

1 Characterization of **fluidized nanoparticle** agglomerates by using **adhesive** 2 **CFD-DEM simulation**

3 Daoyin Liu^{1,2}, Berend G. M. van Wachem³, Robert F. Mudde², Xiaoping Chen¹, J. Ruud van
4 Ommen^{2*}

5 * Corresponding author. E-mail address: j.r.vanommen@tudelft.nl

6 1 Key Laboratory of Energy Thermal Conversion and Control of Ministry of Education, School of Energy and Environment,
7 Southeast University, Nanjing 210096, Jiangsu, China

8 2 Department of Chemical Engineering, Faculty of Applied Sciences, Delft University of Technology, Julianalaan 136, 2628
9 BL Delft, The Netherlands

10 3 Thermofluids Division, Department of Mechanical Engineering, Imperial College London, Exhibition Road, London SW7
11 2AZ, United Kingdom

12 **Abstract**

13 Nanoparticles form **multi-stage** agglomerates when they are fluidized: simple agglomerates (a
14 few dozens of micrometres), and complex agglomerates (a few hundreds of micrometres). This paper
15 studies nanoparticle fluidization by using adhesive CFD-DEM (Discrete Element Model) simulation,
16 which includes an adhesive contact model having the ability to model the contact and bounce/stick
17 between elastic-plastic and cohesive particles. The discrete elements represent the simple
18 agglomerates of nanoparticles. The effects of particle density, fluidizing gas velocity, and van der
19 Waals force on gas-solid flow and agglomerate behaviors are investigated. It is shown that the
20 particles, representing simple agglomerates, form large agglomerates during fluidized process. As the
21 agglomerates move around in the bed, they continuously break and recombine. The solid movement is
22 evaluated by a dispersion coefficient **and it decreases with increase in cohesion**. By monitoring the
23 evolution of contacts, the agglomerate breakage is visualized and compared for the different
24 conditions. Increasing fluidizing gas velocity and reducing particle adhesion both increase
25 agglomerate breakage rate. The **structure parameters of agglomerates are further analyzed**: averaged
26 coordination number of agglomerates around 3.0, packing **fraction** around 0.2-0.3, and fractal
27 dimension around 1.9-2.3, **which can be used to indicate structures and formation mechanisms of**
28 **agglomerates**.

29 **Keywords:** Nanoparticle Fluidization; Agglomerate; Discrete Element Model; Adhesion

1. Introduction

Nanoparticle fluidization is a promising technique to disperse and process nanoparticles [1-4]. Extensive experimental studies have been performed on nanoparticle fluidization [1, 5]. Two aspects are of key importance: fluidization behavior and agglomerate structure, since nanoparticles are not fluidized individually but rather as large, very porous agglomerates.

To characterize the fluidization behavior, the minimum fluidization velocity (u_{mf}) and bed expansion ratio (h/h_{mf}) are usually used for measuring macro fluidization quality [6-9]. **The reduction of cohesive forces or the increase of separating forces can result in a reduced u_{mf} , usually along with an increased h/h_{mf} .** For example, $u_{mf}(\text{TiO}_2) > u_{mf}(\text{SiO}_2)$, since the van der Waals forces (vdW) are larger for materials with higher surface energy [7]. u_{mf} can be reduced by pre-treatment of the anti-static surfactant, drying, or providing particles with an apolar coating, because of reduced cohesive forces (electrostatic force, capillary force, or hydrogen bond force) [7, 10]. Also, u_{mf} can be reduced by fluidization assistance methods including mechanical vibration, auxiliary magnetic fields, and ultrasound fields, **because of increased separating forces** [8, 11-13].

Regarding the agglomerate structure, information including the agglomerate size and fractal dimension, are investigated. It is generally accepted that the complex agglomerates are around a few hundreds of micrometres [6, 14, 15]. The fractal dimension can be used to characterize agglomerates, but its variation with various parameters still needs more investigations [16]. The agglomerate information can be obtained in different ways, but it remains difficult to obtain the agglomerate properties in-line [17].

The numerical simulations have also been used for studying nanoparticle fluidization. In order to make the following description consistent, we restate the concept of “agglomerate” here. In most studies, the commercial metaloxide nanopowders are considered, with primary particle sizes of 5-50 nm. Fluidized agglomerates of such materials have a multi-stage structure: aggregates of nanoparticles sintered during production (~200 nm), simple agglomerates (a few dozens of micrometres), and complex agglomerates (a few hundreds of micrometres) [6, 7, 18]. In literature, most of the simulation studies consider the complex agglomerates as ‘particles’ **with the equivalent sizes and densities.** The

1 simulations can predict bubbling dynamics and bed pressure expansion [19-21]. However, the model
2 cannot reveal the structure dynamics of the complex agglomerates, e.g. breakage and reunion, which
3 are also critical for the overall gas-solid contact efficiency. In our previous study [22], we have
4 developed an adhesive CFD-DEM model, where the simple agglomerates are assumed as ‘particles’,
5 which are the building blocks of the larger complex agglomerates. It is shown that this model can
6 reveal agglomerate breakage and reunion, as well as fluidized dynamics. In that paper [22], we have
7 presented details of the model and tested its reliability for studying fluidization; the fluidized bed
8 undergoes uniform fluidization to defluidization by increasing the cohesive force.

9 In this study, the gas and solid flow dynamics, the agglomerate break-up and reunion behaviors,
10 and the statistics of agglomerate properties (coordination number, packing fraction, fractal dimension)
11 are investigated. The effect of particle density, cohesion, and fluidizing gas velocity are also compared.
12 Through this effort, our aim is to show that the current adhesive CFD-DEM model is able to study the
13 dynamic properties of the complex agglomerates as well as the fluidization behaviors.

14 **2. Numerical Model**

15 **2.1 Adhesive CFD-DEM model**

16 The adhesive CFD-DEM model is used for simulating nanoparticle agglomerate fluidization. The
17 simple agglomerates are represented by cohesive and plastic particles. Since the model details are
18 presented in our previous paper [22], only a general description is given here. The current adhesive
19 CFD-DEM model is a combination of the conventional CFD-DEM model, an adhesive contact model,
20 and a drag force scale method. The simulation is performed using MultiFlow, a parallel multi-phase
21 flow solver [23]. The general algorithm at each time step is similar to the conventional CFD-DEM
22 model, but the contact model is substituted by the adhesive contact model and the drag force is
23 reduced by a scale factor. The governing equations with sub-models are summarized in Table 1.

24 Like the solver in the conventional CFD-DEM model, the motion of the gas phase is calculated
25 by local averaged Navier-Stokes equations while the motion of each particle is calculated by Newton’s
26 second law. The interaction between the particles is modelled by the adhesive contact model.
27 Basically, it is a hysteresis linear spring–dashpot contact model, which has the ability to model

1 different collision regimes, from bounce to stick, between elastic-plastic and cohesive particles. The
2 elastic-plastic property is expressed by a plasticity index (ψ_p) which indicates the ratio between the
3 loading and the unloading spring stiffness, $\psi_p = 1 - k_{n,load} / k_{n,unload}$ [24]. The cohesive force is expressed
4 by a non-dimensional Bond number, e.g. $Bo_{vdw} = F_{vdw,0} / mg$.

5 The interaction between the gas and particles is modelled by a semi-empirical drag model with
6 the drag force scale factor (ξ). In the current simulation, Wen&Yu drag force correlation is used [25,
7 26]. ξ is used to consider drag reduction when particles are agglomerated, an approach coming from
8 simulation studies of Geldart A particles [27-29]. When particles are agglomerated, the particles inside
9 the agglomerate hardly experience the gas flow, which results in a reduction of the inter-phase surface
10 and thus the drag force. However, the accurate calculation of ξ is still an unresolved problem. In the
11 previous study [22], we determined different values of ξ for cases with $Bo_{vdw} = 0, 20, 50,$ and 100 by a
12 “predictor-corrector method”. It is shown that $\xi = 0.2$ is reasonable for $Bo_{vdw} = 20$ case. In this study,
13 since the simulation conditions are close to $Bo_{vdw} = 20$, $\xi = 0.2$ is used as constant for all cases for
14 simplification.

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Table 1

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19 2.2 Computational Setup

20 Simulations are performed on a three-dimensional fluidized bed with size of [4, 0.8, 20] mm,
21 which contains particles representing the simple agglomerates. The fluidizing gas is air at room
22 temperature, with density 1.205 kg/m^3 and viscosity $1.8\text{e-}5 \text{ kg/(m.s)}$. At the inlet the uniform gas
23 velocity is specified, and at the outlet the gas pressure–outlet boundary condition is adopted. At the
24 walls, the no–slip wall condition for the gas phase is used. Considering the bed size is limited, the
25 interaction between particles and wall is treated by noncohesive collision.

26 Based on experimental measurements, the size of the simple agglomerates is usually around 5–45
27 μm [18]. To avoid too heavy computation [30], the particle diameter (d_{sa}) is chosen as $40 \mu\text{m}$ to

1 represent the simple agglomerates. For the particle density (ρ_{sa}), we have compared two values, e.g.
2 the heavy one, $\rho_{sa} = 250 \text{ kg/m}^3$ estimated based on simple agglomerates of TiO_2 nanoparticles, and
3 the light one, $\rho_{sa} = 137 \text{ kg/m}^3$ agglomerates of SiO_2 .

4 The effect of vdW force is studied by changing Bo_{vdw} . In our previous study [22], Bo_{vdw} is
5 changed from 0 to 100 for TiO_2 agglomerates, representing the vdW force 0–8 nN, which is based on
6 literature study. The cases with $Bo_{vdw} = 0$ and 20 show uniform and agglomerating fluidization, but
7 for the cases with $Bo_{vdw} = 50$ and 100, the bed makes the transition to a fixed bed regime. Thus, in
8 this study the range of Bo_{vdw} is narrowed and varied from 5 to 20. Other input parameters of the
9 contact model are kept constant: coefficient of restitution due to the visco-elastic nature $e_{vis} = 0.8$,
10 frictional coefficient $\mu_f = 0.35$, unloading spring stiffness $k_{n,unload} = 2 \text{ N/m}$, plasticity index $\psi_p =$
11 0.2, cohesive forces due to physical or chemical adsorption $F_{bond,0} = 0$ [22]. The Young's modulus of
12 the agglomerates is a function of porosity, primary particles and surface energy, and estimated around
13 the order of 100 KPa for the nanoparticle agglomerates [31]. Correspondingly, the stiffness is around
14 the order of 1 N/m in the linear spring contact model. The kinetic energy dissipation due to plastic
15 deformation of the nanoparticle agglomerates is mainly varied between 0.2 and 0.5, according to
16 measurements by the nanoindentation [24]. The effect of fluidizing gas velocity (u_g) is also studied.
17 Since the experimental studies in literature usually cover u_g from $\sim 1 \text{ cm/s}$ up to a few cm/s and
18 minimum fluidization velocity (u_{mf}) is reported around or below 1 cm/s . In the current simulation, u_g
19 is changed from 2 to 4 cm/s .

20 In a summary, the effects of particle density, fluidizing gas velocity, and vdW force are
21 investigated. The simulation cases are summarized in Table 2. Cases 1 to 3 are used to study effect of
22 u_g on fluidization with light particle agglomerates, Cases 4 to 6 to study effect of u_g on fluidization
23 with heavy particle agglomerates, and Cases 6 to 8 to study effect of Bo_{vdw} on fluidization.

24
25 Table 2
26

1 2.3 Definition of agglomerate characteristics

2 The DEM particles, representing the simple agglomerates, will build larger complex
3 agglomerates when fluidized. The complex agglomerate radius, coordination number, packing fraction,
4 and fractal dimension are evaluated. Their definitions are listed in the following.

5 The agglomerate radius (R_{agg}) is defined by $m_{agg}R_{agg}^2 = \sum_1^{n_p} m_{sai}l_i^2$, where n_p is the particle
6 number contained in an agglomerate, m_{agg} and m_{sa} the agglomerate mass and particle mass, and l_i
7 the distance between the centers of the particle i and the agglomerate [32].

8 The packing fraction (p) is defined as the ratio between the sum of the particles' volume and the
9 agglomerate's volume [33]. The coordination number (C_N) is defined as the number of touching
10 neighbors for a given particle [33]. The fractal dimension (D_f) describes the relation between the
11 agglomerate size and the agglomerate density and is determined from the equation
12 $n_p = (R_{agg}/r_{sa})^{D_f}$ [34].

13 2.4 Identification of agglomerates

14 The identification of agglomerates is not trivial in a dense particle system, since the short-term
15 contacts (collisions) and long-term contacts (agglomeration) should be distinguished. In our previous
16 paper, the DEM particles in contact whose relative velocity is smaller than a certain limit and
17 coordination number larger than 2, are recognized as agglomerates, but the limit of the velocity could
18 influence the results and should be chosen carefully [22]. In this paper, we identify the agglomerates
19 by monitoring the breakage of contacts. If a contact holds for a given period of time, then it is treated
20 as an agglomerate. The longer time period allows to identify more stable agglomerates. This method is
21 more objective, but the computation is expensive. There are other criteria could be possible for
22 agglomerate identification by image analysis methods. An evaluation of various criteria is beneficial
23 and currently ongoing in our group. We will now briefly describe the agglomerate identification
24 procedure for this paper:

25 Step 1, Choose the reference time (t_0), and one comparison time (t_1). The contacts at t_0 and t_1
26 are compared, and the shared particles are considered as candidates of agglomerate's elements and
27 stored as a list.

1 Step 2, Build the agglomerates from the shared contact list. The particles in the shared list of t_0
2 and t_1 are linked using a Last In First Out (LIFO) data structure. For example, if particle A contacts
3 with B1, B2 and B3, B1 again contacts with C1 and C2, and B2 again contacts with D1, then the
4 agglomerate is built in the sequence of A-B1-C1-C2-B2-D1-B3.

5 Step 3, Choose another comparison time (t_2), and repeat Steps 1 and 2.

6 Therefore, by choosing a reference time t_0 and a series of comparison times, the evolution of
7 agglomerate breakage can be obtained, as illustrated in Figure 1 for the case psa250vdw20ug4. The
8 particles in the same agglomerate are shown with the same color. Here, the reference time is 1.00 s.
9 Initially, at time 1.01 s, the agglomerates identified are very large, because the particles just touching
10 briefly are included in agglomerates. With an increase in the time interval, the agglomerates gradually
11 become smaller and the number of agglomerate decreases. In order to represent typical agglomerates,
12 the agglomerates with only a few particles are filtered out. Here, we use 15 as the lower limit. **The**
13 **limit setting is like image analysis of agglomerates during experiment studies. It could change results**
14 **quantitatively, but it does not change the trend of the effects of parameters on agglomerates breakage,**
15 **because it influences equally to each of the case. In preliminary studies, we compared the limit values**
16 **of 15 and 10, they give very similar quantitative results about agglomerate breakage rate and**
17 **probability distribution of agglomerate properties.**

18 The disappearance of agglomerates with time indicates that the ‘old’ agglomerates identified at t_0
19 completely break up. Thus, by monitoring the evolution of contacts, the stability of agglomerates can
20 be characterized. This information cannot be obtained by examining contacts separately for each
21 instantaneous image. Note that although the process of agglomerate recombination is not examined in
22 this paper, the recombination and breakage of agglomerates maintain a dynamic equilibrium during
23 normal fluidization.

24

25

Figure 1

26

3. Results

The simulation results include two aspects: flow pattern at the macro scale and agglomerate dynamic at the meso scale. In this paper, the flow pattern is described briefly to check the model's reliability and show the overall picture of fluidization. Prediction of agglomerate dynamic is the unique of the current model and thus it is focused.

3.1 General flow pattern

Figure 2 shows the instantaneous solid flow pattern for all the cases listed in Table 2. Generally, the particles are fluidized in the form of agglomerates. The agglomeration depends on particle density, fluidizing gas velocity and vdW force. A quantitative analysis of the agglomeration will be given in Sections 3.2 and 3.3. The dynamics of the solid flow patterns are shown by an animation in the supplementary materials of this paper. From Figure 2 and the supplementary animation, the dynamic breakage and reunion processes of the agglomerates can be observed. As the agglomerates move around in the bed, they break into smaller ones. Meanwhile the smaller ones recombine continuously to new, larger agglomerates.

Figure 3 shows the instantaneous gas velocity vector with contours of solid volume fraction for four of the cases. Only four cases are selected for shown, since the others give similar information. Generally, some channelling with higher gas velocity is observed. In the cases of $\rho_{sa}137vdw20ug2$ and $\rho_{sa}250vdw5ug4$, the distribution of solid and gas velocity is relatively more uniform, because of the weaker particle cohesion. Compared with the case $\rho_{sa}250vdw20ug2$, the channeling in the case $\rho_{sa}250vdw20ug4$ is more obvious, which indicates that channeling increases by increasing fluidizing gas velocity under the same particle cohesion.

Figure 4a shows the time-series of bed pressure drop (ΔP) for different cases. The solid straight lines indicate the ideal bed pressure drop calculated by the weight of bed materials per bed cross section area. It is shown that ΔP fluctuates around the theoretical values. The fluctuation amplitudes are expressed by the standard deviation ($\sigma_{\Delta p}$) of ΔP , as shown in Figure 4b. As expected, by increasing particle density, u_g from 2 cm/s to 4 cm/s, and Bo_{vdw} from 5 to 20, $\sigma_{\Delta p}$ increases.

1 Figure 2

2 Figure 3

3 Figure 4a

4 Figure 4b

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6 In order to quantify the particle movement, the solid dispersion coefficient is calculated based on
7 statistics of particle displacement [35-37]. Given trajectory of a single particle, the local solid
8 dispersion (D_n) of particle n can be calculated as following

9
$$D_n = \frac{(\Delta r_n)^2}{2\Delta t} \quad (1)$$

10 where Δr_n is the displacement of particle n during a time interval Δt . The displacement can be
11 along horizontal or vertical direction. The averaged solid dispersion coefficient (D_s) is calculated by

12
$$D_s = \frac{1}{N} \sum_{n=1}^N \frac{(\Delta r_n)^2}{2\Delta t} = \frac{1}{\Delta t} \frac{1}{N} \sum_{n=1}^N \frac{(\Delta r_n)^2}{2} = \frac{1}{\Delta t} S^2 \quad n = 1, \dots, N \quad (2)$$

13 where S^2 is the mean of the squared displacement of all the particles, and N the particle number.
14 Based on the curve of $S^2 - \Delta t$, D_s is determined. As an example, Figure 5a shows the curve of
15 $S^2 - \Delta t$ along z direction for four of the cases. It is shown that S^2 increases with time and then
16 fluctuates around some steady value. The value of D_s is calculated from the maximal slope of the
17 secant line from its initial time to a selected time.

18 Figure 5b summarizes D_s along x and z directions (D_{sx} and D_{sz}) for different cases. The D_s along
19 y direction is not presented here, since it is much smaller as a result of the limited bed depth. It is
20 shown that D_{sz} is much larger than D_{sx} , which means the vertical solids mixing is dominant. D_{sz}
21 decreases by increasing particle density and particle cohesion, while D_{sz} increases obviously with
22 increase in fluidizing gas velocity. It is interesting to note here that the qualitative trends of D_{sz} with
23 the parameters are the same for the agglomerate breakage rate, which will be presented in the
24 following section.

25

26 Figure 5a

Figure 5b

3.2 Agglomerate breakage

Before further analyzing the agglomerate breakage, we emphasize that the agglomerate breakage and reunion maintain a dynamic equilibrium. This point is proved by plotting the variation of the coordination number (C_N) with time. Figure 6a shows the averaged C_N with time, which is calculated by averaging C_N over all the particles at each instantaneous time. It is shown that C_N increases quickly from 0 to 2-3 within 0.3 seconds, and then fluctuates around one steady value, which indicates that the agglomerate breakage and recombination are in a dynamic balance.

Figure 6b shows the probability distribution of C_N for one selected case, $\rho_{sa250vdw20ug4}$, at different times. The distribution curve quickly becomes stable, with the range of C_N from 0 to 7. Again, the stable distribution of C_N indicates that the agglomerates breakage and reunion are balanced.

From Figures 6a and 6b, it is interesting to note that the averaged C_N is a bit larger than that of normal fluidized beds, which fluctuates around 1.7 in a fluidized bed with particle diameter of 100 μm (Figure 22 in Ref. [38]). Also, the distribution shape is very similar to the normal fluidized conditions, with dominant coordination ranging from 1 to 4, but there are some quantitative differences in frequency of accounts (Figure 18 in Ref. [38]).

Figure 6a

Figure 6b

The agglomerate breakage for different cases can be compared by both visualization and statistical methods. Figure 7 visualizes the agglomerate breakage by highlighting the evolution of one selected agglomerate. The particles in one agglomerate are tagged with the black color at time 1.01 s, and then traced in the following times. For a good visualization of the agglomerate breakage, here only 1/5 of the background particles are shown. It is clear that the case $\rho_{sa250vdw20ug2}$ shows the slowest agglomerate breakage rate, while the case $\rho_{sa250vdw5ug4}$ shows the fastest rate. Thus, increasing fluidizing gas velocity and decreasing particle adhesion both increase agglomerate breakage rate.

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Figure 7a
Figure 7b
Figure 7c
Figure 7d

7 To quantitatively compare the agglomerate breakage rate of the different cases, the agglomerate
8 number (N_{agg}) and agglomerate diameter (d_{agg}) are calculated during the evolution of contacts
9 following the approach shown in Figure 1. Figure 8a compares N_{agg} as a function of time interval for
10 the different cases. For the **light particle** cases, N_{agg} first decreases rapidly and then decreases
11 gradually. The rapid decrease is caused by breakage of weak contacts, while the slow decrease is
12 caused by breakage of stable contacts. **By comparison of the two fractions**, it can be seen that the
13 fraction of the weak contacts is dominant.

14 For the **heavy** particle cases, the decrease of N_{agg} is slower, and becomes even slower by
15 increasing the vdW force or decreasing u_g . Furthermore, it can be seen that N_{agg} first increases within
16 a very short time. The increase in N_{agg} indicates a different breakage mechanism from that during the
17 decrease period, which will be discussed in the following along with the variation of d_{agg} .

18 Figure 8b shows the mass-averaged d_{agg} as a function of time interval for the different cases.
19 Generally, d_{agg} for the **heavy particle** cases are larger than **light particle** cases. For all the cases,
20 during the evolution of contacts, d_{agg} first decreases rapidly, and then decreases gradually to a value
21 around 0.2 mm. After some period with stable values, d_{agg} decreases to zero before the time interval
22 reaching 1.5 s for most cases. However, d_{agg} stays intact for the case `psa250vdw20ug2`, which means
23 the agglomerate is more stable. Again, it indicates that increasing particle cohesion or decreasing
24 fluidizing gas velocity leads to decrease in agglomerate breakage rate.

25 **The above different behaviors of N_{agg} and d_{agg} with agglomerate breakage are further analyzed.**
26 The process with an increase in N_{agg} **but** decrease in d_{agg} , is caused by agglomerate fragmentation,

1 i.e. one agglomerate breaks down into more agglomerates. As d_{agg} increases, it takes longer time for
2 N_{agg} to reach maximum, which indicates the fragmentation lasts longer. This is because large
3 agglomerates tend to fragment. **However**, the process with a decrease in both N_{agg} and d_{agg} is mostly
4 a result of agglomerate attrition, i.e. the agglomerate becomes smaller by losing small fragments (i.e.,
5 one or a few DEM particles) from it. When this process continues, the agglomerate will become
6 smaller and smaller, and is no longer counted when small enough (here the lower limit value
7 $N_{agg}=15$).

8 **Therefore**, we can conclude that there are two kinds of breakage, i.e., fragmentation and attrition.
9 Generally, the fragmentation leads to an increase in N_{agg} while the attrition leads to a decrease in
10 N_{agg} . Also, there are different contacts in an agglomerate, i.e. weak contacts, strong contacts, and
11 sometimes permanent contacts, which result in different stabilities of agglomerates.

12 Figure 8c compares critical times for different cases, which are determined from the data shown
13 in Figures 8a and 8b. The critical times are $\tau_{N_{max}}$, τ_{N15} , and τ_{ds} , defined as the time when N_{agg} is
14 maximum, N_{agg} less than 15, and d_{agg} becomes steady, respectively. By comparing $\tau_{N_{max}}$, τ_{N15} , and
15 τ_{ds} , it is clear that the agglomerates break faster in case of **light particles** than for **heavy particles**. The
16 characteristic times decrease **by** decreasing the particle cohesion and increasing the fluidizing gas
17 velocity. **It is note that the agglomerate breakage rate has the similar trend with the solids dispersion**
18 **coefficient from the macro method.**

19

20 Figure 8a

21 Figure 8b

22 Figure 8c

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24 **3.3 Agglomerate properties**

25 In this section, statistical analysis is performed on the agglomerate properties. Figure 9 shows the
26 mass-averaged d_{agg} , C_N , p , and D_f for different cases, at the three critical times of $\tau_{N_{max}}$, τ_{N15} , and
27 τ_{ds} . Please note that the average C_N presented in Section 3.2 is for all the particles in the bed, while

1 here C_N is averaged only over particles in agglomerates. As time increases from τ_{Nmax} to τ_{N15} ,
2 decrease is found for d_{agg} , C_N and D_f of different cases and increase is found for p , which is a result
3 of the evolution of the agglomerate breakage.

4 It is shown that C_N is around 3.0 and p around 0.2-0.3 for different cases. C_N and p can reflect
5 agglomerate structure. Based on a review on the coordination [33], for the random loose packing of
6 mono-sized spheres, C_N ranges from 4.4 to 6 and p from 0.55 to 0.62. Lower packing density and
7 coordination are observed when surface forces are dominant. The minimal stability of agglomerates
8 under gravity is 3.0 to 3.5 contacts and packing fraction of 0.30. Therefore, the agglomerates here are
9 roughly close to the structure with minimal stability.

10 It is shown that D_f is around 1.9-2.3 and it is generally smaller for light particle cases than heavy
11 particle cases. It is extensively reported by experimental studies that the fluidized nanoparticle
12 agglomerates have D_f about 2 [16]. A low D_f usually indicates open cluster structure. The formation
13 of agglomerates by particle-cluster and cluster-cluster agglomeration mechanisms leads to large
14 difference of D_f . For example, agglomerates made by diffusion-limited cluster-cluster agglomeration
15 (DLCA) and ballistic cluster-cluster agglomeration (BCCA) are open clusters, with $D_f = 1.8$ and $D_f =$
16 1.9 , while agglomerates made by diffusion-limited particle-cluster agglomeration (DLA) and ballistic
17 particle-cluster agglomeration (BPCA) are dense clusters, with $D_f = 2.5$ and $D_f = 3.0$ [39]. Therefore,
18 the fluidized agglomerates have a dominant BCCA or DLCA formation mechanism. Figure 10 shows
19 the probability distribution of d_{agg} , C_N , p , and D_f for one selected case, $\rho_{sa250vdw20ug4}$, at times of
20 τ_{Nmax} , τ_{N15} , and τ_{ds} . It is shown that the distribution ranges of these properties are not wide. The
21 combination of the parameters of d_{agg} , C_N , p and D_f characterizes the agglomerate structure, which
22 can highly affect gas diffusion within the agglomerate; this is a very important parameter when
23 carrying out gas phase reactions on the primary particle surface within the agglomerate. A
24 combination of simulation and experimental validation is a useful tool for getting more insight in the
25 behaviour of fluidized nanoparticle agglomerates.

26 Figure 9a

27 Figure 9b

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Figure 9c
Figure 9d
Figure 10

4. Discussion

In the above sections, we have demonstrated the adhesive CFD-DEM model can reveal properties of complex agglomerates during fluidization process. The probability distribution or the instantaneous evolution of agglomerate breakage-reunion can be directly explored. In future, quantitative validation of the model from the meso scale is required. The agglomerate structure reflected by the parameters, e.g. coordination, packing fraction, fractal dimension, can be compared with experimental measurements and used to improve understanding on fluidized nanoparticle agglomerates. Another unresolvable problem, we point here again, is that how to calculate the drag force reduction as a result of agglomeration. A presumed averaged scale factor used currently could be refined by considering agglomerate size and density locally but the computation will be very expensive.

From a broader view on modelling and understanding on nanoparticle fluidization, we should keep in mind that the current CFD-DEM model covers scales only from the simple agglomerates to the fluidized bed with size of a few millimeters. On the one hand, scale-up modelling of a fluidized bed with size of a few centimeters is encouraged for a direct comparison with experiments. On the other hand, micro simulation is encouraged to build relation among the scales of the primary nanoparticles, aggregates, and simple agglomerates. Towards this, a multi-scale modelling strategy will be developed for studying the relation between micromechanical nanoparticle properties and fluidization process.

5. Conclusions

Agglomerate dynamics are investigated in a nanoparticle fluidized bed with different particle density, cohesion, and fluidizing gas velocity, by using adhesive CFD-DEM simulation. The main conclusions that can be drawn from the simulations are:

1 (1) As the nanoparticle agglomerates move around in the bed, they break and recombine
2 repeatedly. Gas channelling is observed in case of strong agglomeration.

3 (2) The averaged solid dispersion coefficient along bed height (D_{sz}) is much larger than that
4 along bed width. D_{sz} increases with increasing fluidizing gas velocity, and decreases with increasing
5 particle cohesion. A strong correlation is observed between the dispersion coefficient and the
6 agglomerate breakage rate.

7 (3) The instantaneous coordination number averaged over all the particles in the bed is constant
8 around 2~3 during fluidization, which is a bit larger than normal fluidized beds.

9 (4) The evolution of agglomerates are visualized and the agglomerate breakage is compared for
10 different cases. Increasing fluidizing gas velocity or reducing particle adhesion can increase
11 agglomerate breakage rate. Also, there are different contacts in an agglomerate, i.e. weak contacts,
12 strong contacts, and sometimes permanent contacts, which result in different stabilities of
13 agglomerates.

14 (5) Statistics of agglomerates show that, the averaged coordination number (C_N) is around 3.0,
15 packing fraction (p) around 0.2-0.3, and fractal dimension (D_f) around 1.9-2.3. The combination of C_N
16 and p indicates that the agglomerates are roughly close to the structure with minimal stability, while
17 D_f indicates the agglomerates have a dominant BCCA or DLCA formation mechanism.

18 Finally, we point out improvements required by the current adhesive CFD-DEM model. We
19 expect that the DEM model, together with the models of macro and micro scales, will be developed as
20 a powerful tool for exploring relation between micromechanical nanoparticle properties and
21 fluidization process.

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25 Programme (FP/2007-2013)/ERC Grant (279632). We thank Lilian de Martín for discussions on
26 agglomerate identification.

27

1 Nomenclature

2 Roman symbols

B_{ovdw}	Ratio of vdW force over particle gravity (-)
C_N	Coordination number (-)
D_f	Fractal dimension (-)
d_{agg}	Complex agglomerate diameter (mm)
d_{sa}	Simple agglomerate diameter (μm or mm)
D_{sx}	Solid dispersion coefficient along x axis (mm^2/s)
D_{sz}	Solid dispersion coefficient along z axis (mm^2/s)
h/h_{mf}	Bed expansion ratio (-)
m_{agg}	Complex agglomerate mass (g)
N_{agg}	Complex agglomerate number (-)
p	Packing fraction (-)
R_{agg}	Complex agglomerate radius (mm)
r_{sa}	Simple agglomerate radius (μm or mm)
u_g	Fluidizing gas velocity (cm/s)
u_{mf}	Minimum fluidization velocity (cm/s)

3 Greek symbols

ε_g	Bed voidage (-)
ψ_p	Plasticity index of simple agglomerate (-)
ξ	Drag force scale factor (-)
ρ_{sa}	Simple agglomerate density (kg/m^3)
ΔP	Bed pressure drop (Pa)
Δt	Time interval (s)
$\sigma_{\Delta p}$	Standard deviation of ΔP (Pa)
τ_{ds}	Critical time: when R_{agg} begins steady (s)
τ_{N15}	Critical time: when N_{agg} less than 15 (s)
τ_{Nmax}	Critical time: when N_{agg} maximum (s)

4 Acronyms

vdW	van der Waals force
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5

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10

11

12

13 **Figure Titles**

14

15 Figure 1 Illustration of identification of agglomerates with increasing time intervals, for the case
16 psa250vdw20ug4. The reference time is 1.00 s, and the comparison time is 1.01 s, 1.11 s, 1.21 s, 1.41
17 s, and 1.61 s, respectively. The particles in the same agglomerate are shown with the same color.

18 Figure 2 Solid flow pattern at 2.5 s for the different cases.

19 Figure 3 Gas velocity vectors with contours of solid volume fraction at 2.5 s. (a) case
20 psa137vdw20ug2; (b) case psa250vdw20ug2; (c) case psa250vdw20ug4; (d) case psa250vdw5ug4

21 **Figure 4 (a) Time-series and (b) time-averaged and standard deviation of bed pressure drop for the
22 different cases.**

23 **Figure 5a Curve of squared displacement along vertical direction with time ($S^2 - \Delta t$) for four of the
24 cases.**

25 Figure 5b Averaged solid dispersion coefficient along the horizontal and vertical directions.

26 Figure 6a Averaged coordination number as a function of time for the different cases.

27 Figure 6b Probability distribution of coordination number for the case ppsa250vdw20ug4 at different
28 times.

29 Figure 7 Agglomerate breakage by tagging one agglomerate. (a) case psa137vdw20ug2; (b) case
30 psa250vdw20ug2; (c) case psa250vdw20ug4; (d) case psa250vdw5ug4. The particles in one
31 agglomerate are tagged with the black color at time 1.01 s, and then traced in the following times. For
32 a good visualization of the agglomerate breakage, here only 1/5 of the background particles are shown.

- 1 Figure 8a Agglomerate number with time interval (Δt), by monitoring breakage of the contacts.
- 2 Figure 8b Mass-averaged agglomerate diameter with time interval (Δt), by monitoring breakage of the
- 3 contacts.
- 4 Figure 8c Critical times determined from the curves of agglomerate number and agglomerate with the
- 5 time interval.
- 6 Figure 9 (a) Mass-averaged agglomerate diameter, (b) coordination number, (c) packing fraction, and
- 7 (d) fractal dimension for the different cases.
- 8 Figure 10 Probability distribution of agglomerate diameter, coordination number, packing density, and
- 9 fractal dimension of case of psa250vdw20ug4.

10 **Table Titles**

- 11 Table 1 Governing equations and sub-models of the adhesive CFD-DEM model
- 12 Table 2 Simulation cases and description

13

Figure 1
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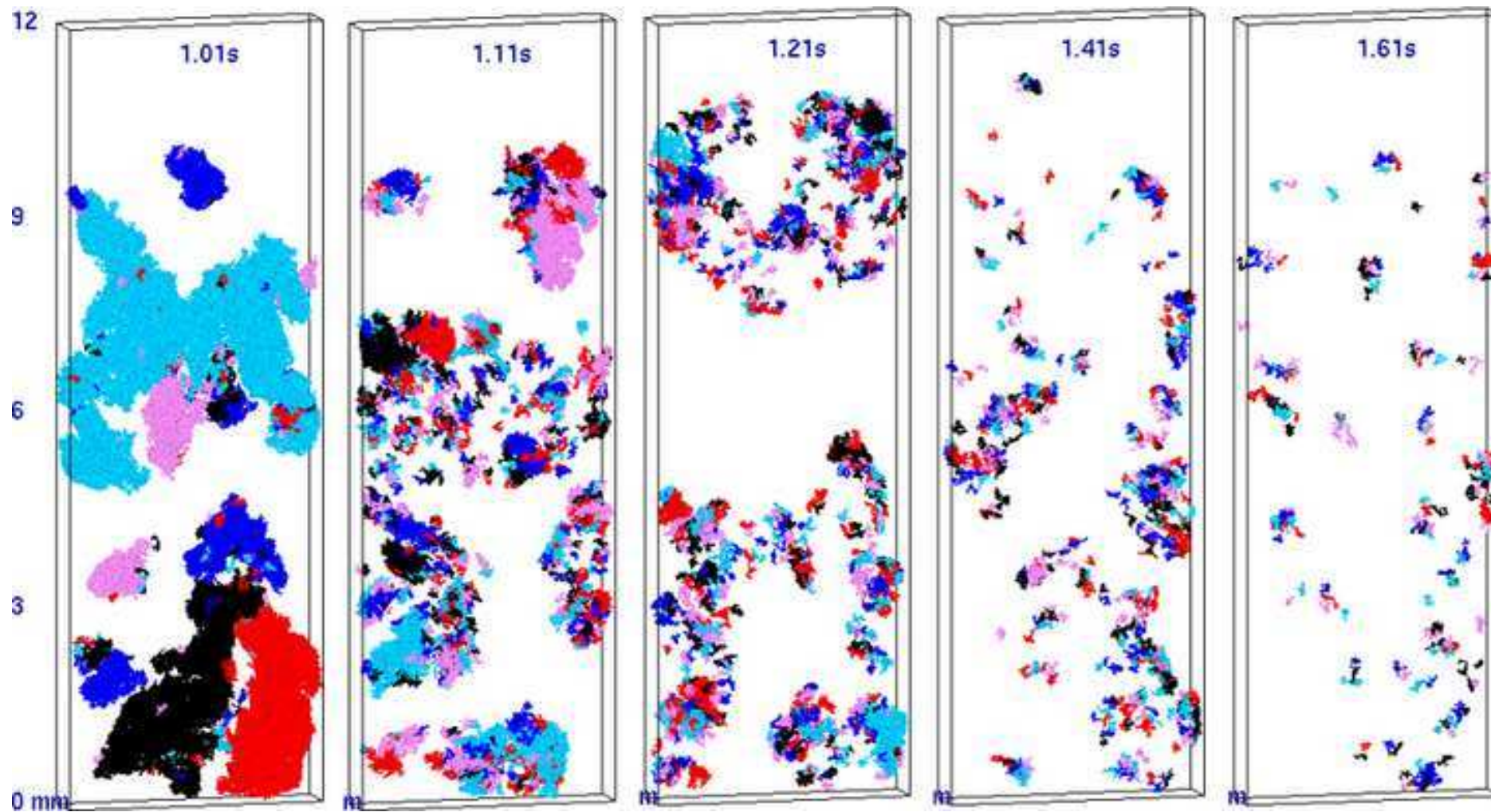


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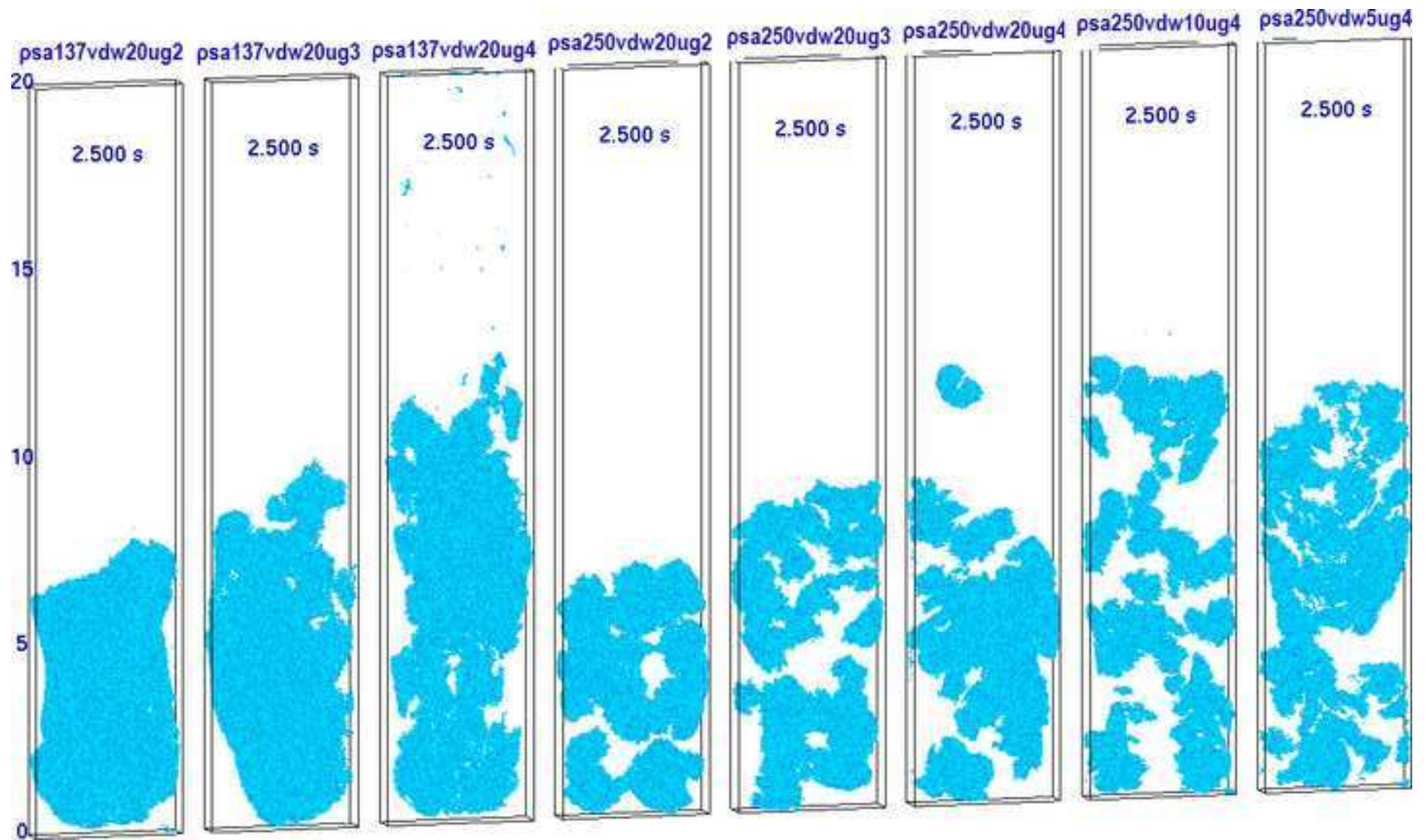


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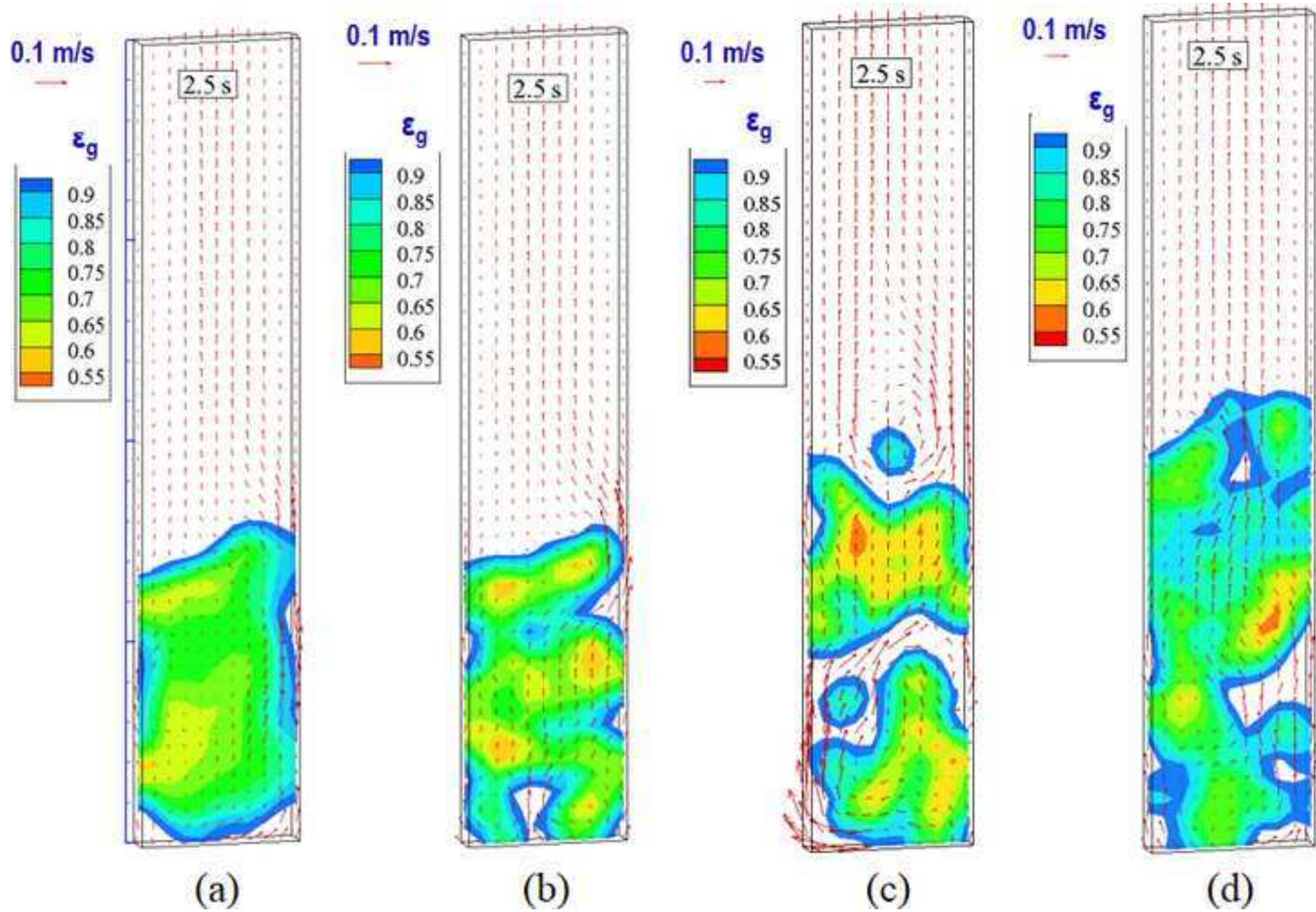


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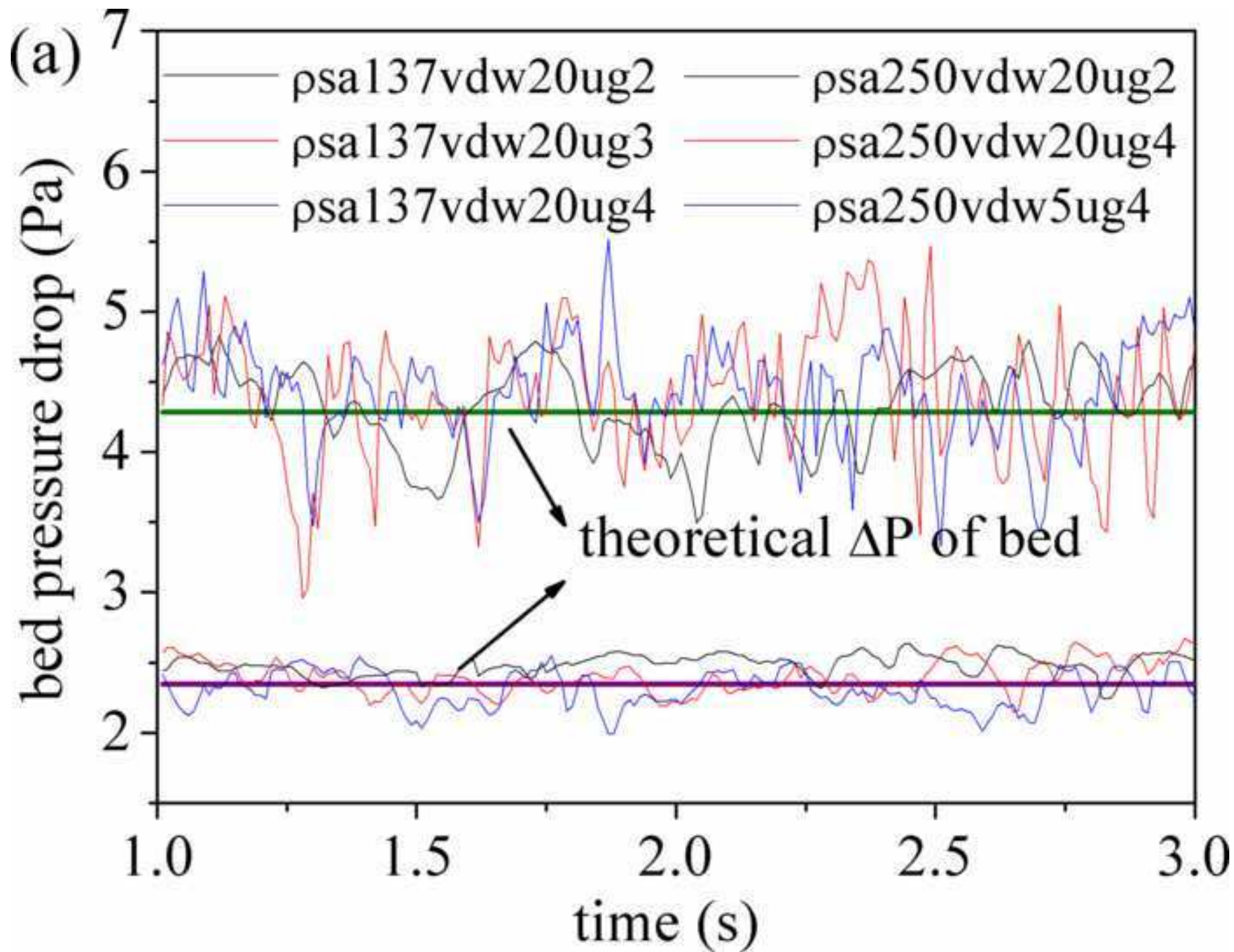


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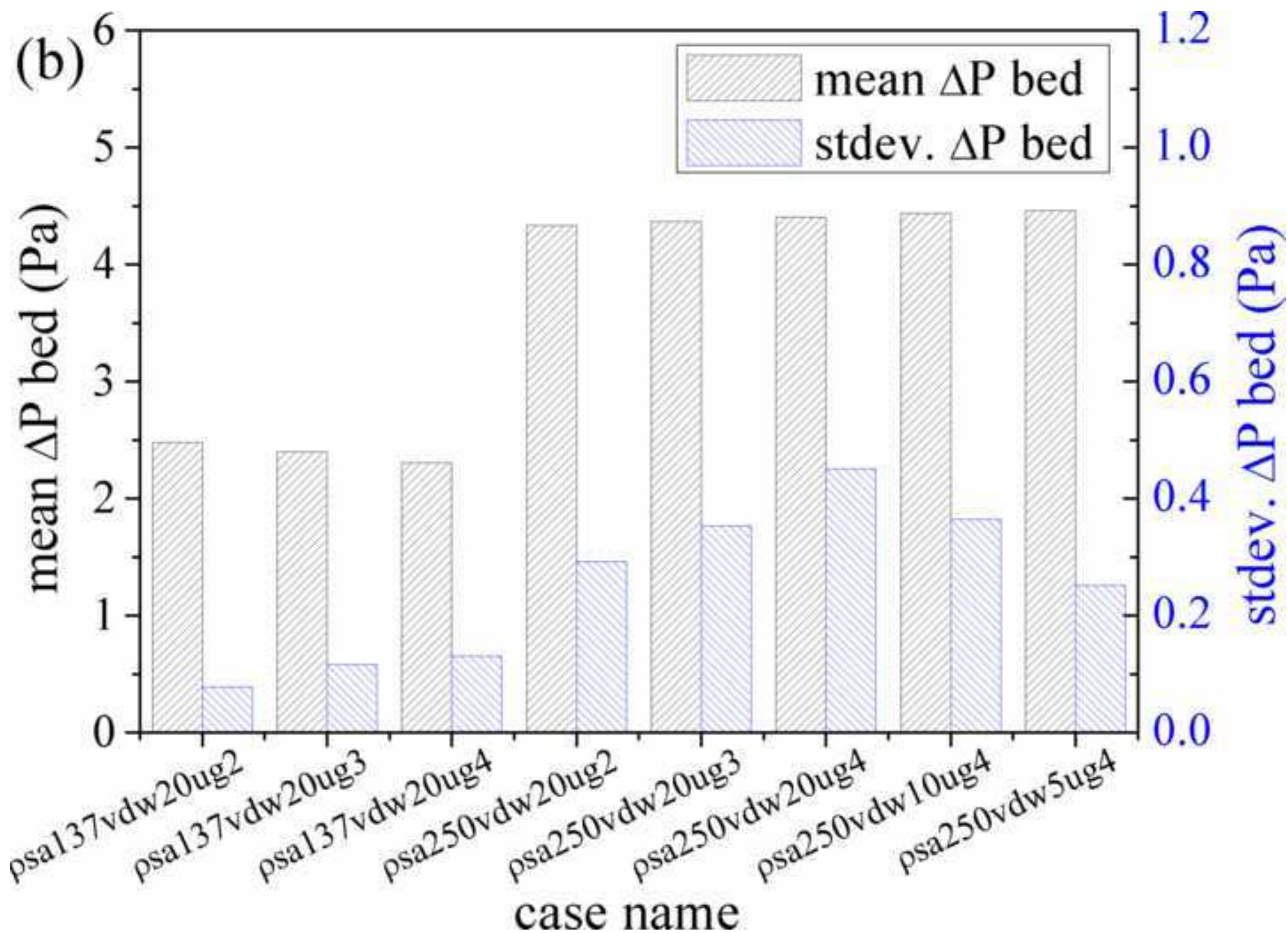


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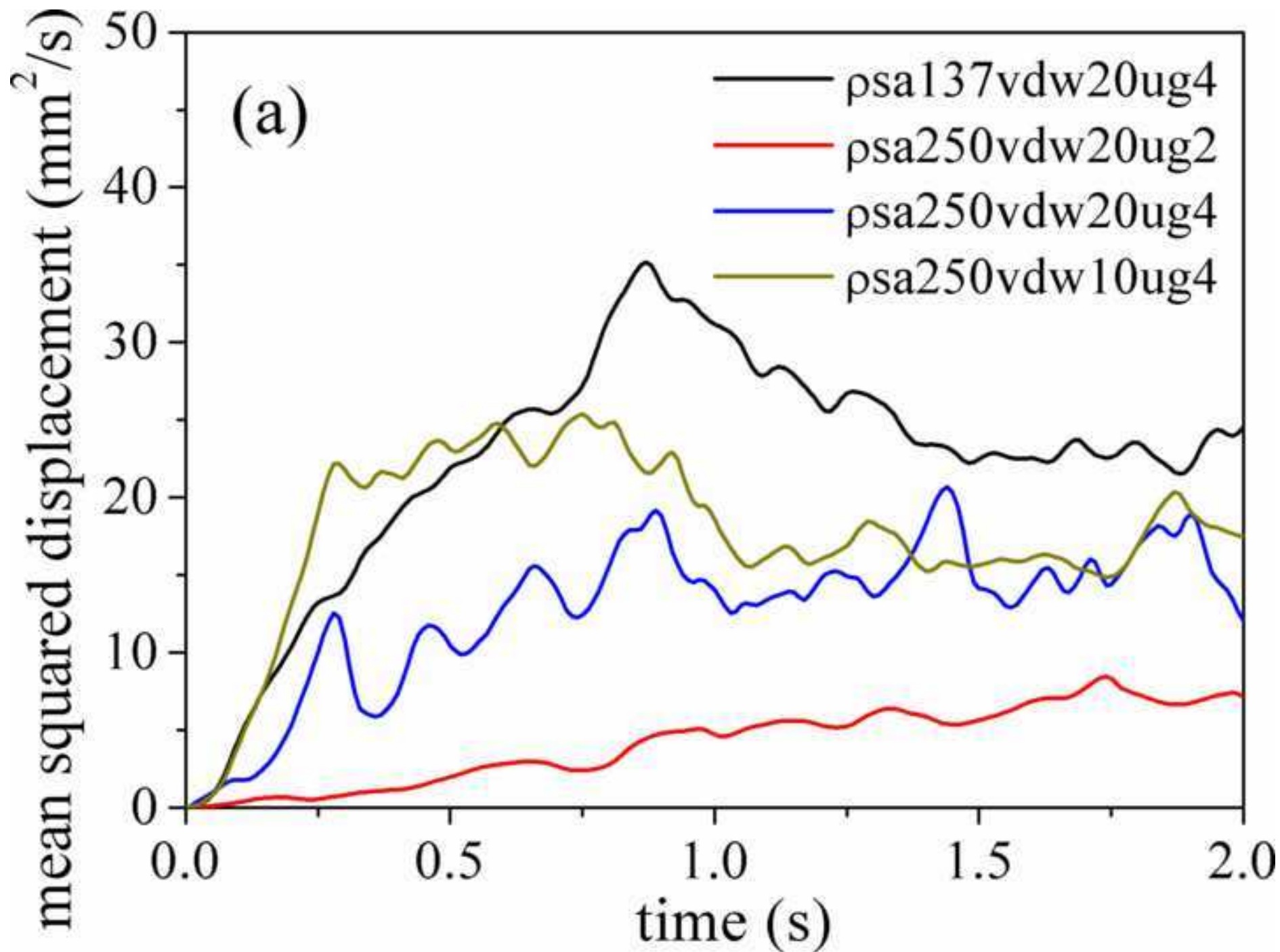


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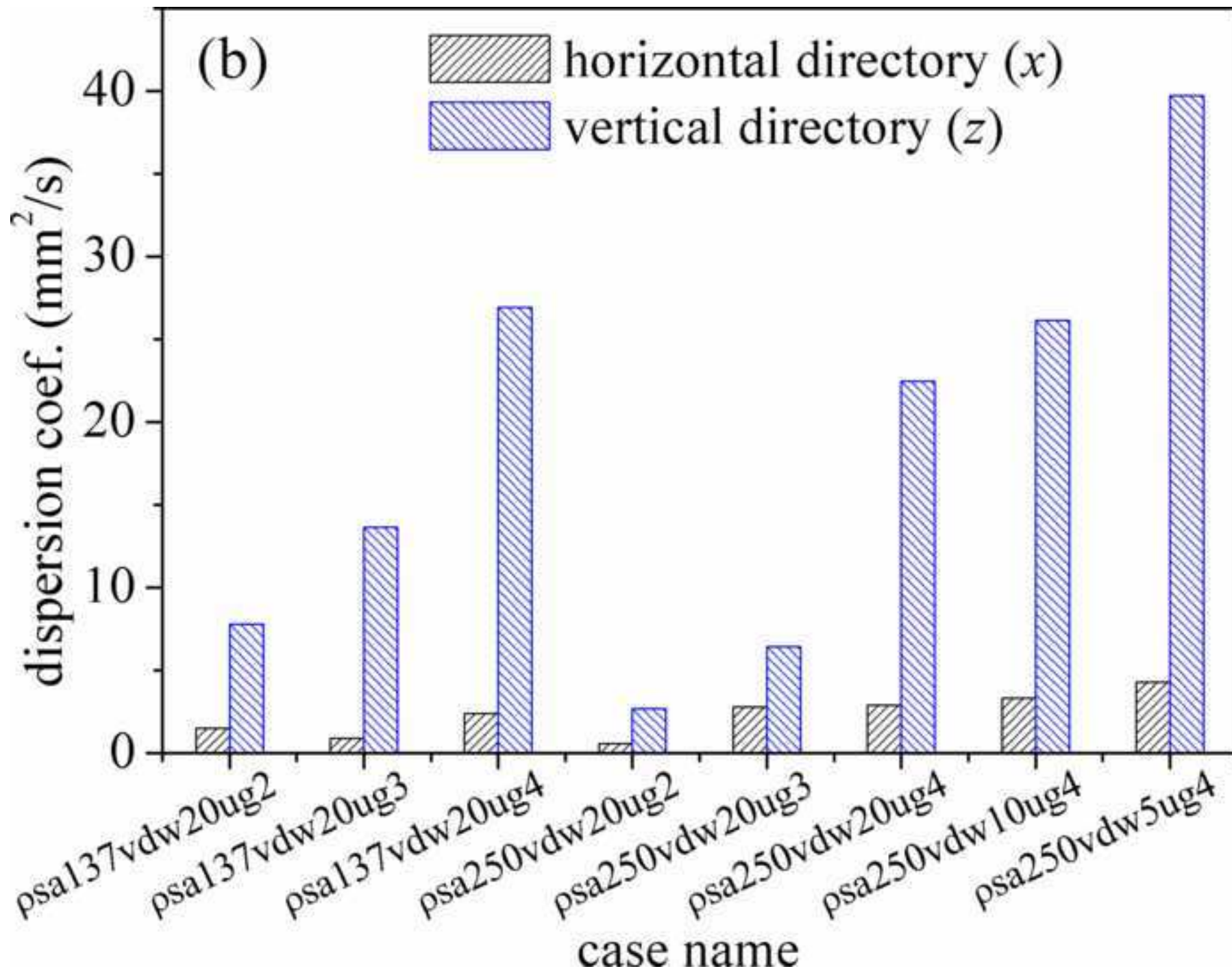
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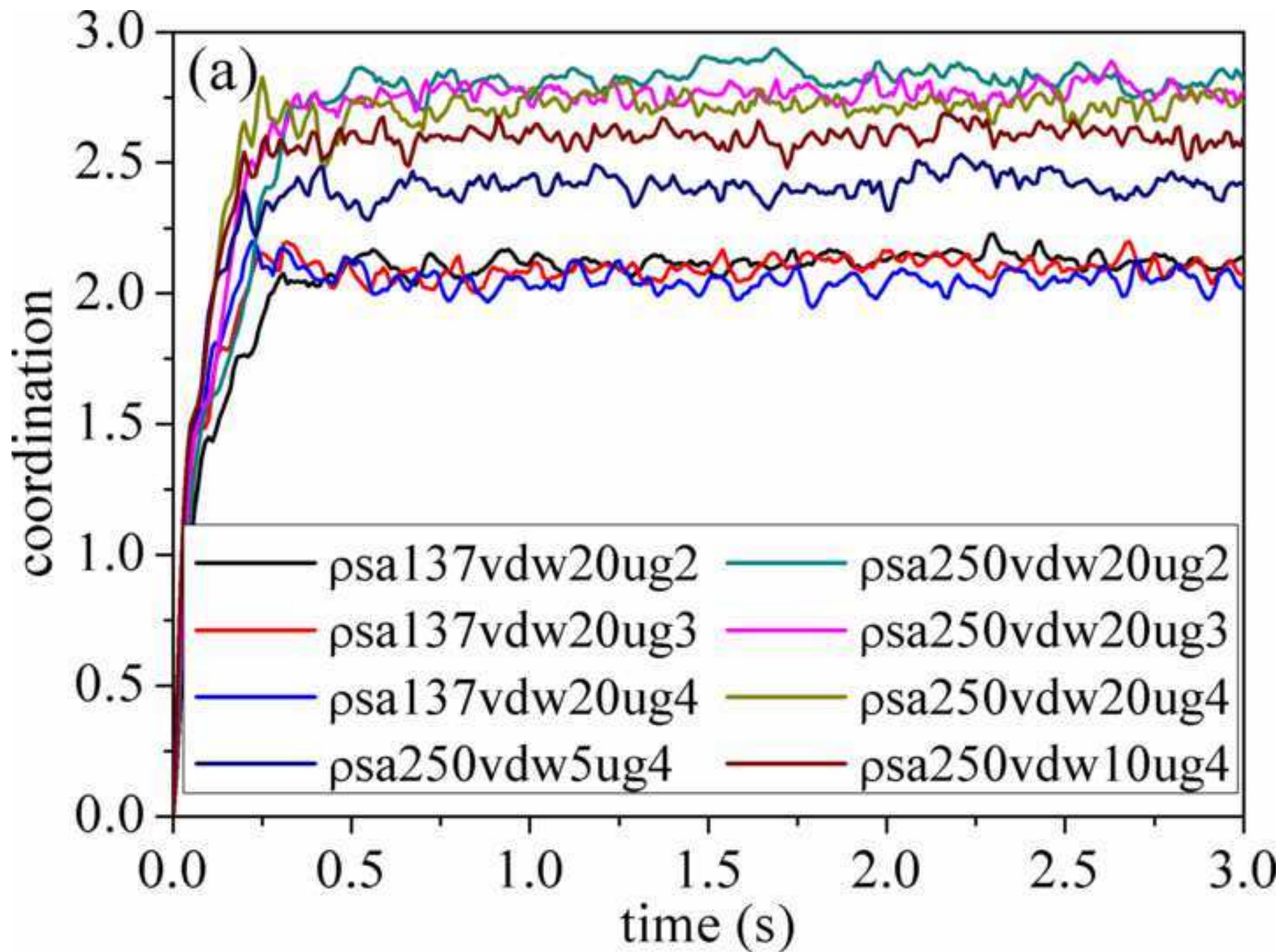


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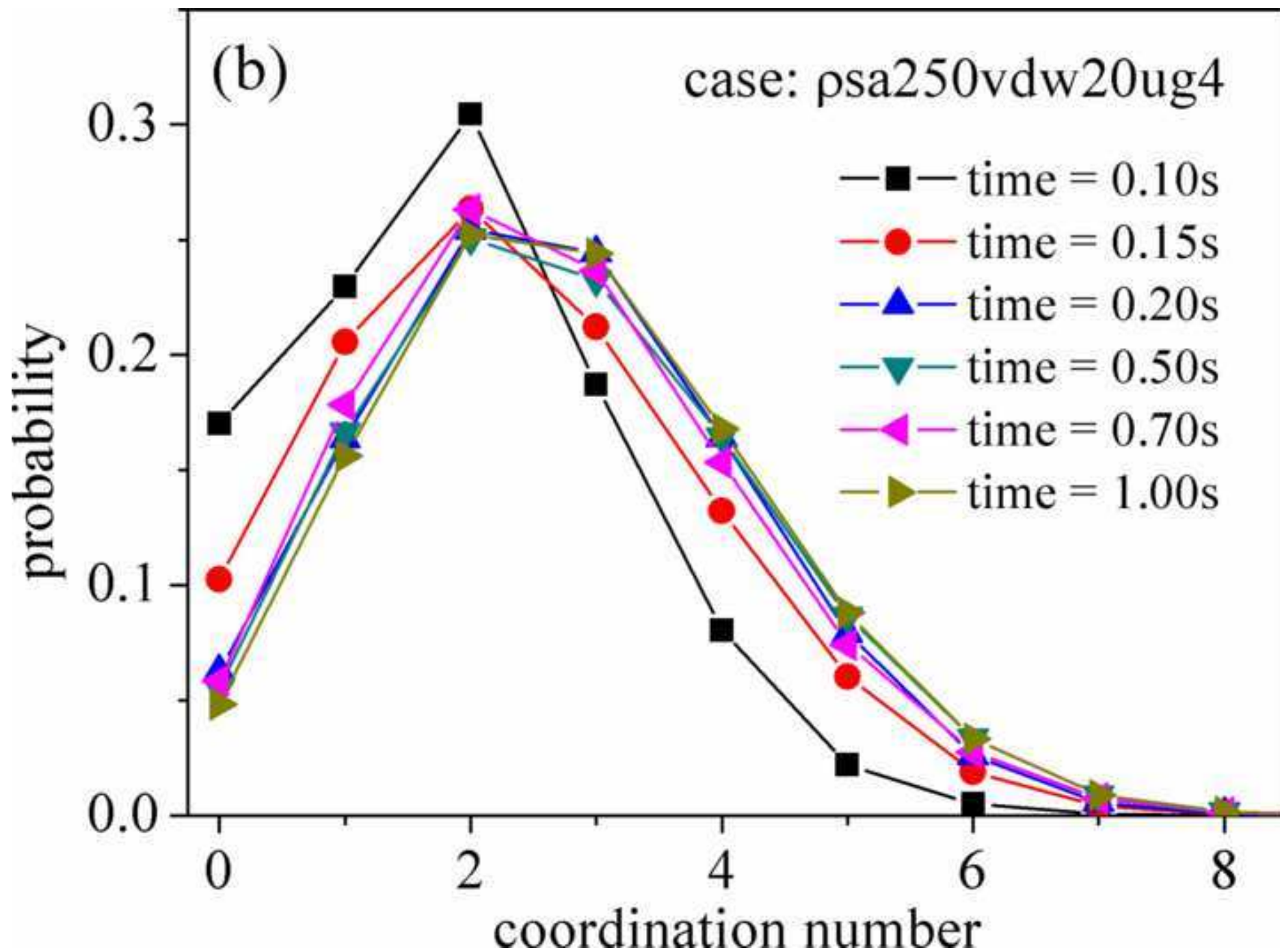
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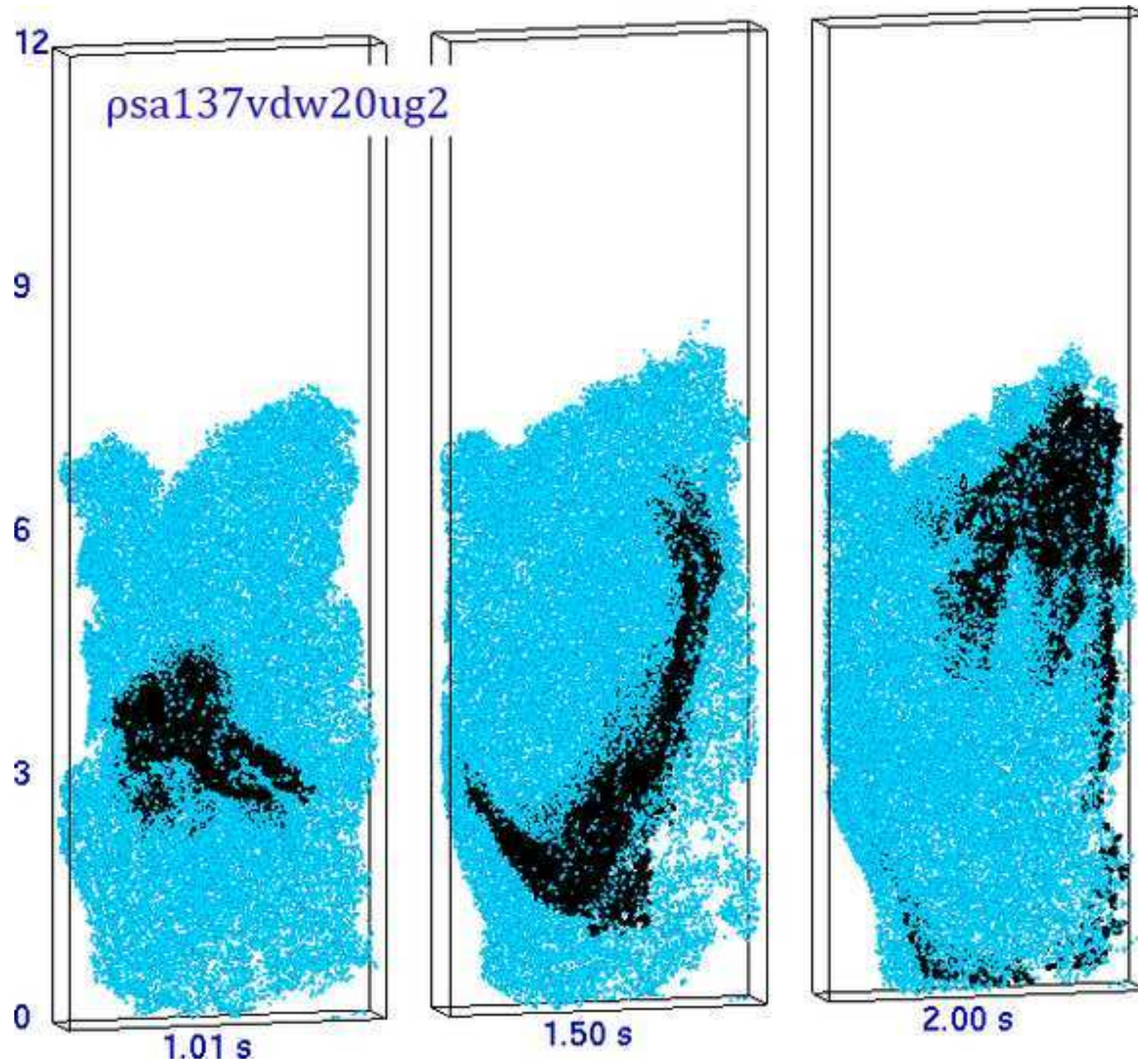


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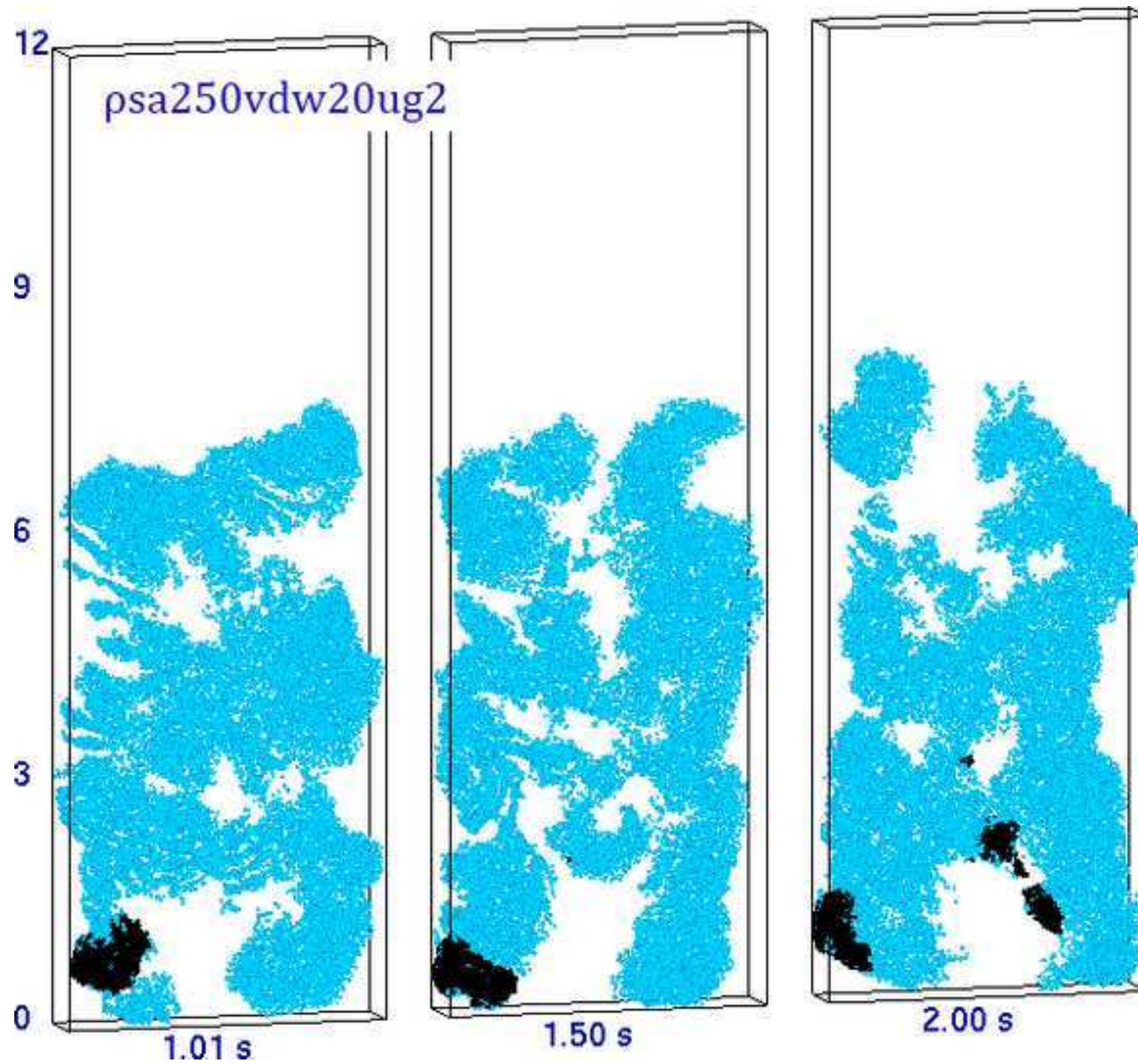


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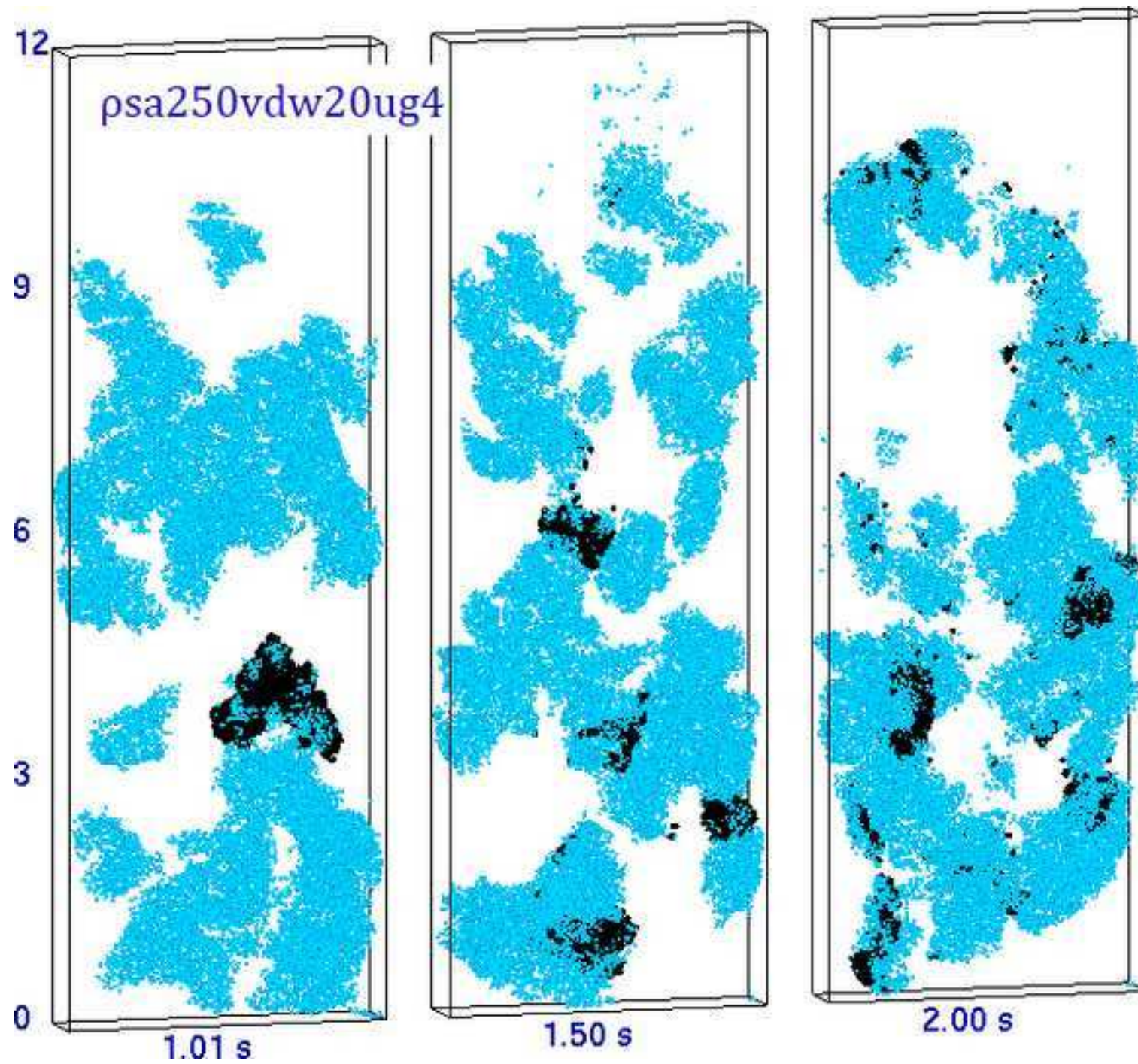


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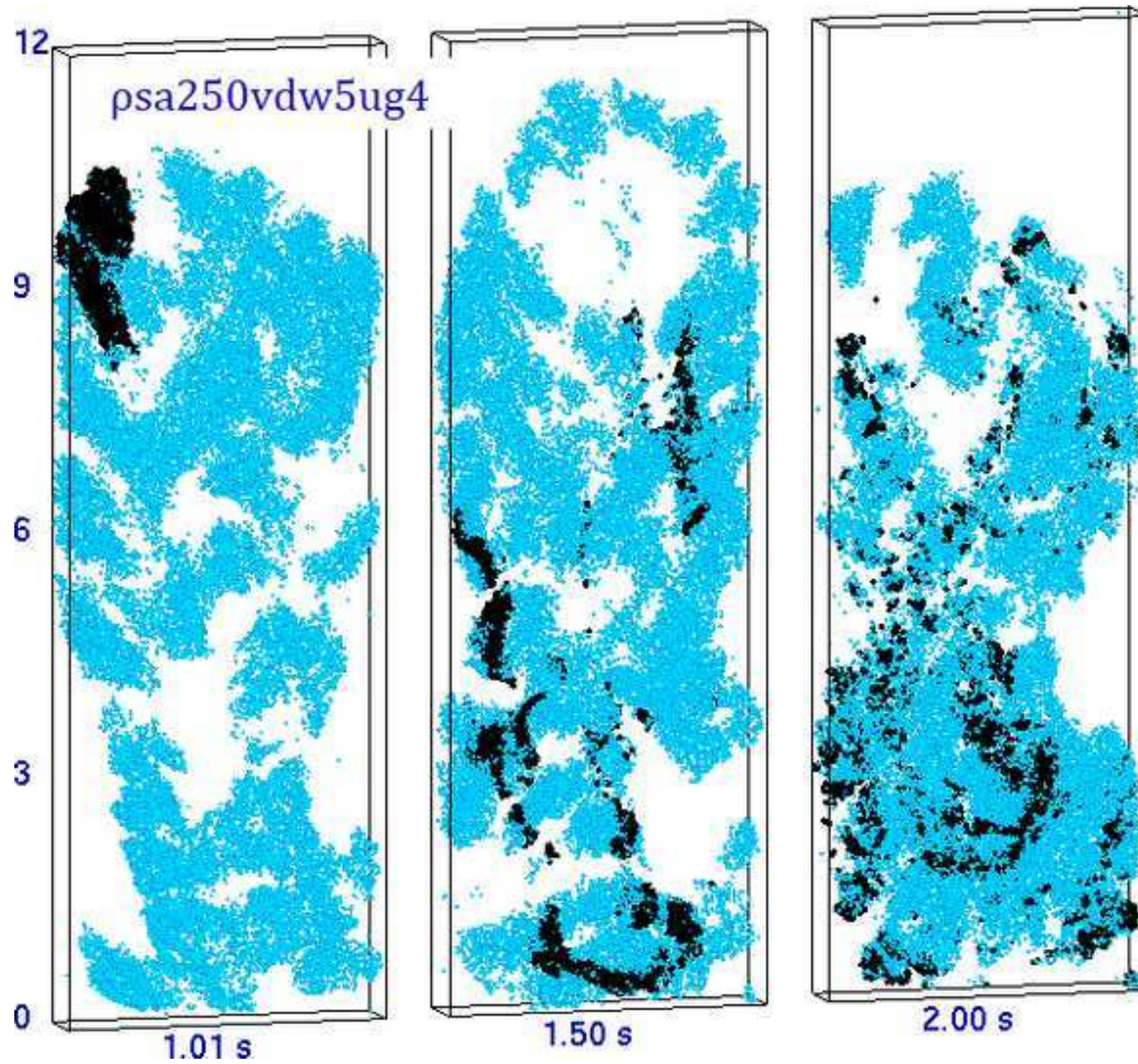


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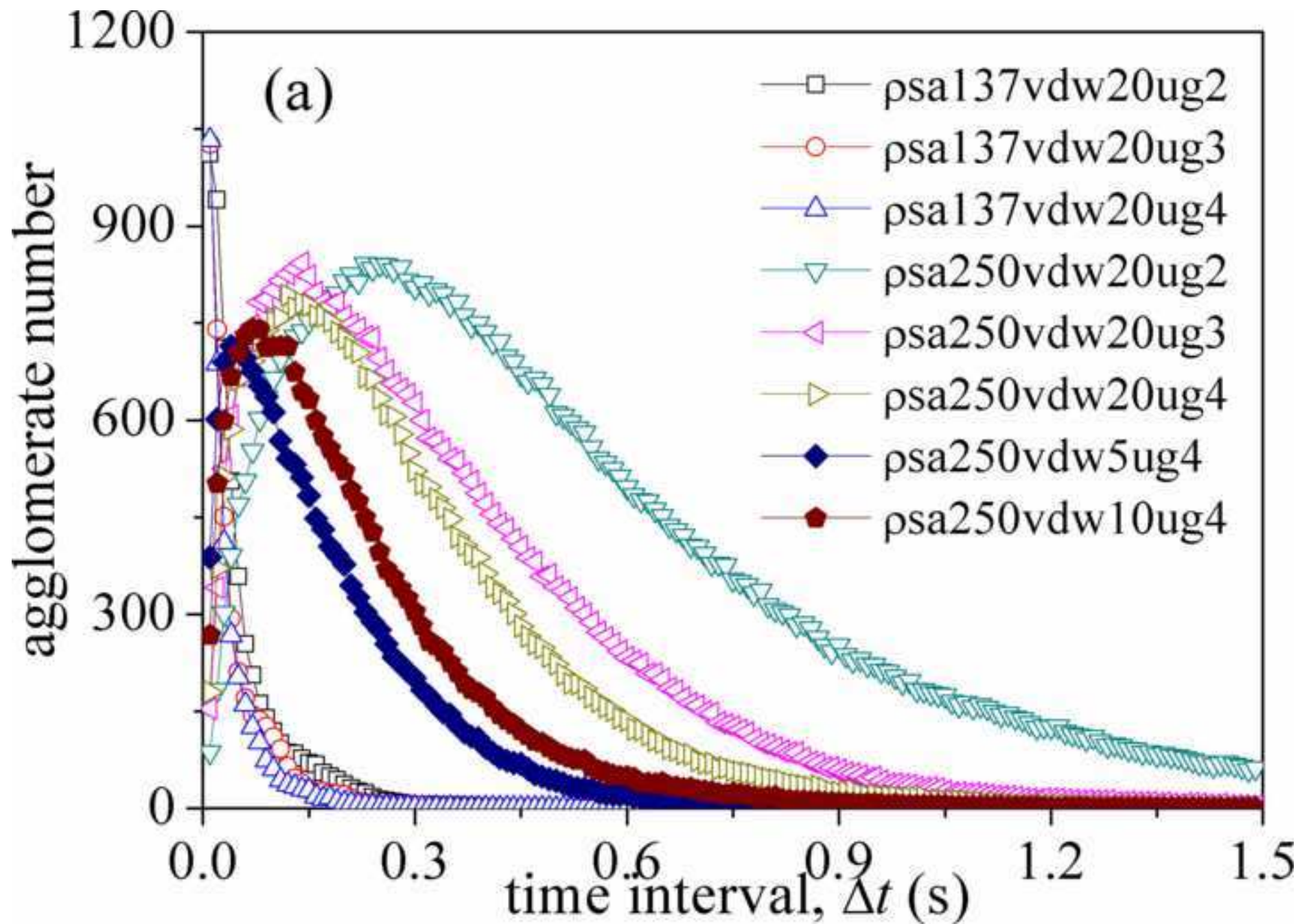
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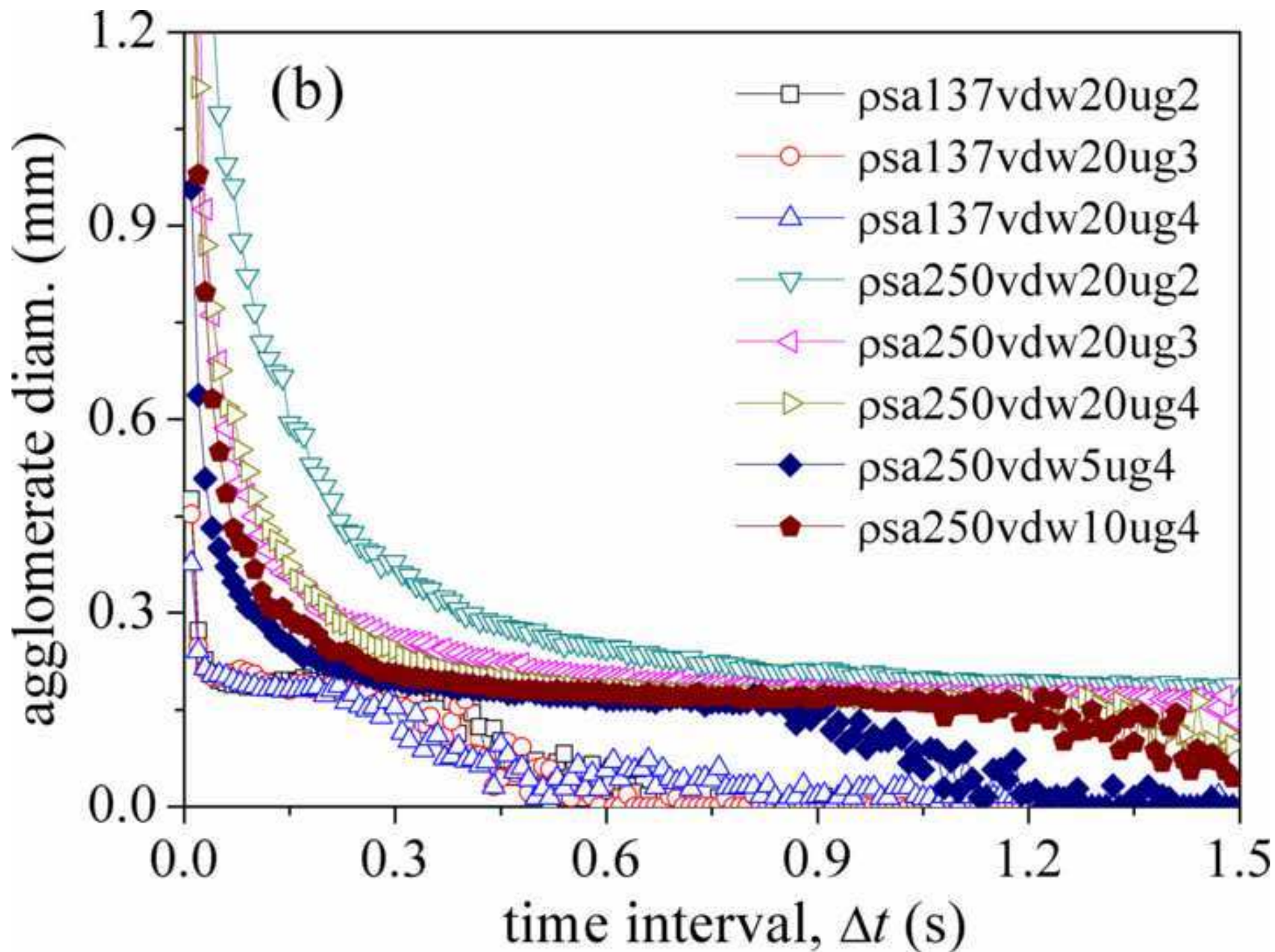
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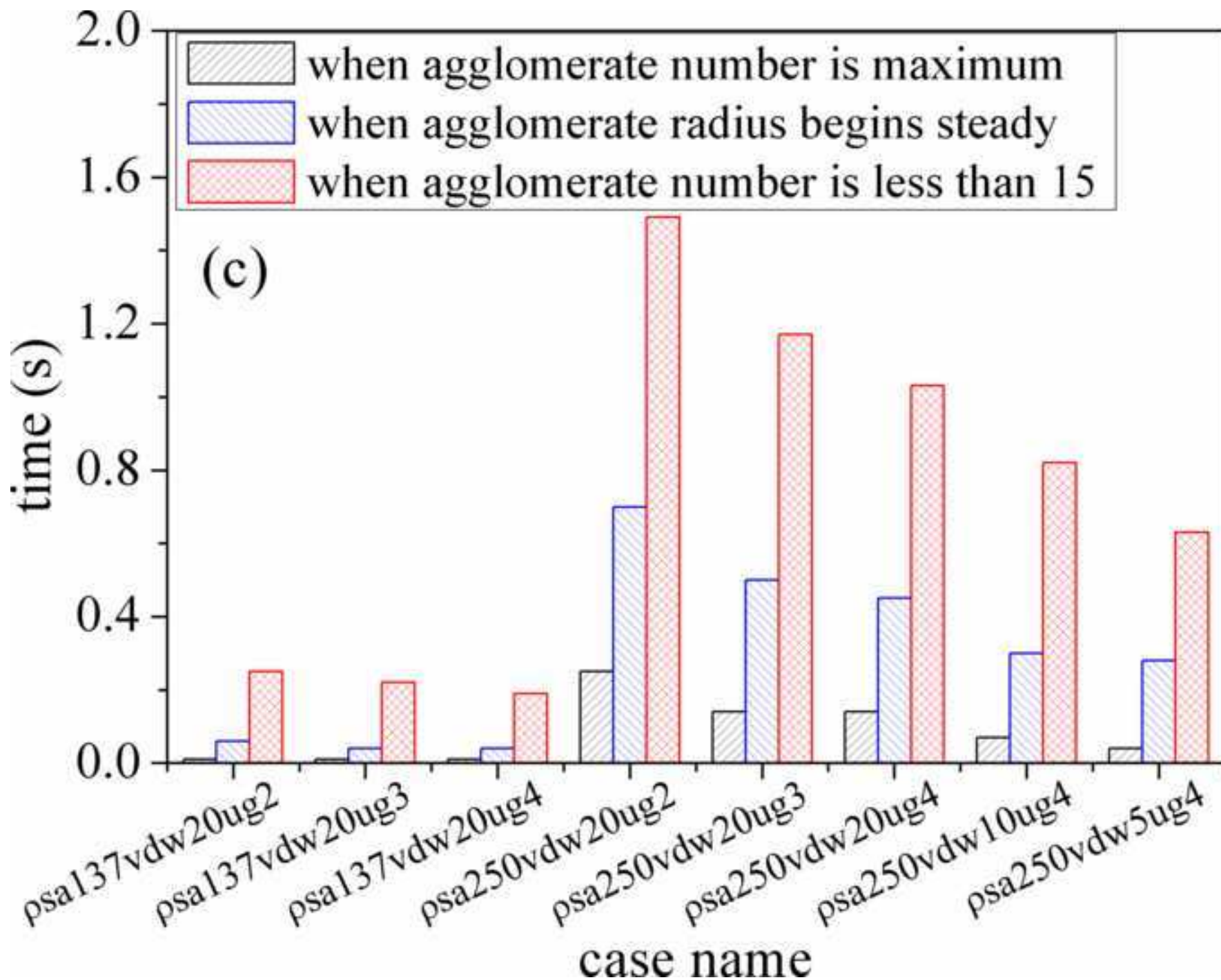


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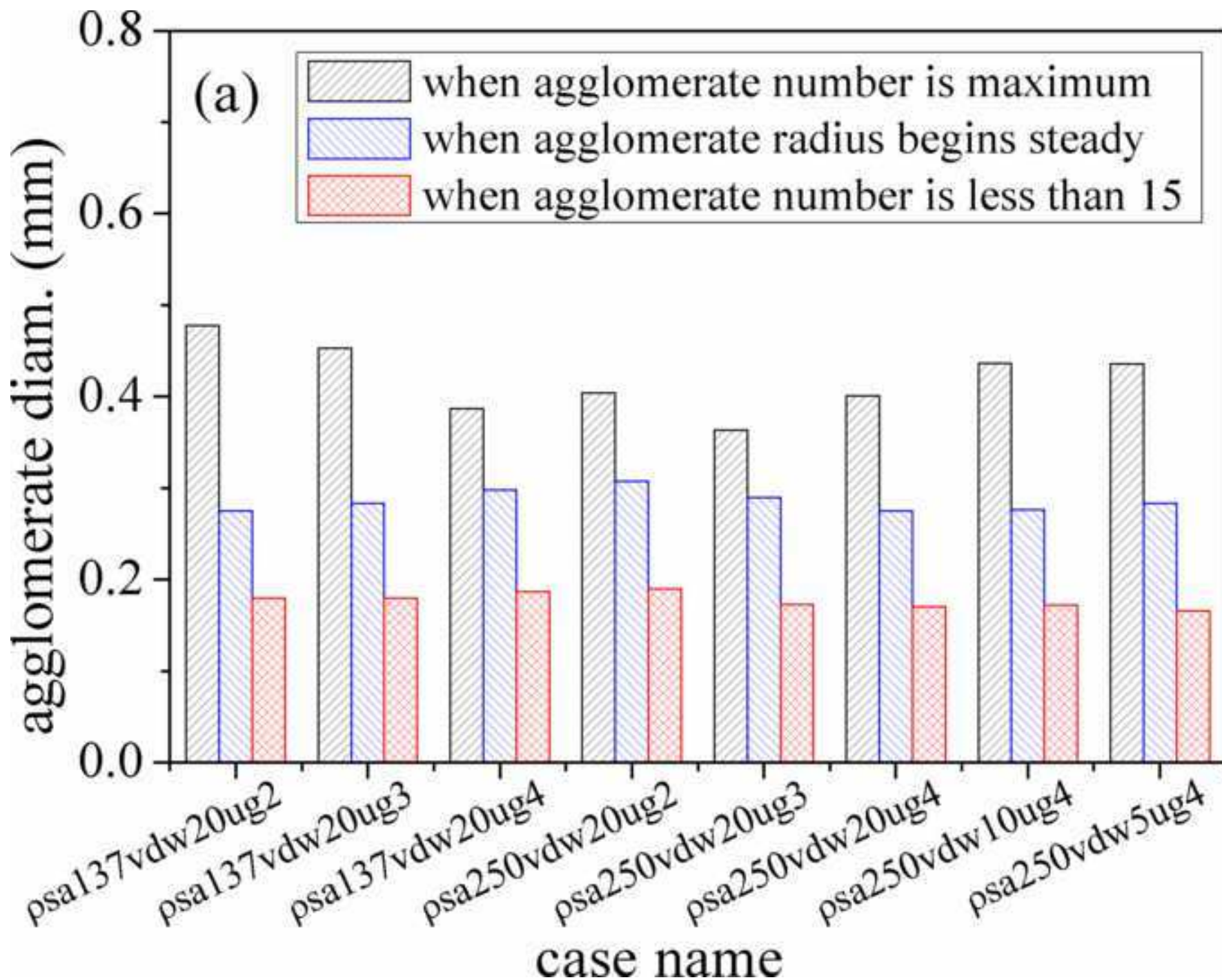


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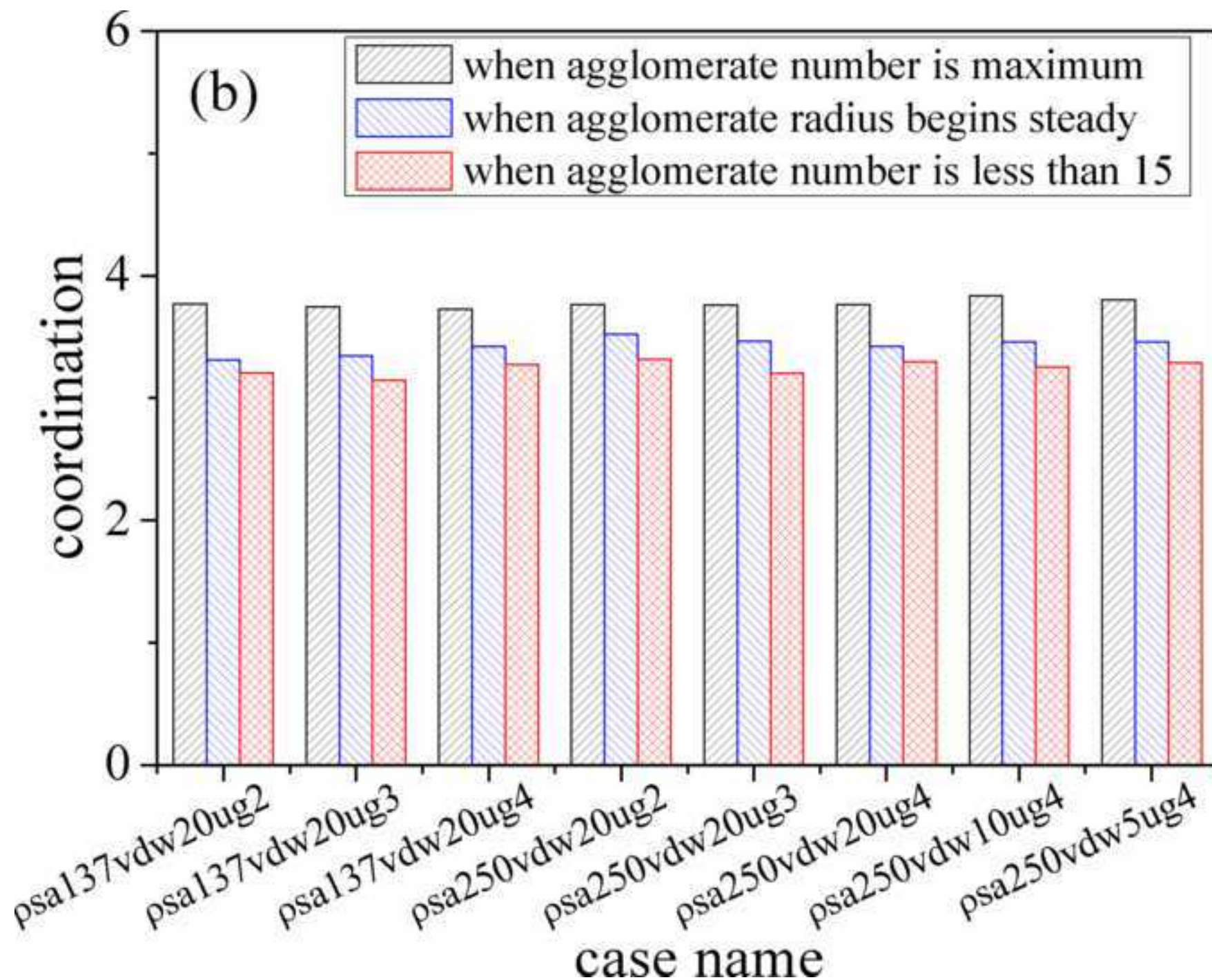


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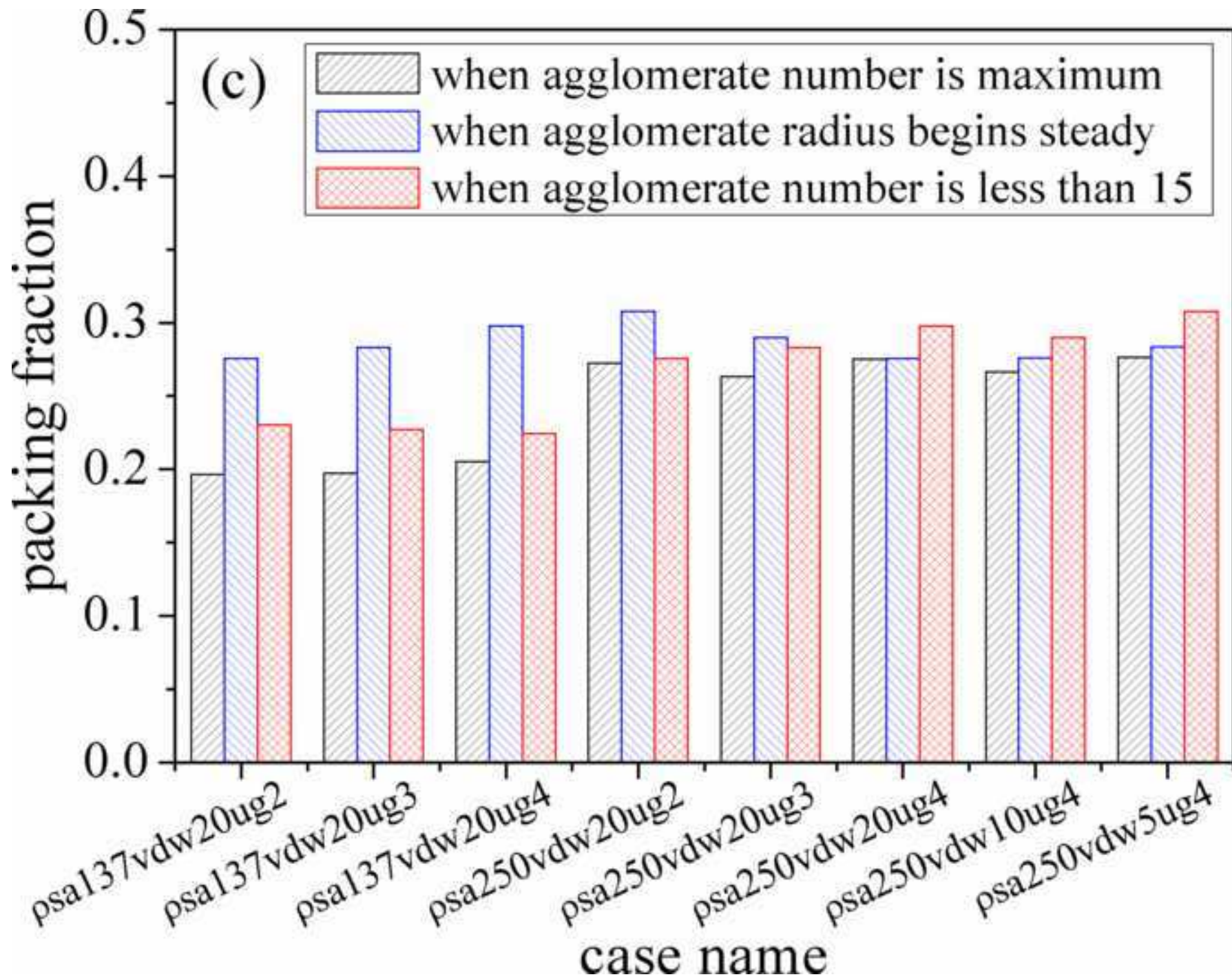
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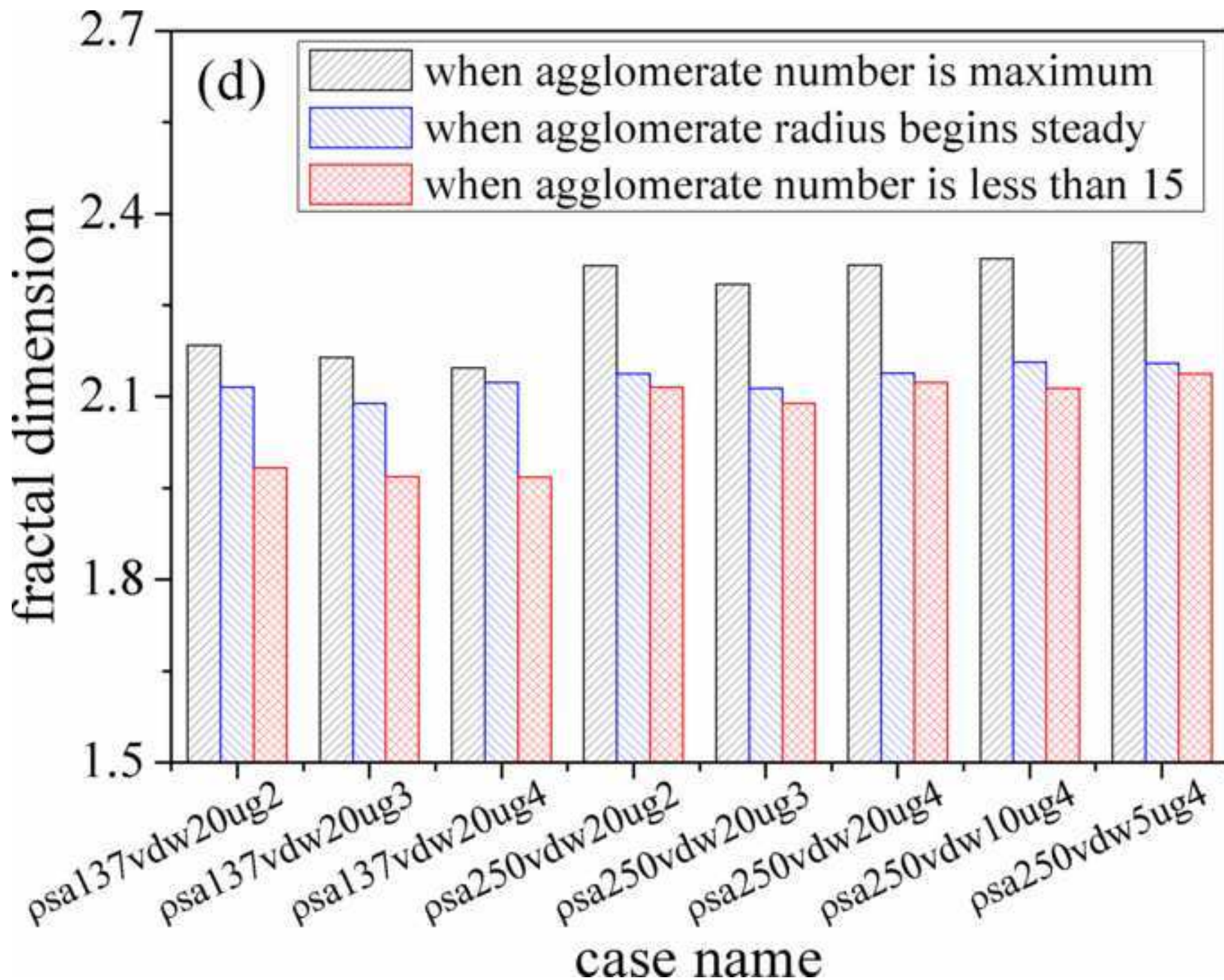


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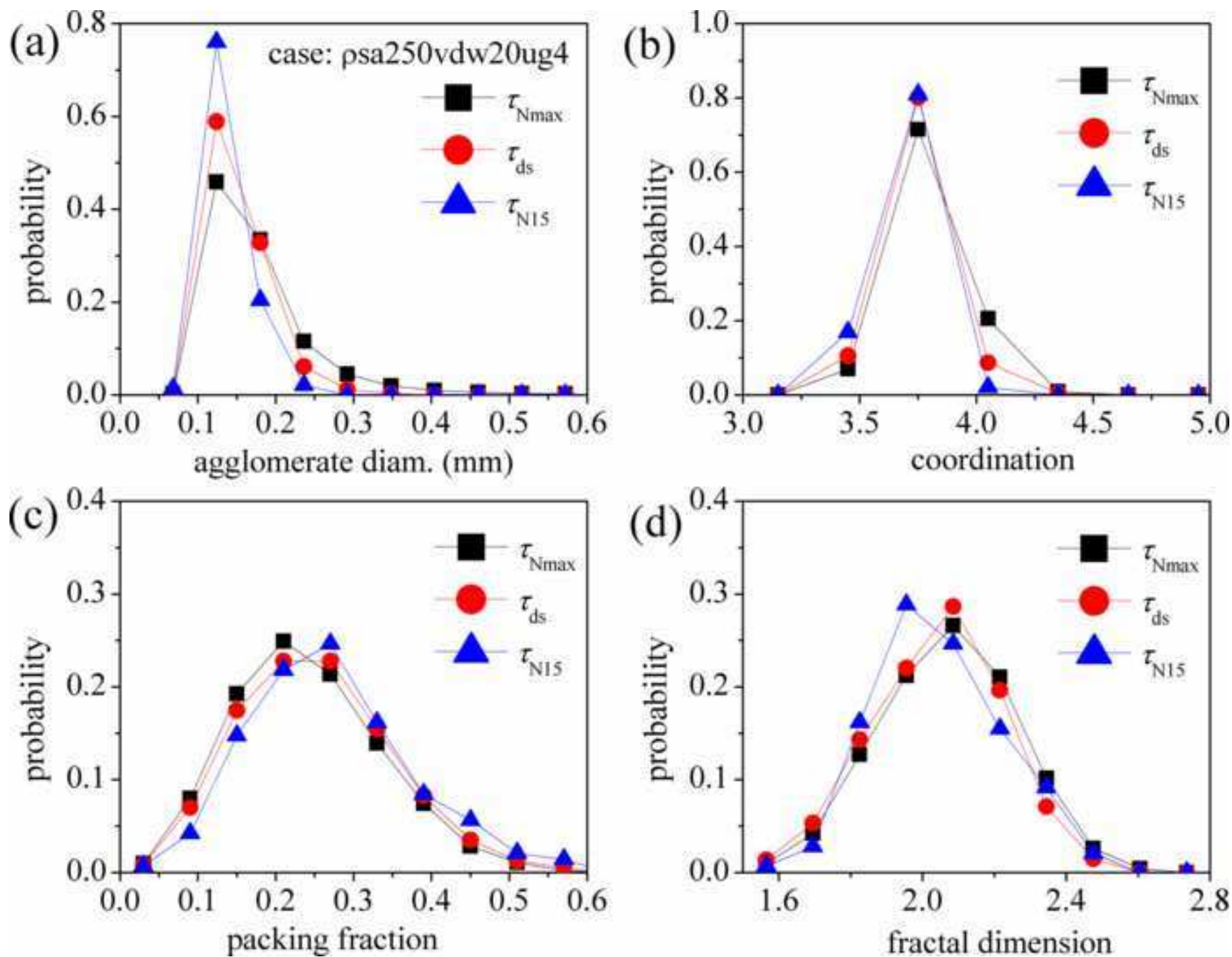
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Table 1

mass and momentum conservation equations of fluid phase	
$\frac{\partial}{\partial t}(\varepsilon_g \rho_g) + \nabla \cdot (\varepsilon_g \rho_g \mathbf{u}_g) = 0$	$\frac{\partial}{\partial t}(\varepsilon_g \rho_g \mathbf{u}_g) + \nabla \cdot (\varepsilon_g \rho_g \mathbf{u}_g \mathbf{u}_g) = -\varepsilon_g \nabla p_g - \nabla \cdot (\varepsilon_g \boldsymbol{\tau}_g) + \mathbf{S}_p + \varepsilon_g \rho_g \mathbf{g}$
$\varepsilon_g = 1 - \sum_{n=1}^{N_p} V_{pn} / V_{cell}$	$\mathbf{S}_p = -\frac{1}{V_{cell}} \sum_{n=1}^{N_p} \frac{V_a \beta}{(1 - \varepsilon_g)} (\mathbf{u}_g - \mathbf{v}_p) \cdot \boldsymbol{\xi} \quad \boldsymbol{\xi} \text{ drag force scale factor}$
motion equations of each particle	
$m_a \frac{d\mathbf{v}_a}{dt} = -V_a \nabla p_g + \mathbf{F}_d \boldsymbol{\xi} + m_a \mathbf{g} + \sum_{b=1}^N (\mathbf{F}_{ab,n} + \mathbf{F}_{ab,t})$	$I_a \frac{d\mathbf{w}_a}{dt} = \sum_{b=1}^N (R_b \mathbf{n}_{ab} \times \mathbf{F}_{ab,t})$
$\mathbf{F}_d = \frac{V_a \beta}{(1 - \varepsilon_g)} (\mathbf{u}_g - \mathbf{v}_p)$	
Wen&Yu drag coefficient	
$\beta = \frac{3}{4} C_D \frac{\rho \mathbf{u}_g - \overline{\mathbf{v}}_p (1 - \varepsilon_g)}{d_p} \varepsilon_g^{-2.7}$	$Re = \rho_g \varepsilon_g \mathbf{u}_g - \overline{\mathbf{v}}_p d_p / \mu_g$
adhesive particle contact model	
$\mathbf{F}_{ab,n} = -F_n(d) \mathbf{n}_{ab} - \eta_n \mathbf{v}_{ab,n} \quad F_n(d) \text{ includes cohesive force}$	
during the approach stage $F_n(d) = 0 \quad d(t) > H_0 \quad H_0$ minimal separation distance	
during the initial loading stage $F_n(d) = k_{n,load} \delta_n - F_{vdw,0}$	
during the reloading and unloading stages $F_n(d) = \begin{cases} k_{n,unload} (\delta_n - \delta_p) - F_{vdw,0} & F_n \geq f_{ad,min} \\ (1 - x_1) (-F_{vdw,0} - F_{bond,0}) + x_1 f_{ad,min} & F_n < f_{ad,min} \end{cases}$	
$F_{bond,0}$ other cohesive forces, e.g. the hydrogen bond force. $F_{bond,0} = 0$ in this paper	
maximum magnitude of the cohesive force $f_{ad,min} = (\delta_0 / d_{p,NP} + 1) (-F_{vdw,0} - F_{bond,0})$	
$d_{p,NP}$ primary nanoparticle size; $x_1 = d / \delta_{ad,min}$, linear interpolation fraction between $d = 0, \delta_{ad,min}$	
permanent plastic deformation $\delta_p = (1 - k_{n,load} / k_{n,unload}) \delta_{max}$	
If $\mathbf{F}_{ab,t} > \mu_f \mathbf{F}_{ab,n} $ then $\mathbf{F}_{ab,t} = -\mu_f \mathbf{F}_{ab,n} \mathbf{t}_{ab}$, and reset $\boldsymbol{\delta}_t = \mathbf{F}_{ab,t} / k_t$	
relative velocity, direction unit vector, and overlap	
$\mathbf{v}_{ab} = (\mathbf{v}_a - \mathbf{v}_b) + (R_a \mathbf{w}_a + R_b \mathbf{w}_b) \times \mathbf{n}_{ab}$	$\mathbf{v}_{ab,n} = (\mathbf{v}_{ab} \cdot \mathbf{n}_{ab}) \mathbf{n}_{ab} \quad \mathbf{v}_{ab,t} = \mathbf{v}_{ab} - \mathbf{v}_{ab,n}$
$\mathbf{n}_{ab} = (\mathbf{r}_b - \mathbf{r}_a) / \mathbf{r}_b - \mathbf{r}_a \quad \mathbf{t}_{ab} = \mathbf{v}_{ab,t} / \mathbf{v}_{ab,t} $	
$\delta_n = R_a + R_b - \mathbf{r}_a - \mathbf{r}_b $	$\boldsymbol{\delta}'_t = \boldsymbol{\delta}_{t0} + \mathbf{v}_{ab,t} \Delta t \quad \boldsymbol{\delta}_t = \boldsymbol{\delta}'_t - (\boldsymbol{\delta}'_t \cdot \mathbf{n}_{ab}) \mathbf{n}_{ab}$
damping coefficient	
$\eta_n = 2\sqrt{m_{ab} k_n} \ln e_{vis} / \sqrt{\pi^2 + \ln^2 e_{vis}}$	$\eta_t = 2\sqrt{m_{ab}' k_t} \ln e_{vis} / \sqrt{\pi^2 + \ln^2 e_{vis}}$

Table 2

case No.	case abbreviation	ρ_{sa} (kg/m ³)	description	
			Bo_{vdw} ($F_{vdw,0}$, nN)	u_g (cm/s)
1	psa137vdw20ug2	137	20 (0.9)	2
2	psa137vdw20ug3	137	20 (0.9)	3
3	psa137vdw20ug4	137	20 (0.9)	4
4	psa250vdw20ug2	250	20 (1.7)	2
5	psa250vdw20ug3	250	20 (1.7)	3
6	psa250vdw20ug4	250	20 (1.7)	4
7	psa250vdw10ug4	250	10 (0.8)	4
8	psa250vdw5ug4	250	5 (0.4)	4