equal; the order of authors is alphabetic by family name.

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Table 1

'Geometry' in Course of Study for Grade 8 in Japan [source: MEXT, 2008]

(1) Through activities like observation, manipulation and experimentation, to be able to find out the properties of basic plane figures and verify them based on the properties of parallel lines.

(a) To understand the properties of parallel lines and angles and basing on it, to verify and explain the properties of geometrical figures.

(b) To know how to find out the properties of angles of polygons based on the properties of parallel lines and angles of triangle.

(2) To understand the congruence of geometrical figures and deepen the way of viewing geometrical figures, to verify the properties of geometrical figures based on the facts like the conditions for congruence of triangles, and to foster the ability to think and represent logically.

(a) To understand the meaning of congruence of plane figures and the conditions for congruence of triangles.

(b) To understand the necessity, meaning and methods of proof.

(c) To verify logically the basic properties of triangles and parallelograms based on the facts like the conditions for congruence of triangles, and to find out new properties by reading proofs of the properties of geometrical figures.

Table 2

Grade 8 mathematics textbook content

Chapter 1 Calculations with algebraic expressions (pp. 6-29)	16 lessons
Chapter 2 Polynomial equations (pp. 30-51)	14 lessons
Chapter 3 Systems of equations (pp. 52-87)	19 lessons
Chapter 4 Parallelism and congruence (pp. 88-115)	15 lessons
Chapter 5 Triangles and quadrilaterals (pp. 116-145)	19 lessons
Chapter 6 Probability (pp. 146-162, 10 lessons)	10 lessons
Total	93 lessons
	plus 12 more lessons are available

Table 3

Grade 8 textbook content in geometry

Sections in Chapters 4 and 5	Number of lessons (each being 50 minutes)
Chapter 4 Parallelism and congruence	
Section 1 Parallel lines and angles	7
Section 2 Congruent figures	8
Chapter 5 Triangles and quadrilaterals	
Section 1 Triangles	8
Section 2 Parallelograms	11
	Total = 34
	four additional lessons can be used flexibly by the teacher

Table 4

Analysis codes – Performance expectations for RP

Category	Code description	
Block type	1 Central instructional narrative 2 Related instructional narrative 3 Unrelated instructional narrative 4 Related graphic 5 Unrelated graphic 6a Exercise Set with diagram 6b Exercise Set without diagram 7a Unrelated Exercise Set with diagram 7b Unrelated Exercise Set without diagram 8a Activity with diagram 8b Activity without diagram 9a Worked example with diagram 9b Worked example without diagram 10 Other	
Content (subject matter topic)	1.1. Geometry: Position, visualisation, and shape	1.1.1. 2-D geometry: Co-ordinate geometry 1.1.2. 2-D geometry: Basics (point, line, and angles) 1.1.3. 2-D geometry: Polygons and circles 1.1.4. 3-D geometry 1.1.5. Vectors
	1.2. Geometry: Symmetry, congruence, and similarity	1.2.1. Transformation 1.2.2. Symmetry 1.2.3. Congruence 1.2.4. Similarity 1.2.5. Constructions using straightedge and compass
	1.3. Measurement	1.3.1. Perimeter, area, and volume 1.3.2. Angle and bearing
Performance Expectations for RP	2.1. Knowing	2.1.1. Representing 2.1.2. Recognising equivalents 2.1.3. Recalling properties and theorems 2.1.4. Consolidating notation and vocabulary 2.1.5. Developing notation and vocabulary (proof) 2.1.6. Recognising aims of lessons
	2.2. Using routine procedures	2.2.1. Using equipment 2.2.2. Performing routine procedures 2.2.3. Using more complex procedures
	2.3. Reasoning-and-proving	2.3.1. Identifying patterns 2.3.2. Conjecturing and discovering 2.3.3. Non-proof argument; empirical 2.3.4. Non-proof argument; rationale 2.3.5. Proof argument; direct proof 2.3.6. Proof argument; other reasoning

Table 5

Example of lesson analysis using the TIMSS framework

Block number & types	Description of textbook	Content
1 (central narrative)	Let's think about what conditions must be satisfied for a triangle to be an isosceles triangle.	1.1.3: 2-D geometry: Polygons and circles
2 (activity)	Q: If you fold a strip of paper as shown below, what kind of triangle is the triangle formed by the overlapping parts? (there is also a diagram)	1.1.3: 2-D geometry: Polygons and circles
3 (related narrative)	This is the same way we folded the paper in 3 on page 117, isn't it?	1.1.3: 2-D geometry: Polygons and circles
4 (unrelated graphic)	Character used throughout the textbook.	NA
5 (graphic)	Two overlapping triangles	1.1.3: 2-D geometry: Polygons and circles
6 (narrative)	In triangle ABC shown above, angle ABC = angle ACB. This can be concluded by noting that the edges of the paper strip are parallel lines.	1.1.2: 2-D geometry: Basics 1.1.3: 2-D geometry: Polygons and circles
7 (exercise set)	Prob. 1: In triangle ABC above, explain the reason why angle ABC = angle ACB.	1.1.2: 2-D geometry: Basics 1.1.3: 2-D geometry: Polygons and circles
8 (worked example)	It has already been proven that when 2 sides of a triangle are equal, then the 2 angles must also be equal. Conversely, can it be said that if 2 angles are equal, 2 sides of the triangle must also be equal? Let's prove "If 2 angles in a triangle are equal, then 2 sides are also equal." In order to do so, we must show that in triangle ABC, from the supposition Angle B = angle C … (the rest of the proof with a diagram follows)	1.1.3: 2-D geometry: Polygons and circles 1.2.3: Congruence
9 (narrative)	From this proof (block 8), we obtain the following theorem. Conditions for isosceles triangles Theorem: if 2 angles in a triangle are equal, then the triangle is an isosceles triangle with the 2 equal angles as the base angles.	1.1.3: 2-D geometry: Polygons and circles

Table 6

Example of lesson analysis using the RP framework

Block reference	Description of textbook	Analysis
Lesson 212 Block 4 Exercise set p. 90	The ideas Yuto and Sakura have for determining the sum of the angles in polygons are shown below. Complete their tables and write the expressions that can be used to calculate the sum of the angles for a pentagon, a hexagon, ...	Recalling properties and theorems Identifying definite patterns (*because a general pattern which will fit a given data is expected to be formed by students*). Conjecturing and discovering (*because it is expected that students realise a pattern they found should be true for all polygons*). Non proof argument – Empirical (*because a proper subset of all the possible cases covered by the claim*).
Lesson 216 Block 1 Narrative p. 98	In elementary school, we learned that the sum of the interior angles in a triangle is 180 by actually measuring the angles or by rearranging the angles of a triangle …	Recalling properties and theorems Non proof argument - Rationale
Lesson 224 Block 2 Activity p. 122	Q If you fold a strip of paper as shown below, what kind of triangle is the triangle formed by the overlapping parts?	Recalling properties and theorems Conjecturing and discovering (*because by this activity students are expected to form a statement but they are not 100% sure this statement is true or not*)
Lesson 224 Block 7 Exercise set p. 122	Prob. 1 In triangle ABC above, explain the reason why angle ABC = angle ACB.	Recalling properties and theorems Proof argument - Direct proof (*because students are expected to prove this by referring to the properties of parallel lines*).
Lesson 2211 Block 2 Worked example p. 132	Ex. 1 Let O be the point of intersection of the diagonals of a quadrilateral ABCD. On diagonal BD, select points E and F such that OE = OF. Prove that quadrilateral AECF is a parallelogram. (a proof follows)	Recalling properties and theorems Proof argument - Direct proof
Lesson 2216 Block 7 Exercise set p. 140	Prob. Is it true that "quadrilaterals that have perpendicular diagonals are rhombi"? If it is not true, give a counterexample.	Recalling properties and theorems Proof argument – Other reasoning such as refutation (*because students are expected to investigate this statement by finding a counterexample).*

Table 7
Click here to view linked References

Table 7

	Narrative			Graphic		Exercise set		Unrelated exercise set		Activity		Worked example		Other
Block type	1	2	3	4	5	6a	6b	7a	7b	8a	8b	9a	9b	10
%	27.8	15.4	0	13.4	11	14.7	2.0	0	0	5.7	1.3	7.7	1.0	0

Numbers indicate percentage per blocks (out of 299 blocks)

Table 8
Click here to view linked References

Table 8

Performance expectations frequency

Knowing						Using routine procedures			Reasoning-and-proving					
2.1.1	2.1.2	2.1.3	2.1.4	2.1.5	2.1.6	2.2.1	2.2.2	2.2.3	2.3.1	2.3.2	2.3.3	2.3.4	2.3.5	2.3.6
5.0	0	69.6	29.8	7.0	9.4	0	7.7	1.0	1.0	8.4	2.0	2.0	20.4	1.7

Numbers indicate percentage per blocks (out of 299 blocks)

Table 9

Click here to view linked References

Table 9

Performance expectations in non-exercise blocks

	Knowing						Using routine procedures			Reasoning-and-proving					
	2.1.1	2.1.2	2.1.3	2.1.4	2.1.5	2.1.6	2.2.1	2.2.2	2.2.3	2.3.1	2.3.2	2.3.3	2.3.4	2.3.5	2.3.6
1	2.4	0	60.2	53	9.6	32.5	0	0	0	0	2.4	4.8	4.8	0	0
8	0	0	95.2	14.3	4.8	0	0	4.8	0	4.8	62	0	0	14.3	4.8
9	3.8	0	92.3	7.7	11.5	0	0	23.1	0	0	0	0	0	65.4	0

Numbers indicate percentage per blocks (block type 1 consisted of 83 blocks, block type 8 was 21 blocks, and block type 9 was 26 blocks)

Table 10

Performance expectations in exercise blocks

Knowing						Using routine procedures			Reasoning-and-proving					
2.1.1	2.1.2	2.1.3	2.1.4	2.1.5	2.1.6	2.2.1	2.2.2	2.2.3	2.3.1	2.3.2	2.3.3	2.3.4	2.3.5	2.3.6
16	0	100	20	14	0	0	30	6	4	24	4	2	70	8

Numbers indicate percentages (out of 50 blocks)