MISO Networks with Imperfect CSIT: A Topological Rate-Splitting Approach

Chenxi Hao and Bruno Clerckx

Abstract—**Recently, the Degrees-of-Freedom (DoF) region of multiple-input-single-output (MISO) networks with imperfect channel state information at the transmitter (CSIT) has attracted significant attention. An achievable scheme, known as Rate-Splitting (RS), integrates common-message-multicasting and private-message-unicasting. In this paper, focusing on the general K-cell MISO IC with an arbitrary CSIT quality of each interfering link, we firstly identify the DoF region achieved by RS. Secondly, we introduce a novel scheme, so called Topological RS (TRS), whose novelties compared to RS lie in a multi-layer structure and in transmitting multiple common messages to be decoded by groups of users rather than all users. The design of TRS is motivated by a novel interpretation of the K-cell IC with imperfect CSIT as a weighted sum of a series of partially connected networks. We show that the DoF region achieved by TRS yields the best known result so far, and we find the maximal sum DoF via hypergraph fractional packing. Lastly, for a realistic scenario where each user is connected to three dominant transmitters, we identify the sufficient condition where TRS strictly outperforms conventional schemes, and show that TRS is optimal for some CSIT qualities.**

I. Introduction

Channel state information at the transmitter (CSIT) is crucial in downlink multi-user transmissions. However, acquiring accurate CSIT is challenging in practical systems. In wireless systems like LTE, the CSIT is obtained by uplink-downlink reciprocity in the Time Division Duplex setup, or by user feedback in the Frequency Division Duplex setup. In multi-cell scenarios, the CSIT has to be shared among the transmitters in order to perform coordinated beamforming and/or joint transmission. Those procedures result in imperfect CSIT due to the channel estimation error, quantization error and the latency in the feedback link and backhaul link. Performing interference mitigation techniques designed for perfect CSIT using imperfect CSIT results in undesirable multi-user interference, which deteriorates the system performance. Hence, the fundamental question that should therefore be addressed is how to design proper transmission strategies for the imperfect CSIT setting.

Recent work [1] found the optimal DoF region of a two-user multiple-input-single-output (MISO) broadcast channel (BC) with a mixture of perfect delayed CSIT and imperfect instantaneous CSIT. However, one corner point of the optimal DoF region is achieved by a Rate-Splitting (RS) approach which does not rely on delayed CSIT and is applicable to the scenario with only imperfect instantaneous CSIT. Reminiscent to the Han-Kobayashi scheme [2], [3], each user's message in RS is split into a common and a private part. The private messages are unicast to their respective intended users along Zero-Forcing (ZF) precoders using a fraction of the total power. The common messages are encoded into a super common message, and the super common message is multicast using the remaining power. At the receiver side, each user firstly decodes the super common message and proceeds to decode the desired private message afterwards using successive interference cancellation (SIC). This RS approach can be easily applied to the K-user MISO BC. Considering that the CSIT error of user k decays with signal-to-noise-ratio (SNR) as $\mathrm{SNR}^{-\alpha_k}$ where $0\leq\alpha_1\leq\cdots\leq\alpha_K\leq1$ is commonly termed as the CSIT qualities, the sum DoF achieved by RS is $1+\sum_{k=1}^{K-1}\alpha_k$. The optimality of this result was shown in [4].

Since then, there have been extensive researches on RS. The sum rate analysis in the presence of quantized CSIT and the precoder optimization were investigated in [5] and [6], [7], respectively. Literature [8] extended the idea of RS into the massive Multiple-Input-Multiple-Output (MIMO) deployment and proposed a Hierarchical RS (HRS) which tackles the multi-user and multi-group interference using a two-layer RS approach. Other related works on MISO BC can be found in [9]–[13]. The application of RS to the two-cell MISO interference channel (IC) was reported in [14]. The scheme was later on extended to the MIMO case with asymmetric number of antennas in [15].

However, designing a scheme suitable for the K-cell IC is a non-trivial step, because the interference overheard by a single user comes through $K-1$ different links and the CSIT qualities may vary across links. A promising idea can be drawn from the HRS designed under massive MIMO setting [8]. In HRS, users are clustered based on the similarity of their transmit correlation matrices. Then, the users in different groups are separated by statistical Zero-Forcing beamforming (ZFBF) using long term CSIT, while the users in the same group are separated by ZFBF using instantaneous CSIT. Due to the imperfect grouping and imperfect instantaneous CSIT, there exists residual intra- and inter-group interference that impacts the system performance. To deal with this problem, RS is enhanced into HRS by integrating an outer RS and an inner RS. The outer RS tackles the inter-group interference by multicasting a system common message to be decoded by all users, while the inner RS tackles the intra-group interference by transmitting a group common message for each group. Using SIC, each user decodes the system common message, the group common message of the corresponding group and the desired private message sequentially.

A similar problem occurs in the K-cell IC if the users can be categorized into groups such that there are identical

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intra-group CSIT qualities, and the intra-group CSIT quality is smaller than the inter-group CSIT qualities. Then, the users belonging to the same and different groups are separated by ZFBF using intra- and inter-group CSIT, respectively. The residual inter- and intra-group interference is tackled by the outer- and inner-layer RS, respectively. Although such a user-grouping method is only applicable to a very limited class of CSIT quality topologies, the concepts of transmitting group common messages in a multi-layer structure sheds light on the central idea to design a transmission strategy for the general K-cell IC with an arbitrary CSIT quality topology. The main contributions are stated as follows.

1) Achievable DoF region of RS: Focusing on the K-cell MISO IC where the CSIT of each interfering link is of an arbitrary quality, we firstly consider a logical extension of the RS designed for two-cell MISO IC. Each transmitter divides the message intended for the corresponding user into a common and a private part. Each private message is unicast using an arbitrary fraction of the total power, while the remaining power at each transmitter is employed to multicast the common message to be decoded by all users. We characterize the resultant DoF region and show that it covers the DoF region achieved by conventional ZFBF (private message transmission only) with power control.

2) Topological RS with weighted sum interpretation: We propose a novel scheme so called Topological RS (TRS), that is suitable for the general K-cell MISO IC with an arbitrary CSIT quality topology. Unlike RS, each user's message in TRS is split into N parts, i.e., $\mathcal{W}_k \triangleq \{w_k^1, w_k^2, \cdots, w_k^N\}$, where w_k^1 is a private message to be decoded by user k, while $w_k^i, i \geq 2$, is a common message to be decoded by a group of users $\mathcal{R}_k^i$. The power allocated to the common messages and the user group $\mathcal{R}_k^i$ are determined based on the specific CSIT quality topology, so that the group common message w_k^i is drowned into the noise at other users via ZFBF. Compared to RS, this operation reduces the number of common messages to be decoded by each user, thus yielding a DoF region no smaller than that achieved by RS. Besides, an upper-bound of the DoF region is derived using the key results of [4]. This result shows that TRS (and RS) is optimal in the two-cell case.

The TRS scheme is inspired by a novel interpretation of the K-cell MISO IC with imperfect CSIT as a weighted sum of a series of partially connected networks superposed in the power domain. The weights of the partially connected networks stand for their separations in the power domain. This weighted sum interpretation explicitly shows whether or not a user is interfered with one another, thus helping us generating group common messages. Moreover, the DoF region achieved by TRS is interpreted as a weighted sum of that achieved in those networks, thus allowing us to employ methodologies applicable for partially connected networks to analyze the DoF region achieved with imperfect CSIT.

3) Sum DoF using graph theory tools: As a consequence of the weighted sum interpretation, studying the sum DoF achieved by TRS is equivalent to studying the sum DoF in each obtained partially connected network. Then, for each partially connected network, we propose two common message group-casting methods using graph theory tools. These two methods called *orthogonal groupcasting* and *maximal groupcasting*[1] are respectively built upon the packing and fractional packing of the hypergraph defined by the network topology. The maximal groupcasting method yields the maximal sum DoF in each partially connected network, thus giving the maximal sum DoF achieved by TRS. This sum DoF is no less than that achieved by RS and ZFBF with power control.

4) Results in realistic scenarios: As it has been shown that in many practical deployments each user has two dominant interferers [16], we consider a realistic setting where each user is connected to its closest three transmitters. We design TRS for a class of CSIT quality topology, which is featured by that the two incoming interfering links associated with each user have unequal CSIT qualities a and b where $0 \leq a \leq b \leq 1$. With maximal groupcasting, we characterize the sum DoF achievable by TRS and show that it is within the range $\left[\frac{K}{3}(1+\frac{b}{2}+\frac{3a}{2}), \frac{K}{3}(1+b+a)\right]$. For a cyclic CSIT quality topology, we find that the proposed TRS approach strictly outperforms ZFBF with power control as long as $b+3a > \frac{6}{K}\lfloor\frac{K}{2}\rfloor - 2$, where $\lfloor\frac{K}{2}\rfloor$ is the maximum integer that is not greater than $\frac{K}{2}$. Moreover, the tightness of the sum DoF achievable by TRS is evaluated. The key findings are two-fold: 1) TRS is optimal when $b=1$, and 2) for nearly half of the values of a and b, TRS achieves more than 90% of the upper-bound for all the CSIT quality patterns with $K=6,7,8,9$.

At the time of submission, another multi-layer RS approach has been proposed in [17] focusing on a K-cell quasi-static SISO IC with almost no CSIT. Unlike our scheme where the power allocation and rate-splitting is designed based on imperfect CSIT, the proposed scheme in [17] is carried out with an even power allocation for all the messages, and each user decides the message set to be jointly decoded and the message set to be treated as noise according to the local CSIR. This scheme is shown useful in minimizing the outage probability and improving the fairness.

The rest of the paper is organized as follows. The system model is introduced in Section II. In Section III, we revisit ZFBF with power control and characterize the DoF region achieved by RS with common message multicasting. In Section IV, we propose the generalized framework of TRS approach together with its weighted sum interpretation, and study its achievability. Section V studies the sum DoF achieved by TRS in realistic scenarios and evaluates the tightness. Section VI concludes the paper.

Notations: Bold upper and lower letters denote matrices and vectors respectively. A symbol not in bold font denotes a scalar. $(\cdot)^H$, $(\cdot)^T$ and $(\cdot)^\perp$ respectively denote the Hermitian, transpose and the null space of a matrix or vector. $\|\cdot\|$ refers to the norm of a vector. $\mathrm{rowrk}(\mathbf{A})$ stands for the row rank of matrix $\mathbf{A}$, while $\mathrm{span}(\mathbf{A})$ refers to the subspace spanned by $\mathbf{A}$. The term $\mathbf{1}_M$ refers to a $M \times 1$ vector with all 1 entries. For a set $\mathcal{A}$, $|\mathcal{A}|$ represents its cardinality; for a complex number a, $|a|$ stands for its absolute value. The term 1_C is the indicator function, it is equal to 1 if condition C holds; otherwise, it is equal to 0. $\mathbb{E}[\cdot]$ refers to the statistical expectation. $(a)^+$ stands

[1]When a common message is to be decoded by a subset of all users, it is referred as a common message groupcasting.

for $\max(a,0)$. $a \bmod n$ calculates the modulus of integer a with the respect of integer n. $\lfloor a \rfloor$ refers to the maximal integer that is no greater than a.

II. System Model

A. K-cell Interference Channel

In this paper, we consider a K-cell interference channel, where each transmitter is serving one user in each cell. We assume that there is a sufficient number of antennas, i.e., K, at each transmitter, while there is a single antenna at each user. In the presence of perfect CSIT, this antenna configuration allows each user to have an interference-free reception of its desired signal. Thus, it is convenient for us to study the fundamental impact of having imperfect CSIT so as to derive an effective transmission strategy. Given this setting, the signal transmitted by a certain transmitter is denoted by $\mathbf{s}_k \in \mathbb{C}^{K\times 1}, \forall k \in \mathcal{K}$ where $\mathcal{K} \triangleq \{1,\cdots,K\}$, and it is subject to the power constraint P. Then, the received signals write as

$$y_k = \sum_{j=1}^{K} g_{kj} \mathbf{h}_{kj}^H \mathbf{s}_j + n_k, \forall k \in \mathcal{K}, \tag{1}$$

where n_k is the additive white Gaussian noise with zero mean and unit variance; $\mathbf{h}_{kj} \in \mathbb{C}^{K\times 1}$ represents the channel between transmitter j and user k, whose entries are i.i.d Gaussian with zero mean and unit variance; $g_{kj} \in \{0,1\}$, $\forall k,j \in \mathcal{K}$, is a binary variable. When $g_{kj}=1$, it means that transmitter j is connected to user k. When $g_{kj}=0$, it means that the signal sent out by transmitter j is drowned into the noise at user k due to the path loss. For convenience, let us use $\mathcal{G} \triangleq \{g_{kj}\}_{\forall k,j \in \mathcal{K}}$ to denote the network topology.

Throughout the paper, we consider $g_{kk}=1$, $\forall k \in \mathcal{K}$, and thus P is referred to as the SNR. For the interfering links, we consider that

- in Section III and IV, we have $g_{kj}=1$, $\forall k \in \mathcal{K}$ and $\forall j \in \mathcal{K} \setminus j$. This indicates a fully connected network where the interference-to-noise-ratio (INR) is equal to SNR;
- in Section V, if $j=(k-1) \bmod K, (k+1) \bmod K$, we have $g_{kj}=1$; otherwise $g_{kj}=0$. This corresponds to a homogeneous cellular network where user k is only connected to three dominant transmitters [16], i.e., transmitter k, $k-1$ and $k+1$. Note that a cyclic setting is assumed such that user 1 is connected to transmitter K, 1 and 2, while user K is connected to transmitter $K-1$, K and 1.

B. CSIT Quality Topology

We consider that the channel vector is expressed as $\mathbf{h}_{kj} = \hat{\mathbf{h}}_{kj} + \tilde{\mathbf{h}}_{kj}$, where $\hat{\mathbf{h}}_{kj}$ is the imperfect CSIT and $\tilde{\mathbf{h}}_{kj}$ represents the CSIT error, drawn from a continuous distribution.

For the link with $g_{kj}=1$, following the classical model firstly introduced in [1], [18], we define the CSIT quality as

$$a_{kj} \triangleq -\lim_{P\to\infty} \frac{\log_2 \mathbb{E}\left[|\mathbf{h}_{kj}^H \hat{\mathbf{h}}_{kj}^{\perp}|^2\right]}{\log_2 P}, \quad \forall k \in \mathcal{K}, \forall j \in \mathcal{K} \setminus k, g_{kj}=1, \tag{2}$$

where the quantity $\mathbb{E}\left[|\mathbf{h}_{kj}^H \hat{\mathbf{h}}_{kj}^{\perp}|^2\right]$ represents the strength of the residual interference resulting from the use ZFBF with imperfect CSIT. The expectation is taken over both the imperfect CSIT $\hat{\mathbf{h}}_{kj}$ and the channel vector $\mathbf{h}_{kj}$. This expression is equivalent to $\mathbb{E}\left[|\mathbf{h}_{kj}^H \hat{\mathbf{h}}_{kj}^{\perp}|^2\right] = P^{-a_{kj}} + o(P^{-a_{kj}})$ when $P \to \infty$. This quantity implies that if transmitter j unicasts a ZF-precoded private message using power $P^{a_{kj}}$, then the residual interference at user k is drowned into the noise. From a DoF perspective, when $a_{kj} \geq 1$, it is equivalent to having perfect CSIT because the interference can be forced within the noise level and the full DoF K can be achieved by ZFBF [1], [18]; when $a_{kj}=0$, it is equivalent to the case without CSIT [1], [18], because the interference term is received with the same power level as the desired signal and the resultant sum DoF is 1. Hence, in this paper, we only focus on the case $0 \leq a_{kj} \leq 1, \forall k \in \mathcal{K}, \forall j \in \mathcal{K} \setminus k, g_{kj}=1$.

However, in Section V, the CSIT quality of the link with $g_{kj}=0$ is not defined, as the strength of the signal sent by transmitter j is drowned into the noise at user k even without performing ZFBF.

Moreover, we consider that the CSIT qualities vary across the links. This leads to a CSIT topology defined by $\mathcal{A} \triangleq \{a_{kj}\}_{\forall k \in \mathcal{K}, \forall j \in \mathcal{K} \setminus k, g_{kj}=1}$. Note that the CSIT qualities of the direct links a_{kk}, $\forall \in \mathcal{K}$, is assumed to be no smaller than the CSIT qualities of the interfering links, i.e., $a_{kk} \geq a_{kj}$. The CSIT quality of the direct links are not included in the CSIT topology because their values only offer beamforming gain, which does not make a difference on the DoF performance. A CSIT quality topology $\mathcal{A}$ can be also defined using a table (see the fully connected IC in Figure 1(a) for example), where each row stands for the CSIT qualities of the incoming links of a certain user, while each column represents the CSIT qualities of the outgoing links of a certain transmitter.

C. Rate-Splitting

The message of each user is assumed to be split into N parts, i.e., $\mathcal{W}_k \triangleq \{w_k^1, w_k^2, \cdots, w_k^N\}$, where w_k^1 is the private message to be decoded by user k only, while $w_k^i, i \geq 2$ is a common message to be decoded by a group of users $\mathcal{R}_k^i$. We consider that each transmitter only has the message intended for its corresponding user. With imperfect local CSIT, the knowledge of the network topology $\mathcal{G}$ defined in Section II-A, and the CSIT quality topology $\mathcal{A}$ defined in II-B, the encoding function for each transmitter can be expressed as

$$\mathbf{s}_k = f(\mathcal{W}_k, \hat{\mathbf{h}}_{kk}, \{\hat{\mathbf{h}}_{kj}\}_{\forall j \in \mathcal{K} \setminus k}, \mathcal{G}, \mathcal{A}), \forall k \in \mathcal{K}. \tag{3}$$

At the receiver side, we consider that there is *perfect local CSIR*, namely user k perfectly knows the effective channels, i.e., the multiplication of the precoders and the channel vectors, so as to decode the desired signal. Let R_k^i denote the rate of message w_k^i. A rate tuple $\left(\{R_k^1\}_{k\in\mathcal{K}}, \cdots, \{R_k^N\}_{k\in\mathcal{K}}\right)$ is said achievable if private message w_k^1 is decoded by user k, and common message $w_k^i, i \geq 2$ is decoded by the group of users $\mathcal{R}_k^i$, with an arbitrary small error probability. Then, the achievable DoF of a certain message w_k^i is defined as $d_k^i \triangleq \lim_{P\to\infty} \frac{R_k^i}{\log_2 P}$. The achievable DoF of user k is computed by $d_k = \sum_{i=1}^{N} d_k^i$.

Throughout the paper, the terminology *common message groupcasting* means that a common message w_k^i is to be decoded by a group of users $\mathcal{R}_k^i$. When the group contains all users, i.e., $\mathcal{R}_k^i{=}\mathcal{K}$, the common message groupcasting becomes *common message multicasting*. When the group is formed by only one user, i.e., $\mathcal{R}_k^i{=}\{k\}$, it actually refers to a *private message unicasting*.

III. Achievable DoF Regions by ZFBF and RS

In this section, focusing on a fully connected network with equal SNR and INR, we revisit two benchmark schemes, i.e., conventional ZFBF with power control and the RS approach with common message multicasting. For RS, we also derive an achievable DoF region in the fully connected K-cell MISO IC with imperfect CSIT.

A. ZFBF with power control

In conventional ZFBF with power control, transmitter k delivers a private message w_k to the corresponding user using power P^{r_k}, $r_k{\le}1$, along a ZF-precoder $\mathbf{p}_k{\subseteq}\,\mathrm{span}(\{\hat{\mathbf{h}}_{jk}^{\perp}\}_{\forall j\in\mathcal{K}\setminus k})$. The signal received by user k can be expressed as

$$y_k{=}\underbrace{\mathbf{h}_{kk}^H\mathbf{p}_k w_k}_{P^{r_k}}+\sum_{j\in\mathcal{K}\setminus k}\underbrace{\mathbf{h}_{kj}^H\mathbf{p}_j w_j}_{P^{r_j-a_{kj}}}+\underbrace{n_k}_{P^0}. \tag{4}$$

By treating the undesired private message as noise, the DoF achieved by each private message writes as

$$d_k{\le}\left(r_k{-}\max_{j:j\in\mathcal{K}\setminus k}(r_j{-}a_{kj})^+\right)^+,\forall k{\in}\mathcal{K}. \tag{5}$$

This expression specifies the DoF region achieved by ZFBF with power allocation policy $\mathbf{r}{\triangleq}(r_1,\cdots,r_K)$. The DoF region achieved by ZFBF with power control, denoted by $\mathcal{D}_{ZF}$, is the union of the DoF regions achieved with all the possible power allocation $\mathbf{r}$ where $r_k{\le}1,\forall k{\in}\mathcal{K}$.

Notably, by performing ZFBF, the expression in (4) can be regarded as the received signal in an IC where the direct links have unit gain, while the strength of the interfering link is $P^{-a_{kj}}$, $\forall k{\neq}j$. Hence, a concise expression of $\mathcal{D}_{ZF}$ by eliminating the variables $\mathbf{r}$ can be obtained using [19, Theorem 5].

B. Rate-Splitting with common message multicasting

The RS approach was firstly introduced focusing on a 2-cell MISO IC with a symmetric CSIT setting, i.e., $a_{12}{=}a_{21}{=}a$. In [14], one user's message is split into a common and a private part, while the other user's message has a private part only. By unicasting the private messages along ZF-precoders using power P^a, and multicasting the common message using the remaining power $P{-}P^a$, the RS scheme achieves the sum DoF $1{+}2a$.

The key ingredient of RS lies in forcing the residual interference caused by ZFBF with imperfect CSIT to the very weak interference regime, while introducing a strong interference, i.e., the common message, which is decodable by treating the private messages as noise. However, the achievability of RS in the general K-cell MISO IC remains an open problem. Here, we propose a logical extension of RS to the K-cell MISO IC. We consider that a certain group $\mathcal{S}{\subseteq}\mathcal{K}$ of users are active, while the remaining users are made silent. This assumption allows us to obtain an achievable DoF region by taking the union of all the possible subsets $\mathcal{S}{\subseteq}\mathcal{K}$ of users.

Specifically, let us consider a general RS approach where each active user's message is split into a private part w_k^p and a common part w_k^c, $\forall k{\in}\mathcal{S}$. These two messages are transmitted using power P^{r_k} and $P{-}P^{r_k}$, respectively, where $r_k{\le}1$. The common messages $\{w_k^c\}_{k\in\mathcal{S}}$ are to be decoded by all the active users. The transmitted signal and received signal are expressed as

$$\mathbf{s}_k{=}\underbrace{\mathbf{p}_k^c w_k^c}_{P-P^{r_k}}+\underbrace{\mathbf{p}_k^p w_k^p}_{P^{r_k}},\forall k{\in}\mathcal{S}, \tag{6}$$

$$y_k{=}\sum_{\forall j\in\mathcal{S}}\underbrace{\mathbf{h}_{kj}^H\mathbf{p}_j^c w_j^c}_{P}+\underbrace{\mathbf{p}_k^p w_k^p}_{P^{r_k}}+\sum_{\forall j\in\mathcal{S}\setminus k}\underbrace{\mathbf{h}_{kj}^H\mathbf{p}_j^p w_j^p}_{P^{r_j-a_{kj}}}+\underbrace{n_k}_{P^0}, \tag{7}$$

respectively, where $\mathbf{p}_k^p{\subseteq}\,\mathrm{span}(\{\hat{\mathbf{h}}_{jk}^{\perp}\}_{\forall j\in\mathcal{S}\setminus k})$ are ZF-precoders, while $\mathbf{p}_k^c$ are random precoders. Note that in the transmitted signal given by (6), the quantities underneath represent the exact power of the corresponding terms, whose summation is subject to the power constraint P. However, in the received signal given by (7), the quantities underneath are the dominant part of the received power at infinite SNR, which are convenient for DoF calculation. In the rest of this paper, we reuse these notations for all the transmitted and received signals.

Each user firstly decodes all the common messages, and secondly recovers the desired private message after removing the common messages using SIC. Then, the DoF tuple achieved by the private messages and the common messages, denoted by $(d_1^p,\cdots d_K^p)$ and $(d_1^c,\cdots d_K^c)$ respectively, are such that

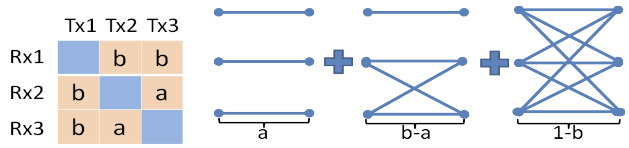
$$\begin{aligned}&\sum_{k\in\mathcal{S}}d_k^c{\le}1{-}\max_{j\in\mathcal{S}}r_j,\\ &d_k^p{\le}\left(r_k{-}\max_{j:j\in\mathcal{S}\setminus k}(r_j{-}a_{kj})^+\right)^+,\forall k{\in}\mathcal{S};\\ &d_j^c{=}d_j^p{=}0,\forall j{\in}\mathcal{K}\setminus\mathcal{S}.\end{aligned} \tag{8}$$

The achievable DoF region by RS with active user set $\mathcal{S}$ and power allocation policy $\mathbf{r}$, denoted by $\mathcal{D}_{RS}(\mathcal{S},\mathbf{r})$, is the set of all DoF tuple $(d_1,\cdots,d_K){=}(d_1^c,\cdots,d_K^c){+}(d_1^p,\cdots,d_K^p)$, for which (8) holds.

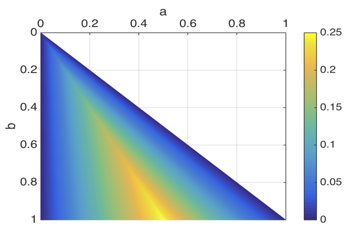
Then, the DoF region achieved by RS results from the union of the DoF regions achieved with all possible subsets $\mathcal{S}$ and power allocation policy $\mathbf{r}$, i.e., $\mathcal{D}_{RS}{\triangleq}\bigcup_{\forall\mathcal{S}\subseteq\mathcal{K},\forall\mathbf{r}}\mathcal{D}_{RS}(\mathcal{S},\mathbf{r})$. To present the closed form expression of the DoF region achieved by RS, let us introduce user set $\mathcal{U}$ whose power allocation policy is defined as

$$r_k{\le}0,k{\in}\mathcal{S}\setminus\mathcal{U};\quad r_k{-}\max_{j:j\in\mathcal{S}\setminus k}(r_j{-}a_{kj})^+{\ge}0,k{\in}\mathcal{U}. \tag{9}$$

Then, with the proof presented in Appendix A, we can eliminate the variable $\mathbf{r}$ and state the DoF region achieved by



(a) CSIT quality table (b) Weighted sum interpretation



(c) Sum DoF gain offered by TRS over RS, i.e., $\frac{\min\{a,b-a\}}{\max\{1+2a,1+b\}}$

Fig. 1: 3-cell IC with hierarchical CSIT quality topology, where $0\le a\le b\le 1$.

RS as $\mathcal{D}_{RS}=\bigcup_{\forall\mathcal{S}\subseteq\mathcal{K},\forall\mathcal{U}\subseteq\mathcal{S}}\mathcal{D}_{RS}(\mathcal{S},\mathcal{U})$, where $\mathcal{D}_{RS}(\mathcal{S},\mathcal{U})$ is the set of $(d_1,\cdots,d_K)=(d_1^c,\cdots,d_K^c)+(d_1^p,\cdots,d_K^p)$ such that

$$d_k^p=0,\forall k\in\mathcal{K}\backslash\mathcal{U};\, 0\le d_k^p\le 1,\forall k\in\mathcal{U};$$
$$\sum_{l=1}^{m} d_{i_l}^p\le\sum_{l=1}^{m} a_{i_{l-1}i_l},\forall(i_1,\cdots,i_m)\in\Pi_{\mathcal{U}}; \tag{10}$$

$$d_k^c=0,\forall k\in\mathcal{K}\backslash\mathcal{S};\, 0\le d_k^c\le 1,\forall k\in\mathcal{S};\, 0\le d_k^p+\sum_{j\in\mathcal{S}} d_j^c\le 1,\forall k\in\mathcal{U};$$
$$\sum_{j\in\mathcal{S}} d_j^c+\sum_{l=1}^{m} d_{i_l}^p\le 1+\min_{j=1,\cdots,m}\sum_{l=1,l\neq j}^{m} a_{i_{l-1}i_l},$$
$$\forall(i_1,\cdots,i_m)\in\Pi_{\mathcal{U}}, \tag{11}$$

where $\Pi_{\mathcal{U}}$ is the set of all possible cyclic sequences[2] of all subsets of $\mathcal{U}$ with cardinality no less than 2.

In (11), we see that for a certain set $\mathcal{U}$, by setting $d_k^c=0,\forall k\in\mathcal{K}\backslash\mathcal{S}$, $\mathcal{D}_{RS}(\mathcal{K},\mathcal{U})$ becomes $\mathcal{D}_{RS}(\mathcal{S},\mathcal{U})$. Then, it is immediate that $\mathcal{D}_{RS}(\mathcal{S},\mathcal{U})\subseteq\mathcal{D}_{RS}(\mathcal{K},\mathcal{U})$. This fact allows us to settle the achievable DoF region by RS as follows.

Proposition 1. *In a fully connected K-cell MISO IC with equal SNR and INR and with CSIT quality topology $\mathcal{A}$, the DoF region achieved by RS with common message multicasting is*

$$\mathcal{D}_{RS}=\bigcup_{\forall\mathcal{U}\subseteq\mathcal{K}}\mathcal{D}_{RS}(\mathcal{K},\mathcal{U}), \tag{12}$$

where $\mathcal{D}_{RS}(\mathcal{K},\mathcal{U})$ is obtained by taking $\mathcal{S}=\mathcal{K}$ in (10).

Remark 1. *Note that the DoF region achieved by ZFBF with power control can be obtained by removing the inequalities related to the common messages, i.e.,* (11)*, and setting $d_k=d_k^p$.*

[2]A cyclic sequence is a cyclically ordered subset of user indices without repetitions [19]. For a certain subset $(i_1,\cdots,i_m)$, there are $(m-1)!$ distinct cyclic orders. For a user set $\mathcal{U}$, there exist $\sum_{m=2}^{|\mathcal{U}|}\binom{|\mathcal{U}|}{m}$ different subset $(i_1,\cdots,i_m)$ with $m\ge 2$. Hence, $\Pi_{\mathcal{U}}$ have $\sum_{m=2}^{|\mathcal{U}|}\binom{|\mathcal{U}|}{m}(m-1)!$ cyclic sequences. For instance, let $\mathcal{U}=\{1,2,3\}$, then $\Pi_{\mathcal{U}}=\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,3,2\}$.

To better understand this achievable region, let us look at the example illustrated in Figure 1(a), where $0\le a\le b\le 1$. For convenience, we let $d^c=\sum_{k=1}^{3}d_k^c$. For $\mathcal{U}=\{1,2,3\}$, $\{2,3\}$, $\{1,3\}$, $\{1,2\}$, $\{3\}$, $\{2\}$ and $\{1\}$, the corresponding $\mathcal{D}_{RS}(\mathcal{U})$ are given by

$$\begin{aligned}\mathcal{D}_{RS}(\{1,2,3\})=\{&0\le d_k^c\le 1,0\le d_k^p\le 1,0\le d_k^p+d^c\le 1,\\&\forall k\in\{1,2,3\},d_1^p+d_2^p\le 2b,d_1^p+d_2^p+d^c\le 1+b,\\&d_1^p+d_3^p\le 2b,d_1^p+d_3^p+d^c\le 1+b,d_2^p+d_3^p\le 2a,\\&d_2^p+d_3^p+d^c\le 1+a,d_1^p+d_2^p+d_3^p\le 2b+a,\\&d_1^p+d_2^p+d_3^p+d^c\le 1+b+a\},\end{aligned} \tag{13}$$

$$\begin{aligned}\mathcal{D}_{RS}(\{2,3\})=\{&d_1^p=0,0\le d_k^c\le 1,\forall k\in\{1,2,3\},0\le d_k^p\le 1,\\&0\le d_k^p+d^c\le 1,\forall k\in\{2,3\},d_2^p+d_3^p\le 2a,\\&d_2^p+d_3^p+d^c\le 1+a\},\end{aligned} \tag{14}$$

$$\begin{aligned}\mathcal{D}_{RS}(\{1,3\})=\{&d_2^p=0,0\le d_k^c\le 1,\forall k\in\{1,2,3\},0\le d_k^p\le 1,\\&0\le d_k^p+d^c\le 1,\forall k\in\{1,3\},d_1^p+d_3^p\le 2b,\\&d_1^p+d_3^p+d^c\le 1+b\},\end{aligned} \tag{15}$$

$$\begin{aligned}\mathcal{D}_{RS}(\{1,2\})=\{&d_3^p=0,0\le d_k^c\le 1,\forall k\in\{1,2,3\},0\le d_k^p\le 1,\\&0\le d_k^p+d^c\le 1,\forall k\in\{1,2\},d_1^p+d_2^p\le 2b,\\&d_1^p+d_2^p+d^c\le 1+b\},\end{aligned} \tag{16}$$

$$\begin{aligned}\mathcal{D}_{RS}(\{3\})=\{&d_1^p=d_2^p=0,0\le d_k^c\le 1,\forall k\in\{1,2,3\},\\&0\le d_3^p\le 1,0\le d_3^p+d^c\le 1\},\end{aligned} \tag{17}$$

$$\begin{aligned}\mathcal{D}_{RS}(\{2\})=\{&d_1^p=d_3^p=0,0\le d_k^c\le 1,\forall k=\{1,2,3\},\\&0\le d_2^p\le 1,0\le d_2^p+d^c\le 1\},\end{aligned} \tag{18}$$

$$\begin{aligned}\mathcal{D}_{RS}(\{1\})=\{&d_2^p=d_3^p=0,0\le d_k^c\le 1,\forall k\in\{1,2,3\},\\&0\le d_1^p\le 1,0\le d_1^p+d^c\le 1\},\end{aligned} \tag{19}$$

respectively. Using (13) through to (19) and Remark 1, it can be verified that the maximum sum DoF achieved by RS and ZFBF with power control are $\max\{1+2a,1+b\}$ and $\max\{2b,\min\{1+2a,2b+a\}\}$, respectively. Then, we see that RS offers DoF gain except in the case $1+b\le 1+2a\le 2b+a$.

Next, considering ZFBF with power control and RS with common message multicasting as benchmark schemes, we move on to propose a novel transmission strategy that yields a greater DoF region in the fully connected K-cell MISO IC with equal SNR and INR and with an arbitrary CSIT quality topology.

IV. Topological Rate-Splitting

In this section, we firstly introduce the idea of Topological Rate-Splitting focusing on the example in Figure 1(a). Secondly, we propose the generalized framework of the TRS motivated by a novel weighted sum interpretation of the fully connected MISO IC with CSIT quality topology $\mathcal{A}$. Then, the sum DoF achieved by the TRS scheme is studied using graph theory tools. Lastly, an upper-bound on the DoF region is proposed.

A. Toy Example

Focusing on the example in Figure 1(a), we propose a simple TRS scheme that yields a greater sum DoF than RS and ZFBF with power control. We use the notation w_k^i to represent a message transmitted by transmitter k using the ith power level. The power levels are defined in an increasing order based on the scaling behavior at infinite SNR. When $i{=}1$, it is a private message as we consider that the private messages are transmitted using the lowest power level, while $i{>}1$ implies a common message. Using this notation, we design a TRS transmission block for the considered example as follows.

Similar to RS, we consider that each transmitter uses power P^a to unicast the private messages w_k^1 along ZF-precoders. Unlike RS, the remaining power $P{-}P^a$ is further split into two levels, i.e., $P^b{-}P^a$ for common message groupcasting and $P{-}P^b$ for common message multicasting.

With power $P^b{-}P^a$, we see that the interference from transmitter 1 to user 2 and user 3, the interference from transmitter 2 to user 1 and the interference from transmitter 3 to user 1, can all be forced within the noise power via ZFBF, because the CSIT qualities of those links $a_{21}{=}a_{31}{=}a_{12}{=}a_{13}{=}b$ are sufficiently good. By doing so, the MISO IC becomes a partially connected network with two cross links $\mathbf{h}_{23}$ and $\mathbf{h}_{32}$ as illustrated in Figure 1(b) (see the figure in the middle). In such a network, transmitter 1 can deliver one message w_1^2 to user 1 without mixing with the messages transmitted by the other transmitters. At the same time, transmitter 2 and 3 are able to deliver *group common messages* to be decoded by user 2 and user 3, without mixing with w_1^2. Here, as we design TRS to enhance the sum DoF, for convenience, we consider that transmitter 2 delivers a group common message w_2^2 while transmitter 3 does not transmit any group common message.

With the remaining power $P{-}P^b$, since the CSIT qualities are not good enough, we see that no interference can be drowned into the noise at any user via ZFBF. This leads to a fully connected network shown in Figure 1(b) (the right-most figure). Then, we consider that transmitter 1 multicasts one common message w_1^3 to be decoded by all users.

Accordingly, the transmitted signals write as

$$\mathbf{s}_1{=}\underbrace{\mathbf{p}_1^3 w_1^3}_{P-P^b}+\underbrace{\mathbf{p}_1^2 w_1^2}_{P^b-P^a}+\underbrace{\mathbf{p}_1^1 w_1^1}_{P^a}, \tag{20}$$

$$\mathbf{s}_2{=}\underbrace{\mathbf{p}_2^2 w_2^2}_{P^b-P^a}+\underbrace{\mathbf{p}_2^1 w_2^1}_{P^a}, \tag{21}$$

$$\mathbf{s}_3{=}\underbrace{\mathbf{p}_3^1 w_3^1}_{P^a}, \tag{22}$$

where $\mathbf{p}_1^2{=}\mathbf{p}_1^1{\subseteq}\,\mathrm{span}(\hat{\mathbf{h}}_{21}^{\perp},\hat{\mathbf{h}}_{31}^{\perp})$, $\mathbf{p}_2^2{\subseteq}\,\mathrm{span}(\hat{\mathbf{h}}_{12}^{\perp})$, $\mathbf{p}_2^1{\subseteq}\,\mathrm{span}(\hat{\mathbf{h}}_{12}^{\perp},\hat{\mathbf{h}}_{32}^{\perp})$, and $\mathbf{p}_3^1{\subseteq}\,\mathrm{span}(\hat{\mathbf{h}}_{13}^{\perp},\hat{\mathbf{h}}_{23}^{\perp})$. The received signals are expressed as

$$\begin{aligned}\mathbf{y}_1{=}&\underbrace{\mathbf{h}_{11}^H\mathbf{p}_1^3 w_1^3}_{P}+\underbrace{\mathbf{h}_{11}^H\mathbf{p}_1^2 w_1^2}_{P^b}+\underbrace{\mathbf{h}_{11}^H\mathbf{p}_1^1 w_1^1}_{P^a}+\\&\underbrace{\mathbf{h}_{12}^H\mathbf{p}_2^2 w_2^2{+}\mathbf{h}_{12}^H\mathbf{p}_2^1 w_2^1{+}\mathbf{h}_{13}^H\mathbf{p}_3^1 w_3^1}_{P^0}+n_1,\end{aligned} \tag{23}$$

$$\begin{aligned}\mathbf{y}_2{=}&\underbrace{\mathbf{h}_{21}^H\mathbf{p}_1^3 w_1^3}_{P}+\underbrace{\mathbf{h}_{21}^H\mathbf{p}_1^2 w_1^2{+}\mathbf{h}_{21}^H\mathbf{p}_1^1 w_1^1}_{P^0}+\\&\underbrace{\mathbf{h}_{22}^H\mathbf{p}_2^2 w_2^2}_{P^b}+\underbrace{\mathbf{h}_{22}^H\mathbf{p}_2^1 w_2^1}_{P^a}+\underbrace{\mathbf{h}_{23}^H\mathbf{p}_3^1 w_3^1}_{P^0}+n_2,\end{aligned} \tag{24}$$

$$\begin{aligned}\mathbf{y}_3{=}&\underbrace{\mathbf{h}_{31}^H\mathbf{p}_1^3 w_1^3}_{P}+\underbrace{\mathbf{h}_{31}^H\mathbf{p}_1^2 w_1^2{+}\mathbf{h}_{31}^H\mathbf{p}_1^1 w_1^1}_{P^0}+\\&\underbrace{\mathbf{h}_{32}^H\mathbf{p}_2^2 w_2^2}_{P^b}+\underbrace{\mathbf{h}_{32}^H\mathbf{p}_2^1 w_2^1}_{P^0}+\underbrace{\mathbf{h}_{33}^H\mathbf{p}_3^1 w_3^1}_{P^a}+n_3,\end{aligned} \tag{25}$$

where all the undesired messages are drowned into the noise. The decoding procedure starts from the messages with the highest received power level and then progresses downwards using SIC. Specifically, user 1 decodes common messages w_1^3, w_1^2 and private message w_1^1; user 2 decodes common messages w_1^3, w_2^2 and private message w_2^1; user 3 decodes common messages w_1^3, w_2^2 and private message w_3^1. The DoF achieved by the common messages are $d_1^2{=}d_2^2{=}b{-}a$ and $d_1^3{=}1{-}b$. Counting the DoF achieved by the private messages, the sum DoF achieved by TRS is $1{+}b{+}a$. The sum DoF gain offered by TRS over RS is $1{+}b{+}a{-}\max\{1{+}b,1{+}2a\}{=}\min\{a,b{-}a\}$, and is illustrated in Figure 1(c).

Remark 2. *The key ingredient of the TRS approach above lies in the multi-layer structure. With ZF-precoders and properly assigned power levels, the CSIT quality topology in Figure 1(a) is interpreted as a series of network topologies in Figure 1(b). As shown, for power level up to a, the network appears as a non-interfering network; for power level a to b, and for power level b to 1, the networks appear to a partially connected network and a fully connected network, respectively. This procedure is called weighted sum interpretation. It explicitly shows that 1) each common message is decoded by a subset of users, and 2) each user needs to decode a subset of all common messages. These observations essentially reveal the effectiveness of the TRS scheme compared to RS. Indeed, in RS, each common message has to be decoded by all users and each user has to decode all the common messages due to the common message multicasting.*

B. Building the Generalized Transmission Block

Motivated by the toy example, we present the generalized transmission block of TRS. We describe the TRS approach focusing on the active user subset $\mathcal{S}{\subseteq}\mathcal{K}$, while the remaining users are made silent.

In TRS, each active transmitter divides the message intended for its corresponding user into $N{=}L{+}2$ parts, i.e., $\mathcal{W}_k{\triangleq}\{w_k^1,w_k^2,\cdots,w_k^N\}$, $\forall k{\in}\mathcal{S}$. The definition of L will be introduced later on. Letting $\mathbf{p}_k^i$ denote the precoder and $P_{k,i}$ denote the power chosen for a certain message w_k^i, the signal transmitted by transmitter k can be expressed as

$$\mathbf{s}_k{=}\sum_{i=1}^{L+2}\underbrace{\mathbf{p}_k^i w_k^i}_{P_{k,i}},\forall k{\in}\mathcal{S}. \tag{26}$$

Private message layer: Private message w_k^1 is intended for user k and is to be decoded by user k only. It is transmitted along a ZF-precoder and is unicast with a fraction of the total power as

$$\mathbf{p}_k^1{\subseteq}\,\mathrm{span}(\{\hat{\mathbf{h}}_{jk}^{\perp}\}_{\forall j\in\mathcal{S}\setminus k}),\quad P_{k,1}{=}P^{r_k},\forall k{\in}\mathcal{S}. \tag{27}$$

Common message layer: The remaining power $P{-}P^{r_k}$ at each user is employed to deliver the $L{+}1$ common messages

$w_k^i, i{=}2,\cdots,L{+}2$. The power allocated to each common message w_k^i and its precoder are obtained based on the CSIT qualities.

Firstly, as only the users in $\mathcal{S}$ are active, we obtain a subset $\mathcal{A}(\mathcal{S})\subseteq\mathcal{A}$ such that $a_{kj}\in\mathcal{A}(\mathcal{S})$ if and only if $k,j\in\mathcal{S}$. Secondly, let $\mathcal{A}(\mathbf{r},\mathcal{S})\subseteq\mathcal{A}(\mathcal{S})$ denote the set formed by all the elements of $\mathcal{A}(\mathcal{S})$ that are greater than $r_0\triangleq\max_{k\in\mathcal{S}} r_k$, i.e., $\mathcal{A}(\mathbf{r},\mathcal{S})\triangleq\{a_{kj}\}_{\forall a_{kj}\in\mathcal{A}(\mathcal{S}),a_{kj}>r_0}$. Thirdly, letting L denote the number of different values of $\mathcal{A}(\mathbf{r},\mathcal{S})$, we represent these L values by $a_{\pi(1)},\cdots,a_{\pi(L)}$, which satisfy $a_{\pi(1)}<a_{\pi(2)}<\cdots<a_{\pi(L)}$. Besides, for convenience, we define $a_{\pi(L+1)}{=}1$. Using these $L{+}1$ variables $a_{\pi(1)},\cdots,a_{\pi(L+1)}$, we divide the remaining power $P{-}P^{r_k}$ at each transmitter into $L{+}1$ power levels, i.e., $P^{a_{\pi(1)}}{-}P^{r_k}$, $P^{a_{\pi(2)}}{-}P^{a_{\pi(1)}}$, $\cdots$, $P^{a_{\pi(L+1)}}{-}P^{a_{\pi(L)}}$.

Then, we assign these $L{+}1$ power levels to the common messages w_k^i, $2\leq i\leq L{+}2$, and choose a ZF-precoder for each of them as

$$P_{k,2}{=}P^{a_{\pi(1)}}{-}P^{r_k},$$
$$P_{k,i}{=}P^{a_{\pi(i-1)}}{-}P^{a_{\pi(i-2)}}, 3\leq i\leq L{+}2; \tag{28}$$
$$\mathbf{p}_k^i\subseteq \text{span}\left(\{\hat{\mathbf{h}}_{jk}^{\perp}\}_{\forall j\in\mathcal{S}\setminus\mathcal{R}_k^i(\mathcal{S},\mathbf{r})}\right), \text{ where}$$
$$\mathcal{R}_k^i(\mathcal{S},\mathbf{r})\triangleq\{j{:}j{\in}\mathcal{S}\setminus k, a_{jk}{<}a_{\pi(i-1)}\}\cup k. \tag{29}$$

Such a precoder and power allocation policy suggests that w_k^i is a *group common message* to be decoded by the group of users $\mathcal{R}_k^i(\mathcal{S},\mathbf{r})$, while it is drowned into the noise at other users $\forall j\in\mathcal{S}\setminus\mathcal{R}_k^i(\mathcal{S},\mathbf{r})$.

With the precoders and power allocation policy given in (27) through (29), the signal received by user k writes as

$$y_k{=}\sum_{j\in\mathcal{S}}\sum_{i=1}^{L+2}\mathbf{h}_{kj}^H\mathbf{p}_j^i w_j^i{+}n_k \tag{30}$$
$$={\sum_{i=2}^{L+2}}\left(\underbrace{\mathbf{h}_{kk}^H\mathbf{p}_k^i w_k^i}_{P^{a_{\pi(i-1)}}}+\sum_{\substack{j:\forall j\in\mathcal{S}\setminus k,\\ a_{kj}<a_{\pi(i-1)}}}\underbrace{\mathbf{h}_{kj}^H\mathbf{p}_j^i w_j^i}_{P^{a_{\pi(i-1)}}}+\sum_{\substack{j:\forall j\in\mathcal{S}\setminus k,\\ a_{kj}\geq a_{\pi(i-1)}}}\underbrace{\mathbf{h}_{kj}^H\mathbf{p}_j^i w_j^i}_{P^{a_{\pi(i-1)}-a_{kj}}}\right)+ \tag{31}$$
$$\underbrace{\mathbf{h}_{kk}^H\mathbf{p}_k^1 w_k^1}_{P^{r_k}}+\sum_{\forall j\in\mathcal{S}\setminus k}\underbrace{\mathbf{h}_{kj}^H\mathbf{p}_j^1 w_j^1}_{P^{r_j-a_{kj}}}+\underbrace{n_k}_{P^0}, \tag{32}$$

As expressed in (31), if the CSIT quality of the cross link $\mathbf{h}_{kj}$ is greater than or equal to the allocated power level, i.e., $a_{kj}{>}a_{\pi(i-1)}$, the common message $w_j^i, i{\geq}2$, is drowned into the noise at user k due to ZFBF; otherwise, w_j^i is received by user k with power $P^{a_{\pi(i-1)}}$. As expressed in (32), the undesired private message $w_j^1,\forall j{\in}\mathcal{S}\setminus k$ is received by user k with power $P^{r_k-a_{kj}}$. If $a_{kj}{\geq}r_k$, w_j^1 is drowned into the noise; otherwise, w_j^1 becomes an undesirable interference overheard by user k.

The decoding procedure is based on SIC. Let us focus on the received signal in (30). Firstly, user k decodes common messages w_k^{L+2} and $\{w_j^{L+2}\}_{j:a_{kj}<a_{\pi(L+1)}}$ by treating all the other messages as noise. Secondly, after removing those recovered messages, user k decodes w_k^{L+1} and $\{w_j^{L+1}\}_{j:a_{kj}<a_{\pi(L)}}$, by treating all the other messages with lower received power as noise. This procedure runs for $L{+}1$ rounds till all the common messages are recovered. At last, user k decodes its desired private message w_k^1 by treating the undesired private messages as noise.

For convenience, let us denote the set of common messages decoded by user k in ith round of SIC by

$$\mathcal{T}_k^i(\mathcal{S},\mathbf{r})\triangleq w_k^i\cup\{w_j^i\}_{j:\forall j\in\mathcal{S}\setminus k,a_{kj}<a_{\pi(i-1)}}, \quad \text{where } 2\leq i\leq L{+}2. \tag{33}$$

Then, the DoF region achieved by the proposed TRS scheme, denoted by $\mathcal{D}_{TRS}$, is stated below.

Proposition 2. *In a fully connected K-cell IC with equal SNR and INR and with CSIT quality topology $\mathcal{A}$, the DoF region achieved by the proposed TRS scheme lies in*

$$\mathcal{D}_{TRS}{=}\bigcup_{\forall\mathcal{S}\in\mathcal{K},\forall\mathbf{r}}\mathcal{D}_{TRS}(\mathcal{S},\mathbf{r}), \tag{34}$$

where $\mathcal{D}_{TRS}(\mathcal{S},\mathbf{r})$ is achievable with active user subset $\mathcal{S}$ and power allocation policy $\mathbf{r}$ for the private messages. It is the set of the DoF tuples $(d_1,\cdots,d_K){=}\sum_{i=1}^{L+2}(d_k^i,\cdots,d_K^i)$ such that

$$d_k^i{=}0, i{=}1,\cdots,L{+}2,\forall k{\in}\mathcal{K}\setminus\mathcal{S}; \tag{35}$$
$$0{\leq}d_k^1{\leq}\left(r_k{-}\max_{j\in\mathcal{S}\setminus k}(r_j{-}a_{kj})^+\right)^+, \forall k{\in}\mathcal{S}; \tag{36}$$
$$0{\leq}d_k^2, \sum_{\forall j:w_j^2\in\mathcal{T}_k^2(\mathcal{S},\mathbf{r})}d_j^2{\leq}a_{\pi(1)}{-}\max\{r_k,\max_{j\in\mathcal{S}\setminus k}r_j{-}a_{kj}\},\forall k{\in}\mathcal{S}; \tag{37}$$
$$0{\leq}d_k^i, \sum_{\forall j:w_j^i\in\mathcal{T}_k^i(\mathcal{S},\mathbf{r})}d_j^i{\leq}a_{\pi(i-1)}{-}a_{\pi(i-2)},\forall k{\in}\mathcal{S}, 3{\leq}i{\leq}L{+}2, \tag{38}$$

where $\mathcal{T}_k^i(\mathcal{S},\mathbf{r})$, $2{\leq}i{\leq}L{+}2$, is defined in (33) *as a function of $\mathcal{S}$ and $\mathbf{r}$.*

Proof. See Appendix B. □

We point out that it is cumbersome to obtain a concise expression of $\mathcal{D}_{TRS}$ by eliminating the variables $\mathbf{r}$. This is because the DoF of the common messages transmitted in each power layer are characterized by $|\mathcal{S}|$ different inequalities, which strongly depend on the CSIT quality topologies (see (37) and (38)).

In the rest of this section, we consider an inner-bound $\bar{\mathcal{D}}_{TRS}(\mathcal{S},\mathbf{r})\subseteq\mathcal{D}_{TRS}(\mathcal{S},\mathbf{r})$, obtained by replacing (37) with

$$d_k^2{=}0,\forall k{\in}\mathcal{K}\setminus\mathcal{S}; 0{\leq}d_k^2, \sum_{\forall j:w_j^2\in\mathcal{T}_k^2(\mathcal{S},\mathbf{r})}d_j^2{\leq}a_{\pi(1)}{-}r_0,\forall k{\in}\mathcal{S}; \tag{39}$$

where $r_0\triangleq\max_{k\in\mathcal{S}} r_k$. When there is an even power allocation for the private messages, i.e., $r_k{=}r_j,\forall k,j{\in}\mathcal{S}$, we have $\bar{\mathcal{D}}_{TRS}(\mathcal{S},\mathbf{r}){=}\mathcal{D}_{TRS}(\mathcal{S},\mathbf{r})$. Comparing this inner-bound with the DoF region achieved by RS given in (8), we can reach the conclusion that the DoF region achieved by TRS covers that achieved by RS. To see this, let us express any achievable DoF tuple $(d_1^c,\cdots,d_K^c)$ for which (8) holds as $\sum_{i=2}^{L+2}(d_1^{c,i},\cdots,d_K^{c,i})$, where the DoF tuple $(d_1^{c,i},\cdots,d_K^{c,i})$ are subject to $\sum_{k\in\mathcal{S}} d_k^{c,i}\leq a_{\pi(i-1)}-a_{\pi(i-2)}$ and $d_k^{c,i}=0$, $\forall k\in\mathcal{K}\setminus\mathcal{S}$. Then, it readily shows that the DoF tuple $(d_1^{c,i},\cdots,d_K^{c,i})$ also lies in (39) and (38), because the summation of d_k^i is taken over the set $\forall j{:}w_j^i\in\mathcal{T}_k^i(\mathcal{S},\mathbf{r})$, which is a subset of $\mathcal{S}$. This fact implies that the DoF region achieved by TRS covers that achieved by RS, i.e., $\mathcal{D}_{RS}(\mathcal{S},\mathbf{r})\subseteq\bar{\mathcal{D}}_{TRS}(\mathcal{S},\mathbf{r})\subseteq\mathcal{D}_{TRS}(\mathcal{S},\mathbf{r})$.

C. weighted sum Interpretation

We note that the construction of the TRS scheme is motivated by a novel weighted sum interpretation of the CSIT quality topology as a series of network topologies superposed in the power domain. Specifically, with the power and ZF-precoders chosen for the common messages in (28) and (29), we observe that a transmitter k is only connected to the group of users $\forall j\in\mathcal{R}_k^i(\mathcal{S},\mathbf{r})$. Besides, as shown by the received signal given in (31), the messages $w_j^i\in\mathcal{S}\setminus\mathcal{T}_k^i(\mathcal{S},\mathbf{r})$ are forced within the noise power at user k. This fact implies that user k is only connected to transmitters $\forall j,w_j^i\in\mathcal{T}_k^i(\mathcal{S},\mathbf{r})$. Accordingly, this topology can be expressed using a connectivity matrix $\mathbf{M}^i(\mathcal{S},\mathbf{r})\in\{0,1\}^{|\mathcal{S}|\times|\mathcal{S}|}$, whose element in row k and column j, i.e., m_{kj}, is given by

$$m_{kj}^i=\begin{cases}1 & \text{if } w_j^i\in\mathcal{T}_k^i(\mathcal{S},\mathbf{r});\\ 0 & \text{otherwise.}\end{cases} \tag{40}$$

Note that the value of m_{kj}^i in (40) represents whether or not w_j^i is decoded by user k.

The DoF tuple (39) and (38) achieved by the common messages transmitted with power layer i can be interpreted as $(a_{\pi(i-1)}-a_{\pi(i-2)})\times\hat{\mathcal{D}}_{TRS}^i(\mathcal{S},\mathbf{r})$, where

$$\begin{aligned}\hat{\mathcal{D}}_{TRS}^i(\mathcal{S},\mathbf{r})\colon \hat{d}_k^2=0,\forall k\in\mathcal{K}\setminus\mathcal{S};\, 0\leq\hat{d}_k^2,\forall k\in\mathcal{S},\\ \mathbf{M}^i(\mathcal{S},\mathbf{r})\times\hat{\mathbf{d}}^i\leq\mathbf{1}_{|\mathcal{S}|},2\leq i\leq L+2,\end{aligned} \tag{41}$$

represents the set of DoF tuples $\hat{\mathbf{d}}^i=(\hat{d}_1^i,\cdots,\hat{d}_K^i)$ achieved by common message groupcasting in the partially connected network defined by the connectivity matrix $\mathbf{M}^i(\mathcal{S},\mathbf{r})$. The weights $a_{\pi(i-1)}-a_{\pi(i-2)}$, $i\geq 2$, stand for the fractions of channel use of the partially connected networks in the power domain (Note that we assume $a_{\pi(0)}=r_0$). For clarity, let $\{\hat{w}_k^i\}_{k\in\mathcal{S}}$ denote the common messages transmitted in the partially connected network defined by topology $\mathbf{M}^i(\mathcal{S},\mathbf{r})$. The achievable DoF of $\hat{w}_k^i$ is represented by $\hat{d}_k^i$. Then, the DoF d_k^i of the common message w_k^i transmitted in TRS is obtained by $d_k^i=(a_{\pi(i-1)}-a_{\pi(i-2)})\hat{d}_k^i$. Consequently, the DoF region $\bar{\mathcal{D}}_{TRS}^c$ contributed by all the common messages $\sum_{i=2}^{L+2}(d_1^i,\cdots d_K^i)$ can be expressed by the weighted sum of the DoF region achieved in the $L+1$ partially connected networks, i.e.,

$$\bar{\mathcal{D}}_{TRS}^c=\sum_{i=2}^{L+2}(a_{\pi(i-1)}-a_{\pi(i-2)})\times\hat{\mathcal{D}}_{TRS}^i(\mathcal{S},\mathbf{r}). \tag{42}$$

Similarly, when $r_k\leq\min_{j\in\mathcal{S}\setminus k}a_{jk}$, $\forall k\in\mathcal{S}$, the private message unicasting part is interpreted as a partially connected network formed by $|\mathcal{S}|$ parallel direct links, because all the interference is drowned into the noise.

This weighted sum interpretation bridges the DoF region achieved TRS with the achievable DoF region in partially connected networks, thus allowing us to employ methodologies applicable for partially connected networks to analyze the DoF region achieved by TRS. Motivated by this, we study the sum DoF achieved TRS in the next subsection.

D. Sum DoF Characterization Using Graph Theory Tools

In this part, we aim to find the maximal sum DoF given the DoF region $\bar{\mathcal{D}}_{TRS}(\mathcal{S},\mathbf{r})$ specified by (36), (39) and (38). To do so, it is straightforward that the maximum DoF of the private messages achieved by the TRS scheme is $d_k^1=\left(r_k-\max_{j\in\mathcal{S}\setminus k}(r_j-a_{kj})^+\right)^+$. Then, the work consists in computing the maximum sum DoF contributed by all the common messages. As a consequence of the weighted sum interpretation in (42), this sum DoF maximization is decoupled into a series of optimization problems

$$\mathcal{P}_i\colon\quad \max\quad \hat{d}_s^i(\mathcal{S},\mathbf{r})\triangleq\sum_{k\in\mathcal{S}}\hat{d}_k^i,\forall i=2,\cdots,L+2 \tag{43}$$

$$\begin{aligned}\text{s.t.}\quad & (\hat{d}_k^i)_{k\in\mathcal{S}}\in\hat{\mathcal{D}}_{TRS}^i(\mathcal{S},\mathbf{r})\\ & \Rightarrow 0\leq\hat{d}_k^i,k\in\mathcal{S},\mathbf{M}^i(\mathcal{S},\mathbf{r})\times\hat{\mathbf{d}}^i\leq\mathbf{1}_{|\mathcal{S}|}.\end{aligned} \tag{44}$$

For convenience, we drop the variables $(\mathcal{S},\mathbf{r})$ in the following analysis. As explained in Section IV-C, solving the problem $\mathcal{P}_i$, $i\geq 2$, is related to maximizing the sum DoF achieved by common message groupcasting in a partially connected network. In recent years, the DoF of a partially connected network has received a lot of attention in [20]–[25]. Although all of these works look at symmetric DoF as a figure of merit, graph theory methodologies have been identified as a useful means because of its powerful ability to describe whether or not a user's message is interfered with one another. Motivated by that, we solve our problems in a similar way.

We model the partially connected network with connectivity matrix $\mathbf{M}^i$ as a hypergraph $\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i)$, where $\hat{\mathcal{W}}^i\triangleq\{\hat{w}_k^i\}_{k\in\mathcal{S}}$ is the vertex set of the hypergraph and $\mathcal{T}^i\triangleq\{\mathcal{T}_k^i\}_{k\in\mathcal{S}}$ with $\mathcal{T}_k^i$ defined in (33) is the hyperedge set of the hypergraph. Note that a member of $\mathcal{T}^i$ is actually a subset of $\hat{\mathcal{W}}^i$. If each member of $\mathcal{T}^i$ has two vertices, e.g., $\mathcal{T}_k^i=\{\hat{w}_k^i,\hat{w}_j^i\}$, then $\mathcal{T}_k^i$ actually means an edge between $\hat{w}_k^i$ and $\hat{w}_j^i$, and the hypergraph $\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i)$ is actually a graph. When an element of $\mathcal{T}^i$ has more than two elements, i.e., $|\mathcal{T}_k^i|\geq 3$, then $\mathcal{T}_k^i$ is called an hyperedge with $|\mathcal{T}_k^i|$ vertices.

In the following, focusing on the hypergraph $\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i)$, we interpret the optimization problem $\mathcal{P}_i$ as two classical problems in graph theory, which lead to a sub-optimal solution and the optimal solution.

1) Orthogonal Groupcasting: We firstly propose a sub-optimal solution, so called *orthogonal groupcasting*, by assuming that each user only decodes at most one common message. In other words, no two of the common messages $\{\hat{w}_k^i\}_{\forall k\in\mathcal{S}}$ are received by a single user. This assumption imposes a constraint $\hat{d}_k^i\in\{0,1\}$ to the optimization problem $\mathcal{P}_i$ in (43).

Then, a DoF tuple $(\hat{d}_k^i)_{k\in\mathcal{S}}$ achieved by orthogonal groupcasting defines a subset $\mathcal{X}^i\subseteq\hat{\mathcal{W}}^i$ which contains all the messages with DoF 1, i.e., $\mathcal{X}^i=\{\hat{w}_k^i\}_{\forall k,\hat{d}_k^i=1}$. The sum DoF is identical to the cardinality of $\mathcal{X}^i$, i.e., $|\mathcal{X}^i|$. According to the definition of orthogonal groupcasting, this subset has the property that no two elements of $\mathcal{X}^i$ are together in the same member of $\mathcal{T}^i$. Therefore, this subset $\mathcal{X}^i\subseteq\hat{\mathcal{W}}^i$ is called a *packing* in the hypergraph $\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i)$ [26]. Finding the maximum sum DoF is equivalent to finding the largest size of a packing, and the largest size is defined to be the *packing number* $p(\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i))$ of $\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i)$. Hence, the sum DoF achieved by the orthogonal groupcasting is stated as follows.

Proposition 3. *In a fully connected K-cell MISO IC with equal SNR and INR and with CSIT quality topology $\mathcal{A}$, the sum DoF achieved by TRS designed with orthogonal common message groupcasting is*

$$d_{s,TRS}^{orth}=\max_{\forall\mathcal{S}\subseteq\mathcal{K},\forall\mathbf{r}} d_{s,TRS}^{orth}(\mathcal{S},\mathbf{r}), \tag{45}$$

where $d_{s,TRS}^{orth}(\mathcal{S},\mathbf{r})$ is given by

$$d_{s,TRS}^{orth}(\mathcal{S},\mathbf{r})=\sum_{k=1}^{K}\left(r_k-\max_{j\in\mathcal{S}'\setminus k}(r_j-a_{kj})^+\right)^+ + \sum_{i=2}^{L+2}(a_{\pi(i-1)}-a_{\pi(i-2)})\times p(\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i(\mathcal{S},\mathbf{r}))), \tag{46}$$

where $p(\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i(\mathcal{S},\mathbf{r})))$ refers to the packing number of a hypergraph $\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i(\mathcal{S},\mathbf{r}))$ defined by vertex set $\hat{\mathcal{W}}^i$ and hyperedge set $\mathcal{T}^i(\mathcal{S},\mathbf{r})$ defined in (33).

2) Maximal groupcasting: To find the optimal solution to problem $\mathcal{P}_i$, let us firstly look at the following problem

$$\tilde{\mathcal{P}}_i:\quad \max \sum_{k\in\mathcal{S}}\tilde{d}_k^i \tag{47}$$

$$\text{s.t.}\quad \mathbf{M}^i\times\tilde{\mathbf{d}}^i\leq t\times\mathbf{1}_{|\mathcal{S}|},\tilde{d}_k^i\in\mathbb{Z}^+,\forall k\in\mathcal{S}, \tag{48}$$

where t is a positive integer. A feasible $(\tilde{d}_k^i)_{k\in\mathcal{S}}$ satisfying (48) defines a *multiset* $\mathcal{X}^i$ which contains $\hat{w}_k^i$ if $\tilde{d}_k^i>0$. The multiplicity[3] of $\hat{w}_k^i$ in $\mathcal{X}^i$ is $\tilde{d}_k^i$, and the sum DoF is equal to $|\mathcal{X}^i|$.

In this way, the inequality (48) can be interpreted as follows. For the vertices in the same member of $\mathcal{T}^i$, the sum of their multiplicity in $\mathcal{X}^i$ is smaller than or equal to t. According to [26], a multiset $\mathcal{X}^i$ with such a property is called a *t-fold packing* of the hypergraph $\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i)$. When t=1, the t-fold packing collapses to the packing of the hypergraph $\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i)$ that is introduced in Section IV-D1. Consequently, the optimization problem $\tilde{\mathcal{P}}_i$ is interpreted as finding the largest size of a t-fold packing, and the largest size is defined as the t-fold packing number $p_t(\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i))$.

So far, we are one step closer to our objective. According to [26, Section 1.2], the optimal result of Problem $\mathcal{P}_i$ in (43) can be found using the result of Problem $\tilde{\mathcal{P}}_i$ in (47) by taking $t\to\infty$ as

$$p_f(\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i))=\lim_{t\to\infty}\frac{p_t(\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i))}{t}. \tag{49}$$

This quantity is called *fractional packing number* of the hypergraph $\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i)$. Besides, the DoF of message $\hat{w}_k^i$ is expressed as $\hat{d}_k^i=\lim_{t\to\infty}\frac{\tilde{d}_k^{i*}}{t}$, where $\tilde{d}_k^{i*}$ is the result of the t-fold packing problem $\tilde{\mathcal{P}}_i$.

[3]The multiset $\mathcal{X}^i$ may have multiple identical elements. For instance, one has $\mathcal{X}^i=\{\hat{w}_1^i,\hat{w}_2^i,\hat{w}_2^i\}$, and the multiplicity of $\hat{w}_1^i$ is 1 and the multiplicity of $\hat{w}_2^i$ is 2.

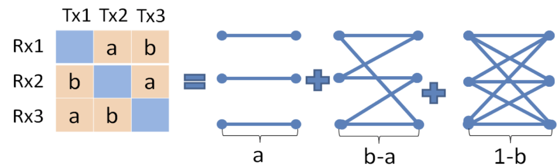


Fig. 2: 3-cell IC with a cyclic CSIT quality topology

Therefore, we may state an achievable sum DoF resulting from the maximal groupcasting as follows.

Proposition 4. *In a fully connected K-cell MISO IC with equal SNR and INR and with CSIT quality topology $\mathcal{A}$, the sum DoF achieved by TRS with maximal groupcasting is*

$$d_{s,TRS}^{\max}=\max_{\forall\mathcal{S}\subseteq\mathcal{K},\forall\mathbf{r}} d_{s,TRS}^{\max}(\mathcal{S},\mathbf{r}), \tag{50}$$

where $d_{s,TRS}^{\max}(\mathcal{S},\mathbf{r})$ is given by

$$d_{s,TRS}^{\max}(\mathcal{S},\mathbf{r})=\sum_{k=1}^{K}\left(r_k-\max_{j\in\mathcal{S}'\setminus k}(r_j-a_{kj})^+\right)^+ + \sum_{i=2}^{L+2}(a_{\pi(i-1)}-a_{\pi(i-2)})\times p_f(\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i(\mathcal{S},\mathbf{r}))), \tag{51}$$

where $p_f(\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i(\mathcal{S},\mathbf{r})))$ refers to the fractional packing number of a hypergraph $\mathcal{H}^i(\hat{\mathcal{W}}^i,\mathcal{T}^i(\mathcal{S},\mathbf{r}))$ defined by the vertex set $\hat{\mathcal{W}}^i$ and the hyperedge set $\mathcal{T}^i(\mathcal{S},\mathbf{r})$ defined in (33).

Note that both common message groupcasting methods suffice to achieve the sum DoF in the example illustrated in Figure 1(a). To highlight the gain offered by the maximal groupcasting, let us focus on the 3-cell scenario with a cyclic CSIT quality topology illustrated in Figure 2.

Following the footsteps presented in Section IV-B, the transmitted signal consists of three power levels, P^a, P^b-P^a and $P-P^b$, which are used for private message unicasting, common message groupcasting and common message multicasting. To highlight the benefit of performing maximal groupcasting, we only discuss the sum DoF achieved by the messages transmitted in the second power level.

With the power P^b-P^a and ZF-precoders, three interfering links can be "removed", and the remaining links form a cyclic partially connected network as illustrated in Figure 2. In this network, with the orthogonal groupcasting method, only one message can be successfully transmitted, e.g., $(d_1^2,d_2^2,d_3^2)=(b-a,0,0)$, $(0,b-a,0)$ or $(0,0,b-a)$. Otherwise, there will be some users receiving a mixture of two common messages, which contradicts the philosophy of the orthogonal groupcasting method. However, the maximal groupcasting method requires each user to decode multiple common messages. By doing so, although the DoF of each common message decreases, the sum DoF can be enhanced since more common messages can be transmitted. Specifically, since each user receives the mixture of two common messages, it is straightforward that the per common message DoF $\frac{b-a}{2}$

is achievable, thus leading to the sum DoF of $\frac{3}{2}(b-a)$, which outperforms $b-a$ achieved by orthogonal groupcasting.

Counting the DoF $3a$ achieved by the private messages and the DoF $1-b$ achieved by common message multicasting with power $P-P^b$, the sum DoF achieved by TRS designed with maximal groupcasting is $1+\frac{b+3a}{2}$. Note that this result outperforms the sum DoF $1+2a$ achieved by RS, and the sum DoF $\max\{a+b,3a\}$ achieved by ZFBF with power control.

On the other hand, due to the complicated expression of the sum DoF achieved by TRS, the general sufficient and necessary condition where TRS strictly outperforms RS and ZFBF with power control is yet to be characterized. In an extreme case where the CSIT of the interfering links associated to a single user have equal qualities, i.e., $a_{kj}=\alpha_k$, $\forall k\in\mathcal{K}$, $\forall j\in\mathcal{K}\setminus k$, following the footsteps presented in Section IV-B, we can see that there always exists a user who has to decode all the common messages. As a result, the sum DoF achieved by TRS is essentially impacted and is no greater than the sum DoF achieved by RS. This observation implies that TRS is more useful in the scenario when the CSIT qualities of the interfering links associated to each user have a larger variance.

E. Upper-bound

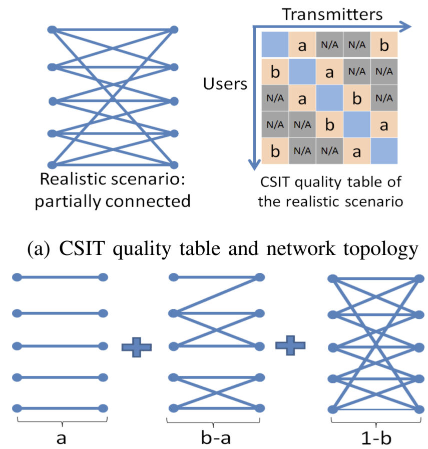
While we discuss the achievability in this paper, it is worth mentioning that the optimal DoF region/sum DoF performance of the considered MISO IC with imperfect CSIT remains an open problem. Tackling this problem is extremely challenging due to the varying CSIT qualities across the interfering links. In this part, using the methodology proposed in [4], we derive an upper-bound on the DoF region and sum DoF to evaluate the tightness of the DoF achievable by TRS.

Let us start with a two-cell case where the CSIT qualities of the two interfering links are equal to a. We enhance the MISO IC by assuming perfect CSIT for the direct links and providing user 1's message to user 2. Then, according to Fano's inequality and through some simple manipulations, we have

$$\begin{aligned} nR_1 &\le I(W_1;\mathbf{y}_1^n|\mathcal{U}) \le n\log P - h(\mathbf{y}_1^n|\mathcal{U},W_1) + no(\log P) \\ &= n\log P - h(\bar{\mathbf{y}}_1^n|\mathcal{U}) + no(\log P), \quad (52) \\ nR_2 &\le I(W_2;\mathbf{y}_2^n|\mathcal{U},W_1) \le h(\mathbf{y}_2^n|\mathcal{U},W_1) + no(\log P) \\ &\le h(\bar{\mathbf{y}}_2^n|\mathcal{U}) + no(\log P), \quad (53) \end{aligned}$$

where $\mathcal{U}\triangleq\{\mathbf{h}_{kj},\hat{\mathbf{h}}_{kj},k,j=1,2,\mathcal{G},\mathcal{A}\}$ denotes the set of perfect global CSI, imperfect global CSI, network topology and CSIT quality topology, while $\mathbf{y}_k^n$ denotes the received signal from the first time slot to the nth time slot and $\bar{y}_k=\mathbf{h}_{k2}^H\mathbf{s}_2+n_k$ is obtained by removing the signal sent by transmitter 1. The equalities are due to the fact that translation (i.e., from $\mathbf{y}_k^n$ to $\bar{\mathbf{y}}_k^h$) does not change the differential entropy.

Then, to derive an upper-bound on the sum DoF, it remains to bound $h(\bar{\mathbf{y}}_2^n|\mathcal{U})-h(\bar{\mathbf{y}}_1^n|\mathcal{U})$. As $\bar{\mathbf{y}}_2^n$ can be considered as the received signals in a two-user BC, it readily shows that this quantity is bounded by $an\log P$ following the footsteps in [4]. Consequently, the DoF region of the considered two-cell MISO IC is bounded by $d_1\le 1$, $d_2\le 1$ and $d_1+d_2\le 1+a$. This upper-bound indicates that TRS and RS are the optimal schemes.



(a) CSIT quality table and network topology

(b) Weighted sum interpretation

Fig. 3: 5-user examples with realistic setting, where $0\le a\le b\le 1$.

However, in the general K-cell case, it is not possible to apply the methods in [4] to obtain an upper-bound on the sum DoF. For example, in the three-cell case, by giving W_1 to user 2 and user 3, and giving W_2 to user 3, the upper-bound on the sum rate contains the following term $h(\mathbf{y}_2^n|\mathcal{U},W_1)-h(\mathbf{y}_1^n|\mathcal{U},W_1)$. After removing W_1 (i.e., the transmitted signal from transmitter 1), the residual terms in $\mathbf{y}_2^n$ and $\mathbf{y}_1^n$ consist of the transmitted signals from transmitter 2 and 3, which cannot be considered as a BC and the method in [4] is no longer applicable. Despite of that, we can obtain an upper-bound on the DoF of any subset of two users, and the upper-bound writes as the following proposition.

Proposition 5. *In a fully connected K-cell MISO IC with equal SNR and INR and with CSIT quality topology $\mathcal{A}$, the DoF region lies in*

$$d_k\le 1, d_j\le 1, d_k+d_j\le 1+\min\{a_{kj},a_{jk}\}, k,j\in\mathcal{K}, k\neq j. \quad (54)$$

Moreover, the upper-bound on the sum DoF $d_1+\cdots+d_K$ can be obtained by solving a linear program given the constraints in (54). For the three-cell scenarios illustrated in Figure 1(a) and 2, the sum DoF $d_1+d_2+d_3$ is upper-bounded by $\frac{3}{2}+b+\frac{a}{2}$ and $\frac{3}{2}+\frac{3a}{2}$, respectively. It can be shown that TRS achieves the upper-bound when $b=1$ in the cyclic scenario illustrated in Figure 2. More tractable and insightful analysis on the tightness is provided in the next section for a particular realistic scenario.

V. Realistic Scenarios

So far, we have identified the achievability of the TRS scheme in the fully connected IC where $g_{kj}=1$, $\forall k,j\in\mathcal{K}$. In this section, we show that the philosophy of the TRS scheme is also applicable to partially connected networks with imperfect CSIT. To see this, we switch our attention to a realistic scenario in the homogeneous cellular network [16], where each user typically only receives the signal sent by its serving transmitter, and is interfered by the signals sent by two adjacent transmitters, i.e., user k only sees $\mathbf{s}_k$, $\mathbf{s}_{k+1}$ and $\mathbf{s}_{k-1}$. The signals sent out by farther transmitters are assumed to be negligible due to the long distance. Note that it is assumed that user 1 is connected to transmitter K, 1 and 2, while user K is connected to transmitter $K-1$, K and user 1.

In the following, we firstly design a TRS approach for a class of CSIT quality topologies, where for each user, one incoming interfering link has CSIT quality b, while the other interfering link has CSIT quality a, i.e., either $(a_{k,k+1},a_{k,k-1})=(a,b)$ or $(a_{k,k+1},a_{k,k-1})=(b,a)$, $\forall k\in\mathcal{K}$. It is assumed that $a\leq b$. A 5-cell example is illustrated in Figure 3(a). Secondly, we find a closed-form expression for the maximal sum DoF achieved by the proposed TRS. Lastly, we compare the results with the sum DoF achieved by ZFBF with power control.

A. TRS scheme

Without sum DoF maximization, we design the TRS by considering that all users are active and each transmitter uses power P^a, i.e., $r_k=a,\forall k\in\mathcal{K}$, to unicast the private message. Following the footsteps presented in Section IV-B, the transmitted signal is expressed as

$$\mathbf{s}_k=\underbrace{\mathbf{p}_k^3 w_k^3}_{P-P^b}+\underbrace{\mathbf{p}_k^2 w_k^2}_{P^b-P^a}+\underbrace{\mathbf{p}_k^1 w_k^1}_{P^a}, \tag{55}$$

where $\mathbf{p}_k^2\subseteq\operatorname{span}(\{\hat{\mathbf{h}}_{jk}^{\perp}\}_{j=k+1,k-1,a_{jk}=b})$, and $\mathbf{p}_k^1\subseteq\operatorname{span}(\hat{\mathbf{h}}_{k+1,k}^{\perp},\hat{\mathbf{h}}_{k-1,k}^{\perp})$ are ZF-precoders, while $\mathbf{p}_k^3$ is a random precoder. The message w_k^1 is a private message intended for user k, w_k^2 is a common message to be decoded by user k and user j for some $j=k+1,k-1,a_{jk}=a$, while w_k^3 is a common message to be decoded by user k, $k-1$ and $k+1$.

The signal received by user k writes as

$$\begin{aligned} y_k=&\sum_{j=k-1}^{k+1}\underbrace{\mathbf{h}_{kj}^H\mathbf{p}_j^3 w_j^3}_{P}+\underbrace{\mathbf{h}_{kk}^h\mathbf{p}_k^2 w_k^2}_{P^b}+\\ &\sum_{j=k-1,k+1,a_{kj}=a}\underbrace{\mathbf{h}_{kj}^H\mathbf{p}_j^2 w_j^2}_{P^b}+\sum_{j=k-1,k+1,a_{kj}=b}\underbrace{\mathbf{h}_{kj}^H\mathbf{p}_j^2 w_j^2}_{P^0}+\\ &\underbrace{\mathbf{h}_{kk}^H\mathbf{p}_k^1 w_k^1}_{P^a}+\sum_{j=k-1,k+1}\underbrace{\mathbf{h}_{kj}^H\mathbf{p}_j^1 w_j^1}_{P^0}+\underbrace{n_k}_{P^0}. \end{aligned} \tag{56}$$

The sets of the common messages that are decoded by user k are defined as $\mathcal{T}_k^1\triangleq\{w_k^1\}$, $\mathcal{T}_k^2\triangleq w_k^2\cup w_j^2$, $j=k+1,k-1,a_{jk}=a$ and $\mathcal{T}_k^3\triangleq\{w_k^3,w_{k+1}^3,w_{k-1}^3\}$. By performing SIC, the achievable DoF lies in

$$d_k^1\leq a,\quad\sum_{j\in\mathcal{T}_k^2}d_j^2\leq b-a,\quad\sum_{j\in\mathcal{T}_k^3}d_j^3\leq 1-b,\forall k\in\mathcal{K}. \tag{57}$$

With the definition of $\mathcal{T}_k^i$, the weighted sum interpretation of the CSIT quality topology in Figure 3(a) is illustrated in Figure 3(b). The left, middle and right figures respectively stand for the partially connected networks where the private message unicasting, common message groupcasting and common message multicasting are performed. Next, given the achievable DoF region in (57), we study the maximal achievable sum DoF.

B. Sum DoF achieved by the proposed TRS

Firstly, it is clear that the maximum sum DoF achieved by the private messages $\{w_k^1\}_{k\in\mathcal{K}}$ is Ka.

Secondly, the maximum sum DoF achieved by the common messages $\{w_k^3\}_{k\in\mathcal{K}}$ can be found as follows. The inequalities in (57) related to w_k^3 can be explicitly written as $d_1^3+d_2^3+d_3^3\leq 1-b$, $d_2^3+d_3^3+d_4^3\leq 1-b$, $\cdots$, $d_{K-1}^3+d_K^3+d_1^3\leq 1-b$ and $d_K^3+d_1^3+d_2^3\leq 1-b$. Summing these K inequalities yields $3\sum_{k=1}^K d_k^3\leq K(1-b)$, leading to the sum DoF $\sum_{k=1}^K d_k^3\leq\frac{K}{3}(1-b)$. The equality holds by simply taking $d_1^3=d_2^3=\cdots=d_K^3=\frac{1-b}{3}$.

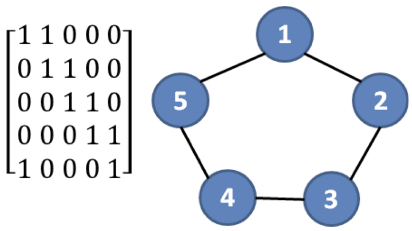
Thirdly, it remains to compute the maximal achievable DoF of the common messages $\{w_k^2\}_{k\in\mathcal{K}}$. To this end, according to the definition of set $\mathcal{T}_k^2$, we obtain a partially connected network with the topology matrix $\mathbf{M}^2$, whose elements are determined following (40). Specifically, if $j=k$ or $a_{kj}=a$, we have $m_{kj}=1$; otherwise, we have $m_{kj}=0$. An example of the obtained partially connected network is illustrated in Figure 3(b). Then, finding the sum DoF achieved by $\{w_k^2\}_{k\in\mathcal{K}}$ subject to (57) is equivalent to computing the sum DoF $\sum_{k\in\mathcal{K}}\hat{d}_k^2$ given $\mathbf{M}^2\hat{\mathbf{d}}^2\leq\mathbf{1}_K$, where $\hat{d}_k^2$ stands for DoF of common message $\hat{w}_k^2$ transmitted in the partially connected network defined by $\mathbf{M}^2$. The DoF d_k^2 achieved in TRS is obtained by $(b-a)\hat{d}_k^2$.

According to the CSIT quality topology mentioned at the beginning of this section, we see that each set $\mathcal{T}_k^2,\forall k\in\mathcal{K}$ has two elements, and each row of $\mathbf{M}^2$ has two "1"s. Then, following the definition introduced in Section IV-D, the hypergraph $\mathcal{H}^2(\hat{\mathcal{W}}^2,\mathcal{T}^2)$ is actually a graph. A member of $\mathcal{T}^2$, i.e., $\mathcal{T}_k^i$, refers to an edge between vertex $\hat{w}_k^2$ and its neighbor $\hat{w}_j^2$ if $a_{kj}=a$. Next, we characterize the sum DoF $\sum_{k\in\mathcal{K}}\hat{d}_k^2$ by evaluating the row rank of $\mathbf{M}^2$.

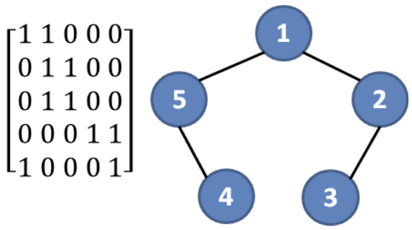
When $\mathbf{M}^2$ has a full row rank, it means that there is no redundant inequality in (57). In other words, there is no overlapping edge in $\mathcal{T}^2$. Moreover, since a vertex $\hat{w}_k^2$ can only have an edge with either $\hat{w}_{k-1}^i$ or $\hat{w}_{k+1}^i$, the graph $\mathcal{H}^2(\hat{\mathcal{W}}^2,\mathcal{T}^2)$ is actually a circuit. An example is illustrated Figure 4(a). It can be verified that the sum DoF of the common messages $\{\hat{w}_k^2\}_{k\in\mathcal{K}}$ is $\frac{K}{2}$ (obtained by adding up all the K inequalities involved in $\mathbf{M}^2\hat{\mathbf{d}}^2\leq\mathbf{1}_K$ and dividing the sum by 2).

When $\mathbf{M}^2$ has a deficient row rank, it means that some edges of the graph $\mathcal{H}^2(\hat{\mathcal{W}}^2,\mathcal{T}^2)$ are redundant. This fact breaks the circuit when $\mathbf{M}^2$ has full row rank into pieces. Clearly, if the row rank of $\mathbf{M}^2$ is $\operatorname{rowrk}(\mathcal{M}^2)=K-1$, the graph is a chain (see Figure 4(b)); if the row rank of $\mathbf{M}^2$ is $\operatorname{rowrk}(\mathcal{M}^2)=K-2$, the graph consists of two separated chains (see Figure 4(c)), then the maximum sum DoF can be computed by adding up the sum DoF achieved in each chain. Hence, when $\operatorname{rowrk}(\mathcal{M}^2)=r$, the graph has $K-r$ separated chains. The remaining work is to characterize the maximum sum DoF for a single chain.

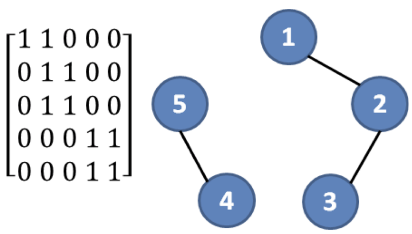
Intuitively, as two connected vertices correspond to a sum DoF constraint $\hat{d}_k^2+\hat{d}_{k+1}^2\leq 1$, the maximum sum DoF for a single chain is equal to the number of disjoint vertices. Hence, denoting the length of a chain by K_n, the sum DoF is $\frac{K_n}{2}$ if K_n is an even number and $\frac{K_n+1}{2}$ if K_n is an odd number. The rigorous proof is presented in Appendix C.



(a) rowrk($\mathbf{M}^2$)=5



(b) rowrk($\mathbf{M}^2$)=4



(c) rowrk($\mathbf{M}^2$)=3

Fig. 4: The hypergraph $\mathcal{H}^2(\mathcal{W}^2,\mathcal{T}^2)$.

In general, when $\text{rowrk}(\mathcal{M}^2)=r$, the sum DoF of common messages $\{\hat{w}_k^2\}_{k\in\mathcal{K}}$ writes as

$$\sum_{k\in\mathcal{K}}\hat{d}_k^2=\begin{cases}\frac{K}{2} & \text{if } r=K\\ \sum_{n=1}^{K-r}\frac{K_n}{2}1_{K_n\text{ is even}}+\frac{K_n+1}{2}1_{K_n\text{ is odd}} & \text{if } r<K\end{cases}\tag{58}$$

$$=\frac{K}{2}+\frac{\epsilon}{2},\tag{59}$$

where ϵ stands for the number of chains that have odd number of vertices. Then, the maximum sum DoF achieved by $\{w_k^2\}_{k\in\mathcal{K}}$ transmitted in TRS is $(b-a)\left(\frac{K}{2}+\frac{\epsilon}{2}\right)$.

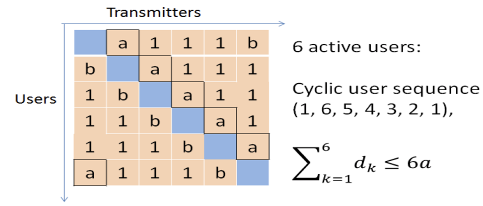
According to the above analysis and counting the sum DoF achieved by $\{w_k^i\}_{k\in\mathcal{K}},i=1,3$, we state the maximum achievable sum DoF in the considered scenario as follows.

Proposition 6. *In a K-cell MISO IC where 1) each user is connected to its closest three transmitters, and 2) the CSIT qualities of the two incoming interfering links associated to each user are a and b with $0\le a\le b\le 1$, the maximum sum DoF achieved by TRS designed by unicasting private messages with power P^a is*

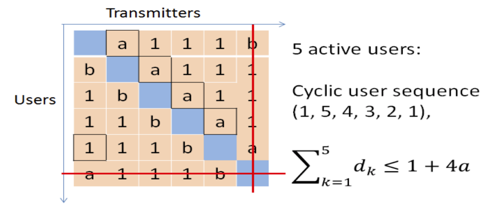
$$d_{s,TRS}^{\max}(\mathcal{K},\mathbf{r}=\mathbf{a})=\frac{K}{3}+\frac{K}{6}b+\frac{K}{2}a+\frac{b-a}{2}\epsilon,\tag{60}$$

where ϵ is defined in (59) *and $\mathbf{r}=\mathbf{a}$ means that $r_k=a,\forall k\in\mathcal{K}$.*

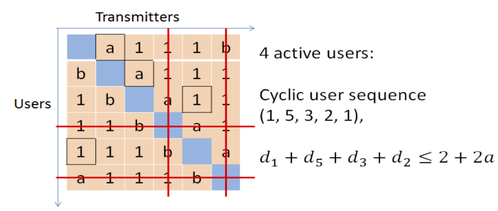
Obviously, the sum DoF achieved by the proposed TRS scheme strongly depends on ϵ, i.e., the number of chains with odd number of vertices. Since there are at least two elements in a chain, the shortest length of a chain with odd number of



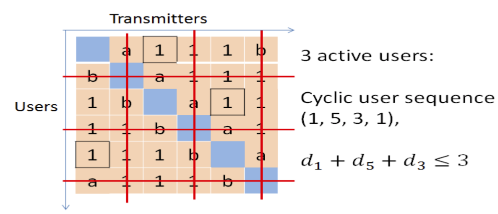
(a) All users are active.



(b) 5 active users, k=1,2,3,4,5.



(c) 4 active users, k=1,2,3,5.



(d) 3 active users, k=1,3,5.

Fig. 5: Illustration of computing the sum DoF achieved by ZFBF with power control

vertices is 3. Hence, the maximal value of ϵ is $\epsilon^*=\frac{K}{3}$, $\frac{K-2}{3}$ and $\frac{K-4}{3}$ when K mod 3=0, K mod 3=2 and K mod 3=1, respectively. This indicates that the best topology that yields the greatest sum DoF has the property that in the generated graph there exist ϵ^* chains with three vertices and $\frac{K-3\epsilon^*}{2}$ chains with two vertices. Then, by substituting ϵ^* into (60), we find that the best sum DoF is $\frac{K}{3}(1+b+a)-\frac{k_m}{3}(b-a)$, where $k_m\triangleq 2K$ mod 3. Besides, the worst topology has the property that all the chains have an even number of vertices. The worst sum DoF is equal to $\frac{K}{3}(1+\frac{b}{2}+\frac{3a}{2})$.

C. Discussion

1) Comparison with conventional schemes: In this part, we compare the sum DoF achieved by the proposed TRS scheme with the sum DoF achieved by preliminary schemes. In RS, each user employs a fraction of the total power to unicast the private message, while employs the remaining power to transmit the common message. However, unlike the received signal presented in (7), in the considered realistic scenarios, each user only decodes three common message transmitted by the dominant transmitters rather than all common messages.

This fact implies that the DoF achieved by the common messages are specified by $|\mathcal{S}|$ different inequalities, rather than the single inequality $\sum_{k\in\mathcal{S}} d_k^c \leq 1-\max_{j\in\mathcal{S}} r_j$ in (8). Hence, the achievable DoF region specified in Proposition 1 cannot be used to evaluate the DoF region achieved by RS in the considered realistic scenarios. Instead, we look at the sum DoF achieved by ZFBF with power control.

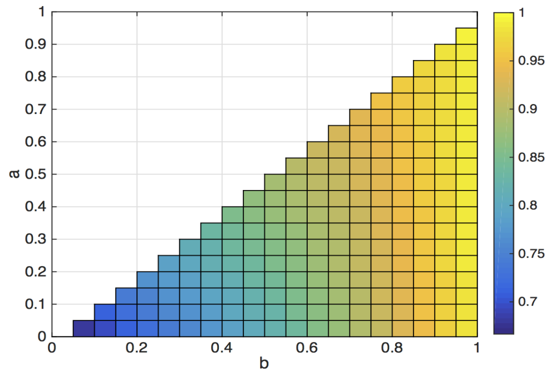
According to Remark 1, finding the maximum sum DoF achieved by ZFBF with power control requires a huge amount of efforts for evaluating the DoF region obtained for all the possible active user set $\mathcal{U}\subseteq\mathcal{K}$. To find a tractable result, we focus on the cyclic CSIT quality topology, i.e., $a_{k,k+1}=a$ and $a_{k,k-1}=b$, $\forall k\in\mathcal{K}$, where the index k is based on modulus K. We evaluate the achievable sum DoF for each possible active user set $\mathcal{U}$ using (10) with $\mathcal{S}=\mathcal{K}$, and the maximum of them yields the sum DoF achieved by ZFBF with power control.

A 6-user example is shown in Figure 5. Note that the 1 in row k and column j with $j\neq k-1,k+1$ is obtained because the DoF achieved by ZFBF in the considered scenario is identical to the DoF achieved by ZFBF in a fully connected MISO IC with CSIT quality $a_{kj}=1$, $\forall j\in\mathcal{K}\setminus\{k,k-1,k+1\}$. When all the users are active, the achievable sum DoF $6a$ is given by inequality $\sum_{l=1}^{m} d_{i_l}^p \leq \sum_{l=1}^{m} a_{i_{l-1}i_l}$, where the cyclic user sequence is $(i_1,\cdots i_6)=(1,6,5,4,3,2)$. Similarly, when there are 5 active users, the achievable sum DoF is $1+4a$, given by the cyclic user sequence $(1,5,4,3,2)$. When there are 4 active users, the best active users set that yields the maximum sum DoF is $\{1,2,3,5\}$ because user 5 does not interfere and is not interfered by the other three users. The achievable sum DoF is $2+2a$. When there are 3 active users, the sum DoF 3 can be achieved by simply scheduling user 1, 3 and 5. Hence, the maximum sum DoF achieved by ZFBF with power control is $\max\{6a,1+4a,2+2a,3\}$.

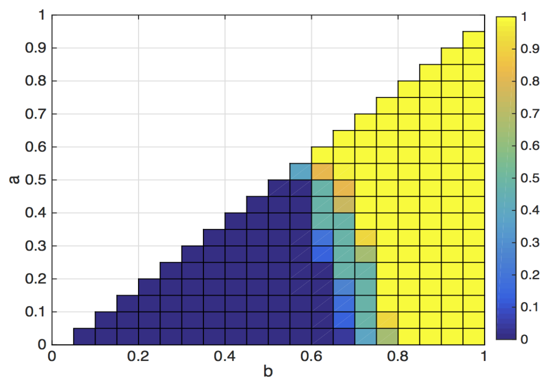
In general, when K is an even number, by applying the same method as above, the maximum sum DoF achieved by ZFBF with power control is $\max_{n:\frac{K}{2}\leq n\leq K}\{K-n+(2n-K)a\}$, where n stands for the number of active users. Thus, if $a\geq\frac{1}{2}$, we have $d_{s,zfbf}=Ka$; otherwise, we have $d_{s,zfbf}=\frac{K}{2}$.

When K is an odd number, the maximum sum DoF is achieved by ZFBF with power control is $\max\{\lfloor\frac{K}{2}\rfloor,\lfloor\frac{K}{2}\rfloor-1+a+b,2\lfloor\frac{K}{2}\rfloor-n+1+(2n-2\lfloor\frac{K}{2}\rfloor-1)a\}$, where $\lfloor\frac{K}{2}\rfloor+2\leq n\leq K$. The number $\lfloor\frac{K}{2}\rfloor-1+a+b$ is the achievable sum DoF when there are $\lfloor\frac{K}{2}\rfloor+1$ active users. It is achieved by scheduling user 1, 3, $\cdots$, $2\lfloor\frac{K}{2}\rfloor-3$ who are not interfered by each other, and scheduling another two adjacent users, i.e., user $2\lfloor\frac{K}{2}\rfloor-1$ and user $2\lfloor\frac{K}{2}\rfloor$. The quantity $2\lfloor\frac{K}{2}\rfloor-n+1+(2n-2\lfloor\frac{K}{2}\rfloor-1)a$ is the achievable sum DoF when there are n active users. It is achieved by scheduling $2\lfloor\frac{K}{2}\rfloor-n$ separated users, and $2n-2\lfloor\frac{K}{2}\rfloor$ adjacent users. Through some calculations, it can be verified that when $a\geq\frac{1}{2}$, we have $d_{s,zfbf}=Ka$; when $1-b\leq a\leq\frac{1}{2}$, we have $d_{s,zfbf}=\lfloor\frac{K}{2}\rfloor-1+a+b$; when $a\leq 1-b$ and $a\leq\frac{1}{2}$, we have $d_{s,zfbf}=\lfloor\frac{K}{2}\rfloor$.

According to the analysis in Section V-B, in the cyclic CSIT quality topology, the sum DoF achieved by the proposed TRS is $d_{s,TRS}^{\max}(\mathcal{K},\mathbf{a})=\frac{K}{3}(1+\frac{b}{2}+\frac{3a}{2})$. Then, it can be verified that $d_{s,TRS}^{\max}(\mathcal{K},\mathbf{a})>d_{s,zfbf}$ as long as $b+3a>\frac{6}{K}\lfloor\frac{K}{2}\rfloor-2$. This implies that the condition $b+3a>\frac{6}{K}\lfloor\frac{K}{2}\rfloor-2$ is a sufficient condition for TRS to yield a sum DoF strictly greater than ZFBF with power control. When this condition does not hold, we can seek for an optimal active user set $\mathcal{S}^*$ and optimal power allocation policy $\mathbf{r}^*$, which maximize the sum DoF achieved by TRS.



(a) Average tightness, i.e., $\frac{d_{s,TRS}^{\max}}{d_{upper}}$



(b) Proportion of CSIT topologies that achieve $\frac{d_{s,TRS}^{\max}}{d_{upper}}\geq 90\%$

Fig. 6: Tightness evaluation for the 6, 7, 8 and 9-cell scenarios

2) Tightness: In this part, we evaluate the tightness of the sum DoF achievable by TRS in the considered scenario. Using Proposition 5, we can obtain an upper-bound on the DoF region as given by (61) at the top of next page.

With the above constraints, one can calculate the upper-bound $d_{upper}=\max\sum_{k=1}^{K} d_k$ on the sum DoF numerically using MATLAB. In Figure 6(a), we evaluate the average tightness, i.e., $\frac{d_{s,TRS}^{\max}}{d_{upper}}$, for all the $2^6+2^7+2^8+2^9=960$ possible CSIT quality patterns of 6, 7, 8 and 9-cell scenarios. As shown, for all the CSIT quality topologies and all the possible values of a and b, TRS achieves more than 60% of the upper-bound, and the tightness increases with the values of a and b. It is worth mentioning that TRS is optimal when $b=1$. Besides, Figure 6(b) illustrates the proportion of the CSIT quality topologies that achieve 90% of the upper-bound. We can see that for nearly half of the values of a and b (the yellow grids), TRS achieves more than 90% of the upper-bound for all the CSIT quality topologies.

$$\begin{cases} d_k \le 1, d_j \le 1, d_k + d_j \le 1 + \min\{a_{kj}, a_{jk}\} & \text{if } (k,j)=(1,K) \text{ or } j=k+1, \forall k=2,\cdots,K-1; \\ d_k \le 1, d_j \le 1, d_k + d_j \le 2 & \text{else.} \end{cases} \quad (61)$$

VI. Conclusion

This paper, for the first time to our knowledge, studies the DoF of a K-cell interference channel with an arbitrary CSIT qualities of the interfering links. We firstly consider a Rate-Splitting approach where each user's data is split into a common part and a private part. The private messages are unicast along ZF-precoders using a fraction of the total power, while the common messages are multicast using the remaining power and are to be decoded by all users. With an arbitrary power allocation for the private messages, we characterize the DoF region achieved by RS, and show that it covers the DoF region achieved by ZFBF with power control. Secondly, we propose a novel scheme called Topological RS. Compared to RS, the novelty lies in splitting the power used to transmit common messages into multiple layers. In each layer, with the properly assigned power level and ZF-precoders, we transmit common messages to be decoded by groups of users rather than all users. This multi-layer structure reduces the number of common messages decoded by each user, thus enhancing the DoF achieved by the common messages. The DoF region achieved by TRS is derived and is shown as a superset of the DoF region achieved by RS and ZFBF with power control. Besides, the sum DoF is studied using Graph Theory tools and the sum DoF of a class of realistic scenarios is characterized.

Apart from that, we would like to emphasize the usefulness of the weighted sum interpretation that is used to design the TRS scheme. It bridges the MISO IC with imperfect CSIT and partially connected networks, thus providing an illustrative view of how many common messages can be transmitted. To this end, graph theory methodologies can be introduced as powerful tools to analyze the DoF performance. This weighted-sum interpretation can be applied to many other scenarios, such as MISO networks with alternating CSIT qualities.

So far, the optimal DoF region and/or sum DoF of a K-cell interference channel with imperfect CSIT remains an open problem due to the lack of tight outer-bound. Our proposed TRS drives the inner-bound one step further, and the obtained insights are transferrable to practical deployments.

Appendix

A. Proof of Proposition 1

The proof follows the footsteps in [19, Section III.B and Appendix D]. It has two steps. The first step is to characterize $\mathcal{D}_{RS}(\mathcal{S},\mathcal{U})$. As it will be shown later on, the union of $\mathcal{D}_{RS}(\mathcal{S},\mathcal{U})$ over $\mathcal{U}$ is a subset of $\mathcal{D}_{RS}(\mathcal{S}) \triangleq \bigcup_{\forall \mathbf{r}} \mathcal{D}_{RS}(\mathcal{S},\mathbf{r})$. The second step is to show $\mathcal{D}_{RS}(\mathcal{S}) \subseteq \bigcup_{\forall \mathcal{U}\in\mathcal{S}} \mathcal{D}_{RS}(\mathcal{S},\mathcal{U})$.

1) Step 1: For user $k\in\mathcal{S}\backslash\mathcal{U}$, we choose $r_k=0$ (Note that this choice is equivalent to $r_k=-\infty$ from a DoF perspective). Besides, we consider a polyhedral relaxation on the DoF tuple specified in (8) by requiring $r_k - \max_{j:j\in\mathcal{S}\backslash k}(r_j - a_{kj})^+$ to be non-negative. Then, the achievable DoF region via polyhedral relaxation is the set of the DoF tuples such that

$$0 \le d_k^p \le r_k - \max_{j:j\in\mathcal{U}\backslash k}(r_j - a_{kj})^+, \forall k\in\mathcal{U},$$
$$\sum_{k\in\mathcal{S}} d_k^c \le 1 - \max_{j\in\mathcal{S}} r_j. \quad (62)$$

The polyhedral relaxation requires that the power exponents $\mathbf{r}$ such that the power of interference overheard by user k is lower than the received power of user k's desired private message. Otherwise, the power exponents $\mathbf{r}$ are regarded as achieving an invalid DoF tuple. However, according to (8), those power exponents actually lead to a valid DoF tuple. Hence, the DoF region is shrinked by the polyhedral relaxation. Now, denoting $d^c = \sum_{k\in\mathcal{S}} d_k^c$, we rewrite (62) as

$$d_k^p \le r_k - (r_j - a_{kj}) \Rightarrow r_j - r_k \le a_{kj} - d_k^p, \forall k\in\mathcal{U}, \forall j\in\mathcal{U}\backslash k, \quad (63)$$
$$d_k^p \le r_k \Rightarrow -r_k \le -d_k^p, \quad (64)$$
$$d_k^p \ge 0, \quad (65)$$
$$d^c \le 1 - r_k \Rightarrow r_k \le 1 - d^c, \forall k\in\mathcal{U}. \quad (66)$$

Following the footsteps in [19, Section III.B], we define a fully connected directed graph $\mathcal{G}(\mathcal{V},\mathcal{E})$, where $\mathcal{V}=\{v_0, v_1, \cdots, v_{|\mathcal{U}|}\}$ is the vertex set and $\mathcal{E}$ is the set of the arcs. The length assigned to the arc from v_j to v_k is $l(v_j, v_k) = a_{kj} - d_k^p$ for $i,j\neq 0$, and the length assigned to the arc from v_k to v_0 is $l(v_k, v_0) = 1 - d^c$, while the length assigned to the arc from v_0 to v_k is $l(v_0, v_k) = -d_k^p$.

As defined in [27], a function f is called a potential if for every two vertices, a and b, such that $l(a,b) \ge f(a) - f(b)$ holds. Then, by setting $f(v_0)=0$ and $f(v_k)=r_k$, we see that any achievable DoF tuple such that (62) holds, corresponds to a potential function for the directed graph. Moreover, the potential theorem [27, Theorem 8.2] suggests that there exists a potential function for a directed graph if and only if each circuit of $\mathcal{G}$ has a non-negative length. Thus, a DoF tuple is said satisfying (62) if and only if each circuit of $\mathcal{G}$ has a non-negative length.

- For the circuits (v_0, v_k, v_0), we have $1 - d^c - d_k^p \ge 0$, yielding $d^c + d_k^p \le 1$, $\forall k\in\mathcal{U}$.
- For the circuits $(v_{i_0}, \cdots, v_{i_m})$ with $i_0 = i_m$, $\forall (i_1, \cdots, i_m)\in\Pi_{\mathcal{U}}$, $\forall m\ge 2$, we have $\sum_{l=1}^m d_{i_l}^p \le \sum_{l=1}^m a_{i_{l-1}i_l}$.
- For the circuits formed by $(v_{i_1}, \cdots, v_{i_j}, v_0, v_{i_{j+1}}, \cdots, v_{i_m})$ with v_0 between v_{i_j} and $v_{i_{j+1}}$, $\forall (i_1, \cdots, i_m)\in\Pi_{\mathcal{U}}$, we have $d^c + \sum_{l=1}^m d_{i_l}^p \le 1 + \sum_{l=1, l\neq j}^m a_{i_{l-1}i_l}$. Considering all the possible positions of v_0, we have the minimum operator in the last inequality of (11).

Consequently, $\mathcal{D}_{RS}(\mathcal{S},\mathcal{U})$ characterized by (10) and (11) is immediate.

2) Step 2: To show $\mathcal{D}_{RS}(\mathcal{S})\subseteq\bigcup_{\forall\mathcal{U}\in\mathcal{S}}\mathcal{D}_{RS}(\mathcal{S},\mathcal{U})$, we firstly introduce $\mathcal{D}'_{RS}(\mathcal{S},\mathcal{U})$ as

$$\mathcal{D}'_{RS}(\mathcal{S},\mathcal{U})=\{(d_1^c,\cdots,d_K^c,d_1^p,\cdots,d_K^p)\in\mathcal{D}_{RS}(\mathcal{S},\mathcal{U}),\quad d_k^p>0,\forall k\in\mathcal{U}\}. \quad (67)$$

Then, it is clear that $\mathcal{D}'_{RS}(\mathcal{S},\mathcal{U})\subseteq\mathcal{D}_{RS}(\mathcal{S},\mathcal{U})$, the remaining work is to show $\mathcal{D}_{RS}(\mathcal{S})\subseteq\bigcup_{\forall\mathcal{U}\in\mathcal{S}}\mathcal{D}'_{RS}(\mathcal{S},\mathcal{U})$. We aim to show that a DoF tuple lying outside $\bigcup_{\forall\mathcal{U}\in\mathcal{S}}\mathcal{D}'_{RS}(\mathcal{S},\mathcal{U})$ also lies outside $\mathcal{D}_{RS}(\mathcal{S})$. Such a DoF tuple has at least one of the following features:

- $d_k^p<0$ or $d_k^p>1$ or $d^c+d_k^p>1$ for some user $k\in\mathcal{S}$.
- $\sum_{l=1}^m d_{i_l}^p>\sum_{l=1}^m a_{i_{l-1}i_l}$ for some cyclic sequence $\forall(i_1,\cdots,i_m)\in\Pi_{\mathcal{U}}$.
- $d^c+\sum_{l=1}^m d_{i_l}^p>1+\sum_{l=1,l\neq j}^m a_{i_{l-1}i_l}$ for some users $\forall(i_1,\cdots,i_m)\in\Pi_{\mathcal{U}}$.

It has been shown in [19] that the DoF tuple satisfying the first and second feature cannot be included in $\mathcal{D}_{RS}(\mathcal{S})$. It remains to show that the DoF tuple satisfying the third feature cannot belong to $\mathcal{D}_{RS}(\mathcal{S})$. To this end, we employ the similar method in [19]. Assuming the DoF tuple satisfying the third feature lies in $\mathcal{D}_{RS}(\mathcal{S})$. Then, there exists some r_{i_l}'s such that

$$d^c+\sum_{l=1}^m r_{i_l}-\max_{i_k\in\mathcal{S}\setminus i_l}(r_{i_k}-a_{i_li_k})^+>1+\sum_{l=1,l\neq j}^m a_{i_{l-1}i_l}$$
$$\Rightarrow d^c-1+\sum_{l=1}^m r_{i_l}-\max_{i_k\in\mathcal{S}\setminus i_l}(r_{i_k}-a_{i_li_k})^+-\sum_{l=1,l\neq j}^m a_{i_{l-1}i_l}>0. \quad (68)$$

Since $\max_{i_k\in\mathcal{S}\setminus i_l}(r_{i_k}-a_{i_li_k})^+\geq r_{i_{l-1}}-a_{i_li_{l-1}}$, the l.h.s. of (68) can be upper-bounded as

$$\begin{aligned}&d^c-1+r_{i_{j-1}}-\max_{i_k\in\mathcal{S}\setminus i_{j-1}}(r_{i_k}-a_{i_{j-1}i_k})^+ +\\&\sum_{l=1,l\neq j}^m r_{i_{l-1}}-\max_{i_k\in\mathcal{S}\setminus i_{l-1}}(r_{i_k}-a_{i_{l-1}i_k})^+-a_{i_{l-1}i_l}\\\leq&d^c-1+r_{i_{j-1}}-\max_{i_k\in\mathcal{S}\setminus i_{j-1}}(r_{i_k}-a_{i_{j-1}i_k})^+ +\\&\sum_{l=1,l\neq j}^m r_{i_{l-1}}-r_{i_{l-1}}+a_{il-1i_l}-a_{il-1i_l}\leq 0,\end{aligned} \quad (69)$$

which contradicts (68). This implies that $\mathcal{D}_{RS}(\mathcal{S})\subseteq\bigcup_{\forall\mathcal{U}\in\mathcal{S}}\mathcal{D}'_{RS}(\mathcal{S},\mathcal{U})$, which completes the proof.

B. Proof of Proposition 2

We firstly show the DoF tuple achieved by common message $\{w_k^i\}_{k\in\mathcal{S}},i\geq2$ in (37) and (38), and secondly show the DoF tuple achieved by private messages $\{w_k^1\}_{k\in\mathcal{S}}$ in (36).

For user $k,\forall k\in\mathcal{K}$, when common messages of set $\mathcal{T}_k^i(\mathcal{S},\mathbf{r})$ are decoded, it is assumed the common messages of set $\mathcal{T}_k^l(\mathcal{S},\mathbf{r}),\forall l>i$, have been successively recovered and removed. Then, denoting the noise plus the interferences within the noise power by $\tilde{n}_k$, the received signal is expressed as

$$\tilde{y}_k=\sum_{j:w_j^i\in\mathcal{T}_k^i(\mathcal{S},\mathbf{r})}\underbrace{\mathbf{h}_{kj}^H\mathbf{p}_j^iw_j^i}_{P^{a_{\pi(i-1)}}}+\sum_{l=2}^{i-1}\sum_{j:w_j^i\in\mathcal{T}_k^i(\mathcal{S},\mathbf{r})}\underbrace{\mathbf{h}_{kj}^H\mathbf{p}_j^iw_j^i}_{P^{a_{\pi(l-1)}}}+\left(\underbrace{\mathbf{h}_{kk}^H\mathbf{p}_k^1w_k^1}_{P^{r_k}}+\sum_{j\in\mathcal{S}\setminus k}\underbrace{\mathbf{h}_{kj}^H\mathbf{p}_j^1w_j^1}_{P^{r_j-a_{kj}}}\right)+\underbrace{\tilde{n}_k}_{P^0}. \quad (70)$$

This system corresponds to a multiple-access-channel (MAC) where user k wishes to decode messages of set $\mathcal{T}_k^i(\mathcal{S},\mathbf{r})$. Following the capacity region of MAC [16], the sum rate of any non-empty subset $\mathcal{M}\subseteq\mathcal{T}_k^i(\mathcal{S},\mathbf{r})$ of messages are given by

$$\begin{aligned}\sum_{j:w_j^i\in\mathcal{M}}R_j\leq& I(\mathcal{M};\tilde{y}_k|\mathcal{T}_k^i(\mathcal{S},\mathbf{r})\setminus\mathcal{M})\\=&h(\tilde{y}_k|\mathcal{T}_k^i(\mathcal{S},\mathbf{r})\setminus\mathcal{M})-h(\tilde{y}_k|\mathcal{T}_k^i(\mathcal{S},\mathbf{r})),\end{aligned} \quad (71)$$

Considering that the input are random Gaussian codes, the entropies in (71) are equal to

$$h(\tilde{y}_k|\mathcal{T}_k^i(\mathcal{S},\mathbf{r})\setminus\mathcal{M})=a_{\pi(i-1)}\log_2 P+O(1),i\geq2, \quad (72)$$
$$h(\tilde{y}_k|\mathcal{T}_k^i(\mathcal{S},\mathbf{r}))=a_{\pi(i-2)}\log_2 P+O(1),i\geq3, \quad (73)$$
$$h(\tilde{y}_k|\mathcal{T}_k^2(\mathcal{S},\mathbf{r}))=\max\{r_k,\max_{j\in\mathcal{S}\setminus k}r_j-a_{kj}\}\log_2 P+O(1), \quad (74)$$

where $O(1)$ refers to the terms that do not change with P. Substituting (72), (73) and (74) into (71) and dividing them by $\log_2 P$ lead to (37) and (38).

When user k decodes private message w_k^1, all the common messages have been recovered and removed. By treating the undesired private messages as noise, the rate of w_k^1 writes as

$$R_k^1\leq I(w_k^1;y_k|\mathcal{T}_k^i(\mathcal{S},\mathbf{r}),i=2,\cdots,L+2) \quad (75)$$
$$=h(y_k|\mathcal{T}_k^i(\mathcal{S},\mathbf{r}),i=2,\cdots,L+2)-h(y_k|\mathcal{T}_k^i(\mathcal{S},\mathbf{r}),i=2,\cdots,L+2,w_k^1) \quad (76)$$
$$=h(\sum_{j\in\mathcal{S}}\mathbf{h}_{kj}^H\mathbf{p}_j^1w_j^1+\tilde{n}_k)-h(\sum_{j\in\mathcal{S}\setminus k}\mathbf{h}_{kj}^H\mathbf{p}_j^1w_j^1+\tilde{n}_k) \quad (77)$$
$$=r_k\log_2 P-\max_{j\in\mathcal{S}\setminus k}(r_j-a_{kj})^+\log_2 P+O(1). \quad (78)$$

Then, (36) is immediate.

C. Proof of the sum DoF of the realistic scenario considered in Section V

Without loss of generality, we consider the case $\text{rowrk}(\mathcal{M}^2)=K-1$ and the edges $\mathcal{T}_k^2=\{\hat{w}_k^2,\hat{w}_{k+1}^2\}$, $\forall k=1,K-1$, and $\mathcal{T}_K=\mathcal{T}_{K-1}$. Clearly, in this scenario, there is one chain with length K. The inequality $\mathbf{M}^2\hat{\mathbf{d}}^i\leq1$ is explicitly expressed as $\hat{d}_1+\hat{d}_2\leq1$, $\hat{d}_2+\hat{d}_3\leq1$, $\hat{d}_3+\hat{d}_4\leq1$, $\cdots$, $\hat{d}_{K-1}+\hat{d}_K\leq1$. Adding up the inequalities with odd index yields

$$\text{If } K \text{ is even,}\quad \sum_{l=1}^{\frac{K}{2}}\hat{d}_{2l-1}^2+\hat{d}_{2l}^2=\sum_{k=1}^{K}\hat{d}_k^2\leq\frac{K}{2}; \quad (79)$$

$$\text{If } K \text{ is odd,}\quad \sum_{l=1}^{\frac{K-1}{2}}\hat{d}_{2l-1}^2+\hat{d}_{2l}^2=\sum_{k=1}^{K-1}\hat{d}_k^2\leq\frac{K-1}{2}. \quad (80)$$

Inequality (79) provides an upper-bound on the sum DoF of common messages $\{\hat{w}_k^2\}_{k\in\mathcal{K}}$ when K is an even number. The equality holds with $\hat{d}_1^2{=}\hat{d}_3^2{=}\cdots{=}\hat{d}_{K-1}^2{=}1$. When K is an odd number, we obtain an upper-bound on the sum DoF of common messages $\{\hat{w}_k^2\}_{k\in\mathcal{K}}$ by adding $\hat{d}_K^2$ to both sides of (80) as

$$\sum_{k=1}^{K}\hat{d}_k^2\leq\frac{K-1}{2}+\hat{d}_k^2\leq\frac{K+1}{2}. \qquad (81)$$

The inequality (81) is obtained due to the fact that $\hat{d}_K^2\leq 1$. Then, using (81) we can obtain the maximum sum DoF of common messages $\{\hat{w}_k^2\}_{k\in\mathcal{K}}$ as $\frac{K+1}{2}$ by taking $\hat{d}_1^2{=}\hat{d}_3^2{=}\cdots{=}\hat{d}_{K-2}^2{=}\hat{d}_K^2{=}1$.

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