

The Cross-Section of Asia-Pacific Mortality Dynamics: Implications for Longevity Risk Sharing*

Enrico Biffis[†]

Yijia Lin[‡]

Andreas Milidonis[§]

November 8, 2016

Abstract

We study the dynamics of longevity risk across a subset of countries in the Asia-Pacific (APAC) region. We use hand-collected and existing data on age-specific mortality rates from emerging and developed economies, to understand how secular changes in mortality vary within and across APAC countries. We use our results to identify cross-hedging opportunities among longevity risk exposures in the APAC region. We also introduce k -forward contracts, which offer natural risk sharing opportunities to hedgers in different countries. We consider the example of Korea and Japan as a case study.

1 Introduction

We study the dynamics of longevity risk¹ across a subset of populations in the Asia-Pacific (APAC) region. Using a new dataset constructed from both hand-collected and existing data from emerging and developing economies in APAC (see Milidonis, 2015), we conduct an extensive analysis of the balanced panel resulting from the new dataset to understand how age-specific mortality improvements vary within the APAC region. The objective is to explore the existence of cross-hedging opportunities among a subset of longevity risk exposures in the APAC region. We therefore also provide an application of longevity index design, using the Li and Lee (2005) multi-population model (henceforth LL model) as a reference framework. The LL model offers a compelling approach to modeling the structure of mortality improvements, by extracting a common APAC time-series factor and a set of individual country-specific factors modulated by age-dependent coefficients. The model performs well in our

*We are grateful to the Insurance Risk and Finance Research Centre (IRFRC: www.irfrc.com) at Nanyang Business School for generous financial support, and for providing the Asia-Pacific mortality dataset. We also thank Maria Efthymiou and Francesca Rigoni for excellent research assistance.

[†]Biffis (e.biffis@imperial.ac.uk) is at the Finance Department of Imperial College Business School, Imperial College London, South Kensington Campus, London SW7 2AZ, UK, and at the Risk Management and Insurance Department at the J. Mack Robinson College of Business, Georgia State University, 35 Broad Street, Atlanta, GA 30303, USA.

[‡]Lin (yijialin@unl.edu) is in the Department of Finance at the University of Nebraska–Lincoln, Lincoln, NE 68588-0490, USA.

[§]Milidonis (andreas.milidonis@ucy.ac.cy) is at the Department of Accounting and Finance, School of Economics and Management of the University of Cyprus, P.O. Box 20537, CY-1678, Nicosia, Cyprus. A large part of the paper was completed while he was at the IRFRC, Nanyang Business School, NTU, Singapore.

¹By longevity risk we mean the risk of systematic mortality improvements.

sample relative to competing models widely used in the literature, as well as more recent Generalized Dynamic Factor Models (GDFMs).²

The APAC region is important for longevity risk management for at least three reasons. First, the market for longevity risk has so far revolved around pension and insurance liabilities originating in Europe and North America.³ Hedging solutions for defined benefit (DB) pension plans and books of annuities have mainly taken the form of pension buy-outs, pension buy-ins, and longevity swaps⁴. The gradual shift from DB to defined contribution (DC) retirement plans means that these longevity risk transfer agreements will deal by and large with ‘legacy’ pension assets and liabilities. The APAC region presents a different environment, as a number of APAC countries are relatively young, and social security and pension systems are often not very well developed. At the same time, insurance is growing strongly in the region,⁵ and may provide natural hedging opportunities for domestic and global (re)insurers. These include partial offsetting of longevity exposures with mortality protection products,⁶ longevity-driven modulation of new business across different economies, as well as design of longevity indices that may bring together hedgers and hedge suppliers from different countries within the APAC region. This work provides some results in this direction, and explores data and statistical approaches that can help making these concepts operational.

Second, the market for longevity risk solutions has so far been dominated by indemnity based products with a focus on micro longevity risk. Hedging instruments have been structured mainly as insurance contracts indemnifying the hedger against her own mortality experience, rather than making payments based on a reference longevity index.⁷ This represents a formidable barrier to product standardization and liquidity, which the market is slowly trying to overcome via indexed solutions⁸ and securitization of pools of longevity exposures.⁹ The heterogeneity in APAC life expectancy trends and age structures suggests that the region could represent an important source of origination of longevity exposures that are weakly correlated with the bulk of exposures pooled by (re)insurers and other institutions operating in the traditional pension buyout market. This origination market

²See Forni et al. (2005), Alessi et al. (2007), Gao and Hu (2009), French and O’Hare (2013).

³See Blake et al. (2008), Lane Clark & Picock (2012), Blake et al. (2013), Cox et al. (2013), Biffis et al. (2016a), Lin et al. (2014), for example.

⁴Buy-outs entail the transfer to another institution of some or all the liabilities of a pension plan, together with the responsibility to meet them. Buy-ins entail the purchase of bulk-annuities to insure some or all the liabilities of the pension plan, while retaining responsibility to meet them. Bespoke longevity swaps provide floating payments linked to the mortality experience of specific pension plans or annuity providers. Indexed swaps make floating payments linked to the evolution of a reference index of mortality/longevity. See Biffis and Blake (2010a), Biffis and Blake (2013), and Biffis and Kosowski (2014) for an overview.

⁵See Swiss Re (2013), for example.

⁶See Cox and Lin (2007); Gatzert and Wesker (2014), for example.

⁷See Blake et al. (2008); Biffis et al. (2016a).

⁸See, for example, Coughlan et al. (2011); Cairns (2013); Fetiveau and Jia (2014).

⁹See Biffis and Blake (2010b, 2013).

could therefore help hedge suppliers diversify their risk and facilitate intermediation with capital market investors.¹⁰ Moreover, APAC hedgers from emerging economies face the challenge of limited statistical information, which drives up the costs of longevity risk transfers. This means that multi-population models are relevant, if not essential, for countries where longevity risk management has to benchmark its underlying population against more mature populations. Our work contributes to a better understanding of these issues, by identifying relevant benchmarks for mortality improvements within the APAC region, and by quantifying co-movement in the longevity risk faced by different APAC populations.

Third, with the exception of more mature economies within the APAC region, the contribution of APAC to the economic costs of global mortality improvements is poorly understood. A recent study by the IMF suggests that demographic changes represent a significant threat to global growth and to the sustainability of social security systems, because of the fiscal burden of an aging population (see IMF, 2012). For example, the IMF indicated that old-age dependency ratios¹¹ are expected to double from 24 to 48 percent in developed economies over the period 2010-50, and nearly treble from 13 to 33 percent in emerging economies. The latter figures, based on United Nations (2011) data, are subject to considerable uncertainty regarding longevity risk (see IMF, 2012, Chapter 4). Our work on the region relies on novel data originating directly from APAC countries (in most cases the statistics office of the relevant country), and therefore contributes to the understanding of mortality improvements in an area of strategic importance for the global economy.

On the methodological side, we develop our analysis of APAC mortality from the perspective of the LL model, which studies multiple populations jointly, and ensures that mortality forecasts for individual populations do not diverge in the long run (e.g., Hyndman et al., 2013). The use of the LL model is supported by a horse race between several competing models reported in Biffis et al. (2016b). We apply the model to all countries in our sample simultaneously, and disentangle common mortality improvement factors from country-specific risk factors that show considerable variation in terms of trend and volatility. For some pairs of APAC countries, we find country-specific factors that are strongly negatively correlated over the entire sample period. The most notable example is the Republic of Korea (henceforth Korea), which features the fastest decreasing mortality trend in the APAC region.¹² The severity of longevity risk in that country has resulted in regulators

¹⁰See Biffis and Blake (2013, 2014).

¹¹A common measure of aging, the old-age dependency ratio is the ratio of the population aged 65 and older to the population aged 15 to 64.

¹²Over a large part of our sample period, Korea was classified as an emerging economy (see Milidonis and Efthymiou, 2016, section 3.2).

“considering using a longevity risk measure when weighing risk-based capital ratios, given the country has a population aging faster than Japan”.¹³ Once an APAC wide longevity risk factor is taken into account, the residual longevity risk of Korea evolves quite differently from the country-specific longevity risk of other countries, such as Japan and Australia, for which the secular decline in mortality is slower. These observations suggest that a mortality index could be designed to allow hedgers in two different countries to mitigate their individual longevity risk by gaining opposite exposure to the same index. The basic idea is that, if country-specific drivers of longevity are heterogeneous enough, then short and long hedging positions in a common index become possible. Incentives to trade of this sort directly address the issue of longevity being a one-way risk, in the sense that there is no natural long counterparty.¹⁴ This crucial aspect is poorly explored in the literature on longevity risk management, which typically focuses on hedging demand, and the trading of fictitious longevity products, while ignoring supply-side considerations.¹⁵ Our study therefore provides an important contribution to the existing literature, by identifying indexes associated with positive and negative hedging demands, which could therefore be practically traded.

As an example, we study extensively the case of Korea and Japan, examining how hedgers in the two countries would benefit from portfolios of forward contracts suitably written on a country-specific mortality risk index. To distinguish them from other contracts studied in the literature, in particular q -forwards, we refer to these instruments as k -forward contracts. The reason why we focus on Korea and Japan is because they have the largest populations in our dataset, and their country-specific longevity risk factors are systematically and strongly negatively correlated over time. The approach, however, could be equally applied to country pairs such as Korea and Taiwan, and Korea and Australia, to name a few. The use of forward contracts is just for simplicity. Our results are clearly relevant for indexed longevity swaps, as well as longevity trend bonds¹⁶ and other insurance-linked securities (ILS). To elaborate on the latter, consider the example of the Kortis bond issued by Swiss Re in 2010,¹⁷ which exploits cross-country differentials in mortality improvements to offer payoffs appealing to both the issuer and the investors. On the supply side, the instrument provides simultaneous hedging

¹³See “Samsung Life Told to Brace for 96-Year Life Spans: Korea Markets” (*Bloomberg*, October 29, 2014; available at <http://www.bloomberg.com/news/print/2014-10-30/samsung-life-told-to-brace-for-96-year-life-spans-korea-markets.html>, last accessed on December 30, 2014).

¹⁴Although pharmaceuticals companies and firms delivering healthcare services have long been indicated as natural longevity hedge suppliers, they have not shown strong appetite for this risk. In any case, their hedging capacity is dwarfed by global longevity hedging needs (e.g., IMF, 2012).

¹⁵Notable exceptions are represented by Biffis and Blake (2010b) and Biffis and Blake (2013).

¹⁶The bonds we have in mind are not to be confused with capital intensive hedging instruments such as the EIB longevity bond (e.g., Blake et al., 2008; Biffis and Blake, 2010a).

¹⁷Indexed on England & Wales and US population, Kortis was the first ever longevity trend bond: it would reduce payments to investors in the case of a large divergence between the mortality improvements experienced by male lives aged 75 – 85 in England & Wales and by male lives aged 55 – 65 in the US.

opportunities to mortality and longevity risk in two different countries. On the demand side, the use of a ratio in mortality improvements offers a wide enough range of potential outcomes over a relatively short time horizon, the sort of payoff delivering a risk-return profile desirable to investors operating in the ILS space. Following this line of reasoning, the patterns of mortality co-movements that we identify for several APAC countries can provide the basis for the design of multi-population products trading off the rate of increase/decrease in mortality in different countries. Similarly, the example of contract design we consider in our hedging examples is relevant for the design of indexes appealing to ILS investors.

The paper is organized as follows. In the next section, we offer a brief overview of the APAC mortality dataset developed by the Insurance Risk and Finance Research Centre (IRFRC) at Nanyang Business School. In section 3, we carry out statistical analyses based on the LL model. In section 4, we apply our results to the design of a mortality indexed instrument, identify natural long and short counterparties, and quantify the gains from trade to an annuity provider in two different countries. The case of Korea and Japan is analyzed in detail. We also provide a detailed example of hedge portfolio construction with k -forwards, demonstrating how hedgers on both sides of the trade can benefit from the transaction. Finally, section 5 concludes. An online appendix (see Biffis et al., 2016b) provides a number of additional results, including a horse race among different mortality models, including GDFMs, which provide support for the use of the LL model in our dataset, as well as for the factor structure on which the model relies.¹⁸

2 Data

The IRFRC dataset was assembled by gathering information hierarchically from the following sources: (a) Human Mortality Database¹⁹ (HMD), (b) each country’s Department of Statistics, (c) Human Life-table Database²⁰ (HLT), and (d) other sources including direct communication with local government offices (see Milidonis, 2015).²¹ The variable of interest for our analysis is the one-year probability of death,²² $q(x, t, i)$, of a person belonging to age group x (or simply aged x) in country i , computed

¹⁸In particular, the relevant number of factors supporting a model should be estimated from the data (e.g., Bai and Ng, 2002, 2007; Hallin and Liška, 2007).

¹⁹See <http://www.mortality.org>.

²⁰See <http://www.lifetable.de>.

²¹The IRFRC has compiled two datasets on APAC. This paper uses the Asia-Pacific Age Gender (APAG) dataset. The second dataset is called APA (Asia-Pacific Aggregate) dataset, and reports mortality data at the aggregate population level. The APAC dataset is used by Milidonis and Efthymiou (2016) to explore intertemporal relationships in mortality within the APAC region. Both datasets can be downloaded from the IRFRC website (www.irfrc.com).

²²In case $q(x, t, i)$ is not reported, but death rates are available, we estimate $q(x, t, i)$ by following the HMD protocol. In order to check the accuracy of the results, we also estimate the $q(x, t, i)$ ’s for all data points for which death rates are available. We then measure the estimation error as the difference between estimated and reported $q(x, t, i)$ ’s. The

at time t . For the analysis carried out in the next sections, we require a balanced panel dataset, meaning that for all the countries that we analyze we need the same number of age groups, over the same number of years. We focus on female population. The resulting balanced panel includes 7 countries over a 31-year period (1980-2010), each with 15 five-year age groups, ranging from $[0 - 4]$ to $[70 - 74]$. We will refer to ‘age group x ’ for the age class $[x - (x + 4)]$, with $x = 0, 5, \dots, 70$. The seven countries are: Australia (AUS), Hong Kong (HKG), Japan (JAP), New Zealand (NZL), Singapore (SGP), Korea (KOR), and Taiwan (TWN). Figure 1 shows the time-series evolution of the average (across all age-groups) death probabilities. As expected, we observe an overall decreasing trend in mortality across all countries. There is considerable heterogeneity, however. For example, Korea has the fastest decreasing trend in mortality,²³ while Japan has the slowest.

3 Methodology and Analysis

The model developed by LL provides a transparent and widely employed method to study the joint evolution of mortality in different populations. It builds on the Lee-Carter model, which has been applied in several settings and extended in different directions (e.g., Li and Hardy, 2011; Milidonis et al., 2011; Lin et al., 2014), and provides a pragmatic approach to model mortality risk across time, age, and different countries.²⁴

Let us denote by $q(x, t, i)$ the one-year death rate²⁵ for age (group) x (for $x = 1, \dots, N$), in year t (for $t = 1, 2, \dots, T$), for population i (for $i = 1, 2, \dots, M$). The LL approach assumes a log-affine structure for $q(x, i, t)$, and distinguishes between a common factor capturing the overall secular decline in mortality, and country-specific longevity risk factors:

$$\ln q(x, t, i) = a(x, i) + B(x)K(t) + b(x, i)k(t, i) + \epsilon(x, t, i), \quad (3.1)$$

where $a(x, i)$ is an age-specific parameter equal to the average population mortality level at age x in country i ,

$$a(x, i) = \frac{\sum_{t=1}^T \ln q(x, t, i)}{T},$$

estimation error is found to be negligible. For the very few cases where we have missing data, we interpolate between adjacent values of non-missing data.

²³This aspect has been discussed extensively by demographers (see Park, 1998, for example, and references therein).

²⁴The online appendix (Biffis et al., 2016b) compares the goodness-of-fit of the LL model with alternative models. The use of an explanation ratio as performance metric shows that the LL model fits the historical data better than competing models.

²⁵For consistency with standard actuarial notation used in the previous section, $q(x, t, i)$ can be understood as the quantity ${}_1q_x^i(t)$ indexed on population i .

and $K(t)$ is a common risk factor shaping the mortality evolution of all populations, and modulated by the age-specific parameter $B(x)$. The term $b(x, i)k(t, i)$ allows for differences among short-term death rate changes in different countries, and again relies on an age-time multiplicative structure, in line with the Lee-Carter approach.

Following LL, we model $K(t)$ as a random walk with drift,

$$K(t) = c + K(t - 1) + \sigma_K e(t), \quad e(t) \sim N(0, 1), \quad (3.2)$$

where the error terms $e(t)$ are i.i.d. standard Normal. The country-specific factor $k(t, i)$ is modeled as an AR(1) process:

$$k(t, i) = r_{0,i} + r_{1,i}k(t - 1, i) + \sigma_{k,i}e_i(t), \quad e_i(t) \sim N(0, 1). \quad (3.3)$$

If the country-specific parameter $r_{1,i}$ does not satisfy $|r_{1,i}| < 1$, $k(t, i)$ can be modeled as a random walk with drift or richer time series process, as suggested by Li and Lee (2005). For some data subsamples, for example, we rely on an AR(2) model; see table 1.

We estimate model (3.1) using the female population mortality data for age groups $0, 5, \dots, 70$ for different sampling periods for the seven APAC countries in our dataset, following the two-step procedure outlined in Li and Lee (2005). We use log mortality rates weighted by the total populations of the different countries.²⁶ Our estimation shows that the common risk factor $K(t)$ is downward sloping, implying a long-term trend of mortality improvement. Interestingly, for some countries the risk factors $k(t, i)$ move in opposite directions (see figure 2). For example, the Japanese factor $k(t, \text{JAP})$ is upward sloping, as opposed to the Korean one, $k(t, \text{KOR})$. This suggests a possible longevity risk hedging opportunity between the two countries. For example, pension plans in either country could enter a longevity forward/swap contract written on $k(t, \text{JAP})$ or $k(t, \text{KOR})$ to hedge the risk of their pensioners living longer than currently expected. Similar opportunities are available for other countries (such as Korea and Australia or Taiwan), but we focus our attention on Korea and Japan for three reasons. First, Korea seems to have a pivotal role in our APAC dataset, showing the fastest average mortality improvement relative to the other countries. Second, Korea and Japan have the widest gap in average mortality improvements, and hence provide an interesting case study magnifying any hedging benefits which might be available on a smaller scale for different countries. Third, with a population of over 50 million and 125 million, respectively, Korea and Japan are the largest countries

²⁶We check the robustness of our results with the mortality rates weighted by the populations of each age group across the sample countries. The results are qualitatively similar.

in our sample, and hence offer the most sizeable opportunities in terms of potential longevity risk market development. These opportunities are discussed more in detail in the next section.

The parameter estimates for models (3.2)-(3.3) are reported in table 1 for different sampling periods. The positive estimate of $\hat{r}_{0,\text{JAP}}$ is consistent with an upward sloping trend of the estimated country-specific common-risk factor $k(t, \text{JAP})$. In contrast, the negative value of $\hat{r}_{0,\text{KOR}}$ of the country-specific risk factor $k(t, \text{KOR})$ is consistent with the estimated downward trend. To do mortality forecasting, we simulate the realizations of factors $K(t)$ and $k(t, i)$ beyond the end of the sampling period, as discussed in the online appendix (see Biffis et al., 2016b).

4 Longevity Risk Sharing Implications

The intuition behind the LL model is that total mortality risk can be split into a common risk factor, $K(t)$, modulated by the age dependent coefficient $B(x)$, and a country specific factor, $k(t, i)$, which is modulated by the age dependent coefficient $b(x, i)$. The common risk factor is undiversifiable from a multi-country longevity risk management perspective, but country-specific risk factors can be (partially) diversified away when they are weakly or negatively correlated.²⁷ As discussed in section 2, mortality decreases at different rates in different countries, hence the heterogeneity in behavior of the country-specific factors obtained through the LL model and depicted in figure 2. As a practical application, let us consider in detail the case of Japan and Korea. As observed from the results in figure 2, there seems to be a well-defined directional relationship among the three factors $K(t)$, $k(t, \text{JAP})$ and $k(t, \text{KOR})$ since 1980, with Japan and Korea having consistent, opposite trends. In addition, Korea is positively correlated with the APAC factor for the reasons explained above. Since Korea experiences the fastest decreasing mortality trend among the APAC countries in our sample, its country-specific time-series factor is negative and trends downwards, in order to account for the additional mortality decrease over and above the APAC time-series factor. On the other hand, Japan's country specific mortality risk factor trends upwards, to compensate the impact of the APAC common mortality risk factor. In the online appendix (Biffis et al., 2016b) we analyse more in detail the evolution over time of the APAC common component and the residual component for the two countries, demonstrating that trends and correlations are stable over the sampling period.

If hedgers based in Japan and Korea face the same type of risk (e.g., longevity risk in pension liabilities), then the negative correlation in country-specific factors would allow them to partially

²⁷Note that, from the point of view of individual populations, we are dealing with systematic risk in both cases, in the sense that longevity risk is the risk of improvements in aggregate mortality. When considering multiple countries, however, some diversification in aggregate risk may be achieved by having exposures spread over different countries.

hedge their exposure by taking opposite positions on a similar instrument written on a proxy for country-specific risk; see table 2. As an example, let us suppose that the forward contract is written at time t on the index $k(t+1, \text{KOR})$, where we assume that counterparties are risk neutral (i.e., we abstract away from forward longevity/mortality risk premiums), and there is no counterparty default risk (or deals are fully collateralized, and collateral posting is costless; see Biffis et al., 2016a).

This means that in the case that $k(t+1, \text{KOR})$ declines relative to the forward price set at inception, the Korean hedger will realize a net gain on the contract (forward price minus realized value of the index), which will then be used by the hedger to offset the increase in pension liabilities induced by the increase in longevity risk. Symmetrically, the Japanese hedger taking the other side of the trade will experience a net loss on the contract (realized value of the index minus forward price), which will be partly offset by the negative correlation of the pension liabilities with the index.

Before examining more in detail the design of a k -forward, an instrument written on a country-specific index $k(t, j)$, we devote the next section to discussing the impact of country-specific risk on the valuations of an annuity provider.

4.1 Valuation of a 20-year temporary-life annuity

Consider an immediate annuity with a term of twenty years,²⁸ which is sold in Japan and Korea to females aged 55 at the end of 2010. Our objective is to quantify the impact of mortality changes on the value of the annuity quantified in terms of Actuarial Present Value (APV). To do so, we forecast the annual death probabilities, conditional on survival at the beginning of each year, over the period 2011-2030. We only focus on the impact of mortality changes, and hence assume zero interest rates for our analysis.

4.1.1 Baseline case

First, we compute the APV of the annuity product using the mean annual forecast for each of the three time series $K(t)$, $k(t, \text{JAP})$, and $k(t, \text{KOR})$. The results are given in table 3, under the heading “Baseline”. To measure the impact of Japan’s country-specific risk factor on the product’s APV, we compute the difference between the APV computed with and without the country-specific factor (i.e., we set $k(t, \text{JAP}) = 0$). In the case of Japan, the APV is USD 16.725 when both the APAC

²⁸Immediate life annuities that pay survival benefits throughout the lifetime are more common in these markets. To simplify our analysis and focus on inter-countries hedging opportunities, we use immediate twenty-year, temporary-life annuities as our examples. A term of twenty years is likely to underestimate the duration of annuity liabilities; more realistic assumptions would only strengthen our results, which are particularly compelling given the mild assumptions we make.

common factor and the Japan-specific factor are included. When $k(t, \text{JAP})$ is set to zero, the country-specific effect that reduces the APAC common effect goes away. Accordingly, the APV increases to USD 17.078, that is, an absolute (relative) increase in APV of USD 0.353 (2.11%). The results for the Korean annuity go in the opposite direction: starting from an APV of USD 16.496, this figure decreases to USD 15.632 when the Korea-specific factor is set to zero, yielding a decrease in APV of USD 0.864 (-5.24%). These figures demonstrate that the contribution of country risk to pension/annuity-like liabilities is significant for both Japan and Korea.

4.1.2 Deviations from baseline

To measure the impact of changes in mortality trends in the LL model, we examine several scenarios reflecting increasingly large deviations from the baseline scenario. Hence, we quantify risk as the deviation from the expected values of the three time-series factors $K(t)$, $k(t, \text{JAP})$ and $k(t, \text{KOR})$. In table 3, we show how country-specific risk affects the APV of annuity liabilities when the common risk factor remains at the mean forecasted value, but the country-specific factors move one to three standard deviations away from their forecasted value. The results we obtain strengthen the conclusion that country-specific risk factors are material for annuity and pension liabilities. In the case of a two-standard-deviation scenario (0.183), the APV of the Japanese annuity increases to 16.908, where all the change from the baseline (16.725) is attributed to the common risk factor, a 1.08% rise. For the Korean annuity, on the other hand, the percentage change in APV is negative at -0.87%. As expected, these values increase in absolute terms as we move into the tail of the distribution. They are 1.65% and -1.36% for Japan and Korea, respectively, when we focus on three standard deviations above the mean forecasted value of the country specific factors.

4.2 Hedging with k -forwards

We now consider the design of a forward contract indexed on the country-specific risk factor k , and show how it could be used by hedgers in two countries to hedge their longevity risk exposures. We begin with a simple illustration of the main ideas, and then provide a more detailed hedging example.

In line with Biffis and Blake (2010b), let $q(x, t, i)$ represent a simple proxy for the net assets of a pension plan or annuity provider in country i . This can be justified by assuming that a longevity exposure can be proxied by the survival probability $p(x, t, i) = 1 - q(x, t, i)$, and noting that in the fully funded case it is backed by one unit of money, resulting in net assets equal to $q(x, t, i) = 1 - p(x, t, i)$. The case of a fully funded liability and single age-time pair is clearly just for illustration. According

to the LL model, the net assets admit the representation

$$q(x, t, i) = \exp(S(x, t)) \exp(C(x, t, i)), \quad (4.1)$$

where $S(x, t) = B(x)K(t)$ represents the systematic, APAC-wide component, and $C(x, t, i)$ is the country-specific residual given by

$$C(x, t, i) = a(x, i) + b(x, i)k(t, i) + \varepsilon(x, t, i).$$

Let us now consider $j \in \{\text{JAP}, \text{KOR}\}$. As illustrated in figures 3-4 for age groups $x = 55, 60$, the quantities $C(x, t, \text{JAP})$ and $C(x, t, \text{KOR})$ are negatively correlated, with the first one increasing, and the second one decreasing over time. The resulting effect is to mitigate longevity risk (relative to the APAC wide component) for Japan, and to increase it for Korea, again relatively speaking. As the Japanese counterparty benefits from its own country-specific factor, which is negatively correlated with the Korean one, there is an incentive for Japan to take on some exposure to $C(x, t, \text{KOR})$ in exchange for a premium. The Korean counterparty, on the other hand, has an incentive to pay that premium, as it allows its net assets to be partially hedged against the effects of $C(x, t, \text{KOR})$, which are gaining strength over time. Note that we are not just looking at a possible hedging instrument written on some plausible index, with no explanation for who might take the other side of the trade; here we identify a specific index for which two specific counterparties have an incentive to trade. Let us illustrate a possible index design by considering a forward contract written on $f(C(x, t, \text{KOR}))$, with $f(\cdot)$ some nonnegative, nondecreasing payoff function. After entering a position of notional size $n > 0$ at time 0, the Japanese hedger's net assets at maturity $t > 0$ are given by

$$\begin{aligned} \tilde{q}(x, t, \text{JAP}) &= q(x, t, \text{JAP}) + n [F(t) - f(C(x, t, \text{KOR}))] \\ &= \exp(S(x, t)) \exp(C(x, t, \text{JAP})) + n [F(t) - f(C(x, t, \text{KOR}))] \end{aligned} \quad (4.2)$$

where $F(t)$ is the t -maturity forward price of index $f(C(x, t, \text{KOR}))$. The choice of the identity function for f would yield a plain vanilla k -forward. An alternative, simple choice of index design is given by $f(c) = \exp(c)$, and can be used to refine intuition on the value of the instrument to the counterparties. Assume for the sake of illustration that the notional amount happens to coincide with $\exp(S(x, t))$ at time t . The payoff from the position would in this case simplify to

$$\tilde{q}(x, t, \text{JAP}) = \exp(S(x, t)) [\exp(C(x, t, \text{JAP})) - \exp(C(x, t, \text{KOR})) + F(t)] \quad (4.3)$$

Symmetrically, the Korean hedger's position would be given by

$$\tilde{q}(x, t, \text{KOR}) = \exp(S(x, t)) [\exp(C(x, t, \text{KOR})) + \exp(C(x, t, \text{KOR})) - F(t)]. \quad (4.4)$$

Expressions (4.3)-(4.4) make it clear that by entering the position, the Korean hedger commits to paying a fixed amount $F(t)$ in exchange for a floating payment $\exp(C(x, t, \text{KOR}))$ that will partially mitigate any reduction in the country-specific component, and hence the associated reduction in net assets. On the other hand, the Japanese hedger agrees to make a floating payment $\exp(C(x, t, \text{KOR}))$ in exchange for a fixed amount $F(t)$. As the term $C(x, t, \text{KOR})$ is trending downwards, it has an adverse effect on the Japanese hedger's net assets, but that is mitigated by the Japanese-specific component, which trends in the opposite direction.

To provide some examples of the hedging strategy, let us assume that counterparties enter a portfolio of k -forward contracts at the end of 2000. The contracts are written on the index $\exp(C(x, t, \text{KOR}))$, for age groups $x \in \{55, 60\}$, and maturities of 1 to 10 years. Using as baseline the case in which the term structure of forward prices is simply equal to the expected value of the index at different maturities, $F(t; x, \text{KOR}) = E[\exp(C(x, t, \text{KOR}))]$, we use expressions (4.3)-(4.4) to simulate the net assets of the counterparties over time. Figure 5 shows the dramatic decrease in standard deviation per individual age group (and both age groups) and time horizons for the two counterparties. The results show that transferring a country-specific risk component from one party to the other can improve predictability of cashflows and reduce capital charges for *both* hedgers. As the results are based on the assumption of a zero forward risk premium, we see that trading would be valuable for the Japanese hedger even if the forward prices embedded a negative risk premium.

We explore the robustness of the hedging instrument by allowing the LL model to be re-estimated during the hedging period to determine the country-specific residual. The procedure can also be used to understand how the instrument's value would behave following a regular marking to model of the position. For simplicity, we consider a single re-estimation/valuation date (end of 2005). After re-estimating the LL model at the end of year 2005, we project forward the new Korean country-specific residuals based on the new estimates. The results are presented in table 4. The upper part of the table shows what happens to the counterparties' net assets, in terms of changes in both the average net assets and in their standard deviation. We see that the reduction in standard deviation remains substantial, while the impact on average values is inhomogeneous across age groups. For example, the Japanese hedger experiences an increase in average net assets at the expense of the Korean hedger, who experiences the opposite. However, both hedgers benefit from an increase in average net assets for

age group 60 . The results are driven by two main effects: i) re-estimation of the LL model results in a different forward curve for the unexpired forward contracts, and ii) the (age-dependent) notional hedge amount fixed at inception may result in over/under-hedging in light of the re-estimation procedure. As the reduction in the net assets' volatility is remarkably robust to the re-estimation procedure, the results suggest that a richer term structure of risk premia could support the trading of k -forwards. Moving away from the assumption of a zero risk premium, in the lower part of table 4 we present a simple example of decreasing negative risk premia for age group 55, and increasing positive risk premia for age group 60. The overall effect is to deliver an increase in average net assets for both hedgers, while delivering a substantial reduction in the standard deviation of the net assets. As the term structures of risk premia for the two age groups are exactly the opposite, one can appreciate how the positive risk premia associated with some age buckets may be funded by the negative risk premia associated with other age buckets. This shows that in addition to cross-country hedging opportunities, country-specific mortality risk factors may offer interesting hedging opportunities along the age cross-section.

5 Conclusion

In this paper we have examined a recently constructed dataset on APAC mortality (available through the IRFRC), and have carried out an analysis of seven populations based on the Li and Lee (2005) model. The approach allows us to distinguish a common risk component from country-specific factors driving the evolution of mortality. A country that stands out is Korea, as its mortality improvements have consistently outpaced those of other countries, Japan in particular. We use the results to identify longevity risk sharing opportunities, the idea being that, if country-specific drivers of longevity are heterogeneous enough, then short and long hedging positions on a common index become possible. Korea and Japan provide a compelling example, as they are the largest populations in our sample, and have the widest spread in speed of mortality improvements over the sample. We use the two countries as a case study to show how an indexed mortality forward could be designed to offer natural trading incentives. For robustness, the online appendix (Biffis et al., 2016b) reports an analysis of the dataset based on different mortality models, including GDFMs, providing support for the use of the Li and Lee (2005) model and its appealing operational advantages when it comes to interpreting mortality risk factors. In this paper, we use only a portion of the new APAC dataset constructed by the IRFRC. A more extensive analysis of the dataset in future research will allow us to better understand the co-movement of APAC mortality rates, and hence to identify further opportunities for longevity risk sharing and intermediation in the APAC region.

References

- Alessi, L., Barigozzi, M., and Capasso, M. (2007). Dynamic factor GARCH: Multivariate volatility forecast for a large number of series. Technical report, LEM Working Paper Series.
- Bai, J. and Ng, S. (2002). Determining the number of factors in approximate factor models. *Econometrica*, 70(1):191–221.
- Bai, J. and Ng, S. (2007). Determining the number of primitive shocks in factor models. *Journal of Business & Economic Statistics*, 25(1).
- Biffis, E. and Blake, D. (2010a). Mortality-linked securities and derivatives. In Bertocchi, M., Schwartz, S., and Ziemba, W., editors, *Optimizing the Aging, Retirement and Pensions Dilemma*. John Wiley & sons.
- Biffis, E. and Blake, D. (2010b). Securitizing and tranching longevity exposures. *Insurance: Mathematics & Economics*, 46(1):186–197.
- Biffis, E. and Blake, D. (2013). Informed intermediation of longevity exposures. *Journal of Risk and Insurance*, 80(3):559–584.
- Biffis, E. and Blake, D. (2014). Keeping some skin in the game: How to start a capital market in longevity risk transfers. *North American Actuarial Journal*, 18(1):14–21.
- Biffis, E., Blake, D., Pitotti, L., and Sun, A. (2016a). The cost of counterparty risk and collateralization in longevity swaps. *Journal of Risk and Insurance*, 87(2):387–419.
- Biffis, E. and Kosowski, R. (2014). Managing capital market risk for retirement. In Olivia S. Mitchell, Raimond Maurer, P. B. H., editor, *Recreating Sustainable Retirement: Resilience, Solvency, and Tail Risk*, chapter 9. Oxford University Press, USA.
- Biffis, E., Lin, Y., and Milidonis, A. (2016b). Online appendix to “The cross-section of Asia-Pacific mortality dynamics: Implications for longevity risk sharing”. Technical report.
- Blake, D., Cairns, A., Coughlan, G., Dowd, K., and MacMinn, R. (2013). The new life market. *Journal of Risk and Insurance*, 80(3):501–558.
- Blake, D., Cairns, A., and Dowd, K. (2008). The birth of the Life Market. *Asia-Pacific Journal of Risk and Insurance*, 3(1):6–36.
- Cairns, A. J. G. (2013). Robust hedging of longevity risk. *Journal of Risk and Insurance*, 80(3):621–648.
- Coughlan, G., Khalaf-Allah, M., Ye, Y., Kumar, S., Cairns, A., Blake, D., and Dowd, K. (2011). Longevity hedging 101: A framework for longevity basis risk analysis and hedge effectiveness. *North American Actuarial Journal*, 15(2):150–176.
- Cox, S. H. and Lin, Y. (2007). Natural hedging of life and annuity mortality risks. *North American Actuarial Journal*, 11(3):1–15.
- Cox, S. H., Lin, Y., Tian, R., and Yu, J. (2013). Managing capital market and longevity risks in a defined benefit pension plan. *Journal of Risk and Insurance*, 80(3):585–619.
- Fetiveau, C. and Jia, C. (2014). Longevity risk hedging with population based index solution: A study of basis risk based on England & Wales population. Technical report, Deutsche Bank, London.
- Forni, M., Hallin, M., Lippi, M., and Reichlin, L. (2005). The generalized dynamic factor model. *Journal of the American Statistical Association*, 100(471).

- French, D. and O'Hare, C. (2013). A dynamic factor approach to mortality modeling. *Journal of Forecasting*, 32(7):587–599.
- Gao, Q. and Hu, C. (2009). Dynamic mortality factor model with conditional heteroskedasticity. *Insurance: Mathematics and Economics*, 45(3):410–423.
- Gatzert, N. and Wesker, H. (2014). Mortality risk and its effect on shortfall and risk management in life insurance. *The Journal of Risk and Insurance*, 81(1):57–90.
- Hallin, M. and Liška, R. (2007). Determining the number of factors in the general dynamic factor model. *Journal of the American Statistical Association*, 102(478):603–617.
- Hyndman, R., Booth, H., and Yasmineen, F. (2013). Coherent mortality forecasting: The product-ratio method with functional time series models. *Demography*, 50:261–283.
- IMF (2012). The financial impact of longevity risk. *Global Financial Stability Report*.
- Lane Clark & Picoock (2012). Pension buy-ins, buy-outs, and longevity swaps 2012. Technical report, Lane Clark & Picoock LLP.
- Li, J. and Hardy, M. (2011). Measuring basis risk in longevity hedges. *North American Actuarial Journal*, 15:177–200.
- Li, N. and Lee, R. (2005). Coherent mortality forecasts for a group of populations: An extension of the lee-carter method. *Demography*, 42:574–594.
- Lin, Y., Tan, K. S., Tian, R., and Yu, J. (2014). Downside risk management of a defined benefit plan considering longevity basis risk. *North American Actuarial Journal*, 18(1):68–86.
- Milidonis, A. (2015). Multi-population mortality risk in Asia-Pacific. Technical report, IRFRC, Nanyang Business School. Retrieved from www.irfrc.com.
- Milidonis, A. and Efthymiou, M. (2016). Mortality leads and lags. *The Journal of Risk and Insurance*, forthcoming.
- Milidonis, A., Lin, Y., and Cox, S. H. (2011). Mortality regimes and pricing. *North American Actuarial Journal*, 15:266–289.
- Park, K.-A. (1998). Recent trends and patterns of mortality in Korea. *Development and Society*, 27:67–81.
- Swiss Re (2013). World insurance in 2012. *SIGMA*, 2013.
- United Nations (2011). *World Insurance in 2010*. Department of Economic and Social Affairs, Population Division, United Nations, New York City.

A Tables and figures

1980-2000								
\hat{c}	$\hat{r}_{0,\text{JAP}}$	$\hat{r}_{1,\text{JAP}}$	$\hat{r}_{2,\text{JAP}}$	$\hat{r}_{0,\text{KOR}}$	$\hat{r}_{1,\text{KOR}}$	$\hat{\sigma}_K$	$\hat{\sigma}_{k,\text{JAP}}$	$\hat{\sigma}_{k,\text{KOR}}$
-0.4692	0.1037	0.9383	-	-0.3820	0.9877	0.1948	0.3296	0.1920
1980-2005								
\hat{c}	$\hat{r}_{0,\text{JAP}}$	$\hat{r}_{1,\text{JAP}}$	$\hat{r}_{2,\text{JAP}}$	$\hat{r}_{0,\text{KOR}}$	$\hat{r}_{1,\text{KOR}}$	$\hat{\sigma}_K$	$\hat{\sigma}_{k,\text{JAP}}$	$\hat{\sigma}_{k,\text{KOR}}$
-0.4623	0.2092	0.5638	0.4752	-0.3427	0.9656	0.1719	0.3254	0.2175
1980-2010								
\hat{c}	$\hat{r}_{0,\text{JAP}}$	$\hat{r}_{1,\text{JAP}}$	$\hat{r}_{2,\text{JAP}}$	$\hat{r}_{0,\text{KOR}}$	$\hat{r}_{1,\text{KOR}}$	$\hat{\sigma}_K$	$\hat{\sigma}_{k,\text{JAP}}$	$\hat{\sigma}_{k,\text{KOR}}$
-0.4471	0.2297	0.7051	0.4158	-0.3268	0.9589	0.1635	0.3668	0.3259

Table 1: Estimated parameters for the random walk with drift of the common risk factor $K(t)$, and for the country-specific factors models for $k(t, \text{JAP})$ and $k(t, \text{KOR})$ based on different sampling periods. When restricting the dataset to periods 1980-2000 and 1980-2005, we rely on an AR(2) process for Japan: $k(t, \text{JAP}) = r_{0,i} + r_{1,\text{JAP}}k(t-1, \text{JAP}) + r_{2,\text{JAP}}k(t-2, \text{JAP}) + \sigma_{k,\text{JAP}}e_{\text{JAP}}(t)$, with $e_{\text{JAP}}(t) \sim N(0, 1)$.

$k(t+1, \text{KOR})$ relative to forward price		Long side (JAP) positions at $t+1$	Short side (KOR) positions at $t+1$
Up	Liabilities	Higher	Lower
	Forward	Gain	Loss
Down	Liabilities	Lower	Higher
	Forward	Loss	Gain

Table 2: Stylized effects of changes in the underlying index $k(t+1, \text{KOR})$ for hedgers trading a forward contract over $[t, t+1]$.

	Baseline JAP	Baseline KOR	1 sigma JAP	1 sigma KOR	2 sigma JAP	2 sigma KOR	3 sigma JAP	3 sigma KOR
APV (with Total Risk)	16.725	16.496	16.771	16.461	16.908	16.353	17.005	16.274
APV (with Common only)	17.078	15.632	17.078	15.632	17.078	15.632	17.078	15.632
APV of Country Factor	0.353	-0.864						
% of Country to APV	2.111%	-5.240%						
APV of Total Risk			0.046	-0.035	0.183	-0.142	0.281	-0.222
APV of Common Risk			-	-	-	-	-	-
APV of Country Risk			0.046	-0.035	0.183	-0.142	0.281	-0.222
% of Country to APV			0.273%	-0.214%	1.082%	-0.871%	1.650%	-1.365%

Table 3: Deviations of country risk with common risk kept constant at mean forecasted value.

		Ages 55-59				Ages 60-64				
Horizon	Risk Premium	Spread JAP	Spread KOR	Change in s.d. JAP	Change in s.d. KOR	Risk Premium	Spread JAP	Spread KOR	Change in s.d. JAP	Change in s.d. KOR
2001	0%	0.00%	0.00%	-7.9%	-27.8%	0%	0.00%	0.00%	-6.31%	-15.07%
2002	0%	0.00%	0.00%	-8.1%	-28.3%	0%	0.00%	0.00%	-6.46%	-15.47%
2003	0%	0.00%	0.00%	-8.9%	-28.6%	0%	0.00%	0.00%	-7.06%	-15.66%
2004	0%	0.00%	0.00%	-8.9%	-28.6%	0%	0.00%	0.00%	-7.03%	-15.62%
2005	0%	0.00%	0.00%	-9.6%	-29.6%	0%	0.00%	0.00%	-7.73%	-16.29%
2006	0%	8.71%	-8.04%	-1.8%	-19.0%	0%	6.15%	7.73%	-2.40%	-33.14%
2007	0%	10.49%	-7.75%	-2.5%	-20.0%	0%	7.15%	8.17%	-3.38%	-24.50%
2008	0%	11.67%	-7.46%	-2.2%	-20.3%	0%	7.74%	8.59%	-3.07%	-20.59%
2009	0%	13.02%	-7.13%	-1.8%	-20.7%	0%	8.43%	9.03%	-2.73%	-18.77%
2010	0%	15.09%	-6.80%	-1.4%	-20.6%	0%	9.52%	9.46%	-2.15%	-18.01%
		Average	Average	Average	Average		Average	Average	Average	Average
		5.90%	-3.72%	-5.32%	-24.34%		3.90%	4.30%	-4.83%	-19.31%

		Ages 55-59				Ages 60-64				
Horizon	Risk Premium	Spread JAP	Spread KOR	Change in s.d. JAP	Change in s.d. KOR	Risk Premium	Spread JAP	Spread KOR	Change in s.d. JAP	Change in s.d. KOR
2001	-1%	-1.36%	1.03%	-7.9%	-27.8%	1%	1.54%	-1.03%	-6.31%	-15.07%
2002	-1%	-1.37%	1.06%	-8.1%	-28.3%	1%	1.57%	-1.06%	-6.46%	-15.47%
2003	-1%	-1.39%	1.09%	-8.9%	-28.6%	1%	1.60%	-1.09%	-7.06%	-15.66%
2004	-1%	-1.41%	1.12%	-8.9%	-28.6%	1%	1.64%	-1.12%	-7.03%	-15.62%
2005	-1%	-1.43%	1.15%	-9.6%	-29.6%	1%	1.67%	-1.15%	-7.73%	-16.29%
2006	-6%	1.25%	-1.97%	-1.8%	-19.0%	6%	14.82%	1.70%	-2.40%	-33.14%
2007	-6%	2.90%	-1.50%	-2.5%	-20.0%	6%	16.01%	1.96%	-3.38%	-24.50%
2008	-6%	3.95%	-1.02%	-2.2%	-20.3%	6%	16.79%	2.19%	-3.07%	-20.59%
2009	-6%	5.16%	-0.49%	-1.8%	-20.7%	6%	17.69%	2.44%	-2.73%	-18.77%
2010	-6%	7.08%	0.05%	-1.4%	-20.6%	6%	18.99%	2.67%	-2.15%	-18.01%
		Average	Average	Average	Average		Average	Average	Average	Average
		1.34%	0.05%	-5.32%	-24.34%		9.23%	0.55%	-4.83%	-19.31%

Table 4: Average relative spread between expected net assets with and without hedging, $E[\tilde{q}(x, t, j)]/E[q(x, t, j)] - 1$, and reduction in the net assets' standard deviation through hedging, for age groups $x = 55, 60$ and different risk premia (RP) defined via the expression $F(t; x) = (1 + RP(t))E[\exp(C(x, t, \text{KOR}))]$.

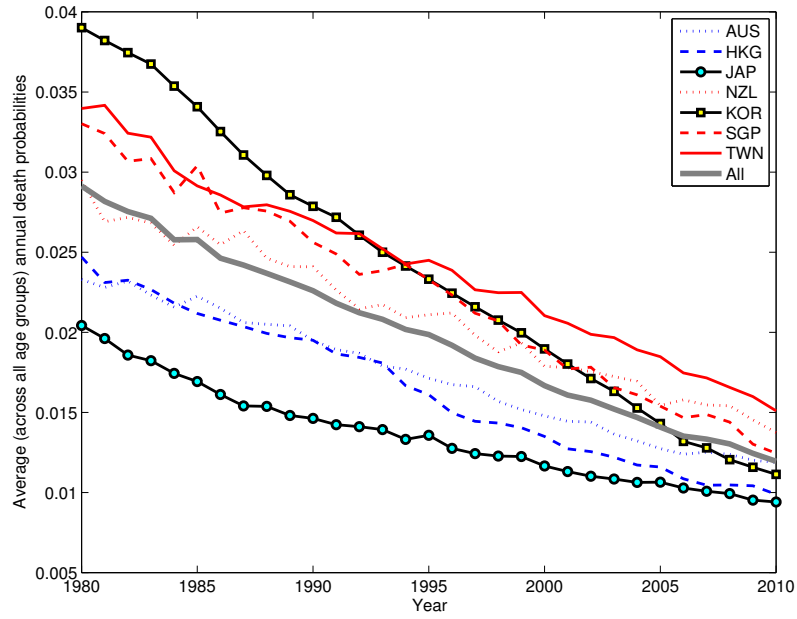


Figure 1: Time-series evolution of the average (across all age-groups) yearly death probability by country over 1980-2010. The seven countries are: Australia (AUS), Hong Kong (HKG), Japan (JAP), New Zealand (NZL), Singapore (SGP), Korea (KOR), and Taiwan (TWN).

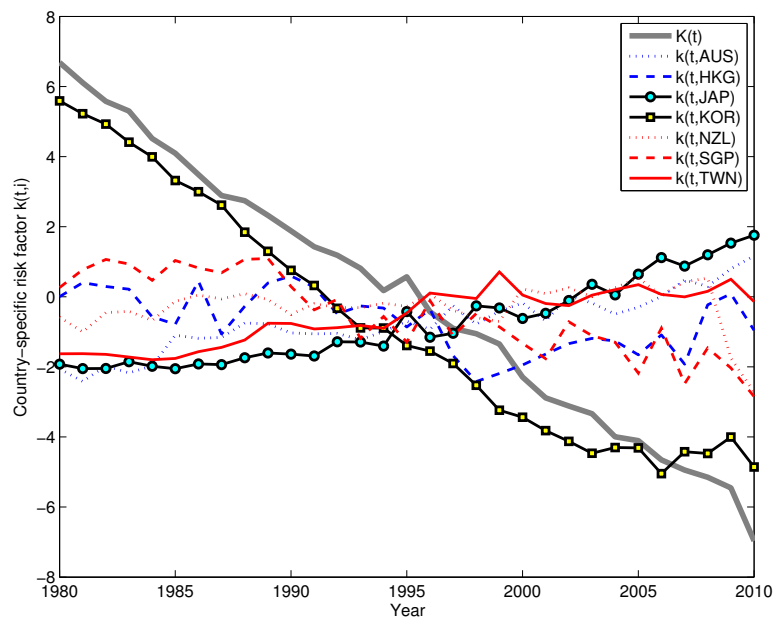


Figure 2: Parameter estimates of $K(t)$ and $k(t, i)$ for female populations from the LL model.

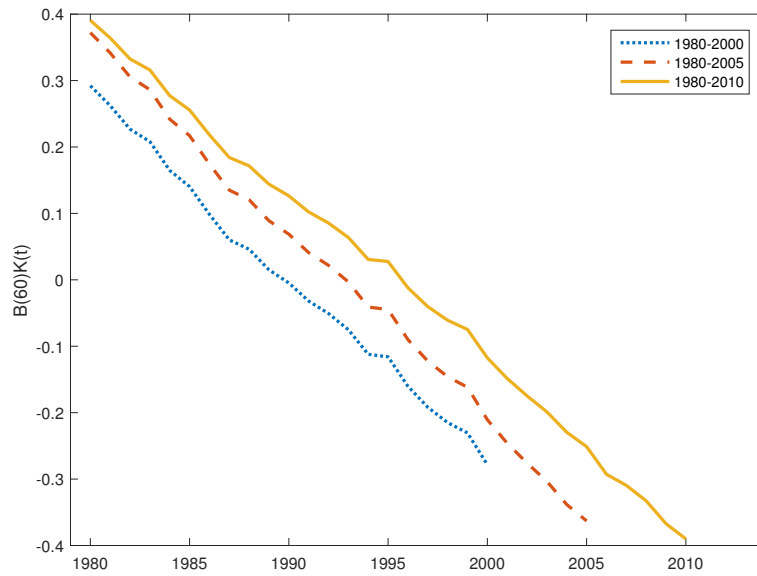


Figure 3: Estimates for the APAC common component $B(x)K(t)$ for age group $x = 60$ and sampling periods 1980-2000, 1980-2005, and 1980-2010.

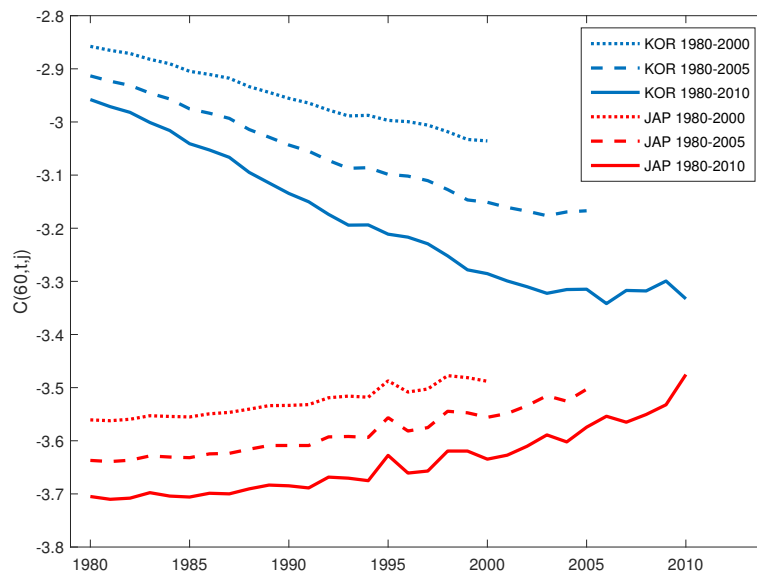


Figure 4: Estimates for the average residual country-specific component $E(C(x, t, j)) = a(x, j) + b(x, j)k(t, j)$ for countries $j \in \{JAP, KOR\}$, age group $x = 60$, and sampling periods 1980-2000, 1980-2005, and 1980-2010.

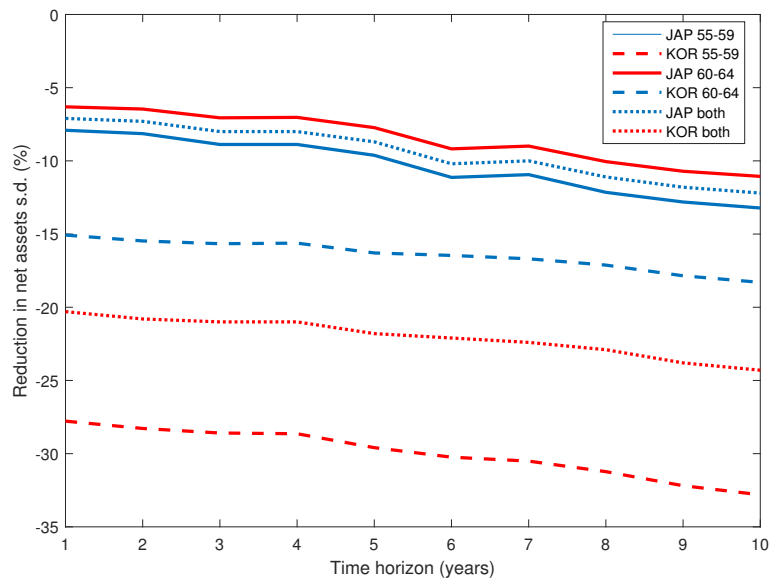


Figure 5: Percentage reduction in standard deviation of $q(x, t, j)$ after hedging, for countries $j \in \{\text{JAP}, \text{KOR}\}$, age groups $x \in \{55, 60, \{55\} \cup \{60\}\}$, and maturities $t = 2001, \dots, 2010$.